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LIGHTNING ARRESTER SPARK GAPS

Their Relation to the Problem of Protecting Against Impulse Voltages

BY CHESTER T. ALLCUTT

Abstract of Paper

This paper describes a new form of high-voltage lightning arrester gap which has been called the "impulse protective gap" because of its particular effectiveness in protecting against line disturbances of steep wave front.

The paper opens with a brief resumé of some of the results of previous investigations of the subject of impulse voltages. A discussion of the points involved in securing adequate protection against transient voltages of steep wave front follows. Particular reference is made to some of the conditions that may exist when a high-frequency impulse is superimposed on a low-frequency wave. In this connection is shown the desirability of a gap having a selective action, making it sensitive to steep wave fronts, and a number of forms of gap having this selective property are described.

Methods employed in testing these gaps are described and the results of a large number of experiments are tabulated. Tests on the action of a high-frequency impulse combined with a 60-cycle wave are included in the experimental work. From the experimental data a number of curves are plotted showing the discharge characteristics of the impulse protective gap under many different conditions.

The results of the tests are highly favorable and indicate that the new gap may have a wide application in the field of lightning protection.

IN GENERAL, a high-voltage lightning arrester consists of two distinct parts: First, a spark gap for discharging abnormal voltages; and second, means for preventing the normal line voltage from maintaining a power arc across the gap. In past years the satisfactory interruption of the power arc has been the most serious part of the lightning arrester problem, and, in consequence, the attention of manufacturers has been concentrated on the development of this feature. It is only comparatively recently that the gap itself has been receiving its due share of attention.

It is quite a simple matter to provide an electrical "safety valve" in the form of a spark gap to discharge abnormal voltages of low frequency, but when we have to deal with steep wave fronts, or impulse voltages, as they have been termed, there are certain complications due to dielectric spark lag. The existence of the phenomenon of dielectric spark lag was first brought to the attention of the Institute by E. E. F. Creighton[1] more than ten years ago. A later paper by Steinmetz and Hayden[2] gave the results of certain tests on the transient breakdown voltage of different gaps as a function of the energy available. Other papers[3] on impulse tests on porcelain shed some further light on the subject of dielectric spark lag but still left our knowledge of the subject in a rather incomplete state. It remained for F. W. Peek, Jr.[4] to present to the Institute the first quantitative data on the breakdown strength of different forms of gap under the action of transient voltages of carefully predetermined characteristics.

Mr. Peek's extensive investigations have definitely established the fact that some forms of spark gap require a very much higher voltage to discharge a high-frequency impulse than is required to discharge a continuously applied e.m.f. The name "impulse ratio" has been given to the ratio of the impulse breakdown voltage of a gap to the continuously applied breakdown voltage. The impulse ratio was found to vary with the shape of the gap electrodes, the length of the gap and the shape of the impulse wave applied. In case of the needle gap, for example, the impulse ratio was found to be considerably greater than two under some conditions. The sphere gap, on the other hand, was found to have an impulse ratio of substantially unity through a wide range of gap setting. The general conclusion reached by Peek was that a gap in which the discharge is preceded by corona will, in general, have an impulse ratio greater than unity, the impulse ratio increasing with the non-uniformity of the field and with the steepness of the wave front applied.

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1. E. E. F. Creighton. Methods of Testing Electrical Porcelain. A. I. E. E. TRANSACTIONS, Vol. 25, p. 365, May 1906.

2. J. L. R. Hayden and C. P. Steinmetz. Disruptive Strength with Transient Voltages. A. I. E. E. TRANSACTIONS, Vol. 29, p. 1125, June, 1910.

3. A. Chernyschoff and C. A. Butman. Different Methods of Testing Electrical Porcelain. *Electric Journal*, Vol. 12, p. 282, 1915.

L. E. Imlay and P. H. Thomas. High-Frequency Tests of Line Insulators. A. I. E. E. TRANSACTIONS, Vol. 31, p. 2233, December, 1912.

4. F. W. Peek, Jr. The Effect of Transient Voltages on Dielectrics. A. I. E. E. TRANSACTIONS, Vol. 34, p. 1695, September, 1915.

The relation of his experimental work on transient voltages to the subject of lightning protection was discussed by Peek in a later paper[5]. In this paper he indicates the desirability of a low impulse ratio in a lightning arrester gap and shows the superiority of the sphere gap as a protective device, over the simple horn gap usually employed in connection with high-voltage arresters.

PROTECTION AGAINST IMPULSE VOLTAGES

The papers referred to above have resulted in a great advance towards the solution of the problems involved in securing adequate protection against line disturbances of steep wave front. By the simple expedient of using a lightning arrester gap having spherical electrodes, the operating man may be perfectly sure that the voltage to ground at the point at which the arrester is installed will never appreciably exceed the 60-cycle discharge voltage of the gap. This is a great step in advance, but the question at once arises: Does it give the best possible protection against steep wave fronts? In this connection it must be pointed out that the danger to apparatus from an impulse voltage may be all out of proportion to the actual magnitude of the impulse. Consider, for example, a transformer winding designed for 50,000 volts. If 100,000 volts, at normal frequency, be applied to the winding the stresses on the insulation will only be doubled. But, on the other hand, if a high-frequency impulse of 100,000 volts be applied, the stresses on certain parts of the insulation may be many times normal because of the "piling up" of the steep wave front on the end turns of the winding. Consequently, if we are to protect the winding by means of a spark gap shunting it, the gap should be selective in its action; *i.e.*, it should discharge a high-frequency impulse at a *lower* voltage than an abnormal e.m.f. of line frequency.

The need for the selective action referred to above is emphasized by a consideration of the possible combinations of a high-frequency impulse with the normal line-voltage wave. Let us consider the three combinations shown in Fig. 1, and their effect on a winding protected by a gap.

Case A.—Impulse Occurring at Zero Point of Line E.M.F. Wave. In this case the presence of line voltage does not affect the action of the impulse. The insulation stresses in a winding and the discharge characteristics of a gap protecting the winding will be the same as if the impulse alone were applied.

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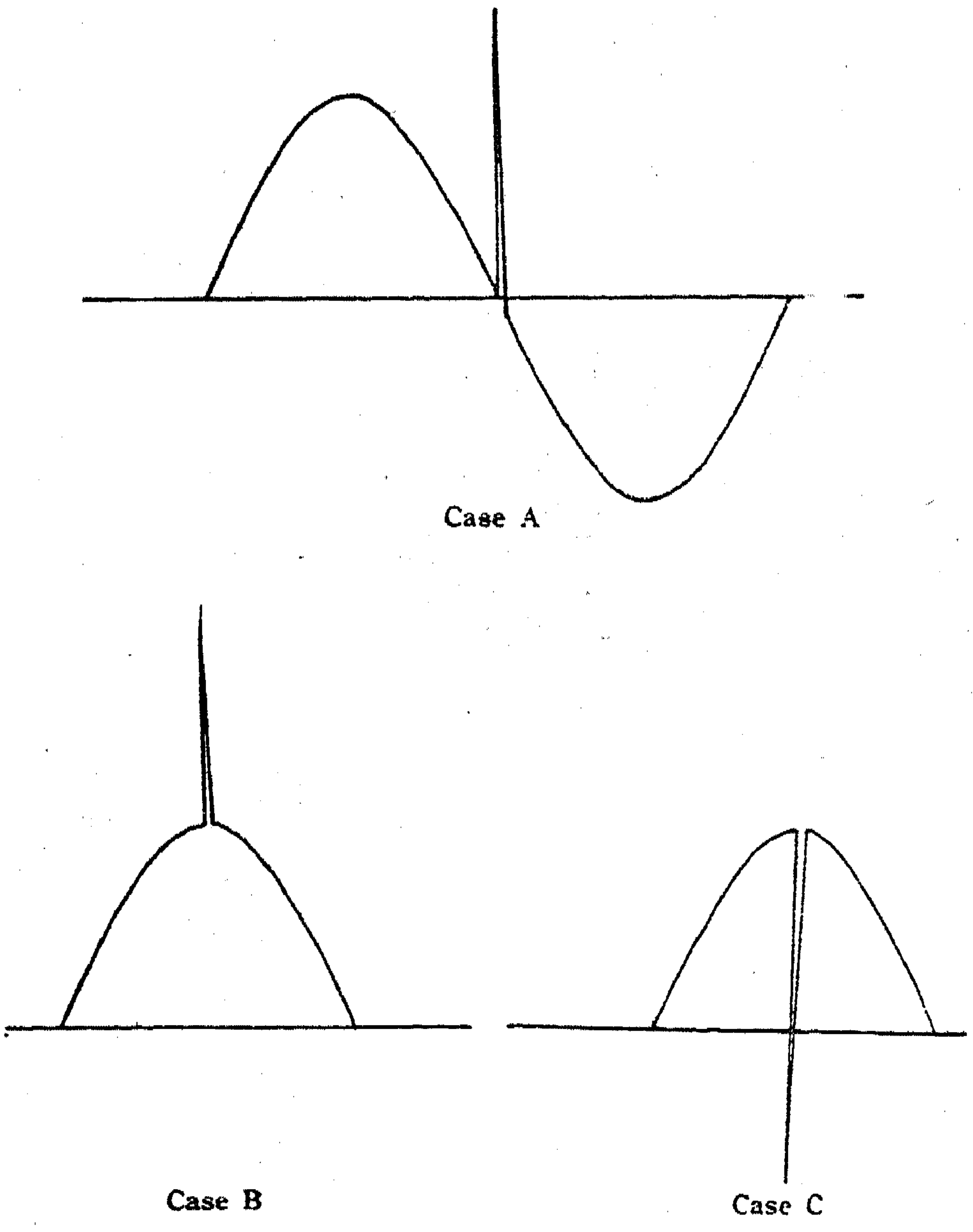
[5] F. W. Peek, Jr. Lightning, *General Electric Review*, Vol. 19, p. 586, July, 1916.

Case B.—Impulse Occurring at the Peak of the Line E.M.F. Wave and Adding Thereto. In this case the total e.m.f. impressed on the gap and tending to discharge it is equal to the sum of the line voltage and the impulse voltage. The insulation stresses between turns of the winding, on the other hand, are largely dependent on the value of the impulse alone, since line frequency does not cause a high voltage between turns. It would appear that, in this case, the gap affords a better protection than in Case A. For example, suppose the gap to be set at double line-voltage. In Case A, an impulse of double line-voltage may be impressed on the winding without the gap functioning, while in Case B the maximum impulse that can be applied to the winding without causing the gap to discharge is just equal to line voltage.


Case A

Case B

Case C

FIG. 1—IMPULSE VOLTAGES SUPERIMPOSED ON 60-CYCLE WAVES

Case C.—Impulse Occurring at the Peak of the Line E.M.F. Wave and Subtracting Therefrom. In this case it is obvious that the gap furnishes much less effective protection than in the preceding two cases. A very considerable impulse voltage may be impressed on the winding without causing any rise in voltage across the gap. The presence of the line-frequency wave prevents the impulse from causing a rise in voltage across the gap but does not prevent the full destructive effect of the impulse from being felt by the winding. Assuming, once more, that the gap is set at double line voltage, it will be seen that an impulse of double line voltage may be applied without causing any rise whatever in the voltage across the gap, while it will require an impulse of *three times* line voltage to cause the gap to discharge.

The fact that such a condition is possible has not been generally appreciated, although it is clear that it may be a source of grave danger. Adequate protection against such a combination of high-frequency and line-frequency demands the use of a gap which is highly sensitive to steep wave fronts. It is true that, if the impulse is oscillatory, the second half-cycle may cause an ordinary gap to discharge, but such a discharge is too late to protect the winding against the destructive effects of the first half-cycle of the impulse.

THE IMPULSE PROTECTIVE GAP

A rather extensive experimental investigation of impulse phenomena has lead to the development of a new type of gap for lightning arrester service, which possesses in a marked degree the selective properties referred to in the preceding discussion. This gap has been termed the "impulse protective gap" because of its particular effectiveness in discharging line disturbances of steep wave front. The general principle underlying the action of the impulse gap is comparatively simple. In its most elementary form it consists of two gaps in series, each gap being shunted by a relatively high impedance. At line-frequency these impedances are proportional to the respective discharge voltages of the gaps which they shunt, but they are designed to change at different rates with changes of frequency so that, under the action of a high-frequency impulse, one of the impedances becomes much greater than the other and causes most of the high-frequency voltage to be impressed on one of the gaps. The breakdown of this gap will result in the total voltage being impressed on the remaining gap, which will break down in turn.

Fig. 2 is a diagrammatic representation of a gap constructed according to the above principle. One of the two equal gaps, g and g', is shunted by a condenser C' and the other by an equal condenser *C* and an inductance *L*. At line-frequency, the inductive reactance of *L* is inappreciable compared with the condensive reactances and, therefore, causes no unbalance in the voltage across the two gaps. To a high-frequency impulse, however, the condensers offer very little impedance and most of the high-frequency voltage appears across the inductance *L* and thus across the gap *g*. Discharge of the gap *g* is, in general, immediately followed by a discharge across *g′*.

It has been found by experiment that better results are obtained by using a single gap, having an intermediate electrode, rather than two distinct gaps. Fig. 3 shows a number of different gaps constructed in this manner. *A* shows a horn gap having an auxiliary electrode mounted midway between the horns. The auxiliary electrode is connected to one of the horns through a condenser *C′* and to the other through a condenser *C* and an inductance *L*. The action is similar to that of the gap shown in Fig. 2.

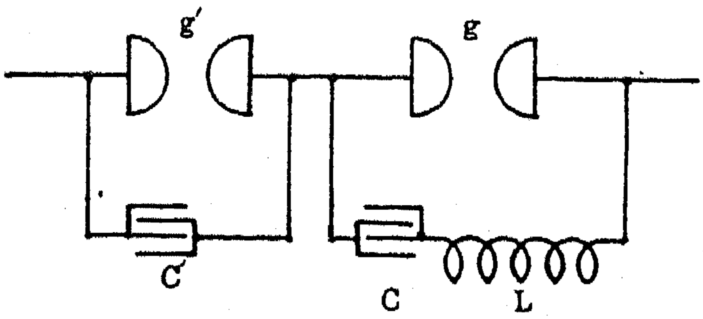


FIG. 2

In place of the reactance *L*, a resistance may be employed, as shown in Fig. 3B. This is the preferred form of gap, from a commercial standpoint, since the inductance used in *A* must necessarily be quite bulky, unless condensers of unreasonably high capacity be employed. With the structure shown in *B*, it has been found possible to secure excellent results with capacities as low as 10^{-11} farads.

Fig. 3C shows another form in which a reactance coil is used as an auto-transformer to produce a larger voltage between the auxiliary electrode and the main horn than is produced by the simple reactance shown in *A*.

Fig. 3D shows another modification, in which two intermediate electrodes are employed. R_1 is of greater resistance than R_2 so that a high-frequency impulse causes the right-hand gap to break down first, followed successively by the two others. For extremely long gaps it is probable that this form would give better results than *B*, but for commercial voltages it has been found possible to secure a gap of ample protective power by using but one auxiliary electrode.

Preliminary tests were made on a number of different gaps similar to those shown in Fig. 1. The early experiments included tests to determine the effect of different shapes of electrodes, both for the main horns and for the auxiliary electrode. The results of these preliminary experiments indicated that the scheme shown in Fig. 3B was the most promising for commercial development. A wire of small diameter (practically a pointed electrode) was found to be most satisfactory for the auxiliary electrode. The pointed auxiliary electrode gave results so much superior to a spherical or cylindrical electrode that further experiment with these latter forms was dropped.

As a result of these preliminary experiments it was decided to make complete tests on the two forms shown in the illustrations, Fig. 4 and Fig. 5. These two gaps differed only in the shape of the horn electrodes. The gap shown in Fig. 4 had plain horn electrodes made of 3/8-in. (0.95-cm.) diameter brass rod, while the other gap had brass hemispheres 6.25 cm. in diameter clamped to the horns, this presenting spherical discharge surfaces. In both gaps, the auxiliary electrode consisted of a blunt point of copper wire 0.025 in. (0.064 cm.) in diameter. During the tests, each horn was supported on a pillar insulator composed of three porcelain insulator units. These pillars rested on a wooden table which brought the gap to a height of about 5 ft. (1.5 m.) above ground. The porcelain units composing the pillars were of special design, having an unusually high electrostatic capacity, approximately 2×10^{-10} farads. Two of these units in series were used as the condensers between

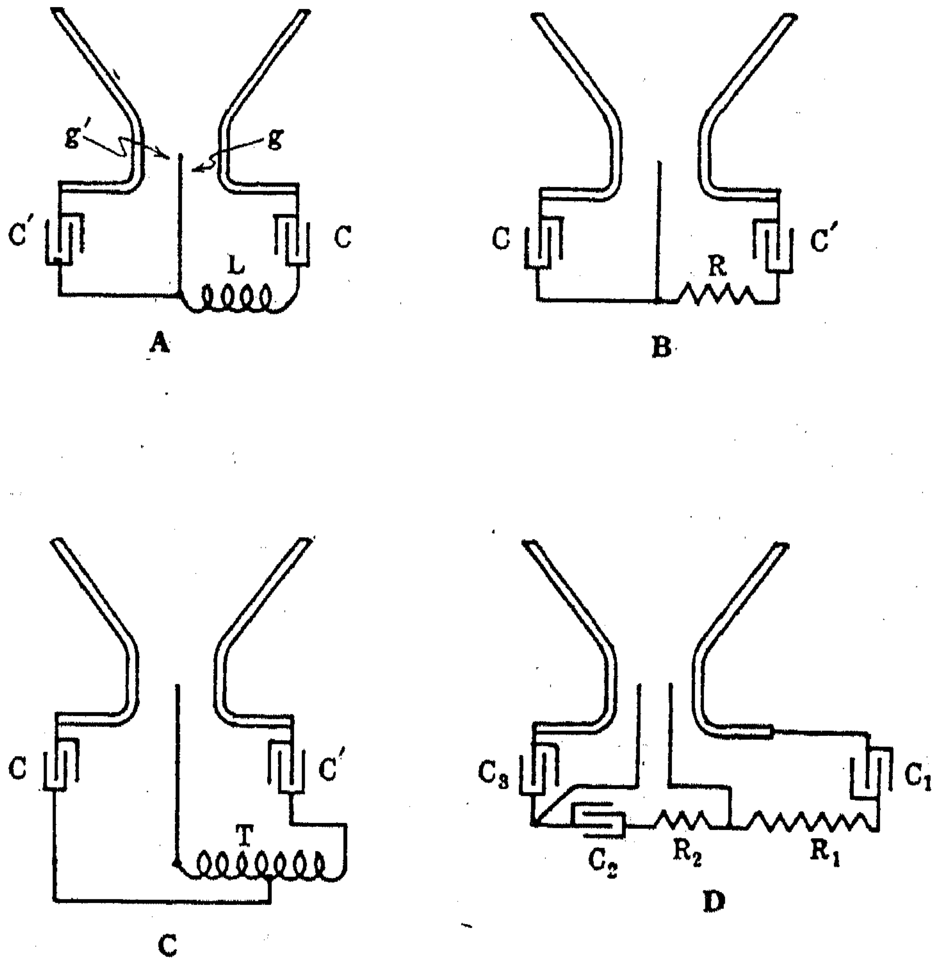


FIG. 3—IMPULSE PROTECTIVE GAPS

the auxiliary electrode and each of the horns (corresponding to C and C' in Fig. 3B). A water tube was used for the resistance. In most of the tests its value was approximately 100,000 ohms.

TEST METHODS

The method employed in obtaining impulse voltages of predetermined characteristics was practically identical with that described by Peek in his 1915 paper.[6] Fig. 6 shows a diagram of connections. When the voltage of the transformer T is gradually raised, the sphere gap G_r will eventually discharge. At this instant, the condenser C is charged up to the breakdown voltage of the gap. When G_r breaks down, the condenser discharges through the inductance L and the resistance R. The wave form of this discharge current may be accurately calculated from the values of L, R and C. The shape of the voltage wave across R is, of course, identical with the shape of the current wave. The gap G under test is connected across the resistance R. A sphere gap G_m is also connected across R in order to obtain a direct measurement of the impulse voltage. The voltage of the impulse applied to the gap under test may be varied by changing the setting of the gap G_r. For a more complete discussion of this method of obtaining impulses the reader is referred to Peek's paper.

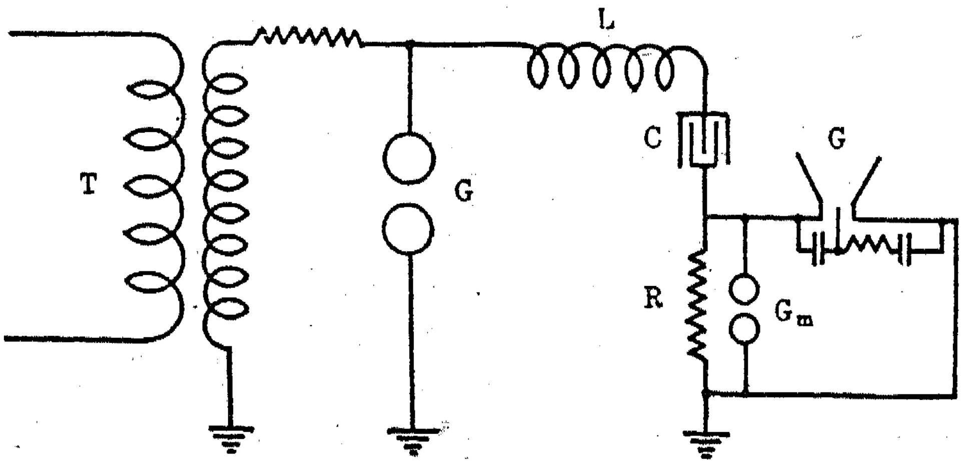


FIG. 6—CONNECTIONS FOR IMPULSE TESTS

The wave adopted as a standard for these tests was a critically damped impulse having a wave front corresponding to a 500,000-cycle sine wave. This wave is shown in Fig. 7. The constants of the oscillating circuit used in producing this impulse were approximately, $C = 10^{-9}$ farads, $L = 0.25$ millihenrys and $R = 1000$ ohms.

The procedure adopted in making the tests was as follows: First, the gap under test was adjusted to the desired setting.

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[6] Loc. cit. 4.

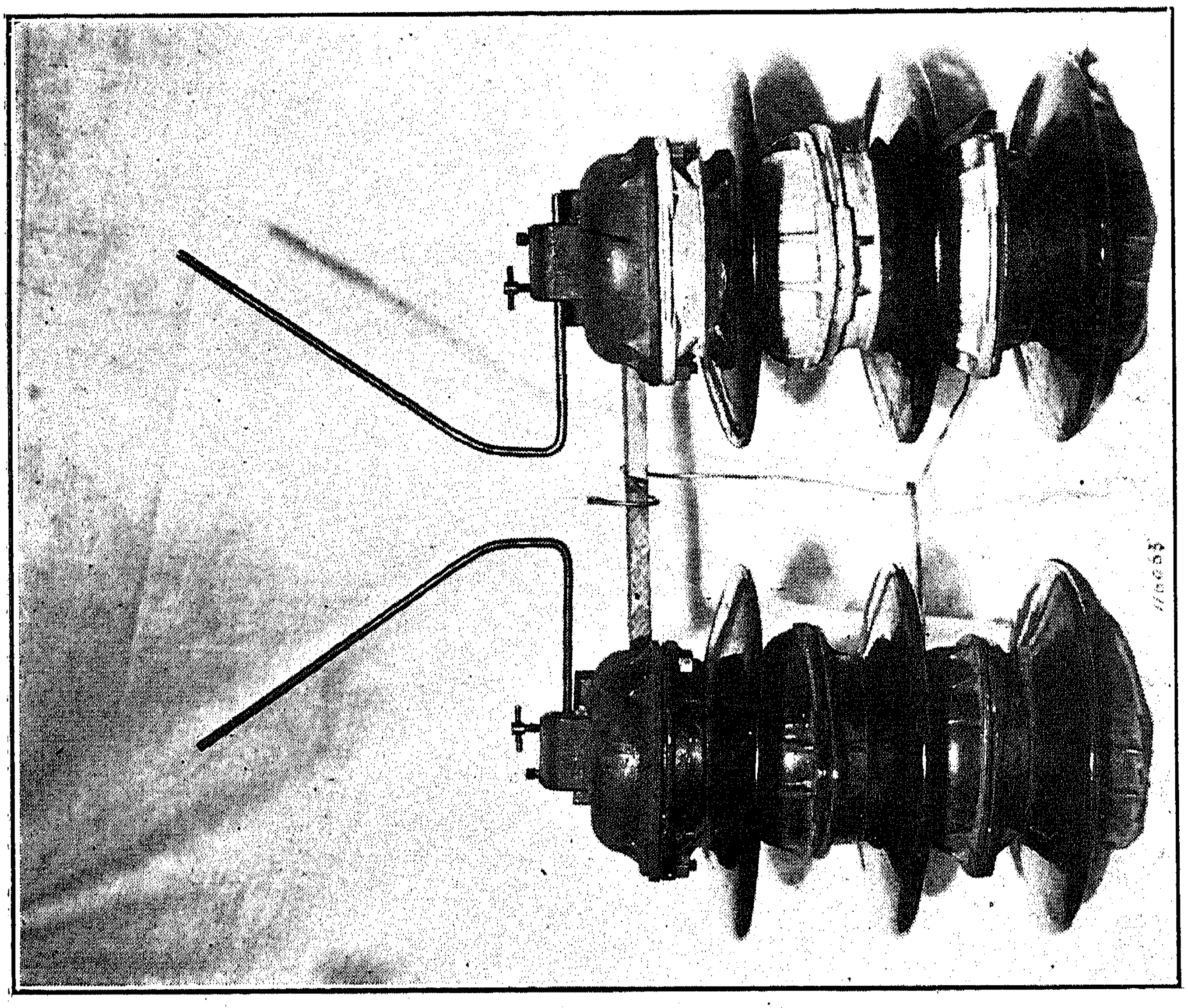

FIG 4

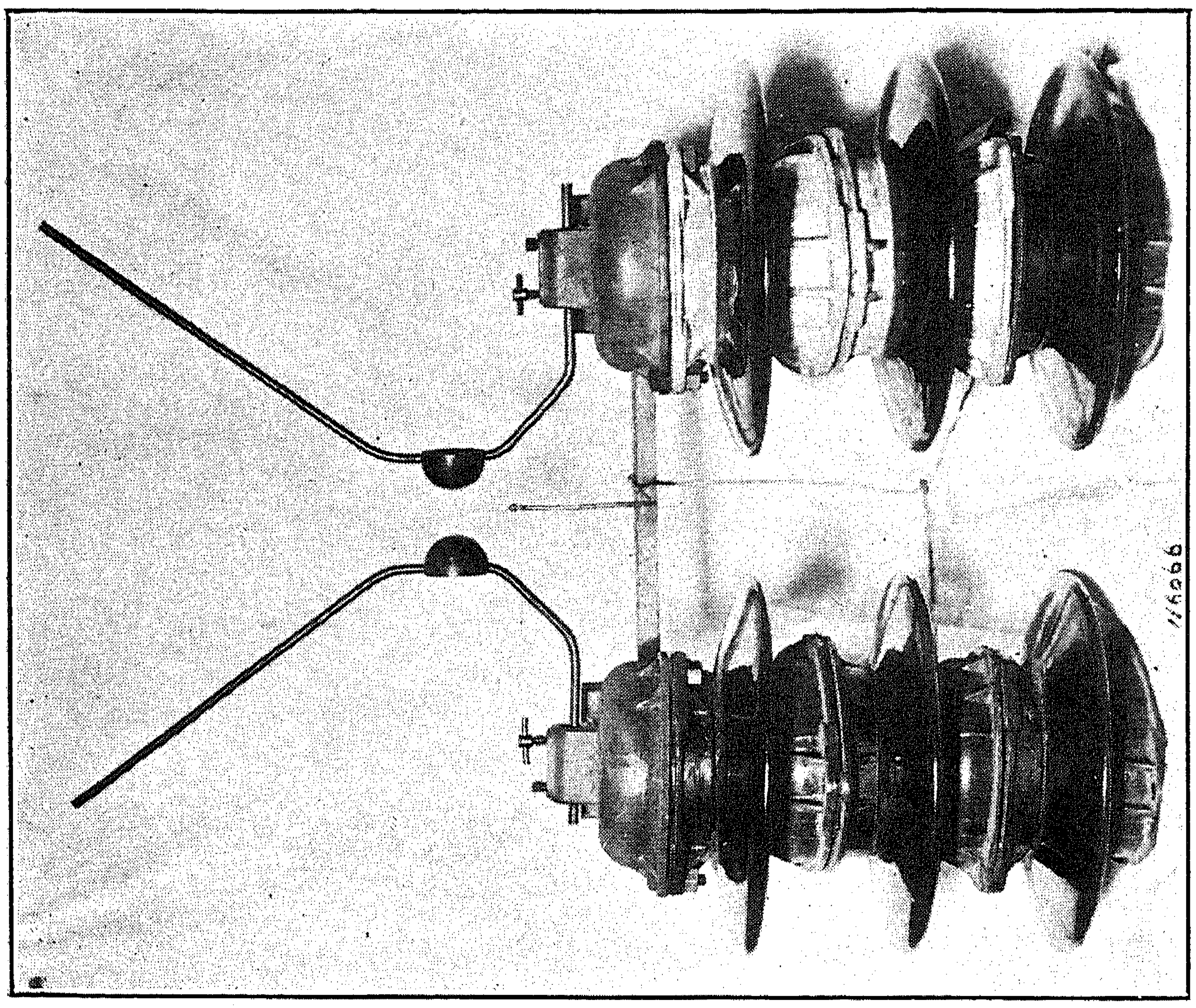

FIG. 5

[ALLCUTT]

Then successive impulses were applied at intervals of from 15 to 30 seconds and the gap G_r adjusted until a setting was found at which about 50 per cent of the impulses would discharge across the gap being tested. Then the measuring gap G_m was adjusted until the impulse discharges were shared equally by it and the gap under test. The setting of G_m that fulfilled this condition was taken as a measure of the impulse discharge voltage of G. Thus it will be seen that the impulse discharge voltage of the gap under test was in every case determined by direct comparison with a sphere gap.

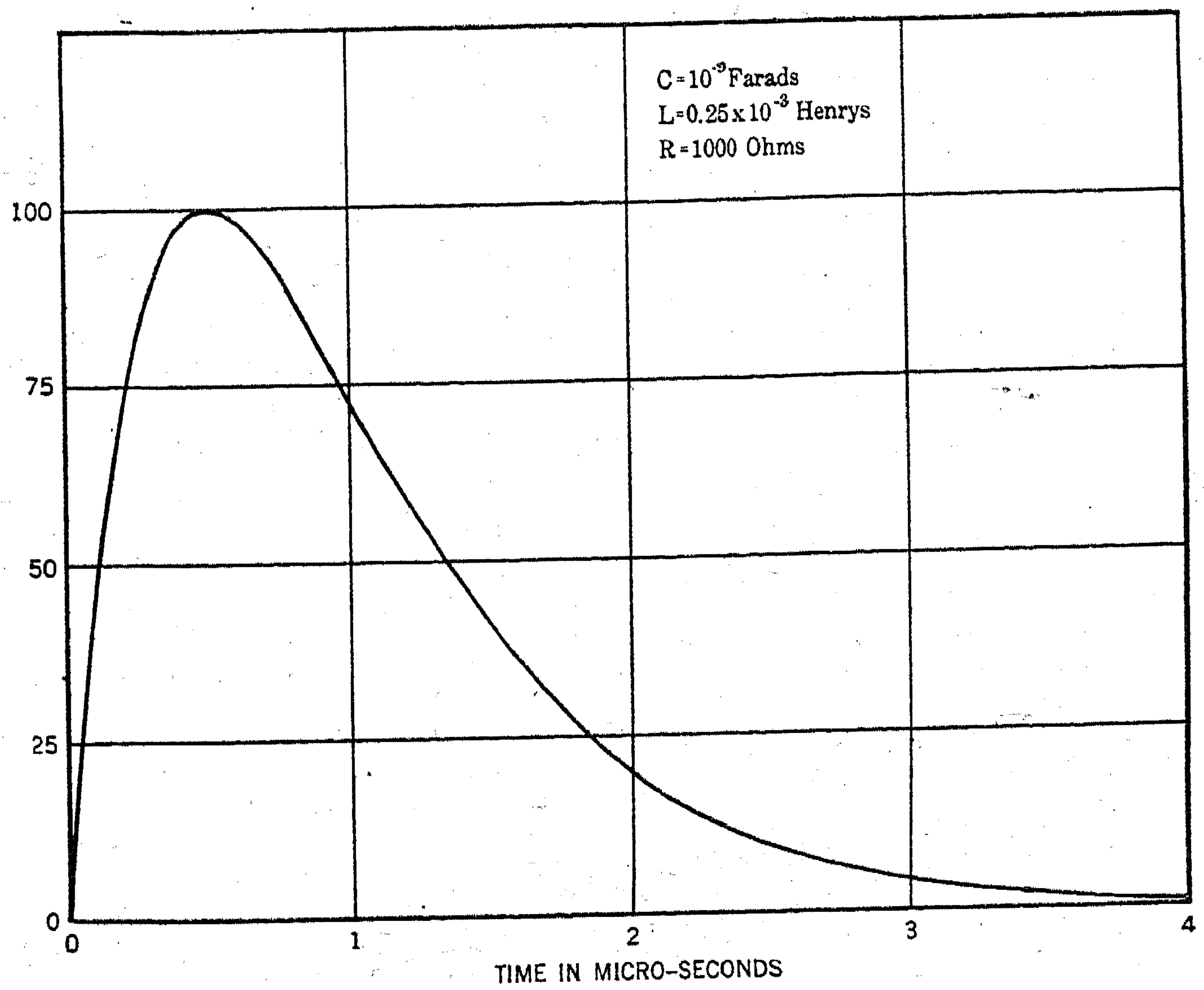

FIG. 7—500,000-CYCLE CRITICALLY DAMPED IMPULSE WAVE

The circuit used in testing the effect of an impulse voltage superimposed on a 60-cycle wave is shown in Fig. 8. Two high-tension transformers T_1 and T_2 are employed. These transformers are excited from the same 60-cycle supply circuit. T_1 applies a 60-cycle voltage to the gap while T_2 excites the impulse circuit which comprises a condenser C, an inductance L and a resistance R. As in Fig. 6, G_r is a spark gap used to regulate the impulse voltage. A condenser K of relatively large capacity is connected across the transformer T_1 external to the protective reactance L_p. This condenser presents a negligible impedance in comparison with the impedances shunting the gap G, so the full value of an impulse voltage across the resistance *R* may be regarded as being impressed on the gap under test. If the voltage of the transformer T_2 be slowly raised, the impulse resulting from the discharge of G_r will occur at the peak of the 60-cycle wave. By proper selection of the relative polarities of T_1 and T_2 this impulse may be madeto add to, or subtract from, the 60-cycle wave produced by T_1, thus giving a resultant voltage wave, impressed on·the gap *G*, similar to *B* or *C* in Fig. 1.

In making tests with the outfit described above, the voltage of T_1 was first set at the desired value. This voltage was carefully determined by means of a sphere gap, since the presence of the condenser *K* caused the terminal voltage of T_1 to be considerably above the value determined from transformer ratio. Then successive impulses were applied and, as before, a setting of G_r was found which would cause about 50% of the impulses to break down the gap *G*. When testing the effect of the wave shown in Fig. 1B, where the impulse adds to the 60-cycle wave, the measuring gap G_m was connected in parallel with the gap under test and a direct comparison obtained between the two. In testing case *G*, where the impulse subtracts from the 60-cycle wave, the value of the impulse voltage was computed from the setting of G_r and checked by an occasional measurement with a sphere gap connected across the resistance *R*.

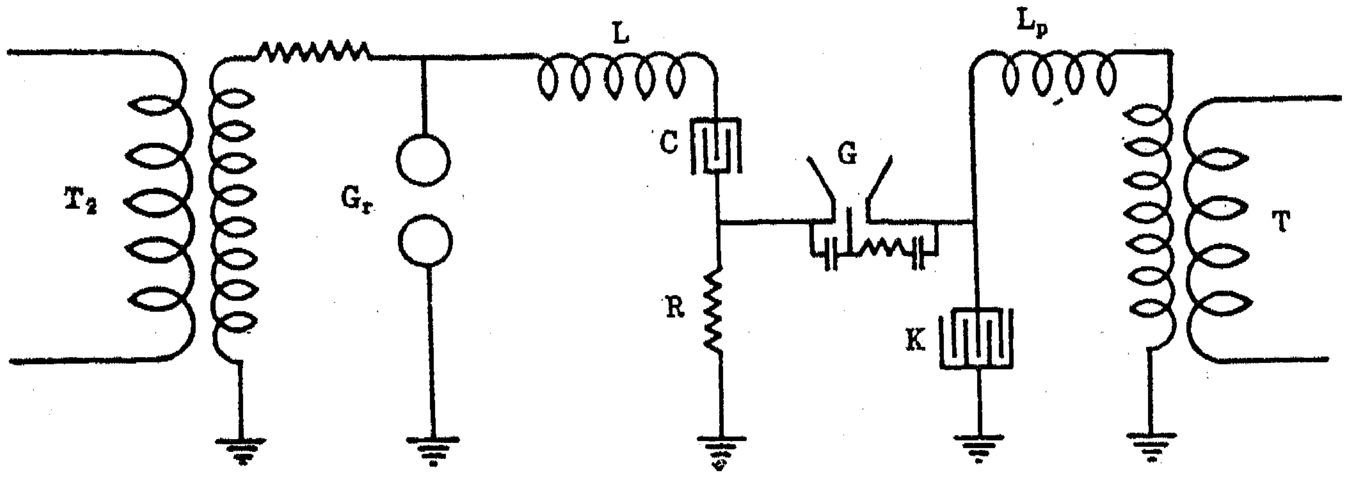

FIG. 8—CONNECTIONS FOR APPLYING IMPULSE VOLTAGE SUPERIMPOSED ON 60-CYCLE WAVE

The 60-cycle discharge voltage of the gaps tested was determined in the usual way by comparison with a standard 25-cm. sphere gap.

RESULTS OF IMPULSE TESTS

The first series of experiments undertaken was for the purpose of determining the effect of the presence of the auxiliary electrode on the 60-cycle discharge voltage of the two forms of gap tested. Careful tests were made at gap settings ranging from ½ in. (1.27 cm.) to 3 in. (7.6 cm.). The results of these tests indicated that the presence of the auxiliary electrode does not appreciably lower the 60-cycle discharge voltage. With the auxiliary electrode in place, breaks were somewhat less consistent than with the plain gap, but the irregularities were not of sufficient magnitude to indicate an appreciable lowering of the breakdown voltage.

TABLE 1. IMPULSE PROTECTIVE GAP

3/8-in. Diameter Horn Electrodes.

Gap setting. inches	60-Cycle discharge. kv. max.	500,000 Impulse		Impulse ratio
		Discharge one side only kv. max.	Discharge both sides kv. max.	
0.5	37	16	16	0.45
1.0	60	28	28	0.47
1.5	73	40	40	0.55
2.0	81	53	55	0.68
2.5	88	67	73	0.83
3.0	93.5	80	84	0.90

Preliminary tests with impulse voltages were then made in order to determine the best values for the capacitances and resistances used in the circuits connecting the auxiliary electrode with each horn. The results of these experiments showed that the capacitances and resistances could be varied through wide

TABLE II. IMPULSE PROTECTIVE GAP

6.25-Cm. Spherical Electrodes Mounted on Horns.

Gap setting. inches	60-Cycle discharge. kv. max.	500,000 Impulse		Impulse ratio
		Discharge one side only kv. max.	Discharge both sides kv. max.	
0.5	39	12.5	12.5	0.31
1.0	71	30	30	0.42
1.5	99	47	49	0.50
2.0	121	60	65	0.54
2.5	137	74	82	0.60
3.0	147	85	93.5	0.64

limits without affecting the impulse discharge voltage. The values of the capacitances employed in the succeeding experiments were, therefore, largely determined by convenience. In practically all the experiments discussed in the following pages, the condenser connected between the auxiliary electrode and each horn had a capacity of approximately 10^{-10} farads. As previously stated, each of these condensers consisted of two pillar-type porcelain insulator units in series (see Figs. 4 and 5). Using capacitances of the value given above, tests made with resistances varying from 50,000 ohms to several megohms showed no variation in the impulse discharge voltage of the gap. Accordingly, 100,000 ohms was selected as a convenient value of resistance to use in the remainder of the tests. This resistance is high enough to cause a very great unbalance in the voltage across the two sides of an impulse gap under the action of high frequency, and, on the other hand, is low enough to present a negligible impedance to 60-cycles in comparison with the capacitances employed.

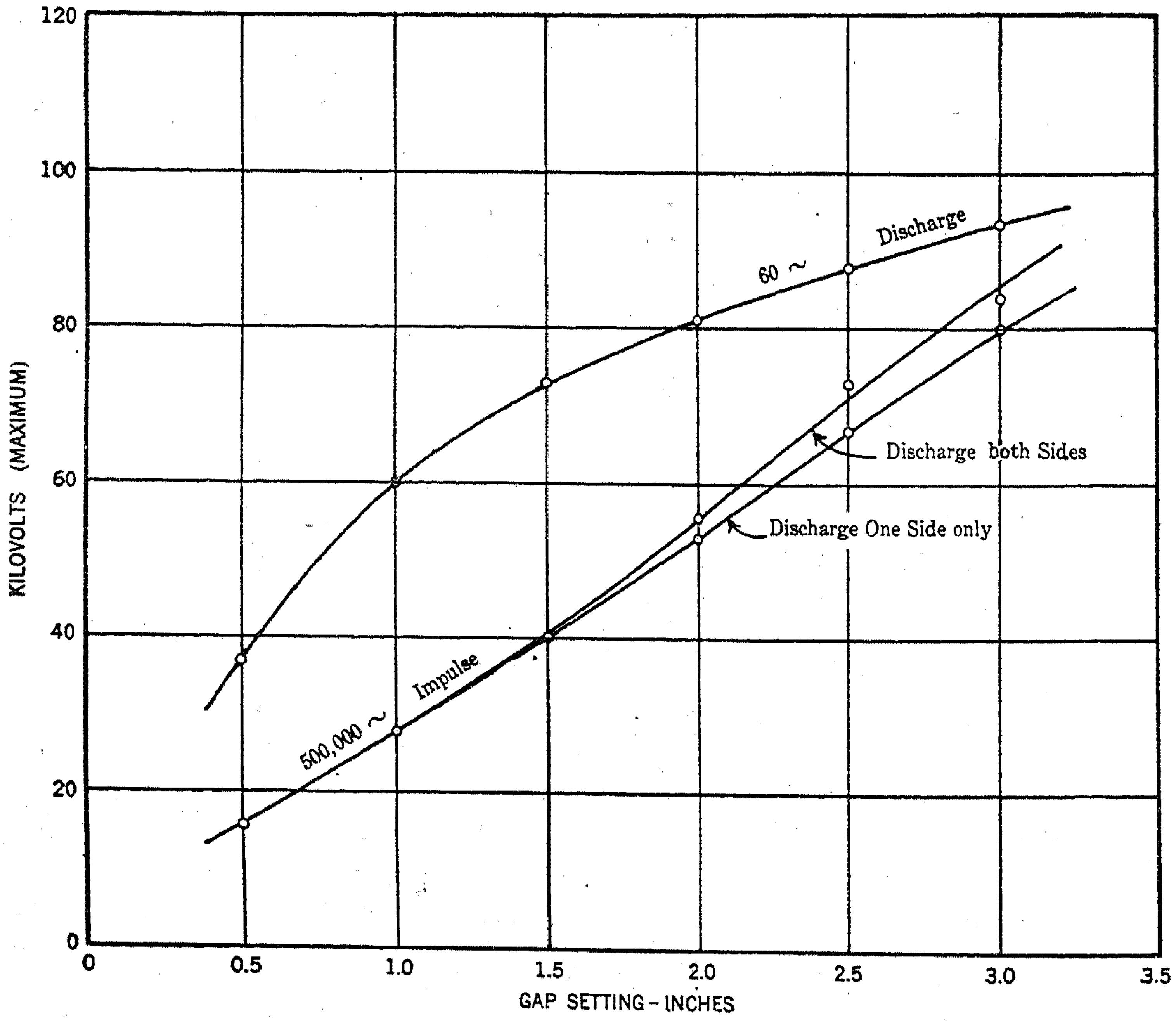

FIG. 9—DISCHARGE CURVES OF IMPULSE PROTECTIVE GAP ⅜-INCH DIAMETER HORN ELECTRODES

Having decided on the constants given above, the two forms of impulse protective gap shown in Figs. 4 and 5 were tested for both 60-cycle breakdown and 500,000-cycle impulse breakdown, with various gap settings ranging from ½ in. (1.27 cm.) to 3 in. (7.6 cm.). The results of these tests are given in Tables I and II.

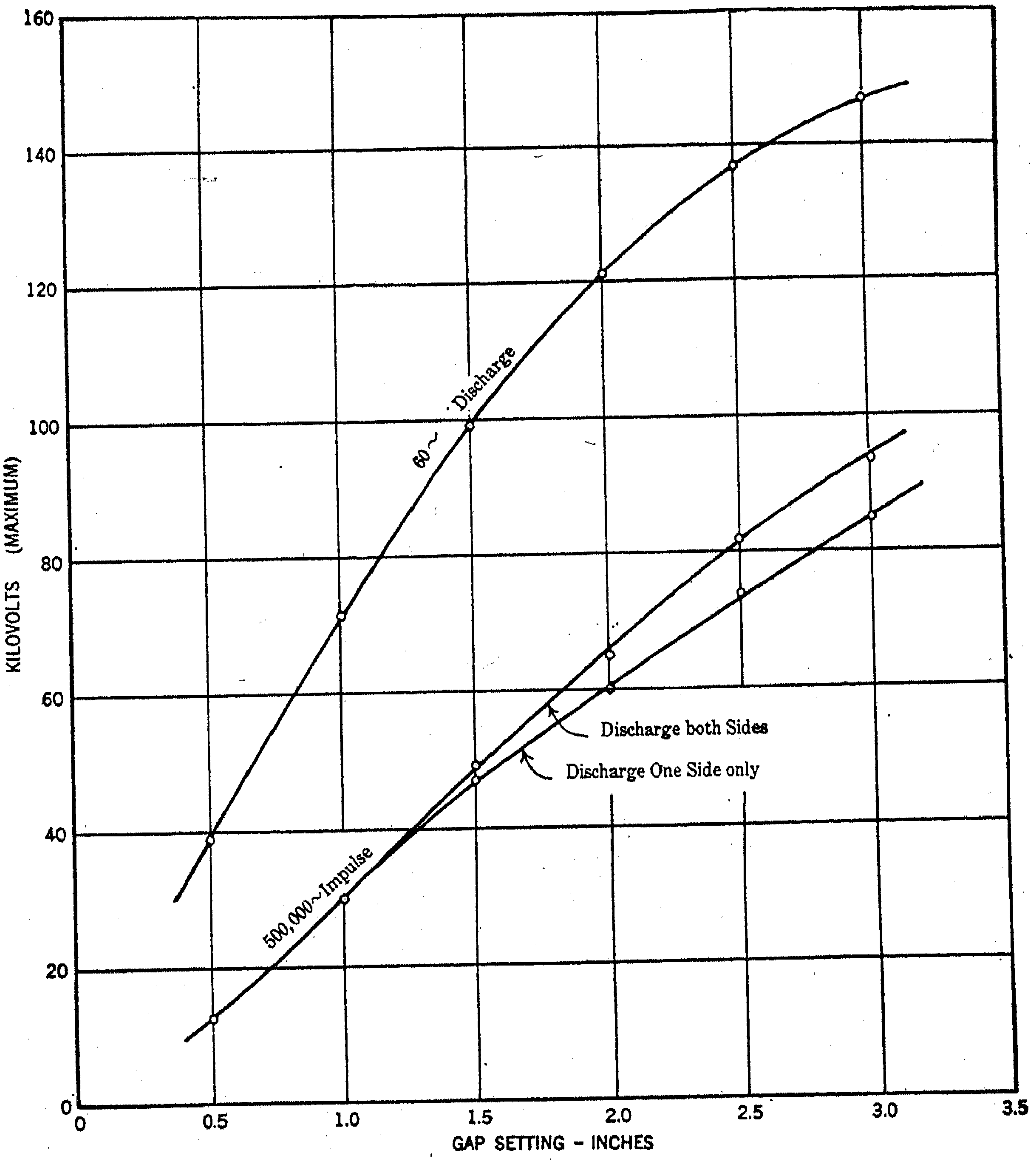

FIG. 10—DISCHARGE CURVES OF IMPULSE PROTECTIVE GAP 6.25-CM. SPHERICAL ELECTRODES MOUNTED ON HORNS

It will be noted that, under the heading "500,000 cycle impulse discharge voltage", there are two columns, one giving the impulse voltage which would discharge across one side of the gap only, and the other giving the impulse voltage required to cause a discharge across the whole gap. For small gap settings the difference between these two voltages was found to be too small to measure; that is to say, successive applications of an impulse of just sufficient voltage to cause an occasional discharge, resulted in part of the discharges being across the whole gap and part of them being between the auxiliary electrode and one horn only. In computing the impulse ratio, given in column 5, the value of the impulse voltage required to discharge across both sides of the gap was used. This value of the impulse discharge voltage was also used in all the curves given hereafter except where the contrary is specifically stated.

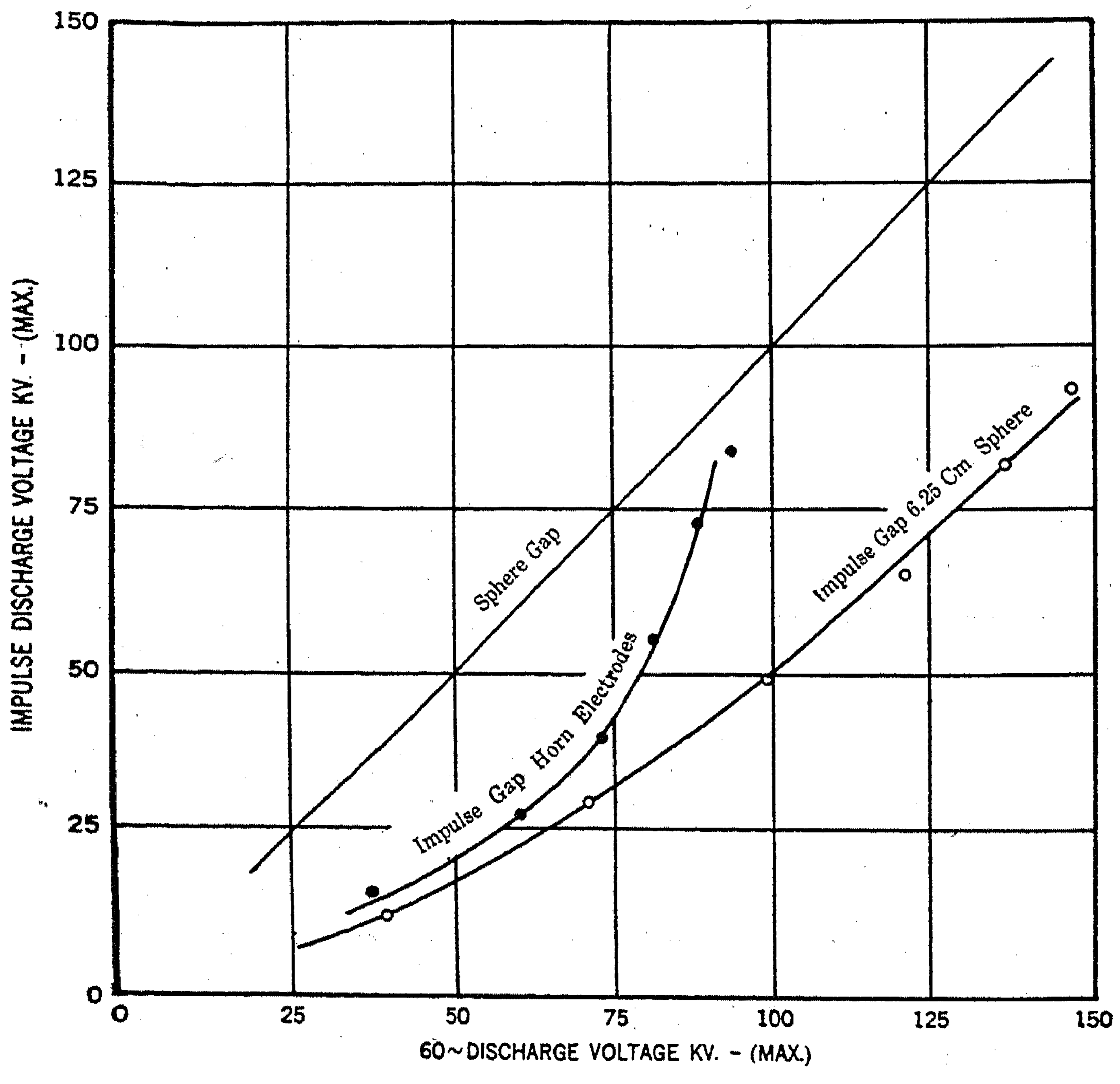

FIG. 11—CURVES SHOWING RELATION BETWEEN 60-CYCLE DISCHARGE VOLTAGE AND 500,000-CYCLE IMPULSE DISCHARGE VOLTAGE

The data given in Tables I and II are presented graphically in Figs. 9 to 12 inclusive. Figs. 9 and 10 give the 60-cycle and impulse discharge voltages as a function of gap setting for the two forms of gap shown in Figs. 4 and 5 respectively. It will be seen that in every case the impulse discharge voltage is *less* than the 60-cycle breakdown, indicating an impulse ratio of less than unity. In Fig. 11, the impulse discharge voltages are plotted against the 60-cycle discharge for the two forms of impulse gap and for the plain sphere gap. Fig. 12 shows the impulse ratio of the two gaps as a function of gap setting. It will be noted that with the form of gap having the plain horn electrodes, the impulse ratio increases rapidly with the larger gap settings and will probably become higher than unity for gap settings much in excess of 3 in. (7.6 cm.). Where spherical electrodes are used, it is evident from the curves that the impulse ratio will remain well under unity even for gap settings considerably greater than 3 in. The reason for the superiority of the gap equipped with

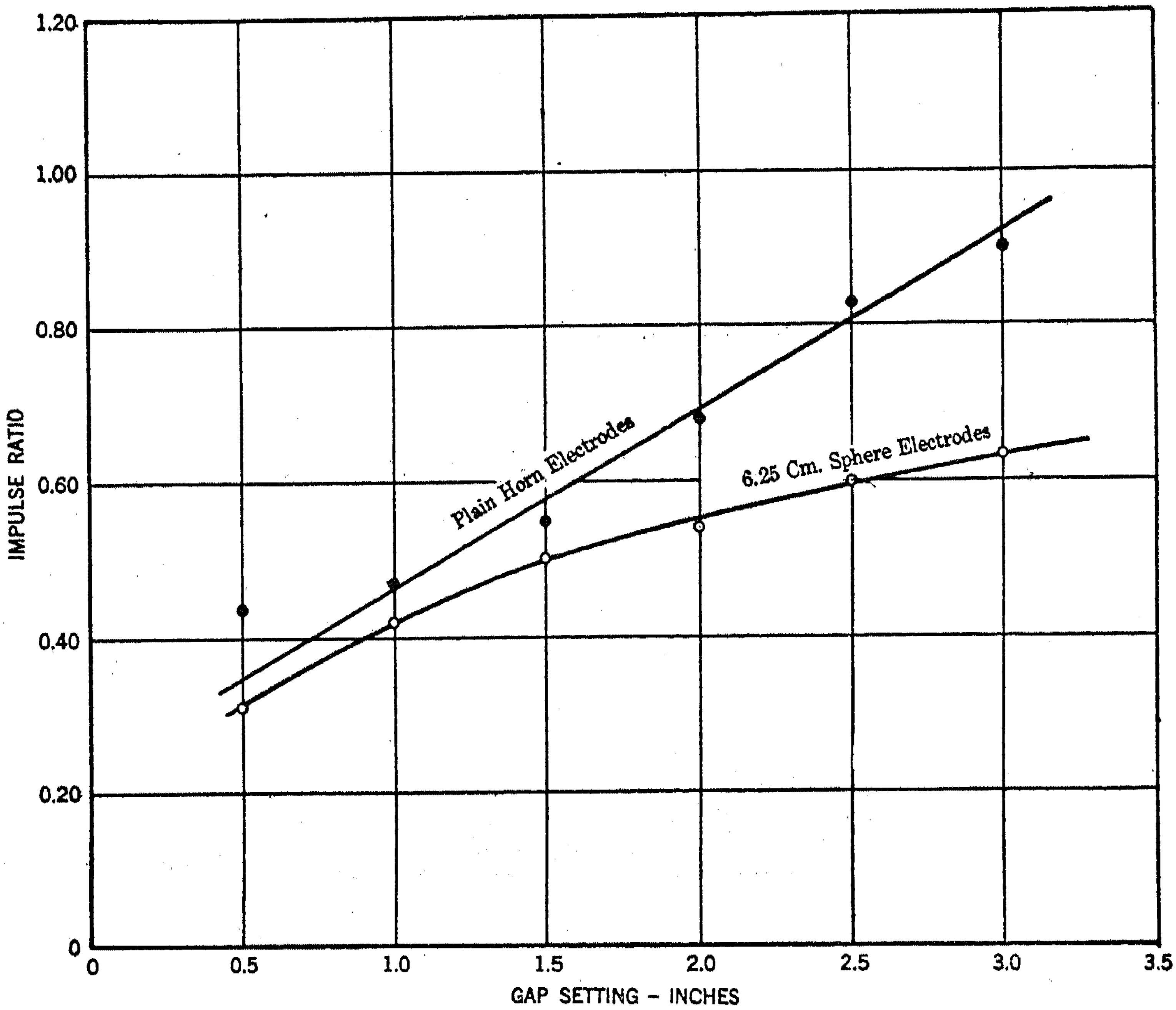

FIG. 12—IMPULSE PROTECTIVE GAPS—IMPULSE RATIO AS FUNCTION OF GAP SETTING

spherical electrodes may be made clear by reference to Figs. 9 and 10. By comparison of these curves it appears that, for a given gap setting, the impulse discharge voltages of the two gaps do not differ greatly. That is to say, a very great change in the shape of the main electrodes does not affect the impulse discharge voltage to a corresponding degree. We know, however, that the shape of the main electrodes will very materially modify the 60-cycle discharge. In view of these facts it is obvious that the lowest impulse ratio will be obtained by so shaping the main electrodes as to give the highest possible 60-cycle breakdown voltage. This condition is approximated with spherical electrodes.

A consideration of the foregoing results seems to lead to the conclusion that the impulse gap has great possibilities as a protective device. In themselves, however, these results are by no means conclusive. The action of the new gap under more adverse conditions must be studied before its protective value can be regarded as definitely established. While it has already been shown that the impulse gap will discharge a much lower impulse voltage than a sphere gap having the same 60-cycle discharge, there remains the possibility that the gap may exhibit large time lag when an impulse considerably in excess of its discharge voltage is applied, thus permitting the voltage to rise to a dangerous value before discharge takes place. This condition was tested by determining the "equivalent sphere gap" of an impulse gap subjected to a transient voltage greater than its discharge voltage. The results of two series of tests made are

TABLE III. IMPULSE PROTECTIVE GAP

3/8 in. Diameter Horn Electrodes
Gap Setting, 1½ in. (3.8 cm.); 60-cycle Discharge 73 kv. max.

Impulse applied kv. max.	Equivalent Sphere gap	
	cm.	kv. max.
40	1.30	40
47	1.45	43.5
57	1.55	46
82	1.63	48
100	1 75	51

TABLE IV. IMPULSE PROTECTIVE GAP

6.25 Cm. Spherical Electrodes Mounted on Horns.
Gap Setting, 1 in. (2.54 cm.); 60-cycle Discharge, 71 kv. max

Impulse applied kv. max.	Equivalent sphere gap	
	cm.	kv. max.
30	0.95	30
45	1.05	32.5
71.	1.15	35
86	1.20	37
100	1.20	37

given in Tables III and IV and in the curves in Figs. 13 and 14. In obtaining the above results a sphere gap was connected in parallel with the impulse gap under test. Then a setting of the sphere gap was found that would permit the two gaps to share the discharges equally when a given impulse voltage was applied. If we assume the time lag of the sphere gap to be negligible, this

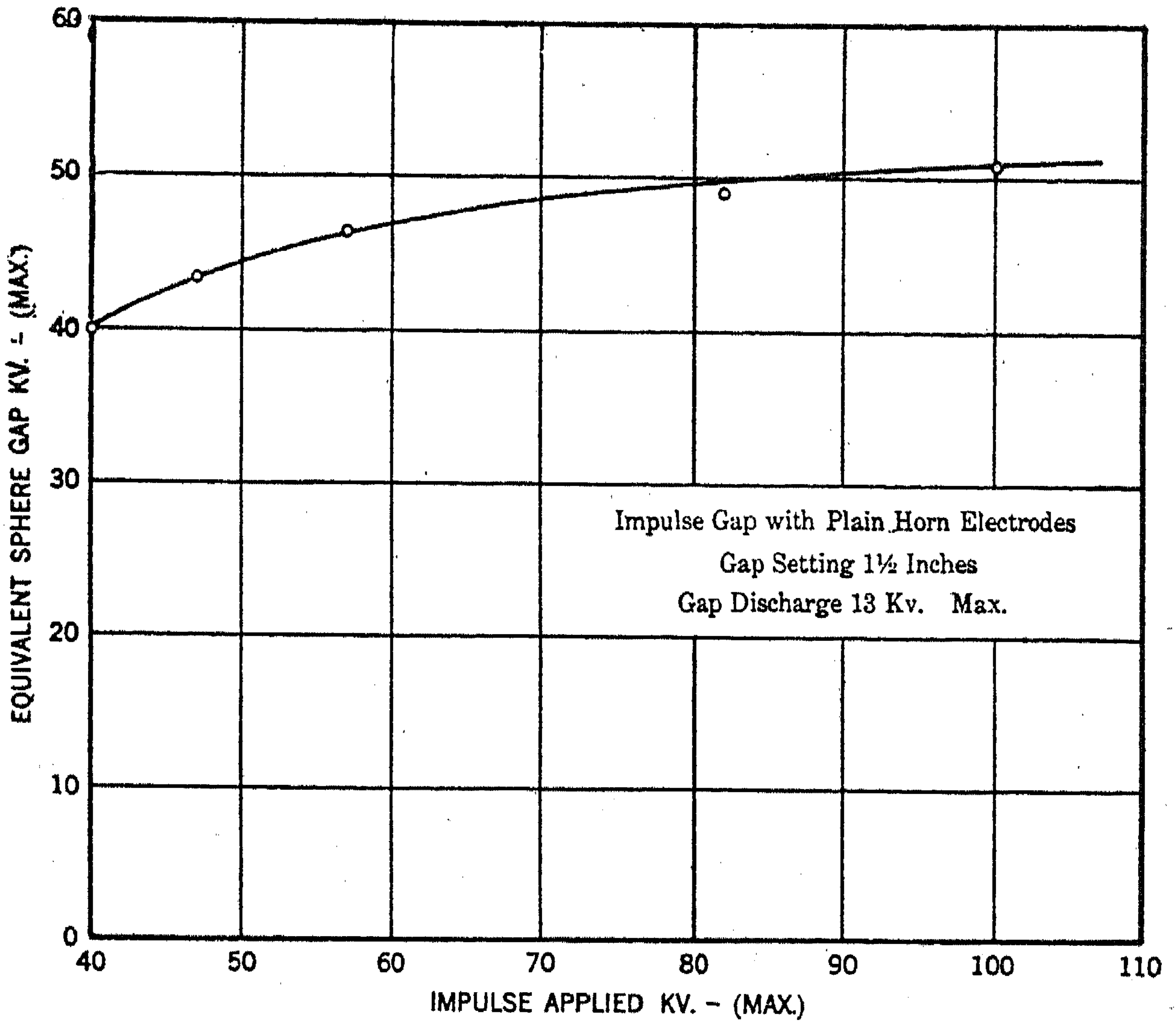

FIG. 13—SHOWING EFFECT OF IMPULSES HIGHER THAN THE IMPULSE DISCHARGE VOLTAGE OF THE GAP

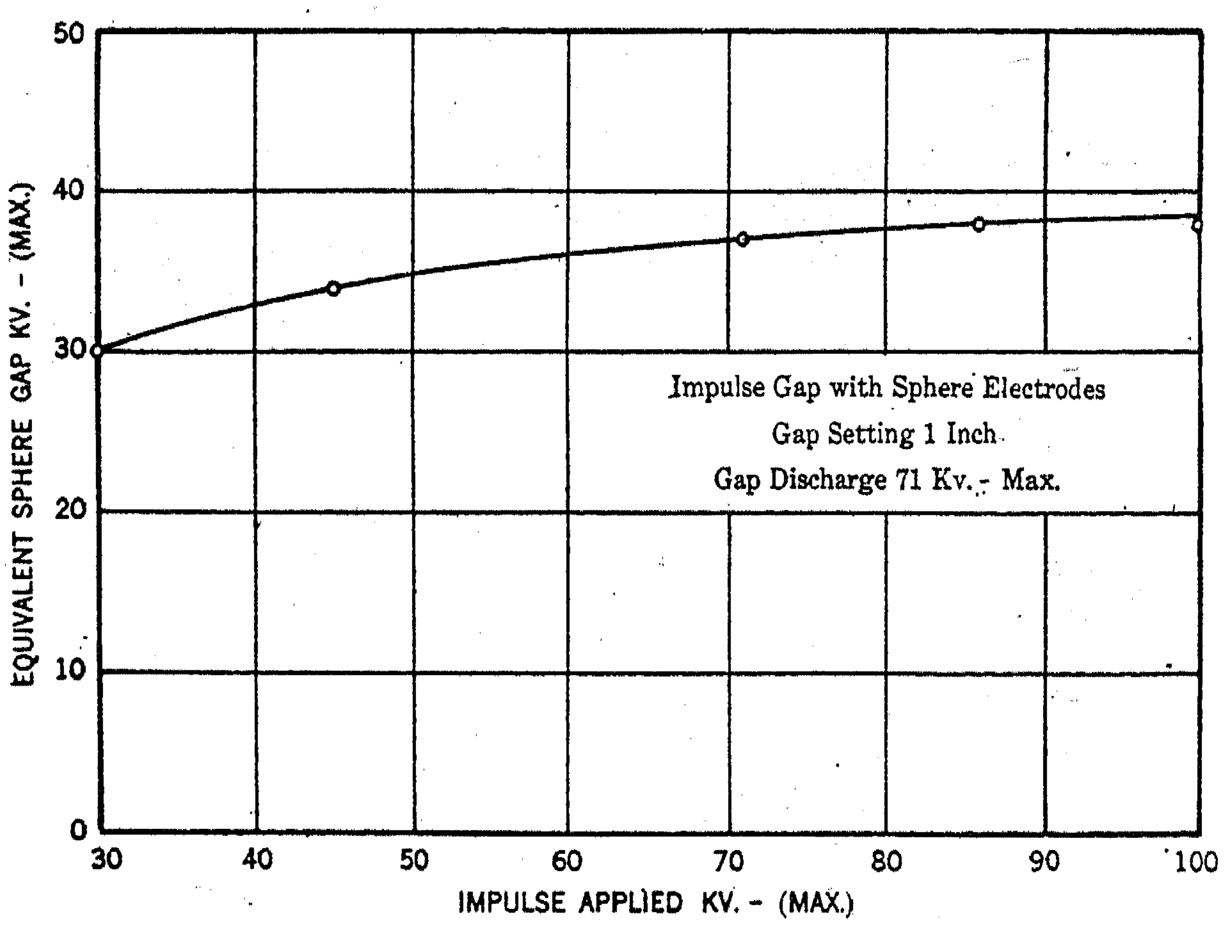

FIG. 14—SHOWING EFFECT OF IMPULSES HIGHER THAN THE IMPULSE DISCHARGE VOLTAGE OF THE GAP

"equivalent sphere gap" setting is a measure of the actual voltage to which the impulse gap permits an impulse of super-discharge voltage to rise. Figs. 13 and 14 show that the two forms of impulse gap do permit some rise in voltage above the minimum impulse discharge value, but this rise is not of sufficient magnitude to cut down seriously the degree of protection afforded. Even with an impulse applied, having a value of three times the minimum discharge voltage, breakdown occurs at very much less than the 60-cycle discharge voltage.

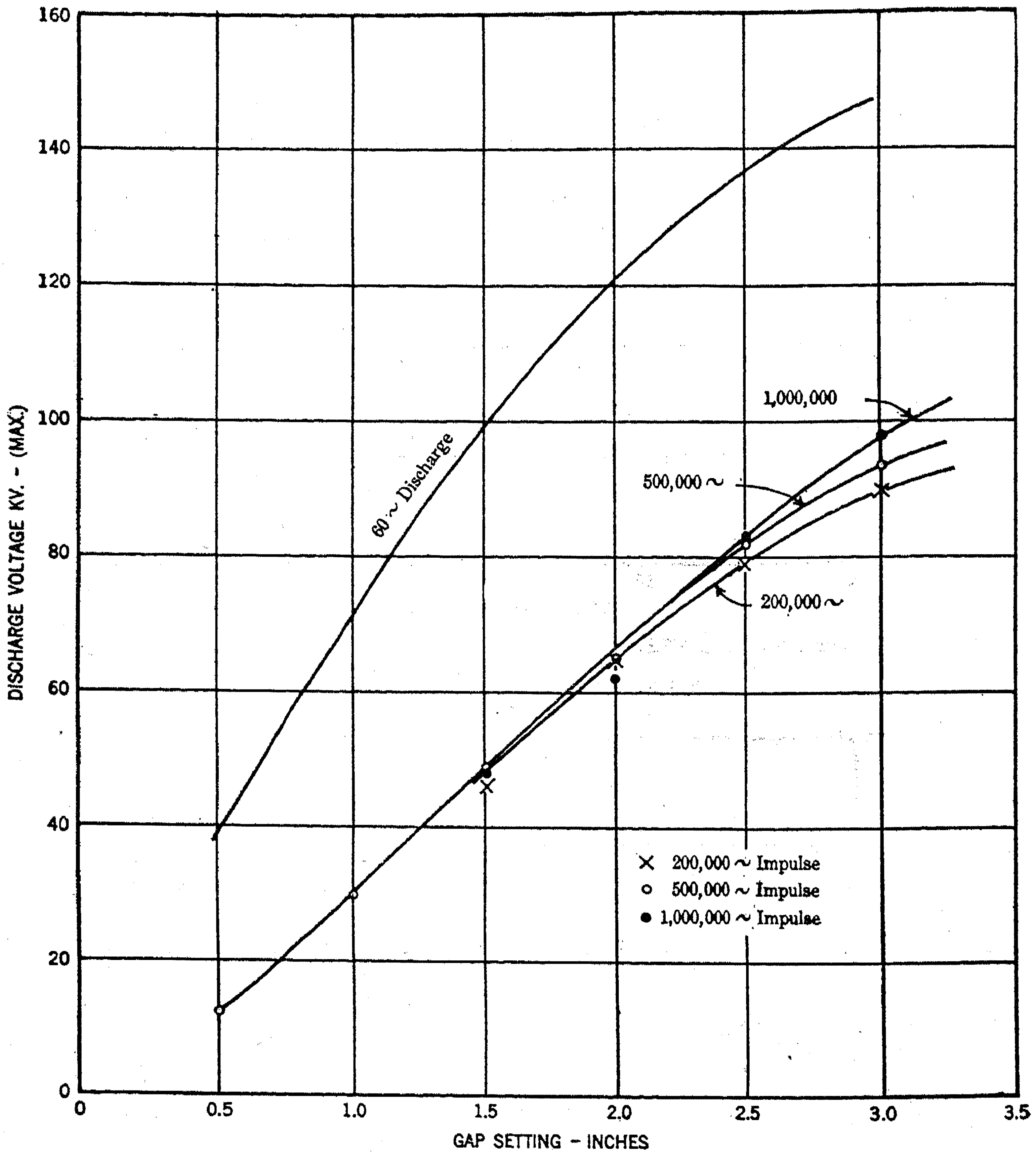

FIG. 15—TESTS ON IMPULSE PROTECTIVE GAP WITH SPHERE ELECTRODES SHOWING EFFECT OF IMPULSES OF DIFFERENT FREQUENCIES

In addition to the above tests, it was thought desirable to undertake further experiments to determine the discharge characteristics of the impulse gap with frequencies of other than 500,000 cycles. Fig. 15 shows the results of some of these tests. It will be seen that the impulse discharge voltage increases slightly with increasing steepness of wave front applied. This increase in discharge voltage with increasing frequency is not very great, so it was not deemed necessary to make a complete series of tests at other than 500,000 cycles. It is a reasonable assumption that the tests made at this latter frequency will represent a fair average value of the discharge characteristics of the gap under the action of high-frequency transients liable to occur in practise.

TABLE V.

Impulse Protective Gap with ⅜-in. diameter Horn Electrodes
Tests with 500,000 Impulse Superimposed on 60-Cycle Wave

Gap setting. Inches	60-Cycle discharge. kv. max.	60-cycle voltage applied. kv. max.	Impulse required to cause discharge		Equivalent line voltage kv. r. m. s.
			Case B kv. max.	Case C kv. max.	
1.0	60	30	9	30	36.5
1.5	73	36.5	16	45	44.5
2.0	81	40.5	20	52	49.5
2.5	88	44	40	71	54
3.0	93.5	47	41	90	57

TABLE VI.

Impulse Protective Gap with 6.25 cm. Spherical Electrodes
Tests with 500,000 Impulse Superimposed on 60-Cycle Wave

Gap setting. Inches	60-cycle discharge. kv. max.	60-cycle voltage applied. kv. max.	Impulse required to cause discharge		Equivalent line voltage kv. r. m. s.
			Case B kv. max.	Case C kv. max.	
1.0	71	36	9	32	44
1.5	99	49.5	18	47	60.5
2.0	121	60	22	68	73.5
2.5	137	68	32	84	83
3.0	147	73	45	96	89

Results of Tests

Impulse Voltages Superimposed on 60-Cycle Wave

In making tests with an impulse voltage superimposed on a 60-cycle wave, the 60-cycle voltage applied was equal to one-half the discharge voltage of the gap. With this 60-cycle voltage applied, the value of the impulse voltage required to cause discharge was determined both for an impulse adding to the 60-cycle wave and for an impulse subtracting therefrom. This test may be regarded as closely simulating conditions that may actually occur in practise. Having the gap set for a discharge voltage equal to double the 60-cycle voltage applied, was taken as a fair approximation of the usual practise with regard to the setting of lightning arrester gaps.

The results of the tests made are given in Tables V and VI.

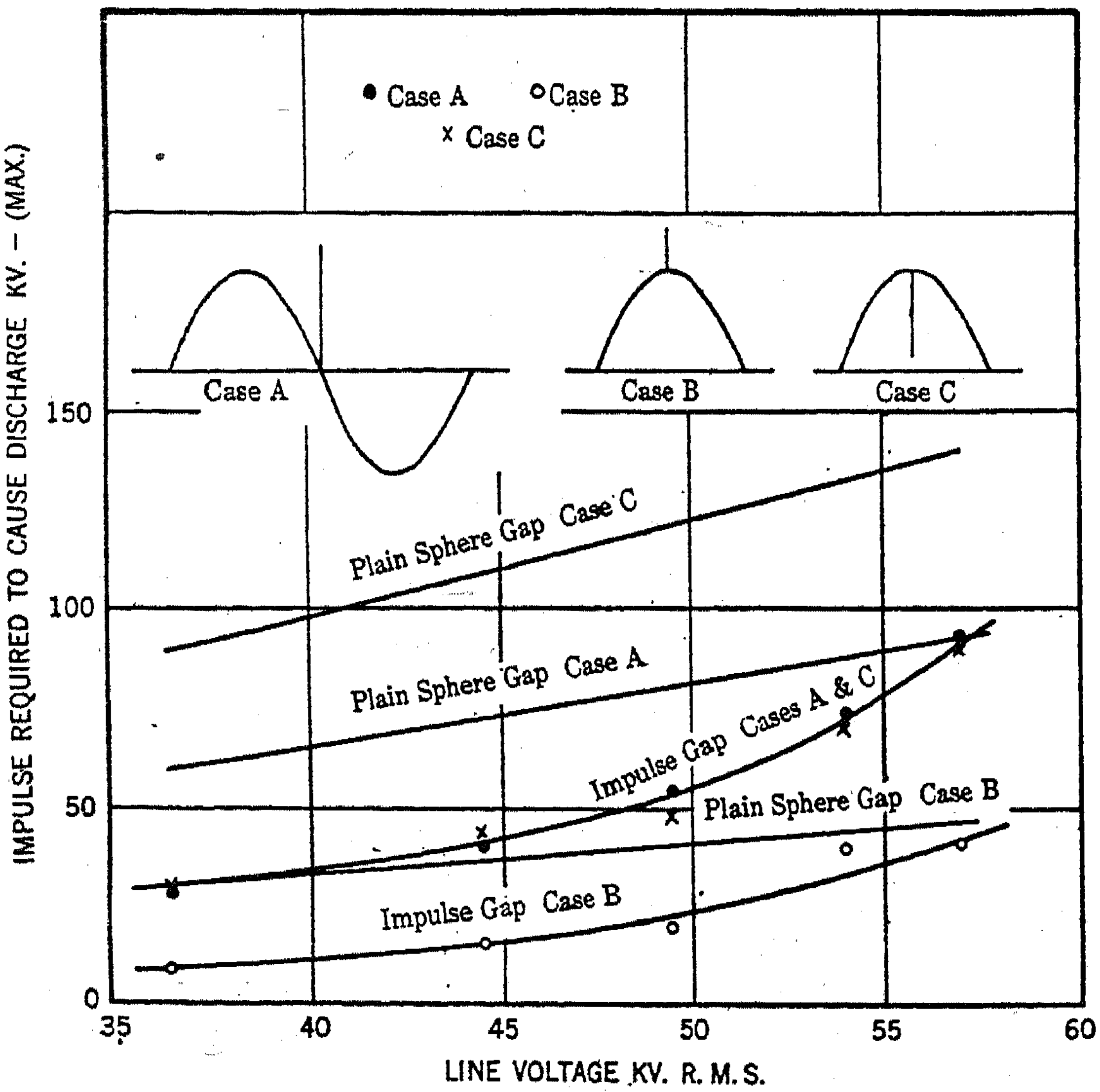

FIG. 16—TESTS ON IMPULSE PROTECTIVE GAP WITH PLAIN HORN ELECTRODES SHOWING EFFECT OF IMPULSES SUPERIMPOSED ON 60-CYCLE WAVE—GAP SET AT TWICE 60-CYCLE VOLTAGE TO GROUND

In column 3 are given the real peak values of the 60-cycle voltages applied at the different gap settings. This is, of course, equivalent to the peak value of voltage from one line to ground, which is applied to a lightning arrester gap in service. In order to form a more convenient basis for correlating the test results with service conditions, the root-mean-square value of the three-phase line voltage, equivalent to this peak voltage to ground, is given in the last column. Figs. 16 and 17 show, graphically, the impulse voltage required to cause discharge, as a function of this equivalent line voltage. The curves given for Case A are taken from the results, given in Tables I and II, for the impulse voltage alone. It is interesting to note that, for both forms of impulse gap, the results for Cases A and C are practically identical. In addition to the curves for the impulse gaps, the characteristics of a plain sphere gap are shown for purpose of comparison. The curves for the sphere gap are computed, assuming an impulse ratio of one. It might be stated here that actual tests on a sphere gap subjected to an impulse subtracting from a 60-cycle wave (Case C) closely checked the computed curves given.

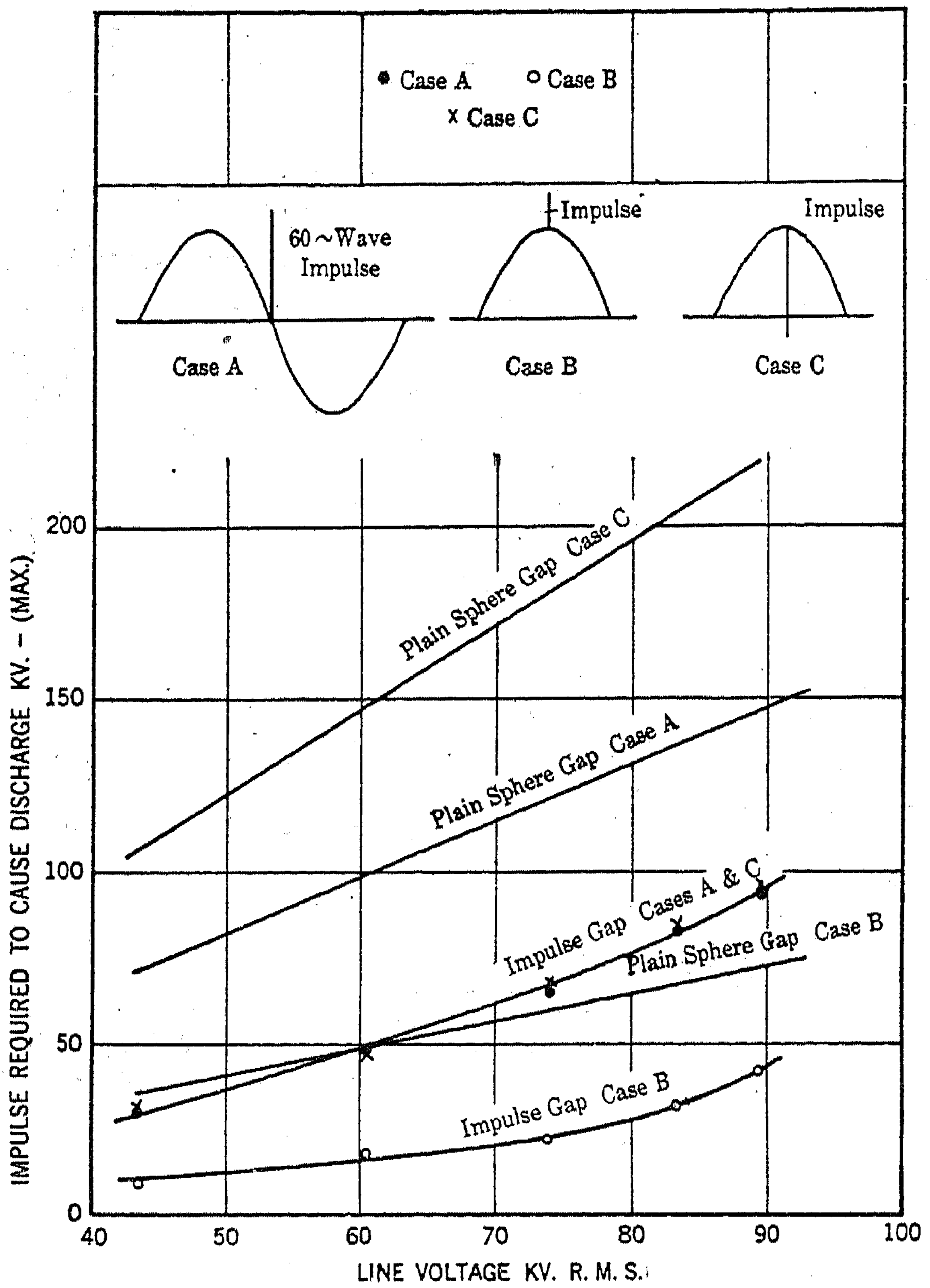

FIG. 17—TESTS ON IMPULSE PROTECTIVE GAP WITH SPHERICAL ELECTRODES SHOWING EFFECT OF IMPULSE VOLTAGE SUPERIMPOSED ON 60-CYCLE WAVE—GAP SET AT TWICE 60-CYCLE VOLTAGE TO GROUND

Figs. 16 and 17 are particularly effective in showing the very superior protection afforded by the impulse protective gap when subjected to an impulse subtracting from the 60-cycle wave (Case C). For example, on a 66,000-volt line an impulse protective gap would discharge such an impulse having a value of but 56 kv. max. (See Fig. 16), while a plain sphere gap would require an impulse of 161 kv. max. to cause a discharge under the same conditions.

CONCLUSION

The conclusions arrived at from the foregoing experiments may be summarized as follows:

1. In order to secure adequate protection against line disturbances of steep wave front, it is important that a protective gap having the lowest possible impulse ratio be used. A gap having an impulse ratio of less than unity is particularly desirable.

2. There are certain combinations of a high-frequency impulse with a wave of line frequency for which the degree of protection afforded by an ordinary spark gap is greatly lessened. The need for a protective gap having selective properties, rendering it sensitive to steep wave fronts, is emphasized by these conditions.

3. A protective gap, having a selective discharge for steep impulses, may be constructed without greatly complicating the usual horn gap structure.

4. Exhaustive tests of such a gap have shown it to be superior to the ordinary sphere gap under all conditions. Even under the most unfavorable conditions the new gap gives a high degree of protection.

DISCUSSION ON "LIGHTNING ARRESTER SPARK GAPS—THEIR RELATION TO THE PROBLEM OF PROTECTING AGAINST IMPULSE VOLTAGES" (ALLCUTT), ATLANTIC CITY, N. J., JUNE 27, 1918.

Paul M. Lincoln: In the design of lightning arresters, we must recognize two distinct functions. First is the necessity of discharging the surges which come in from outside sources. They must be taken care of, and discharged through a path where they can do no harm instead of being allowed to go through apparatus where their discharge may result in disaster to the apparatus; that is one function. The second function is to let go of the dynamo current after it has started through the lightning arrester.

This piece of analysis apparatus proposed by Mr. Alcutt I want to point out, is of advantage on both of these scores. First, it allows a setting of the arrester gap which will permit the surges of lower value to go through and second, it allows the spacing of the horns to be increased, so that if the dynamo current of the order of 60 cycles or so starts through the lightning arrester, the dynamo current will let go easier; that is, the spacing of the horn gaps is permitted to be larger for the frequencies of the dynamo current. Therefore for both of these functions the suggestion put forth in this paper is of very material assistance.

F. W. Peek, Jr.: Before discussing directly Mr. Allcutt's results, I will briefly review some of my own work which has a bearing on this paper; this review will, I hope, make clearer what I have to say in the direct discussion.

The over-voltages that cause insulation failures in practise may be divided into three classes:

1. Gradual increase of voltage on the line, due to static or low-frequency surges. Such gradually applied voltages readily discharge over an arrester gap.
2. Very high-frequency oscillations of voltages too low for any gap arrester to discharge, but which may cause very high internal voltages in apparatus.
3. The form of over-voltage with which we are at present concerned,—lightning impulses of very steep wave front, when the voltage across the apparatus increases from normal to a very high value in perhaps a millionth of a second.

In 1913 I became interested in determining means of producing impulses of known wave shape and voltage, and in measuring impulse voltages. This work was stimulated by lightning troubles that I was asked to investigate on the lines of one of the large operating companies. In this particular case certain switch bushings with a 60-cycle arc-over voltage of about 100 kv. were protected by aluminum arresters with horn gaps set at about 50 kv., 60 cycles. Lightning always arced over the bushings, and never arced over the arrester gaps. The 60-cycle arc-over voltage of a gap or insulator was evidently no criterion of the lightning arc-over voltage. It was obvious that a lightning arrester gap should be designed, if possible, for a low lightning arc-over voltage, and insulators and apparatus for a high lightning arc-over voltage. This difference between the lightning and 60-cycle arc-over voltage, illustrated by the above practical case was caused by the "lag" of the gap.

It had long been known that there was some sort of a time lag between the application of voltage and the discharge of a gap, and an interesting paper by Hayden and Steinmetz showed that energy was necessary. The importance of the shape of the electrode on the arc-over voltage was however not suspected. Lightning arresters were, in fact, at that time given the E. N. G. or *equivalent needle gap* test. That the needle gap was just the wrong one to use will appear later. There were no definite data in regard to time, the reason being that there was no generator for producing known impulses or a means of measuring them.

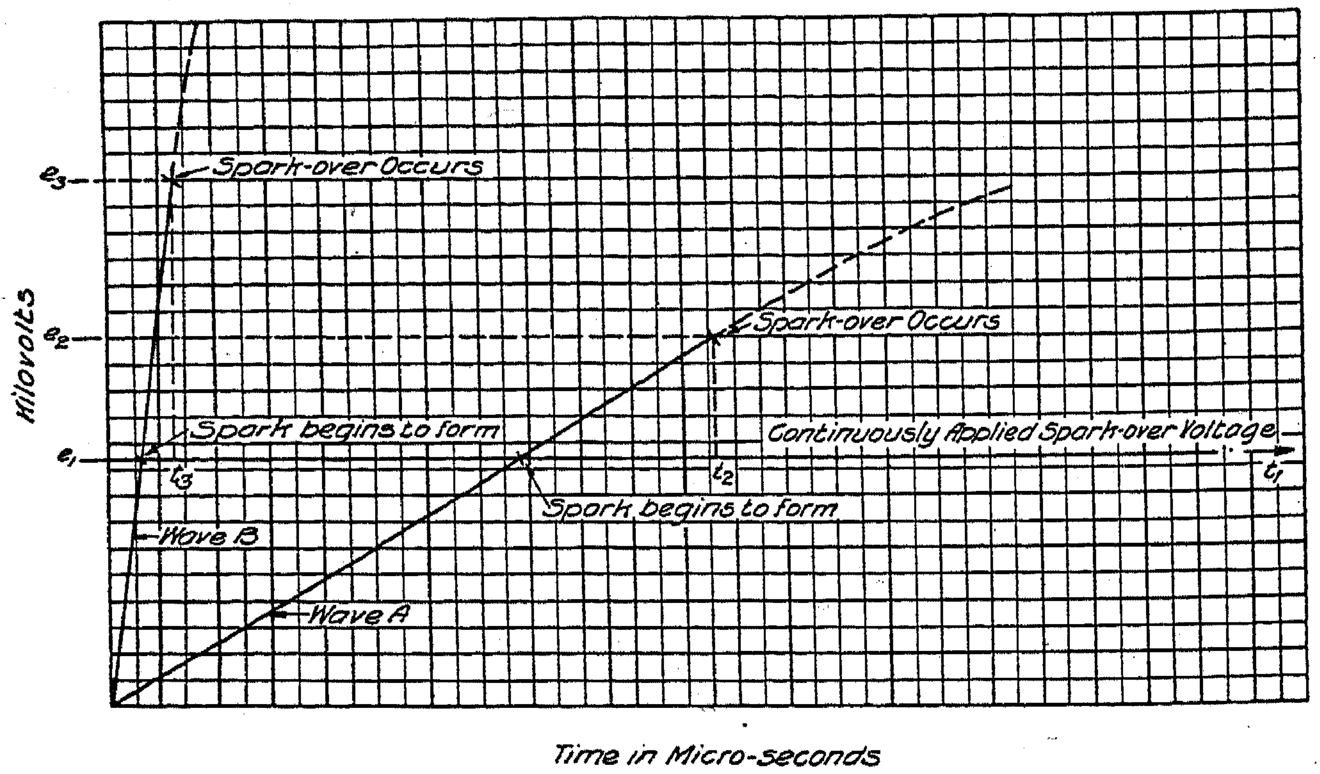

FIG. 1—DIAGRAMMATIC ILLUSTRATION OF WHY THE "LIGHTNING" OR IMPULSE SPARK-OVER VOLTAGE IS HIGHER THAN THE CONTINUOUSLY APPLIED

From theoretical considerations it seemed to me that the lag or time to break down a gap should depend upon the dielectric field and, therefore, upon the shape and spacing of the electrodes. A gap of a shape and spacing to cause a uniform field should be "fast"; a gap producing a non-uniform field should be slow, for reasons that will be given later. A sphere gap should, therefore, be fast, and a needle gap slow. Time lag manifests itself by an increase in the arc-over voltage as the rate of application of or steepness of the wave front is increased. The mechanism of breakdown is as follows:

When a 60-cycle voltage is slowly applied to a gap, and gradually increased, spark-over will occur at some definite voltage. This is the minimum voltage that will cause sufficient ionization for the gap to discharge, and it requires a relatively long time.

Lightning voltages, or voltages of steep wave front, start at zero, or line voltage, and increase at the very rapid rate of millions or billions of volts per second. When such a voltage is applied across a gap or insulator, spark-over does not occur at the instant the minimum or 60-cycle spark-over voltage is reached as a considerable time is required at this voltage. When this voltage is reached, the spark begins to form but is only completed after the rapidly rising voltage has reached some higher value. The slower the gap the higher the voltage will rise. (See Fig. 1.). In a uniform field break-down takes place over a relatively short path everywhere at the same time. In the case of a non-uniform field represented, for instance, by the needle gap, corona forms around the electrodes before arc-over. A vast amount of air must be ionized. The condition is equivalent to putting the corona or arc resistance in series with an ever increasing capacity represented by the unbroken dielectric. Time is thus required to bring all of the space between the electrodes up to the breakdown gradient. (See Fig. 2).

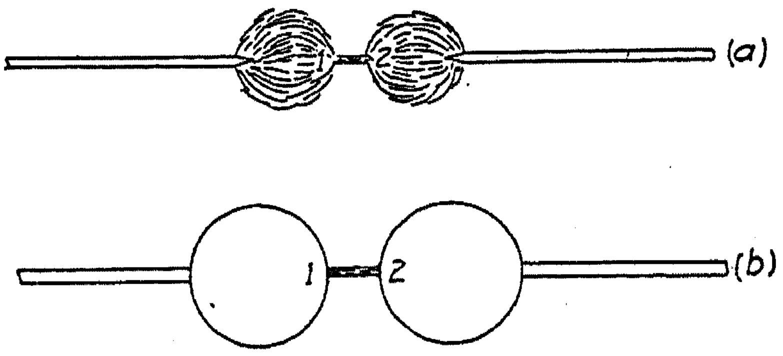

FIG. 2—DIAGRAM ILLUSTRATING SPARK FORMATION ON SPHERES AND ON POINTS

In order to illustrate this theory experimentally, the sphere horn was devised. The original sphere horn shown in Fig. 13 was built in March 1914. The first tests were made with an oscillator, this oscillator did not give any definite or known wave shape, but illustrated the principle.

It was realized that in order to make any progress a systematic experimental investigation, to supplement the theoretical one, was necessary. It was only possible to get definite results by means of an impulse generator giving waves of known voltage, wave front, shape, duration, etc. Such a generator was designed and built, and made definite data possible. The results of this investigation were published by the A. I. E. E. in 1915.* A few of the laws resulting are as follows:

1. Two gaps, or insulators, with equal 60-cycle spark-over

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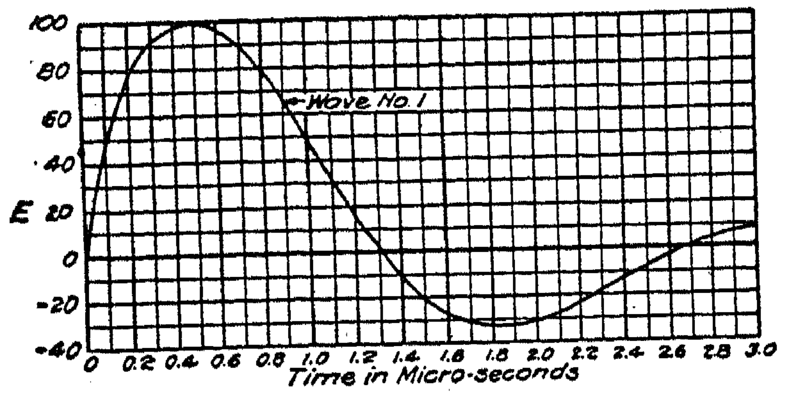
*F. W. Peek, Jr. "The Effect of Transient Voltages on Dielectrics." TRANS. A. I E. E. Aug. 1915. Vol. XXXIV., Part II, p. 1857.

F. W. Peek, Jr., "Lightning," *G. E. Review*, July 1916.

voltages, may have entirely different lightning or impulse spark-over voltages because of the time lag.

2. The time lag is greatest in a non-uniform field or for electrodes and a gap setting where corona precedes arc-over; it is minimum for a uniform field.



Approximate Single Half Sine Wave Impulse (Only First Half of the Wave Need be Considered as the Voltage is Comparatively Low After the First Crest)

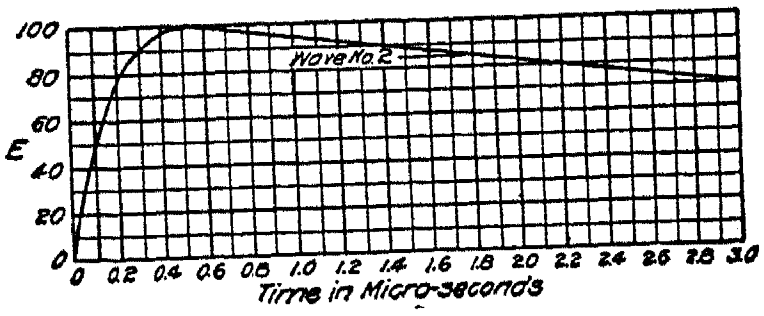


Fig. 3—Impulse with Steep Front but of Comparatively Long Duration

3. The time lag for any given electrodes and spacing is not constant, but depends upon the steepness of the wave or the rate at which the voltage is applied. The arc-over voltage increases and the time lag decreases with increasing steepness of wave front. The ratio of the 60-cycle to the impulse spark-over voltage was termed the "impulse ratio."

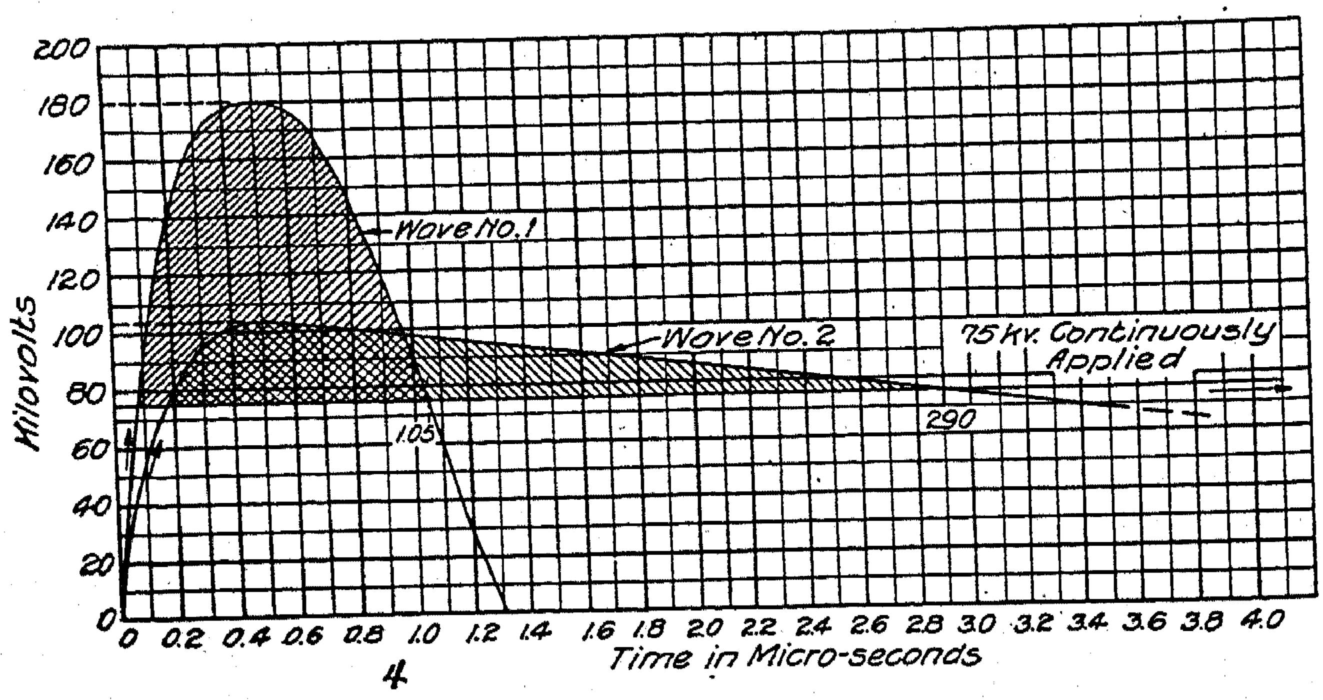


Fig. 4—Continuously Applied and Impulse Voltages to Just Cause Spark-Over of a 10-cm. Gap Between Needles

Some of the other interesting facts noted were:

The impulse spark-over voltage, unlike the 60-cycle spark-over voltage, of an insulator or gap is not greatly affected by rain. Wet and dry insulations have practically the same impulse breakdown strength. The corona starting point is practically the same for impulse voltages as it is for operating voltages. When two fast gaps are placed in series there is appreciable lag unless the arrangement is such that both gaps spark-over simultaneously. Corona produced by voltages lasting less then a micro-second can be quite readily seen. In some tests, voltages were used of such a steep wave front that a "drop" of 25 kv. was obtained over a 2-in. copper tube a yard long. Figs. 3 to 8 give some actual curves obtained in the tests.

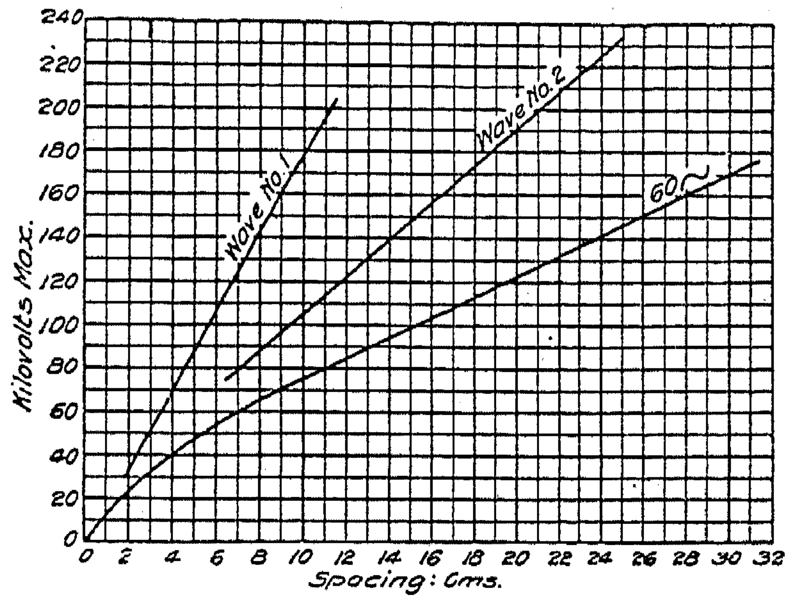


FIG. 5—NEEDLE GAP SPARK-OVER CURVES FOR 60 CYCLES AND FOR IMPULSE WAVES NOS. 1 AND 2 SHOWN IN FIGS. 3 AND 4

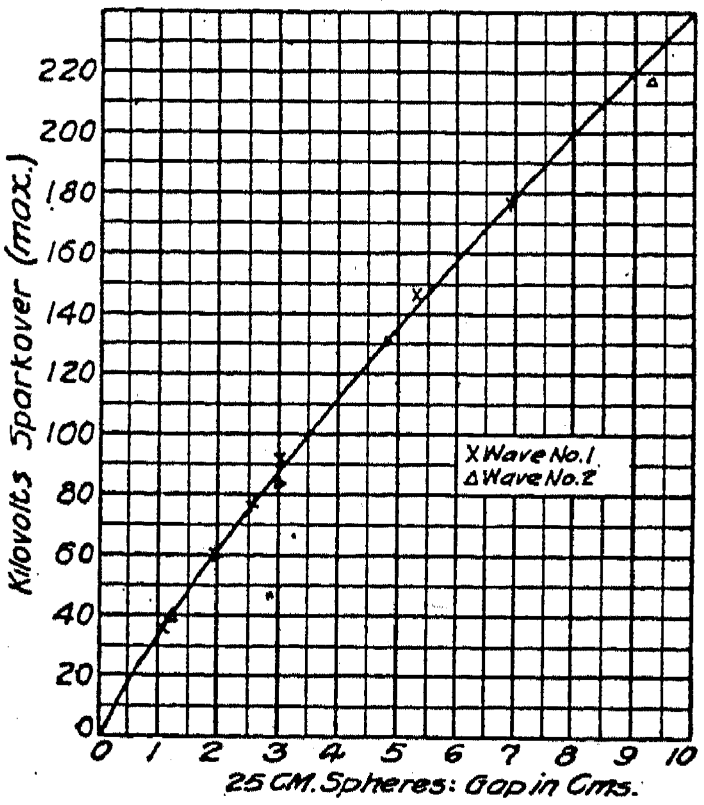


FIG. 6—SPHERE-GAP SPARK-OVER CURVES FOR 60 CYCLES AND FOR IMPULSE WAVES NOS. 1 AND 2, SHOWN IN FIGS. 3 AND 4 (DRAWN CURVE 60 CYCLES—POINTS, WAVES NOS. 1 AND 2)

It will be of interest to take a specific example to illustrate time lag:

At 10 cm. spacing between needle gaps, the 60-cycle spark-over voltage is 75 kv.; the impulse spark-over voltage for a given wave is 180 kv.; the time lag is of the order of 0.5 micro-seconds. For spheres set at 75 kv., 60 cycles, the spark-over voltage for this impulse is 75 kv. and the time lag is so small it cannot be detected, it is probably from 0.001 to 0.0001 that of a needle.

Referring to the special form of sphere-horn gap which is the subject of Mr. Allcutt's paper:

During my early work I found that while the 60-cycle and impulse spark-over voltages of a sphere gap were approximately equal, the impulse spark-over voltage of two spheres in series was much higher than the 60-cycle spark-over voltage.

In other words, there was no measurable lag for a single sphere gap, but considerable lag when two sphere gaps were placed in series. The reason for this is that the two gaps do not break down simultaneously unless the voltage divides between them in proportion to their respective strengths. One gap sparks over and puts the resistance of its arc in series with the other gap. The resistance of an arc depends upon the current flowing in the arc. The resistance remains high because the current is limited by the capacity of the sphere that has not yet broken down. Time is required to charge this capacity up to breakdown voltage through the resistance. This can be illustrated by experiment.

I found that this time lag could be eliminated if the gaps were made to divide their voltages in proportion to their respective strengths so that simultaneous arcs occurred over both gaps and thus acted as a single arc, as in a uniform field.

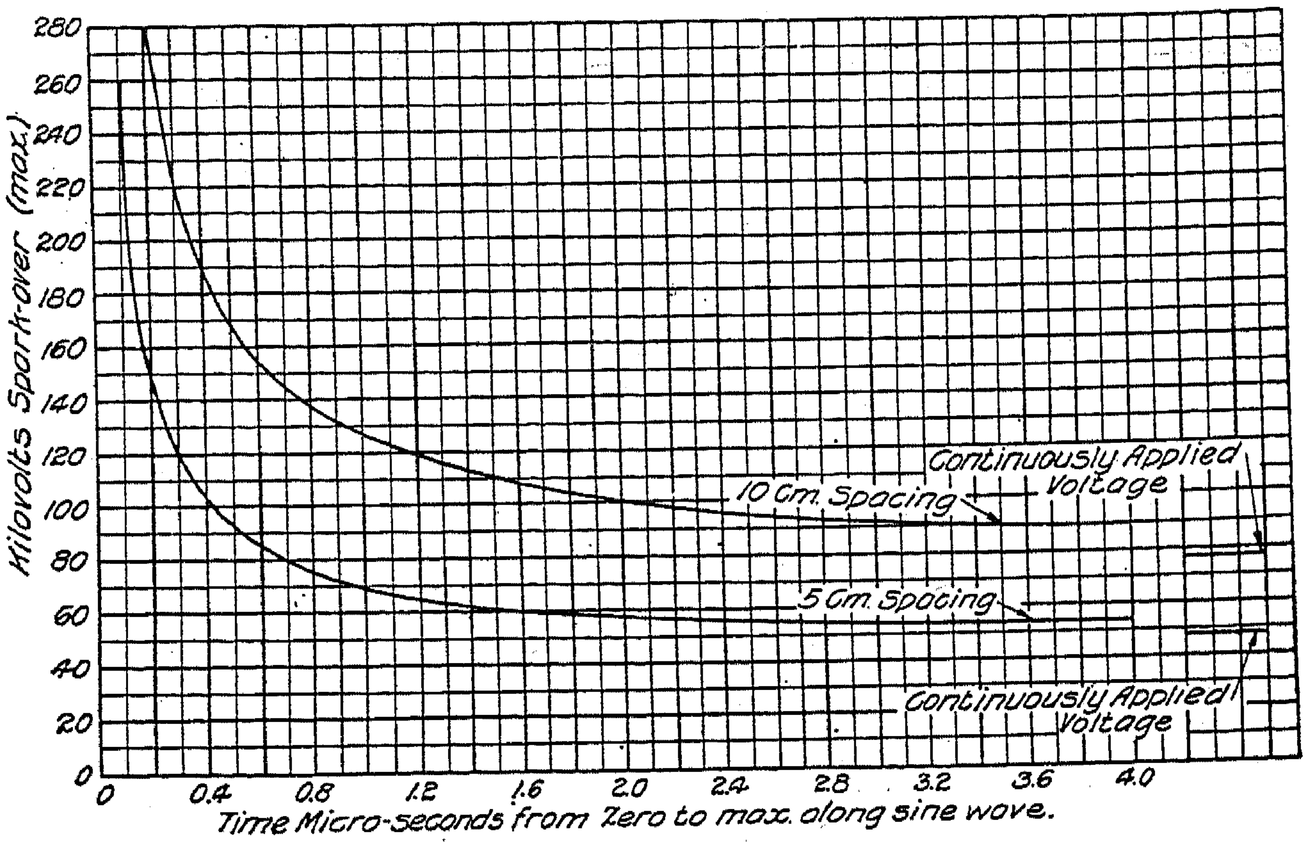


FIG. 7—TIME AND VOLTAGE TO SPARK-OVER NEEDLE GAPS (SINE WAVE IMPULSE)

In the gap shown by Mr. Allcutt, the division of voltage is not greatly affected under ideal conditions at 60 cycles by the auxiliary electrode. The auxiliary electrode is held at mid-potential because it is connected at the mid-point between the two equal condensers. (See Fig. 9). The capacity current is too small at 60 cycles to cause any appreciable "drop" across the resistance. If the condenser circuit were opened on one side, the gap on that side would break down at 60 cycles at about half voltage. This is exactly what happens under impulse. For steep wave fronts the resistance has the effect of opening the condenser circuit on that side. (See Fig. 10). The gap on that side breaks down. The voltage does not immediately disappear across the arc. The gap has lag. Corona precedes spark-over. The complete spark-over voltage is higher. Whether it is above or below the 60-cycle setting depends upon the impulse. The effect is the same as that which would result from a needle gap which could be set at, for instance, 100 kv. for 60-cycle operation and instantly and automatically reduced to a 50 kv. 60-cycle setting whenever an impulse came on the line. For moderately steep wave fronts the spark-over voltage would be greater than 50 kv. and less than 100 kv., but for very steep wave fronts, the impulse ratio of the 50-kv. gap would be greater than two, or the spark-over voltage would be greater than 100 kv. This lag is shown in Mr. Allcutt's data, Figs. 13 and 14. Super-voltages are equivalent to voltages of increased wave fronts and increase the spark-over voltage of the gap.

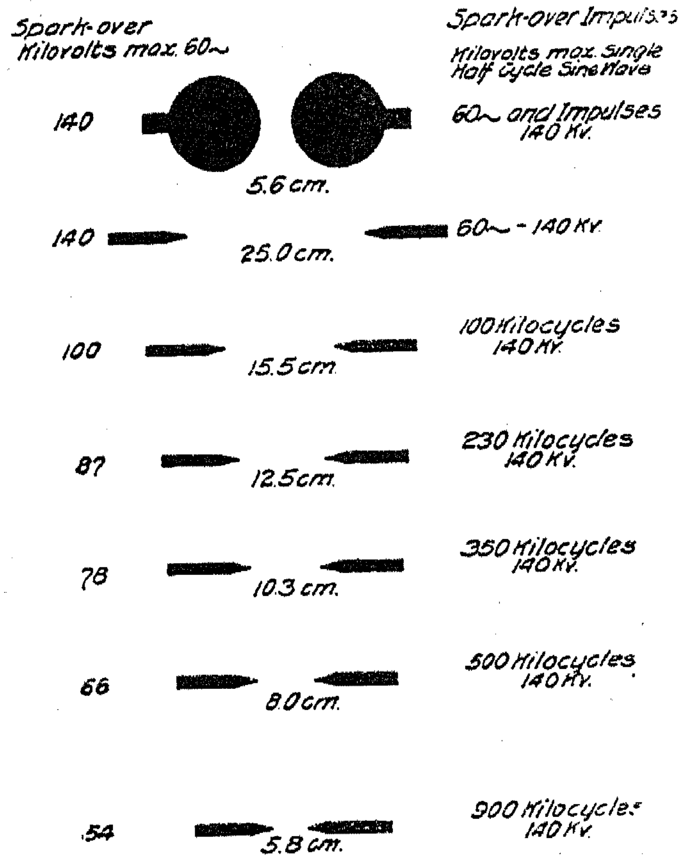


Fig. 8—Settings of Needle and 12.5 cm. Sphere Gaps for Equal Impulse Spark over Voltages of Various Wave Fronts
(Impulse Voltage Setting 140 kv. Max.—Waves Single Half Sine—100 Kilocycles and 900 Kilocycles)

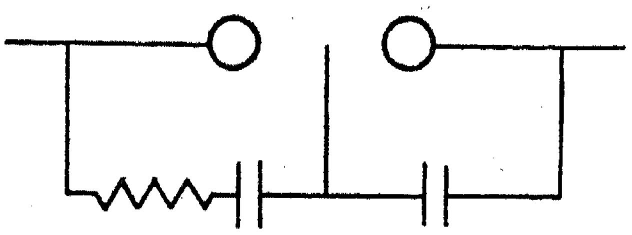


Fig. 9

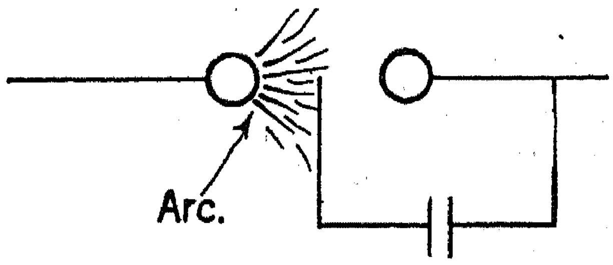


Fig. 10

Fig. 11 gives data which I have obtained on such a gap at 3-in. and ½-in. spacings. You will note the characteristics are as discussed above. Starting with a sphere and a selective sphere at the same 60-cycle setting, 3 in., the sphere arc-over voltage remains practically constant for any wave front. At a moderate wave front the selective-gap has about 5 to 10 per cent lower arc-over voltage than the sphere, while for the steeper wave fronts the voltage is higher. *For steep wave fronts the sphere is the superior gap.* My tests show less advantage for the selective gap than Mr. Allcutt's. (See tests, Tables I and II. *In making such tests it is important to be sure that the impulse generator is giving a pure impulse and there are no secondary oscillations*, otherwise the results will be quite different.

Mr. Allcutt points out the important fact that the impulse voltage necessary to discharge a given gap will depend upon the point on the 60-cycle voltage wave at which the impulse reaches the gap.

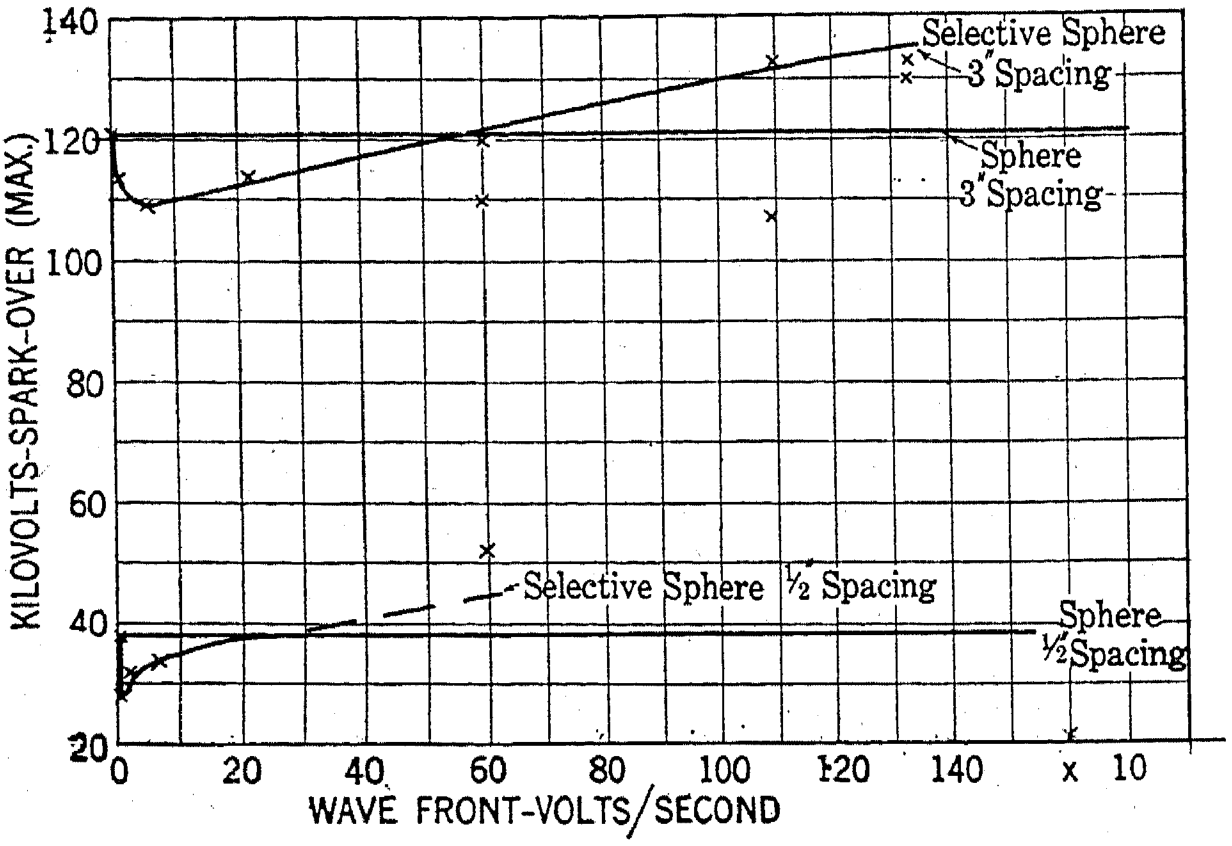


FIG. 11

There are three cases which require consideration:

Case A—where the impulse occurs at the zero point of the 60-cycle wave and is thus not affected thereby. The impulse voltage required to produce discharge may be denoted by A.

Case B—where the impulse occurs at the maximum of the 60-cycle wave and is additive. In this case it would be expected that the gap would discharge when the sum of the two voltages reached approximately the A discharge voltage of the gap. Thus, if L is the maximum of the 60-cycle line voltage, approximately,

$$B = A - L$$

Case C—where the impulse occurs at the maximum of the 60-cycle voltage wave, but in the opposite direction. In this case it would be expected that it would be necessary for the impulse voltage to reduce the 60-cycle voltage to zero and rise to a value A in the opposite direction, or approximately,

$$C = A + L$$

For Case C, the impulse voltage would rise to a higher value than in case A and B before the arrester discharged. End turns of transformers would, therefore, be subjected to the highest stresses for Case C.

TABLE I

Selective Sphere-Horn

(6-¼-cm. Spheres with Auxiliary Electrode)

One side grounded.

Gap setting		60-cycle spark-over kv. max.	60-cycle spark-over ½ gap kv. max.	Impulse spark-over total gap	Apparent impulse ratio	Impulse ratio
In.	Cm.					
				83 Kilocycles (Single half cycle)		
½	1.27	38	23	28	0.74	1.21
2	5.08	105	67	81	0.77	1.20
3	7.62	121	88	113	0.94	1.29
				500 Kilocycles (Single half cycle)		
½	1.27	38	23	32	0.84	1.40
2	5.08	105	67	87	0.83	1.30
3	7.62	121	88	109	0.90	1.24
				2000 Kilocycles (Single half cycle)		
½	1.27	38	23	34	0.90	1.48
3	7.62	121	88	114	0.94	1.30
				Impulse spark-over total gap	Apparent impulse ratio	Wave front volts/sec.
3	7.62	121	88	113	0.94	0.4×10^{11}
3	7.62	121	88	109	0.90	5×10^{11}
3	7.62	121	88	114	0.94	22×10^{11}
3	7.62	121	88	110 – 120*	0.91 – 1.00	60×10^{11}
3	7.62	121	88	106 – 133*	0.88 – 110	110×10^{11}
3	7.62	121	88	130 – 132*	107 – 110	133×10^{11}

Above data for Case A—Impulse applied at zero of 60-cycle wave.

*Range of equal spark-over.

Tests which I have made for both the plain sphere and the selective sphere check approximately with the conclusions which were arrived at above from theoretical considerations.

Mr. Allcutt came to the same theoretical conclusions as outlined above for the sphere, for A, B and C and these theoretical results are plotted accordingly in his Fig. 17. He found from tests on the selective gap that for Case B the impulse voltage was reduced as would be expected, but for Case C, quite contrary to expectations, the impulse required to spark-over the gap was the same as in Case A. It is difficult to reconcile his tests B and C, since if an additive wave reduces the voltage, a subtractive wave should increase the voltage. My tests show this to be the case for both gaps—in other words, that the *relative* protective values of the sphere and the selective sphere are for an impulse of a given wave front approximately the same for all three cases.

The results of my tests are given in Table III. These tests were all made with the impulse at which the selective gap discharges at the lowest voltage as shown in Fig. 11. Attention is called to the great difference between my tests and Mr. Allcutts. I found an average difference of only about 15 per cent between the sphere and selective gap for Case C *with the impulse giving the minimum break-down on the selective gap.* As the steepness of the impulse wave front is decreased this difference disappears and the discharge voltages become equal; as the steepness is increased the difference disappears and finally the break-down of the selective gap becomes greater than that of the sphere. (See Fig. 11.)

TABLE II

Impulse and 60-Cycle Spark-Over Voltages at the Impulse Frequency of Minimum Discharge for the Selective Gap.

Gap setting		Impulse frequency kilocycles	Sphere 60-cy. or any impulse kv.max.	Selective sphere-horn					
				Pittsfield tests			Mr. Allcutt's test		
In.	Cm.			60-cy. kv. max.	Impulse kv.max	Apparent impulse ratio	60 cy. kv. max.	Impulse kv. max.	Apparent impulse ratio
½	1.27	500	38	*38	32	0.84	†39	12.5	0.31
2	5.08	500	105	105	87	0.83	121	65	0.54
3	7.62	500	121	121	109	0.90	147	93.5	0.64

*60-cycle values for one sphere grounded as in an arrester.

†60-cycle values for spheres not grounded.

Above data for Case A—impulse applied at zero of 60-cycle wave.

The impulse ratio figures given by Mr. Allcutt do not correspond to what I originally termed the impulse ratio, in my paper. The object of this ratio was to indicate the relative speed of different gaps. A gap without time lag has an impulse ratio of one; this ratio cannot be less than one. Mr. Allcutt gives values of less than one, although the gap is slow. The

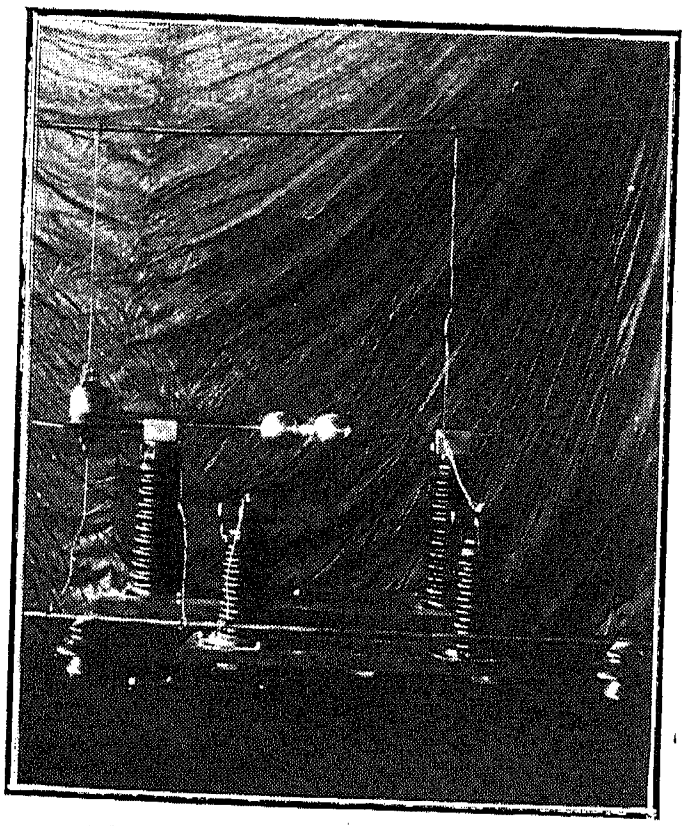


FIG. 12A—RELATIVE PROTECTIVE VALUES OF SPHERES AND POINTS

60-cycle spark-over voltage setting in kilovolts maximum.

Insulator	Spheres	Points
120	77	77

The spheres protect by discharging the impulse. The points have the same *60-cycle* voltage setting but do not discharge.

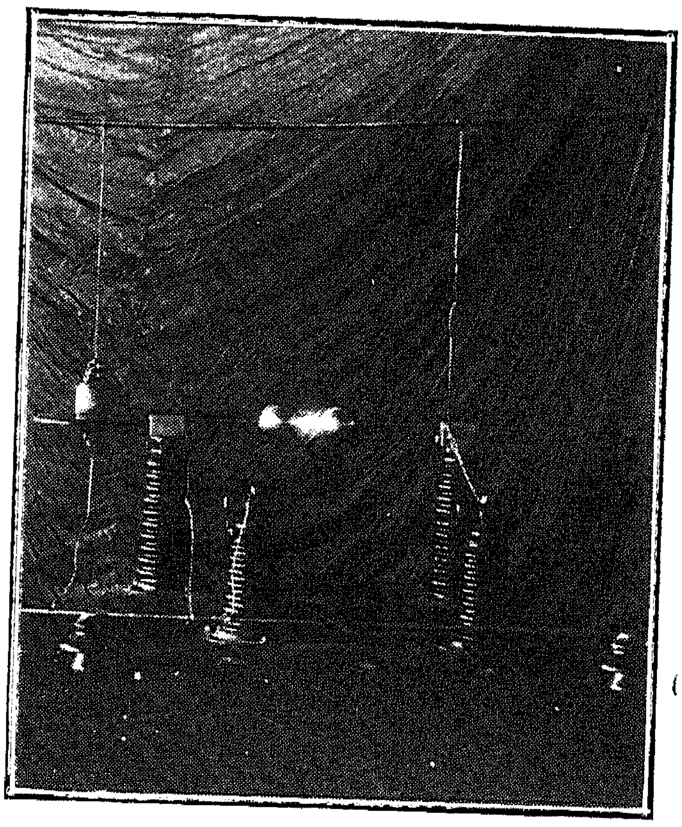
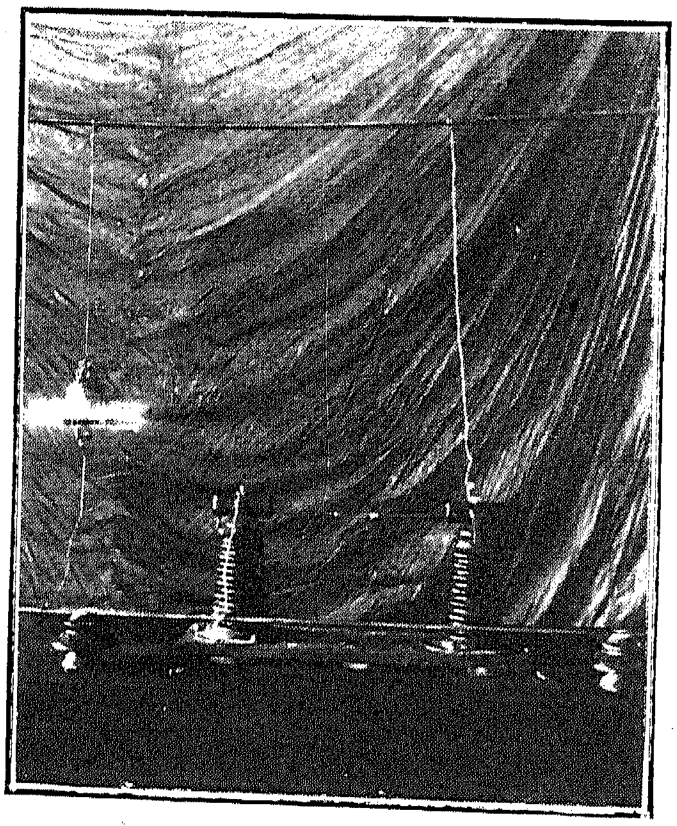


FIG. 12B—RELATIVE PROTECTIVE VALUES OF SPHERES AND POINTS

60-cycle spark-over voltage setting in kilo volts maximum.

Insulator	Spheres	Points
120	137	77

Although the sphere *60-cycle* spark-over voltage setting is, in this case, higher than that of either the insulator or the points, the spheres protect because the sphere "lightning" spark-over voltages is the lowest of the three.



[PEEK]

FIG. 12C—RELATIVE PROTECTIVE VALUES OF SPHERES AND POINTS

60-cycle spark-over voltage settings in kilovolts maximum.

Insulator	Spheres	Points
120	——	77

The points do not protect because of their very high "lightning" spark-over voltage.

The spark-overs shown are due to a succession of steep wave point impulses applied to the line.

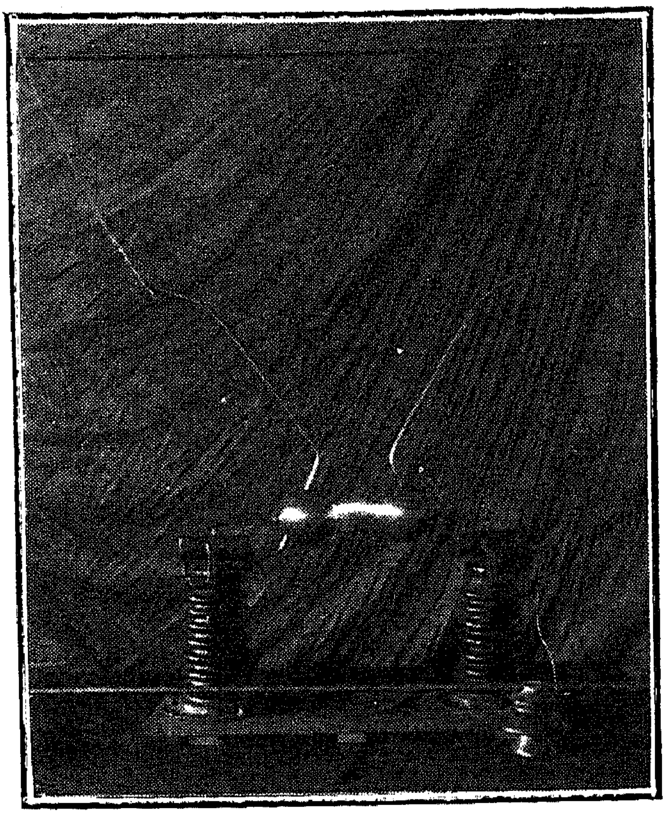
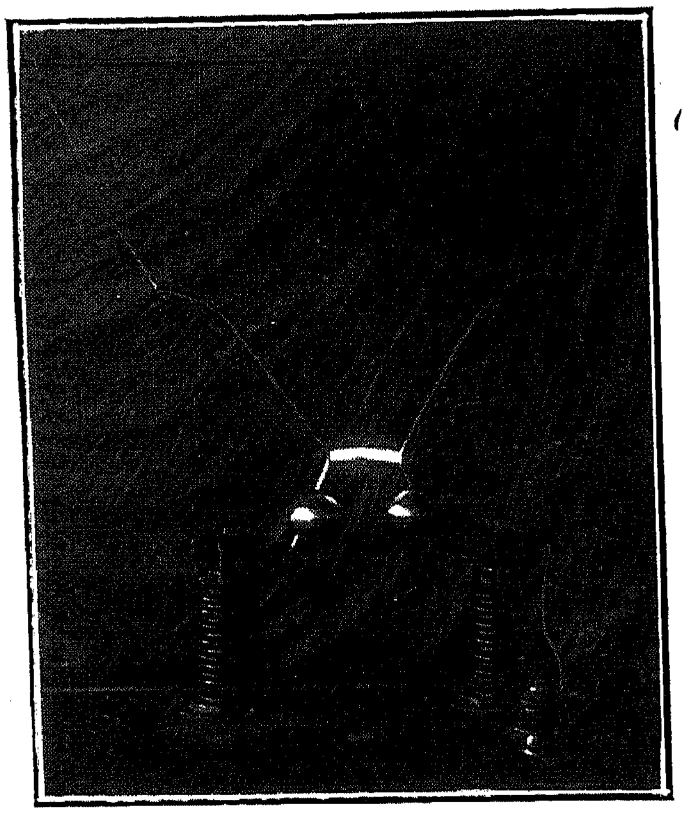


FIG. 13A—SPHERE AND HORN GAPS HAVING A STEEP WAVE-FRONT VOLTAGE APPLIED

60-cycle spark-over voltage settings (kv-max.)
Spheres 250 Horns 140



[PEEK]

FIG. 13B—SPHERE AND HORN GAPS HAVING A SLANTING WAVE-FRONT VOLTAGE APPLIED

60-cycle spark-over voltage settings (kv.-max.)
Spheres 250 Horns 140

impulse arc-over voltage divided by the 60-cycle voltage of half the gap would give this figure for the selective gap. See Table II. I mention this merely to prevent confusion in regard to my original use of the term. Mr. Allcutt's ratio is an *apparent impulse ratio*. As an instance, take the 3-in. setting for a 500-kc. wave; Mr. Allcutt finds an impulse spark-over voltage of 93, while my tests give an impulse voltage of 109. My tests give an *apparent* impulse ratio of 0.90; Mr. Allcutt's 0.64. Part of this discrepancy is due to the fact that Mr. Allcutt takes the non-grounded value of 121 kv. for the 60-cycle voltage while the grounded value of 121 kv. should be used.

Any gain due to the selective sphere-horn is, in effect, that of raising the operating voltage and not of time lag. Any apparent laboratory gain is thus lost if practise makes a large increase in spacing necessary. Greater gains of a similar nature may be made in other ways. Rain lowers the 60-cycle arc-over voltage of a gap. It is, for this reason necessary to set a gap installed outside, at about double the voltage of a gap installed inside. A fast gap protected from rain has been devised which makes a much greater practical gain for all wave fronts than that obtained by the selective gap, at its minimum discharge voltage. This is obvious since the average apparent gains due to *impulses* under the best conditions with the selec-

TABLE III

Effect on the 60-Cycle Voltage on the Impulse Spark-Over

Comparison of Sphere and Selective Sphere-Horn at the Impulse Frequency of Minimum Discharge for the Selective Gap.

Spacing		Selective gap, impulse kv. max.			Sphere gap impulse kv. max.			*Ratio Selective/Sphere		
In.	Cm.	A	B	C	A	B	C	A	B	C
		Pittsfield Tests								
1	2.54	49	54	97	68	52	108	0.73	1.04	0.90
2	5.08	87	54	115	105	60.5	148	0.83	0.90	0.78
3	7.62	113	67	147	121	86	160	0.94	0.78	0.92
		Mr. Allcutt's Tests								
1	2.54	30	9	32	71	35	107	0.42	0.25	0.30
2	5.06	65	22	68	121	67	181	0.54	0.36	0.37
3	7.62	93.5	45	96	147	74	220	0.63	0.61	0.44

*Tests made at 500 kc., or at the wave front giving the minimum discharge on the selective gap.

Applied 60-cycle voltage = half 60-cycle discharge voltage of the gap.

Case A—Impulse applied at zero of 60-cycle wave.

Case B—Impulse applied at maximum of 60-cycle wave-additive.

Case C—Impulse applied at maximum of 60-cycle wave-subtractive.

tive gap, Figs. 9 and 11, are only 10 to 15 per cent, while a covered gap cuts the voltage in half. *A covered fast gap is of greatest importance.*

So far the problem has only been discussed from the arrester standpoint. While it is important to design an arrester gap of low impulse arc-over voltage, it is equally important to design leads, insulators, and insulation for high impulse puncture and arc-over voltages. Great strides have been made in this direction.

Percy H. Thomas (Read by L. W. Chubb): Mr. Allcutt has devised an apparatus which will cause a discharge over the spark gap more easily as the result of a voltage impulse of steep wave front than as a result of commercial frequency current. Furthermore, the effectiveness of the arrangement does not vary numerically with the frequency but rather may be said to be effective for all frequencies above a critical intermediate range, which is the author's intention and the desired characteristic. From a certain point of view the apparatus described by Mr. Allcutt has characteristics especially adapted to the protection of commercial electrical apparatus from high-frequency discharges. From certain other points of view, however, considerations might be urged to minimize its importance. For example, experience may show that it is not desirable to cause the spark gap to discharge with very high-frequency impulse. In commercial circuits there may be very many relatively harmless impulses of sufficient abruptness to cause Mr. Allcutt's apparatus to act when no actual protection is required.

It might happen, for example, with the device adjusted in such a way that high-frequency impulses of voltages not too high to be safe did not discharge that an adjustment would result which did not discharge commercial frequency impulses easily enough. In other words, is not the ideal condition a discharge ratio of unity? The abrupt discharge to ground of a voltage impulse on the terminal of a transformer may be far more severe on the end turns than the original impulse. This results not only because the swing of voltage is greater with the discharge to ground but because it may be of even a higher frequency than that of the original impulse.

It may also be said that it is only under the most favorable and very unusual conditions that a disturbance of a frequency as high as 500,000 cycles can be met with in transmission lines. The size of conductors and all apparatus connected with a commercial system are of such high capacity and inductance that practically all impulses will be of much lower frequency. Again, these very high-frequency discharges are of very small energy content, so that they would ordinarily be absorbed in any commercial system or any commercial apparatus without producing any considerable rise in potential. Especially if their initial voltage be so high as to break down any commercial lightning arrester.

It is not safe, however, to object to such new devices as are proposed by Mr. Allcutt on purely theoretical grounds, for actual trial often reverses conclusions based on theoretical considerations.

It would seem to me that a particularly favorable field for the trial of this devise would be distribution systems carrying house lighting transformers from 2000 to 5000 volts. These transformers suffer frequent injury, and adequate arc suppressing devices are available in case of the use of very sensitive lightning arresters.

L. W. Chubb: This work of Mr. Allcutt's is very interesting and the tests in all cases made in shunt with the sphere-gap, leave no doubt regarding the quickness of the gap and the fact that it limits the voltage to a reasonable figure.

Some time ago we ran impulse tests on electrical porcelain and other dielectric materials, and the results show distinctly that voltage impulses of short duration far above the ordinary breakdown voltage of the insulator may be applied repeatedly without rupture. Each impulse, however, produces a permanent injury and eventually the insulation will give way under the repeated stress. High-frequency impulses coming in and striking the end turns of a transformer will not break it the first time, or for many times. Each time such an impulse comes it will cause a certain permanent injury, and repeated disturbances will eventually cause failure.

The selective properties of Mr. Allcutt's gap seem to me to be of great importance for it takes care of the first half cycle of a wave train which starts in opposition to the instantaneous value of line voltage. The ordinary gap, under the same conditions, set to double line voltage will require three times normal voltage to discharge on the first half cycle of the wave train, which will seriously injure the insulation of inductive end turns. The usual gap will, of course, discharge on the second alternation in this case, but this is too late as the voltage rise has not been restricted and the harm has been done.

C. T. Allcutt: I am very glad to hear from Mr. Peek on this subject and to find that he is conducting some experiments to check the results I have obtained. It would appear from the data and curves that Mr. Peek has included in his discussion, that there is a serious lack of agreement in our results. It is unfortunate that Mr. Peek has neglected to give any data concerning the construction of the gaps tested by him or the test methods employed. If such information had been included in his discussion I feel confident that the cause of the apparent discrepancies could be readily pointed out.

The particular forms of gap for which I have presented test data are but two of a large number of forms of gap, based on the same principle, that have been tested in our laboratory. The gaps described are, naturally, the preferred forms, *i.e.*, those that give the best results consistent with a commercially practicable construction. Any considerable modification of the construction described may result in a large increase in the impulse discharge voltage.

I do not believe that Mr. Peek appreciated the importance of having the auxiliary electrode of such small dimensions so as to be, in effect, a point electrode. Perhaps the fault is mine for having insufficiently emphasized the importance of this point in my paper. For example, Mr. Peek states that the 60-cycle discharge voltage between the auxiliary electrode and one of the main electrodes is about one-half the discharge voltage of the main gap. This is clearly *not* the case where a point auxiliary electrode is used. In the sphere-horn gap tested by me, the 60-cycle discharge voltage between the auxiliary electrode and one of the main electrodes was approximately *one quarter* the discharge voltage of the whole gap. (See Fig. 14). In the gap

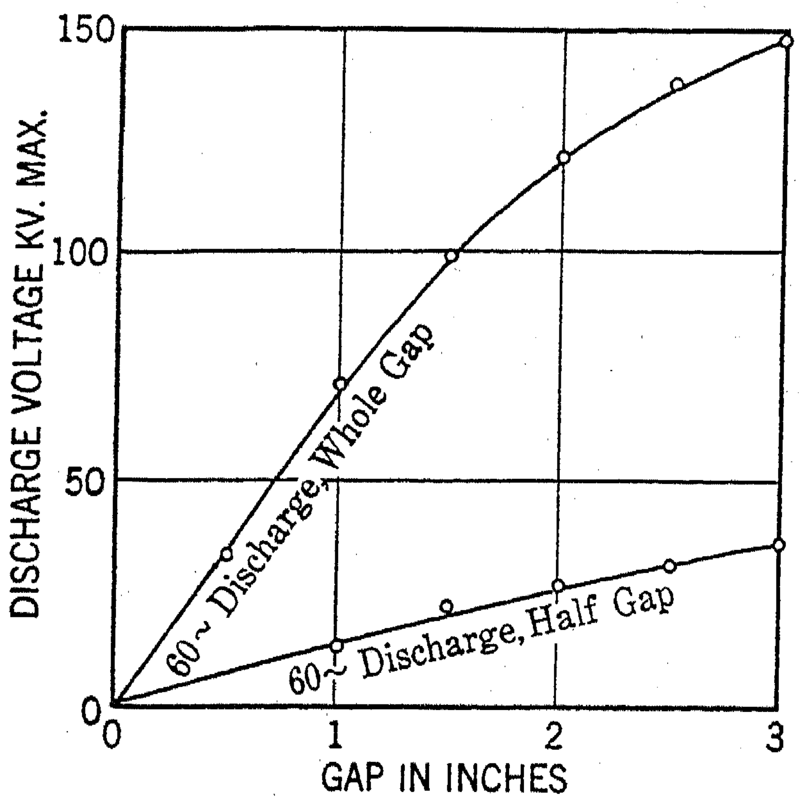


FIG. 14—IMPULSE GAP

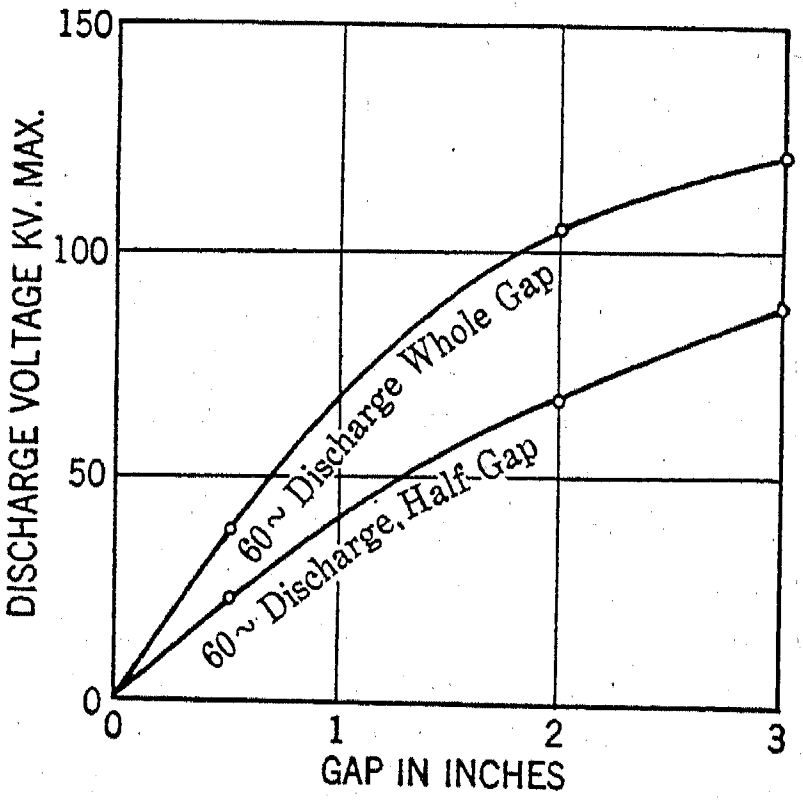


FIG. 15—PEEK'S GAP

tested by Mr. Peek, conditions were quite different, as is shown by Fig. 15, which was plotted from the data given in Table I of his discussion. It is evident that Mr. Peek used an auxiliary electrode of quite different dimensions from that described in my paper. This alone is sufficient to account for his failure to secure results comparable with my own, as the impulse discharge voltage is largely dependent on the 60-cycle discharge voltage of the half gap.

In the footnote to his Table II and again in the text of his discussion, Mr. Peek intimates that I used the standard ungrounded sphere gap calibration curve as representing the 60-cycle discharge characteristics of the gap under test. As I stated in my paper, the 60-cycle discharge voltage of the gap under test was obtained by comparison with a standard 25-cycle sphere gap. All tests were made with one electrode grounded. The presence of the horns and the height of the gap above ground both tended to make the 60-cycle discharge curve somewhat higher than the standard grounded calibration curve for 6.25-cm. spheres. Mr. Peek apparently was misled by the fact that the experimentally determined 60-cycle discharge curve for the sphere horn gap with one side grounded, happened to correspond quite closely with the standard *non-grounded* curve for 6.25-cm. spheres.

In another part of his discussion Mr. Peek objects to my use of his own very apt expression "impulse ratio." Mr. Peek introduced this term some three years ago with these words: "We have termed the ratio of the impulse breakdown voltage to the continuously applied breakdown voltage the "Impulse Ratio." In my paper I defined impulse ratio practically in Mr. Peek's own words and attempted to use the term in strict accordance with his own definition.

Presented at the 34th Annual Convention of the American Institute of Electrical Engineers, Atlantic City, N. J., June 27, 1918.

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THE OXIDE FILM LIGHTNING ARRESTER

BY CHARLES P. STEINMETZ

Abstract of Paper

A short history of lightning protection of electric systems is given, as relating to the three successive types of electric circuits; the communication circuits, the power circuits of negligible electrostatic capacity, and the high-power circuits containing distributed capacity and inductance and capable of electric oscillation, leading to the three problems of discharging over-voltage to ground, opening the power current which follows the discharge and discharging so that no power current follows even for a fraction of a half wave. It is shown that these problems are solved by the spark gap to ground, by the use of non-arcing metals in the multigap arrester, which opens the circuit at the end of a half wave of current, and by the so-called "counter e.m.f." type of arrester, represented by the aluminum cell and the oxide film arrester.

It is shown that the necessity of taking care of recurrent discharges in high-power systems had led to the universal adoption of the aluminum cell arrester in such systems, in spite of its disadvantage of requiring daily attendance in charging, and of containing an electrolyte and oil.

In the oxide film arrester a type of arrester is presented which has the same characteristics and therefore the same advantages as the aluminum cell arrester, but does not require daily attendance and contains no liquids.

Its method of operation is explained, and its difference from the aluminum cell arrester; the dielectric film, which punctures under the discharge, and reseals after the discharge, is formed from the solid materials between the terminal plates, compressed PbO_2, and therefore no spontaneous chemical action occurs which dissolves the film, as in the aluminum cell, in which the film forms from the aluminum electrode, gradually dissolves, and therefore requires daily charging.

A short description of the construction of the oxide film arrester is given, a record of its operation in industrial service for over three years, and oscillograms showing the performance of this arrester under recurrents, oscillations and under high-power impulses.

THREE periods can be distinguished in the development of lightning arresters, corresponding to the three periods of the use of electricity:

(1) Electric circuits of negligible power, telegraph and telephone.

(2) Electric power circuits of negligible electrostatic capacity;

d-c. lighting and railway, a-c. secondary and 2300-volt primary distribution.

(3) High-voltage electric power circuits, transmission lines, etc.

(1) In electric circuits of negligible power, such as telegraph and telephone circuits, a simple minute spark gap to ground afforded protection by discharging lightning to ground, and was sufficient until the recent years, when with the general introduction of electric power circuits the problem arose, in electric communication circuits to take care of crosses with power circuits.

(2) In electric power circuits, a simple spark gap to ground became insufficient for protection, since the power current, following the lightning discharge as arc, short-circuited the system and burned up the arrester. The problem then arose, to safely open the short circuit of the machine current to ground, through the lightning arrester, after the lightning discharge has passed, and to leave the arrester in operative condition to receive following lightning discharges.

Of the various devices developed heretofore, the magnetic blow-out lightning arrester still is used in direct-current railway circuits.

The first scientific investigation of this problem, is recorded in the paper[1] by A. J. Wurts, presented before the A. I. E. E. at the meeting of May 1894. Since that time all lightning arresters for alternating-current power circuits of negligible electrostatic capacity are based on the multigap principle between non-arcing metals, whatever constructive forms the arrester may assume—as the present compression chamber lightning arrester. The multigap arrester operates on the principle, that the lightning discharge over the multigaps closes the circuit to ground, but the power arc following the discharge extinguishes at the end of the half wave of the alternating current, as the non-arcing character of the gaps does not permit the reverse current of the next half wave to start. The multigap arrester thus short-circuits for a part of a half wave. It obviously is suited only for alternating currents.

For years difficulties were met with the question of resistance; without series resistance, in large systems the short-circuit power even of a part of a half wave may be sufficient to disable and destroy the arrester, while the use of a series resistance, while limiting the power current and thereby protecting the arrester, also limited the discharge capacity and thereby reduced the protection. This problem was solved by the use of multigaps shunting the series resistance, so that moderate discharges passed over the resistance, while high power lightning discharges found a path without series resistance, over the shunted gaps, and at the same time, the shunting resistance made the power arc at the shunted gaps unstable and thus assisted in the extinction of the short circuit at the end of the half wave.[2]

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[1] TRANS. A. I. E. E., 1894, Vol. XI, p. 337, "*Discriminating Lightning Arresters and Recent Progress in Means for Protection Against Lightning*", by A. J. Wurts.

(3) As soon however, as circuits came into use, which had considerable electrostatic capacity, such as high-voltage transmission circuits, extended underground cable systems, or lower voltage circuits (including generator circuits) inductively connected with such circuits, the multigap arrester failed by frequently, or even usually destroying itself by the discharge, burning up.

In such circuits, oscillations between capacity and inductance may occur, started by a lightning discharge or any internal disturbance such as switching etc., resulting in recurrent high-frequency oscillations, of which the arcing ground on a transmission line is typical and probably best known. With such continual discharges, often several per half wave, the multigap arrester short-circuits at the first oscillation, for the remainder of the half wave, and while the multigap functions properly and opens the short circuit at the end of the half wave, the oscillation of the next half wave again short-circuits, and so on, so that the effect is that of a continuous short circuit, and no lightning arrester, no matter how large, can dissipate the short-circuit power of a big system for any appreciable time.

For such systems, in which recurrent high frequency oscillations, as arcing grounds, may occur, a lightning arrester is necessary, which does not short-circuit the machine current even for a fraction of a half wave, but merely discharges the over voltage, the oscillation which, however high in voltage it may be, is small in energy compared with the short-circuit power of the system, as it represents only the stored energy of capacity and inductance. The only arrester of this character heretofore was the electrolytic, or aluminum cell lightning arrester, developed by E. E. F. Creighton, J. L. R. Hayden, F. W. Peek and others. It acts towards an over-voltage discharge like a counter e. m. f. equal to the normal circuit voltage, and the discharge current passing through the arrester thus is the short-circuit current of the over voltage, while the normal machine voltage does not discharge, is held back and not disturbed. The aluminum cell arrester thus can discharge continual disturbances, over-voltage oscillations occurring at every half wave, for a considerable time, half an hour to several hours, before it is endangered by the temperature rise due to the accumulated energy of these discharges.

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[2] See A. I. E. E. TRANS., 1907, Vol. XXVI, p. 425, "*Protection Against Lightning, and the Multigap Lightning Arrester*" by D. B. Rushmore and D. Dubois.

The aluminum cell arrester comprises a series of cells—usually conical and stacked into each other—of aluminum electrodes with an electrolyte, of which neither the salt nor its ions appreciably dissolve alumina. In "forming" the cell, by an alternating current passing through it, the electrodes are coated by a thin non-conducting film of alumina, which grows in thickness until it holds back the impressed voltage. Any over-voltage punctures this film, but the current passing through the puncture holes, again forms alumina and closes the holes. Thus the aluminum cell acts like a self repairing electrostatic condenser of a disruptive strength equal to the impressed voltage: about 250 to 300 volts per cell.

The practical experience of the last ten years has proven the aluminum cell arrester as the only type capable of affording protection in modern high-power circuits, and proven this so conclusively, as to lead to its universal adoption in such circuits in spite of the inconveniences incident to the need of daily attention in charging, the use of a liquid electrolyte, and the difficulty of testing the arrester without taking it apart, except by watching the appearance of the charging arc, or measuring the charging current.

These inconveniences incident to the aluminum cell arrester were well realized however, and as soon as the minor troubles met with the aluminum cell arrester in the early years had been overcome engineers went energetically to work on the problem of developing a lightning arrester of the characteristics of the aluminum cell arrester, but which does not require any attention beyond that given to every apparatus in a well managed system, that is, an occasional inspection, at least once or twice a year.

Numerous researches were made by the engineers whose splendid work I here acknowledge: Messrs. H. D. Brown, E. E. F. Creighton, Crosby Field, V. E. Goodwin, J. L. R. Hayden, N. A. Lougee, and G. B. Phillips. Some of these researches have not yet led to results ready for communication, but I am glad to present to you here as the result of the work of these men, and more particularly of E. E. F. Creighton, Crosby Field and N. A. Lougee, in the *Oxide Film Lightning Arrester* a new type of lightning arrester, which has all the characteristics and advantages of the aluminum cell arrester, but does not require any charging and thus requires no special attention, contains no liquid electrolyte, no inflammable material, and like the aluminum cell arrester, can be located outdoors as well as indoors.

The oxide film arrester, like the aluminum arrester, acts like a counter e. m. f. equal to the normal circuit voltage, freely discharging any over voltage, but holds back the normal machine voltage. Thus the discharge is limited to the energy of the over voltage, as in the aluminum arrester, and like the latter, the oxide film arrester can continuously discharge recurrent surges, such as arcing grounds etc., without endangering itself, for a considerable time, sufficiently long to notice and eliminate the disturbance.

Compared with the almost entire absence of knowledge of lightning phenomena in electric circuits, under which Mr. Wurts had to work in developing the non-arcing metal multigap lightning arrester, our present knowledge of lightning phenomena is very great. Nevertheless, there are so many disturbances in large electric systems, which we cannot or only incompletely reproduce in our laboratories, that the final decision on the success, that is, the effectiveness and permanence of a lightning arrester, still is best given by the experience in industrial systems.

Therefore, after extensive laboratory tests had been completed and had proven the oxide film arrester as of the same characteristic as the aluminum arrester, but requiring no special attention, a number of industrial installations were made, and more added the next year and the third year. Now, however, when a considerable number of installations of these arresters, for voltages from 110 to 33,000, have been in successful operation, some for over three years, and have proven their protective value and their permanence, I consider it desirable to bring the arrester to your attention.

Of the numerous tests made on the performance of the arrester, it may be sufficient here to give only two, by the oscillograms Figs. 1 to 4, showing the action on a recurrent oscillation, in Figs. 1 and 2, and on a single high-power impulse, Figs. 3 and 4.

The tests, oscillograms Figs. 1 and 2, were made in the usual manner: a surge or continual oscillation was produced by a large condenser, connected to an alternating-current supply and discharging over a spark gap through an inductance. The latter was chosen so as to give a frequency of 1200 cycles to the oscillation, and thereby bring it well within the range of the oscillograph. This surge was impressed upon the apparatus to be protected, a transformer energized by another alternating-current circuit. Fig. 1 shows the oscillogram without protection of the transformer, and Fig. 2 the oscillogram with an oxide film cell shunting the transformer and thereby protecting it.

In Fig. 1, the lowest curve shows the voltage of the 320-volt 47-cycle power supply circuit of the transformer, with the oscillations superimposed on it, rising to surge peaks of 2800 volts. The upper curve shows the oscillating currents passing through the transformer, rising to current peaks of 40 amperes. The middle curve is absent, as no arrester is used in this test. In Fig. 2 however, where an oxide film cell is shunted across the transformer, the middle oscillogram shows the current oscillations passing through the arrester, with peaks of 35 to 41 amperes. The lower curve in Fig. 2 then shows again the circuit voltage wave impressed upon the transformer, with the oscillations cut down by the oxide film cell. This voltage wave, the lower curve in oscillogram Fig. 2, well illustrates the characteristic action of this type of "counter-e. m. f. arrester"—to which the oxide film arrester and the aluminum cell belong. The oscillation peaks are sharply cut off at a maximum voltage of 60 per cent above circuit voltage: the value for which the spark gap was set. As the result, the oscillations are very greatly cut down from the high values which they have in the unprotected circuit Fig. 1 (2800 volts), and become unsymmetrical. The half waves of oscillation in the same direction as the circuit voltage are greatly reduced, by the limitation of the voltage to 60 per cent above-normal, while the reverse half waves—which lower the instantaneous circuit voltage— are less affected. Corresponding thereto the discharge current through the arrester, the middle oscil-

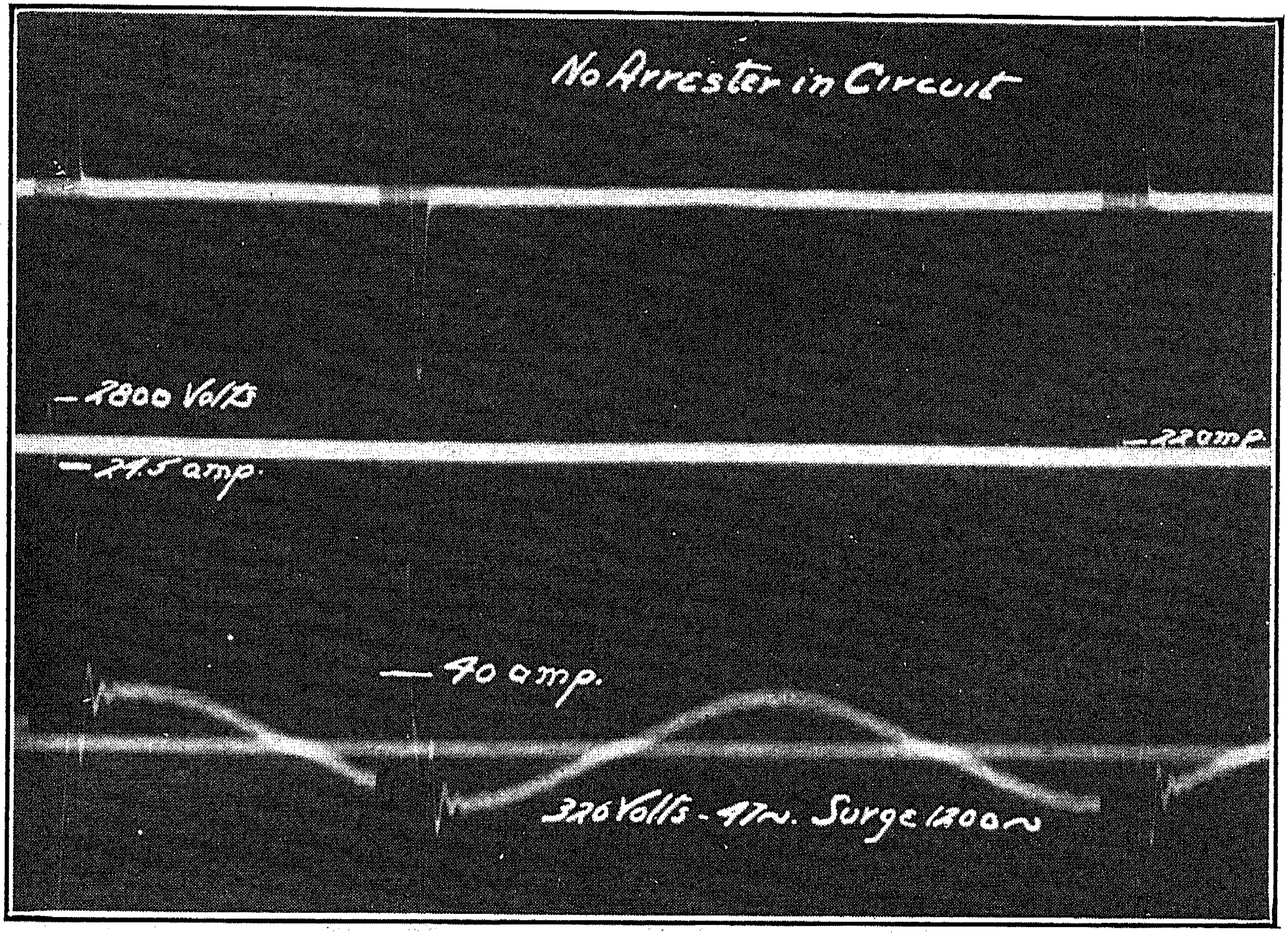
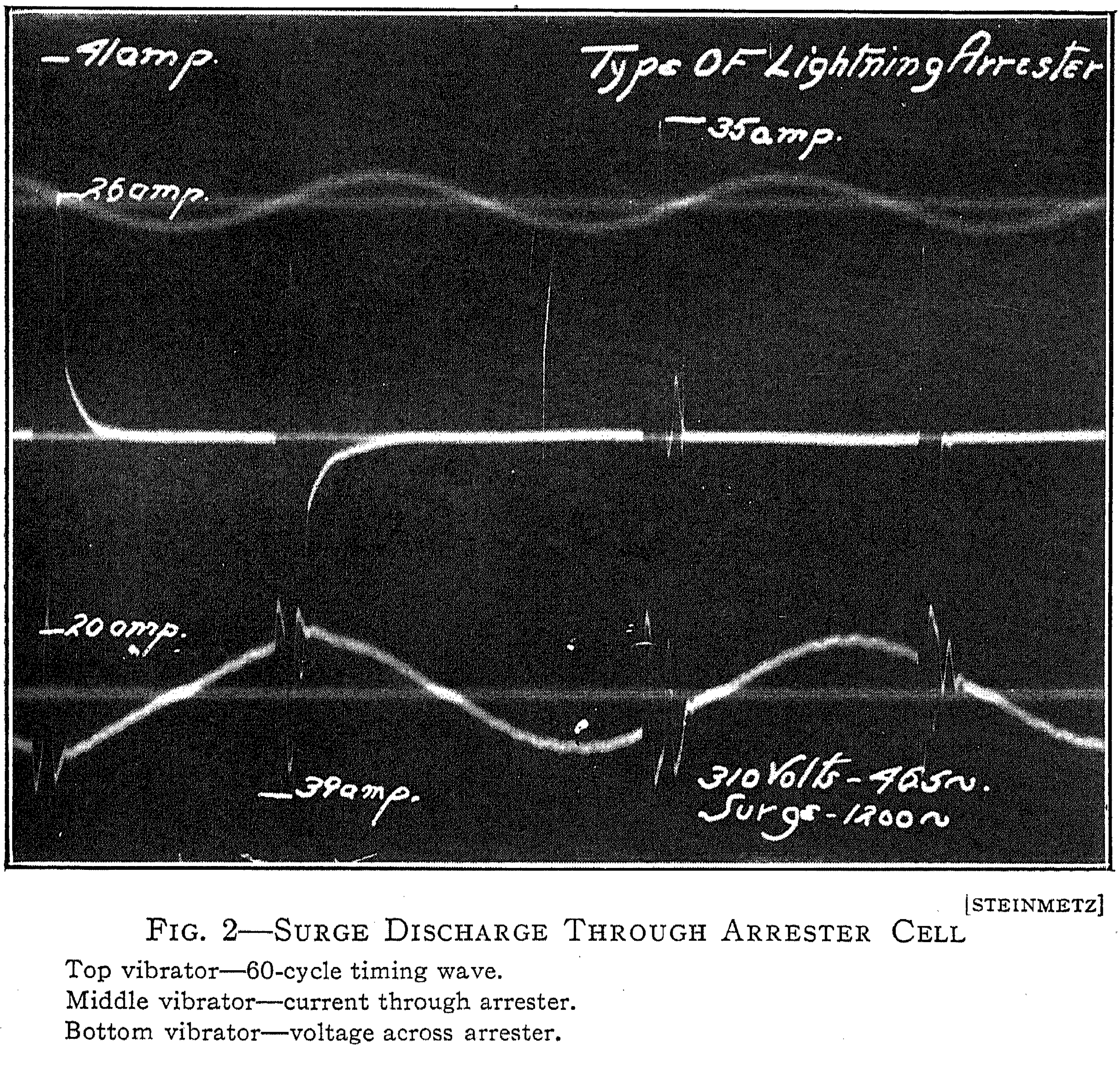


FIG. 1—SURGE DISCHARGE AS IN FIG. 2—NO ARRESTER IN CIRCUIT
Top vibrator—current through transformer.
Bottom vibrator—circuit voltage.



[STEINMETZ]

FIG. 2—SURGE DISCHARGE THROUGH ARRESTER CELL
Top vibrator—60-cycle timing wave.
Middle vibrator—current through arrester.
Bottom vibrator—voltage across arrester.

PLATE XXXV.
A. I. E. E.
VOL. XXXVII, 1918

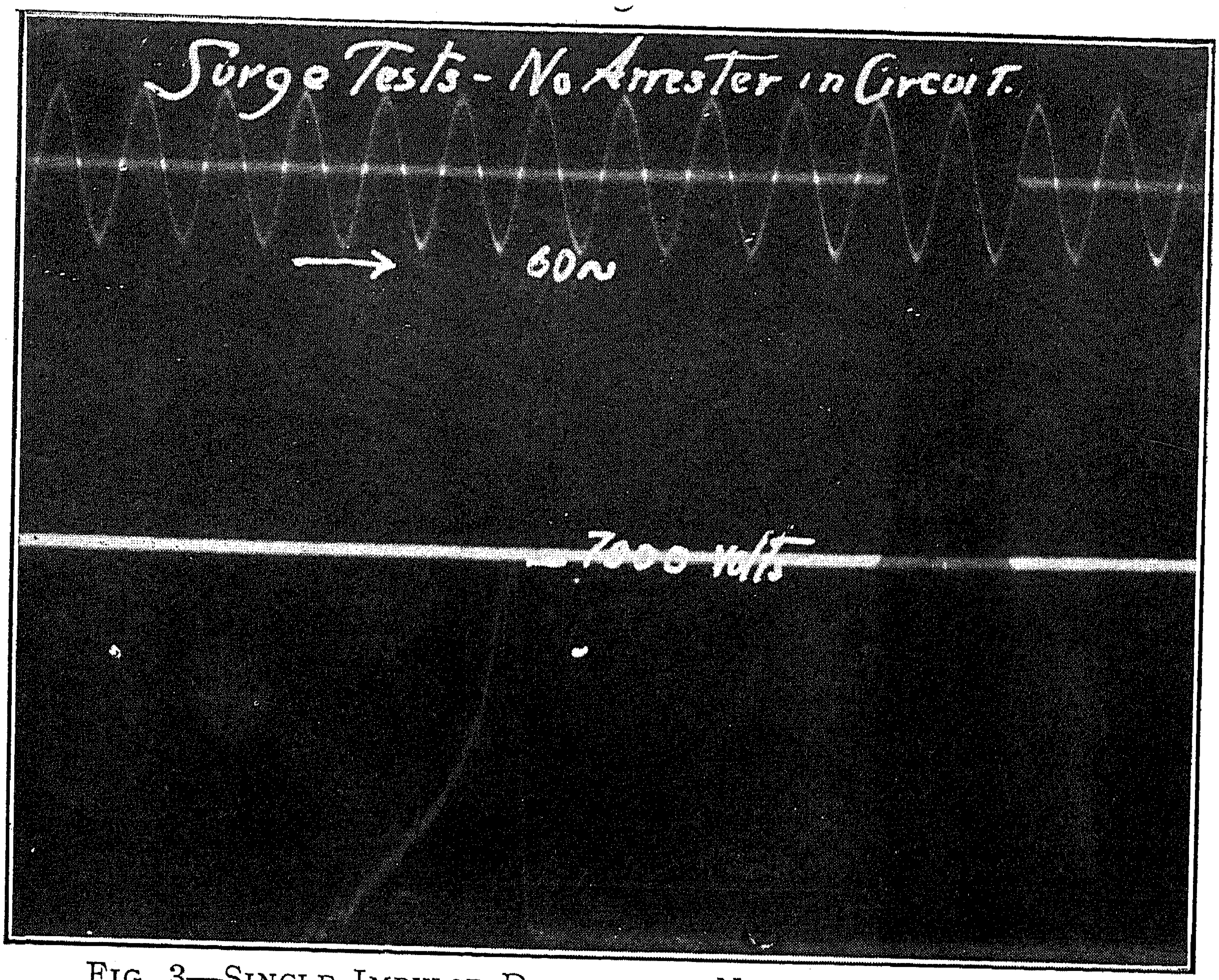
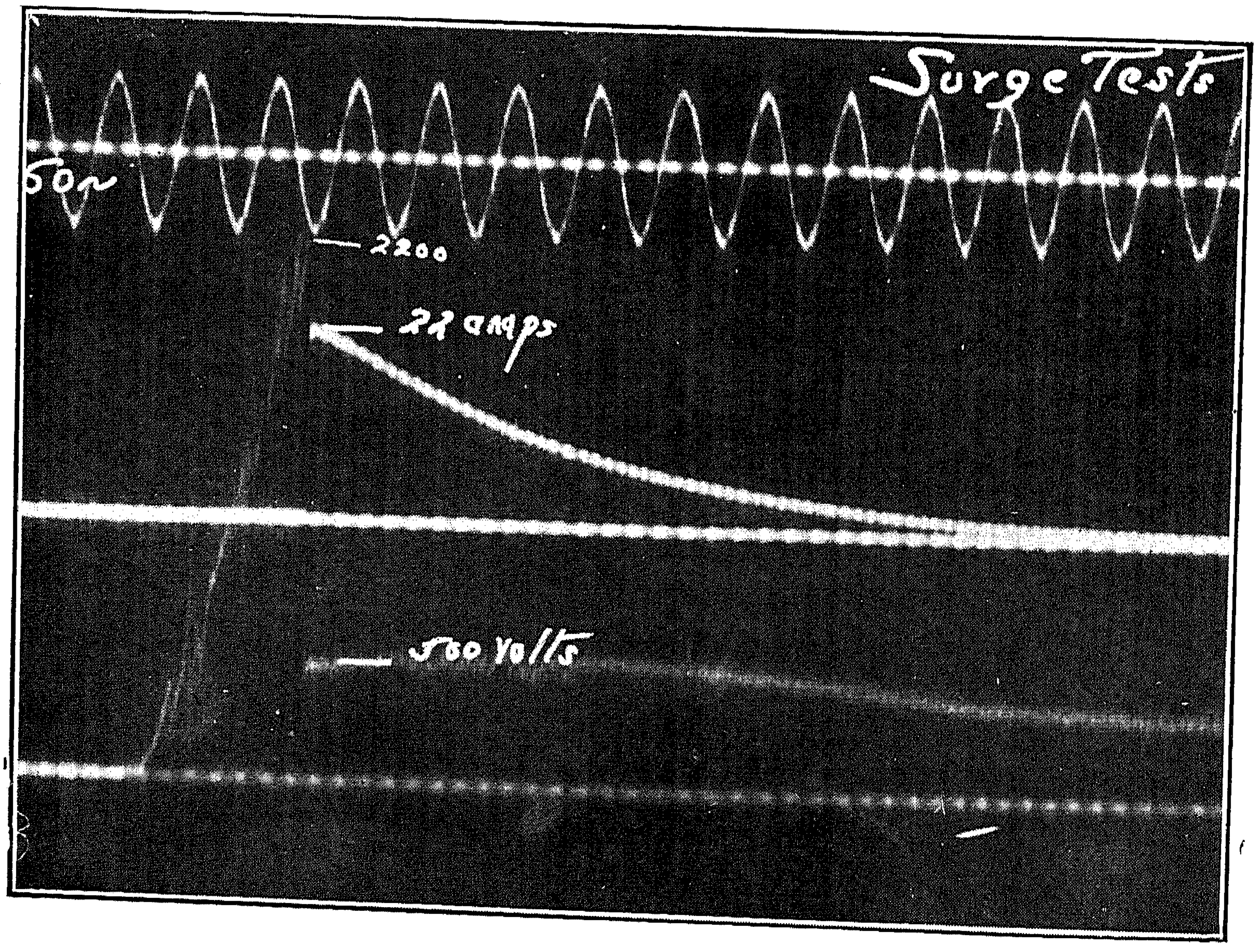


FIG. 3—SINGLE IMPULSE DISCHARGE—NO ARRESTER IN CIRCUIT
Top vibrator—60-cycle timing wave
Bottom vibrator—circuit voltage.



[STEINMETZ]

FIG. 4—SINGLE IMPULSE DISCHARGE AS IN FIG. 3—ARRESTER IN CIRCUIT
Top vibrator—60-cycle timing wave.
Middle vibrator—current through arrester.
Bottom vibrator—voltage across arrester.

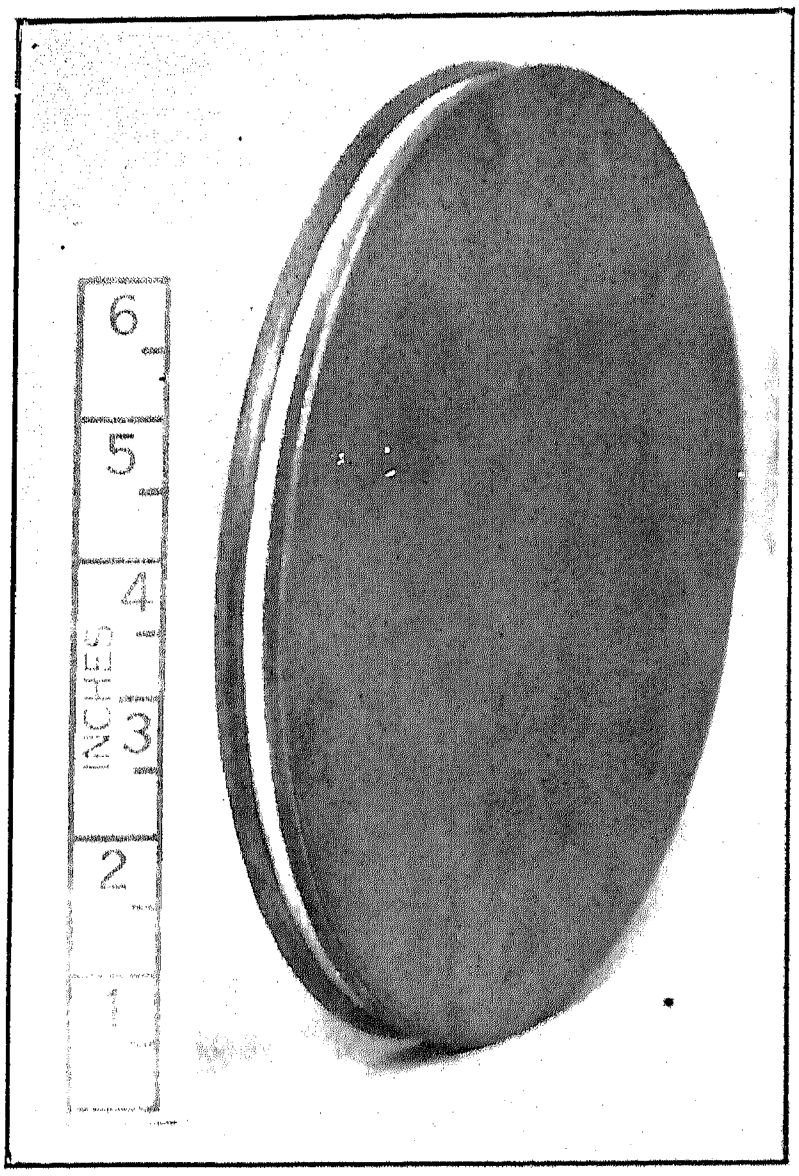
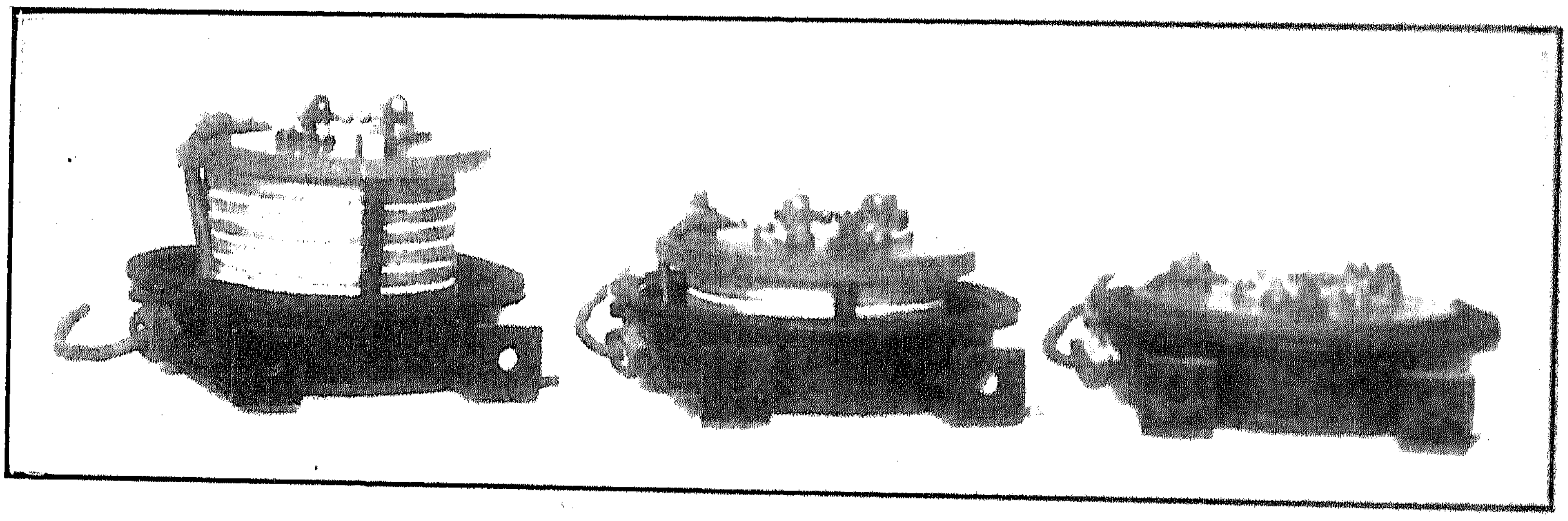
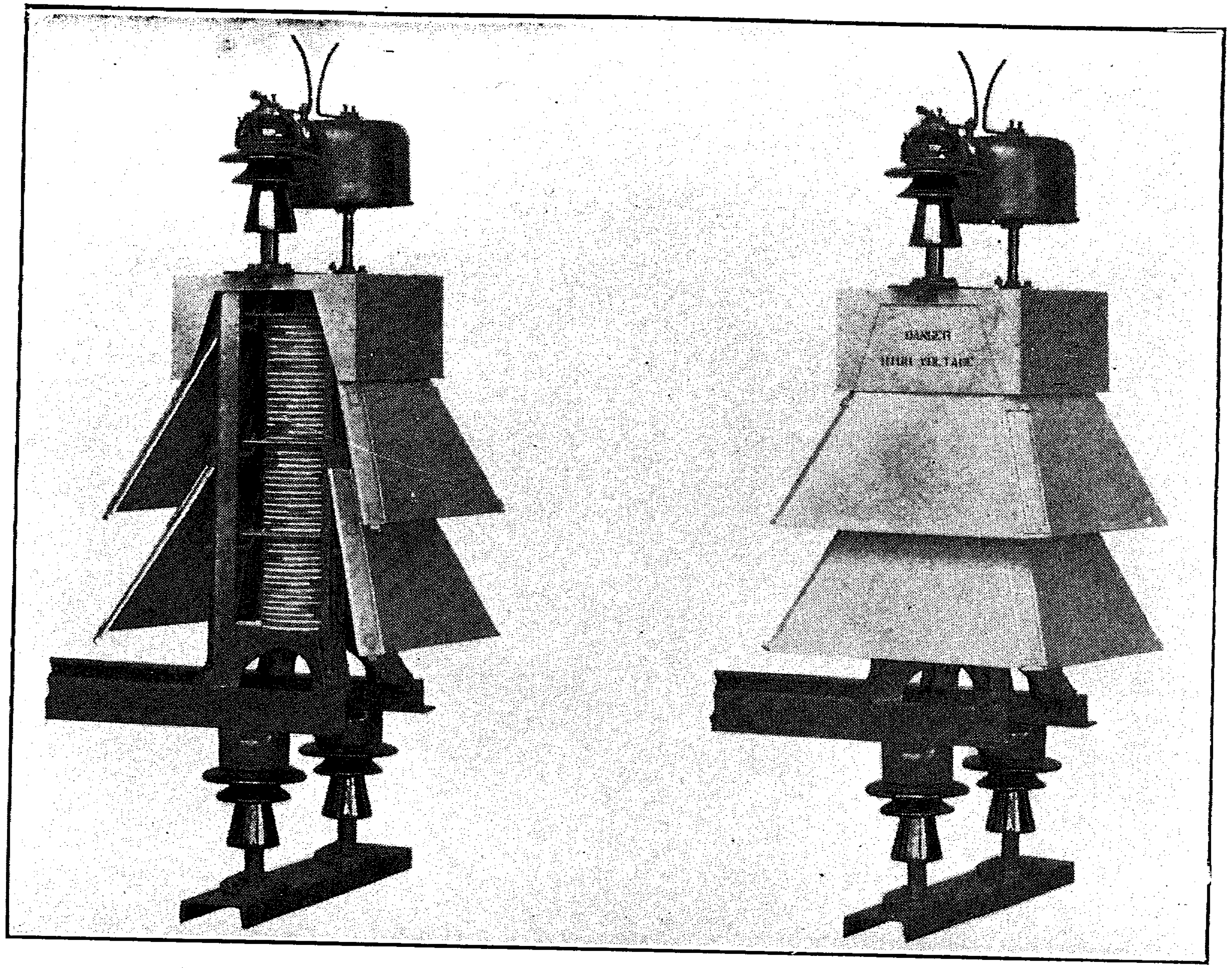


Fig. 5—Type "O F" Lightning Arrester Cell



[STEINMETZ]

Fig. 6—Type "O F" Lightning Arresters With Covers Removed Ratings 325-650, 900-1350, 2100-2600-Volt Alternating Current or Direct Current Respectively



[STEINMETZ]

FIG. 7—PHASE SECTION OF 15,000-25,000-VOLT OUTDOOR TYPE "O F" LIGHTNING ARRESTER WITH SIDE OF HOUSING REMOVED TO SHOW INTERIOR ARRANGEMENT

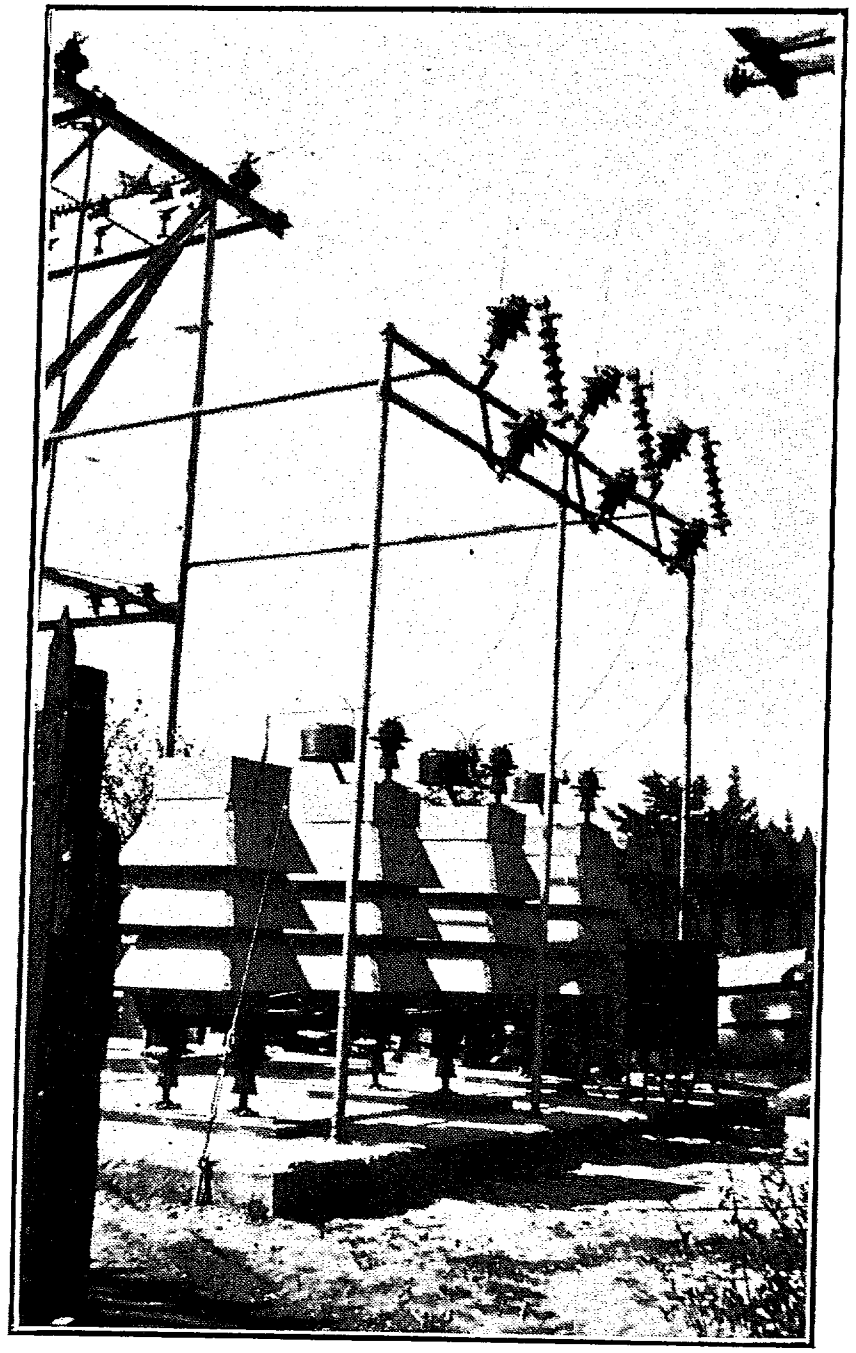


FIG. 8—TYPE "O F" LIGHTNING ARRESTER FOR OUTDOOR SERVICE INSTALLED ON A 33,000-VOLT CIRCUIT

logram in Fig. 2, is unsymmetrical also: in the first and second oscillation of Fig. 2, the first and third half wave of oscillating voltage is cut off, and the first and third half wave of the oscillating discharge current therefore higher than the second half wave of the oscillating discharge, which latter corresponds to a half wave of oscillating voltage in opposition to the circuit voltage, therefore not raising the circuit voltage. The third oscillation of Fig. 2 happens to start with a half wave of oscillating voltage in opposition to the circuit voltage, and the first half-wave oscillating discharge current through the arrester, in the middle curve, thus is smaller than the second half wave; the second half wave of the oscillation is cut down in the voltage and therefore gives the maximum discharge current, 35 amperes in this case.

This feature is well brought out by the oscillograms Figs. 1 and 2, due to the use of a different frequency, 60 cycles, for the power supplying the oscillator. This caused the successive oscillations to occur at different parts of the half waves of the 47-cycles circuit which was to be protected.

Fig. 3 and 4 then show the protective action of the oxide film arrester on a 550-volt direct-current circuit, against a single (non-oscillatory) impulse produced by opening a highly inductive circuit (railway motor). Fig. 3 shows on the lower curve the oscillogram of the impulse in the 550-volt circuit, rising to 7000 volts. The upper curve merely is a 60-cycle timing wave, to enable measuring the duration of the impulse. Fig. 4 shows the same circuit, with an arrester shunting it. The impulse voltage rises to the value for which the discharge gap of the arrester is set, in this case 2200 volts. Then the arrester discharges, and the voltage instantly drops back to normal, while a slowly decreasing discharge current through the arrester dissipates the magnetic energy of the impulse.

The cell of the oxide film arrester, shown in Fig. 5, consists of two circular metal plates as electrodes, which are kept apart by a porcelain ring, as shown in the figure. The space between the electrodes inside of the porcelain ring, is filled with the active material, lead peroxide PbO_2, which is put in under moderate pressure. This active material is a good conductor, but has the characteristic, that by the action of an electric discharge, it is converted in the path of the discharge, into a lower oxide, which is an insulator. Thus when an alternating current is passed through such a cell, the active material at the electrodes gradually converts into a non-conductor, and forms a thin insulating film at the electrode. This grows in thickness, until it cuts off the further flow of current and holds back the voltage, about 250 to 300 volts per cell. Then only a small leakage current, of a few milliamperes, passes at normal voltage, but if an over voltage of any kind appears at the cell, the insulating film of lead oxide punctures and freely discharges through the lead peroxide, but in doing so, converts the surface of the lead peroxide in the path of the discharge into the lower non-conducting oxide, and thereby closes the puncture holes, repairs or reseals the film.

In manufacture, naturally, just as in the aluminum cell arrester, the insulating film is not produced after assembly of the cell by the slow process of passing a current through, but the film is put on before assembly, in the oxide film arrester, by dipping the plates in a suitable insulating varnish, which gives them a coating just thick enough to hold back the circuit voltage. Then after assembly, voltage is put on the cell for testing it and sealing any holes or defects which may exist in the varnish film.

In the oxide film arrester, the electrodes have nothing to do with the arrester action, and any suitable material can be used. First we used brass, but now use sherardized iron, the latter having a higher melting point and thus standing high power discharges, which would melt holes in the brass electrodes.

In this arrester, the action, which holds back the normal voltage but passes freely an over voltage, thus resides in the active material between the electrodes, and it is this material which forms and reforms the film. As this material is a solid, no chemical action occurs such as the gradual dissolution of the alumina film in the aluminum cell arrester, but the film remains intact permanently, and thus no daily "charging", that is, repairing of the film, is required.

A number of such cells, depending on the voltage of the circuit, are piled on top of each other, with a spark gap in series, and, for low and moderate voltages, incased as shown in Fig. 6.

As the cells are hermetically sealed, by the metal of the electrodes being spun over the porcelain separating ring, the cells can be installed outdoors as well as indoors, requiring in outdoor installation merely some protection by petticoats, as shown in Fig. 7, to keep the rain from short-circuiting the cells. Fig. 8 shows such an outdoor installation of a 33-kv. arrester, with three-phase stacks and the ground stack of cells, protected against the weather by metal petticoats. Fig. 8 shows also the spark gaps on the line side of the arresters. They are protected sphere gaps, to give instantaneous discharge, with a horn attachment to allow the arc to flare up and thereby help in its extinction.

As well known, the plain horn gap has the disadvantage of requiring an appreciable —though a very short (microseconds) —time for discharge, and an extremely sudden high voltage, as a very steep wave front, thus may pass the horn gap and flash over elsewhere. Therefore in modern high-voltage lightning arresters the horn gap is shunted by a properly proportioned sphere gap, the latter being "instantaneous" in its action. In outdoor use however, rain lowers the discharge voltage of the sphere gap, and thus requires a setting which gives a higher discharge voltage in dry weather than necessary. Therefore a protected sphere gap has been designed, which overcomes this disadvantage in the open sphere gap, and is shown in Fig. 8.

The need of using this spark gap in series with the arrester, is the only still remaining undesirable feature which the oxide film arrester shares with the aluminum cell arrester, the multigap arrester and other types. While by the work of Mr. F. W. Peek, on the time lag of electric discharges[3], the means have been given to make the discharge gap "instantaneous", that is, faster than any other discharge path over gaps or through insulation in the system, so that the arrester takes care effectively of any over voltage above its discharge voltage, it does not discharge voltages lower than the discharge voltage of its spark gap, even if these lower voltages may involve some danger to the system by their high frequency. Such low voltages, while they cannot endanger the main insulation between circuit and ground, may, if of sufficiently high frequency, lead to local accumulations of voltage across inductive parts of the circuit, as regulators, current transformers, end turns and coils of generators and transformers, and there cause damage by puncturing insulation between turns and causing internal short circuits.

Against these high frequency disturbances of moderate voltage, the only existing protection is the addition to the arrester of a capacity discharge path permanently connected from the circuit to ground. Such capacity path should be without resistance to flatten steep wave fronts, and contain a moderate series resistance, to dissipate high-frequency energy and stop cumulative oscillations in their beginning. Before I leave the field of electrical engineering, I hope still to see an arrester, of the type of the oxide film or the aluminum cell, which has no spark gap, but is permanently shunted across the circuit, and thus capable of taking care not only of over voltages, but equally well of steep wave fronts and high-frequency oscillations, even if of lower than the circuit voltage. Such an arrester then would give universal protection.

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[3] A. I. E. E. Trans. 1915, Vol. XXXIV, Part II, p. 1857, *The Effect of Transient Voltages on Dielectrics* by F. W. Peek, Jr.

Presented at the 34th Annual Convention of the American Institute of Electrical Engineers, Atlantic City, N. J., June 27, 1918.

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THE OXIDE FILM LIGHTNING ARRESTER

BY CROSBY FIELD

ABSTRACT OF PAPER

The oxide film arrester is a new type of lightning arrester made up of a film of insulation in contact with a conducting powder. Upon the application of over-voltage, the insulation will be pierced, but the powder will very rapidly turn into insulation and plug any holes punctured in the original insulation by the over-voltage, thus forming in substance a resealing insulation. A brief description of this arrester is given together with the principles underlying its action and a comparison with other types of lightning arresters. Mention is also made of other characteristics of this combination which are not used in the present arrester, but which are being applied in other developments. A few notes of tests on the commercial arrester complete this paper. With the exception of the basic patents issued to the author this is the first time any disclosure has been made of this arrester.

THIS paper will be confined to a brief statement of the scientific principles underlying a new type of lightning arrester called the "oxide film arrester."* The functioning of this arrester depends upon the fact that certain dry chemical compounds can be changed with extreme rapidity from very good conductors of electricity to almost perfect non-conductors by the application of a slight degree of heat. Lead peroxide is a good example of such a substance. It has a specific resistance of the order of one ohm per inch cube. The resistance varies with the pressure to which it has been compressed. At a temperature of about 150 deg. cent. the lead peroxide (PbO_2) will be reduced to red lead, commercially known as minium (Pb_3O_4). This has a specific resistance of about 24 million ohms per inch cube. At slightly higher temperatures this minium will be reduced through the sesquioxide (Pb_2O_3) to litharge (PbO), which last named is practically an insulator. [A megger reading of infinity is obtained on a column 3 millimeters long (0.11 in.) and 5 square millimeters area (0.2 sq. in.)].

Again the oxides of bismuth give similar characteristics. There are, furthermore, several other compounds and mixtures of compounds that will give these same results.

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*U. S. Patent No. 1,238,660—Crosby Field.

Lead peroxide is normally in the physical state of a powder If this powder be placed between two electrodes and a current passed, the temperature due to the resistance at the contact of the peroxide and the metal will cause heat to be generated locally at the surface. When this heat is sufficient to create a temperature of about 150 deg. cent. a film of the lower oxides of lead forms, producing a film of insulation which stops the current. This method of film formation over any large area is rather irregular, and of course the oxide is not used in such a fashion in the commercial arrester. Instead of this formation of litharge film any insulating film may be put on the electrodes initially. As insulating film spread on the metal plates there have been used thin layers of the following; glass, water glass, halowax, cloth, balsam, shellac, oil, paints, lead paints, varnishes, and lacquers of all available kinds. In all cases the results are sim-

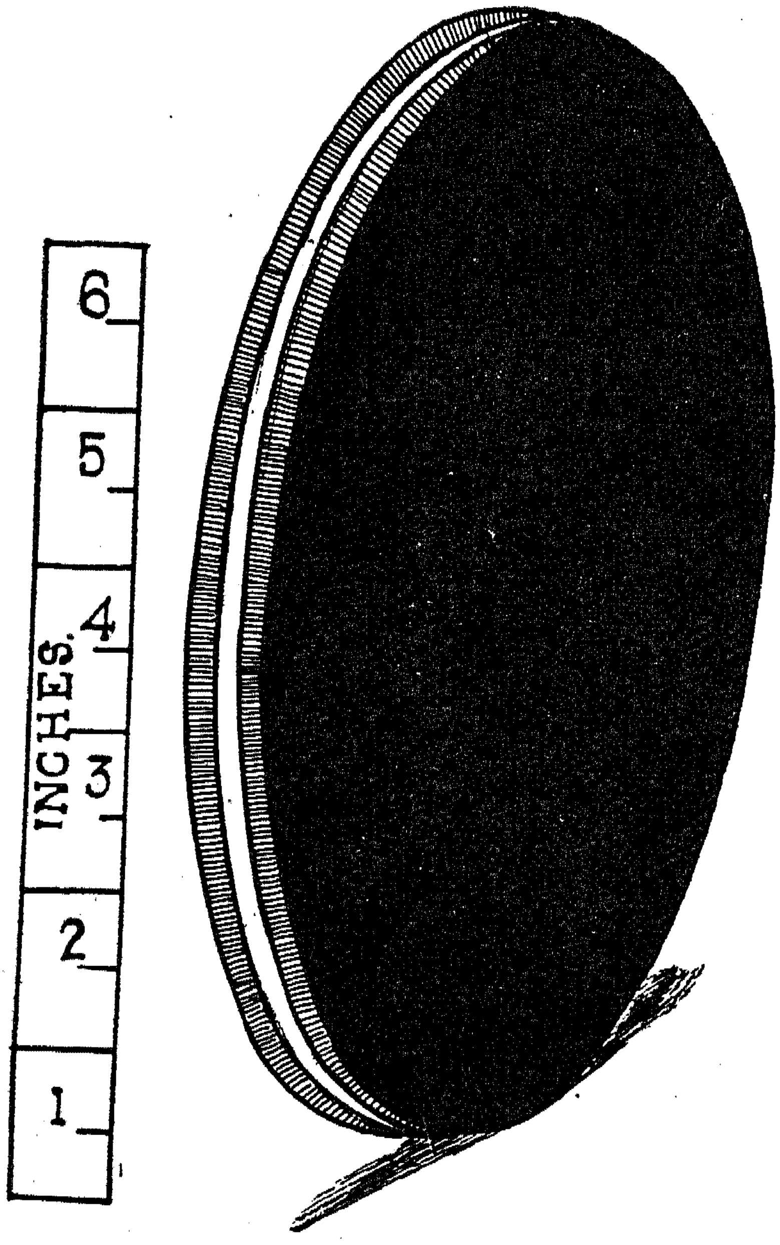

FIG. 1

A single cell of the oxide film lightning arrester consisting of a ring of porcelain with two circular steel disks, one spun on each side of the porcelain and insulated by the porcelain. The inside of the ring of porcelain between the plates is filled with peroxide of lead.

ilar, varying only with the voltage at which puncture of the film of insulation occurs.

The foregoing statements define the principle of the commercial oxide film arrester. Fig. 1. It comprises two sheet metal electrodes, set about 0.5 in. (12.7 mm.) apart, one or both covered with a thin insulating film and the space between the plates filled with some such substance as that described above, as, for example, lead peroxide. Fig. 2 shows the disassembled parts of a single cell. At a permissible voltage of 300 volts per cell the insulating film prevents any appreciable current flowing under

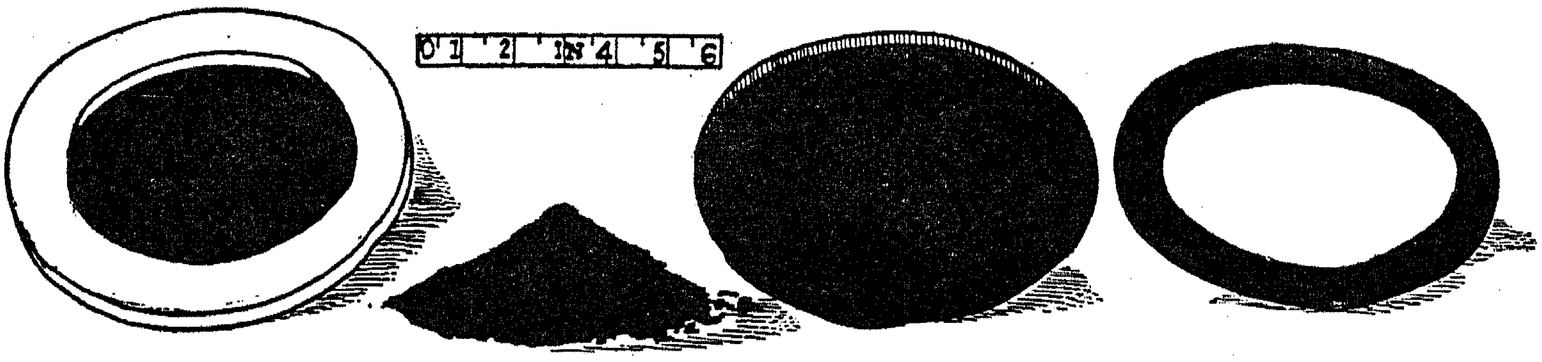

FIG. 2

Shows the disassembled parts of a single cell of the oxide film lightning arrester. From left to right are a steel disk spun on a ring of porcelain, a pile of brown peroxide of lead, the other steel disk and an asbestos washer.

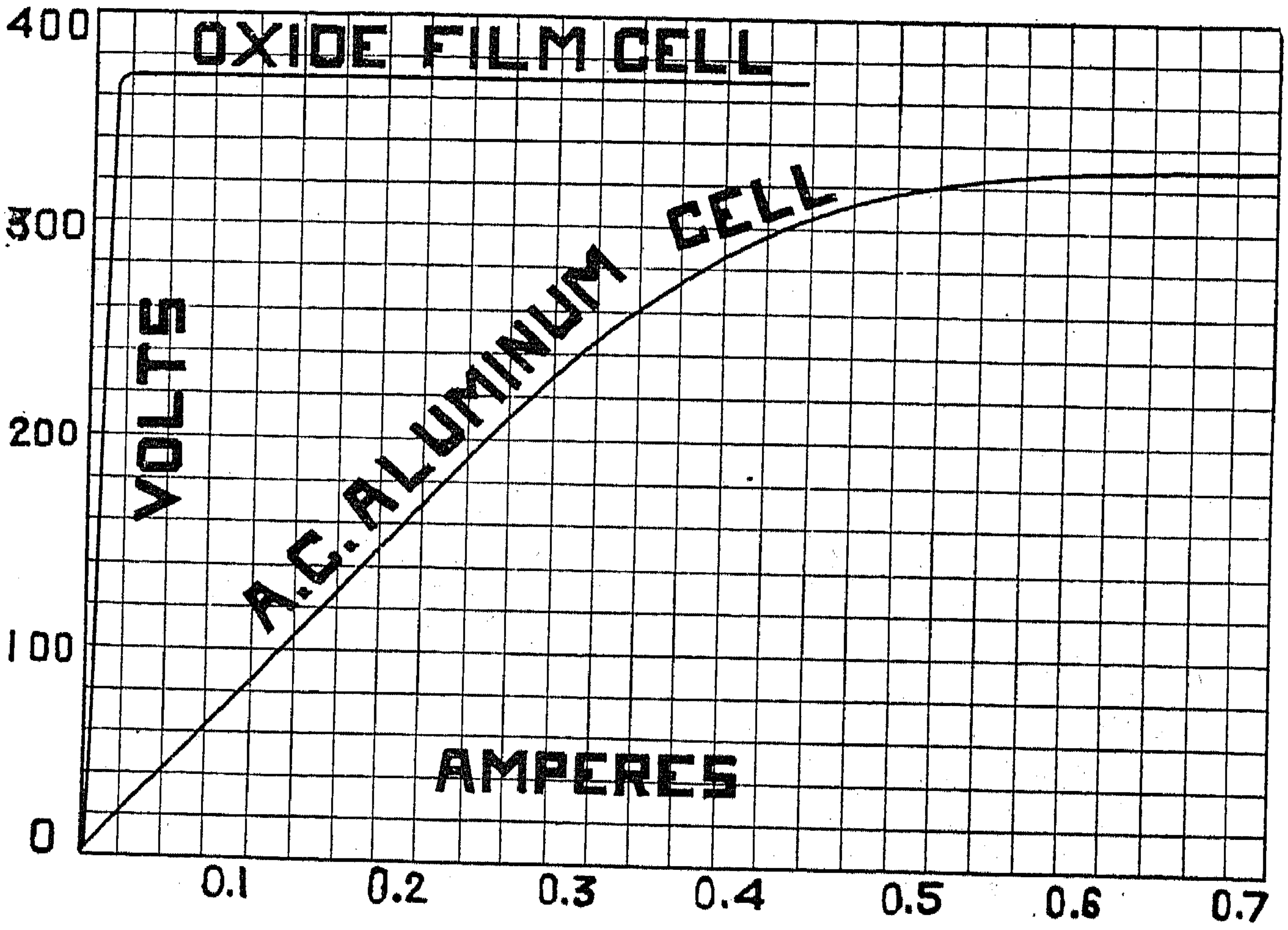

FIG. 3

The comparative volt-ampere characteristics of the oxide film cell and the a-c. aluminum arrester cell. The oxide film has only a few milliamperes of leakage current up to the critical film voltage when the film gives way more suddenly than the film in the aluminum cell. The critical voltage of the oxide film can be made approximately as low as the hydroxide film on the aluminum cell.

normal conditions. As soon as the voltage rises slightly above normal the film punctures in one or more microscopic points, the lightning charge meets with practically no resistance and flows to earth. Fig. 3. The dynamic current starts to follow but because of the fact that the insulation was punctured in such fine points, the current density near these points is exceedingly great. This results in a localized heating which speedily raises the temperature to a value sufficient to change to insulating litharge all the conducting peroxide in this minute path of the current flow in contact with the electrodes. The film consequently reseals, stopping the further flow of dynamic current. This action is so rapid that its duration cannot be measured on an oscillograph giving two thousand cycles per second, that is to say, the action of resealing occurs in less than one four thousandth part of a second after the excess of lightning voltage has ceased. Fig. 5 shows the visible spots of insulating litharge plugs on the surface of an electrode.

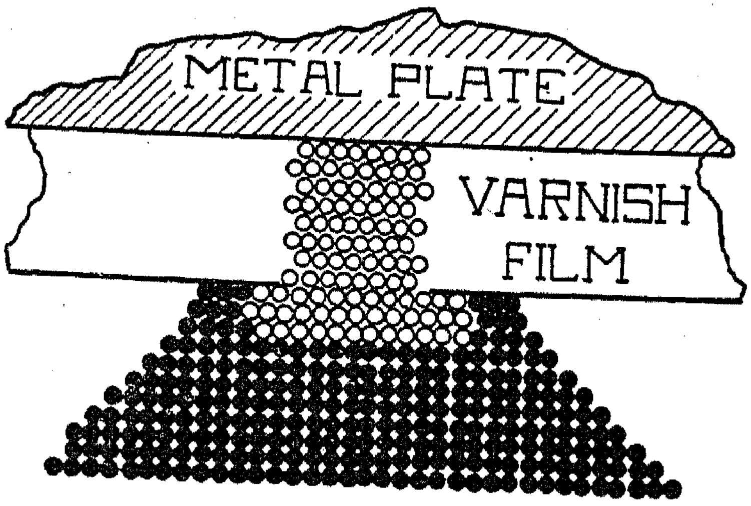

FIG. 4

A magnified, imaginary representation of one of the films on one metal plate. As shown the film is punctured by a spark and filled with a litharge plug which is represented by the open circles. The cross-section in the discharge path, a short distance away from the metal electrode, is sufficiently expanded to make the current of low enough density not to heat the peroxide to a temperature of reduction to litharge. The peroxide is represented by the solid dots and only those in the path of the discharge are shown. At the other electrode, not shown in this magnified diagram, a similar effect may be taking place although there is a difference between the positive and negative crators.

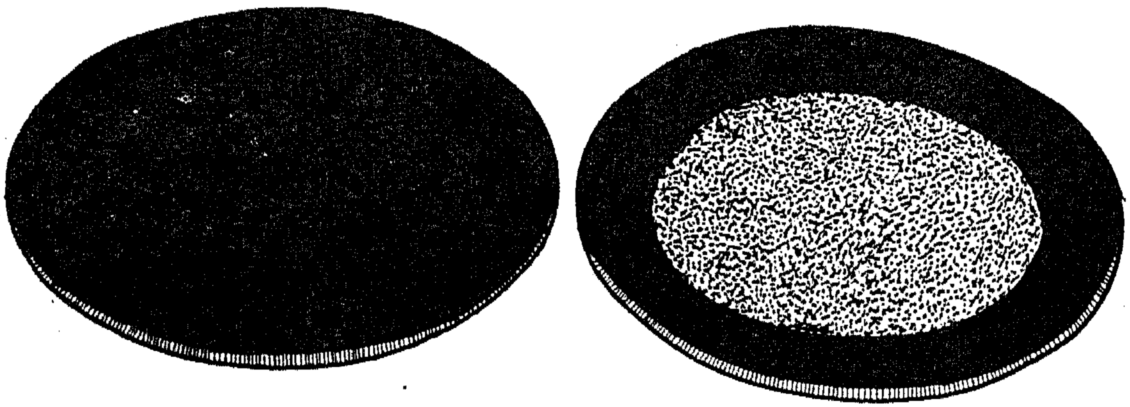

FIG. 5

New electrode and also one from a cell having had passed through it several thousand discharges. The light colored plugs of litharge are plainly visible in the many spots on the surface of the plate.

This film can be made of litharge itself, as well as any of the insulating materials above named. For example, metal plates may be inserted in any of the well-known lead electroplating solutions, and thus a very thin lead peroxide film (measuring a few hundred thousandths of an inch) formed. By proper heating this will be changed to litharge and this form of electrode can be used. Peroxide may also be sprinkled over any metal plate and the plate heated, which will reduce the peroxide to litharge. Again, the metal chosen for the electrode itself may be lead and if heated in the air a thin film of litharge will be formed on the surface. Again, an aluminum electrode may be put in any of the common electrolytes, and a thin aluminum film be built up. This may be used with the peroxide powder. Of these methods of forming the film the most preferable is by dipping in varnish or lacquer highly burnished surfaces of brass, steel, or copper, and is consequently used in the commercial arrester. The ohmic resistance of the arrester during discharge is quite low (less than 1 ohm per cell). Thus, when the insulating film is punctured the arrester offers very slight impedance to the flow of energy at abnormal voltages.

There is a certain range of voltage necessary to pierce any given insulation. The exact voltage depends not only upon the thickness of the insulation and its dielectric strength but also on the relation of the dielectric spark lag to the duration of the super-spark potential and the frequency of alternations of the transient surge.

If an arrester is to give protection of insulation in shunt with it, the arrester must relieve the abnormal electric pressure before damage is done to the insulation. Although tests are frequently made with the arrester and the insulation it is to protect in parallel, a more convenient method has been standardized and is known as the equivalent sphere gap test. Both the insulation and the arrester are compared by comparing each to the equivalent sphere gap.

The equivalent sphere gap of the oxide film arrester may be analyzed, as in other cases, into separate and distinct parts. First, there is the equivalent sphere gap of the main gap in series with the cells. Second, the equivalent sphere gap to initiate a discharge through the insulating film on the plate surface of the cell. Third, there is the equivalent sphere gap of the resistance drop of the current discharging through the powdered peroxide in its path. Fourth, there is the equivalent sphere gap of the inductance of the arrester.

Commenting on these factors in their relation to this arrester, the main gap is itself a sphere gap which has the fastest spark of any practical gap. The gap setting, like that of the aluminum arrester, is only slightly above that of the normal voltage of the circuit.

The equivalent sphere gap of the film is several times greater than the thickness of the film because solid material has a greater dielectric spark lag than air, but with this multiple of the thickness of the film the equivalent sphere gap is still low. Since peroxide is a good conductor, the series resistance in the path of the discharge is insufficient to give an undesirable voltage drop. As to the inductance of the arrester, it has a minimum value due to the fact that each cell is only 0.5 in. long, as shown in Fig. 1, and these cells are placed one on top of another. In other words, the total length of the arrester (which constitutes the inductance) is short as compared to the necessary length of conductor from line to earth.

One of the obstacles that had to be overcome in the making of this arrester was the increase in the resistance after a great many heavy discharges had passed through it. The predominant reason for the increase seems to be explained by the following theory. The current passing through this small puncture in the film heats up very rapidly not only the powder but also the air contained within the interstices of the powder. The particles are thereby thrown out of contact with each other, thus producing a fluffiness. The decrease in the number of contacts decreases the actual cross sectional area of conduction, hence increases the resistance. This raises the equivalent sphere gap. This action is accelerated, of course, by the giving off of the oxygen itself evolved in the reduction from lead peroxide to the lower oxide. If, however, this same arrester be violently jarred or the filling powder be compressed, or any other method utilized to restore the particles to their previous intimate contact, the equivalent needle gap will fall again. While increased fluffiness appears to be the predominant cause of change of the equivalent sphere gap, the increased thickness of the film of litharge at the point of puncture of the film is finally a factor of moment. The total area of the film must be sufficient to give a reasonable number of years of life to the arrester. There are other factors relating to the details of manufacture which give a limited degree of control over this change in equivalent sphere gap.

In all the commercial oxide film arresters used for alternating current the power factor is nearly unity. For special purposes however, the power factor can be made anything desired from 10 per cent. to unity. This is obtained by combining with the conducting oxide other non-conducting materials. This principle has been made use of for condensers but it has not been found desirable to incorporate it in the arrester.

To summarize—an arrester operating under a new principle has been made which comprises in essence one or more metal electrodes covered with an insulating film, and separated by a conducting powder, which has the peculiar characteristic of becoming a non-conducting powder upon the application of heat. Voltage higher than that which can be withstood by the insulating film punctures it in one or more points of about 0.005 cm. diameter. Dynamic current flowing gives a high current density in the conducting powder adjacent to these punctures which in turn heats it up rapidly, reducing the powder to a non-conductor, and sealing the holes in the film. The powder being a poor heat conductor localizes this action, so that very little more powder is reduced than is actually necessary to seal up these minute punctures.

The critical spark voltage and that part of the equivalent sphere gap controlled thereby is a function of the thickness and kind of material used for the film.

COMPARISON OF THE "OXIDE FILM" WITH WELL KNOWN ARRESTERS

The earliest form of non-electrolytic film arrester was known as the dry aluminum arrester.* It was a direct attempt to utilize the dry film which forms on the surfaces of pure aluminum immediately after it comes in contact with the oxygen of the air. The hydroxide film is easily formed in electrolyte and on drying becomes a dry film which gives sufficient action to prevent a discharge up to a given critical voltage, depending upon the thickness of the film. The film can also be formed by a spark or arc of a conductor in contact with a plate. Naturally this conductor should be of a non-metallic nature. In the earliest form tried powdered carbon was used mixed with dioxide of manganese which gives a liberal supply of oxygen at the heated point.

One of the objects of the development of this arrester was to decrease the cost of manufacture and it was found with the new principle involved in the oxide film arrester, where the powder furnishes the film rather than the plate, that the aluminum could be replaced by a cheaper metal, such as steel, and, as already described, the initial film known in the early stages of development as the "paint skin" type could be furnished by a layer of varnish. On first sight, knowing the extreme thinness of the hydroxide film on wet aluminum cell it might not seem that the dry cell would give the same general characteristics as the wet cell. But a comparison of the volt-ampere curves shows the same general characteristics. For a-c. voltages of 300 volts average per cell the current in the dry cell is of negligibly small value up to 40 milliamperes. The power factor is nearly unity and the current flow is due to very slight leakages through the films. In the case of the aluminum electrolytic cells there is an equivalent condition, the d-c. leakage current of the order of one milliampere being due to leaks through the hydroxide film. In the a-c. aluminum arrester the leakage current on the plate area used is much greater due to the destructive action of the alternating current on the hydroxide film. Furthermore, the wet cell with its thinner film is a condenser of appreciable capacitance which takes a charging current of about 0.5 ampere at 60 cycles. When the voltage reaches a certain critical value which is between 300 and 400 volts for the wet aluminum cell and between 300 and 500 for the oxide film cell (or higher if the paint film is made thicker) the current is allowed to pass freely through the cells, limited only by the ohmic resistance of the cell independent of the film. Since the oxide film arrester has no dissolution of the film, as occurs in the wet aluminum cell, charging is not only unnecessary but undesirable. This extends the use of the oxide film arrester to localities where there are no attendants.

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* U. S. Patent, E. E. F. Creighton.

Although the wet aluminum plate becomes frosted to an appreciable thickness by the passage of current in long use, the actual thickness of the film, as represented by the critical voltage, is not changed. In the oxide film arrester, however, the film less than one 1 mil thick (0.025 mm.) initially thickens up by the addition of successive spots of litharge for each successive discharge. This represents the wear on the arrester and limits its total life. Fig. 3 shows comparative volt-ampere characteristics of the oxide film arrester and the a-c. aluminum arrester. Since both of these arresters have a leakage current which wears the plates of the cells when alternating current is supplied, it is necessary, as previously stated, to place a spark gap in series with the cells. This spark gap is set at a value slightly above the normal potential of the circuit so that nothing but abnormal voltages will cause a discharge.

The foregoing data show that the oxide film arrester has general characteristics closely like the standard aluminum electrolytic arrester. It has the obvious advantage which comes from being dry rather than wet, it will not congeal and needs no daily charging.

In making the characteristic volt-ampere discharge curves the oxide film arrester does not lend itself as readily to the test as the wet aluminum cell. While its critical film voltage is evident, the change from no conduction to full conduction is more sudden. Therefore, the discharge rate at double potential is best shown by throwing double potential on the cell and subsequently reducing the voltage to its normal value per cell.

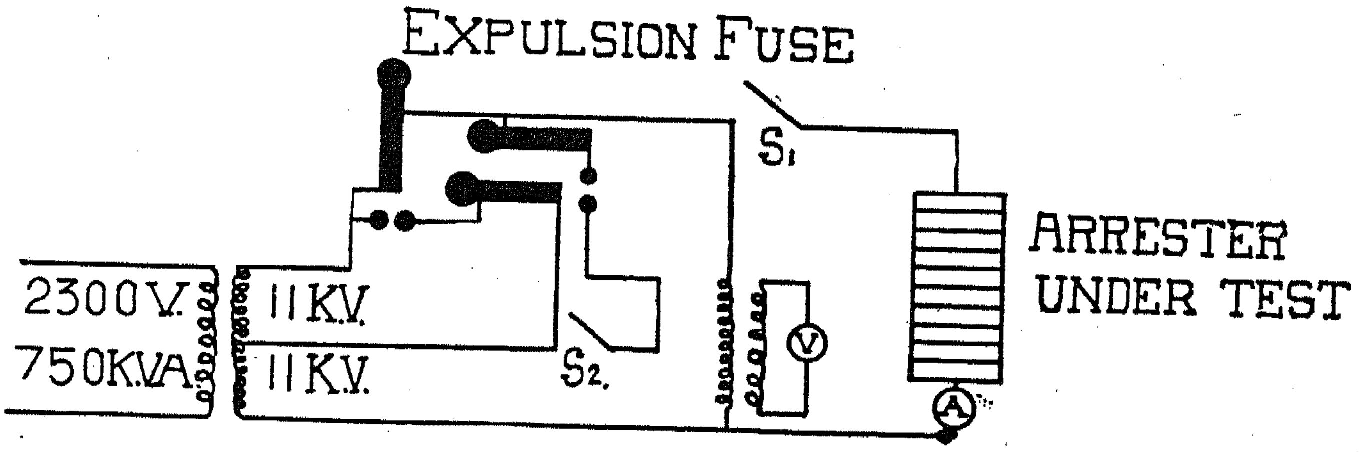

FIG. 6

Arrangement of three expulsion fuses which short-circuit gaps in the path of discharge to first throw double voltage on an arrester under test and then reduce the voltage to normal value by shifting the contacts on a transformer from full coil winding to half coil winding.

This gives a very considerable quantity of electricity through the cells and is a severe test.

In order to conveniently alter the voltage from double value to normal value the circuit is arranged as shown in Fig. 6. The object of the connections may be briefly stated: A transformer with two coils in series impresses voltage on the arrester and then the contacts of the arrester are automatically shifted to one coil giving half the voltage. A heavy pendulum closes switch *S*-1 and sets the oscillograph into operation. The full voltage on the transformer is thrown on to the oxide film arrester under test, marked *O-F*, which has a number of cells sufficient only for half the voltage of the transformer. In other words this throws double voltage on to the arrester and the heavy current passing through the cells causes fuse *F*-1 to blow. The operation of this fuse short-circuits half the transformer and throws the other half across the arrester. This is done by means of gaps and fuses as follows: When the expulsion fuse *F*-1 blows, the conducting gases are shot into the open gap *G*-1 which closes the circuit through fuse *F*-2 to the mid-point of the transformer. This short circuit on half the transformer causes fuse *F*-2 to blow and the hot gases discharging from fuse *F*-2 close the gap *G*-2 which throws the mid-point of the transformer on to the arrester through the switch *S*-2 which is closed just previous to starting the test. These several operations occur with a rapidity depending upon the size of fuses used. It is possible by this means to throw momentarily 22,000 volts on an 11,000 volt arrester and note the character attending its discharge and recovery. Fig. 7 shows such an operation on an oxide film arrester. The initial discharge current is 40 amperes during the first half-cycle due to the point it strikes in the descending wave during the second half-cycle it is 106 amperes. After the third half-cycle the litharge film has so completely sealed up the path of discharge that the current is too small to show on the oscillogram. The leakage current with no series gap is of the order of a few milliamperes.

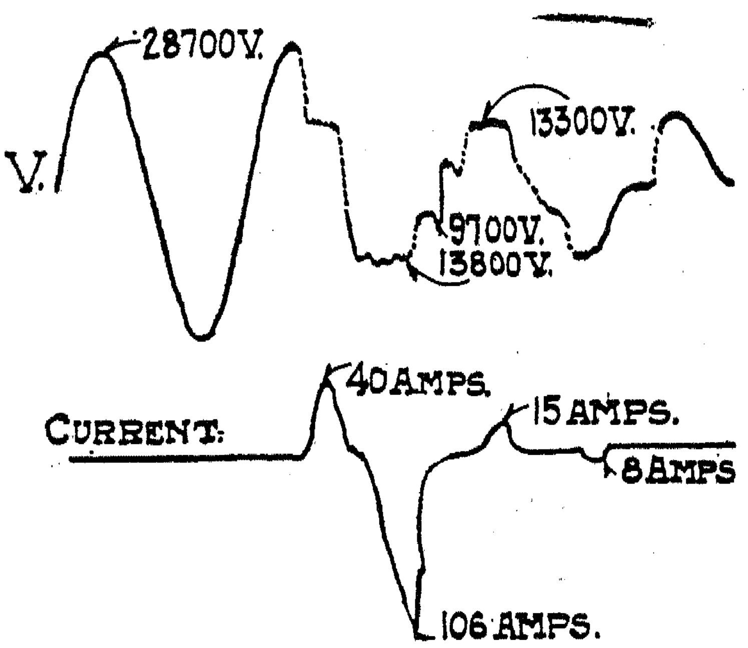

FIG. 7

Operation of an oxide film arrester on double dynamic voltage initially and its recovery on normal voltage. The upper record with a peak voltage of 28,700 volts shows the arrester connected to the circuit as the voltage wave starts to decrease on its third peak. The voltage immediately drops to the critical value of the cells—about 13,500. On the lower record the current is seen to rise to 40 amperes. On reversal of the voltage to 13,800 in the negative direction as shown by the upper record, the current as shown by the lower record rises to 106 amperes. The switching operations produce several electro-magnetic kicks as shown by the irregular voltage wave as it rises in the next cycle to 13,300 peak value. At this lower applied voltage the current in the series of cells, as shown by the lower record, is 15 amperes. In the subsequent half cycle the current rises only to 8 amperes. In the last half cycle of voltage shown, where the switching operation is complete, and the wave assumes its normal smooth form, the current in the cells is too small to be registered by the oscillograph. Its value is of the order of milliamperes. This figure is a copy of an oscillogram. The copy was made desirable by overlapping of the discharge on the two ends of the film.

DISCUSSION ON "THE OXIDE-FILM LIGHTNING ARRESTER" (STEINMETZ) AND "THE OXIDE-FILM LIGHTNING ARRESTER" (FIELD), ATLANTIC CITY, N. J., JUNE 27, 1918.

C. P. Steinmetz: As Mr. Field's paper represents the state of the work when he left this field a few years ago, I wish to bring it up to date in a few words.

On the first page of Mr. Field's paper he refers to the resistivity of the filler powder as one ohm per inch cube. At present this resistivity has been reduced to 0.1 of an ohm per centimeter, or 0.4 ohms per inch cube, that is, 25 times the conductivity, and correspondingly increased discharge rate, and as a result thereof, the resistance is materially less than 0.1 ohm per cell.

At the time that Mr. Field left it was found by laboratory experiment that there is an increase in resistance, after a great many heavy discharges made on the arrester in the laboratory, resulting in a gradual, though noticeable, increase of the equivalent sphere gap, and resulting decrease of the protective action. This, Mr. Field has explained by the production of gas by the discharge causing a fluffiness, and as a contributing cause by the reduction of the actual section, by the increase of thickness of the film and also the wearing action of the leakage current.

This feature was, indeed, the one which led me in my paper to refer to the necessity of industrial experience in existing plants, as a final criterion in lightning protective apparatus, since with all our skill and ability and thought, we cannot perfectly reproduce the actual conditions, and this was one of the reasons which led us to hold back the arrester in order to get fuller information on its life, and I am glad to say that after two to three years of industrial service, and after the arrester has received many thousands of heavy discharges, cells have been taken off and retested, and we have found no increase of resistance and no increase of the equivalent spark gap.

Although we have thus not found how long a life the arrester has, we have evidence that the life of the arrester, if it is not accidentally destroyed, is so long that in two or three years' industrial service no appreciable or measurable deterioration occurs.

After finding this out by experience, we could realize, why our tests were not representative of industrial practise. We passed continuously and in rapid succession very many thousands of heavy discharges through it, and since every discharge produces a little gas, gradually gas must accumulate, and finally result in separation of the particles, increasing the resistance and the equivalent spark gap. In nature, however, we have individual discharges, or even a large number of discharges following each other, but then followed by a period of rest, when the gas produced can be dissipated, and that is the reason why the increase of equivalent spark gap did not occur, and so far we have found in two or three years no evidence of deterioration of this arrester under industrial service.

With regard to the wear produced by the leakage current. Such a wear exists in the aluminum arrester, where the leakage current is an electrolytic current, but it does not exist in the oxide film arrester, as the conducting material is of that class of conductors which I have called pyro-electric conductors, having a definite metallic conductivity at ordinary temperature, which, only at high temperature changes to electrolytic conductivity, and therefore this leakage current does not deteriorate.

As regards the discharge rate, I want to call attention to the oscillogram on the last page of Mr. Field's paper, which, as he states, shows the characteristic of the arrester in holding back the normal circuit voltage, and discharging with practically no resistance the over-voltage. In this oscillogram double voltage has been thrown on the arrester, taking the discharge, dropped the voltage, not to nothing, as a short-circuiting arrester would do, thus not short-circuiting the machine power, but to the "counter electromotive force" of the cell, discharging only the over voltage. As this was of limited power, the current discharge is only about 100 amperes, but this does not represent what we call the discharge rate of the arrester, which is the current which would pass at double voltage, because the current in this oscillogram is not the current taken at double voltage, but at very little above normal voltage.

The test to really find the discharge rate of the arrester is to impress double voltage with so much power back of it, that the voltage does not drop, and it is a difficult test, because it requires an enormous power supply back of the arrester, and therefore I cannot give you entirely definite values, but only the information that the discharge current at double voltage is materially above 1000 amperes, and apparently higher than that of the aluminum arrester and the protection apparently greater than or at least fully as high, as that given by the aluminum arrester.

In a future paper, Mr. Lougee, who continued the development after Capt. Field, expects to give more complete information on tests and performances of this arrester.

N. A. Lougee: The oxide-film arrester, although a new arrester, has been shown by tests and operating experience to be very reliable and to give excellent protection. Very many tests have been made on this arrester duplicating as far as possible in a laboratory all the operating conditions it might be subjected to, in order to be as certain as possible that its operation would be successful before it left the laboratory. Moreover, complete data have been obtained by means of discharge alarm recorders attached to the majority of the arresters which have been installed to date. We hope in a later paper to give an account of the extensive laboratory tests and operating results which we have obtained.

Table I, I believe, will be of interest in showing briefly how and to what extent the high-voltage units of oxide film arresters have been installed.

TABLE I.

			1915	1916	1917	1918 up to June 1st	Total up to June 1, 1918
Number of cells installed		Indoors	200	2932	1448	1544	6124
		Outdoors	. . .	. . .	1440	2900	4340
		Total	200	2932	2888	4444	*10,464
Cells used for life runs, Tests and Development:			3488	2436	1712	465	8101
Number of Arresters installed	11—15 Kv.	Indoors	2	5	6	10	23
		outdoors	. .	. .	5	6	11
		total	2	5	11	16	34
	22—25 Kv.	indoors	. .	7	4	2	13
		outdoors	. .	. .	3	4	7
		total	. .	7	7	6	20
	33—38 Kv.	indoors	. .	4	. .	. .	4
		outdoors	. .	. .	1	4	5
		total	. .	4	1	4	9
	Total	indoors	2	16	10	12	40
		outdoors	. .	. .	9	14	23
		total	2	16	19	26	63
Number of recording devices installed:			1	9	16	19	45
Number of arresters destroyed:				1	1		2
Cause				Isolated transformer stations destroyed by fire.			

*In addition hereto, 10,000 cells are in production and will be installed during the present season.

In addition to these cells, we installed in 1914 about forty 650 d-c. "O F" arresters on electric cars and car circuits. These arresters were of a preliminary type and of different design than is used at present. These first experimental units gave excellent results and many are still in operation.

As noted from the table, we installed the first high-voltage arresters in 1915 and since then we have gradually increased the number of installations and added an outdoor design. I wish also to call attention to the great number of cells we have used for development work. These 8000 are of the present cell construction—before this design of the present cell was perfected, a great number of preliminary units, of course, were used in the development. About three-quarters of the installations at present are equipped with a discharge alarm recorder so that we have definite information that the arresters are successfully taking care of all kinds of disturbances.

I think it will be of interest at this time to say a little more concerning the life of this arrester. This fact was something impossible to determine from laboratory tests alone, and consequently we were anxious to learn what effect actual operating conditions would have on the cells, before we considered it desirable to publicly announce the development of this arrester.

One cell from each of the four stacks of two different 11,000-volt "O F" arresters were returned to us for test after these arresters had been in operation about a year. Very careful tests on all these cells showed that the initial equivalent sphere gap had not increased in any case, that the 60-cycle voltage breakdown had not increased, and oscillograms showed these cells to give excellent protection against a surge voltage. The interior of the cells, which were opened, showed no deterioration of the filler, in fact the cells as a whole were exactly as good as new cells. The arresters from which these cells were taken were equipped with discharge alarm recorders and these recorders showed an advance in counter reading of about 3000 discharges in each case since the arrester was installed.

Last winter one complete phase from each of the two arresters mentioned above was returned for further tests. These arresters during the two years they had been installed showed several thousand discharges from the discharge alarm recorder, and moreover operators in each station had kept a careful record so these particular arresters were known to have had more than normal operation. Initial equivalent sphere gap measurements on a complete phase as a whole showed a value similar to what is obtained with new cells. Some of the cells were opened and no deterioration at all could be seen.

Cells from a few other installations have been returned to us and in every case to date no deteriorations in the protective value of the cells or of the interior of the cells has been found.

By having cells returned from arresters every once in awhile, we will eventually learn what the average life of an arrester under normal service should be, and at present there is every indication that the life of the oxide-film lightning arrester is at least as long as that of any other lightning arrester.

L. W. Chubb: I would like to know whether tests have been made to find out whether one point heals up and an entirely different point breaks down, on a heavy discharge, or whether the first place that breaks partially heals up and progressively breaks down along the edge causing the point of discharge to travel along like the wandering of an arc on an electrode. The reason I ask is because, if the original puncture heals completely there must be an interruption and rise of voltage in order to break down a new point of the film. If the electrostatic charge of the film is not sufficient to heal one spot the voltage must drop and rise with each successive puncture giving an interrupter action and dangerous conditions similar to an arcing ground. The electrolytic arrester has enough capacitance and stored energy to heat a scintillation or breakdown without an appreciable drop of voltage and thus gives the true counter e.m.f. effect.

The oscillograms Figs. 3 and 4 in Dr. Steinmetz's paper, show the discharge of an electromagnetic field, without and with the arrester in shunt. In the first case the voltage rises to 7000 volts, in the second case the voltage rises to the gap voltage of 2200 volts and then drops to 560 volts, after the arc is established. The gap, of course, has an arc drop of approximately 50 volts, leaving about 510 volts creating a current of 22 amperes. I cannot understand how the resistance per cell can be as low as one ohm or 0.1 ohm per cell in view of this result. The oscillograph is too slow to follow very high-frequency oscillations and therefore gives only a smooth mean curve.

One of the advantages of the electrolytic type of arrester is the presence of high capacitance. In many cases high-frequency and steep-wave-front impulses can be discharged across the gap and absorbed without great rise of voltage or piercing of the film. I would like to know whether the new arrester has sufficient capacitance to obtain this condensive effect.

In both papers the equivalent spark gaps of different parts of the arrester are mentioned, but no figures are given. I would like to know what the equivalent spark gap of the arrester is. It seems that a varnish film which will hold line voltage so as to extinguish the gap after discharge will require several times line voltage before it pierces on an impulse, as the impulse ratio of solid dielectric films is known to be very high.

C. T. Allcutt: About the only point I want to mention in connection with Dr. Steinmetz's paper is the matter which Mr. Chubb brought out regarding the electrostatic capacity. As far as I can figure from the dimensions given, and from certain figures regarding the charging and leakage current of the arrester given by Capt. Field, the capacity of the oxide film arrester is in the neighborhood of 0.01 of the capacity of the aluminum arrester, and as a condenser, its effect in discharging impulse voltages would not be very great. Apparently, it would be necessary, in discharging an impulse, to have that impulse actually pierce the dielectric film of the arrester, which might be expected to involve some time lag and some decrease in voltage. The difference in capacity is very great, and in that point, certainly the oxide film arrester does not possess all of the advantages of the electrolytic arrester, as it would seem.

C. P. Steinmetz: I will answer the three questions asked. Whether the puncture occurs at the film or a sealed-up puncture hole, is merely a question of probability. In resealing after a discharge the litharge spot thickens up so as to hold back the circuit voltage, and it is merely a question of probability whether the next discharge which punctures goes through an unbroken film or through a litharge spot, since just as in the aluminum cell, the resealed puncture hole is of the same strength as the rest of the film.

As to the capacity effect: The capacity of the cell is higher than it was in Capt. Field's time, due to the decreased resistance of the film, but it is less than that of the aluminum cell. The capacity at normal frequency is of less magnitude than the resistance, but it is, as you see from the construction, sufficiently large to act as a capacity path for those extremely steep wave fronts, and extremely high frequencies at which the time lag of the insulation comes into consideration, though at 60 cycles the capacity is practically negligible.

The oscillograms of the impulse voltage given in my paper as Figs. 3 and 4, are identically the same kind as would be given by an aluminum cell, that is, the impulse voltage rises to the breakdown voltage of the gap, and at that point the voltage drops to the circuit voltage, and the 560 volts is not the resistance drop across the arrester, but is what you might call the "counter electromotive force of the arrester", which holds back the normal circuit voltage, the voltage of the direct-current railway generator which was on the circuit. This would occur in identically the same way with the aluminum cell, and it is a characteristic of this type arrester not to discharge the normal machine voltage, but only over-voltage. But from this follows nothing regarding the discharge resistance. The discharge resistance is obtained by dividing the excess voltage over the normal circuit voltage, by the discharge current.

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THE DESIGN OF TRANSPOSITIONS FOR PARALLEL POWER AND TELEPHONE CIRCUITS

BY HAROLD S. OSBORNE

ABSTRACT OF PAPER

This paper presents the results obtained in a recent design of transposition systems for telephone circuits exposed to induction from circuits of other kinds, particularly from three-phase power circuits, and the coordinate arrangements of transpositions in the power circuits. The new arrangements are the result of a systematic investigation of the degree of flexibility which could be obtained in the coordination of telephone transpositions and transpositions in outside disturbing circuits, particularly three-phase power circuits.

In presenting the results obtained, a discussion is first given of the requirements which must be met by systems of transpositions for telephone circuits in general, and by the new "exposed line" system in particular. An outline is given of the methods used in the design work and the theory upon which it is based. The diagrams in the paper show the arrangements of transpositions for all circuits on eight crossarms of telephone line. The results to be obtained from the use of these diagrams are outlined, and the suitable locations of coordinate transpositions in parallel power circuits are discussed.

The application of these and other arrangements of coordinated transpositions to specific cases must take into account the variations in separation and other changes in the power and telephone circuits. It is considered to be beyond the scope of the present paper to go into a discussion of the problems involved in these specific applications.

INTRODUCTION

IN a well-engineered telephone system the power transmitted into a long-distance telephone circuit is almost entirely absorbed in line losses. The circuit terminates in that marvelous little electric motor, the telephone receiver, which requires only a few millionths of a watt for its operation. As the power delivered to the line by the telephone transmitter may be measured in hundredths of a watt, only a fraction of one per cent of the transmitted energy need be received at the far end of the line. This is fortunate, for as the telephone currents are of relatively high frequency, the energy losses suffered by transmission over a given circuit are very much greater than would be the losses for 60-cycle current. Moreover, the telephone currents must be carried over very great distances. It has therefore been necessary to do an enormous amount of work in developing means for increasing the efficiency of long telephone circuits in order that country-wide service may be given with a network of conductors which should not be prohibitively expensive.

The large ratio between the energies at the transmitting end and at the receiving end of a long distance telephone line imposes very severe limitations on the amount of inductive effect which can be permitted between different telephone circuits. If the transfer of energy between two circuits is more than a millionth of the energy transmitted there is danger that one can overhear on one circuit what is said on the adjacent end of the other. This must of course be avoided, and an even stricter limitation of the transferred energy is desirable.

The method which is used to prevent excessive transfer of energy between telephone circuits on the same pole line is the installation of a systematic arrangement of transpositions of the conductors of each circuit.

Present standard telephone practise for open-wire circuits provides ten wires spaced about one foot (0.3 m.) apart on a crossarm and any number of crossarms spaced two feet (0.6 m.) apart on a pole. The currents carried by these circuits are of very high frequency compared with those used in power practise and tend to make much of small mutual impedances or admittances between the circuits. It is small wonder then that when in 1886 the first long distance telephone line was built between New York and Philadelphia it was found that, listening at the distant end, an observer could not tell which circuit was used for transmission but could hear equally well on all of them. The systematic study of telephone transpositions began at that moment and has continued until the present day.

One factor which has much complicated the study of telephone transpositions is the common use of phantom circuits. These are more accurately described by the French terms *derived* or *superposed* circuits. A phantom circuit is formed by superposing a third circuit on two 2-wire telephone circuits called the side circuits of the phantom. The phantom and its side circuits thus use the same copper conductors, and it is evident that great care in balancing the inductive relations is necessary to prevent the transfer of energy between circuits so intimately related.

Long distance telephone lines have not grown up without neighbors, and in the design of telephone transposition systems it has been necessary to take them into account. Telegraph systems and power transmission systems have been contemporaries with the telephone system in the process of development. These systems have not in general been obliged, for any reason of their own, to restrict their electric and magnetic fields as has been necessary with telephone circuits. From the first, however, the limitation of the effect of these outside fields has been a factor in the design of telephone transposition systems.

It is evident that when a telephone circuit is paralleled by an electric power circuit carrying hundreds or even thousands of kilowatts, the mutual inductive connection between the two circuits must be extremely minute if we are to avoid induction in the telephone circuit of power of the same magnitude as that of the voice currents or even much more. In fact, this difficult problem would probably be quite impractical of solution if we had to rely alone upon balance produced by transpositions. Fortunately, another very weighty factor works for the solution of the problem. The fundamental frequency of commercial power circuits is out of the easily audible range and quite out of the range of frequencies which the telephone line must transmit in order to reproduce intelligible speech. As a matter of fact the noise produced by power circuits in telephone circuits is practically not at all due to the fundamental but due to the harmonics or high-frequency components of the power circuit which lie within the range of telephone frequencies. The energy represented by these harmonics is represented by kilowatts or even by watts rather than by thousands of kilowatts. In spite of this fact the degree of balance required in cases of close parallel is very precise indeed.

Of all the means used to reduce the inductive effect between power circuits and telephone circuits the coordinate transposition of both classes of circuit is probably the most generally important. The application of this means to parallels between telephone and power circuits is in practical cases attended with difficulties because instead of two circuits running accurately parallel for long distances the separation usually varies at frequent intervals and very irregularly.

The usual case, in which the power circuit is three-phase, presents another complication which is fundamental to this type of circuit. The telephone circuit is a single-phase circuit and its conductors have only two relative arrangements on the pins.

The three-phase circuit has three possible arrangements of its conductors on the pins, and three sections of line, each with a different arrangement, are necesssary for what is called a "barrel" of the power circuit. The combinations of relative pin positions of the two circuits when both are transposed are, therefore, six in number. The difficulties introduced by this relation have been discussed in an earlier paper* before the Institute.

Although the importance of neutralizing as far as practicable the effect of outside circuits has always been a factor in the design of transposition systems, the possibilities of coordinating the telephone transpositions with the transpositions in three-phase power circuits were not at first fully developed. With the great growth in both telephone circuits and three-phase power circuits this matter has been growing rapidly more important in telephone transmission.

In connection with the very extensive investigations of the Joint Committee on Inductive Interference in California it was therefore considered advisable to undertake the development of a new system of telephone transpositions which would embody the maximum possibilities of coordination of transpositions in the two classes of circuit. There has resulted from this work a system of transpositions known as the "exposed-line" transposition system.

OUTLINE OF METHODS OF DESIGN

The design of a transposition system requires the mastery of a very difficult and complicated technique. It is attempted here to give only a general outline of the methods employed and some of the difficulties encountered and overcome. Some of the mathematical work is given in the appendices.

Conductive and Inductive Connections. The mutual connection between two circuits may be of any of the following characters

Resistance
Leakage
Inductance
Capacitance

Two circuits may be said to be unbalanced with respect to each other by a mutual connection of any of the four above characters when the mutual connection enters into the circuit in such a way that current in one circuit tends to cause a current in the other circuit.

**Inductive Interference as a Practical Problem.* Messrs. Griswold & Mastick, TRANSACTIONS, A. I. E. E., Vol. XXXV, 1916, p. 1051.

By resistance and leakage we denote conductive connections respectively series and shunt. Although two metallic circuits which have no resistance in common cannot have a mutual resistance unbalance, care is necessary to avoid a mutual resistance unbalance between a phantom and its side circuits. This is however a matter apart from transpositions. The leakage from telephone circuits is almost all directly to ground. This is normally kept very low because of the large effect which high leakage would have upon the efficiency of the circuits. For these reasons leakage is not an appreciable factor in the interference between telephone circuits, except under abnormal conditions, such as those produced by baling wire thrown over a telephone line.

Two circuits strung side by side have both mutual inductance and mutual capacity unbalances. The two terms represent the coefficients by which are measured the unbalanced action of the mutual electromagnetic and electrostatic fields respectively, and both represent inductive rather than conductive effects. It is the purpose of transpositions to correct for these as largely as may be.

Inductive Effects Between Telephone Circuits. The general equations for the mutual inductive effects between long parallel circuits are hopelessly complex. For the benefit of those interested a discussion is given in Appendix A of the mathematics by which these inductive effects may be computed in some simple special cases. In Appendix B are given equations for the most simple general case, that is, the case of inductive effects between two long symmetrical parallel wires with ground return. Examination of equations (**7**) to (**10**) in this appendix shows that the current or voltage in either conductor is represented by the sum of two complex exponential terms. Little consideration is required to convince one that similar general equations for forty parallel wires would be impracticable either to establish or to use.

However, in studying the inductive effects between telephone circuits (commonly called cross-talk), use can fortunately be made of a beautiful simplification. Although the circuits are very closely associated on the same pole line, the requirement for good operation after the circuits are transposed is that the transfer of energy from one circuit to another shall be exceedingly small. This being the case, the reaction of the induced current on the inducing circuit is evidently negligible. The problem can therefore be studied by computing the current and voltage of the inducing circuit as though no currents whatever were induced in other circuits and then computing the inductive effect of these currents and voltages on each adjacent circuit under the assumption that no current flows in any of the other circuits.

In order that this computation may be carried out, it is necessary that the coefficients of induction between the circuits be determined. The mutual inductance unbalance between all pairs of circuits on a 40-wire pole lead can be readily computed. It was estimated, however, that to compute the direct capacities between circuits would require the time of one man for at least ten years. It was therefore found much cheaper and more practical to make measurements of the capacities between the wires in a short section of line under construction. The results obtained in this way are given in Table I.

Resultant Unbalance between Long Circuits. With the data and by the use of the approximations outlined above, it is possible to compute the inductive effect between any two short lengths of telephone circuit on the pole lead. In carrying out the computation of the effects for long circuits, however, a complication arises due to the fact that the current and voltage in the disturbing circuit are different both in magnitude and phase angle at different points of the line. The change in magnitude, because of attenuation effect, varies from 0.05 per cent to over 0.5 per cent per kilometer on open wire telephone circuits of different construction, and the change in phase angle varies from a small value up to nearly 10 degrees per kilometer. Also, in the propagation of induced current along the disturbed circuit to its terminal, this current undergoes a similar change in magnitude and in phase angle. Therefore, the current induced between two circuits in one kilometer length of line cannot be perfectly balanced against that induced in another kilometer length, and no matter how often or under what arrangement the transpositions are placed, they cannot serve to perfectly balance out the induced effects. There is always a resultant effect which is known as the "type unbalance". The transposition system must be so designed that this type unbalance is kept below the desired limits for all combinations of the circuits on the lead.

A brief mathematical discussion of type unbalance is given in Appendix C.

0-WIRE POLE LEAD.

25	24	23	22
26	27	28	29
32	27	25	32
32	25	22	26
41	31	25	27
61	42	30	29
81	58	40	35
35	25	20	24
48	30	19	17
86	45	24	19
178	91	43	25
339	182	87	46
45	30	24	26
63	35	21	17
134	65	30	
336	149		
853			
		Wires	40
			39
			38
			37
			36

The type unbalance does not measure the entire inductive effect between two telephone circuits, for this is contributed to also by another factor which is sometimes very important. It is impracticable to locate the telephone transpositions at exactly the theoretically correct points. In laying out a new line it is sometimes possible to set poles exactly at the theoretical transposition point, and when this cannot be done the nearest pole is generally chosen as the transposition pole. However, the transposition itself occupies two pole spans. Moreover, small changes in separation of wires cannot be avoided, even going round a curve introduces a certain amount of irregularity in the separation of wires. These and other irregular factors increase the inductive effects above those due to the type unbalances and must be taken into account in designing the transposition system.

Phantom Circuits. One factor which introduces a large complication into the design of transposition systems is the general use of phantomed circuits in long-distance telephone lines. This is standard practise for long distance telephone circuits, and makes it possible to obtain three circuits from each two pairs of wires instead of two. The principle involved in phantoming is generally understood. The arrangement is indicated diagrammatically in Fig. 1.

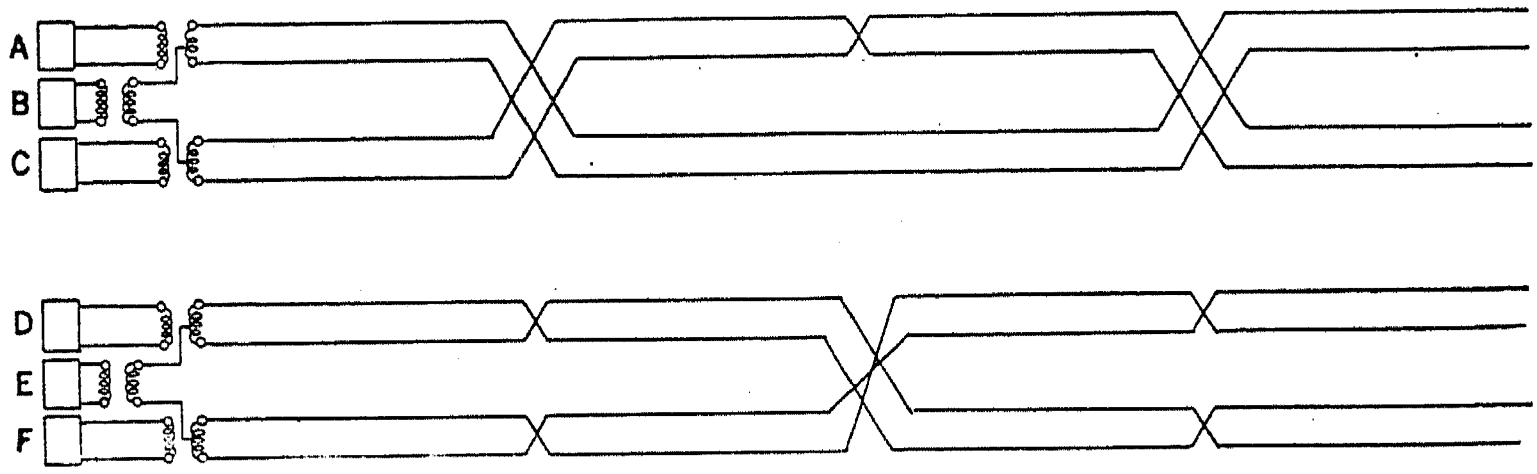


FIG. 1—PHANTOM TELEPHONE CIRCUITS

By the transposition of a phantom circuit the pin positions of its side circuits are interchanged, and the coefficients of inductive effect between the side circuit and other circuits on the lead are changed. In order that the phantoming may be possible without interference, it is therefore necessary not only to very carefully balance the terminal transformers and other apparatus connected to the telephone lines, but also to use a transposition system that provides for a very high degree of balance of the inductive effects between the phantom and its side circuits and also between these circuits and the other circuits on the pole line.

It is evident that in designing the transpositions in the side circuits it is necessary to take into account the transpositions in the phantom. For example, in the figure, side circuit *C* would be considered balanced to side circuit *F* if the transpositions in these two circuits only were to be considered. However, the transpositions in the phantom circuits change the coefficients of induction between circuits *C* and *F* in different parts of the section shown and therefore prevent the transpositions in the side circuits from balancing this section.

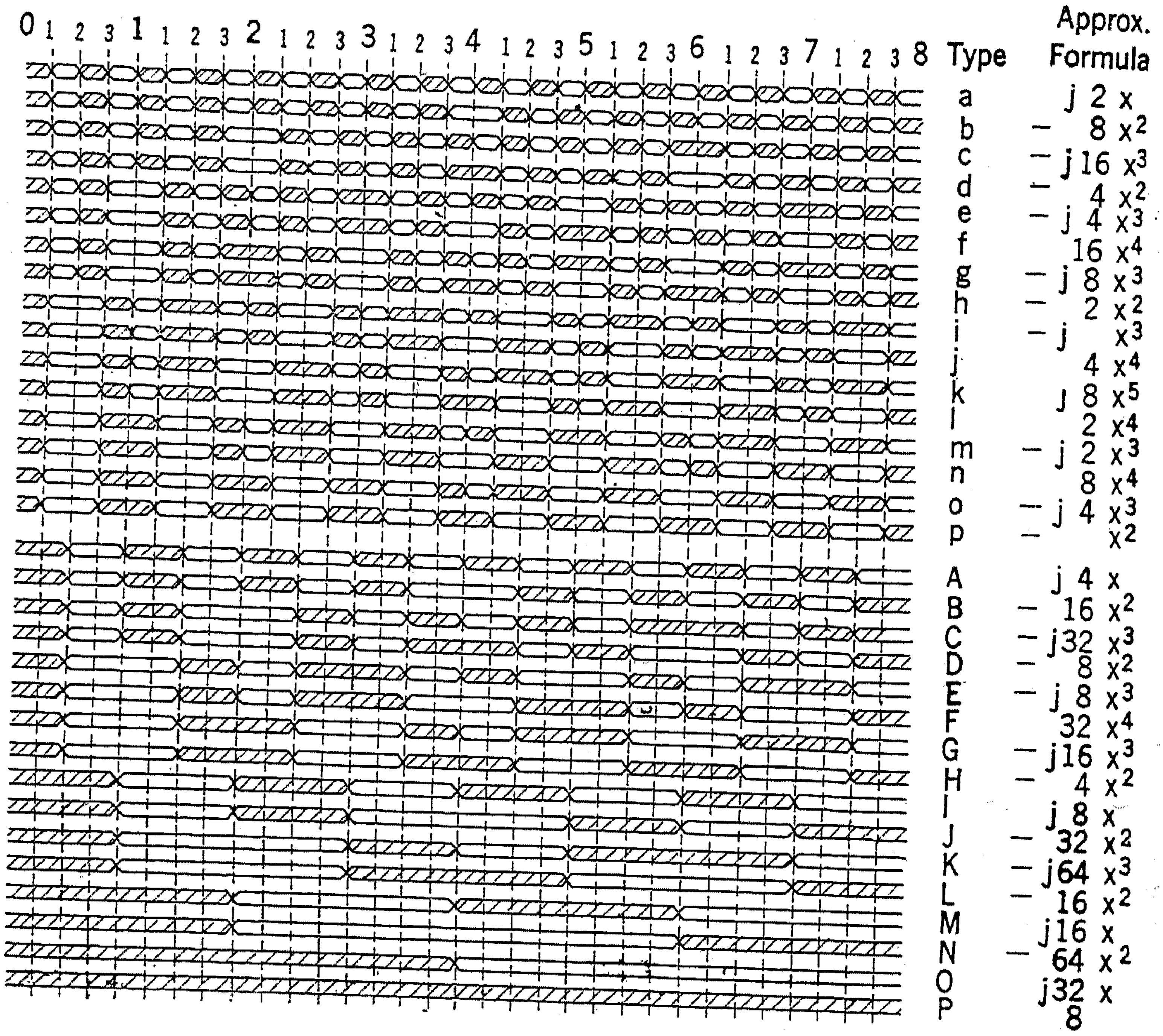


FIG. 2—TYPICAL ARRANGEMENTS OF TRANSPOSITIONS

Equations showing the method of computing the equivalent exposure and the type unbalance between circuits which are phantomed are given in Appendix *D*.

In order to fit in with standard telephone practises a transposition system must provide for the phantoming of any two horizontally adjacent pairs of circuits and any two vertically adjacent pole pairs. The system must also provide that any of these combinations of circuits may be phantomed and any others non-phantomed.

Typical Arrangements of Transpositions. In carrying out the design of a transposition system it is convenient and economical to arrange the transpositions for each circuit in accordance with one or another of a list of selected typical arrangements. In Fig. 2 are shown the types, 32 in number, suitable for use in a transposition section made up of 32 parts of equal length. Reading from bottom to top, the types are arranged in the drawing so that each type has one more transposition per section than the preceding type.

The type unbalance between two telephone circuits is determined by the relative transposition of the two circuits, that is, the points at which one circuit is transposed but not both. The typical arrangements of transpositions have the characteristic that the exposure between any two circuits so transposed can be represented by some other type; that is, the exposure between any two types is the same as the exposure between some other type and a circuit without transpositions. For example, exposure f to $k =$ exposure F to $K = A$, exposure J to $m = k$. For transposition sections divided into 64 equal parts, 32 more typical arrangements can be added to the 32 shown in the figure by adding a transposition at the midpoint of each elemental length of each type.

Typical Arrangements for Exposed Lines. The purpose of the exposed-line transposition system is particularly to provide increased facility in balancing the induction from outside circuits in so far as this can be done by transpositions. The effect of outside circuits in inducing voltages between the telephone wires and ground which may be productive of very serious interference cannot be remedied by transpositions in the telephone circuits, but can frequently be greatly reduced by transpositions in the outside circuits. The telephone circuit transpositions must be designed to equalize the voltages induced in the two sides of each telephone circuit with the greatest possible flexibility in the location of the transpositions in the outside circuits. This imposes great limitations on the typical arrangements of transpositions which can be used.

The outside disturbing circuits can be classified as follows:

1. Non-transposed circuits such as telegraph circuits. This applies also to the residual components of current and voltage in power circuits.

2. Two-wire transposed circuits such as single-phase power circuits.

3. Three-phase power circuits.

In order that the disturbing effect of circuits of the first class may be reduced as far as possible, no typical arrangement of telephone transpositions can be used whose type unbalance factor to non-transposed circuits exceeds a certain value. This eliminates a few of the types having the least numbers of transpositions.

Two-wire outside circuits can best be transposed in accordance with some of the typical arrangements for telephone circuit transpositions. A balance to these circuits requires then that certain types be set aside for their use and that the exposures between the transposed telephone circuits and the outside circuits transposed in accordance with these types be kept below the limiting values set for type unbalance for these cases.

The inherent complication in balancing the inductive effects between telephone circuits and three-phase power circuits arises, as has been noted, from the fact that a three-phase circuit has three different arrangements of conductors on the pin positions and a two wire telephone circuit has but two. There are, therefore, six combinations of relative pin positions of the two circuits, all to be used for equal lengths of line.

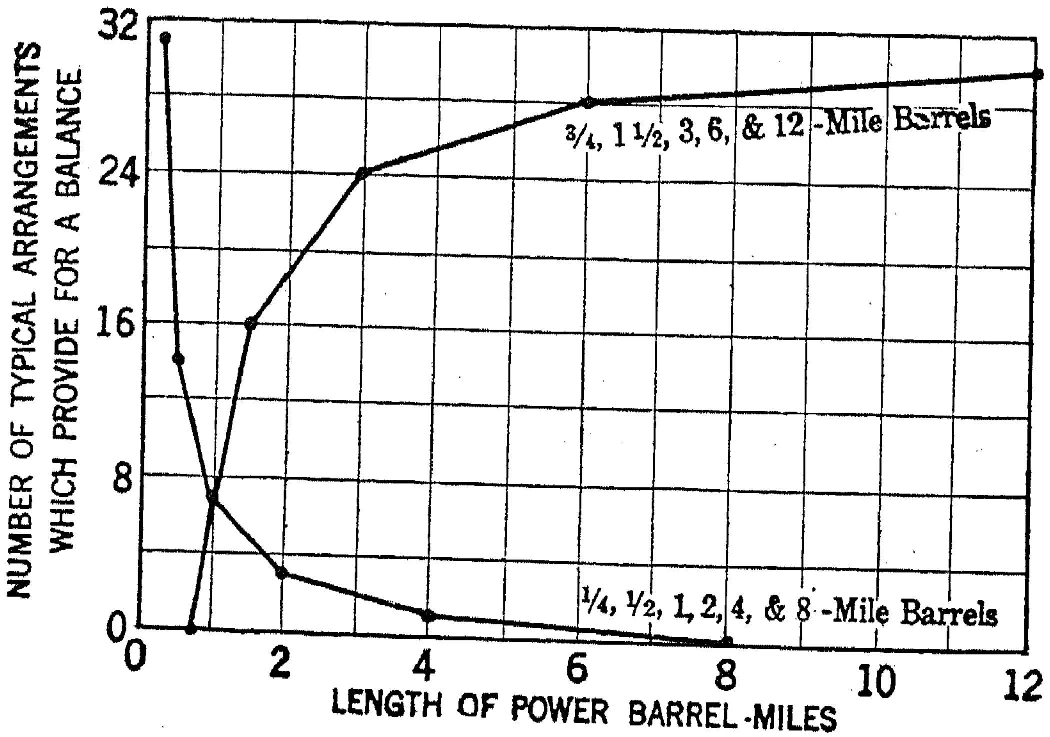


FIG. 3—NUMBER OF TYPICAL ARRANGEMENTS OF TELEPHONE TRANSPOSITIONS WHICH COORDINATE WITH THREE-PHASE POWER LINES TRANSPOSED WITH DIFFERENT LENGTHS OF BARREL

In considering the severe restrictions placed on the selection of telephone transposition types by the necessity of balancing to transposed three-phase power circuits, the following conditions may be noted:

1. If the barrels of the power line are very long, they can be made to coordinate with almost all of the typical arrangements by having each

non-transposed section of power circuit, that is, each third of a barrel, equal in length to one-half or one-fourth of the telephone transposition section, for most of the typical arrangements balance within this length. This is shown in Fig. 3, assuming 12.8 km. (8 miles) to be the length of the telephone transposition section. Thus with a 19.2-km. (12-mile) power barrel the telephone circuits may be balanced for each 6.4-km. (4-mile) non-transposed length of the power circuit. However, as the length of the power barrel is progressively decreased to 9.6 km. (6 miles), 4.8 km. (3 miles) 2.4 km. (1 ½ miles) etc., the number of typical arrangements of telephone transposition which can be used very rapidly decreases.

2. If the barrels of the power line are very short, they may be made to coordinate with all the typical arrangements by having a complete barrel between telephone transpositions. In the case shown in Fig. 3 this requires a 0.4-km. (¼ mile) barrel. As the length of barrel is increased, however, to 0.8 km. (½ mile) 1.6 km. or 3.2 km. (1 to 2 miles) etc., the number of typical arrangements of telephone transposition which can be used, very rapidly decreases.

3. Barrels in the power line of relative lengths other than those mentioned do not in general coordinate with telephone transposition arrangements.

As a chief requirement of the exposed-line system was that it should provide a maximum possible flexibility in the location of transpositions in parallel three-phase power circuits, this was made a matter of most careful study. The results obtained are outlined in the description of the system given in the next section of this paper.

Determination of Limiting Permissible Exposures. Before the attempt can be made to select, from among the typical arrangements of transpositions which can be used, a type for each telephone circuit on the lead, a table is made up of the limiting type unbalances which can be permitted between pairs of circuits, assuming that a given limit of the induced current between circuits cannot be exceeded. The procedure is as follows:

1. A computation is made of the total amount of unbalance which could be permitted between each combination of circuits on the lead, taken two at a time. This unbalance is expressed in terms of the length of untransposed exposure between the circuits which would give the limiting value of induced current, no current being induced in the rest of the circuit.

2. The total allowable unbalance for each pair of circuits is reduced by a figure representing the unbalance which must be expected to arise from irregularities with the type of construction to be employed.

3. The remaining unbalance represents the limiting permissible effect of type unbalance in a long length of line.

4. From data regarding the attenuation of currents along the telephone circuits, and from the total limiting effect of type unbalance, a com-

putation is made of the limiting type unbalance per transposition section. This sets a direct limit on the relative exposure type between each pair of circuits.

The Design of a Transposition System. With the information regarding the requirements of the proposed transposition system prepared as indicated above, the creation of a design which shall meet the requirements of maximum economy and efficiency is a problem requiring a great deal of skillful and conscientious application on the part of a trained expert. The problem is of course one of selecting from the typical arrangements which can be used, the best type for each circuit such that the type unbalance between no two circuits will exceed the established limit.

In order that a design may be complete it must provide for any number of crossarms on the telephone lead. However, it is generally desirable to take into consideration only four crossarms at the beginning of the design. Considering all the circuits, phantom and two-wire, which may be placed on the lead, taken in combinations of two there are about 2500 such combinations on a standard 40-wire pole line.

In carrying out the design of the exposed-line transposition system it was desired to make the requirements both of efficiency and economy as exacting as could possibly be met. These requirements were set, therefore, at the beginning of the work, so high that no design could meet them, and then gradually reduced until it was possible to complete the design. As the design depends on a number of interdependent requirements, this process not only requires a very large amount of work but demands the continual exercise of trained judgment on the part of the designer to bring about the mutual adjustment of requirements so as to produce the best final result.

The process of determining whether or not a design can be established to meet a given set of requirements is very technical and will not be described here in detail. It involves a systematic consideration of all the possible arrangements. These possibilities are very great in number—about seven million in the case of the *E* section design—and are, of course, not considered individually, but in groups, and successive groups of possibilities are rejected when it has been proved that they could not meet the requirements.

The systematic grouping of the possibilities, and the means adopted for the examination and elimination of the groups depends a great deal upon the experience and skill of the designer. The general method which has been successfully employed can be roughly described as a consideration of the design in steps. First the attempt is made to carry through a preliminary design in which pairs of circuits which cannot be exposed to each other untransposed for the entire length of the transposition section (P exposures) are provided with different typical arrangements of transpositions, but in which no other requirements are considered. If this design is possible, the other balance limitations of the types selected are added, one by one.

When a design is finally reached it is usually found that any one of a considerable number of arrangements will meet the balance requirements almost equally well. The most economical of these is chosen for use. In determining upon the relative economy of different designs, account is taken of the extent to which circuits on different pin positions will exist in the telephone plant and the extent to which it might be possible, in retransposing existing lines, to use the present transpositions as a part of the transpositions called for in accordance with the new system.

EXPOSED-LINE TRANSPOSITION SYSTEM

Special Electrical Requirements. The exposed-line transposition system was designed to meet the following special electrical requirements:

1. The greatest practicable flexibility for the arrangement of transpositions in power circuits paralleling the telephone circuits, the power circuit transpositions co-ordinating with those in the telephone circuits so as to produce a balance of the inductive effects. This requirement to be met for transpositions both in three-phase circuits and in single-phase circuits.

2. A suitable balance against the inductive effects of telegraph circuits or other ground return circuits paralleling the telephone circuits.

3. A degree of freedom from interference between the telephone circuits (crosstalk) suitable for modern high grade telephone circuits used in connection with loading and with telephone repeaters.

General Requirements. In addition to these special requirements, it must meet the following requirements for any satisfactory transposition system:

1. *Neutral Points.* The system must be arranged so that neutral points, that is, points at which a maximum degree of balance is obtained between all circuits on the telephone line are

provided where there are discontinuities in line construction or in the parallel. In the case of open wire toll lines of the Bell telephone system, there must be neutral points at regular intervals of about 12.8 km. (actually 7.88 miles). In addition there must be neutral points wherever there is an important junction between telephone lines and at other points of discontinuity. It is, therefore, desirable that the system include a design intended for a maximum length of 12.8 km. and another design or designs for shorter maximum distances.

2. *Cost.* The completed section must be designed for as low a cost as practicable without sacrifice of the electrical requirements. This means that as far as other conditions of design permit, the typical arrangements having the least numbers of transpositions should be placed on the circuits most often used. In general, it is desirable that phantom circuits have fewer transpositions than two-wire circuits as phantom circuit transpositions are the more expensive.

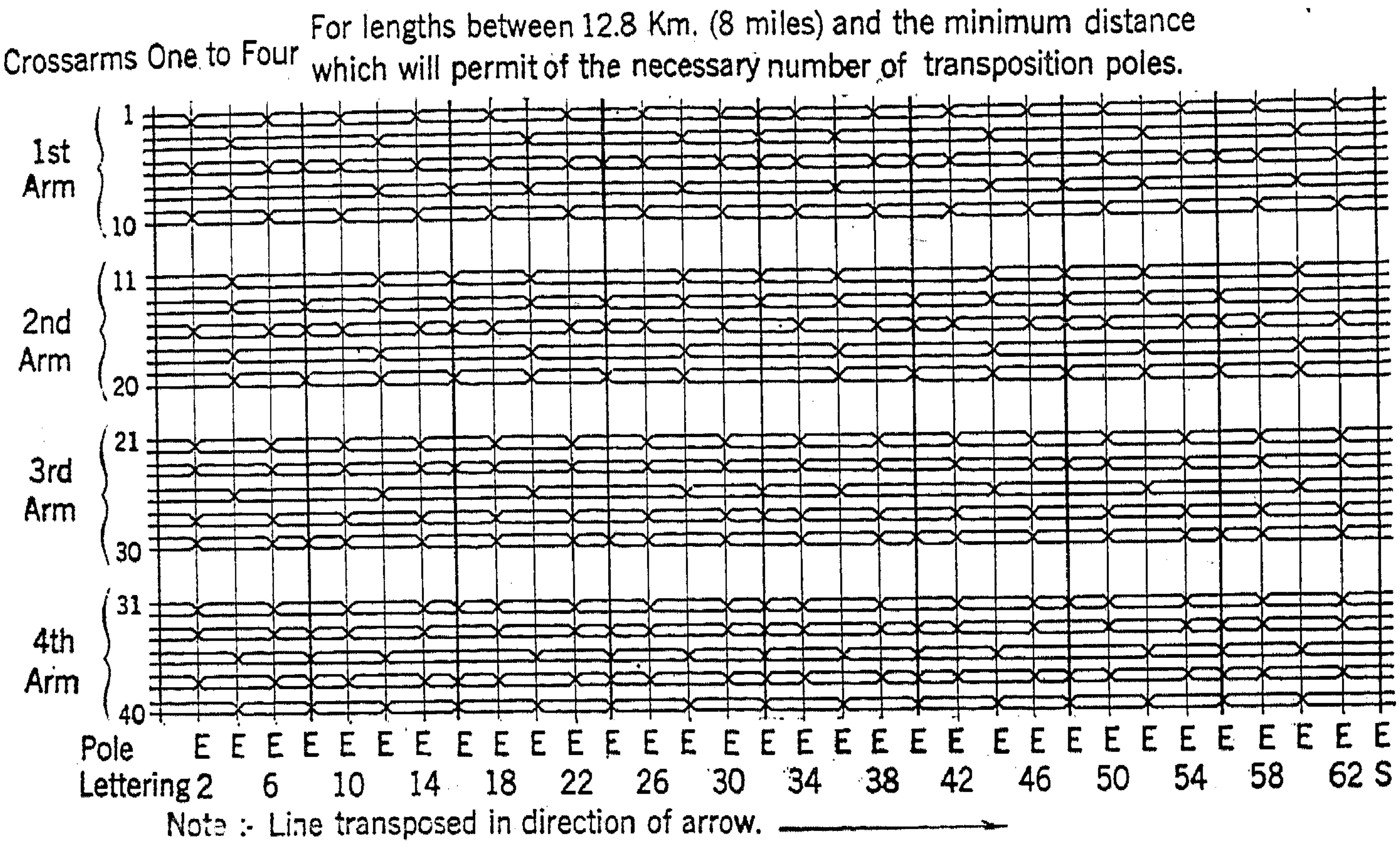

Crossarms One to Four
For lengths between 12.8 Km. (8 miles) and the minimum distance which will permit of the necessary number of transposition poles.
1st Arm
1
10
2nd Arm
11
20
3rd Arm
21
30
4th Arm
31
40
Pole Lettering
E E E E E E E E E E E E E E E E E E E E E E E E E E E E E E E E
2 6 10 14 18 22 26 30 34 38 42 46 50 54 58 62 S
Note :- Line transposed in direction of arrow.

FIG. 4—*E* SECTION—TRANSPOSITION SYSTEM FOR NON-PHANTOMED CIRCUITS ON EXPOSED TELEPHONE LINES

3. *Transposition Poles.* It is desirable that the transposition poles be as few in number as practicable not only on account of expense but because the use of large numbers of transposition poles limits the minimum length of a transposition section. The use of many poles may also tend to increase the induction between telephone circuits due to irregularities.

E Section and L Section. As a result of the study, two designs have been completed which are called respectively the *E* section and the *L* section. Diagrams showing transpositions according to the *E* section for crossarms 1 to 8 of a pole line are shown in Figs. 4 to 9, inclusive, and according to the *L* section in Figs. 10 to 15, inclusive. It will be noted by examination of the figures that the designs provide for the phantoming of any two horizontally adjacent pairs or of any two vertically adjacent pole pairs on the lead. In carrying out the designs provision has been made for extension if necessary to any number of crossarms.

The *E* section is designed for a maximum length of approximately 12.8 km. It was found impossible to meet the electrical requirements for this case with the use of 32 transposition poles per section. It was possible, however, to design the *E* section so that in the first four crossarms the non-phantom circuits and the phantom circuits which are most commonly used require only 32 transposition poles, so that in some toll lines not exceeding four crossarms, the larger number of transposition poles will not be necessary. For circuits on crossarms below the fourth and for some infrequently used phantomed circuits on the upper crossarms, 64 transposition poles are required.

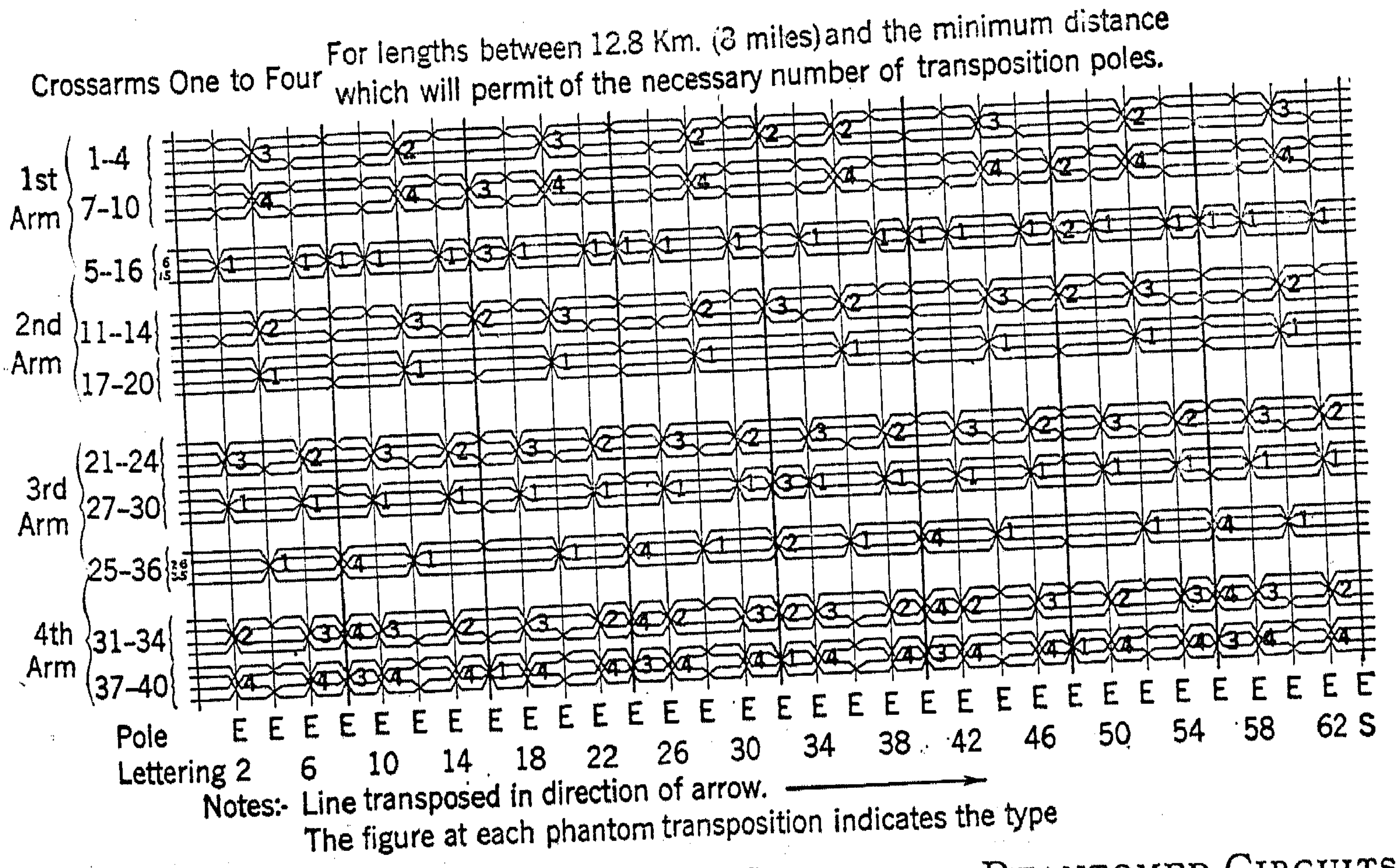

Crossarms One to Four
For lengths between 12.8 Km. (8 miles) and the minimum distance which will permit of the necessary number of transposition poles.
1st Arm
1-4
7-10
5-16
2nd Arm
11-14
17-20
3rd Arm
21-24
27-30
25-36
4th Arm
31-34
37-40
Pole Lettering
E E E E E E E E E E E E E E E E E E E E E E E E E E E E E E E S
2 6 10 14 18 22 26 30 34 38 42 46 50 54 58 62
Notes:- Line transposed in direction of arrow.
The figure at each phantom transposition indicates the type

FIG. 5—*E* SECTION—TRANSPOSITION SYSTEM FOR PHANTOMED CIRCUITS ON EXPOSED TELEPHONE LINES—ARRANGEMENTS FOR PHANTOMING ALL CIRCUITS

The *E* section may be used for as short a length of line as supplies the required number of transposition poles. For distances less than 6.4 km., however, it is economical to use the

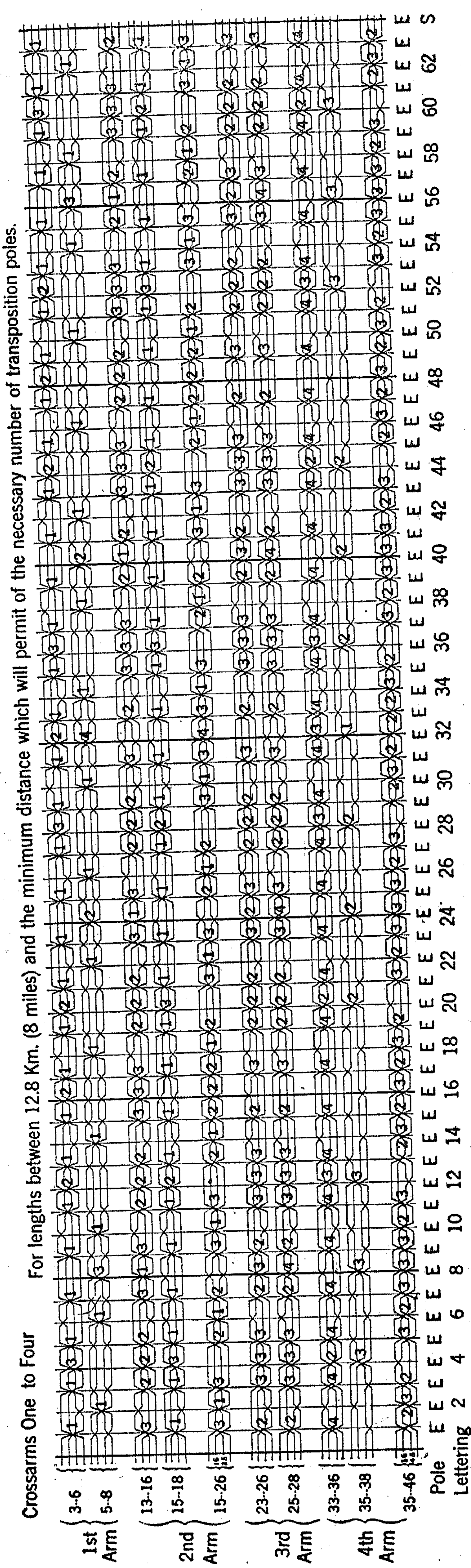
L section. It has just half the number of transposition poles for the corresponding circuits.

When two or more *L* sections are used consecutively in the same loading section, junction transpositions should be installed at the *S* poles between them in order to reduce the induction between the telephone circuits. These junction transpositions are indicated in the drawings for as many as four consecutive *L* sections.

Use of E and L Sections on Exposed Lines. The results obtained in meeting the special electrical requirements regarding balance to the inductive effects of outside circuits are indicated in the following table:

	E Section	*L Section*
1. Element of section in which each circuit is transposed at least once, *i. e.* balances to a parallel non-transposed disturbing circuit.	8th Section	Quarter-Section
2. Neutral points suitable for the location of transpositions in paralleling three-phase power circuits, separation of approximate centers of disturbing and disturbed groups of wires *more* than 6 meters.	8th Section points; *i. e.*, barrels having nominal length of 4.8 km. (3 miles) or 9.6 km. (6 miles) can be used. Five points dividing distance between any two 8th-section points into six equal parts, all five transpositions to be rotated in the same direction; *i. e.*, nominal 0.8 km. (0.5 mile) barrels can be used also.	Quarter-section points; *i.e.* nominal 4.8 km. (3 mile) barrels can be used.
3. Neutral points suitable for the location of transpositions in paralleling metallic single-phase power circuits—separation as defined above *more* than 6 meters.	8th-section points. 3 points dividing distance between any two 8th-section points into four equal parts.	Quarter-section points.
4. Neutral points suitable for the occurrence of discontinuities—separation as defined above *more* than six meters.	8th-section points	Quarter-section points
5. Neutral points suitable for the location of transpositions in disturbing power circuits or the occurrence of discontinuities in disturbing power or telegraph circuits—separation as defined above *less* than 6 meters.	Mid-section point	Mid-section point
6. Neutral points suitable for the occurrence of discontinuities in telephone circuits.	S-poles	S-poles

The results given in the table are further illustrated in Figs. 16 and 17 which show premissible relative locations of the tele-



For lengths between 12.8 Km. (8 miles) and the minimum distance which will permit of the necessary number of transposition poles.

Notes:- The figure at each phantom transposition indicates the type
Line transposed in direction of arrow. ⟶

Fig. 6—*E* Section—Transposition System for Phantomed Circuits on Exposed Telephone Lines—Arrangements for Horizontal Pole Pair Phantoms and Odd Vertical Phantoms

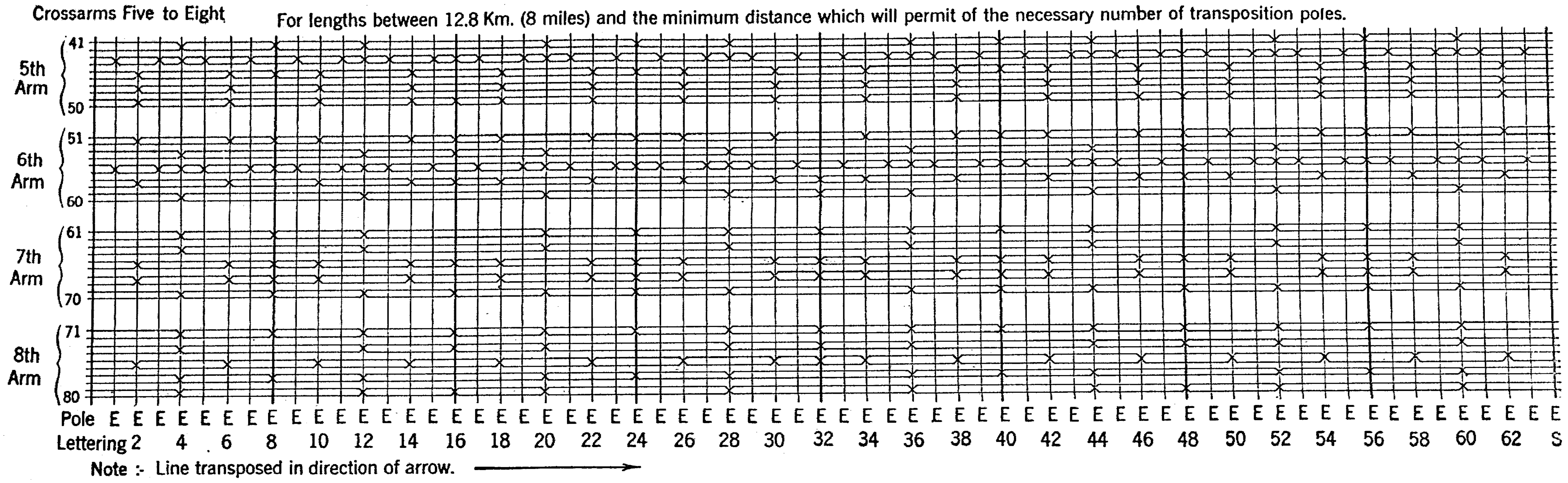
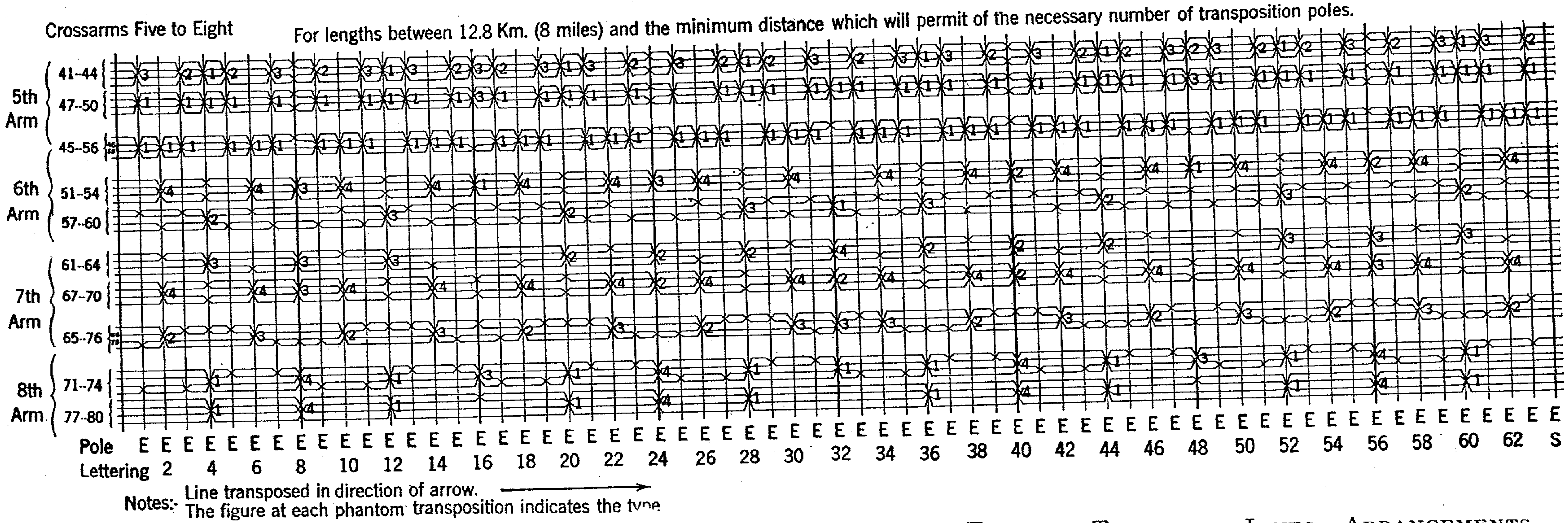

Crossarms Five to Eight
For lengths between 12.8 Km. (8 miles) and the minimum distance which will permit of the necessary number of transposition poles.
5th Arm
41
50
6th Arm
51
60
7th Arm
61
70
8th Arm
71
80
Pole E E E E E E E E E E E E E E E E E E E E E E E E E E E E E E E E E E E E E E E E E E E E E E E E E E E E E E E E E E E E E E E E
Lettering 2 4 6 8 10 12 14 16 18 20 22 24 26 28 30 32 34 36 38 40 42 44 46 48 50 52 54 56 58 60 62 S
Note :- Line transposed in direction of arrow.

FIG. 7—*E* SECTION—TRANSPOSITION SYSTEM FOR NON-PHANTOMED CIRCUITS ON EXPOSED TELEPHONE LINES



Crossarms Five to Eight

For lengths between 12.8 Km. (8 miles) and the minimum distance which will permit of the necessary number of transposition poles.

Notes:- Line transposed in direction of arrow. ⟶
The figure at each phantom transposition indicates the tvne

Fig. 8—*E* Section—Transposition System for Phantom Circuits on Exposed Telephone Lines—Arrangements for Phantoming all Circuits

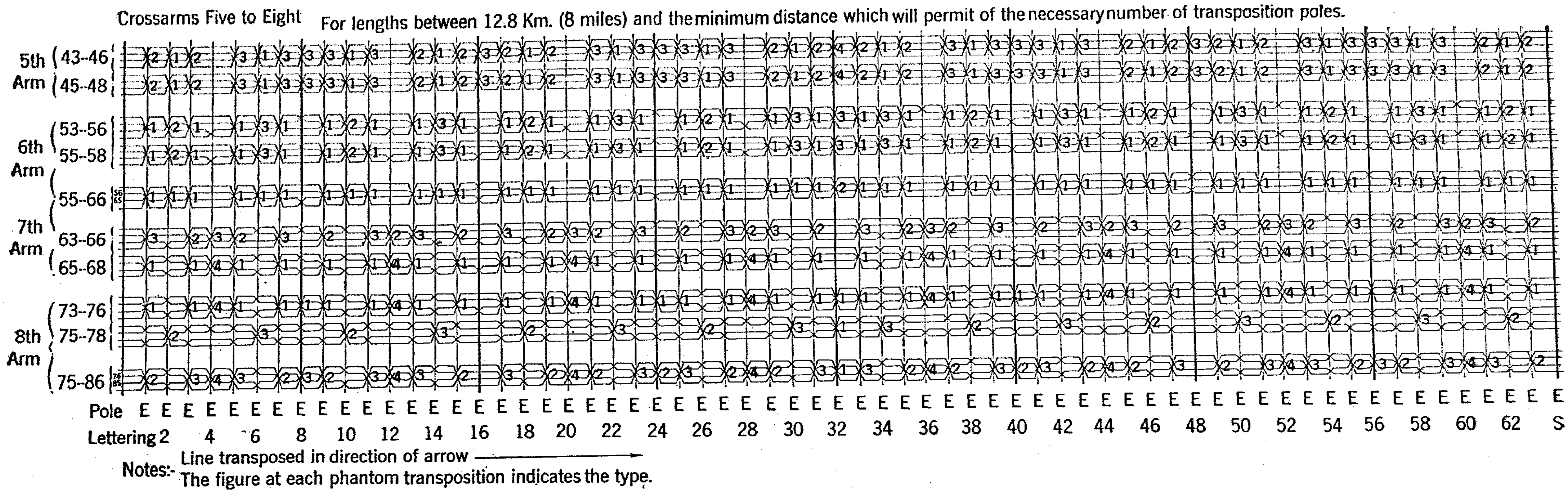

Crossarms Five to Eight
For lengths between 12.8 Km. (8 miles) and the minimum distance which will permit of the necessary number of transposition poles.
5th Arm
43-46
45-48
6th Arm
53-56
55-58
55-66
7th Arm
63-66
65-68
8th Arm
73-76
75-78
75-86
Pole E E E E E E E E E E E E E E E E E E E E E E E E E E E E E E E E E E E E E E E E E E E E E E E E E E E E E E E E E E E E E E E
Lettering 2 4 6 8 10 12 14 16 18 20 22 24 26 28 30 32 34 36 38 40 42 44 46 48 50 52 54 56 58 60 62 S
Notes:- Line transposed in direction of arrow
The figure at each phantom transposition indicates the type.

FIG. 9—*E* SECTION—TRANSPOSITION SYSTEM FOR PHANTOMED CIRCUITS ON EXPOSED TELEPHONE LINES—ARRANGEMENTS FOR HORIZONTAL POLE PAIR PHANTOMS AND ODD VERTICAL PHANTOMS

phone transposition poles and of transpositions and discontinuities in the outside parallel circuits or changes in separation between the two classes of circuit.

It will be noted that the *E* section can be used to coordinate with transpositions in three-phase power circuits which create for the maximum length of *E* section 0.8 km. (½ mile), 4.8 km. 3 miles) or 9.6 km. (6 miles) barrels in the power circuit. As it was considered very important to obtain the maximum practicable degree of flexibility in this respect, this matter was given very thorough study and a large amount of work was required to arrive at this result. It should be noted, moreover, that in the use of a 0.8-km. barrel, two barrels occupying a 1.6-km. section of the transposition system are required to obtain a balanced condition.

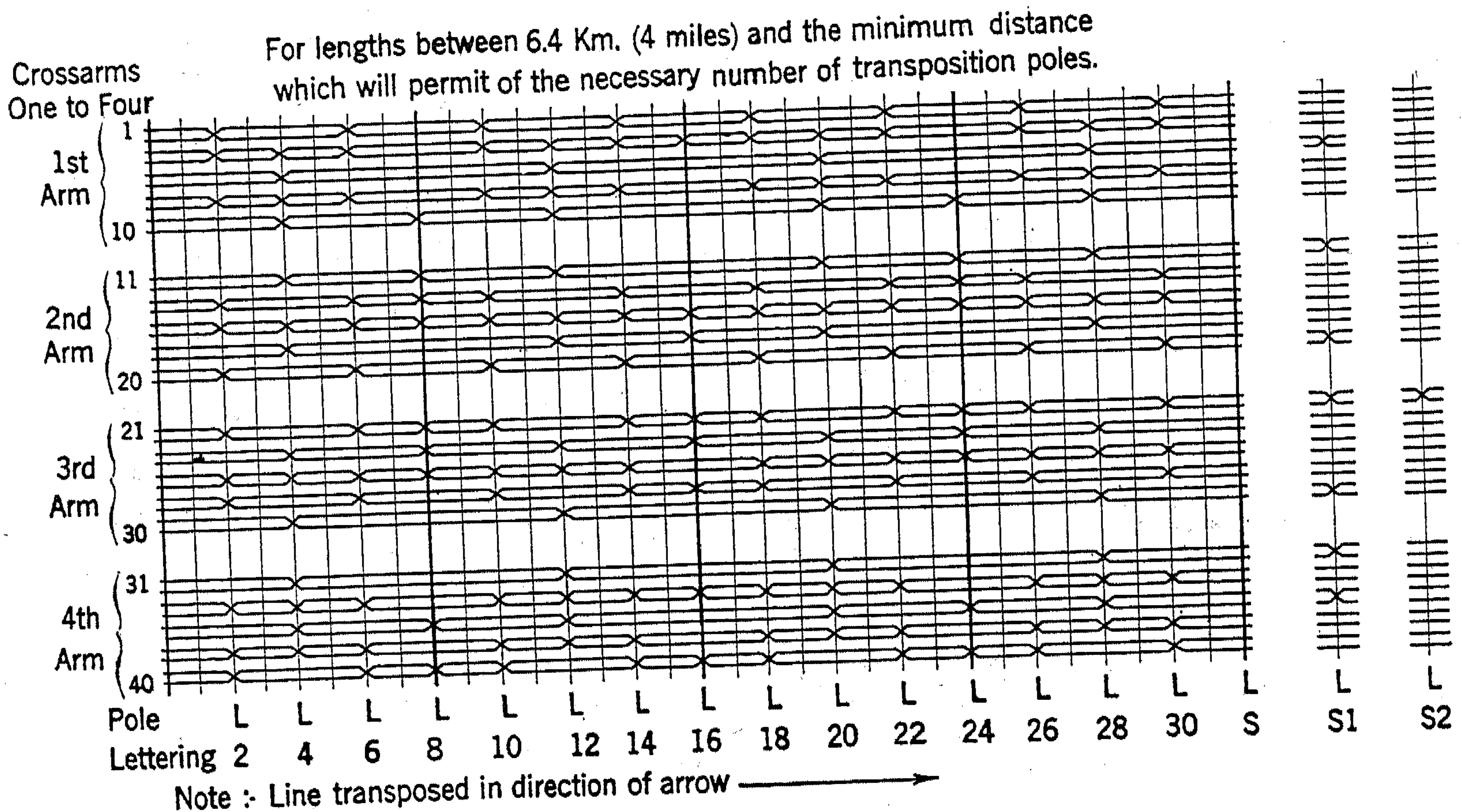
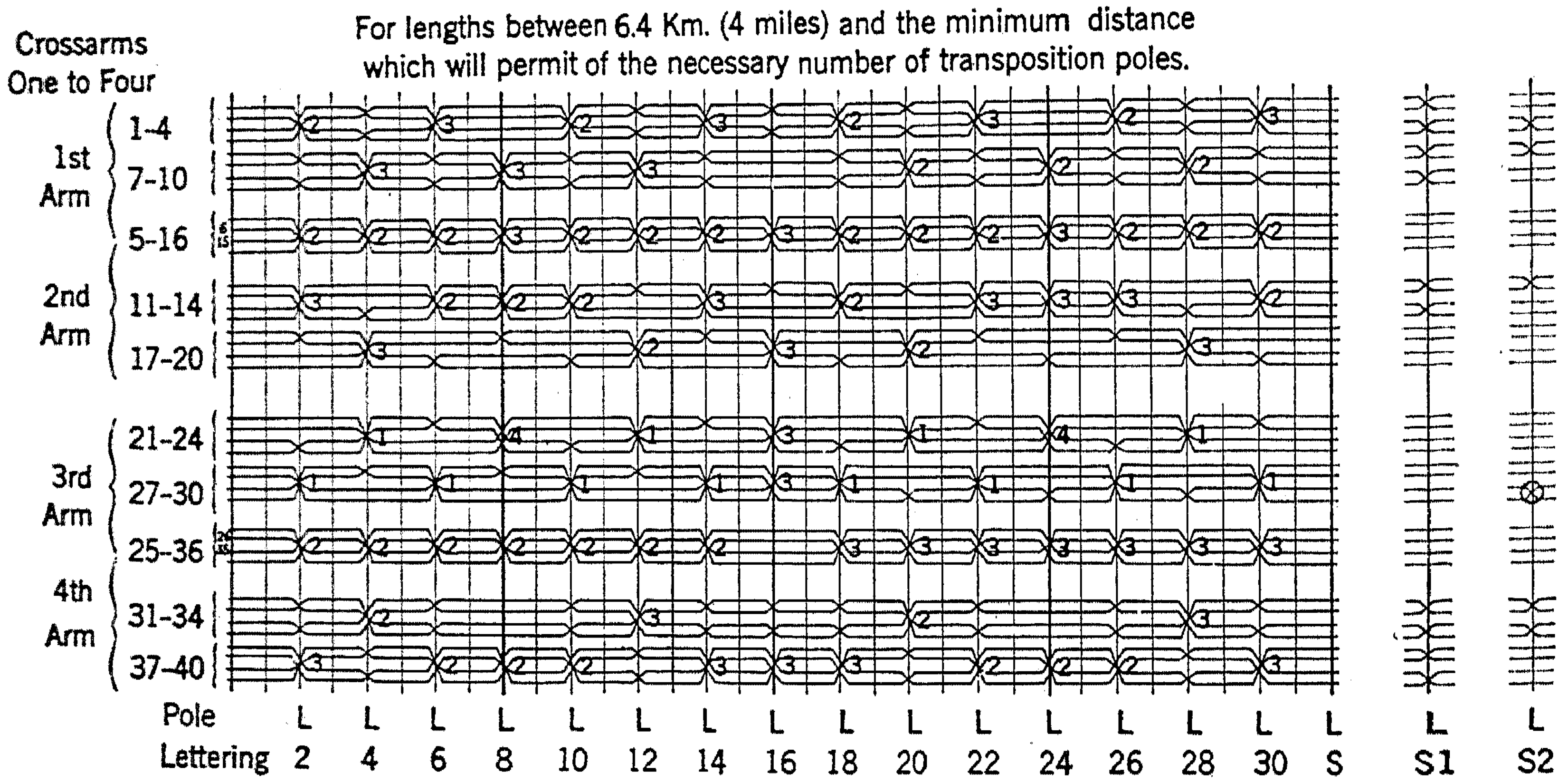

For lengths between 6.4 Km. (4 miles) and the minimum distance which will permit of the necessary number of transposition poles.
Crossarms One to Four
1st Arm
1
10
2nd Arm
11
20
3rd Arm
21
30
4th Arm
31
40
Pole Lettering
L 2
L 4
L 6
L 8
L 10
L 12
L 14
L 16
L 18
L 20
L 22
L 24
L 26
L 28
L 30
L S
L S1
L S2
Note :- Line transposed in direction of arrow

Fig. 10—*L* Section—Transposition System for Non-Phantomed Circuits on Exposed Telephone Lines

In the *L* section the requirements for the avoidance of excessive interference between the telephone circuits are more exacting than in the *E* section because the use of a smaller number of transposition poles makes available a correspondingly smaller number of typical arrangements of transpositions. It was found possible, however, to design this section so that it balances to a three-phase power line transposed at the quarter section points.

The exposed-line transposition system has been designed primarily for use on toll lines and not for use on joint lines with power distribution circuits. It is, however, somewhat better for this case than the other transposition systems in use.

When large inductive effects are experienced from circuits less than 20 ft. (6 m.) away from a side circuit, the variation in separation between the telephone circuit and the exposing circuits as the side circuits shifts from one pair of pin positions to another must be taken into consideration. The *E* section is not designed to give a high degree of balance under this condition for all circuits in each eighth-section, but does give balance to uniform outside induction effects for most circuits in each eighth-section and with one exception, for all circuits in each half-section. Under similar conditions the *L* section gives a balance for most circuits in each quarter section and for all circuits in each half section.



Notes: Line transposed in direction of arrow. ⟶

The figure at each phantom transposition indicates the type.

Transposition marked ⊗ to be cut in only when 3 consecutive L's are used.

FIG. 11—*L* SECTION—TRANSPOSITION SYSTEM FOR PHANTOMED CIRCUIT ON EXPOSED TELEPHONE LINES—ARRANGEMENTS FOR PHANTOMING ALL CIRCUITS

It should be clearly pointed out that in the use of the telephone transposition sections to coordinate with transpositions in power circuits, it is important that the transpositions in the two lines be coordinated both as regards relative length and as regards relative location of telephone transposition section and transpositions in the power circuits. The locations of the transpositions in either line must, therefore, be made with reference to the requirements of the other. It is frequently the case that the locations of the transpositions in the power circuit are to a large degree determined by the changes in separation and other characteristics of the parallel.

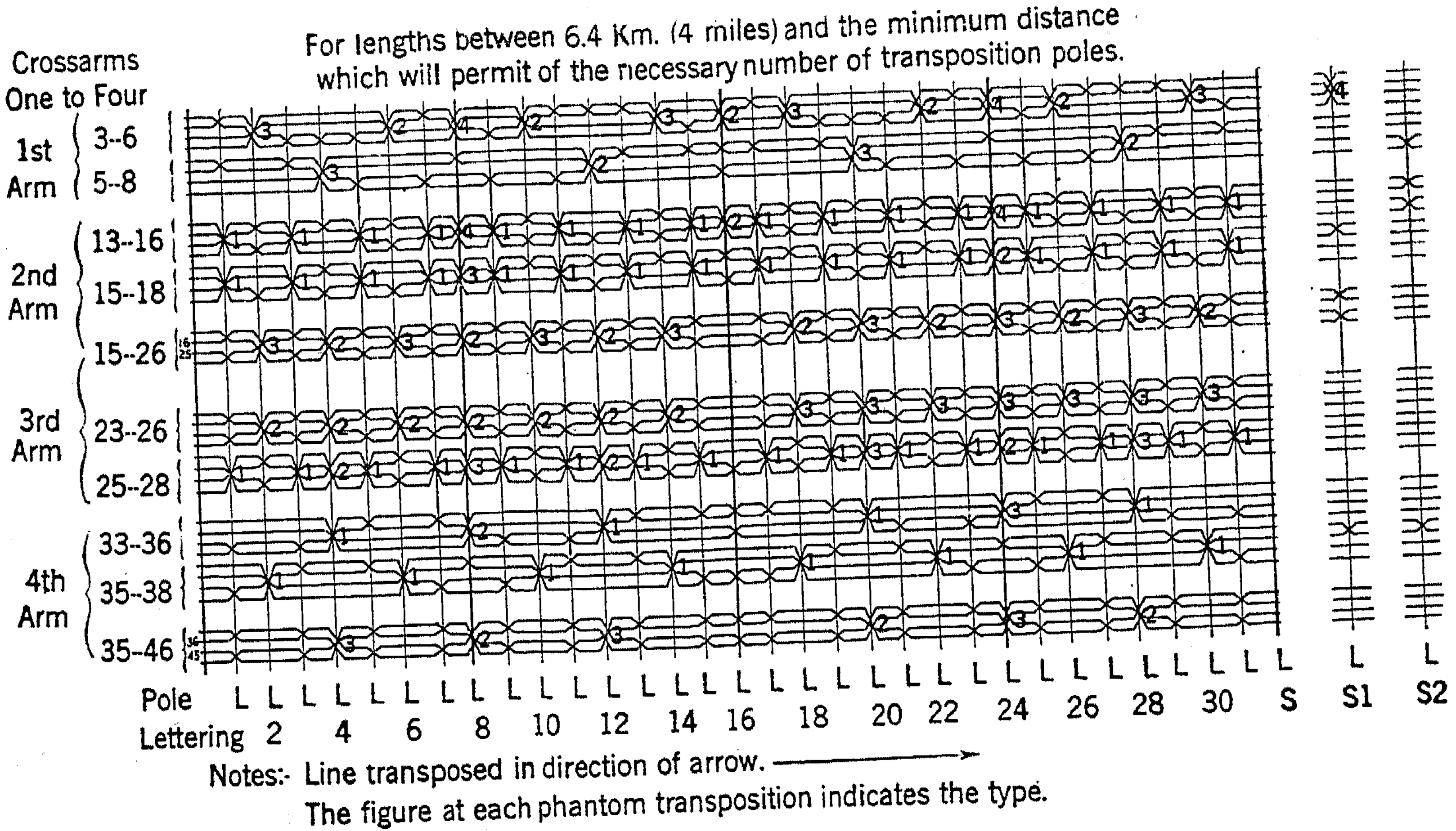

For lengths between 6.4 Km. (4 miles) and the minimum distance which will permit of the necessary number of transposition poles.
Crossarms One to Four
1st Arm
3-6
5-8
2nd Arm
13-16
15-18
15-26
3rd Arm
23-26
25-28
4th Arm
33-36
35-38
35-46
Pole Lettering
L 2 4 6 8 10 12 14 16 18 20 22 24 26 28 30 S
L S1
L S2
Notes:- Line transposed in direction of arrow.
The figure at each phantom transposition indicates the type.

FIG. 12—*L* SECTION—TRANSPOSITION SYSTEM FOR PHANTOMED CIRCUITS ON EXPOSED TELEPHONE LINES—ARRANGEMENTS FOR HORIZONTAL POLE PAIR PHANTOMS AND ODD VERTICAL PHANTOMS

The *E* and the *L* section are both designed to give a high degree of balance of the inductive effects between the telephone circuits. In using them, therefore, it is important that the transposition poles be spaced as accurately as practicable in

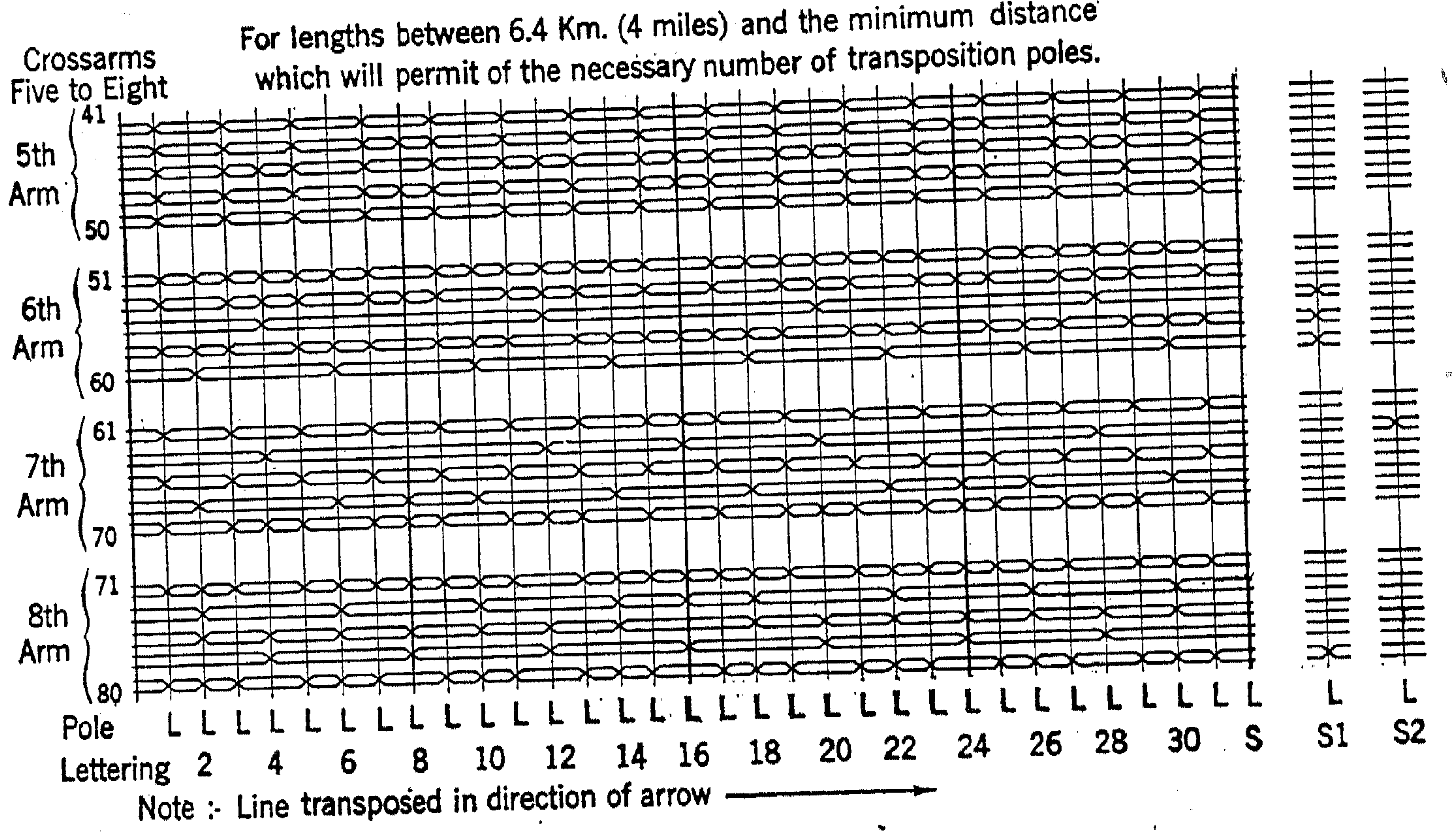
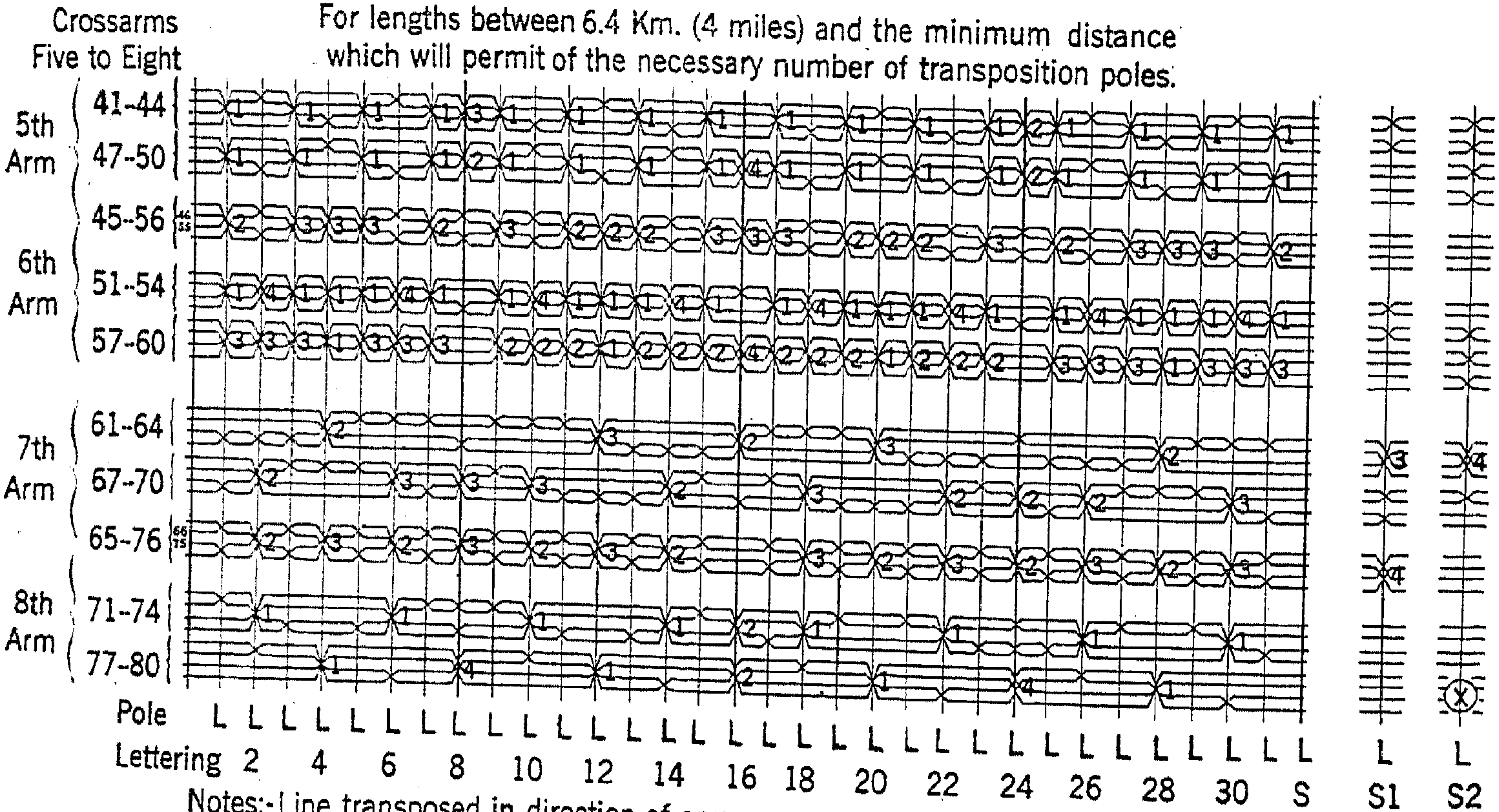

For lengths between 6.4 Km. (4 miles) and the minimum distance which will permit of the necessary number of transposition poles.
Crossarms Five to Eight
5th Arm
41
50
6th Arm
51
60
7th Arm
61
70
8th Arm
71
80
Pole Lettering
L 2 4 6 8 10 12 14 16 18 20 22 24 26 28 30 S
L S1
L S2
Note :- Line transposed in direction of arrow

FIG. 13—*L* SECTION—TRANSPOSITION SYSTEM FOR NON-PHANTOMED CIRCUITS ON EXPOSED TELEPHONE LINES

order to avoid diminishing the effectiveness of the system in this respect because of irregularities.

The design work led to a large number of possible arrangements of approximately equal electrical efficiency. In choosing from these the most economical, account was taken both of the cost of transposing when stringing new circuits and of the cost of retransposing to the new system telephone lines now transposed according to other arrangements standard with the Bell Telephone Companies. Although a most careful study was made of economy it was necessary because of the special characteristics required in this system to allow in the final sections a cost of telephone transpositions 25 to 50 per cent greater than that of the section which is standard in the Bell Telephone System for use on non-exposed lines.



For lengths between 6.4 Km. (4 miles) and the minimum distance which will permit of the necessary number of transposition poles.

Notes:-Line transposed in direction of arrow. ⟶

The figure at each phantom transposition indicates the type.

Phantom transposition marked ⊠ is a No. 3 type with 3 L's and a No. 4 type with 4 L's.

FIG. 14—*L* SECTION—TRANSPOSITION SYSTEM FOR PHANTOMED CIRCUITS ON EXPOSED TELEPHONE LINES—ARRANGEMENTS FOR PHANTOMING ALL CIRCUITS

CONCLUSION

The results obtained in the design of the *E* and *L* sections were possible only through a systematic study of the problem. The work directed particularly to the design of these two sections occupied about two man-years' time of the specially trained experts who carried out the work. The investigation has been so thorough that the conclusion seems warranted that the *E* section and the *L* section are substantially the best arrangement of transpositions which could be devised to meet the requirements established for them.

The work of many members of the engineering department of the American Telephone and Telegraph Company has contributed to the results. The theory of induction between telephone circuits and the special analytical methods of design have been developed in the department during a considerable number of years. Special mention should be made of the work of Mr. G. A. Campbell who, years ago, laid the foundations for the theory of induction between telephone circuits, and of Messrs. A. G. Chapman and R. E. Leonard, who perfected the methods of systematic search and carried out the design of the *E* and *L* sections.

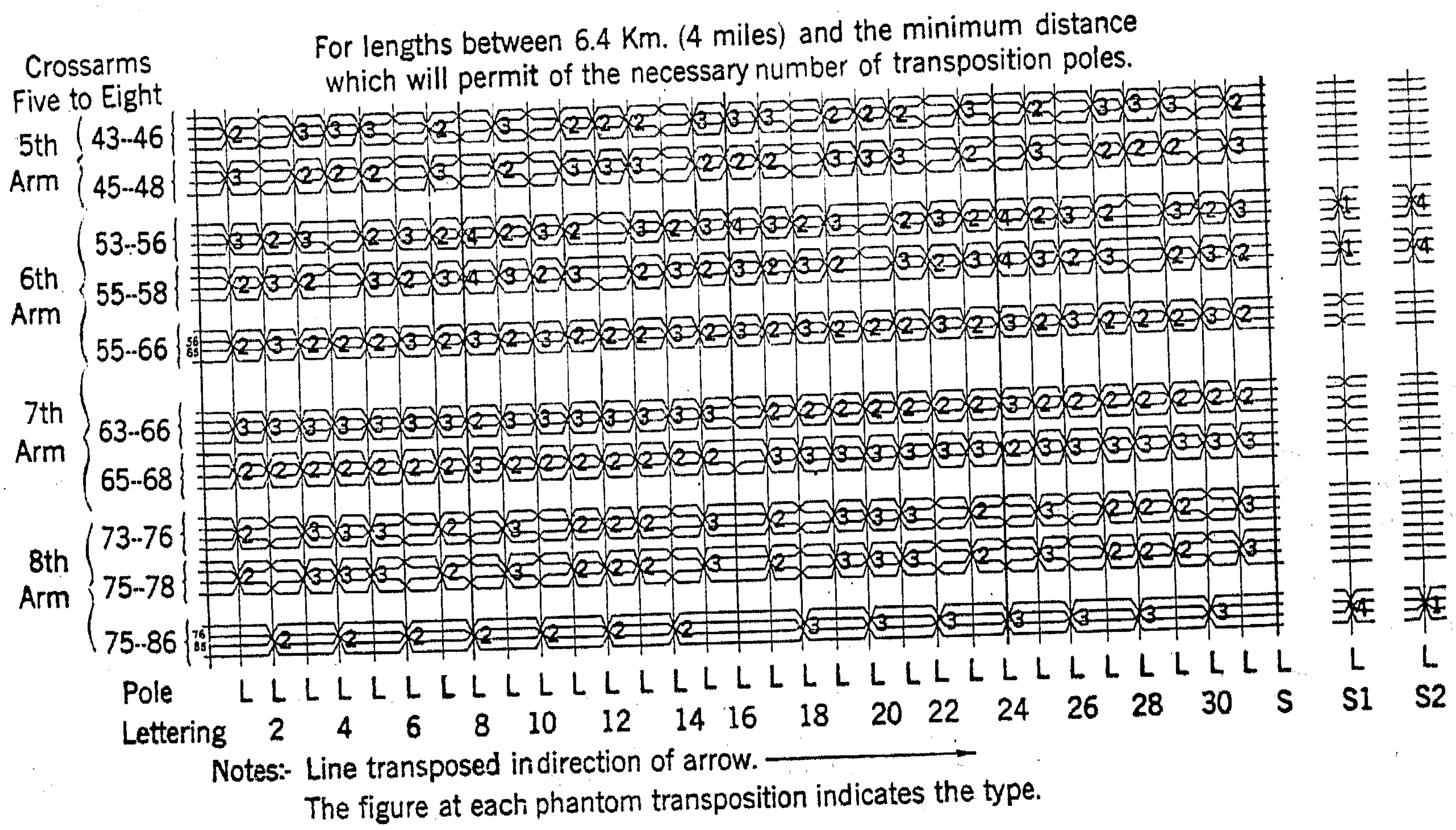

For lengths between 6.4 Km. (4 miles) and the minimum distance which will permit of the necessary number of transposition poles.
Crossarms Five to Eight
5th Arm
43-46
45-48
6th Arm
53-56
55-58
55-66
7th Arm
63-66
65-68
8th Arm
73-76
75-78
75-86
Pole Lettering
L 2 4 6 8 10 12 14 16 18 20 22 24 26 28 30 S
L S1
L S2
Notes:- Line transposed in direction of arrow.
The figure at each phantom transposition indicates the type.

FIG. 15—*L* SECTION—TRANSPOSITION SYSTEM FOR PHANTOMED CIRCUITS ON EXPOSED TELEPHONE LINES—ARRANGEMENTS FOR HORIZONTAL POLE PAIR PHANTOMS AND ODD VERTICAL PHANTOMS

APPENDIX A. INDUCTION BETWEEN PARALLEL WIRES

1. *Two Wires with Ground Return—Short Length of Exposure.* If the current and voltage in a short element of circuit No. 1 (Fig. 18) are represented by I and E, the inductive effect which this element of circuit No. 1 exercises on parallel circuit No. 2 can be represented as follows:

The voltage induced in circuit No. 2 because of current I is

$$V_2 = -j\,\omega\,I\,M\,l \qquad (1)$$

where ω equals 2π times the frequency, l the length of element of circuit considered and $M\omega$ the mutual impedance between the circuits per unit length. In the case of a perfectly conducting earth surface, M is made up entirely of the mutual inductance caused by the linkage of flux from circuit No. 1 with circuit No. 2. With a perfectly conducting earth surface this mutual inductance can be readily computed by the method of images. Under actual conditions the mutual inductance is affected by the resistance of the earth, which increases the linkages between the two grounded circuits because it causes the return current from circuit No. 1 to distribute more or less widely throughout the earth rather than to flow in a current sheet on the earth surface. In many cases, also, the fact that the two circuits both use the earth return having a finite resistance introduces an appreciable resistance term into the mutual impedance. Sometimes this term is very important.

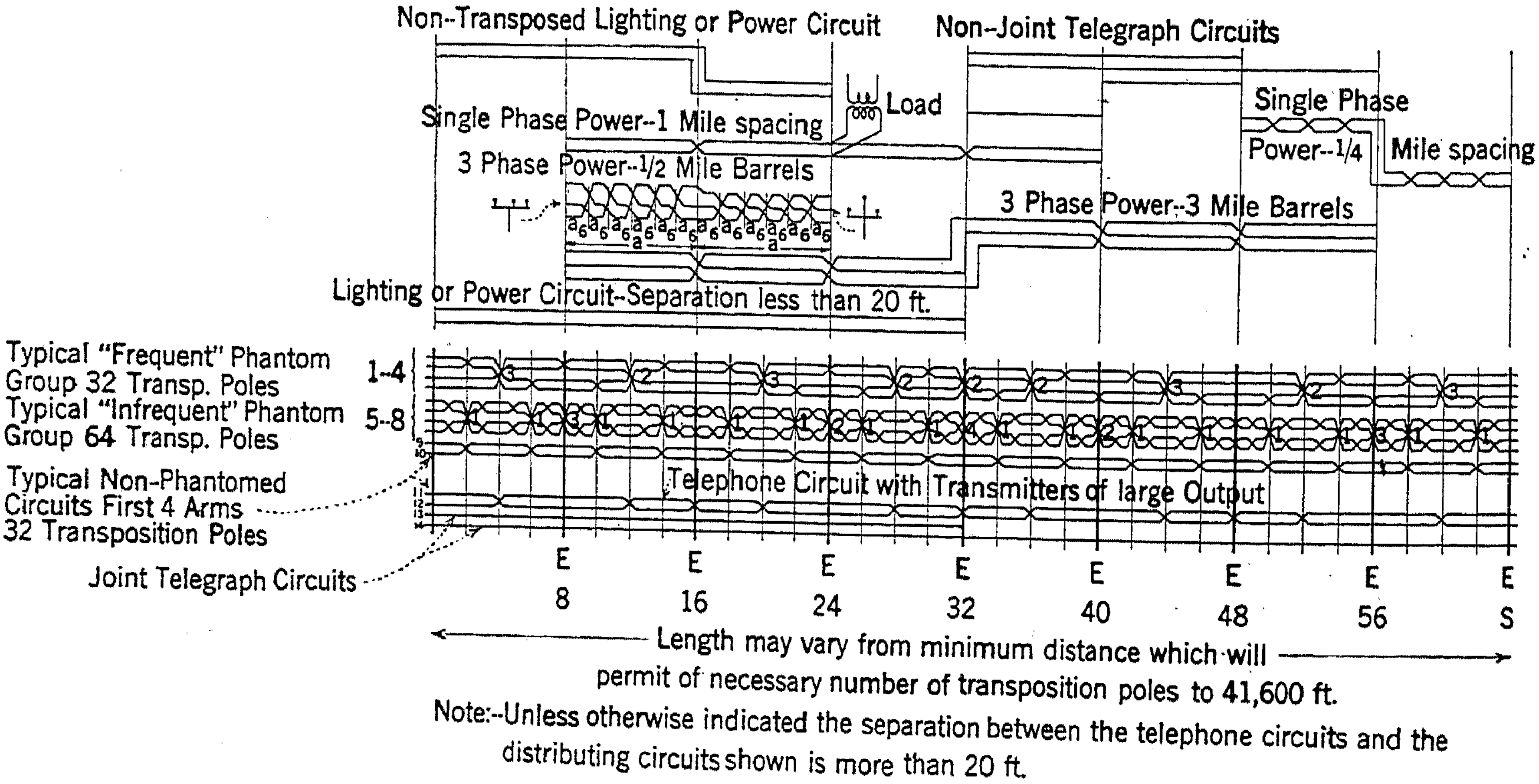

Non-Transposed Lighting or Power Circuit
Non-Joint Telegraph Circuits
Single Phase Power-1 Mile spacing
Load
Single Phase Power-1/4 Mile spacing
3 Phase Power-1/2 Mile Barrels
3 Phase Power-3 Mile Barrels
Lighting or Power Circuit-Separation less than 20 ft.
Typical "Frequent" Phantom Group 32 Transp. Poles 1-4
Typical "Infrequent" Phantom Group 64 Transp. Poles 5-8
Typical Non-Phantomed Circuits First 4 Arms 32 Transposition Poles
Joint Telegraph Circuits
Telephone Circuit with Transmitters of large Output
E 8
E 16
E 24
E 32
E 40
E 48
E 56
E S
Length may vary from minimum distance which will permit of necessary number of transposition poles to 41,600 ft.
Note:-Unless otherwise indicated the separation between the telephone circuits and the distributing circuits shown is more than 20 ft.

FIG. 16—CHARACTERISTICS OF EXPOSED LINE *E* SECTION —DIAGRAM ILLUSTRATES LENGTH, NUMBER OF TRANSMISSION POLES AND PERMISSIBLE LOCATIONS OF TRANSPOSITIONS AND DISCONTINUITIES IN EXTRANEOUS DISTURBING CIRCUITS

The voltage induced in a short section of wire 2 because of the voltage between wire 1 and ground is

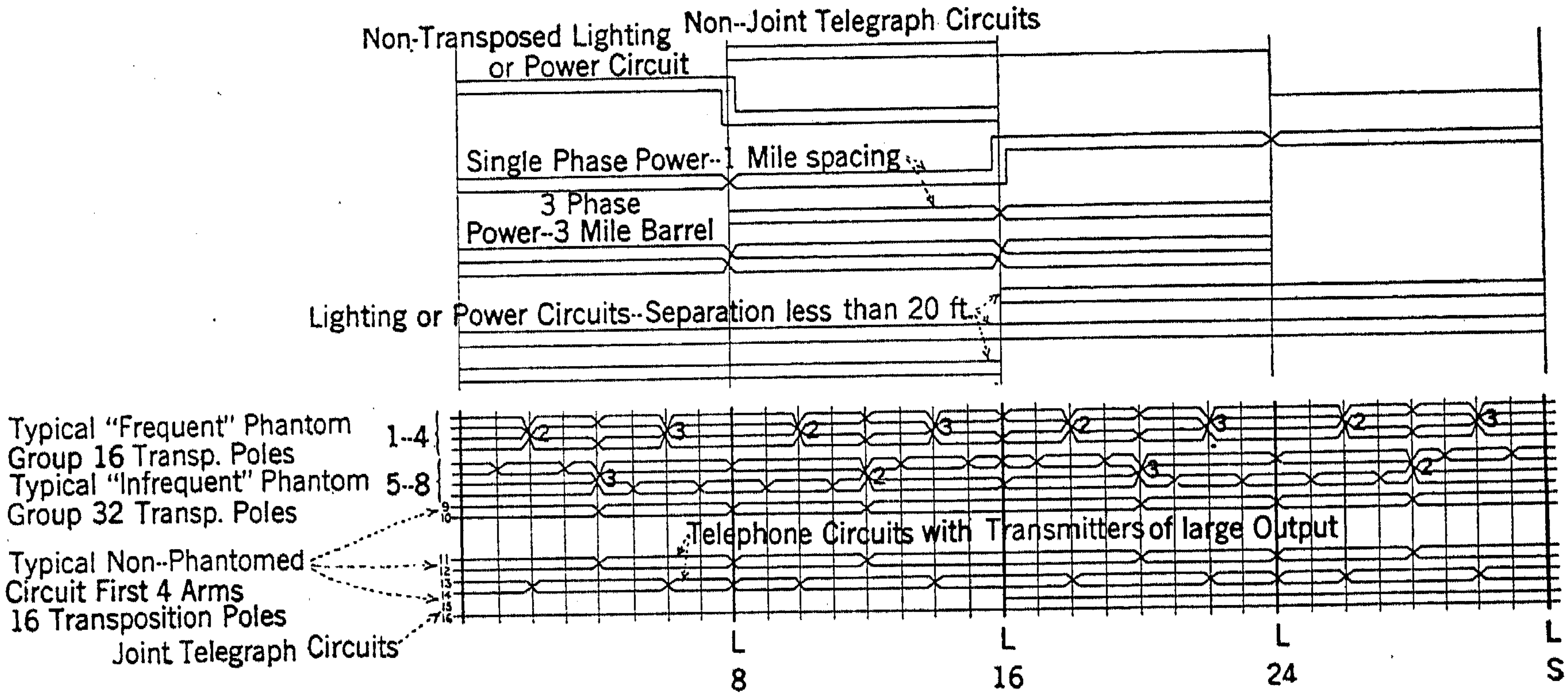
$$e_2 = E\frac{C_M}{C_2 + C_M} \tag{2}$$

In this equation C_M and C_2 are the direct capacities from wire 2 to wire 1 and to the ground, respectively; that is, they are the capacities measured by the charges on wire 1 and on the earth when wire 1 is connected to the earth and unit potential is applied between wire 2 and the combination of wire 1 and earth.

If the short section of wire 2 is connected to the earth, the charging current flowing to ground is

$$i_2 = j\,\omega\,l\,E\,C_M \tag{3}$$

That is the induced voltage acts like a generator having a voltage e_2 connected between wire 2 and ground through a condenser whose capacity is equal to the total grounded capacity of the exposed section of wire 2. This is indicated in Fig. 19.


Non-Transposed Lighting or Power Circuit
Non-Joint Telegraph Circuits
Single Phase Power-1 Mile spacing
3 Phase Power-3 Mile Barrel
Lighting or Power Circuits-Separation less than 20 ft.
Typical "Frequent" Phantom Group 16 Transp. Poles
1-4
Typical "Infrequent" Phantom Group 32 Transp. Poles
5-8
Typical Non-Phantomed Circuit First 4 Arms 16 Transposition Poles
Joint Telegraph Circuits
Telephone Circuits with Transmitters of large Output
2
3
L 8
L 16
L 24
L S

← Length may vary from minimum distance which will → permit of necessary number of transposition poles to 20,800 ft.

Note:-Unless otherwise indicated the separation between the telephone circuits and the distributing circuits shown is more than 20 ft.

FIG. 17—CHARACTERISTICS OF EXPOSED LINE *L* SECTION—DIAGRAM ILLUSTRATES LENGTH, NUMBER OF TRANSPOSITION POLES, AND PERMISSIBLE LOCATIONS OF TRANSPOSITIONS AND DISCONTINUITIES IN EXTRANEOUS DISTURBING CIRCUITS

2. *Small Induced Effects.* In cases where the induced currents and voltages are small compared with the inducing currents and voltages so that their reaction on the inducing circuit can be neglected, the magnitude of the induced effects can be obtained by solving for the values of current and voltage in the disturbing circuit, neglecting the presence of the other circuit, and then applying equations similar to those given above for each element of the parallel between the two circuits.

3. *A number of Parallel Wires.* The computation of induced currents is much complicated when there are several disturbing wires or several disturbed wires or both.

The computation of the electromagnetic induction may be complicated by the reaction between the induced currents in different disturbed circuits. In cases where the induced effect is small, however, the electromagnetic induction can be computed for this case as for a single circuit.

In determining electrostatic induction with a large number of parallel circuits, it is evident that it becomes exceedingly complicated to compute the direct capacities which are used. It is possible to greatly simplify this work in cases in which the disturbed and disturbing circuits are separated so that each group has little effect on the direct capacities of the other group. Under these conditions the direct capacities within the disturbing circuits can be computed and from this computation a determination can be made of the charge of electricity on each of the disturbing wires taking account of the voltages on all wires. From these charges it is possible to compute the potentials between the disturbed wires and ground if isolated in space.

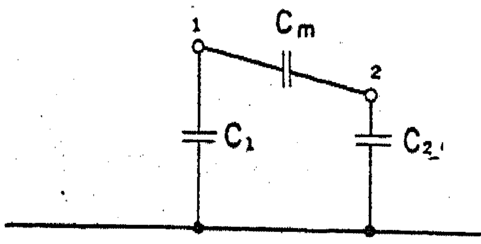


FIG. 18

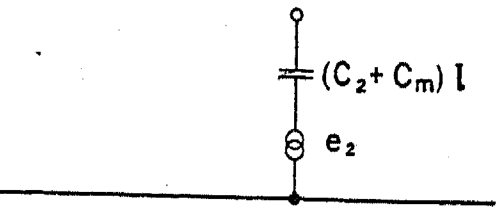


FIG. 19

These potentials may then be assumed to regulate through the direct capacities of the disturbed circuits as indicated in the above discussion of induction between two parallel wires. In computing the flow of current with many disturbed wires the effect on one wire of the other wires must, of course, be considered.

This outlines briefly the method which is in use for computing the induction between three-phase power circuits and telephone circuits. The method sometimes described in the text books is incorrect in that it does not take account of the mutual reactions between the three conductors of the three-phase circuit which materially modify their resultant electrostatic field.

4. *Long Parallel Circuits.* In the case of two long circuits which closely parallel each other without means for neutralizing inductive effects, the reaction of the induced currents on the disturbing circuit cannot be neglected. If the circuits are very long there tends to be the same distribution of current and potential at the receiving ends of the disturbing and disturbed

circuits as there would be if equal voltages were applied to the two circuits at the transmitting end. The simplest case of this sort is when the two circuits are symmetrical. The equations for this case are given in Appendix B. It may be seen that even in this case the equations are so complicated that their use in practise would be burdensome. Where there are many long parallel wires the problem becomes one of utmost complexity.

5. *Parallel Telephone Circuits.* In the case of telephone circuits many conductors are run very closely parallel for long distances. It would seem, then, that the general solution of the inductive effect between telephone conductors would be so complicated as to be hopeless for practical use.

In attacking this problem, however, we fortunately are not interested in the general case. If telephone circuits are suitable for commercial service they must be so arranged that the inductive effects between any two circuits are exceedingly small.

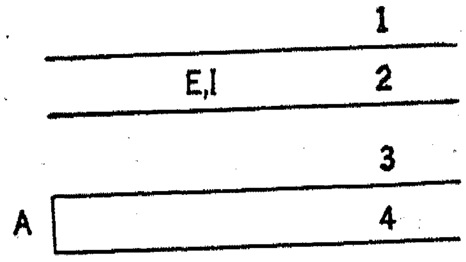


FIG. 20

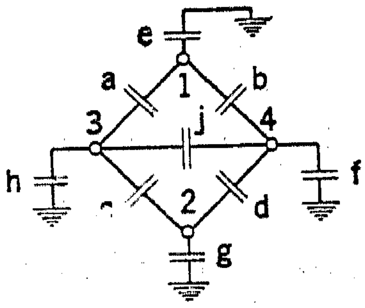


FIG. 21

This is, therefore, a case in which the reaction of the induced currents on the disturbing circuit can be neglected.

If one considers then a short section of close parallel between a disturbing circuit 1-2 carrying a voltage E and current I and a disturbed circuit 3-4 as indicated in Fig. 20 the voltage due to electromagnetic induction can be expressed as in equation (4)

$$e = j\,\omega\, I\, M\, l \tag{4}$$

In computing the current flowing at A due to electrostatic induction one may represent the direct capacities in the form of a Wheatstone bridge as in Fig. 21. In this figure the direct capacities a, b, c and d represent the Maxwell capacity coefficients. Capacities e, f, g and h represent the sum of the direct capacities between the wires, 1, 4, 2 and 3 respectively and all other wires and ground. This grouping is permissible because the induced effects all being small, the other circuits on the pole line are practically at ground potential, and have the same effect on the mutual capacity unbalance between the circuits 1, 2 and 3, 4 as they would if actually connected to earth.

The induced current is represented approximately by equation (5)

$$i = j\,\omega\,E\,\frac{a'\,d' - b'\,c'}{a' + b' + c' + d'} \qquad (5)$$

$a' = a + \frac{h\,e}{s}$, $b' = b + \frac{e\,f}{s}$, etc., s being equal to $e + f + g + h$.

When the circuits are transposed so that the electrostatically induced current is small it results that in a balanced transposition section a', b', c' and d' are all very nearly equal. Under these conditions the magnitude of the electrostatically induced current in one small section of line is represented with a sufficient degree of accuracy by equation (6).

$$i = j\,\omega\,E\,\frac{a' - b' - c' + d'}{4} \qquad (6)$$

The equations given above indicate the coefficients of induction which must be determined in order to make possible computations of the induced currents between telephone circuits. The mutual inductance unbalance between any two circuits can be computed from the geometrical positions of the conductors. The mutual capacity unbalance can be computed readily from the table of measured values of direct capacity for all wires on a 40-wire pole lead. (Table I.) The capacities of course vary with the number and to a small extent with the size of conductors on the lead. Forty wires is a typical condition and has been used as representative. When desirable, correction has been made for the use of wires of larger size.

The above equations therefore present a thoroughly practical method of computing the current induced between short sections of two telephone circuits which are well transposed against each other. The correctness of the results obtained in this way has been checked by experiment.

APPENDIX B—THE INDUCTION BETWEEN TWO LONG PARALLEL SYMMETRICAL CIRCUITS

Assume two long symmetrical wires, A and B, as shown in Fig. 22, operating with ground return. Assume the linear self impedances of the circuits to be z_1 and z_2 respectively, the linear mutual impedance to be m, and the linear direct admittances to ground and linear mutual admittance to be y_1', y_2', and y_m respectively.

The differential equations for inductive effect between the two circuits are as follows:

$$\frac{-d\,v_1}{d\,x} = i_1\,z_1 + i_2\,m \qquad \frac{-d\,v_2}{d\,x} = i_2\,z_2 + i_1\,m$$

$$\frac{-d\,i_1}{d\,x} = v_1\,y_1 - v_2\,y_m \qquad \frac{-d\,i_2}{d\,x} = v_2\,y_2 - v_1\,y_m$$

In the above formulas, $y = y_1' + y_m$, $y_2 = y_2' + y_m$, and m and y_m are both written as positive quantities.

For the case of two symmetrical wires let

$$z_1 = z_2 = z$$

$$y_1 = y_2 = y$$

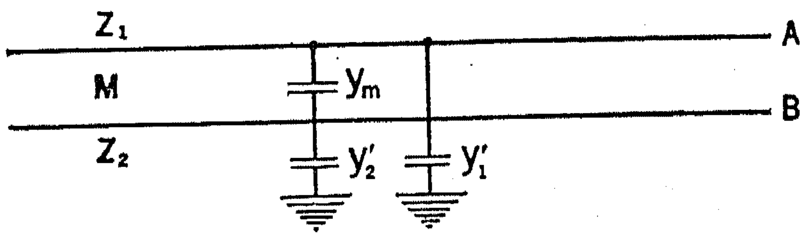


FIG. 22—INDUCTION BETWEEN TWO SYMMETRICAL GROUNDED CIRCUITS

Use is made of the following auxiliary equations:

$$A = 2\sqrt{\frac{z - m}{y + y_m}} \qquad B = \frac{1}{2}\sqrt{\frac{z + m}{y - y_m}}$$

$$\gamma_s = \sqrt{(z - m)\,(y + y_m)} \qquad \gamma_p = \sqrt{(z + m)\,(y - y_m)}$$

Equations are given below for a number of conditions which are of interest. In these equations, v_1 and i_1 represent the voltage and current on the disturbing circuit A at any distance x from the transmitting end, and v_2 and i_2 represent the voltage and current on the disturbed circuit B.

Case I. Voltage impressed on circuit A at $x = 0$ is E_1
Circuit B connected to earth at $x = 0$ through impedance W
Both circuits infinite in length.

$$v_1 = E_1\,\frac{2\,B\,(A + 2\,W)\,\epsilon^{-\gamma_p x} + A\,(2\,B + W)\,\epsilon^{-\gamma_s x}}{W\,(A + 4\,B) + 4\,A\,B} \tag{7}$$

$$i_1 = E_1\,\frac{(A + 2\,W)\,\epsilon^{-\gamma_p x} + (4\,B + 2\,W)\,\epsilon^{-\gamma_s x}}{W\,(A + 4\,B) + 4\,A\,B} \tag{8}$$

$$v_2 = E_1 \frac{2\,B\,(A + 2\,W)\,\epsilon^{-\gamma_p x} - A\,(2\,B + W)\,\epsilon^{-\gamma_s x}}{W\,(A + 4\,B) + 4\,A\,B} \tag{9}$$

$$i_2 = E_1 \frac{(A + 2\,W)\,\epsilon^{-\gamma_p x} - (4\,B + 2\,W)\,\epsilon^{-\gamma_s x}}{W\,(A + 4\,B) + 4\,A\,B} \tag{10}$$

Case II. If the resistance, leakance, and internal reactance are negligible and the equivalent height of the wires above ground is the same for static and magnetic induction, $\gamma_s = \gamma_p$ and

$$\frac{A}{4\,B} = \frac{z - m}{z + m} = \frac{y - y_m}{y + y_m} \tag{11}$$

Then, if $W = \infty$, it follows that

$$v_2 = v_1 \frac{y_m}{y} \tag{12}$$

Under these conditions, ($\gamma_s = \gamma_p$ and $W = \infty$) the sending end impedance of the disturbing circuit is

$$z_0 = \sqrt{\frac{z\,y}{y^2 - {y_m}^2}} \tag{13}$$

This equation shows that under these conditions there is no reaction from circuit B at the terminal of circuit A.

Case III. If $z\,y_m + y\,m = 0$ **(14)**

Then $B = A/4$ and it follows that

$$v_2 = \frac{E_1}{2}\,(\epsilon^{-\gamma_p x} - \epsilon^{-\gamma_s x})$$

$$i_2 = \frac{E_1}{A}\,(\epsilon^{-\gamma_p x} - \epsilon^{-\gamma_s x}) \tag{16}$$

that is, at $x = 0$, the induced voltage and current are 0.

If y_m is negligible, equation (**14**) requires that $m = 0$. This may be approximated by the use of compensating transformers. The above relation suggests, however, that by properly adjusting the value of m an approximate neutralization of induced voltage might be obtained in some special cases in which the electrostatic induction is considerable.

Case IV. When $W = \infty$, then at $x = O$,

$$v_2 = E_1 \frac{y\,(D-1) + y_m\,(D+1)}{y\,(D+1) + y_m\,(D-1)} \tag{17}$$

where $\frac{\gamma_p}{\gamma_s} = D$.

The ratio of this voltage to that computed by equation (**12**) gives an idea of the error involved in the assumption which leads to equation (**12**).

Case V. If at $x = O$,

$$W = \frac{v_1}{i_1},$$

then

$$W = \sqrt{A\,B}$$

and

$$v_2 = E_1 \frac{2\sqrt{B} - \sqrt{A}}{2\sqrt{B} + \sqrt{A}} \tag{19}$$

This impedance, $\sqrt{A\,B}$, is

$$\sqrt{A\,B} = Z_0 = z_0 \sqrt[4]{\left[1 - \left(\frac{m}{z}\right)^2\right]\left[1 - \left(\frac{y_m}{y}\right)^2\right]} \tag{20}$$

Where $z_0 = \sqrt{\frac{z\,y}{y^2 - y_m^2}}$ = the infinite line impedance for a (**13**) single wire.

Case VI. At a long distance from the origin,

$$v_1 = v_2 = \frac{E_1\, e^{-\gamma_p x}}{2} \text{ if } W = O. \tag{21}$$

$$\text{Also } v_1 = v_2 = E_1 \frac{4\,B}{A + 4\,B}\, e^{-\gamma_p x} \text{ if } W = \infty. \tag{22}$$

It will be noted that the voltages on the disturbed and disturbing wires tend to become equal at a considerable distance from the origin, and that the two expressions for v_1 and v_2 with maximum and with minimum shielding are nearly the same.

APPENDIX C—TYPE UNBALANCE BETWEEN TRANSPOSED TELEPHONE CIRCUITS

The induction between long transposed telephone circuits on the same lead is the vector sum of the induction between all short elements of the circuits, taking into account the attenuation and phase change of the inducing currents and voltages from the transmitting point, and the attenuation and phase change of the induced currents in their propagation to the end of the circuit from the point at which the inductive effect occurs.

A simple typical arrangement of transpositions between two telephone circuits in a short balanced section is indicated in Fig. 23 in which similar telephone circuits P and Q are divided into four sections of lengths A, B, C and D by the transpositions in the two circuits. The resulting induction between the two circuits in this section is the sum of two terms:

a. Unbalance Due to Irregularity in Length. As this is a short section of line, one may consider as a first approximation that the current and voltage in the inducing circuit P are the same throughout this section. Currents induced in sections B and D, therefore, tend to neutralize currents induced in sections A and C, provided the length $A + C$ is equal to the length $B + D$. When this is not the case there is a resultant induced current in circuit Q due to irregularities approximately proportional to the resultant unbalanced length $A - B + C - D$.

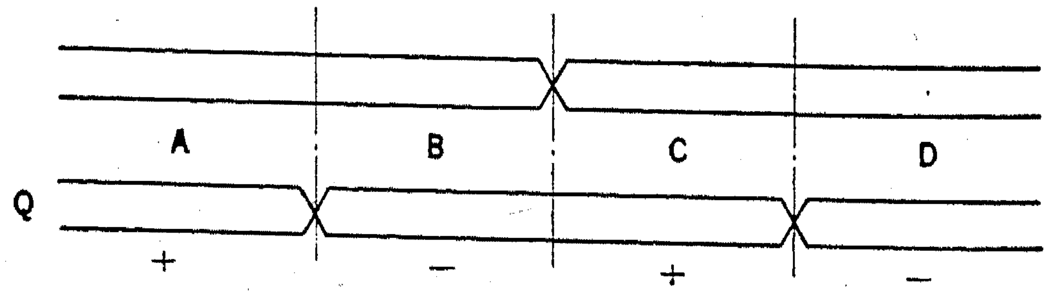


FIG. 23

b. Type Unbalance. If lengths A, B, C and D are all exactly equal there will nevertheless be a resultant induced current in circuit Q because of the fact that the currents and voltages in either circuit P or Q change in magnitude and phase as they are propagated along the circuit.

In this appendix are given equations illustrating the method of computing type unbalance.

If P is a section of a long telephone circuit the current and voltage in it can be represented by equations (**23**) and (**24**).

$$I = I_0 \, e^{-\alpha l} \sin (\omega t - \beta l) \qquad (\mathbf{23})$$

$$E = E_0 \, e^{-\alpha l} \sin (\omega t - \beta l) \qquad (\mathbf{24})$$

In these equations α and β are the attenuation constant and the phase-change constant of the telephone circuit and of course vary with the frequency of the current.

The induced current in a short element of circuit a distance L from the end of the circuit, can be represented as equation (**25**).

$$d\,X = K\,d\,l\,e^{-\alpha l} \sin(\omega\,t - \beta\,l) \qquad \textbf{(25)}$$

where $K\,d\,l$ is the induction between short elements at the end of the circuits and is computed with the help of equations (**4**) and (**6**) for the electromagnetic and electrostatic components. (Appendix A.) This induced current will be propagated on circuit Q back to the beginning of the circuit and under the condition for transmission without reflection, that is, terminal impedance equal to line impedance, will have when it reaches the beginning of the circuit, the value given in equation (**26**)

$$d\,X_0 = K\,d\,l\,e^{-2\alpha l} \sin(\omega\,t - 2\ \beta\,l) \qquad \textbf{(26)}$$

A similar equation can be written for each element of the circuit. The induction due to length A is obtained by integrating expression (**26**) for the length between zero and A as shown in equation (**27**)

$$X_A = \int_0^A K\,d\,l\,e^{-2\alpha l} \sin(\omega\,t - 2\ \beta\,1) \qquad \textbf{(27)}$$

The general expression for this integral is rather complicated. In practical telephone circuits, however, the effect of the attenuation in the short section of circuit between successive transpositions is very much less than the effect of the change in phase angle. Under these conditions the attenuation can be neglected for short lengths and the effective value of the expression (**27**) reduces to that given in equation (**28**)

$$X_A = K\,A\ \frac{\sin\beta\,A}{\beta A}\ \overline{/\beta\,A} \qquad \textbf{(28)}$$

Similarly, the crosstalk at the end of the circuit due to section B of the exposure between the lines can if B is equal in length to A be represented as in equation (**29**).

$$X_B = -\ K\,A\ \frac{\sin\beta\,A}{\beta\,A}\ e^{-2\alpha A}\ \overline{/3\ \beta\,A} \qquad \textbf{(29)}$$

In practical cases the ratio $\dfrac{\sin\beta\,A}{\beta\,A}$ is practically unity and the exponential expression $e^{-2\alpha A}$ is also practically unity. Under these conditions the induced current at the end of the circuit caused by sections A and B of the parallel can be represented approximately as in equation (**30**).

$$X_{AB} = 2\,K\,A \sin \beta A \,\underline{/\pi/2 - 2\,\beta A} \tag{30}$$

Similarly, the resultant induction in circuit Q from sections A, B, C and D can be represented as in equation (**31**)

$$X_{AD} = 4\,K\,A \sin \beta A \cos 2\,\beta A \,\underline{/\pi/2 - 4\,\beta A} \tag{31}$$

This simple example corresponds to one of the typical arrangements of transpositions, namely, that denoted by M in Fig. 2. A similar equation can be worked out for each typical arrangement, and numerical values worked out for any given values of β and A.

For much work a sufficiently good approximation can be made in the formulas for type unbalance by assuming that

$$\sin n\,\beta A = n\,\beta A$$

$$\cos n\,\beta A = 1$$

The resulting simple formulas give an easy means of judging of the approximate relative effectiveness of different arrangements. They are given for different typical arrangements under the heading *Approximate Formulas* in Fig. 2. When X represents the phase change per mile these formulas represent numerically the type unbalance for a 12.8 km. (8 mile) section. The angles of the type unbalance for different types of the same length differ only by 90 deg. or multiples of 90 deg., and these differences only are indicated in the approximate formulas.

The type unbalance varies, of course, with the frequency. In the design of telephone transpositions type unbalances are computed for a frequency found to represent the average crosstalk effect.

It remains to point out the relation between resultant unbalance due to type for a long length of line and type unbalance for one transposition section. Considering current of a single frequency, the ratio between these two for N sections is

$$\frac{1 - e^{-2N\gamma L_0}}{1 - e^{-2\gamma L_0}} \tag{32}$$

where L_0 is the length of the section.

Equation (**32**) assumes that the wires have the same relative pin positions at the beginning of each section If the pin positions are the same in alternate sections and the opposite in successive sections formula (**32**) becomes

$$2 \sin \beta L_0 \frac{1 - e^{2N\gamma L_0}}{1 - e^{-4\gamma L_0}} \tag{33}$$

For a large number of sections, and for the attenuations prevailing on good telephone toll lines equations (**32**) and (**33**) become approximately

$$\frac{1}{2 \gamma L_0} \tag{34}$$

and

$$\frac{\sin \beta L_0}{2 \gamma L_0} \tag{35}$$

Actually, results obtained by these simple formulas must be corrected for the fact that the telephone current is a compound of currents of different frequencies, for which the values of β and γ are different. In the case of loaded lines a correction can be made rather simply. The change in phase angle from one loading section to another is large and varies widely for different components of the telephone current. Good results are therefore obtained by assuming that the effects of successive sections add together by a root-sum-square law. The resulting ratio between long line unbalance and type unbalance for a single section is

$$\sqrt{\frac{1 - e^{-4N\alpha L_0}}{1 - e^{-4\alpha L_0}}} \tag{36}$$

Since N is large and αL_0 small, this becomes approximately

$$\frac{1}{2 \sqrt{\alpha L_0}} \tag{37}$$

APPENDIX D, FORMULAS FOR TYPE UNBALANCE IN PHANTOMED CIRCUITS

Under the heading, *Phantom Circuits*, in the paper it is pointed out that the use of phantoms in telephone line construction complicates the computation of type unbalance because each transposition of a phantom circuit interchanges the pin positions of its two side circuits and, therefore, changes the coefficients of induction between these and other circuits.

The equations for computing the resultant induction per section due to type unbalance are given below. These equations are given for the case in which a typical arrangement of transpositions is associated with each side circuit of the phantom independent of its interchange of pin positions with the other side circuit of the phantom. This is the arrangement which has been found most suitable for use in the exposed line transposition system. Transposition systems can be designed with a typical arrangement associated with the wires on a given pair of pin positions. With this arrangement, each side circuit of the phantom is transposed partly in accordance with one typical arrangement and partly in accordance with another as it changes from one pair of pin positions to the other with transpositions of the phantom circuit. Under some conditions, this arrangement has advantages in the economy of retransposing non-phantomed circuits to form phantoms. It is used in the transposition system which is standard with the Bell Telephone Companies for non-exposed lines.

The formulas for the resultant induction per section due to type unbalance are as follows:

Suppose two circuits, 1 and 2, to be combined in a phantom R and two circuits, 3 and 4, to be combined in a circuit Q.

Denote by T_1, T_2, T_R, etc., the type unbalances corresponding to the typical arrangements of transpositions on circuits 1, 2, R, etc. Further, let K_{1R}, K_{2R}, etc. represent the induction per unit untransposed length between circuits 1 and R, between circuits 2 and R, etc.

For Side Circuit to Phantom of Which it is a Part.

$$X_{1R} = S\,T_{1R} + D\,T_1 \tag{38}$$

$$X_{2R} = S\,T_{2R} - D\,T_2 \tag{39}$$

$$\text{Where } S = \frac{K_{1R} + K_{2R}}{2}$$

$$D = \frac{K_{1R} - K_{2R}}{2}$$

T_{1R} represents the type unbalance for the typical arrangement corresponding to the relative exposure between type 1 and type R.

T_{R} is similarly defined.

For Side Circuit to a Non-phantomed Circuit or a Phantom Other Than its Own.

$$X_{1Q} = S\,T_{1Q} + D\,T_{1RQ} \tag{40}$$

$$X_{2Q} = S\,T_{2Q} - D\,T_{2RQ} \tag{41}$$

When
$$S = \frac{K_{1Q} + K_{2Q}}{2}$$

$$D = \frac{K_{1Q} - K_{2Q}}{2}$$

The nomenclature is the same as before.

T_{1RQ} represents the type unbalance corresponding to the relative exposure between the arrangement giving unbalance T_{1R} and typical arrangement Q.

For example, if circuit 1 is transposed to Type A
" R " " F
" Q " " L

$T_{1RQ} = T_{(AF)L} = T_{KL} = T_O$

That is, T_{1RQ} is in this case the type factor for arrangement O, which is obtained by combining successively types A, F, and L.

For the Two Side Circuits of a Phantom

$$X_{12} = K_{12}\,T_{12} \tag{42}$$

For Side Circuits in Two Different Phantoms

$$X_{13} = S\,T_{13} + D_a\,T_{13R} + D_b\,T_{13Q} + D_c\,T_{13RQ} \tag{43}$$

$$X_{14} = S\,T_{14} + D_a\,T_{14R} - D_b\,T_{14Q} - D_c\,T_{14RQ} \tag{44}$$

$$X_{24} = S\,T_{24} - D_a\,T_{24R} - D_b\,T_{24Q} + D_c\,T_{24RQ} \tag{45}$$

$$X_{23} = S\,T_{23} - D_a\,T_{23R} + D_b\,T_{23Q} - D_c\,T_{23RQ} \tag{46}$$

Where

$$S = \frac{K_{13} + K_{14} + K_{24} + K_{23}}{4}$$

$$D_a = \frac{K_{13} + K_{14} - K_{24} - K_{23}}{4}$$

$$D_b = \frac{K_{13} - K_{14} - K_{24} + K_{23}}{4}$$

$$D_c = \frac{K_{13} - K_{14} + K_{24} - K_{23}}{4}$$

In the above equations the nomenclature is as before.

T_{13RQ} denotes the type unbalance corresponding to the relative exposure between the types giving type unbalances of T_{13} and T_{RQ}.

The D terms in the above equations are generally not negligible compared with the S terms. This fact requires that in designing, for example, transpositions between side circuits in two different phantoms, it is necessary to take account not only of the type unbalance of the relative exposure between the two circuits, but also of type unbalances formed by building up combinations of the typical arrangements of transpositions on the two side circuits and on the phantoms of which they form a part.

DISCUSSION ON "DESIGN OF TRANSPOSITIONS FOR PARALLEL TELEPHONE AND POWER CIRCUITS" (OSBORNE), ATLANTIC CITY, N. J., JUNE 27, 1918.

V. Karapetoff: In the appendix, Mr. Osborne uses the expression "mutual impedance." This term is new to me, and I should like to have a definition of it. If this quantity is equal to $2\pi f$ times the mutual inductance, then it would seem proper to call it "mutual reactance," in the same way in which the usual reactance is equal to $2\pi f$ times the coefficient of self induction.

It is also pointed out that the ground may modify this coefficient of mutual impedance and this may be a justification for using the term "impedance" instead of "reactance," to indicate that it is composed in a quadratic relation of resistance and reactance. But then the difficulty arises that the mutual impedance seems to be a coefficient which characterizes the two circuits, and if the resistance is to be taken into account, it is not clear the resistance of which of the two circuits is to be used in the expression for mutual impedance. Our Standardization Rules define "mutual impedance" in application to telephone work but I am not sure that it is applicable here. I also notice that another author (Mr. Fortescue) in a paper presented at this convention uses the expression "mutual impedance." It seems to me that we ought to get a general definition of this quantity, and introduce it into the Standardization Rules so that there may be no doubt about its meaning and use.

Charles Fortescue: In the event of a power company having installed a transmission line without transpositions, what would be considered the ethical arrangement between the power company and the telephone company, in case of inductive interference? If transpositions of the power lines are determined on before the installation is made, I can easily see that the cost would be a very small item, but after the power line has been installed the cost of transpositions might be quite a large item, and I should think it would be a case for a compromise between the telephone company and the power company under such circumstances.

Another condition that one may run across is, quite frequently the power transmission line is not symmetrically designed. Of course, a perfectly symmetrical three-phase transmission line would have lines at the apexes of an equilateral triangle, but quite frequently the lines are equally spaced in the horizontal plane, and quite frequently they are unequally spaced, depending on conditions which may have existed before the line was installed. I am very much interested in this, as it happens to be part of my duty to look into these cases of trouble which arise, from such causes as this, and they are sometimes very difficult to adjust. There is always a great deal of controversy as to who is to blame, and who should be the one to correct the trouble.

I might refer to the question that Prof. Karapetoff brings up. I have myself used the expression "mutual impedance" in some cases. In alternating-current circuits where we have currents which have a common part in a part of the circuit, and are purely mutually inductive in other parts of the circuit. and which it is convenient to consider separately, the expression "mutual impedance" comes in very handily, and is a very useful quantity to use. As far as I know, it has never been defined, but I think it is a perfectly definite and well understood quantity, and it is commonly used among telephone engineers.

A. G. Chapman: Certain experimental data relating to mutual interference between telephone circuits may be of interest. Mr. Osborne shows in Fig. 2 of this paper typical arrangements of transpositions for telephone circuits and in Appendix C discusses the method of computing approximately the inductive effects between two telephone circuits transposed in accordance with any two of these arrangements. It is interesting to note that increasing the number of points at which one circuit and not the other is transposed may increase rather than decrease the inductive effects between the two circuits. For example, if the method of computation outlined in Appendix C is followed, the *N* exposure shown on Fig. 2 will be found to be much more effective against the mutual inductive effects commonly termed crosstalk than the *M* exposure, although the latter type of transposition arrangement has three transpositions while the former has but two. The reason for this is clear if it is noted that the second half of the *M* exposure starts out with the wires in the same position as in the first half and, therefore, the inductive effects into the two halves tend to add. With the *N* exposure the second half starts out with the wires reversed and, therefore, the inductive effects into the two halves tend to subtract. Extending this reasoning further, it may be shown that the *A* exposure having 15 transpositions is less effective than the *N* exposure since in the case of the former the small inductive effects in each successive eighth of the length, all tend to add.

In order to check the theory of the effect of different arrangements of transpositions roughly outlined above, tests were made of the relative effectiveness of the *M*, *N*, and *O* exposure types. Fig. 1 shows the method of making the measurements. A single-frequency source of alternating current was used in order to facilitate the comparison with computed values. In order to get the effect of a short exposure in a long telephone line, the distant ends of both circuits were terminated with 600-ohm non-inductive resistances, approximately the impedance of a long circuit of the construction used in the tests. The test end of the disturbed circuit was also terminated with 600 ohms resistance. A measured amount of single-frequency alternating current was sent out on the disturbing telephone circuit. A sensitive receiving circuit of high impedance was then alternately bridged across the 600-ohm termination at the test end of the disturbed circuit and across a comparison circuit which impressed at the terminals of the detector a known voltage of the same frequency as that impressed on the disturbing circuit. By adjusting the voltage of the comparison circuit and listening to the sounds heard in the receiver in its two positions, it was possible to measure the voltage impressed by the disturbing circuit across the 600-ohm termination of the disturbed circuit. The current through this termination could then be readily computed.

The single-frequency tests were made on two adjacent specially transposed No. 12 N. B. S. gauge pairs in a 40-wire toll line. The inductive effects were measured in a transposition section about 7 miles (11.2 km.) long. The transpositions were carefully located so that the crosstalk would be due principally to type unbalance and not due to irregularities in wire and pole spacing except for combinations of circuits having very highly efficient transposition types.

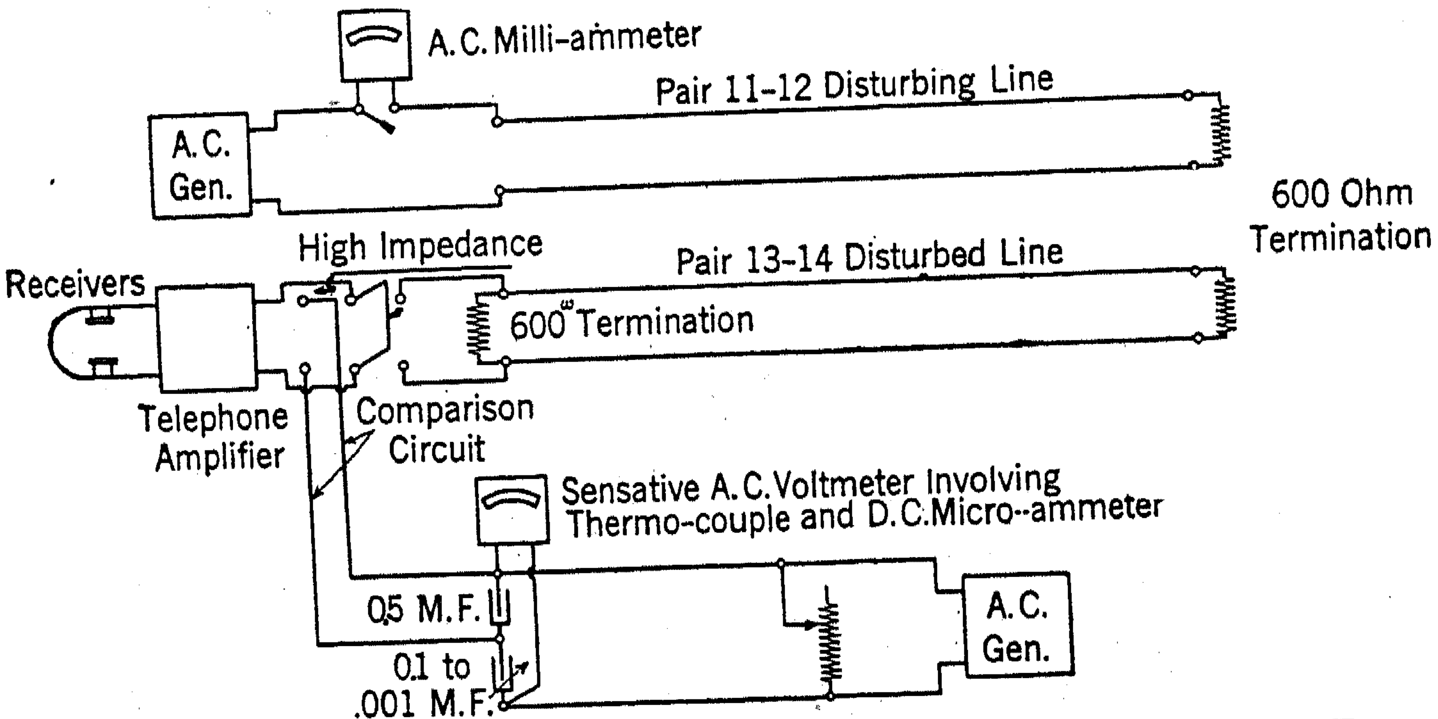


FIG. 1—METHOD OF MEASURING SINGLE-FREQUENCY CROSS-TALK

The crosstalk in a short relatively untransposed length was first tested and then the crosstalk in a length of about 7 miles for the three exposure types was measured. By comparing the latter measurements with the former it was possible to express the exposure of any type in terms of an equivalent untransposed length. The following table shows a comparison between the equivalent untransposed lengths obtained in this manner and corresponding values computed theoretically from the equations of propagation of current on the circuits. From a comparison of the readings of different observers and of successive readings of the same observer, it was estimated that the precision of observations was ordinarily about 10 per cent. In the case of the *N* exposure, the precision was much less since the method was not suitable for accurately measuring the small amount of crosstalk involved. In addition the effect of the irregularities would, of course, be much greater in connection with this exposure since the crosstalk due to the type unbalance is very small.

M, N AND O EXPOSURE TYPES

MEASURED VERSUS COMPUTED VALUES OF EQUIVALENT UNTRANSPOSED LENGTHS

Type exposure	Frequencies tested	No. of readings	Average ratio measured to computed	Computed length 1000 cycles
0	840, 1,000 1,100	12	0.88	0.75 mi.
M	840, 1,000 1,100	12	0.99	0.38 mi.
N*	1,000 2,000	6	0.42	0.044 mi. or 230 feet

*Measured values too small to be read accurately.

Fig. 2 shows the arrangements of transpositions tested and a comparison of computed and measured values of crosstalk for 1000 cycles. It should be noted that the circuits were tested for impedance, insulation and direct-current resistance before making any measurements to insure that they were in good condition. The measured values were obtained from the averages of two sets of readings each involving two observers and taken on different days. The crosstalk diagrams on Fig. 2 show the effect of the transpositions in combining crosstalk at the terminal due to the various eighths of the transposition section. The average measured valve of the ratio of received to disturbing current for the test on the short untransposed length, that is $K\ A$ of the diagram, was 725 millionths while the computed value was 770 millionths.

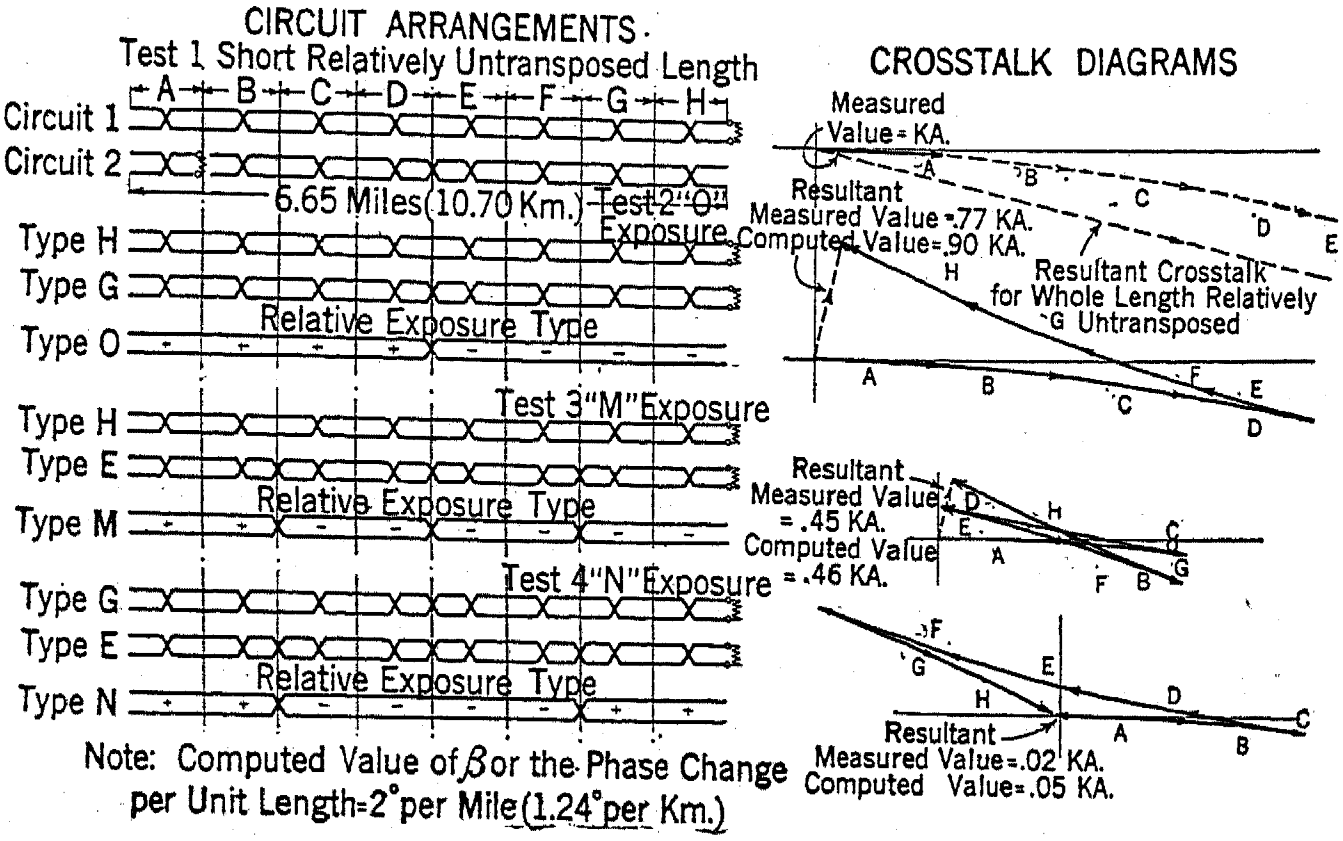

CIRCUIT ARRANGEMENTS
Test 1 Short Relatively Untransposed Length
A
B
C
D
E
F
G
H
Circuit 1
Circuit 2
6.65 Miles (10.70 Km.)
Test 2 "O" Exposure
Type H
Type G
Relative Exposure Type
Type O
+
-
Test 3 "M" Exposure
Type H
Type E
Relative Exposure Type
Type M
Test 4 "N" Exposure
Type G
Type E
Relative Exposure Type
Type N
Note: Computed Value of β or the Phase Change per Unit Length=2° per Mile (1.24° per Km.)
CROSSTALK DIAGRAMS
Measured Value = KA.
Resultant Measured Value = .77 KA. Computed Value = .90 KA.
Resultant Crosstalk for Whole Length Relatively G Untransposed
Resultant Measured Value = .45 KA. Computed Value = .46 KA.
Resultant Measured Value = .02 KA. Computed Value = .05 KA.

FIG. 2—SINGLE-FREQUENCY (100-CYCLE) "CROSSTALK" TESTS

Certain talking tests made later may be of interest since they show absence of crosstalk between circuits having an N exposure which, as previously discussed, is a relatively efficient transposition type compared with the other types here discussed. These tests were made in a manner similar to that shown in Fig. 1 for the single-frequency test, with the difference that the ratio between the received and disturbing currents was measured by shunting off a known fraction of the disturbing current and comparing it with the received current.

Fig. 3 gives data on the telephone lines employed in these talking tests. As indicated about 28 miles (45 km.) of a toll line was transposed by means of four *E* sections. The circuits tested had an *N* exposure in each transposition section and had comparatively large coefficients of induction between them, thus requiring an effective transposition arrangement. Tests made on two successive days indicated that the energy transferred from one circuit to the other was about one hundredth of a millionth of that transmitted into the disturbing circuit. About this amount of energy transfer was estimated due to the measured small irregularities in the locations of the transposition poles. The energy transfer estimated due to type unbalance was negligible in comparison.

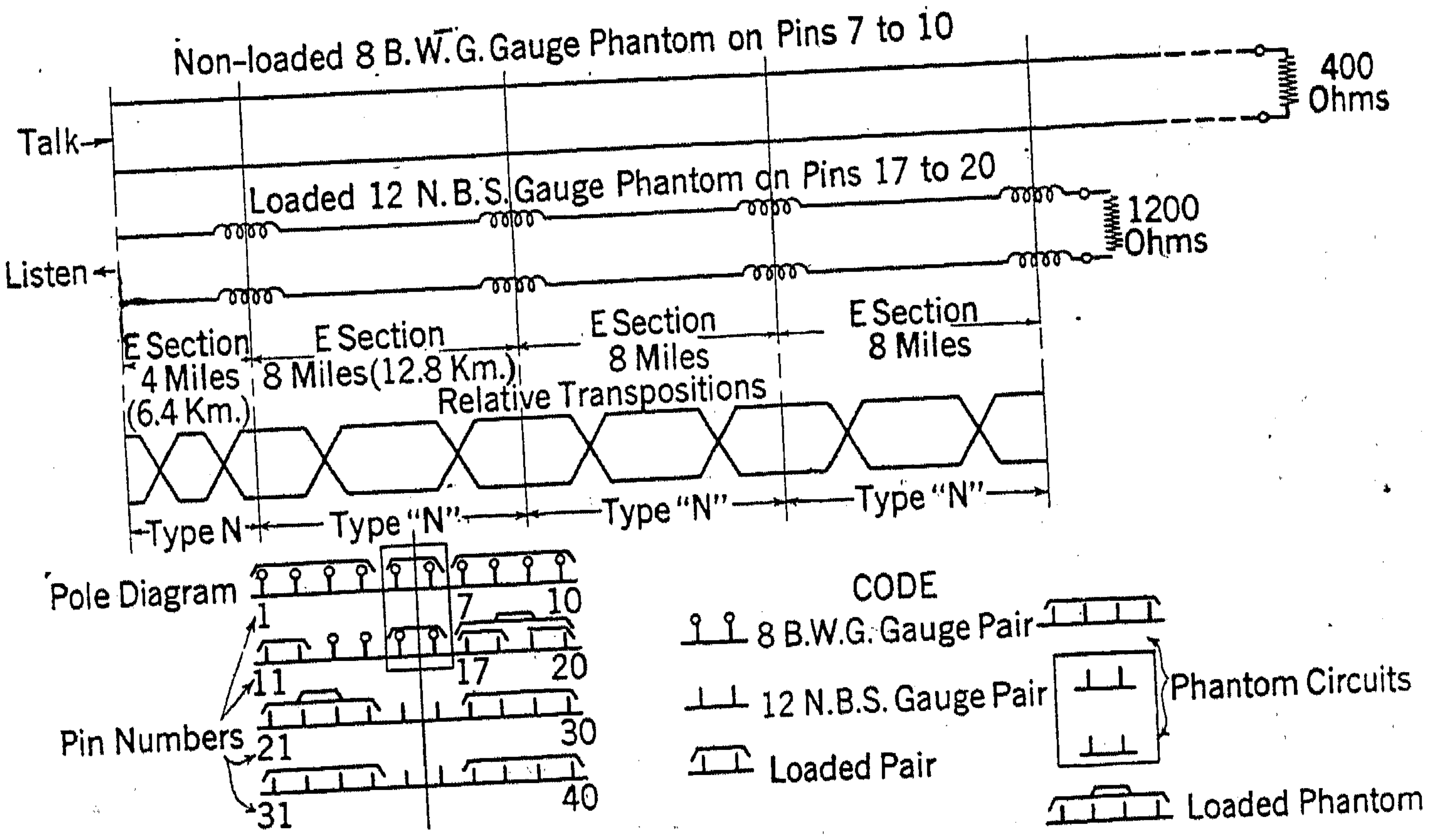


FIG. 3—"CROSSTALK" BETWEEN ADJACENT PHANTOM CIRCUITS HAVING TYPE "N" RELATIVE EXPOSURES

In design of telephone transposition systems, a study must be made of the minimum separation which is permissible between circuits non-transposed to each other. It is, of course, desirable to design arrangements of transpositions for but a few crossarms and then repeat these arrangements on the lower arms. The minimum separation between similarly located relatively untransposed circuits on the first and lower arms is, therefore, of interest.

In Table I of his paper, Mr. Osborne gives a table of the capacities between various combinations of wires and between the wires and ground for a 40-wire pole lead, and in Appendix A discusses the method of computing from these measured values, the coefficients of electric induction between the various circuit combinations. In order to study the question of the repetition of a transposition system on the lower crossarms, it was necessary to make similar capacity measurements on an 80-wire pole lead. The lower curve of Fig. 4 is obtained from the data of these tests and shows the variation with separation of the coefficients of electric induction between a phantom on the end of the first arm and similarly located phantom circuits on lower arms. The magnetic coefficients of induction were readily computed from the known conductor spacings.

H. Mouradian: It may possibly further the understanding of the practical significance of the paper if I should state, briefly, some of my own experiences in the matter of the inductive interference on telephone lines in connection with two specific cases of exposure:

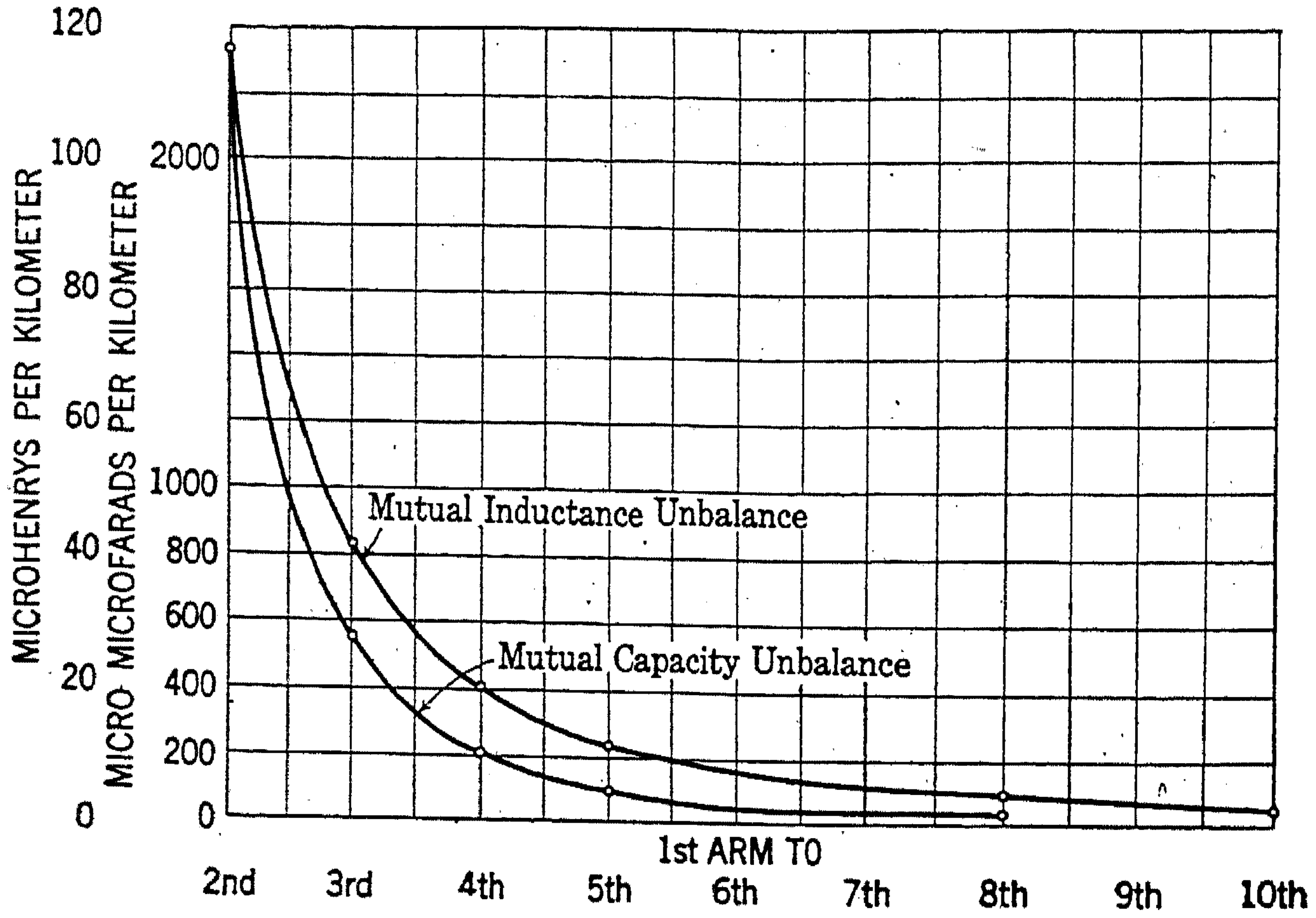


Fig. 4—Electric and Magnetic Coefficients of Induction Between Phantom Circuits on Pins 1 to 4 and Phantoms on Corresponding Pins on Lower Arms with Standard 10-Pin Arms Spaced 2 Feet Apart

(1) In the first case of exposure, of approximately eight miles (12.8 km.) the power circuit was 33,000-volt, three-phase, star grounded neutral. The power line was built in 1905 or 1906. The power company installed on this eight-mile stretch two complete barrels,—a barrel every four miles. Special transpositions were also installed at the time, one special transposition on each telephone toll circuit midway between every two successive cross-talk transposition points on the telephone line throughout the length of the exposure. These two systems of transpositions, one on the power circuits and the other on the telephone circuits, were not, however, co-ordinated. The

result was that the telephone circuits, exposed as described above, were very "noisy", in fact, so noisy that the main industrial company in one of the towns served by the telephone lines actually made a complaint that, if the telephone company did not improve its service, it did not want the service. The next phase in the history of this case brought about a coordination of the telephone transpositions. The situation was further improved by the installation of two special telephone transpositions, instead of one, between every two successive cross-talk transposition points. The noise on the telephone circuits diminished and was found not to exceed 200 to 300 units of noise. Then, by further work on the special telephone transpositions, with a view to locating them as advantageously as possible with due regard to the irregularities in the exposure, the noise on the telephone circuits was finally reduced to approximately 60 to 70 units. At the present time this condition is considered satisfactory from the standpoint of commercial operation.

(2) In the second case I have in mind a direct application was made of one of the systems described in Mr. Osborne's paper. In this case the exposure was for six miles (9.6 km.), the separation fairly uniform at 50 ft. (15.2 m.). The power circuits were three-phase, 23,000-volts, delta. The telephone circuits have just been transposed in accordance with the system described on Fig. 9 of Mr. Osborne's paper. Determinations of the amount of noise were made before and after the installation of the special transpositions with the results summarized in the following table:

	Noise Readings			
	Distant end of telephone line open		Telephone station at distant end of telephone line	
	Before	After	Before	After
Side circuits..........	175	37	172	17
Phantom circuits.....	775	112	887	37

It will be noted from the above table that a material reduction has been obtained through the use of the system of transpositions above referred to. I think I might say, in closing, that the paper discloses several important practical solutions of the rather complex problem of inductive interference.

H. S. Osborne: Professor Karapetoff raises a question about the term "mutual impedance." This term is defined in Rule 916 in the Telephone Section of the Standardization Rules. The definition given there is entirely general, and as Mr. Fortescue points out, the term is used in other than telephone work. It seems that the term is a valuable one and that it should be changed in the Standardization Rules so as to be given there its more general application.

In response to Mr. Fortescue's question regarding the ethical arrangements between power and telephone companies in the case of inductive interference, I would say that my own belief is that, in general, the best practise requires that when either of the two or more schemes for reducing the interference is possible that one should be selected which gives the least total cost, counting up costs of changes in both the power circuits and the telephone circuits. Both systems are run for the benefit of the public and the interest of the public would seem to be best conserved by the application of this principle of least total cost. This principle can be applied independent of the question of the equitable distribution of expense between the two companies.

Mr. Fortescue speaks about the effect of the configuration of conductors of the power circuit. The triangular arrangement of wires is in almost every case materially better than either the horizontal or vertical arrangement in limiting the external field of the line. A great deal of information about this is given in the forthcoming book of Technical Reports of the Joint Committee on Inductive Interference in California.

Address presented at the 34th Annual Convention of the American Institute of Electrical Engineers, Atlantic City, N. J., June 27, 1918.

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ENGINEERS AND THE WAR

BY MAJOR GENERAL WILLIAM M. BLACK
Chief of Engineers, U. S. Army

I FEEL some diffidence in addressing you this evening, partly because, although I might have qualified as an electrical engineer in 1880, I know I could not do it now, and then, I feel that a man who has the privilege of talking to a number of men such as are assembled here at this time has a tremendous responsibility, because the task set before us is one that has been unequalled in the history of the world. We represent a creed, not only in our living but in our Government, which we believe is necessary for the salvation of this world, for bringing to this earth the Kingdom of God.

We are fighting a nation that has a creed directly opposite to ours, a creed that leads them into the belief that Might makes Right, and that anything that is for the supposed good of the State is Right. This attitude has led the people of that nation to break every law that humanity has made for the amelioration of the horrors of war, laws that have gradually grown up through the ages. These laws are being ruthlessly sacrificed by these people in strict accordance with the creed which they hold. That creed and that people must simply be swept from off the face of the earth, and that is the task that we have undertaken.

It is an honor to be privileged to address you on a subject of such importance to our country, the duty of the Engineers in war. Although the part played by Engineers in this war is great and the responsibilities of the profession are correspondingly large, this war, like all wars in the past, is and must be a war carried on in accordance with the principles of the art of war, which are unchanging and which have been recognized and taught ever since organized armies were first created.

Do you realize that almost the only absolutely modern method of warfare now in use is the warfare of the air? The invention of submarines was made during the American Revolution, and submarines were used successfully, though to a limited extent, in our own Civil War. Gas and flame fighting are of ancient origin. Trench fighting is hardly better known today than it was to the veterans of Grant and Lee, of Sherman and Johnson. The advances in human knowledge have caused corresponding improvements to be possible in the weapons of warfare. Increased knowledge of chemistry has produced more powerful explosives and improved methods in metallurgy have enabled these explosives to be utilized, by making possible heavier and more powerful guns. Improvements in the means of transportation have enabled larger bodies to be moved more quickly and more readily and to be subsisted and supplied with greater certainty. The telegraph, the telephone and the wireless have afforded a means of prompt communication and have enabled, larger bodies of men to be given co-ordinated action. With such changes, battles are fought on the same principles and won or lost from the same causes as in the time of Alexander the Great. This war has been called a people's war and so it is in the sense that due to modern facilities the entire resources of the people can be utilized to day as they could not have been utilized in the days of old. It has also been called an Engineer's war because in the quickness of movement and in the works necessitated by these modern inventions the services of Engineers become more conspicuous and perhaps more necessary than in the past. But engineering in warfare has always been essential and it is even doubtful whether the science of engineering does not owe its birth to the works of war. An Engineer myself, I would be the last to belittle the work of our profession. It is a matter of pride that the men of our profession, due to the nature of their employment in time of peace, are, of all the civil professions, most prepared to serve the country in war, but to serve the country adequately in war, the Engineer must add to his peace equipment for professional work. The profession of arms is a profession in itself and it is the profession which deals with the very greatest in magnitude of all the endeavors of men. The effective use of an army which is properly constituted exemplifies the best that men can do in organization, in discipline and in the devotion to duty which causes a man to regard his own life as a thing of small moment toward the attainment of the end sought.

There would be a great amount of effort saved if our people recognized more clearly the existence of the technicalities of the profession of arms. The Government in Washington is simply deluged with suggestions and so-called inventions for the winning of the war. The records show that about 98 per cent of all of these are without military value and that time and labor have been thrown away by men eager to help, but entirely ignorant of the history and conditions of warfare.

An example with which some of you are familiar is the electrical gun. For years the possibility of such a weapon has been a fascinating line of study to electricians. The principle of the solenoid is the germ. If a series of solenoid coils were to be energized and de-energized in succession sufficiently and rapidly, such a series around a tube can be made to impart a movement of translation and rotation to a projectile. But practical results are today impossible. A six-inch service rifle having a length of 20 feet, fires a projectile weighing 110 lb. with a muzzle velocity of 2600 ft. per sec., or in other words, the projectile leaves the muzzle with a kinetic energy of translation of 115,500 ft-lb. This energy has been stored in the projectile during its travel through the bore of the rifle, or say in 1/65th of a second. The average power expended has therefore been at the rate of 7,607,500 ft-lb. per second or about 14,000 h. p. or 10,500 kilowatts. These figures are simply approximations and neglect entirely the power required for imparting velocity of rotation and for overcoming the friction in the bore. You can easily estimate the weight and dimension of the generating equipment which would be required for even a moderately powerful gun were all the mechanical and electrical problems of its manufacture solved, and making due allowance for the short-load periods. You can understand the impracticability of transporting the electrical plants required for any number of such guns, and the impossibility of distributing this power over shell-swept ground to guns whose position must be constantly shifted, and which must be put in action on a few seconds' notice. I think that you will agree that until new discoveries give a much improved method of storing or generating electricity, smokeless powder will continue to be the most compact and convenient form of stored energy for guns.

And yet there has been a good deal of time and money wasted in trying to perfect such a gun by men whose patriotism is undoubted, and whose ignorance, also, is undoubted. In other words, if a man has an invention or an idea of an invention, by all means let him work on it, but before he goes to Washington and takes up the time of men busy trying to devise means to beat the Boches, let him make sure that he knows the conditions of war and what he is trying to do to meet those conditions, and then if he is sure of the means, let him present his ideas and inventions to the people in Washington.

We want all we can get and want the best we can get. We want the inventive power of our country if it can be exercised to do good.

There was a proposition made seriously at Washington recently that the United States should provide a fund, of I do not know how many million dollars, and make a Home for Inventors, where any one who thought he had an invention would be able to go, and work it out at the public expense; and recently, although we had a committee of experts there to pass upon these inventions, the results were so utterly unsatisfactory to the inventors that they came in a perfect horde upon the Secretary of the Navy and the Secretary of War, so much so that they had to make a brand new committee of three men, who could be much better occupied, to go ahead and do this same thing; and I am only waiting for the next drive of inventors to show that this Committee will not suit them one particle better than the old one did.

In addition, without doubt, there are many men in our country of the highest patriotism who are sore-hearted because they are not given something to do directly toward the winning of the war. They do not understand, that some condition peculiar to themselves, possibly age, possibly physical condition, possibly mere ignorance of war and its conditions, compel it that the bit that they must do for their country at this time is to continue in their work in civil life and do their part in keeping up the normal life of the country—in itself a service of importance.

The part which Engineers are now playing in the war is a very great one. The records of the American Institute of Electrical Engineers show that out of a total membership of 9443, there are 973 in the service, or 10.3 per cent of its roster. The American Society of Civil Engineers with 8753 active members has 14⅓ per cent in the service. The American Institute of Mining Engineers 10.4 per cent, and the American Society of Mechanical Engineers 10.1 per cent. But these records are not complete. At the outbreak of the present war there were in the Engineers Corps of the Regular Army about 300 officers and approximately 3500 enlisted men. At the present time, there are about 8000 commissioned officers and 200,000 enlisted men, made up of men formerly engaged in works of an engineering character. It is probable that this does not represent much more than one-half of the number of the profession now serving in the Army.

Let us consider the nature of the work of the Engineer, passing from front to rear of the Army.

First in importance is the work of the sappers. They go before and remove obstacles, clearing away obstructions, building bridges and roads, making the trench systems complete, mining, providing light, water, lines for supply (light railways or roads) and military mapping. In this category enter practically all of the branches of the profession. Further to the rear are found the construction and operation of railways; road and bridge construction; the construction of veritable towns for supply depots, with all their accessories, drainage, sewerage, lighting and water supply; construction of quarters and of hospitals; and furthest to the rear, the construction of the ports of debarkation with their wharves storehouses, railway lines, yards and shops, all with their sanitary systems. Separate from these activities, but necessary for their supply, are the Forestry troops who turn the growing timber into lumber of the dimensions required for the various services. Locomotive and car shop troops are performing essential services. Topographic Corps, Sound Ranging Corps and Camouflage Corps are also among the varied activities of the Engineers.

What preparation is required for the fulfillment of these varied duties? For the actual technical work of construction or installation the civil training of the Engineer should prove sufficient when the plans which embody the military features have been prepared, or when the military technique has been learned and assimilated. A fundamental of this military technique is that the time element is to be considered rather than money cost and that the work must be done with whatever materials are available. This requires clearness of conception of the results required, resourcefulness and organization—factors also required for civil work.

As stated earlier, due to the very small numbers of the personnel of the Corps of Engineers of the Regular Army, reliance had to be placed in the members of the profession in civil life. Confidence in their devotion to country and in their ability has not been misplaced. The results already accomplished prove this fully. Could more have been done? Undoubtedly, had the profession been better prepared for the call.

Will you permit me to say a few words concerning the general training of our Engineers, based on a professional experience of more than forty years? The conviction has been forced upon me that in educational matters, as in many other affairs of life, we Americans are inclined to go too fast. The basis for any professional career where the highest is to be attained must be a sound general education. Does anyone of you regret the lessons gained in your own experience? Is not the experience of humanity as shown in properly written history of almost equal value? Would Russia now be in the sad condition existing had her people known that the experiments she is trying have always resulted disastrously? Yet is history thus considered in an ordinary technical course? Again, do you not find a knowledge of the general principles of law and of the special rules of the laws of contracts of value? Are these considered essentials? What is the handicap of an Engineer who is unable to express his ideas clearly in spoken and written English? Is this taught thoroughly in our technical courses?

It goes without saying, that the study of pure and applied mathematics is found in all technical courses. But, are these subjects well grasped before their application in special technical courses is studied? Is any faculty of an Engineer of greater value than the ability to form a mental picture of his problems and of its solution? Yet is that study which assists most in this faculty—descriptive geometry—properly apprehended? Is there any branch of the profession which in its application is not based on a knowledge of topographical work, on a knowledge of construction materials and of how these should be used? Is the study of these branches of civil engineering insisted upon sufficiently in the mechanical and electrical courses? In effect would not our professional men be better equipped for their civil work were they not in too great a hurry in their youth to enter directly into life's combat? Does not this war teach that without a long and elaborate preparation down to the last details, an attempted "drive" must fail?

These remarks apply to all Engineers, both military and civil. In the rush of war men cannot always be hand-picked for special jobs and frequently it becomes necessary for an available man to be used for the work immediately necessary, irrespective of his previous training. In this supreme test of humanity the best man is he who is prepared to meet any emergency—perhaps not in the most finished way—but to meet it.

There are things that the engineers in this country can do. If they do know enough to give us some ideas for helping along in the killing of Boches, for God's sake let us have them. If they do not, what they can do is to help the supply of men for the winning of the war. We are now short of officers of Engineers, very short, and we are going to be very much shorter. We must have educated engineers for this work, and we must not only have the men for the line work of the army, but we must have mechanicians and artisans and laborers for the special work.

All of you men have spheres of influence—do your best in them, and if you can be used otherwise, and the problem comes up in which we need you, you may be sure you will be called upon. There is this problem now, the supply of men, in which you can help, either by your own personal sacrifice, in going out, or by influencing others.

Now as to soldier work. The movements of drill and the construction methods peculiarly military are easily learned. The knowledge of the art of war which will enable these to be applied promptly and properly is more difficult. But most difficult to acquire is the peculiar mental discipline which makes the soldier. The Army is a huge machine which must work co-ordinately in all of its parts. That competition, which in civil life causes one body to advance further and faster than another, is out of place in an Army. *All* must work together and for one common end. Each man must so subordinate his will and desire to the common good as to work willingly and earnestly in the sphere allotted to him. This does not mean that all initiative is to be suppressed. On the contrary each man must use his initiative to the utmost, but in his own allotted sphere of action. Each must learn to obey and obey from the heart. Through such obedience comes the knowledge of how to command when command becomes a duty. All of this is hard to learn. But each man who is called upon to help in this war must learn it, if he would help effectively.

By all means let us have military training in our schools, but let it be true military training and not tin soldier work.

. There is another line of technical military knowledge which must also be studied hard. The machinery for the organization, training, supply and leadership of troops; the methods of obtaining, accounting for and issuing supplies; of keeping returns of the men; and the channels of command must be studied. To civilians in general this is wholly unknown, but if a man is to be of service in the Army, it must be learned until its use becomes automatic.

What have the Engineers done? War was declared April 6, 1917. By the middle of July, nine regiments of Railroad Engineers had been raised and organized and two had actually started for France. In each regiment were two officers of the Corps of Engineers of the Regular Army, the Colonel and the Regimental Adjutant. The remaining officers were all from the Engineer Reserve Corps, some receiving their commissions only when on the point of sailing. Of course, few of the officers had had any previous military training and the tasks of organization were most difficult. Since then, there have been organized:

Five Corps Regiments consisting of Sapper, Searchlight and Sound Ranging troops; 43 Sapper Regiments and trains; 2 Mounted Battalions and trains; 5 Ponton Trains; 4 Inland Waterway Companies; 40 Railway Regiments and Battalions including all classes of Standard Gage and Light Railway troops necessary for the construction, operation and maintenance of railways; 1 Railway Transportation Corps; 1 Highway Regiment; 1 Gas and Flame Regiment; 1 Gas Training Service; 5 Forestry and Auxiliary Forestry Regiments; 1 Surveying and Printing Battalion; 1 Military Mapping Service; 2 Supply and Shop Regiments; 1 Water Supply Regiment; 1 Quarry Regiment; 1 Mining Regiment; 1 Electrical and Mechanical Regiment; 2 Crane operating Companies; 1 Camouflage Battalion; 18 Truck and Auto Companies and 44 Depot Detachments.

The greater part of these organizations is now overseas. Some are serving with the British Army, some with the French, but the majority is with our own troops, in service both at the front and in the rear.

I wish I could go into greater detail as to the work of these Sound Ranging Corps, because it comprises some new electrical work of the highest character, and the apparatus for it has been perfected in this country. We took the best devised at the beginning of the war, and our physicists went to work and have made marked improvements. Perhaps you do not know what sound ranging is. The artillery is stationed in the rear of the line. There is almost no direct artillery fire any longer—that is, as a rule, the gun is fired from a point where the target cannot be seen at all. The first thing to be destroyed, invariably, is the enemy's artillery, then the trenches are attacked. The obstructions of wire are torn to pieces, the trenches themselves are practically leveled, and after that is done, in the assault, there is what is termed the barrage fire. That, I suppose you know means a fixed or slowly moving curtain of shells dropped on a certain given line and through which passage is almost impracticable.

On both sides the artillery is carefully camouflaged so it cannot be seen from aeroplanes. To show what care is taken, even the tracks that are made in taking the guns to the front are wiped out, the guns themselves are covered, so that neither from an observation balloon nor an airplane from the enemy's line can the position of the gun be seen, and in order that the flashes of the gun cannot be located, there are dummy guns placed at intervals, and flashes from these guns made by electricity, so that the position of the real guns cannot be known.

In order to determine the position of the real guns, there are delicate instruments which have been devised, which are placed at intervals along the line. These instruments are for the purpose of registering the sound of the gun. There is, first of all the sound of the gun in firing. That is preceded frequently, if the range be great, by the sound of the shell passing through the air, and sometimes by the bursting of the shell itself, before the sound of the gun comes. These are all recorded, and the velocity and the direction of the sound is known. By having these instruments at different points on the line, the position of any one gun can be "spotted," and "spotted" so closely that our own artillery fire can be directed and the gun blotted out. That is one of the improvements of modern warfare rendered possible by the advance in general human knowledge, particularly in electrical knowledge, and these instruments are very exact.

This service of the rear is of great importance and magnitude. Picture to yourselves what is required to transport, house, supply and maintain a million men three thousand miles from home, producing nothing and in their work expending enormous amounts of materials.

Taking the question of storage alone, the provision of space required for an army of 1,000,000 for ninety days aggregates 20,000,000 square feet of floor space of covered storage and double that amount of uncovered storage space, with the necessary railway tracks for receipt and shipment and for classification yards, aggregating about 650 miles. Add to this an equal mileage of highways, adequate provision for water supply, sewerage and electric lighting and power and you can realize the work involved in this one item. Add to this the constructions which have been built at the Ports of debarkation (at one of which 375,000 square feet of wharf space had to be provided), the hospitals, barracks, shops, and the lighting, water and sewerage systems required, and some conception of the actual new construction work done, can be formed.

It is estimated that the supply of the army requires the transportation to the front of 25 lb. per man per day. This makes heavy demands on the French railway systems, good as they are. These have had to be supplemented in all but the main line trackage, and a large amount of motive power and of rolling stock has had to be supplied and operated.

Among the special services, the work of the Geologists must be mentioned, and in the line of improved apparatus, it may be stated that new instruments and methods for airplane photography have been devised and introduced. Other new auxiliary aids for fighting have been worked out, some of which have already proved their value on the battle field.

Yes, the Engineers are doing their work well. Be it in constructions in the rear, or under fire, be it in the transportation of ammunition to the firing line, the construction of strong points and obstacles, the construction and destruction of bridges in the face of an enemy, or as in recent instances, under the feet of the enemy, or be it with their rifles in beating back an attack, they are doing and *dying*. All glory to our Comrades in arms in France! There is not a red-blooded American who does not envy them.

But is there not a war duty for us also, for us who are held on this side of the ocean? Yes, undoubtedly. To some is allotted a task in supply, to some a task in manufacture, to some a task in organization. But that is not all. The life of the nation must go on. Her civil machinery must function undisturbed. With so many called away from these civil duties, the onus of the work will be the heavier for those who are left. Let us each then do his bit as and where it presents itself knowing that if each does his best, with love of country and forgetfulness of self as guides, the results are sure.

Gentlemen, I have been in the front line trenches, and a more abominable place for a man to be in, apart from any enemy, you can hardly imagine. It is raining there much of the time. The mud is up to your ankles, even though the trench be drained and the best attempt made to keep the drainage in good condition. The men at the front are distant from their supports, and they are scattered, maybe a dozen at a place, directly in the face of the enemy, and they have to stay there, and have to be prepared to resist any attack that comes, with a certain knowledge that should the attack come in force, or should a real earnest effort be made to take the line, they in the front line are almost sure to perish, and they are not minding it. They are staying there in the face, not of possible death, but in the face sometimes of almost certain death. They are doing their work, and our engineers have the most dangerous part of that work to do.

You remember in that first British drive to Cambrai, the Boches got in the rear of the firing line. They struck a part of one of our Engineer Railway Regiments, building light railways to the front to bring up ammunition. These men had not their arms with them. All they could do was to scatter and get in shell holes, and as soon as the Boches went by to rally and go back, and take the arms thrown down by the wounded or dead, and then pass over, and they formed their part of the line and did their share of the fighting.

The evening of the great drive, I had the honor of dining with Sir Charles Douglas Haig. He then told me of one of the battalions of the Sixth Engineers, which had been with the British about six weeks, and in that time they had built twenty bridges with spans, ranging from 16 to 60 feet, and he spoke of the wonderful work they had been doing. Two days afterward these men took their place and held about a mile of the British line, and held it so that the Boches could not get through.

The same happened with one of the railway regiments, again caught in the same way, and they formed a part of that miscellaneous army that Gen. Carey got together of the supposedly non-combatant troops, and the line which they formed prevented the capture of Amiens. They held the Germans. There, again, the engineers played their part valiantly.

Just two days ago a cablegram came across from the other side, about one of the companies, Company C, I think, of the First Regiment, which had been ordered to do particularly dangerous work in the neighborhood of the Marne. As they started out, two of the officers were killed at once, but the rest went on and did what they were ordered to do.

Then again, you have noted that when the Germans got down to the Marne and were crossing a bridge, some of the engineers waited until the head of the Germans had gotten completely across the bridge, which was filled with the Germans, and they blew up the bridge, and those who got there wished they had not.

Our men are doing their work on the other side wonderfully, and those are our brothers in the profession who have gone over and now wear the uniform.

(General Black then exhibited some moving pictures showing the organization of the Engineers and how they do some of their work, and finally some of the work that they have already accomplished in France, after which he added):

In landing in France, you are struck at once by the number of maimed men you see on the street and by the women in black, and then, as you get to know them, you will see that these people are from their suffering simply the more determined to carry this war through to a successful finish. The same thing is true in England. You are struck by the grim determination to win success at whatever cost. There, due to a difference of temperament, things are taken somewhat differently. I was at a tea in England, and was presented to an English woman of title. She was gowned just as any woman would be for an afternoon reception. In speaking to her I mentioned I had two boys in France. She said, "I did have four, I have only two now," and then seeing me look a little startled, she said, "You know we feel our private griefs must not be allowed to show, that we must not wear mourning, that we must go ahead with our ordinary duties and try to keep up the social life."

We have only just begun to fight. We have not begun to suffer. If you were there, and could see what these people have done, and what they are prepared to do, and then feel as you would feel, how completely they trust us to bring this war to a successful finish, you would feel, too, that it is up to us to equal them in endurance, equal them in sacrifice, and to see that freedom, individual freedom, is preserved for all time in this world. I thank you very much.

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ELECTRIC POWER FOR NITROGEN FIXATION

BY E. KILBURN SCOTT

ABSTRACT OF PAPER

Reference is made to propaganda against processes for making nitrates from air by those interested in keeping the Allies dependant on supplies of nitrate from Chili.

A tabular comparison is made of the operations involved in the *indirect* method and the *direct* method of fixing nitrogen. The indirect method involves the manufacture of carbide of calcium and its combination with nitrogen to form calcium cyanimid, from which ammonia and in turn nitric acid are obtained. The direct method merely consists in combining nitrogen and oxygen of the air in the electric arc.

In the direct method electric energy is the only factor, whereas by the indirect much plant of a very diverse and complicated character is required. Also there are difficulties in connection with the platinum catalyst necessary to convert ammonia into nitric acid. It is claimed that the direct method is better because of the simplicity of plant and of operation, and the possibility of working with off peak power. The suggestion is made that a number of plants for making nitrates by the direct arc process should be erected at existing power houses. Keeping the generating plant more fully employed would improve the load factor and reduce costs.

By making nitrate in a number of centers the transportation of same to the explosive factories would be reduced and the risk of interruption of supplies in case of accident or sabotage would be less than in having a few very large factories.

A diagram is given showing the layout of a battery of by-product coke ovens with an electric power house worked by the surplus gas and a nitrate from air plant to use the electricity. Figures are given showing that the nitric acid made by such a plant is about the right amount to combine with the ammonia to form ammonium nitrate, a compound in great demand at the present time for explosives.

ONE of the most powerful combinations in the world is that connected with the exploitation of Chile Nitrates, and to extend the uses of that material and regulate prices, etc., there is a Chile Nitrate Committee supported by the various interests concerned.

It was created for propaganda work amongst farmers and others, to facilitate the use of nitrate as a fertilizer but since the advent of air nitrates some attention has been given to discrediting the methods of fixing nitrogen from air. This has been done partly by paragraphs in the press throwing doubt on the financial and technical success of such methods, etc. German influence, working through political clubs and the press, also assisted the Chile nitrate propaganda while at the same time German scientists were being assisted in every possible way to develop air nitrate processes in their own country.

Years before the war, some of us saw that the question of supplies of Chile nitrate for the manufacture of explosives would be an important factor, and in 1911, at the Portsmouth meeting of the British Association, and later at a meeting of the Society of Arts in London, I sounded a note of warning.

Immediately after the war started, the German government appointed an electrical engineer, head of the Allgemeine Elektricitäts Gesellschaft, to expedite the manufacture of explosives· Air nitrate plants already in operation were greatly extended, and new nitrate plants and the power houses necessary to supply them with electricity, were put in hand. One such power house built early in the war at Bitterfeld, develops 185,000 kv-a. from lignite coal.

Even after three and a half years of war, the Allies still remain practically dependent for explosives on supplies which have to be brought thousands of miles across the sea from a foreign country.

So far as Great Britain is concerned this policy of dependance on supplies from oversea, appears to have been dictated by the Ministers of munitions, two of whom have been lawyer politicians, and one a doctor of medicine. They were probably acting partly on the advice of the head of the department of explosives supplies, also a member of the legal profession.

Considering how much this is an engineer's war, and how expert are the men in charge of the departments in Germany which handle such matters, does it not seem absurd that any of the technical affairs on the Allied side should depend on persons whose particular ability in life is to make speeches that catch the popular vote.

The transportation of Chili nitrate requires much shipping that might be used for other purposes and also occupies attention on the part of the Navy, in order to keep open the sea routes· In 1917 the nitrate imported to United States from Chili amounted to 1,742,540 tons (see figures of Imported Supplies given toward the end of paper) and presumably at least as much more would go direct to Europe from Chili. The average export taken over every day in the year is at least 10,000 tons, and as a round trip is about three months, an easy calculation shows that about 1,000,000 tons of shipping valued at $150,000,000. is tied up in this Chili nitrate carrying trade.

Huge sums have also to be paid for the nitrate, and to shippers and brokers for transportation, insurance, etc. On the other hand by being independent of Chili, the Germans have kept their money at consequent financial and economic advantage to the country.

With the right men at the head of affairs, possessing expert first hand knowledge of the engineering and chemical problems involved, the Allies could have become equally independent of Chili and much money and man power which have been absorbed in building obsolete chemical and other plants would have been saved.

To their credit, certain scientists and engineers of this country not only saw the danger, but insisted on the authorities taking action by providing money to establish plants for the manufacture of nitrates.

At the same time, in this country as well as in England, there has been time lost, owing to certain parties maneuvering to obtain the adoption of their own process to the exclusion of others. In so large a field as nitrogen fixation there must necessarily arise numerous improvements in the various processes so that it is not possible today for anyone to gage or forecast their future relative economic values.

This is particularly the case with processes in which electrical energy plays a leading part for it is a sort of ingrained habit of the electrical engineer to simplify and revolutionize existing methods that they eventually become essentially electrical. The whole history of electrical progress, and especially of electrochemistry and metallurgy establishes that fact.

Investigation boards or committees dealing with nitrate problems should be largely made up of engineers who have expert first hand knowledge of electric power conditions and of apparatus, etc. It is not right that chemists should have practically sole power to pass upon processes in a field which electrical engineering is capturing so completely as the production of nitrates.

I feel that the merits of the arc flame process for making nitric acid have not been adequately and sympathetically considered, and this paper is written with the special object of stating them. I wish also to remove the misconception that the arc flame process is dependent on water power and that it can only be installed economically on a very large scale. The matter is one of special interest to electrical engineers especially those concerned with the production and supply of electrical energy.

METHODS OF MAKING NITRIC ACID

Atmospheric nitrogen can be fixed either as nitric acid or as ammonia. The first named forms a basis of all explosives, whilst ammonia is used for one explosive viz. ammonium nitrate and for that purpose it must be combined with nitric acid.

The only way to make nitric acid direct from the air is to combine the nitrogen and oxygen of the air in the electric arc, and then combine the nitrous gases so produced with water to form acid. Owing to its simplicity this is called the *direct* method.

To make ammonia from atmosphere nitrogen the most straightforward method is the synthetic which has been adopted at Nitrate Plant I at Sheffield, Ala. By this method pure nitrogen and pure hydrogen are raised to a high pressure and temperature and in the presence of a catalyst they combine to form ammonia.

Another method called the *indirect* is used at Nitrate Plant II at Sheffield and briefly this is to make carbide of calcium and treat it with nitrogen to form calcium cyanamid, the cyanamid being then acted on by steam to produce ammonia.

Having obtained the ammonia it has now to be made into nitric acid, and this is done by the catalytic process of Prof. Ostwald using platinum as the catalyst. The method adopted at Sheffield, Ala. is to take about half the ammonia produced, and convert it into nitric acid and then combine this acid with the remainder of the ammonia to form ammonium nitrate. Clearly this makes just half the quantity of ammonium nitrate that would be possible, if all the ammonia was to be combined with nitric acid made in some other way, as for example by the direct arc process.

For some considerable time there has been great shortage of ammonia, and therefore, to use any of it as raw material for another nitrogen compound—nitric acid—seems short-sighted policy.

The unit value of nitrogen in ammonia, is just as high as the unit value of nitrogen in nitric acid, so it is economically and financially unsound to at any time make nitric acid from ammonia as a raw material. For this reason it is impossible for the manufacture of nitric acid via cyanamid and ammonia to compete under normal conditions with other methods.

Those interested in the indirect method have drawn comparisons between it and the direct electric arc process, with the object of showing that the indirect is the better. A tabular statement of all the operations involved in the two processes as given below is the best way to make the comparison.

	Indirect Method Employing Calcium Cyanamid to Make Ammonia and Oxidising the Ammonia to Acid by a Catalyst	Direct Method Employing the Arc Flame Furnace Only
Factories	1. To make calcium carbide.	1. To make nitric acid.
	2. To make cyanamid.	
	3. To make nitric acid.	
Operations	1. Burning limestone.	1. Blowing air through electric arc flame to produce nitrous gases.
	2. Grinding lime.	2. Absorption of gases in towers to produce acid.
	3. Grinding coke or anthracite.	
	4. Mixing lime and carbon in correct proportions.	
	5. Making calcium carbide in electric furnaces.	
	6. Grinding carbide to fine powder in neutral atmospheres.	
	7. Making liquid air to produce pure nitrogen.	
	8. Packing calcium carbide into retorts.	
	9. Making calcium cyanamid by adding nitrogen and by heat of electric resistors.	
	10. Emptying cyanamid from retorts.	
	11. Grinding cyanamid to a fine powder.	
	12. Hydrating cyanamid to rid it of unchanged carbide.	
	13. Superheated steam.	
	14. Treatment of cyanamid with steam in autoclaves to produce ammonia.	
	15. Cooling the ammonia and elimination of water.	

16. Oxidation of ammonia to produce weak nitrous gases by means of platinum catalyst.
17. Absorption of gases in towers to produce acid.

	INDIRECT METHOD	DIRECT METHOD
Raw Materials and Renewals	1. Lime.	1. Air.
	2. Coke.	2. Water.
	3. Carbon electrodes in carbide furnaces.	3. Metal electrodes.
	4. Carbon resistors in cyanamid retorts.	
	5. Pure nitrogen.	
	6. Superheated steam.	
	7. Air.	
	8. Water.	
Electric Energy for	1. Carbide furnaces.	1. Arc flame furnaces.
	2. Grinding carbide.	
	3. Cyanamid retorts.	
	4. Grinding cyanamid.	
	5. Heating catalyst.	
	6. Motors for power, etc., including several cranes.	
Skilled Labor for	1. Carbide furnaces	1. Arc flame furnaces.
	2. Cyanamid retorts.	2. Absorption plant.
	3. Packing cyanamid.	
	4. Grinding machinery.	
	5. Making pure nitrogen.	
	6. Making ammonia.	
	7. Catalytic process.	
	8. Absorption plant.	

It is frequently stated that the amount of electric energy required for a given quantity of nitric acid produced by the indirect process, is less than that required by the direct, and this is put forward as a strong argument in favor of the indirect method. Clearly, however, the only way to compare two methods is to take into account *all* the factors which go to make up the total cost, and appraise them all at their proper values.

If two processes are to be compared as regards one factor only, then it may with equal justice be claimed that the electric energy represented by a few motors and lights required for a plant making acid from sodium nitrate, is less than the electric energy required by all other processes for making acid. Such a statement does not prove anything, and yet it is similar to the one put forward by the advocates of the *indirect* process.

Even when comparisons are made on an energy basis, they are useless, unless *all* forms of energy are included. For example, the steam for the autoclaves of the *indirect* process has to be at 150 lb. per sq. inch superheated to 350 deg. fahr. and this requires considerable heat energy in the form of burning coal. Steam is also required for evaporation of water from the products of the end process, and for the distilled water for the acid absorption towers. The heat energy required for these various purposes should therefore be added to the electric energy taken by carbide furnaces, the cyanamid retorts, and for heating the catalysts.

On the other hand, the direct arc process raises its own steam, from heat of the hot gases coming from the furnace. This steam is sufficient for the end process and for distilled water and also under certain circumstances, may be used for generating electric power, so that the process can work regeneratively. At the Norwegian plant at Rjukan II, for example, there are three 4000 kw. steam turbo-generators, generating electricity with steam raised by the furnace gases.

The energy which can be regenerated in this way may be over 15 per cent, and this should of course, be deducted from the total electric energy measured into the furnace, when estimating the energy used for a given yield.

Obviously the cost of plant using the indirect method, will be very much greater than that in the case of the direct, for if we assume that the cost of a carbide furnace and its accessories is about the same as that of an air nitrate furnace with its accessories, then, the indirect process embraces in addition:

1. A complete plant for making cyanamid.
2. A liquid air plant for making pure nitrogen.
3. Powerful machinery for grinding the carbide and the cyanamid.
4. Steam boilers and autoclaves for making ammonia.
5. A complete catalytic plant for oxidizing the ammonia to nitric acid.

In the indirect method it is essential to have all the materials, gases, etc., absolutely pure, for example at the cyanamid works at Odda in Norway it was necessary to carry a pipe up the mountain side so as to ensure a supply of pure air to the liquid air plant.

When carbide is converted into cyanamid some of the former remains unchanged and in order to obviate danger of explosion a special treatment of the mixture is necessary to ensure a total decomposition of the remaining carbide.

To convert ammonia into nitric acid a catalyst is employed and this usually takes the form of a net of platinum wire, the surface of the wire being covered with finely divided platinum black. The catalyst is readily poisoned by impurities in the gases etc., when the yield then falls considerably. The process is therefore a very delicate one and requires much skill to operate successfully. To "reactivate" the metal, it is necessary to subject it to an acid treatment and eventually to remelt, in which process it is impossible to avoid loss of this expensive metal.

Russia is almost the sole source of platinum, and whilst our Ally, could be depended upon. Today with Germany practically controlling that country, the position is serious. The various allied Governments have had to commandeer platinum as it is essential for several war purposes. With utmost deliberation and foresight the Germans are working to control the worlds storehouse of platinum in the Ural Mountains and any processes which depend upon this rare metal are going to be very seriously handicapped. I consider that those who have had a hand in starting new processes dependent on platinum are very blameworthy. Non-technical politicians could not be expected to know these things but those who did know should have informed them.

By the direct method the cost of air is nil, and the cost of water is practically that of pumping. On the other hand, the materials required in the indirect method are very expensive and especially difficult to obtain at the present time. Over three fourths of the cost of working the indirect process is represented in materials liable to price fluctuation. These are now much higher than before the war, and will remain at the higher level after the war.

In the direct method less than one-fifth of the total cost is represented in materials dependent on market rates, and the principal item of cost, namely electric power, will, if anything, tend to come down in price.

The direct method is very simple to operate, whilst the indirect requires much skilled and unskilled labor, and some of the operations are dangerous to health. Therefore, the more labor demands increase, the more will the indirect method be handicapped in this respect. There are many separate links involving exact operating, to make the whole run smoothly and the slightest hitch in connection with any one link necessarily holds up the whole system.

The manufacture of cyanamid has to be carried out in retorts of relatively small size involving much labor to set up, etc. This is to enable the nitrogen gas to penetrate to all parts of the contained carbide. The times of the reaction and the cooling down, etc., are definitely fixed and it is quite impossible to work the process with off-peak power; also should there be an accident or failure of current for a time, the cyanamid retorts and the carbide furnaces may be injured.

All nitrate processes have a military bearing as regards preparedness, in which is involved the question of transportation. The heavy and bulky raw materials necessary for the indirect process places it at a serious disadvantage from this point of view, especially at the present time when the railways are so congested. With the direct process there is no carriage of raw materials.

The indirect process has been strongly advocated in that after the war, cyanamid will be much used as a fertilizer. On the other hand a large number of objections have been voiced against such use, as witness the following extract from a book by Dr. Brion: "Cyanamid cannot be used with a large number of soils such as very sandy or moor soils, or with such soils as tend to become acid. Further it cannot be used for growing tobacco nor for some kinds of fodder. It is useless as a top dressing and can be applied only in dry weather when it must be plowed in at once. Cyanamid attacks the eyes of men handling it."

Whilst some of these objections may have been overcome by making the cyanamid granular and probably also some of them are over-emphasized, it still remains true that cyanamid is by no means as good a fertilizer as nitrate.

The effect of the calcium in calcium cyanamid in the presence of moisture is to cause the reversion of phosphoric acid and therefore it can only be used in limited quantities in a combined fertilizer.

In a legal action in the State of Maine between the Armour Fertilizer works and Ellis Logan, in April, 1916, there was expert testimony that calcium cyanamid destroyed a crop of potatoes and that not more than 60 to 70 lb. of it should be used per ton of fertilizer.

ELECTRIC POWER

As a basis load for a power house the direct arc process presents the advantage that it can be established anywhere, because the raw materials being only air and water, considerations of transportation do not enter into the situation.

It is particularly suitable for off-peak or off-season loads, for there is no fused material to solidify, and little to deteriorate in case of stoppage. Some of the furnaces can be switched on and off like an arc lamp, without detriment to brickwork or structural details, or to the process of manufacture.

As there seems to be some doubt as to the possibility of running arc furnaces intermittently on a commercial scale, I would mention that about seven years ago a nitric acid factory was built at Legnano, Italy to utilize 10,000 horse power, especially during the night (see Utilization of Atmospheric Nitrogen by T. H. Norton p. 68). Of course this plant has been considerably extended, especially since the war. I am also credibly informed that in Germany there is a very large arc process plant working with off-peak power. At any rate there is no difficulty in doing it, whereas it is impossible to work intermittently with any other method of fixing atmospheric introgen.

In some ways, it is an advantage to run a plant for 8000 or less hours per year, instead of the full number, because the spare time can be conveniently used for renewals and repairs, Less spare plant is thus required and the plant can be operated by two shifts of men.

Because the plants in Norway are very large and only use hydroelectric power, a mythology has grown up, that the arc flame process can only be worked commercially on a very large scale, and with water power. It is, however, worth while to build plants in units to utilize 10,000 kw.

As a matter of fact hydroelectric power may be a disadvantage because of its distance from industrial centers, for either the factory has to be placed in an out of the way position, or else the power has to be transmitted over a long transmission line. I am of the opinion that electrochemical factories should be placed near the power supply, and the ideal position is alongside the power house especially if off-peak power is used.

In a national emergency it is surely better to bring into immediate use all the surplus equipment that already exists, than to start building new power houses, whether hydraulic or steam, and seeing that the direct-arc flame process is suitable for working with off-peak power, I suggest that a number of nitrate plants be forthwith erected at existing power houses.

By erecting say, ten or more nitrate plants of say 10,000 kw. each at power houses near places where nitrates are required, there would be great saving in transportation and early deliveries of nitrate could be made. Further there would be less risk of temporary interruption of supplies in case of accident or sabotage.

As a matter of fact there are power houses which could easily spare more than 10,000 kw. for over 20 hours a day and through the week end. Also there are power houses fully equipped with steam plant now standing idle which in the present crisis they might just as well be brought into use even if the cost of generation is high.

To give an idea of plant available, the following particulars of power houses will be of interest: From the existing plants in New York City, Mr. F. Hedley has recently estimated that about 115,000 kw. of off peak power might be supplied. In Chicago the existing power houses have a capacity of 470,000 kw. with additional 42,000 kw. now being installed. The maximum peak load of December 26, 1917 was 290,000 kw. for only half an hour, and by shutting off heaters on the surface and elevated cars during this short period, the engineers estimated that at least 90,000 kw. would be available.

There is no doubt that by restricting what may be called the luxurious uses of electricity, a considerable amount of generating plant in various parts of the country can be released for urgent war purposes, and this can be done without much increasing the coal consumption. There are also, the water power plants.

I was in Birmingham, England when for some months the street car service was stopped every day for between meal times, in order to save coal. The people readily adapted themselves to the conditions and the extra walking did the business people good whilst lack of street cars in the afternoon kept the women at home who were not engaged in war work.

In the numerous coal fed power houses of this country, a very large amount of fuel is wasted by having to keep the fires banked in the boilers during the valley periods of the load. Therefore there would be considerable saving by linking such stations to an industrial process which depends essentially on the utilization of electric energy, and the furnaces of which can be started and stopped with the facility of switching on and off an arc lamp.

The direct arc process fills these conditions perfectly, and it is the only chemical or metallurgical process which can be utilized easily with off-peak power. It takes the alternating current exactly as generated, for the furnaces can be built to utilize voltages generally available. There is no fused material_to become frozen when current is shut off as in the case of carbide of calcium furnaces, steel furnaces, etc.

Some electrolytic processes may be run intermittently but they require direct current, and therefore there is great additional cost for transforming machinery etc.

Hitherto power station managers, have been content to merely supply electric energy and have taken little interest in the purposes for which the energy was used, at any rate from a financial point of view. The time seems to have arrived when this policy should be changed, and every power house, should have its own base load factory to utilize off peak power. I know of no better form of such load, than the manufacture of nitrates from the air by the direct arc process.

By filling in the valleys the load factor is much improved and this would have the immediate effect of reducing costs. There has, however, been too much consideration given to questions of cost. With U-boats on the high seas trying to stop supplies of Chile nitrate, the railways congested with traffic and electrical works engaged in making munitions, it is of little use discussing power costs. The thing to do is to make *full* use of plants already installed.

Recently much has been heard of the suitability of Muscles Shoals, Alabama as a site for the manufacture of nitrates because of the water power which is to be developed there, but it will take at least four years to complete the hydraulic works. In the meantime a large steam power house is being built in order that the cynamid process may be put in to early operation. This includes a 60,000-kw. turbo generator and should anything happen to it, the nitrate plant would be stopped as the various steps of the indirect cyanamid process are so interlocked.

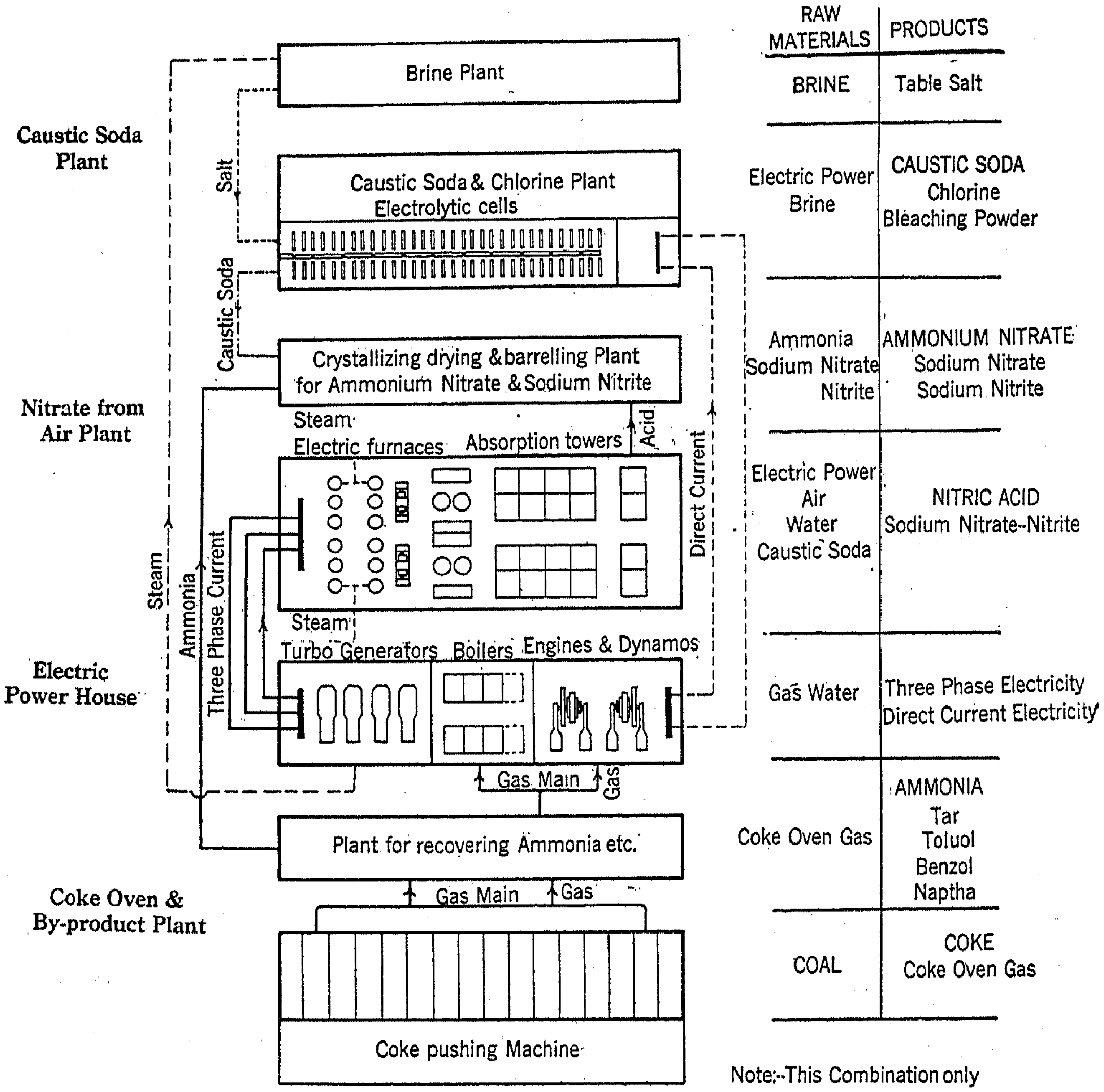
Viewed from this standpoint it would seem to be better in every way to have the manufacture of indispensable materials for explosives manufactured in a number of smaller plants, in widespread centers and by other processes, than the indirect or cyanamid method.

COKE OVEN AND NITRATE PLANTS

At the present time ammonium nitrate is required in very large quantities for burster charges for shells, torpedoes, mines grenades, etc. This is made from two components, viz., nitric acid and ammonia, both of which are difficult to transport, the first because it is a corrosive acid and the second because in every ton of aqua ammonia there are about 2½ tons of water.

An industrial plant capable of furnishing electric energy as well as a supply of ammonia would be ideal, and it so happens that this is the case with a regenerative coke oven plant. Half the total gas made is available and this can be easily turned into electric energy whilst at the same time the nitrogen contained in the coal provides about the right amount of ammonia necessary to combine with the nitric-acid made from the electric energy by the arc flame process.

In order to show how ideal such a system is for making ammonia nitrate, I have prepared following diagram.



Note:-This Combination only requires Coal & Brine to be purchased outside.

DIAGRAM LAYOUT OF NITRATE FROM AIR PLANT WITH ELECTRIC POWER HOUSE USING COKE OVEN GAS

The scheme provides for a combination of a battery of coke ovens with an ammonia recovery plant together with an electric power house in order to utilize the surplus gas. Alongside the power house, there is an electrochemical plant for the manufacture of nitric acid from air by utilizing the three-phase high-tension current. A nitrate house is provided for the purpose of combining the ammonia from the coke ovens with the nitric acid from the elctrochemical plant.

It happens that the by-product of the acid factory is sodium nitrate-nitrite, which is made by combining the gases remaining from the acid towers with caustic soda or soda ash. Electrolytic cells may be laid down as shown in the diagram for purpose of making caustic alkali from brine.

It will thus be seen that the complete project requires only two raw materials viz., coal and brine, and on the other hand, the products which can be made are coke and ammonium nitrate together with toluol, benzol, naptha, tar and sodium nitrate-nitrite.

If electrolytic cells are used there are also the products chlorine and bleaching powder. The chlorine can be combined with the benzol to form chloro-benzol which is an important intermediate in the manufacture of dye-stuffs as well as in the manufacture of picric acid.

Tri-nitrotoluol or T. N. T. is a combination of nitric acid and toluol and it will be noted that the plant supplies both these.

From the point of view of efficient management, and of elimination of transportation charges, the combination is unique, for the ammonia has only to be piped a few yards to the nitrate house and there is no carriage of acid.

As a cheap supply of coal is indispensable for the project, it would be well to locate the plants at industrial centers where this raw material is readily available and which in all probability would be locations where transportation charges are low.

In order to show what can be done with a coke oven plant, the following particulars will be of interest. I take a Koppers type of oven as being the better known, but the figures have general application.

Quality of Coal	Tons per charge	Hours coking time
Low volatile coal	13½	18
Mixture containing 80 per cent high volatile 20 per cent low volatile	12½	16½
High volatile coal	11½	15

A battery of ovens varies in size but we may as well take a round number of 100 for which the average yields are as follows:

Number of ovens	100
Tons of coal per oven	12½
Hours coking time	16
Total yield of coke	72 per cent
Yield small coal and breeze	5 per cent
Net yield good coke	67 per cent
Ammonium sulphate per ton of coal	25 lb.
Reckoned as ammonia per ton of coal	6½ lb.
Tar per ton of coal	9 gal.
Light oil per ton of coal	3 gal.
Total gas per ton, of coal	11,000 cu. ft.
British thermal units	550 per cu. ft.
Surplus gas	55 per cent
Surplus gas per ton of coal	6,000 cu. ft.

Such a battery of ovens, each of which distils 12½ tons of coal in 16 hours, will deal with

$$\frac{100 \times 12.5 \times 24}{16} = 1{,}900 \text{ tons per day}$$

Assuming 6000 cu. ft. of surplus gas per ton of coal and 550 B.t.u. per cu. ft. the total heat value per hour will be

$$\frac{1900 \times 6000 \times 550}{24} = 260{,}000{,}000 \text{ B.t.u.}$$

If employed in gas engines using 13,000 B.t.u. per h.p-hr. the power will be

$$\frac{260{,}000{,}000}{13{,}000} = 20{,}000 \text{ h. p., or say, } 14{,}000 \text{ kw.}$$

If steam boilers and turbines are used instead of gas engines the power will be less, so to be on the safe side, we will take the round figure of 10,000 kw.

We will also assume that electric furnaces utilizing 10,000 kw. for a whole year, can produce 6,300 tons of 100 per cent acid. Nitric acid capable of furnishing theoretically 8000 tons of ammonium nitrate as indicated below:—

	NH_3 +	HNO_3 =	NH_4NO_3
Molecular weights	17	63	80
In short tons	1700	6300	8000

Allowing 25 lbs. of sulphate of ammonia or $6\frac{1}{2}$ lb. of ammonia per ton of coal, a total consumption of 1900 tons of coal per day should give.

$$\frac{1900 \times 365 \times 6.5}{20,000} = 2250 \text{ tons per annum}$$

It will thus be seen that there is plenty of ammonia to combine with the acid made by the surplus gas, even if a higher yield of acid is allowed per kw-yr. and more power is generated.

I purposely leave out of discussion, questions as to types of nitrogen fixation furnaces and of yields obtained. I may say, however, that it is not right to assume that yields are limited to those usually obtained from certain well known furnaces which must of necessity work with single-phase current.

The amount of ammonium nitrate will be less than the theoretical figure because the efficiency of the reaction is not 100 per cent, also it is usual to convert a certain amount of the gas into sodium nitrate-nitrite. A safe figure would be 6000 tons and at this rate it can be shown that with electric energy at 5 mils per kw-hr. and ammonia at 12 cents a pound, the ammonium nitrate can be made at less than half the price the Government is now paying.

In order to show how large a business the nitrogen industry has become, the following figures (compiled by Dr. Paul J. Fox) give the nitrogen balance sheet for the United States for 1917.

IMPORTED SUPPLIES

	Tons of 2,000 lb.	Tons of Nitrogen
Chile saltpetre 95 per cent $NaNO_3$.	1,742,540	272,880
Ordinary saltpetre, potassium, nitrate	4,609	645
Ordinary saltpetre and gunpowder containing 75 per cent KNO_3..	1,500	210
Ammonium sulphate..............	8,135	1,725
Ammonium chloride..............	1,073	280
DOMESTIC SUPPLIES		
Coke oven ammonia—NH_3........	113,760	93,625
Gas works ammonia—NH_3........	12,500	10,288
Calcium cyanamid at 20 per cent nitrogen	12,800	10,534

NITROGEN EXPORTED

	Tons of 2,000 lbs.	Tons of Nitrogen
Nitric acid, 15 per cent nitrogen...	486	73
Picric acid, 18 per cent nitrogen...	26,610	4,790
Dynamite, 12 per cent nitrogen	8,962	1,255
Gunpowder and smokeless powder, 13 per cent nitrogen............	223,270	29,025
Ordinary saltpetre................	875	123

In addition to the above, there are also about 8800 tons represented nitrogen in the following items which are the figures for 1917.

	Value
Loaded cartridges	$42,000,000
Fuses	34,000,000
Shells and projectiles	74,000,000
All other	202,000,000
Total	253,000,000

It will be noticed that ammonium nitrate is not included in these figures, but I assume it would be about 50,000 tons for 1917.

In Great Britain the consumption of ammoniun nitrate is now probably 400,000 tons a year, and the production here will have to be at least as much. To make this, the theoretical proportion of ammonia required is about 85,000 tons and of nitric acid about 315,000 tons.

It will thus be seen that the coke oven plants in the country could supply all the ammonium nitrate required if they were equipped for the purpose. I do not suggest that existing ovens should be so used, because much of the gas is already allocated for various purposes but for new installations of coke ovens it would be worth while to consider the use of the surplus gas and of the ammonia in the way suggested.

The idea of using coke oven gas for generating electric power for general supply is gaining ground and there are a good many such installations in the North of England where electric energy has been sold at lower rates than appears to be general in the United States. When a company owns the power house as well as the coke ovens it can of course sell the surplus gas to itself at a very low rate or even at zero. A battery of coke ovens is a good paying proposition independent of any money received for surplus gas.

The company operating at Fairmont, West Virginia is completing a new power house at the mouth of a coal mine which will have two 10,000-kw. units, and the company has 12,000 kw. in gas engine units which will be worked with gas from coke ovens, so that by 1919 there will be 15,000 kw. to spare.

Until recently most coke oven ammonia was converted into sulphate, but owing to the war demand for nitrate, more and more of it is being made into aqua-ammonia of about 29 per cent strength. In some cases this is being transported many hundreds of miles prior to conversion into ammonium nitrate and since each ton of ammonia necessitates the transportation of about 2½ tons of water, the bearing of this, on the present railway congestion is at once apparent. Tank cars have to be used and they must return empty, so the freight on the actual ammonia carried is extremely high.

There are many coke ovens of the wasteful bee-hive type in operation, which do not recover by-products and the replacement of these by modern coke-ovens would be a great immediate economic gain and meet present war conditions. Coke ovens can be built quicker than large dams for water power and they have the great advantage of giving many products besides coke which are useful for war purposes.

In the present emergency coke ovens are of great value because they give coke for making steel, gas for power purposes, ammonia for nitrate manufacture, and toluol and benzol for explosives.

After the war ammonium nitrate will be in demand for fertilizer as well as for safety explosives and other purposes The high percentage of nitrogen which it contains viz., 35 per cent and the ease with which it can be converted into other compounds makes it especially useful for conveying nitrogen in the fixed form over considerable distances.

It is more profitable to make nitrate than sulphate, because, pound for pound, the nitrate contains nearly twice as much fixed nitrogen and the nitrogen commands a higher price per unit when in the form of ammonium nitrate.

DISCUSSION ON "ELECTRIC POWER FOR NITROGEN FIXATION" (SCOTT), ATLANTIC CITY, N. J., JULY 27, 1918.

C. P. Steinmetz: I decidedly agree with Mr. Scott in the conviction that the final solution of the nitrate problem is to be found in the arc flame process or direct process. I have read numerous reports during the last fifteen years aiming to prove the contrary, but I have not been convinced, and I have always had the feeling that the chemical engineers who made these reports, while endeavoring to be thoroughly impartial, were so much more familiar with chemical methods and relatively less familiar with electrical phenomena, so that the conclusions derived by them have been influenced unintentionally by their familiarity with the one class of work and unfamiliarity with the other class of work.

I agree that the direct process requires more electric power than the cyanamid process, for instance, but at the same time there are other processes, like the ammonia process, which require no electric power, and the reason why the cyanamid process requires much less power is that of the various operations only a very small part, the first quarter, is carried out electrically. We may put the statement the other way and say that in the direct process nothing but electric power is required, and all the work is done by electric power, while in the other, the indirect processes, electric power plays only a secondary part, or no part at all.

The present situation, as we all realize, is peculiar, that we have to face an emergency. It is not a question whether the one process is better or more economical than the other one, but the question is, first, which process can be brought into operation on a larger scale in the quickest manner. I am not sufficiently familiar with all the data which are available, to judge about this, but I am willing to accept the judgment of those who have gone over the field and decided that the cyanamid process offers the best possibility of getting in operation on a large scale in the quickest possible time, but that in addition thereto extensive experimentation and development should be carried out which may lead to more efficient, and therefore final, methods, but I believe a very great mistake is being made by our nation by not considering in this proposed development, or practically not considering, the arc process.

We must realize that without a powerful nitrate industry we are defenselessly at the mercy of any power which controls the ocean, and that we must have a strong nitrate industry indigenous to this country, not merely for the time of this war, but forever after, and such a nitrate industry can subsist on a large enough scale to be useful in emergencies only if an extensive use is found in peace times, and that use must be in the fertilizer industry.

That means an extremely cheap method of production, and that means as far as power is concerned, it must be capable of operation with the cheapest power, that is, secondary power or off-peak power.

Of all the methods of nitrification the only one which is capable of efficient intermittent operation is the arc process, or direct process. The arc is equally as efficient in the second or milli-second after it is started as it is after continuous operation for days. That is not the case, however, with the cyanamid process, where even a very short time interruption means freezing of the carbide furnace and serious loss and interruption.

I saw some time ago a confidential report by a prominent engineer sent by one of our largest utility corporations to Norway to study the Norwegian process, to see whether it could be utilized in the use of off-peak power, and the report in its conclusion said that the Norwegian process,—as I may call the arc process as it was some years ago operated in the Norwegian plants,—if it is operated, not continuously, but half the time, then even if the power cost absolutely nothing, it would not be economical, because the interest on the investment would be greater than the value of the nitrate was at that time. Possibly it is different now, but I have no reason to doubt that the conclusions were correct. I do not know whether the process has been improved at the present time, but that, together with several other features, is a part of the general situation in regard to the Norwegian method.

We hear and have definite information that the Norwegian arc process nitrate plants are commercially successful and profitable, but we also hear that the conditions there are peculiar, such as cannot be found in our country, and the peculiarity is claimed to be the very low cost of power. That is not strictly true in all respects, because off-peak power here can be secured cheaper in many cases than the Norwegian power. The peculiar condition of the Norwegian plant, as it existed some years ago when these reports were made, was a limited power with very cheap capital, so that under those conditions the greatest economy required a development of the highest possible efficiency regardless of the cost of the investment. It is about the same condition which we meet in our largest public utility corporations, like Chicago and Detroit, which, as our President has pointed out, have reached the highest efficiency in electric production, exceeding 20 per cent from the chemical energy of coal to the busbar. An attempt to use apparatus of this high efficiency in an isolated station, supplying a hotel, or in a central station feeding a small city, would be hopelessly uneconomical, for the reason that, because of the size and the load factor the cost of the capital in such a small plant is much higher. We have to balance the efficiency which we can secure against the interest on the investment required to secure the efficiency. This in my opinion, is the characteristic feature of the Norwegian plant, which makes it impossible to transfer it without any change to American conditions, especially to conditions of off-peak power, where not only the higher costs of capital under American conditions—especially under the present conditions—but furthermore, the still higher cost of capital due to its intermittent use, makes a plant aiming at the highest efficiency, at the collection of the last traces of the nitrate, makes such a plant uneconomical. This is the feature which, in my opinion, has very largely been killing the arc process in this country, because no competent electrical engineer realizing this situation was fathering it, but it has rather been done by the attempt to transplant the Norwegian method here under circumstances to which it is not adopted. It means redesign and redevelopment of this method, to a very much lower cost of investment, even at the sacrifice of some efficiency. We can better afford to waste 25 per cent of the nitrate if thereby we can get the cost of investment down to a small fraction of what the Norwegian plant costs, and thereby make it economical for off-peak power.

That is the principal point I want to bring to your attention, and furthermore, there is another matter which I can only give from hearsay, that is, that the financial control of the Norwegian nitrate plants were secured some time ago by German industries, and then afterwards were sold, and since that time we have been given to understand, intentionally or unintentionally, that the new German processes are superior, and that there is nothing in the arc process, because they have given up their control of them.

Whether that is really so, or whether they realize that the conditions in Germany are different from ours with regard to the economy of the arc process—the almost complete absence of larger water powers, the great abundance of skilled chemists and skilled chemical labor bringing about conditions which are very much more in favor of chemical processes and against the arc processes. I leave it to you to think over which may have been the reason.

But there is one feature which has seriously mitigated against the arc process, and that is the following: Some time ago a very careful investigation was made by the famous German chemist Nernst, on the temperature equilibrium of oxygen, nitrogen and nitric oxide, and he showed that at the temperature of the carbon arc the equilibrium corresponds to about 4 per cent nitric oxide. That would mean an efficiency of conversion of 13 per cent, if all this 4 per cent could be saved, but we know at these temperatures dissociation is so rapid, that we can expect to save only a part of the nitric oxide from destruction, in dropping the temperature as rapidly as possible. That means the possible efficiency must be much lower than 13 per cent.

It is true that the temperature equilibrium at the carbon arc temperature corresponds to 4 per cent of nitric oxide, and very much lower percentages at lower temperature. It has been proven however that the nitrification in the arc is not a temperature effect, but is an electrical effect, and the proof of that you can find in the TRANSACTIONS of the A. I. E. E. in the discussion of a paper by my assistant Mr. J. L. R. Hayden, who made an extensive investigation some years back on nitrification, and has given the results in his discussion. This, however, seems to have been forgotten, since at that time the problem was of lesser importance than now. He showed that in low temperature low-voltage arc, nitric oxide can be produced far in excess of the concentration corresponding to the temperature equilibrium at the temperature of the arc, and therefore the effect cannot be a mere temperature effect. He furthermore showed that the relative efficiency of different temperature arcs does not correspond to the temperature. The carbon arc, which is the hottest, is very far from being the most efficient nitrifying arc. The iron arc and the copper arc each have about the same temperature, but not the same nitrifying efficiency, but rather they are on the two extremes of the series investigated. The copper arc is the least efficient, and the iron arc is the most efficient, showing thereby that the effect of the arc in producing the combination of nitrogen and oxygen is not a temperature effect, but a direct electrical effect.

The reason why I have confidence in the arc process is not only due to the absence of the need of any other raw material but air, which makes it specially suitable for use anywhere where power is available—as pointed out by Mr. Scott, at the coke oven for instance—it is not only the possibility of the use of intermittent power, which holds out the hope of developing a powerful nitrate industry in peace times, but it is also the vast possibility of increasing the efficiency of nitric acid production by the arc process, over that reached in the Norwegian plants. Under favorable conditions, I understand, the present efficiency is from 60 to 80 grams per kilowatt-hour. The theoretical efficiency of nitric oxide production is 2500 grams of NO_3H per kilowatt-hour, so that the present best results are an efficiency of 3 per cent.

You can realize, therefore, the vast possibilities there are in increasing the efficiency, and if at an efficiency of only 3 per cent the process has become commercial under favorable conditions, we can see that there are possibilities in this method which will bear considerable efforts being made in studying the development further, and carrying on the development, which I am sorry to say has practically been neglected since the early pioneer days of Charles Bradley, the electrical engineer, who started a plant at Niagara Falls, but failed, due to the cost of investment in his particular method being out of proportion to the returns.

I want to conclude by repeating that my study of the problem—and I have been closely interested in it, not only theoretically, but I have done considerable experimental work during the past twenty years or more,—has led me to the conclusion that while the cyanamid process may have immediate use by being the quickest available, that the process which I consider as the most promising, in the final solution of the problem, is the direct or arc process.

Wm. A. Del Mar (communicated after adjournment): The arc process for the fixation of nitrogen is, in my opinion, the only use to which the off-peak capacity of large power stations can be put, as it is the only electrotechnical process that can thrive under the three characteristic conditions attending the use of such power, namely, absorption of large amounts of power, use of alternating current at the generated voltage and intermittent supply.

The cost of transforming apparatus bars all direct-current electrochemical processes such as electrolytic metal refining, alkali productions, etc. and many thermal processes. Other thermal processes are debarred because they cannot be operated intermittently. Any processes which are exceptional in these respects are too unimportant to absorb the power available from large stations.

Hence if off-peak capacity is ever to be utilized on a large scale, the only promising field for research is the arc-process for nitrogen fixation.

S. Barfoed (communicated after adjournment): As nitric acid is the end product desired it is evident that with the increasing demand upon the nation's man power naturally that system which can produce nitric acid with a minimum of labor is the most logical one to use, other things considered equal. The raw materials needed for the direct method being air and water lend themselves to transport in pipes requiring no attention. Also, the final product, nitric acid, is to be handled in the same manner by means of the new acid proof ferro-alloys. Consequently labor is not needed in this particular part of the cycle of operations.

In the operation of any arc furnaces used by the direct method a row of furnaces may be attended by but two men required for recording and renewing electrodes at intervals.

During the last two and one-half years the writer has been engaged upon the development and operation of a 1000-kv-a. arc furnace using metallic oxide electrodes. The results obtained in this furnace and the simplicity of its operation, also the ease with which it may be kept in repair justifies me in saying that the direct method for making nitric acid as compared with the indirect method has the very advantages which Mr. Scott mentions.

The simple way of getting the end product, nitric acid, with the direct method as the first and only operation must surely be evident to the layman even if he should be a lawyer-politician.

In discussing the proper system to use in nitrogen fixation it must be kept in mind that all the various processes have their proper place of usefulness, all depending upon the raw materials available and the end product it is desired to produce and have the market absorb. The choice of the system is practically decided only by the geographical location of the plant. In places where coal has been obtainable at reasonable prices and where men are rather plentiful but where water power is comparatively expensive, the end product may to advantage be cyanamid. In the greater part of the West coal is not obtainable at prices that would make the indirect method of making acid or even the cyanamid process possible under any circumstances.

The scheme outlined for combining a battery of coke ovens with ammonia recovery plant with an arc furnace plant as outlined by Mr. Scott is entirely feasible and is one to which the writer has given considerable thought, but the conditions for its use during peace times must be favorable. Even in certain sections of the West in spite of the existing water powers such a scheme can be worked out successfully for producing ammonium nitrate. Where districts are equally well off as to coal and water the several methods of producing fixed nitrogen can very well exist side by side and not exclude one another, keeping in mind the suitability of each process with a view as to raw material and the end product.

I endorse the formula "Do it Electrically" when nitric acid is to be produced.

I do not believe that at the present time there are many power stations where surplus power may be had. Also, I believe that the load factor of the majority of stations is so high that the time during which off-peak power is available would be too short for making the operation of arc furnaces satisfactory. In the West there is rather a scarcity of power and new developments by existing companies are not likely to be made in time to be of any use during the war, and that is largely because the power it is thought to develop cheaply is to be of great magnitude. There are however, throughout the West numerous small water power sites of 10,000 h. p. or thereabouts which can very cheaply be developed and completed in a short time and which would be entirely suitable for a nitric acid factory. Statements are often made that such water powers cannot be developed under a certain figure. Such statements of course do not mean anything unless they are qualified as to just what is meant to be included in a development. As generally spoken of I believe what is meant is that the development should include step-up transformers, high-tension switch gear, transmission lines, lowering transformers, substation equipment and distributing lines, etc. For such systems the figures as often given may be correct, but it would be a mistake to draw any conclusions from them when it comes to making a hydroelectric development for use with arc furnaces producing nitric acid.

It is often found that the nitric acid factory can be located in close proximity to the power station and it will be at once evident the great saving that can be made by so doing in the cost of [the plant. The development would include the hydraulic works and the generating equipment which generates power directly at the voltage required by the furnaces.

Looking at the matter in this light I have made several investigations where water power of around 10,000 h. p. can be developed for a figure around $60.00 per h. p. and lower. Where this can be done the power cost will be a minor item compared to the fixed charges on the acid factory.

F. W. Sperr, Jr. (communicated after adjournment): A very important phase of the plan which Mr. Kilburn Scott has developed is that it succeeds in reconciling the interests of two great industries—the nitrogen fixation industry and the by-product coke industry. It cannot be denied that, owing largely to the emphasis that has been placed upon fixation by the cyanamid process, there have hitherto been many elements of mutual antagonism in these two industries.

The by-product coke oven must be considered as an essential factor in our national security and military success. It is the greatest and most dependable source of tri-nitro toluol, the most important high explosive, and must be relied upon for our supply of benzol, creosote, naphthalene, anthracene and a host of other products of military value.

Many people still have the old-fashioned idea that beehive coke works better in a blast furnace than by-product coke. As a matter of fact records from a large number of blast furnaces show a saving of over 200 pounds of coke per ton of pig iron due to the substitution of by-product coke for the beehive product. The significance of this in connection with our fuel shortage and the importance of the steel industry is easy to recognize.

In the establishment of by-product coke ovens, we are still sadly behind both economic and military requirements and every agency should be interested in encouraging the construction of new plants as rapidly as possible.

It is to be regretted that the fear of loss through destructive competition with subsidized ammonia produced by the cyanamid process has operated in certain cases as a discouraging factor. In normal times, ammonia is the principal source of revenue, besides coke, to the by-product coke plant. Ammonia produced by the cyanamid process cannot compete with by-product coke oven ammonia except with the aid of a subsidy. Large quantities of subsidized ammonia or a cheap competing fertilizer like cyanamid, thrown upon the agricultural market might well induce hesitation in the building of by-product coke ovens.

On the other hand, Mr. Scott's proposition to combine the manufacture of nitric acid from the atmosphere with the operation of a by-product coke plant, utilizing the surplus gas from the latter to furnish the requisite power, and the ammonia to combine with the nitric acid forming ammonium nitrate, appears to be as economically sound in time of peace as it is desirable in time of war. From the standpoint of the by-product coke oven owner, it looks entirely feasible and encouraging. Under normal conditions, the nitric acid might be sold separately, or it might still be used together with the ammonia as ammonium nitrate. In this way the expense of purchasing sulphuric acid to make ammonium sulphate would be avoided and a concentrated fertilizer more valuable than ammonium sulphate would be produced. It is to be hoped that arrangements can be made to build a plant for a large scale trial of this very promising proposition.

E. K. Scott: Members who have to deal with electric power-house problems, are naturally interested in improvement of load factor. I know of no load which is more suitable for filling up valleys of a load curve than the arc method of fixing atmospheric nitrogen, for the furnaces can be started and stopped at any time without detriment, as there is no fixed material to freeze when current is off.

A good situation for an air nitrate factory would be on the same plot of land as the power house and the two might well be under the same directorate.

It has often been said that the manufacture of air nitrates is dependent on a very cheap source of power and the implication is made that it must be water power and on a very large scale. I wish to combat this for I consider it feasible to successfully operate moderate sized plants with off peak power from existing power houses.

It is a mistake to unduly emphasize the question of power for there are other equally important factors, *e. g.* transportation of raw materials and finished products. Nitric acid is required in very large quantities for explosives, for dyes, for celluloid and other industries and being difficult to carry, it is best made near to the place where it is to be used.

The nitric acid made in Norway, cannot be exported owing to difficulty of carriage and even when made into nitrate of lime and nitrate of ammonia the cost of carriage to a market abroad is considerable. Take ammonia for example which for many years has been made in Norway with ammonia shipped from England. It has been estimated that the charges for carriage, insurance, transhipment, etc., of the aqua ammonia to Norway and of the finished nitrate of ammonia to England came to more per ton of product than the cost of the electric energy with which the ton was made.

Other things being equal it is often better to be near to a steady market and pay a fair price for power than to go to a considerable distance and get cheaper power. Steam power houses have often an advantage over hydroelectric plants because the latter are usually in inconvenient places.

I fully appreciate the development of hydroelectric power, but think that the building of a coke oven plant with electric power house and nitrate factory is more to the National advantage than if the same money were spent on hydraulic works, etc. The coke oven proposition has certainly wider usefulness.

It may be of some interest to mention that electric energy has been supplied for many years to certain electrochemical factories on the North East Coast in England at prices which compare favorably with Niagara. A considerable amount of power developed in that area is from coke ovens and blast furnace gas and by linking the power houses together it has been found possible to bank some boilers in the large coal fed power houses in Newcastle over the week end and obtain supply from the coke oven and waste heat power houses of Durham and North Yorkshire which of course must be kept going at all hours.

Dr. Steinmetz referred to the investment on an air nitrate plant. A large part of this goes in the absorption towers and the building required to house same. Although chemists have always been accustomed to absorb gases in such towers I think a smaller and less costly method can be developed on engineering lines.

I purposely did not enter into questions regarding reactions, etc., that go on in nitrogen fixation furnaces, but I agree with Dr. Steinmetz that the action is not a purely thermal one. The fact that nitric acid may be formed on the windings of extra high-tension machines is an indication.

Suggestions of limitations in electrical methods are usually traceable to those who are interested in old fashioned chemical methods, as for example, that of getting nitric acid from Chili nitrate, but an audience of electrical engineers will appreciate that when electricity once gets a foothold in any industry it holds on.

Some of us remember the days when electric lighting was struggling against firmly entrenched gas interests. The electric street car was a long time before it finally ousted the horse car, especially in England, whilst the introduction of electric power in mines is still opposed by advocates of compressed air and so on.

Electric engineering is now developing rapidly in the largest fields of all, namely, electrochemistry and electrometallurgy. Very large amounts of energy are being called for which means greater power development and wider opportunities for electrical engineers and manufacturers of electrical equipment. Members will no doubt hear from time to time of other methods of making nitrates which do not require electric energy or require less than the arc process. They need have no fear that electrical methods will not outlast all others and I personally believe the arc process is best of all.

The name of Charles S. Bradley has been mentioned. He was a pioneer in this line of work before the Norwegians or Germans, and I think the fact is worthy of commemoration.

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AMERICA'S ENERGY SUPPLY

BY CHARLES P. STEINMETZ

Abstract of Paper

The gist of the paper is to demonstrate that the economical utilization of the country's energy supply requires generating electric power wherever hydraulic or fuel energy is available, and *collecting the power electrically, just as we distribute it electrically.*

In the first section a short review of the country's energy supply in fuel and water power is given, and it is shown that the total potential hydraulic energy of the country is about equal to the total utilized fuel energy.

In the second section it is shown that the modern synchronous station is necessary for large hydraulic powers, but the solution of the problem of the economic development of the far more numerous smaller waterpowers is the adoption of the induction generator. However, the simplicity of the induction generator station results from the relegation of all the functions of excitation, regulation and control to the main synchronous station. The economic advantage of the induction generator station is, that its simplicity permits elimination of most of the hydraulic development by using, instead of one large synchronous station, a number of induction generator stations and collecting their power electrically.

The third section considers the characteristics of the induction generator and the induction-generator station, and its method of operation, and discusses the condition of "dropping out of step of the induction generator" and its avoidance.

In the appendix the corresponding problem is pointed out with reference to fuel power, showing that many millions of kilowatts of potential power are wasted by burning fuel and thereby degrading its energy, that could be recovered by interposing simple steam turbine induction generators between the boiler and the steam heating systems, and collecting their power electrically. It is shown that the value of the recovered power would be an appreciable part of that of the fuel, and that organized and controlled by the central stations, this fuel power collection would improve the station load factor, give the advantages of the isolated plant without its disadvantages, and produce a saving of many millions of tons of coal.

I. The Available Sources of Energy

A. Coal

THE only two sources of energy, which are so plentiful as to come into consideration in supplying our modern industrial civilization, are coal, including oil, natural gas, etc., and water power.

While it would be difficult to estimate the coal consumption directly, it is given fairly closely by the coal production, at least during the last decades, where wood as fuel had become negligible, and export and import, besides more or less balancing each other, were small compared with the production. Coal has been mined since 1822, and in Fig. 1 is recorded the coal production of the United States, from the governmental reports. The annual production is marked by circles, the decennial average marked by crosses for every five years. Table I gives the decennial averages, in millions of tons per year.

TABLE I

AVERAGE COAL PRODUCTION OF THE UNITED STATES

(decennial average)

Year	Million tons per year	Per cent increase per year
1825	0.11	....
30	0.32	22.4
35	0.83	19.7
40	1.92	17.0
45	4.00	14.5
50	7.46	10.45
55	10.8	8.35
60	16.6	8.72
65	25.9	9.22
70	40.2	8.58
75	56.8	7.42
80	82.2	7.95
85	122	6.80
90	160	5.40
95	206	5.75
1900	281	6.96
05	404	6.60
10	532	....

In Fig. 1 the logarithms of the coal production in tons are used as ordinates. With this scale, a straight line means a constant proportional increase, that is, the same percentage increase per year, and in the third column of Table I are given the average percentage increase of coal production per year.

This Fig. 1 is extremely interesting by showing the great irregularity of production from year to year, and at the same time a very great regularity over a long period of time. Since 1870 the average production may be represented by a straight line, the values lying irregularly above and below the line, which represents an annual increase of 6.35 per cent and thus represents the average coal production[1] C by the equation

$$C = 45.3 \times 10^{0.0267\,(y-1870)} \text{ million tons}$$

or

$$\log C = 0.0267\,(y-1870) + 7.656$$

where y = year.

Before this time, from 1846 to 1884, the coal production could be represented by

$$C = 7.26 \times 10^{.0365\,(y-1850)} \text{ million tons}$$

or

$$\log C = 0.0365\,(y-1850) + 6.861$$

representing an average annual increase of 8.78 per cent.

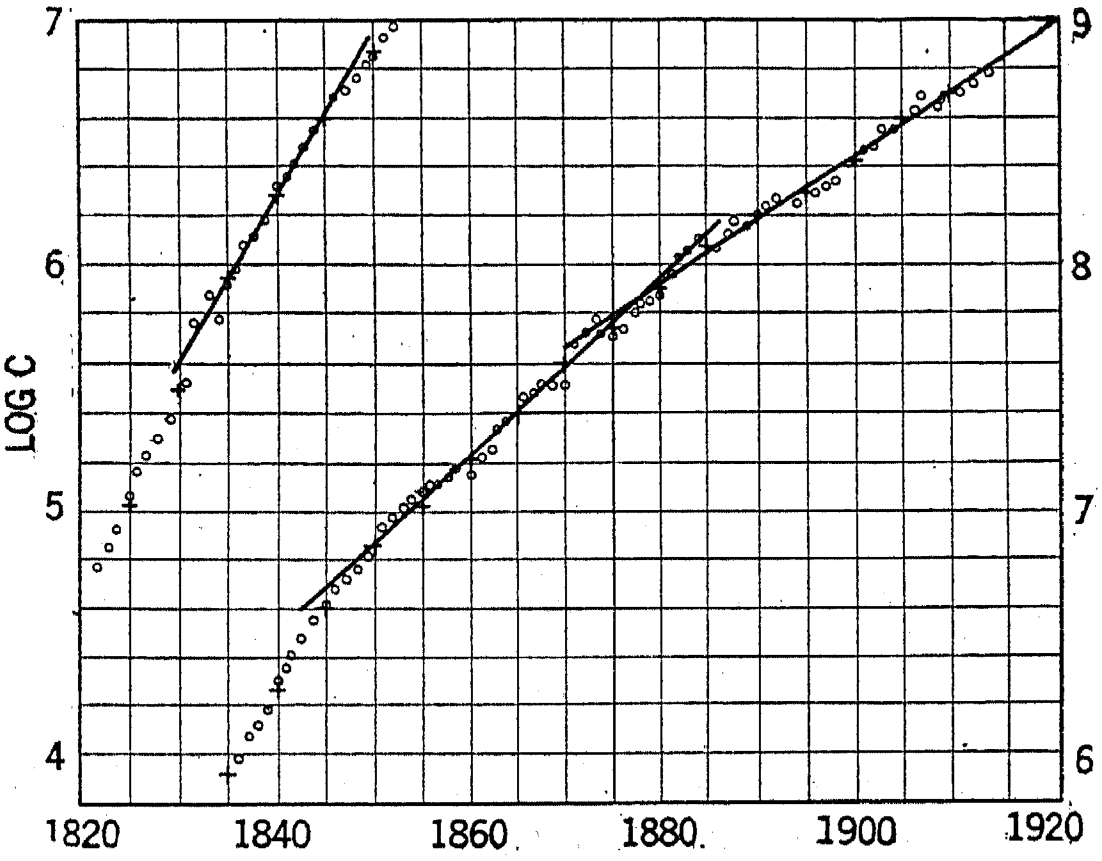

FIG. 1—COAL PRODUCTION OF THE UNITED STATES

It is startling to note how inappreciable, on the rising curve of coal production, is the effect of the most catastrophic political and industrial convulsions, such as the Civil War and the Industrial panic of the early 90's; they are indistinguishable from the constantly recurring annual fluctuations. It means, that the curve is the result of economic laws, which are laws of nature.

Extrapolating from the curve of Fig. 1, which is permissible, due to its regularity, gives 867 million tons as this year's coal consumption. As it is difficult to get a conception of such enormous amounts, I may be allowed to illustrate it. One of the great wonders of the world is the Chinese Wall, running across the country for hundreds of miles, by means of which China unsuccessfully tried to protect its northern frontier against invasion. Using the coal produced in one year as building material, we could with it build a wall like the Chinese Wall, all around the United States, following the Canadian and Mexican frontier, the Atlantic, Gulf and Pacific Coast, and with the chemical energy contained in the next year's coal production, we could lift this entire wall up into space, 200 miles high. Or, with the coal produced in one year used as building material, we could build 400 pyramids, larger than the largest pyramid of Egypt.

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[1] Soft coal and anthracite, and including oil reduced to coal by its fuel value.

It is interesting to note that 100 thousand tons of coal were produced in the United States in 1825; one million tons in 1836; 10 million tons in 1852 and 100 million tons in 1882. The production will reach about 1000 million tons in 1920, and, if it continues to increase at the same rate, it would reach 10,000 million tons in 1958.

Estimating the chemical energy of the average coal as a little above 7000 cal., *the chemical energy of one ton of coal equals approximately the electrical energy of one kilowatt year* (24 *hour service*). That is, one ton of coal is approximately equal in potential energy to one kilowatt-year.

Thus the annual consumption of 867 millions of tons of coal represents, in energy, 867 million kilowatt-years.

However, as the average efficiency of conversion of the chemical energy of fuel into electrical energy is probably about 10 per cent, the coal production, converted into electrical energy, would give about 87 million kilowatts.

Assuming however, that only one half of the coal is used for power, at 10 per cent efficiency, the other half as fuel, for metallurgical work etc., at efficiencies varying from 10 per cent to 80 per cent, with an average efficiency of 40 per cent, then we get 217 million kilowatts (24 hour service) as the total utilized energy of our present annual coal production of 867 million tons.

B. THE POTENTIAL WATER POWERS OF THE UNITED STATES

Without considering the present limitation in the development of water powers, which permits the use of only the largest and most concentrated powers, we may try to get a conception of the total amount of hydraulic energy which exists in our country, irrespective of whether means have yet been developed or ever will be developed for its complete utilization. We therefore proceed to estimate the energy of the total rain fall.

Superimposing the map of rain fall in the United States, upon the map of elevation, we divide the entire territory into sections by rain fall and elevation. This is done in Table II, for the part of our continent between 30 and 50 degrees northern latitude.

TABLE II

TOTAL POTENTIAL WATER POWER OF UNITED STATES

In. rain fall	Ft. elevation	Area $m^2 10^{12} \times$	Avg. elevation m.	Avg. rainfall cm.	Kg-m. per m^2; $10^3 \times$	Kg-m. total $10^{15} \times$
>10	>5000	0.54	2100	12.5	263	142
	1000–5000	0.29	900		112	32.5
10–20	>5000	1.18	2100	37.5	787	930
	1000–5000	1.96	900		338	660
20–30	1000–5000	0.32	900	62.5	563	183
	100–1000	0.97	150		94	91
30–40	1000–5000	0.35	900	87.5	786	275
	100–1000	1.40	150		131	184
40–60	1000–5000	0.27	900	125	1130	305
	100–1000	1.03	150		188	194
						$\Sigma = 2996$ 3000

As obviously only the general magnitude of the energy value is of interest, I have made only few sub-divisions: five of rain fall and four of elevation, as recorded in columns 1 and 2 of Table II[2]. The third column gives the area of each section, in millions of square kilometers, the fourth column the estimated average elevation, in meters, and the fifth column the average rain fall, in centimeters. The sixth column gives the energy, in kilogram-meters per square meter of area, and the last column the total energy of the section, in kilogram-meters, which would be represented by the rain fall, if the total hydraulic energy of every drop of rain were counted, from the elevation where it fell, down to sea level.

As seen from Table II, the total rain fall of the North American Continent between 30 deg. and 50 deg. latitude represents 3000 × 10^{15} kg-m. This equals 950 million kilowatt years (24 hour service). That is, the total potential water power of the United States, or the hydraulic energy of the total

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1. The lowest elevation, < 100 ft., is not included, as having little potential energy.

rain fall, from the elevation where it fell, down to sea level, gives about 1000 million kilowatts.

However, this is not available, as it would leave no water for agriculture; and even if the entire country were one hydraulic development, there would be losses by seepage and evaporation.

An approximate estimate of the maximum potential power of the rain fall, after a minimum allowance for agriculture and for losses is made in Table III, allowing 12.5 cm. rain fall for wastage, and 37.5 and 25 cm. respectively for agriculture where such is feasible.

TABLE III

AVAILABLE POTENTIAL WATER POWER OF THE UNITED STATES

Avg. rainfall cm.	Avg. elevation *m.*	Area m^2 $10^{12}\times$	Wastage cm.	Agriculture, cm.	Available rainfall cm.	Kg. m. per m^2 $10^3\times$	Kg-m. total $10^{15}\times$
12.5	2100	0.54	12.5	....	....	....	....
....	900	0.29	12.5	....	....	....	....
37.5	2100	0.39	12.5	25	....	....	....
....	2100	0.79	12.5	....	25	525	415
....	900	0.98	12.5	25	....	....	....
....	900	0.98	12.5	....	25	225	220
62.5	900	0.21	12.5	37.5	12.5	112	23
....	900	0.11	12.5	....	50	450	50
....	150	0.97	12.5	37.5	12.5	19	18
87.5	900	0.35	12.5	37.5	37.5	337	118
....	150	1.40	12.5	37.5	37.5	56	78
125	900	0.27	12.5	27.5	75	674	182
....	150	1.03	12.5	37.5	75	112	116
							$\Sigma=1220$

This gives about 1200×10^{15} kg-m. as the total available potential energy, which is equal to 380 million kilowatts (24 hour service). Assuming now an efficiency of 60 per cent from the stream to the distribution center, gives 230 million kilowatts (24 hour service) as the maximum possible hydroelectric power, which could be produced, if every river, stream, brook or little creek throughout its entire length, from the Spring to the ocean, and during all seasons, including all the waters of the freshets, were used and could be used. It would mean that there would be no more running water in the country, but stagnant pools connected by pipe lines to turbines exhausting into the next lower pool. Obviously, we could never hope to develop more than a part of this power.

C. Discussion

It is interesting to note that the maximum possible hydraulic energy of 230 million kilowatts, is little more than the total energy which we now produce from coal, and is about equal to the present total energy consumption of the country, including all forms of energy.

This was rather startling to me. It means that the hope that when coal once begins to fail we may use the water powers of the country as the source of energy, is and must remain a dream, because if all the potential water powers of the country were now developed, and every rain drop used, it would not supply our present energy demand.

Thus hydraulic energy may and should supplement that of coal, but can never entirely replace it as a source of energy. This probably is the strongest argument for efforts to increase the efficiency of our methods of using coal.

A source of energy which is practically unlimited, if it could only be used, is solar radiation. The solar radiation at the earth's surface is estimated at 1.4 cal. per cm.^{2} per min. Assuming 50 per cent cloudiness, this would give an average throughout the year (24 hours per day), of about 0.14 cal. per cm.^{2} horizontal surface per min., and on the total area considered in the preceding table, of 8.3 million square kilometers of North America between 30 and 50 latitude, a total of approximately 800,000 million kilowatts (24 hour service), or a thousand times as much as the total chemical energy of our coal consumption; 800 times as much as the potential energy of the total rainfall.

Considering that the potential energy of the rainfall from surface level to sea level, is a small part of the potential energy spent by solar radiation in raising the rain to the clouds, and that the latter is a small part of the total solar radiation, this is reasonable.

Considering only the 2.7 million square kilometers of Table III, which are assumed as unsuited for agriculture, and assuming that in some future time, and by inventions not yet made, half of the solar radiation could be collected, this would give an energy production of 130,000 million kilowatts.

Thus, even if only one-tenth of this could be realized, or 13,000 million kilowatts, it would be many times larger than all the potential energy of coal and water. Here then would be the great source of energy for the future.

II. Hydroelectric Station

A. THE MODERN SYNCHRONOUS GENERATOR STATION

In developing the country's water powers, up to the present time only those of greatest energy concentration have been considered; that is, those where a large volume and a considerable head of water was available within a short distance.

This led to the present type of hydroelectric generating station, as best solving the problem. The equipment of such a station comprises the following apparatus:

Three-phase synchronous direct-connected generators.

Hydraulic turbines of the highest possible efficiency.

Hydraulic turbine speed governing mechanism.

An exciter plant comprising either exciters directly connected to the generators, or several separate exciter machines, connected to separate turbines.

Exciter bus bars.

Voltmeter, and ammeters in exciters and in alternator field circuits.

Field rheostats of the alternators.

Low-tension busbars, either in duplicate, or with transfer or synchronizing bus.

Circuit breakers between generators and busbars, usually non-automatic.

Circuit breakers between transformers and busbars, usually automatic, with time limit.

Voltmeters and potential transformers at the generators.

Synchronoscopes or other synchronizing devices.

Ammeters and current transformers at the generators.

Voltmeter and potential transformer at the busbars.

Ammeters and current transformers at the step-up transformers.

Totaling ammeter for the station output.

Integrating wattmeter.

Relays, interlocking devices etc., etc.

Step-up transformers.

High-tension busbars, possibly in duplicate.

High-tension circuit breakers between transformers and high-tension busbars.

High-tension circuit breakers between high-tension busbars and lines.

Lightning arresters in the transmission lines, with inductances etc.

Ground detectors, arcing-ground or short-circuit suppressors, voltage indicators etc.

Automatic recording devices (multi-recorder), rarely used though very desirable.

Due to the vast amount of energy controlled by modern stations, the auxiliary and controlling devices in these stations have become so numerous as to make the station a very complex structure, requiring high operating skill and involving high cost of installation. At the same time, not only are all these devices necessary for the safe operation of the station, but we must expect that with the further increase of capacity of our electric systems, additional devices will become necessary for safe and reliable operation. One such device I have already mentioned—automatic recording apparatus, such as the multi-recorder.

With this type of station, it is obviously impossible, in most cases, to develop water powers of small and moderate size. A generating station of a thousand horse power will rarely, and one of a hundred horse power will hardly ever be economical.

On the other hand, a hundred horse power motor installation is a good economical proposition, and the average size of all the motor installations is probably materially below one hundred horse power.

Looking over Tables II and III, especially the latter, in the preceding section, it is startling to see how large a part of the potential water power of the country is represented by comparatively small areas of high elevation, in spite of the relatively low rainfall of these areas. As most of these areas are at considerable distance from the ocean, most of the streams are small in volume. That is, it is the many thousands of small mountain streams and creeks, of relatively small volume of flow, but high gradients, affording fair heads, which apparently make up the bulk of the country's potential water power.

Only a small part of the country's hydraulic energy is found so concentrated locally as to make its development economically feasible with the present type of generating station.

Therefore, some different, and very much simpler type of generating station must be evolved, before we can attempt to develop economically these many thousands of small hydraulic powers, to collect the power of the mountain streams and creeks.

B. SIMPLIFICATION OF HYDROELECTRIC STATION

In the following in discussing the simplification of the hydroelectric station to adapt it to the utilization of smaller powers, we limit ourselves to the case where the smaller hydraulic stations feed into a system containing some large hydraulic or steam turbine stations, to which the control of the system may be relegated.

1. We may eliminate the low-tension bus bars, with generator circuit breakers and transformer low-tension circuit breakers and connect each generator directly to its corresponding transformer making one unit of generator and transformer, and do the switching on high-tension busbars, and locating high-tension busbars and circuit breakers outdoors. While it is dangerous to transformers to switch on the high-tension side, due to the possibility of cumulative oscillations, this danger is reduced by the permanent connection of the transformer with the generator circuit, and is less with the smaller units used in small power stations, and thus permissible in this case.

However, the simplification resulting therefrom is not so great, as ammeters, voltmeters and synchronizing devices with their transformers are still retained on the low-tension circuits.

2. As it is not economical to operate at partial load, proper operation of a hydraulic station on a general system is, to operate as many units fully loaded as there is water available, and increase or reduce the number of units (of turbine, generator and transformer, permanently joined together), with the changing amount of available water, thus using all the available energy of the water power.

In this case, the turbine governors, with their more or less complex hydraulic machinery, may be omitted. If then the generators are suddenly shut down by a short circuit which opens the circuit breakers, the turbines will race and run up to their free running speed, until the gates are shut by hand. However, generators, and turbines must stand this, as even with the use of governors, the turbines may momentarily run up to their free speed in case of a sudden opening of the load, before the governors can cut off the water. Where this is not desirable some simple excess speed cut-off may be used.

3. When dropping the governing of the turbines, and running continuously at full load, the question may be raised whether generator ammeters are necessary, as the load is constant, and is all the power the water can give, and it might appear, that am-

meters with their current transformers, etc. could be omitted. However, with synchronous generators, the current depends not only on the load, but also on the power factor of the load and with excessively low power factor due to wrong excitation, the generators may be overheated by excess current, while the power load is well within their capacity. Thus ammeters are necessary with synchronous generators. As soon, however, as we drop the use of synchronous generators, and adopt induction generators, the ammeters with their current transformers may be omitted, since the current and its power factor is definitely fixed by the load. At the same time, synchronizing devices become unnecessary, together with potential transformers, generator voltmeters, etc. A station voltmeter may be retained for general information, but it not necessary either, as the voltage and frequency of the induction-generator station are fixed by the controlling synchronous main station of the system.

4. With the adoption of the induction generator, the entire exciter plant is eliminated, as the induction generator is excited by lagging currents received from synchronous machines, transmission lines and cables existing in the system. This avoids the use of exciter machines, exciter busses, ammeters, voltmeters, alternator field rheostats, etc., in short, most of the auxiliaries of the present synchronous station become unnecessary.

The solution of the problem of the economic development of smaller water powers is the adoption of the induction generator.

Stripped of all unnecessary equipment, the smaller hydroelectric station thus would comprise:

Hydraulic turbines of simplest form, continuously operating at full load, without governors.

Low-voltage induction generators direct connected to the turbines.

Step-up transformers direct connected to the induction generators.

High-tension circuit breakers connecting the step-up transformers to the transmission line. In smaller stations, even these may be dispensed with and replaced by disconnecting switches and fuses.

Lightning arresters on the transmission line where the climatic or topographical location makes such necessary.

A station voltmeter, a totalling ammeter or integrating wattmeter and a frequency indicator may be added for the information of the station attendant, but are not necessary, as voltage, current, output and frequency are not controlled from the induction generator station, but from the main station, or determined by the available water supply.

It is interesting to compare this induction generator station lay-out with that of the modern synchronous station given above. However, it must not be forgotten that *the simplicity of the induction generator station results from the relegation of all the functions of excitation, regulation and control, to the main synchronous stations* of the system, and the induction generator stations thus are feasible only as adjuncts to at least one large synchronous station, hydraulic or steam turbine, in the system, but can never replace the present synchronous generator stations in their present field of application.

C. AUTOMATIC GENERATING STATIONS

With the enormous simplification resulting from the use of the induction generator, it appears entirely feasible to make smaller hydroelectric generating stations entirely automatic, operating without attendance beyond occasional—weekly or daily—inspection.

Such an automatic generating station would comprise a turbine with low-voltage induction generator, housed under a shed, and a step-up transformer, outdoors, connecting into the transmission line with time fuses and disconnecting switches.

It is true that in the big synchronous generating stations of thousands of kilowatts, the cost of the auxiliaries, as exciter plant, regulating and controlling devices, etc., is only a small part of the total station cost, and little would therefore be saved by the use of induction generators. No induction generators would, however, be used for such stations. But the cost of auxiliaries and controlling devices, and the cost of the required skilled attendance, decreases far less with decreasing station size than that of the generators—whether synchronous or induction —or in other words, with decreasing size of the station, *per kilowatt output*, the cost of auxiliaries and controlling devices and of attendance increases at a far greater rate than that of the generators, and very soon makes the synchronous station of the present type uneconomical.

It is also true that in the big modern hydraulic power systems, the cost of the generating station usually is a small part of the cost of the hydraulic development. Therefore any saving in the cost of the generating station would be of little influence in determining whether the hydraulic development would be economical. With decreasing size of the water power the cost of the hydraulic development *per kilowatt output* usually increases so rapidly as very soon to make the development of the water power uneconomical, no matter how simple and cheap the station is.

However, the value of the induction generator is not so much in the reduction of the cost of the generating station, as in the reduction of the cost of the hydraulic development, by making it possible to apply to the electric generator the same principle, which has made the electric motor economically so successful: *Collect the power electrically, just as we distribute it electrically.*

We do not, as in the days of the steam engine, convert the electric power into mechanical power at one place, by one big motor, and distribute the power mechanically, by belts and shafts, but we distribute the power electrically, by wires, and convert the electric power to mechanical power, wherever mechanical power is needed, by individual motors throughout mill and factory.

In the same way we must convert the hydraulic, that is, mechanical power into electrical power by individual generators located along the streams or water courses within the territory, wherever power is available, and then collect this power electrically, by medium-voltage collecting lines and high-voltage transmission lines, and so eliminate most of the cost of the hydraulic development, to solve the problem of the economical utilization of the country's water powers. If we attempt to collect the power mechanically, that is, by a hydraulic development gathering the waters of all the streams and creeks of a territory together into one big station, and there convert it into electric power, the cost of the hydraulic development makes it economically hopeless except under unusually favorable conditions, where a very large amount of power is available within a limited territory, or where nature has done the work for us in gathering considerable power at a waterfall, etc.

It is the old problem, and the old solution: If you want to *do it economically, do it electrically.*

Naturally then, we would use induction generators in these small individual stations just as we use induction motors in individual motor installations; but where large power is available, there is the field of the synchronous generator, and the induction generator is undesirable, just as the synchronous motor is preferable where large power is required—unless the synchronous motor is excluded by conditions of starting torque, etc.

At first, and for some time to come, we would not consider going to anywhere near as small sizes of induction generators, as we do in induction motors. However, there are undoubtedly many millions of kilowatts available in water powers throughout the country, which can be collected by induction-generator stations from 50 h.p. upwards, and that at fair heads, requiring no abnormal machine design (no very low speed).

Consider an instance—a New England mill river with a descent, in its upper course, of about 1100 ft. (335 m.) within five miles (eight km.), of varying gradient. At three places, where the gradient is steepest, by a few hundred feet of cast iron pipe and a small dam of 20 to 30 ft. (6 to 9 m.) length and a few feet height—just enough to cover the pipe intake—an average head of 150 ft. (45 m.) can be secured, giving an average of 75 h.p. each, or a total of 225 h.p. or 170 kw. This would use somewhat less than half the total potential power. The development of the other half, requiring greater length of pipe line, or involving lower heads, would be left to meet future demands for additional power.

The installation of an electric system of 170 kw. would hardly be worth while, but there are numerous other creeks throughout the territory from which to collect power, and within a few miles passes a high potential transmission line, coming from a big synchronous station, into which the power collecting lines coming from the induction generator stations would be tied and from which they would be controlled.

Thus, the large modern synchronous station has its field, and is about as perfect as we know how to build for large concentrated powers; but beyond this, there is a vast field, and therefore *an economic necessity of the development of a different type of hydraulic generating station to collect the scattered water powers* of the country; and that is the *induction generator station*, to which I wish to draw the attention.

I must caution, however, not to mistake small power and low head power. There are on the lower courses of our streams some hydraulic powers, which are relatively small due to their low heads, and which can not be economically developed by the synchronous generator, due to the low head and correspondingly low speed. The designing characteristics of the induction generator, with regard to low-speed machines, are no better—if anything rather worse—than those of the synchronous generator, and the problem of the economical utilization of the low-head water power still requires solution. It is not solved by the induction generator; the latter's characteristic is simplicity of the station, giving the possibility of numerous small automatic generating stations.

III. Induction Generator Station

A. Characteristics of Induction Generator

An induction motor at no load runs at, or rather very close to synchronism. If it is driven above synchronism by mechanical power, current and power again increase, but the electric power is outflowing, and the induction machine consumes mechanical power, and generates electrical power, as an induction generator.

The maximum electrical power, which an induction machine can generate as an induction generator is materially larger than the maximum mechanical power, which the same machine, at the same terminal voltage, can produce as an induction motor.

Resolving the current of the induction machine into an energy component and a wattless or reactive component, the energy current is inflowing, representing consumption of electric power (which is converted to mechanical power) below synchronism. It becomes zero near synchronism, and above synchronism the energy current is in the reverse direction, or outflowing, supplying electric power to the system (which is produced from the mechanical power input into the machine), and the induction machine then is a generator.

The wattless or reactive component is a minimum at synchronism, and increases with the slip from synchronism, and is in the same direction, whether the slip is below synchronism, as a motor, or above synchronism, as a generator. That is, the induction machine always consumes a lagging current (representing the exciting current and the reactance voltages), or, what amounts to the same, produces a leading current. The latter way of putting it is frequently used with induction generators, by saying that the current produced by the induction generator is leading, while the current consumed by the induction motor is lagging. Instead of saying however, that the reactive component of the current generated by the induction generator is leading, we may say, and this makes it often more intelligible, that the induction generator generates an energy current and consumes a lagging reactive current, while the induction motor consumes an energy current and consumes a reactive lagging current.

As with the increasing voltages and increasing extent of our transmission systems the leading currents taken by transmission lines and underground cables are becoming increasingly larger, the induction generator appears specially advantageous, as tending to offset the effect of line capacity. We may thus say that the induction generator (and induction motor) consumes a lagging reactive current, which is supplied by the synchronous generators, synchronous motors, converters and other synchronous apparatus in the system, and by the capacity of lines and cables. Or we may say that the lagging current consumed by the induction generator neutralizes the leading current consumed by the capacity of lines and cables. Or we may say that the leading current produced by the induction generator supplies the capacity of lines and cables: these are merely three different ways of expressing the same facts.

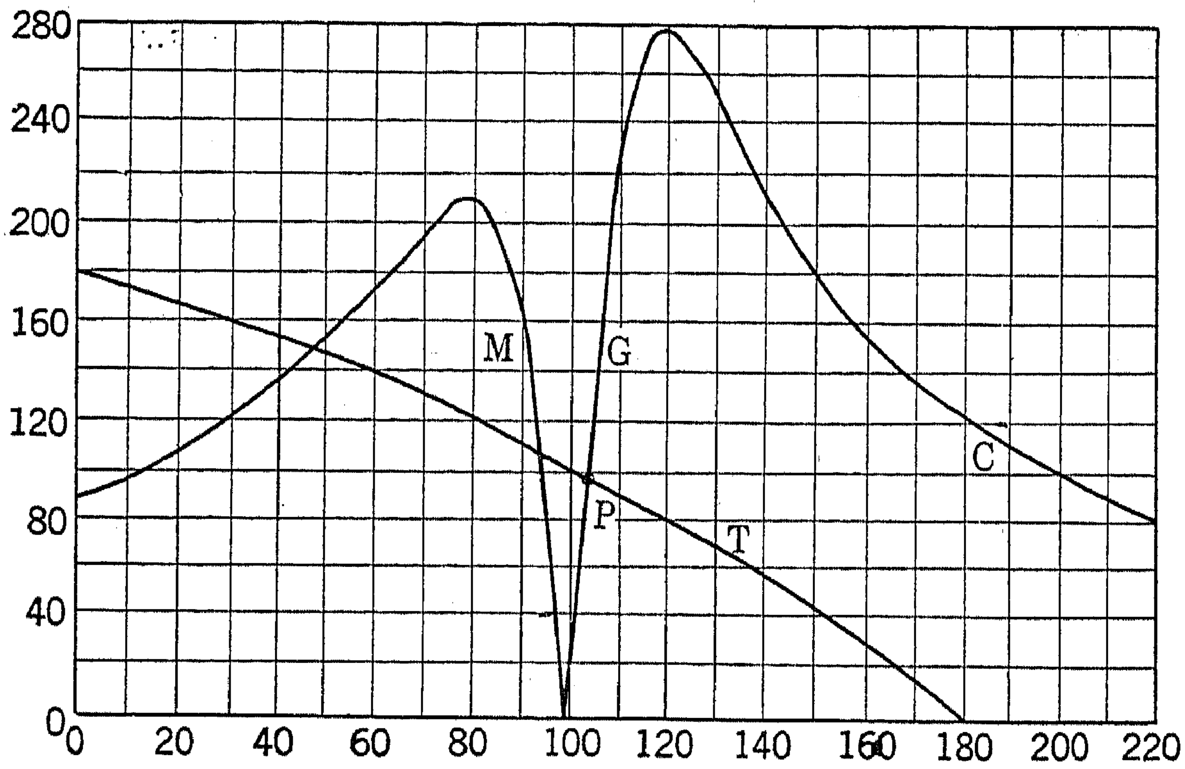

Fig. 2—Small Hydroelectric Induction Generator Plant Constant Terminal Voltage

In Fig. 2 are shown the torque curves, at constant terminal voltage, of a typical moderate size induction machine. *M* is the torque produced as an induction motor below synchronism, and *G* the torque consumed as an induction generator above synchronism, synchronism being chosen as 100 per cent. *T* is an assumed torque curve of a hydraulic turbine.

As seen, the point *P* where *G* and *T* intersect, is 4 per cent above synchronism, and this induction generator thus operates on full load at 4 per cent slip above synchronism or no-load. Assuming now, that the power goes off, by the circuit breakers opening. The turbine then speeds up to 80 per cent above synchronism, where the curve T becomes zero. If at this free running turbine speed the circuit is closed and voltage put on the induction generator, the high torque consumed by the induction generator causes the turbine to slow down, and as at all speeds above 104, the torque consumed by the induction generator is very much higher than that given by the turbine, the machine slows down rapidly, to the speed where the induction generator torque has fallen to equality with the turbine torque, at speed 104, and stable condition is restored.

Inversely, if the flow of water should cease, the induction machine slows down to a little below synchronism, and there continues to revolve as induction motor.

In starting, the circuit may be closed before admitting the water, and the turbine started by the induction machine as a motor, on the torque curve M, running up to speed 100, and then, by admitting the water, the machine is speeded up 4 per cent more and thereby made to take the load as generator. Or the turbine may be started by opening the gates, running up to speed 180, and then, by closing the circuit, the induction machine in taking the power slows the speed down to normal.

With larger machines, the most satisfactory way of starting as involving the least disturbance, probably would be, first to open the gates partly while the turbine speeds up, and when it has reached a speed in the neighborhood of synchronism, say between 95 and 105, the circuit is closed and the water gates opened fully.

B. Instability Conditions of Induction Generator

In Fig. 2, the torque consumed by the induction machine, at all turbine speeds above full load P, is much higher than the torque of the turbine. However, the induction generator torque curve has a concave range, marked by C, and if the induction generator should be such as to bring the generator torque curve at C below the turbine torque curve T, the speed, when once increased beyond the range C, would not spontaneously drop back to normal. While in Fig. 2, C is much higher than T, Fig. C represents the theoretical, but not real case of constant terminal voltage at the induction machine. The voltage however is kept constant at the controlling synchronous main station, and thus must vary with the load in the induction generator station. Assuming an extreme case, of 10 per cent resistance and 20 per cent reactance in the line from the induction machine station to the next synchronism station, we get the modified torque curve shown in Fig. 3. As seen, at full load *P*, there is practically no change; about 4 per cent slip above synchronism. The maximum torque of generator *G* and motor *M*, and the torque at the concave part of the induction generator curve, *C*, have greatly decreased. However, *C* is still above *T*, that is, even under this extreme assumption, the induction generator would pull the turbine down from its racing speed of 180, to the normal full load speed of 104, though the margin has become narrow.

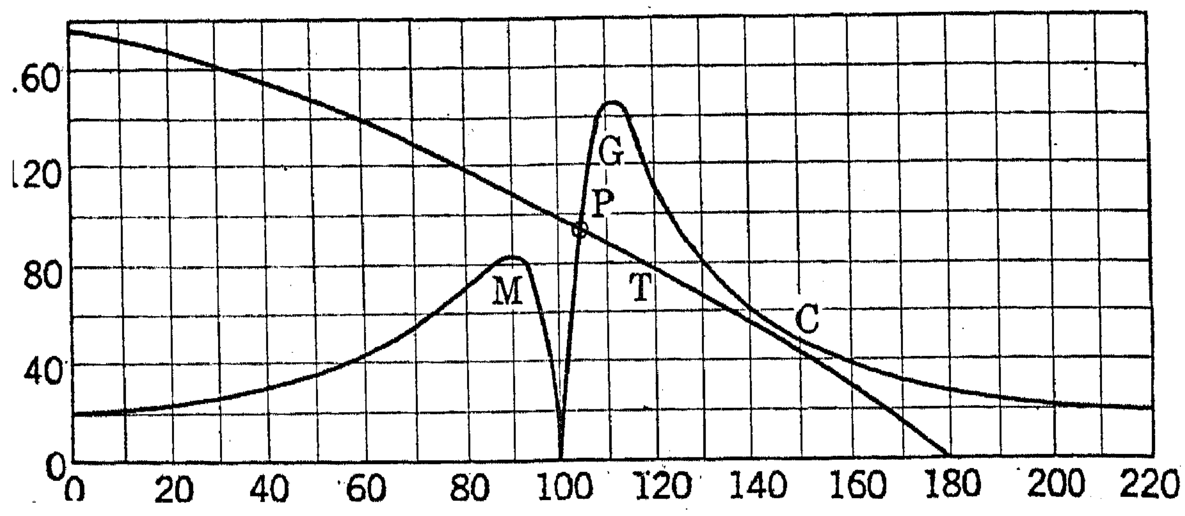

FIG. 3—SMALL HYDROELECTRIC INDUCTION GENERATOR PLANT—CONSTANT VOLTAGE IN SYNCHRONOUS STATION

Assuming however an induction machine with much less slip, with only half the rotor resistance of Figs. 2 and 3. At constant terminal voltage, this gives the curves shown in Fig. 4. The full load *P* is at speed 102, or 2 per cent above synchronism, and while the curve branch *C* is much lower, the conditions are still perfectly stable. Assuming however, with this type of low

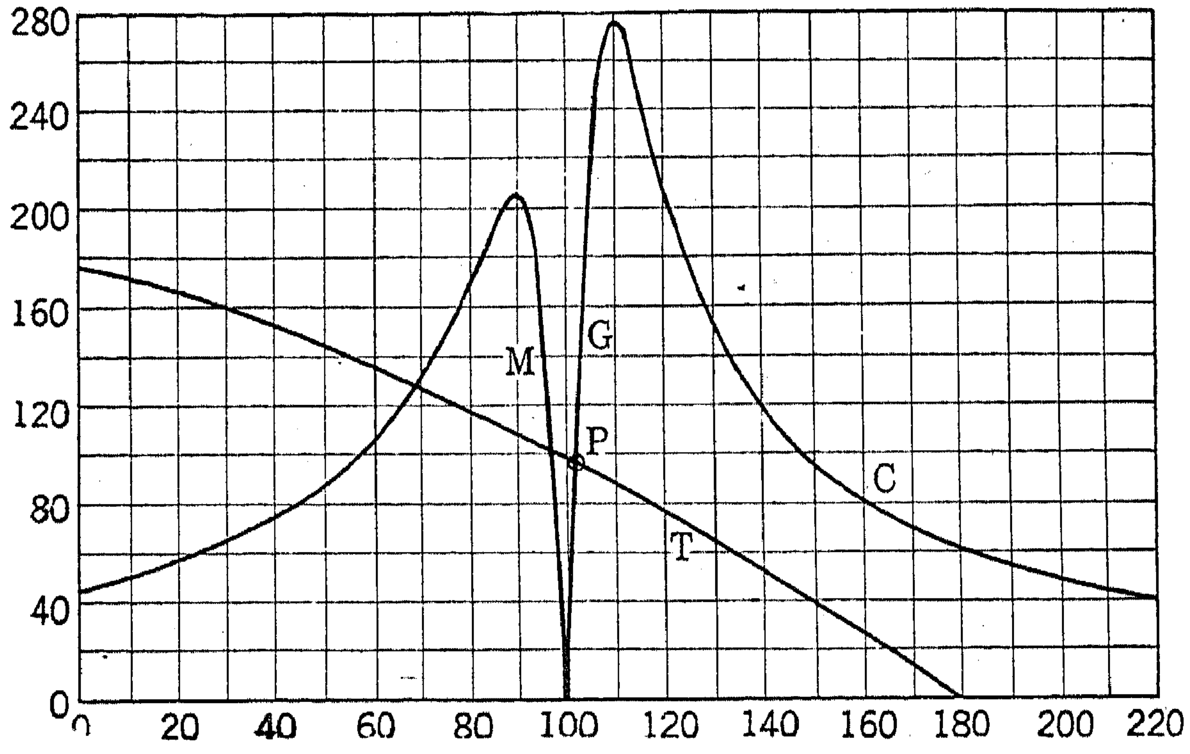

FIG. 4—LARGE HYDROELECTRIC INDUCTION GENERATOR PLANT—CONSTANT TERMINAL VOLTAGE

resistance rotor, a high line impedance, 10 per cent resistance and 20 per cent reactance, as in Fig. 3. We then get the condition shown in Fig. 5. The range *C* drops below *T*, and the induction generator torque curve *G* intersects the turbine torque curve *T* at three points: P, P_1 and P_2. Of these three theoretical running speeds, $P=102$, $P_1=169$ and $P_2=113.5$, two are stable, P and P_1; while the third one, P_2, is unstable, and from P_2, the speed must either decrease, reaching stability at the normal full load point *P*, or the machine speed up to P_1.

If with the conditions represented by Fig. 5, the turbine should—by an opening of the circuit for instance—have speeded up to its free running speed 180, closing the circuit does not bring the speed back to normal, *P*, but the machines slow down only to speed P_1, where stability is reached, at very little output and very large lagging currents in the induction generator. To restore normal condition then would require shutting off the water, at least sufficiently to drop the turbine torque curve *T* below *C*, and then letting the machines slow down to synchronism. They would not go below synchronism, even with the water gates entirely closed, as the induction machine as a motor, on curve *M*, holds the speed.

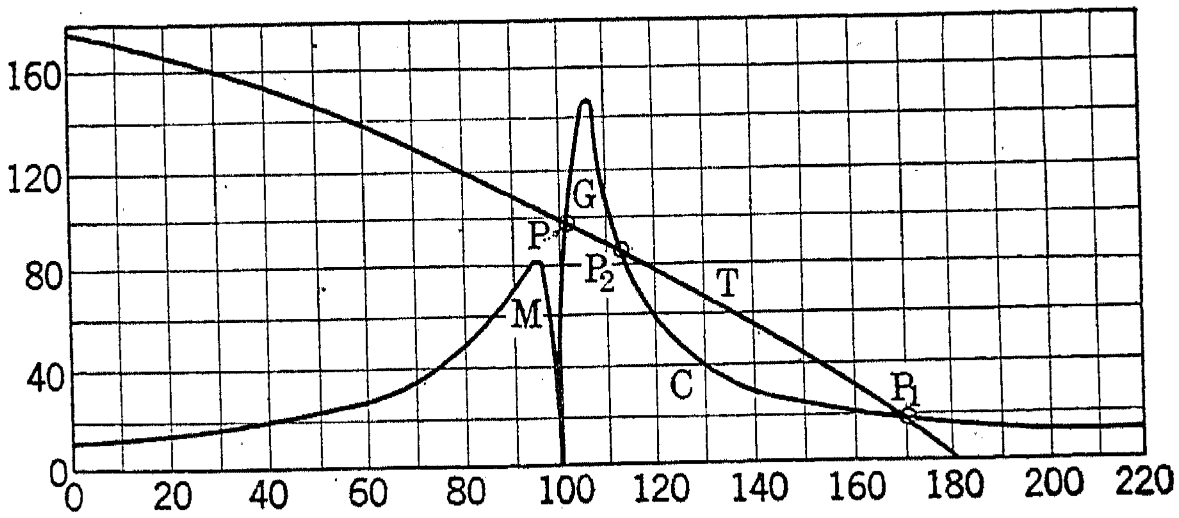

FIG. 5—LARGE HYDROELECTRIC INDUCTION GENERATOR PLANT—CONSTANT VOLTAGE IN SYNCHRONOUS MAIN STATION

A solution in the case Fig. 5 would be the use of a simple excess speed governor, which cuts off the water at 5 to 10 per cent above synchronism.

However, the possibility of difficulty due to the "dropping out of the induction generator" as we may call it in analogy to the dropping out of the induction motor, are rather less real than it appears theoretically. In smaller stations, such as would be operated without attendance, as automatic stations, the torque curve of the induction generator, as a small machine, would be of the character of Figs. 2 and 3, and thus not liable to this difficulty. The low resistance type of induction machines, as represented in Figs. 4 and 5, may be expected only with the larger machines, used in larger stations. In those, some attendant would be present to close the water gates in case of the circuit breakers opening, or a simple cheap excess speed cut-off would be installed at the turbines, keeping them within 10 per cent of synchronism, and within this range, no dropping out of the induction generator can occur.

It is desirable however to realize this speed range of possible instability of the induction generator, so as to avoid it in the design of induction generators and stations.

APPENDIX

Collection of Fuel Power by Steam Turbine Induction Generator

A. THE AUTOMATIC STEAM TURBINE INDUCTION GENERATOR STATION

The same reason which in the preceding led to the conclusion that in the (automatic) induction generator station is to be found the solution of the problem of collecting the numerous small amounts of hydraulic energy, which are scattered throughout our country along creeks and mountain streams, also applies, and to the same extent, to the problem of collecting the innumerable small quantities of mechanical or electrical energy, which are, or can be made available wherever fuel is consumed for heating purposes. Of the hundred millions tons of coal, which are annually consumed for heating purposes, most is used as steam heat. Suppose then, we generate the steam at high pressure—as is done now in many cases for reasons of heating economy—and interpose between steam boiler and heating system some simple form of high pressure steam turbine, directly connected to an induction generator, and tie the latter into the general electrical power distribution system. Whenever the heating system is in operation, electric power is generated, as we may say as "by-product" of the heating plant, and fed into the electric system.

The power would not be generated continuously, but mainly in winter, and largely during the day and especially the evening. That is, the maximum power generation by such fuel power collecting plant essentially coincides with the lighting peak of the central station, thus occurs at the time of the day, and the season when power is most valuable. The effect of such fuel power collection on the central station should result in a material improvement of the station load factor, by cuttnig off the lighting peaks.

The only difference between such steam turbine induction generator stations, collecting the available fuel power scattered throughout the cities and towns, and the hydraulic induction generator stations collecting the powers of the streams throughout the country, is that in the steam turbine plant an excess speed cut-off must be provided, as the free running steam turbine speed is usually not limited to less than double speed, as is the case with the hydraulic turbine. Otherwise however, no speed governing is required. A further difference is, that the greater simplicity and therefore lower investment of the steam turbine plant would permit going down to smaller powers, a few kilowatts perhaps.

It is interesting to note, that even with a very inefficient steam turbine, the electric generation of such fuel power collecting plant interposed between boiler and heating system, takes place with practically 100 per cent efficiency, because whatever energy is wasted by the inefficiency of the steam turbine plant, remains as heat in the steam, and the only loss is the radiation from turbine and generator, and even this in most cases is useful in heating the place where the plant is located. The only advantage of a highly efficient turbine, is that larger amounts of electric power can be recovered from the fuel, and the question thus is that between the investment in the plant, and the value of the recovered power.

If then the total efficiency, from the chemical energy of the fuel to the electric power, were only 3 per cent, it would mean that 3 per cent more coal would have to be burned, to feed the same heat units into the heating system. At an average energy value of 30,000 kj. per kg. of coal, this would give per ton of coal, 900,000 kj. or 250 kw-hr. At a bulk value of ½ cent per kw-hr. it would represent a power recovery value of $1.25 per ton of coal. This is quite considerable, more than sufficient to pay the interest on the investment in the very simple plant required.

At first, the steam turbine induction generator plant, proposed for the collection of fuel power, would appear similar to the isolated plant which, though often proved uneconomical, still has successfully maintained its hold in our northern latitudes, where heating is necessary through a considerable part of the year. However, the difference between the steam turbine induction generator plant and the isolated steam electric plants in our cities, is the same as that between the automatic hydroelectric induction generator station, and the present standard synchronous generator station: by getting rid of all the complexity and complication of the latter, the induction generator station becomes economically feasible in small sizes; but it does so only by ceasing to be an independent station, by turning over the functions of regulation and control to the central main station and so becoming an adjunct to the latter. But by this very feature, the turbo induction generator plant might afford to the central station, the public utility corporation, a very effective means of combatting the installation of isolated plants, by relieving the prospective owner of the isolated plant of all trouble, care and expense and incidental unreliability thereof, supplying central station power for lighting, but at the same time utilizing the potential power of the fuel burned for heating purposes. The simplest arrangement probably would be, that the fuel power collecting plants scattered throughout the city would, as automatic stations, be taken care of by the public utility corporation, their power paid at its proper rates, those of uncontrolled bulk power, while the power used for lighting is bought from the central station at the proper lighting rates.

As this however means a new adjustment of the relation between customer and central station, and is not merely an engineering matter like the hydroelectric power collection, I have placed it in an appendix.

B. Discussion

We realize that our present method of using our coal resources is terribly inefficient. We know that in the conversion of the chemical energy of coal into mechanical or electrical energy, we have to pass through heat energy and thereby submit to the excessively low efficiency of transformation from the low grade heat energy to the high grade electrical energy. We get at best 10 to 20 per cent of the chemical energy of the coal as electrical energy; the remaining 80 to 90 per cent we throw away as heat in the condensing water, or worse still, have to pay for getting rid of it. At the same time we burn many millions of tons of coal to produce heat energy, and by degrading the chemical energy into heat, waste the potential high grade energy which those millions of tons of coal could supply us.

It is an economic crime to burn coal for mere heating without first taking out as much high grade energy, mechanical or electrical, as is economically feasible. It is this feature, of using the available high grade energy of the coal, before using it for heating, which makes the isolated station successful, though it has every other feature against it. To a limited extent, combined electric and central steam heating plants have been installed, but their limitation is in the attempt to distribute heat energy, after producing it in bulk, from a central station. Here again we have the same rule; to do it efficiently, do it electrically. In the efficiency of distribution or its reverse, collection, no other form of energy can compete with electric energy, and the economic solution appears to be to burn the fuel wherever heating is required, but first take out its available high grade energy, and collect it electrically.

Assume we use 200 million tons of coal per year for power, at an average total efficiency of 12 per cent, giving us 24 million kw. (referred to 24-hr. service) and use 200 million tons of coal for heating purposes, wasting its potential power.

If then we could utilize the waste heat of the coal used for power generation, even if thereby the average total efficiency were reduced to 10 per cent, we would require only 240 million tons of coal, for producing the power, and would have left a heating equivalent of 216 million tons of coal, or more than required for heating. That is, the coal consumption would be reduced from 400 million to 240 million of tons, a saving of 160 million tons of coal annually.

Or, if from the 200 million tons of coal, which we degrade by burning it for fuel, we could first abstract the available high grade power, assuming even only 5 per cent efficiency, this would give us 10 million kw. (24-hr. rate), at an additional coal consumption of 10 million tons, while the production of the 10 million kw. now requires 100 million tons of coal, more or less, thus getting a saving of 90 million tons of coal; or putting it the other way, a gain of 9 million kw.—12 million horse power—24-hr. service, or 36 million horse power for an 8-hr. working day.

It is obvious that we never could completely accomplish this; but even if we recover only one-quarter, or even only one-tenth of this waste, it would be a vast increase in our national efficiency.

Thus the solution of the coal problem, that is, the more economic use of fuel energy, is not only the increase of the thermodynamic efficiency of the heat engine, in which a radical advance is limited by formidable difficulties; but is the recovery of the potential energy of all the fuel, by electric collection.

C. Turbo Induction Generator

Assume then that wherever fuel is burned to produce steam for heating purposes, instead of a low-pressure boiler giving a few pounds over-pressure only, we generate the steam at high pressure, at six atmospheres (90 lbs.) or, in larger plants, even at 15 atmospheres (220 lb.) passing the steam through a high pressure turbine wheel directly connected to an induction generator tied into the electric supply system, and then exhaust the steam at 1.25 atmospheres (19 lb.) into the steam heating system, or at 0.48 atmospheres (7 lb.) into a vacuum heating system.

At a fuel value of the coal of 30,000 kj. per kg. we have (see table)

Per ton of coal: Chemical energy, 30 × 10^6 kj: Heat energy of steam from boiler, at 75 per cent. boiler efficiency, 22.5 × 10^6 kj.

	Boiler press		Dist. press		Carnot-efficiency per cent.	Output at 50 per cent. efficiency		Value of power at ½c. per kw-hr. $..	Avg. kw., assuming 25% time of use	Tons of coal per kw.	Size of induction generator, per 100 tons coal annually
	atm:	lb.	atm:	lb.		kj. × 1000	kw-hr.				
Steam Heating	6	90	1.25	19	12.3	1380	385	1.92	0.176	5.7	25 kw.
	15	220	1.25	19	19.8	2230	620	3.10	0.283	3.5	45 kw.
Vacuum Heating	6	90	0.48	7	18.1	2030	565	2.82	0.258	3.9	40 kw.
	15	220	0.48	7	25.0	2820	810	4.05	0.37	2.7	55 kw.

From this it would follow that the average magnitude of the steam turbine induction generator plant for power collection from fuel in heating plants, would be about one-quarter to one-half kw. per ton of coal burned annually, under the assumption, that the use of the heating plant is equivalent to full capacity during one quarter of the time, and the turbine induction generator plant 50 per cent larger, to take care of maximum loads.

As seen, the value of the recovered power would be a substantial percentage of the fuel cost.

With 100 million tons of coal used for heating purposes annually, assuming an average recovery of 600 kw-hr. per ton, this gives a total of 60,000 million kw-hr. per year. One-quarter of this is more electric power than is now produced at Niagara, Chicago, New York and a few other of the biggest electric systems together.

DISCUSSION ON "AMERICA'S ENERGY SUPPLY" (STEINMETZ), ATLANTIC CITY, N. J., JUNE 27, 1918.

A. M. Schoen: In connection with the discussion of the question of our fuel supply and the need of additional power, it occurs to me to mention that about the year 1911, I think it was, I had the honor of representing the Institute at a hearing before the Committee on Agriculture from Congress on the subject of forest conservation. At that time the interests of the Institute lay in the conservation of the waterpower by protecting forests. There was a paper presented at that meeting by Mr. M. O. Leighton, engineer for the U. S. Geological Survey, and in that paper he went comprehensively into a survey he had made of the water powers of the Southern Appalachians. He called attention to the fact that by throwing dams across the ravines at the heads of a number of these streams the water could be impounded, and the low water stage of certain rivers could be materially raised.

The rainfall in the Southern Appalachians is very heavy, I think next to the Puget Sound region the heaviest we have in this country. Mr. Leighton's work was done with a view to improving rivers at that time navigable and rendering navigable others suffering from extreme low water. The same data, however, would indicate that through this method of protecting against freshet and drought, the low water stages could be so improved as to furnish sufficient water to change a considerable part of the power developed at some of the hydroelectric plants from secondary to primary, thus utilizing machinery at present idle during a considerable part of the year. With the present appeal from the Government to help save the coal, now would seem to be the accepted time to make use of this additional power. Mr. Leighton's paper was published in 1908 and will doubtless be found of interest in this connection.

B. A. Behrend: Dr. Steinmetz's interesting suggestion meets with one or the other of the horns of a dilemma which has confronted engineers in regard to the distribution of heat. If Mr. Steinmetz's idea is carried out literally as he wishes it to be, it is necessary to install in each house a boiler, a steam turbine, and an induction generator, so that the latent heat of the steam can be utilized for heating purposes. If this is not done, it is necessary to group together a number of houses in which the old difficulty of distributing heat has to be faced. Mr. Steinmetz does not wish us to consider the second alternative which has been carried out in several plants, particularly in Milwaukee in a station installed there some twelve years ago by Mr. John I. Beggs, for which the speaker designed the electric generating units.

Considering, therefore, the scheme as advocated by Dr. Steinmetz, we have to familiarize ourselves with the idea that dwellings will have to be equipped as indicated above, and whether this is feasible, or whether it is possible to operate such plants without attendants of any kind is a question the consideration of which must be left to the individual judgment of those who have heard Mr. Steinmetz's paper.

In this connection, allow me to point out the suggestion made many years ago by Lord Kelvin to reverse the Carnot cycle and to return the heat thus gained into the building to be warmed. In making this proposal, Lord Kelvin stated that it would not be resorted to until a shortage of fuel was experienced. (See Mathematical and Physical Papers by Lord Kelvin, Vol. I, p. 516.)

S. Barfoed (communicated after adjournment): In large sections of the Pacific Coast one mentioned source of energy supply, namely oil, is being used in an ever increasing amount, in spite of the fact that water power is plentiful in the undeveloped stage.

This oil, which is being burnt for its heat value only is the country's legitimate lubricating stock. It is needed in an ever increasing amount for machinery of all descriptions and should not be used as a source of energy supply merely because it will burn.

The various factors contributing to this misuse are well known to engineers and need no longer be discussed by them.

How the energy sources may be utilized and coordinated to best advantage and with a minimum of waste from an engineering point of view seems almost paltry compared to the vexatious work necessary to bring about an understanding of the problem on the part of the general public and its chosen representatives on the various legislative and executive bodies.

The trained engineer thinks generally not along the lines of least resistance, but he will have all he can do to carry out but a small fraction of Dr. Steinmetz' simple and beautiful dream while our present generation is alive.

Ralph Bennett (communicated after adjournment): In the first section of his article Mr. Steinmetz discusses in a general way the total possible energy to be derived from the use of falling water and of the sun's rays.

The conclusion drawn is that, even with a workable automatic generating station the energy of the water must be supplemented by other sources.

But if it be granted that such stations are feasible and that the electrical network extends for distances so great that local climatical changes may be taken as more or less equalized then other sources not stated become available.

The total energy of the winds is almost incalculable. Efforts to utilize them have been successful only in a small way because the need is for local power at definite times while the wind is a variable source, so far as any small areas are concerned, and cannot be controlled except to reduce the energy delivered by any single power unit.

Two methods of overcoming this difficulty have been generally suggested. Both assume local isolated plants but one stores energy in electrical storage batteries while in the other scheme the energy is stored in water lifted into reservoirs. In both cases the cost of equipment and maintenance is so great that other sources of energy are more reliable and economical.

As simple units feeding into a far extending network wind engines might have a much greater application.

The average wind velocity over the United States is in the vicinity of 10 m. (16 km.) per hr. and 50-ft. (15.2 m.) mills will produce about 5 h. p. at this velocity.

The number of mills which can be used is limited only by the reduction in current velocity which would result from over close spacing. If only one per acre were permissible on the 1,920,000,000 acres of continental United States there would be developed a continuous output averaged over the country of over 8,700,000,000 h. p.

The actual practicable production from this source may really be very considerable in the not distant future for there are known zones of highly sustained wind velocities and commercial mills are already well developed in small sizes. Only a power market capable of paying the high interest charges on the investment is needed to make such a plant possible.

Two other intermittent sources of large amounts of energy are the waves and the tides. Both have been the source of endless experimentation by inventors. But few successful motors have developed. The main difficulty appears to be the extremely rugged construction required as compared with the average energy developed. But connected to networks fed from many and scattered sources the output here might, as in the case of windmills, vary with the power available and not with the local demand.

The coasts of the United States extend for over 21,300 mi. (34,279 km.). Along most of this length there is a tidal range of 5 ft. or better and a constant wave action.

Possibly there may be available over 5000 h. p. per mile of coast from the waves alone and neglecting any energy which might be developed off shore.

This would amount to 106,500,000 h. p. for the entire shore line.

Doubtless successful motors could be built if the market would warrant the expenditure.

Tidal motors are a more difficult problem since the only method considered at all possible is the use of tidal basins and only in rare instances can these be had.

Other sources of stored energy besides coal are in extensive use. At the present time the oil production of the United States equals the equivalent of one-eighth of the coal used.

Vast oil fields are as yet untouched. No complete exploration for new fields has been made.

Oil shales and bitumen charged rocks exist in enormous beds and from them as well as the exhausted sands of old oil fields further supplies of fuel can be drawn as the demand requires.

One half of all the timber cut is wasted, yet the fuel and by-product value of this "refuse" is greater than that of the lumber commercially used.

With the extension of the networks and a policy of free acceptance of offered power the use of these and numerous other sources of smaller or larger supplies of energy will become possible and the necessary use of the coal and high-grade oil reserves can be materially restricted without hardship.

C. P. Steinmetz: In discussing fuel consumption, I have included oil and natural gas, reduced to coal by their approximate fuel value. This is the reason why the tonnage of coal given in my paper is larger than that actually mined. It includes the coal equivalent of the other fuels.

Very interesting and of some importance, is the subject regarding the use of wind power, made by Mr. R. Bennett. At present, due to the great irregularity of the wind, the windmill is of use mainly for such power demands, where great irregularity is permissible, that is, pumping for drainage, etc. Storage of the energy by storage battery is not economical—unless in very exceptional cases—due to the high investment, low efficiency and rapid depreciation of the battery. The only way of utilizing wind power to a greater extent, thus would be by electric collection, feeding whatever power the wind gives into a general system together with hydraulic power, steam power, etc. The only economical way this could be done, appears to me, by the use of an induction generator attached to the wind-wheel. Such low-voltage induction generator would thus be located at the top of the tower and there would also be located the step-up transformer raising to the voltage of the collecting lines. This will probably be about 10,000 volts, that is, low enough not to increase the cost of the transformer and sufficiently high to carry the power for some distance. Even the collecting lines would not come down from the wind-mill tower but jump directly to the next tower, that is, the wind-wheel tower would be used as transmission line tower.

Such a system of collecting wind-power then would comprise a high-voltage trunk line—100,000 volts—main collecting lines issuing at right angles from the trunk line every few miles, at about 10,000 volts, and tied to the trunk line by step-up transformers, and branch collecting lines issuing from the main collecting lines every few hundred feet at right angles, thus about parallel to the trunk line. All these collecting lines would be tower lines and each tower contains a wind-wheel with low-voltage induction generator and a step-up transformer, connecting into the collecting line. Possibly some simple cutout device would also be required, connecting the wind-wheel-induction generator-transformer into the collecting line by centrifugal force and disconnecting it by reverse-power relay.

I believe such an arrangement would be economically feasible, that is, the value of the collected wind-power would pay the interest on the investment and maintenance and a fair profit, even to-day in some localities, where fairly sustained wind velocities exist, power is valuable, and the cost of fuel high.

Presented at the 34th Annual Convention of the American Institute of Electrical Engineers, Atlantic City, N. J., June 27, 1918.

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PRE-CHARGED CONDENSERS IN SERIES AND IN PARALLEL

BY V. KARAPETOFF

ABSTRACT OF PAPER

A condenser is charged from a source of direct voltage, and then is used as a booster in series with this source to charge another condenser. By repeating this process a large number of times the second condenser is finally subjected to twice the voltage of the source. This is the principle of the Delon apparatus for testing cables, and is explained in a numerical example. Then the more general case of two or more "pre-charged" condensers in series is considered, when these condensers are connected to some source of direct voltage; it is shown how to determine the final distribution of voltages among them. A similar problem is solved for pre-charged condensers in parallel. Finally a general network of pre-charged condensers is considered, and equations are derived similar to Kirchoff's laws, from which the final distribution of voltages and charges may be computed knowing the initial distribution.

IN A paper presented before the Institution of Electrical Engineers (British) in February 1916, Mr. O. L. Record described Delon's apparatus for testing the insulation of a high-tension cable.[1] While endeavoring to make clear to himself the theory of this ingenious device the present writer has investigated in general the action of a pre-charged condenser used as a booster in series with some source of e.m.f. for charging another condenser. This led him further to deduce the equations of a general network of such condensers. Before giving these more general relationships it is of interest to explain the principle of Delon's apparatus in a simple numerical example.

The diagram of connections is shown in Fig. 1, in which C is a two-conductor cable under test, T is the testing transformer, R is a synchronous revolving rectifier arm, and C_1, C_2 are two "booster" condensers. The theory of the apparatus, confirmed by actual experience, shows that the cable is subjected to a continuous voltage which is numerically equal to twice the amplitude of the transformer voltage. Thus a cable can be readily tested at quite a high voltage, without having a source of considerable reactive kilovolt-amperes, and without subjecting the cable to dielectric hysteresis or dangerous oscillations.

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[1] Journal Inst. Elec. Engrs. Vol. 54 (1916), p. 610; abstracted in the *Electrical World*, Vol. 67 (1916), p. 665.

Theory of Delon's Apparatus. Let the capacities of condensers C_1 and C_2 each be equal to one quarter of that of cable C, and let the rectifying arm occupy the position shown in Fig. 1 at the instant when the lower terminal of the transformer is positive, and the voltage is at a maximum. Let the instantaneous value of this maximum voltage be 100 kilovolts and let the two condensers and the cable not be previously charged at all. Then at the instant under consideration condenser C_1 is subjected to the full potential difference of 100 kv. between points A_1 and B, and is being "pre-charged" for the next half of the cycle. For the sake of simplicity we shall assume that the duration of the contact is sufficient to charge C_1 to 100 kv.

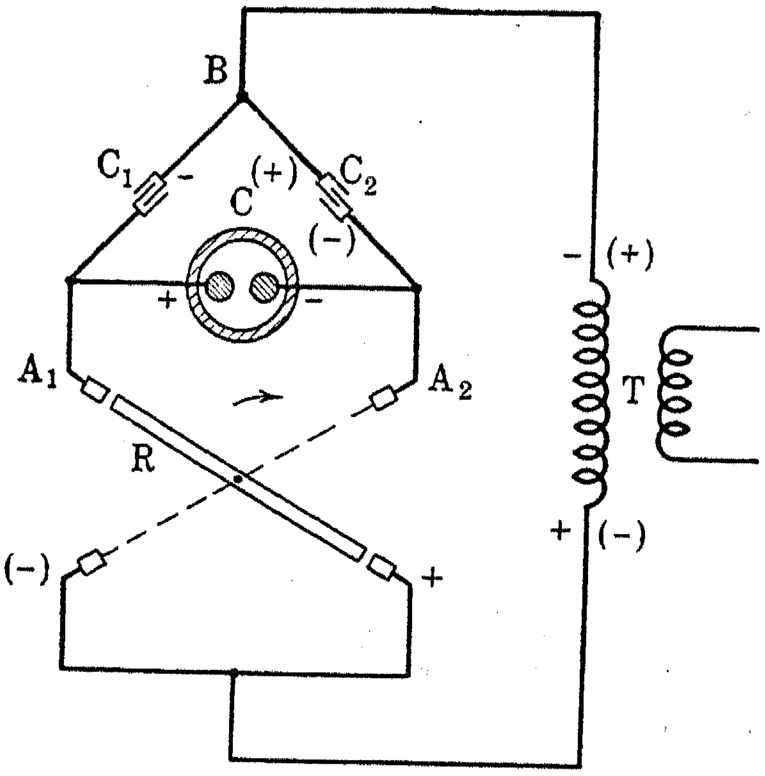

FIG. 1—DIAGRAM OF CONNECTIONS FOR TESTING TWO-CONDUCTOR CABLE

At the same instant the cable C is in series with condenser C_2 and the two together are subjected to 100 kv. Since the capacities C and C_2 are in the ratio of 4 to 1, the voltage across C is 20 kv., that across C_2 is 80 kv. It is important to note that the polarities marked at condensers C_1 and C_2 are those during the process of pre-charging. The polarity of C_2 during the first charge just described is opposite of that shown in the figure.

Half a cycle later the synchronous arm touches contact A_2 and the lower transformer terminal is negative. The corresponding polarities are shown in parentheses. The pre-charged condenser C_1 now serves as a booster in series with the transformer and helps to raise the charge on the cable, while condenser C_2 is being "pre-charged" between A_2 and B for the next impulse. We shall now determine the voltage to which the cable is charged after the second impulse.

Since the capacities C and C_1 are in the ratio of 4 to 1, any gain in voltage X across C corresponds to a loss of $4X$ across

C_1. In other words, if the new voltage across C is $(20 + X)$, that across C_1 is $(100 - 4X)$. But the total instantaneous voltage between A_2 and B is 100 kv., so that

$$(20 + X) - (100 - 4X) = 100,$$

from which $X = 36$ kv. Thus, after the second contact the cable is charged to 56 kv., while the original voltage across C_1 has dropped to -44 kv.

When the synchronous arm comes again in position A_1, the cable is connected in series with condenser C_2 which in the meanwhile has been charged to 100 kv. We thus have the equation

$$(56 + X) - (100 - 4X) = 100$$

rom which $X = 28.8$ kv., so that the cable is now charged to 84.8 kv.

Carrying this reasoning further one readily finds that the voltage across the cable indefinitely approaches the value 200 kv. which is the double of the amplitude of the transformer voltage. Only after this value has been reached do the booster ondensers and the transformer cease to charge the cable further. This is because the total voltage of the transformer plus that of a booster condenser is equal to 200 kv., and a permanent quilibrium is established. Let the cable voltage after the n — th charge be E_n. Then, by analogy with the foregoing xpressions, we have for the $(n + 1)$ th application of voltage

$$(E_n + X) - (100 - 4X) = 100$$

r

$$X = (200 - E_n)/5 \qquad (1)$$

o that

$$E_{n+1} = E_n + X = 40 + 0.8\,E_n \qquad (2)$$

t will thus again be seen that the charging will stop or X will ecome zero when $E_n = 200$ kv.

Applying formula (2) to the first few charges (except the nitial one) we get the following values:

20; 56; 84.8; 107.8; 126.3; 141; 152.8 kv.

Here the first value, 20 kv., does not satisfy the left-hand side f equation (2) when on the right-hand side one puts $E_n = 0$. However, it must be remembered that when the voltage is pplied for the first time the booster condenser is not pre-charged o that equation (1) does not hold true either. Both equations old true for every impulse except the first one.

One could imagine n condensers in place of each booster conenser, and with a proper commutating arrangement these condensers could be pre-charged in parallel and discharged in series. In this case the cable could be subjected to a voltage equal to $n + 1$ times that of the amplitude of the transformer voltage. The idea of charging condensers in parallel and discharging them in series is not new and has been used for other purposes.

Two Condensers in Series. Let us now consider a more general case (Fig. 2) of a condenser of capacity C pre-charged to a voltage E_0 and of a booster condenser C' pre-charged to a voltage E_0'. Let the corresponding charges or displacements of electricity in the condensers be Q_0 and Q_0'. The operations will consist in charging C to a higher voltage by means of pre-charged C' in series with a battery e, then pre-charging C' again, raising the voltage of C, and so forth. The problem is to determine the consecutive values of the voltage across C. These values will be denoted by E_0, E_1, E_2, ...E_n, while the corresponding pre-charge voltages of the booster condenser will be called E_0', E_1', E_2'....E_n'.

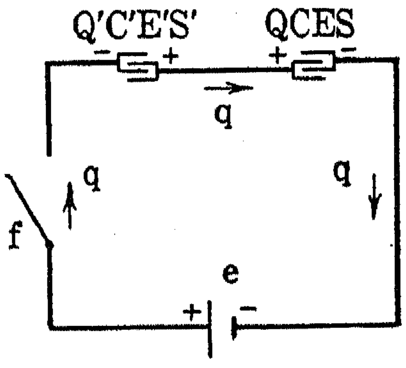

Fig. 2—Two Condensers in Series

We have the fundamental relations

$$Q_0 = C\,E_0 \tag{3}$$

$$Q_0' = C'\,E_0' \tag{4}$$

Since the two condensers are connected in series, it is more convenient to use the reciprocal of capacity namely the so-called *elastance* of a condenser.[2] Introducing the elastances

$$\left.\begin{aligned} S &= 1/C \\ S' &= 1/C \end{aligned}\right\} \tag{5}$$

we get

$$\left.\begin{aligned} E_0 &= Q_0\,S \\ E_0' &= Q_0'\,S' \end{aligned}\right\} \tag{6}$$

Let now condenser C' be connected in series with a constant-voltage battery e, and the switch f closed. With the polarities shown in the sketch the condenser C' assists the battery in further charging condenser C, and a quantity of electricity q

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[2] When a capacity is expressed in microfarads, the corresponding elastance is measured in mega-darafs. See the author's "Electric Circuit" (McGraw-Hill, 1912), p. 148; also his paper "Sur quelques calculs pratiques des champs électrostatiques," *Trans. Congresso Internazionale delle Applicazioni Elettriche*, Torino, 1911.

is displaced through the circuit. The charge on C is increased to $(Q + q)$, that on C' is reduced to $(Q' - q)$. The corresponding voltages now are

$$E_1 = (Q_0 + q)\ S \tag{7}$$

$$E_1' = (Q_0' - q)\ S' \tag{8}$$

An equilibrium is established when

$$e + E_1' = E_1 \tag{9}$$

Substituting in this equation the values from equations (**6**), (**7**) and (**8**) we get

$$e + E_0' - q\ S' = E_0 + q\ S,$$

or

$$e + E_0' - E_0 = q\ (S + S') \tag{10}$$

so that

$$q = (e + E_0' - E_0)/(S + S') \tag{11}$$

All the quantities on the right hand side of this expression are known and q may be computed. Substituting this value of q in equation (**7**) we get

$$E_1 = E_0 + (e + E_0' - E_0)[S/(S + S')] \tag{12}$$

This expression permits one to determine the new voltage of condenser C knowing the initial voltages and the elastances of the condensers.

The larger the booster condenser the more it helps in charging the other condenser, but on the other hand the more expensive it is. Since the result depends only upon the ratio of the elastances we shall denote

$$S'/(S + S') = \alpha \tag{13}$$

$$S/(S + S') = 1 - \alpha \tag{14}$$

In the most efficient and most expensive case of an infinitely large auxiliary condenser ($S' = 0$) we have $\alpha = 0$. In the other extreme, when it is impossible to charge C because $C' = 0$ or $S' = \infty$, the ratio is $\alpha = 1$. In all cases α is a positive regular fraction, and may be called "slack factor." Equation (**12**) becomes

$$E_1 = (1 - \alpha)\ (e + E_0') + \alpha\ E_0 \tag{15}$$

or

$$E_1 = (1 - \alpha)\ V_1 + \alpha\ E_0 \tag{16}$$

Here

$$V_1 = e + E_0' \tag{17}$$

is the total charging voltage applied to condenser C at the instant of closing switch f.

Let now the switch be opened, the condenser C' pre-charged to some other voltage E_1' and the switch closed again. By analogy with equation (**16**), the new voltage to which condenser C will be charged is

$$E_2 = (1 - \alpha)\ V_2 + \alpha E_1,$$

where $V_2 = e + E_1'$ is the new total charging voltage which in a general case may be different from V_1. If this charging process be repeated n times we get

$$\left.\begin{array}{l} E_1 = (1 - \alpha)\ V_1 + \alpha E_0 \\ E_2 = (1 - \alpha)\ V_2 + \alpha E_1 = (1 - \alpha)\ (V_2 + \alpha\ V_1) + \alpha^2 E_0 \\ E_3 = (1 - \alpha)\ V_3 + \alpha E_2 = (1 - \alpha)\ (V_3 + \alpha\ V_2 + \alpha^2\ V_1) \\ \qquad\qquad\qquad\qquad\qquad\qquad + \alpha^3 E_0 \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ E_n = (1 - \alpha)\ (V_n + \alpha\ V_{n-1} + \alpha^2\ V_{n-2} + \dots + \alpha^{n-1}\ V_1) \\ \qquad\qquad\qquad\qquad\qquad\qquad + \alpha^n E_0 \end{array}\right\} \quad (\mathbf{18})$$

No matter how many times the charging process be repeated the voltage across C remains finite because it cannot possibly exceed the highest charging voltage V applied to it.

In practise, the condenser C' would usually be pre-charged every time to the same constant value, say E', so that

$$V_1 = V_2 = \dots = V_n = V = e + E' \quad (\mathbf{19})$$

The preceding expression is then simplified to

$$E_n = (1 - \alpha)\ (e + E')\ (1 + \alpha + \alpha^2 + \dots + \alpha^{n-1}) + \alpha^n E_0$$

or

$$E_n = (1 - \alpha^n)\ V + \alpha^n E_0 \quad (\mathbf{20})$$

Compare this expression with equation (**16**). After an infinite number of applications of voltage, that is $n = \infty$,

$$\alpha^n = 0, \text{ and}$$

$$E_\infty = e + E' \quad (\mathbf{21})$$

independent of α or E_0. However, the values of α and E_0 determine the law according to which the voltage E approaches its ultimate value. With a small slack factor, α, that is with an expensive auxiliary condenser, the final value is practically reached after a few charges, while with an α near unity it may require many thousands of applications.

In the simplest case the same battery e would be used for pre-charging condenser C', so that $E' = e$. Equations (**20**) and (**21**) become

$$E_n = 2\ e\ (1 - \alpha^n) + \alpha^n E_0 \quad (\mathbf{22})$$

$$E_\infty = 2\ e \quad (\mathbf{23})$$

These two relationships apply in Delon apparatus.

Several Condensers in Series. We shall now consider a still more general case of several pre-charged condensers in series. Let the initial voltage, charge, and elastance of the $k-$th condenser be E_{k0}, Q_k and S_k respectively, so that originally

$$E_{k0} = Q_k S_k \tag{24}$$

Let all these condensers be connected in series and also in series with a source of constant voltage e, say a battery, and the switch closed. It is required to find the new voltages across the individual condensers.

We shall consider as positive those voltages E_{k0} which "buck" or oppose the battery voltage, while the voltages which help the battery will be considered negative. Let q be the displacement of electricity or the charge which passes through the circuit when the switch is closed. The new charge in the $k-$th condenser is $Q_k + q$, and the new voltage

$$E_{k1} = (Q_k + q)\, S_k = E_{k0} + q\, S_k \tag{25}$$

But when the equilibrium is established

$$\Sigma\, E_{k1} = e \tag{26}$$

or

$$\Sigma\, E_{k0} + q\, \Sigma\, S_k = e \tag{27}$$

Hence, the additional charge

$$q = (e - \Sigma\, E_{k0})/\Sigma S_k \tag{28}$$

But $e - \Sigma\, E_{k0}$ is the *net* voltage before the switch is closed, and $\Sigma\, S_k$ is the *equivalent* elastance of the whole circuit. We denote

$$e - \Sigma\, E_{k0} = e_n \tag{29}$$

$$\Sigma\, S_k = S_{eq} \tag{30}$$

The preceding expression for q becomes

$$q = e_n/S_{eq} \tag{31}$$

and the new voltage across the $k-$ th condenser

$$E_{k1} = E_{k0} + e_n(S_k/S_{eq.}) \tag{32}$$

Condensers in Parallel. The case of several pre-charged condensers in parallel allows of a very simple solution. Let condensers of capacity C_1, $C_2 \ldots C_k \ldots C_n$ be pre-charged to voltages E_1, $E_2 \ldots E_k \ldots E_n$ respectively and then put in parallel. Let it be required to find the common voltage E after the equalization of the charges. The $k-$ th condenser has lost voltage $E_k - E$, which means that it has lost a charge equal to $(E_k - E)$ C_k. But in the absence of a source of e.m.f. the sum of the charges after the equalization is the same as before, so that the sum of the changes of charge must be zero. We thus have the equation

$$\Sigma\,(E_k - E)\,C_k = 0 \qquad \textbf{(33)}$$

or

$$\Sigma\,E_k\,C_k - E\,\Sigma\,C_k = 0$$

from which

$$E = \Sigma\,E_k\,C_k / \Sigma\,C_k \qquad \textbf{(34)}$$

When using this equation one has to be careful regarding the sign of each individual E_k. If all the positive terminals are connected together and all the negative ones together, then all the E_k's should enter in the equation with the same sign, say plus. But if some positive and other negative terminals are connected together, one should select a positive direction of e.m.f. and enter the individual E_k's in equation (**34**) with the sign plus or minus accordingly. As a practical application of pre-charged condensers in parallel the "method of mixtures" may be mentioned for measuring small capacities.[3]

Network of Condensers. As a most general case treated in this article, let a number of pre-charged condensers be connected in any arbitrary manner so as to form a network similar to a network of conductors. Let the initial voltage of the $k-$ th condenser be E_{k0}, the corresponding charge Q_{k0} and the elastance S_k, so that

$$E_{k0} = Q_{k0}\,S_k \qquad \textbf{(35)}$$

Let various sources of d-c. voltage, e_1, e_2, e_3, etc. be distributed throughout the network. In the beginning a sufficient number of switches are supposed to be opened to prevent an equalization of charges and voltages. Then all these switches are closed, and it is required to find the new voltages and charges on the condensers.

The problem is solved in a manner similar to that in which currents are found in a complicated network of conductors, namely by applying equations analogous to Kirchoff's laws. As soon as the switches have been closed, the distribution of charges on the condensers changes, the $k-$ th condenser receiving an additional charge say q_k. Let q_k be considered positive when a positive displacement moves away from a junction point of three or more condensers. Since electricity behaves like an incompressible fluid, we have a relationship similar to the first Kirchoff law, namely

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[3] See for example V. Karapetoff, Experimental Electrical Engineering, (Wiley) Vol. II, p. 9.

$$\Sigma\, q_k = 0 \tag{36}$$

An independent equation of this kind may be written for each junction point of the network but one.

For each dielectric circuit that can be traced within the network the second Kirchoff law may be applied which in this case simply becomes

$$\Sigma\, e = \Sigma\, E_{k1} = \Sigma\, (Q_{k0} + q_k)\, S_k.$$

But originally we had $Q_{k0}\, S_k = E_{k0}$, so that the preceding expression becomes

$$\Sigma\, e = \Sigma\, E_{k0} + \Sigma\, q_k\, S_k \tag{37}$$

A sufficient number of equations of this form can be written down, together with equations of the kind (**36**), to enable one to solve them as simultaneous equations for the unknown quantities q_k. Knowing a q_k, the corresponding new voltage across the condenser may be computed from the relationship

$$E_{k1} = (Q_{k0} + q_k)\, S_k = E_{k0} + q_k\, S_k \tag{38}$$

In the beginning the directions of q_k may be assumed arbitrarily; then those which actually take place in the opposite direction will come out negative, from the solution of equations (**36**) and (**37**).

DISCUSSION ON "PRE-CHARGED CONDENSERS IN SERIES AND IN PARALLEL" (KARAPETOFF), ATLANTIC CITY, N. J., JUNE 28, 1918.

C. O. Mailloux: I wish to say I saw the apparatus perform at the Electrical Exposition at Versailles in 1908, and, of course, it was at that time one of the most interesting features of the exposition.

L. W. Chubb: Prof. Karapetoff's problem becomes more interesting when resistance load instead of static charges are considered. It is perfectly feasible, instead of using condensers in this apparatus, to use batteries or direct-current generators, in which case the fluctuation of energy can be made very great, and power can actually be utilized at a potential higher than the potential of the source. We have used in certain cases a limited power from transformers giving approximately half of the voltage required, and in the case of one or two hundred kv. it is a great advantage to have a transformer, say, at 100 kv. for a load at 175 kv. When these loads are applied the equations for the apparatus become very much more complicated, because while the connections with the pre-charged condensers are being made, there is, of course, a leakage dependent upon the nature of the load and the time that it is on.

W. V. Lyon (Read by B. A. Behrend): The solution for the distribution of electricity among a network of condensers is, as Prof. Karapetoff points out, readily obtained by applying principles similar to those commonly known as "Kirchoff's Laws". The writer has long used problems involving such networks to emphasize the generality of these principles.*

Another point of view should be both interesting and instructive. The current supplied to a single conductor or to any network of resistances, in which there are no electromotive forces acting, will so distribute itself that the total joule heating loss is a minimum. Mathematically this reduces to the statement that the sum of the resistance drops around a closed circuit is zero.

When a number of condensers that have been pre-charged are reconnected in any form of network and allowed to come to an unrestrained equilibrium†, the charges will so redistribute themselves that the total stored energy is a minimum. Mathematically this is also equivalent to the statement that the voltage drop around any closed circuit is zero. If this principle is combined with a second, namely, the algebraic sum of the charges at any insulated junction is constant, a solution for the redistributed charge is at once obtained. If an electromotive force is *inserted in* any branch of this stabilized network there will be produced an *additional charge* on each condenser equal to that which would have been produced had the condensers originally been discharged. The resultant effect of more than one such electromotive force inserted in the network is determined by calculating the effect of each electromotive force as if it acted alone, all of the others being short-circuited. The net additional charge on any condenser is equal to the algebraic sum of the charges due to the component electromotive forces. The *total* charge on any condenser is the algebraic sum of the stabilized charge and the net additional charge. The stabilized charge should always be calculated for the condition that all of the electromotive forces are short-circuited.

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*Problems in Electrical Engineering (pp, 96, 97, 98) McGraw-Hill Book Co., 1908.

†No electromotive forces acting in the network.

As a means of solution there is little to commend this method. As a method of analysis, however, it emphasizes two important principles. These are the principles concerning the dissipation of energy in a system, and the combination of component effects in producing a resultant condition.

V. Karapetoff: Perhaps one of the reasons why Delon apparatus has not been used in this country is that we have now the possibility of obtaining high voltages of a unidirectional character by means of hot cathode valves, and I hope that a method will be developed, if it has not been already developed, using these hot cathode valves, for producing high unidirectional voltage for testing cables.

Regarding my nomenclature, I do not like the word daraf, but I wanted to create a disturbance among our rigorists in order to induce the Committee on Nomenclature to take some action. To make the situation particularly objectionable, I also spelled the word henry backward, making it yrneh, which word is used in my "Magnetic Circuit". It is not for an individual to coin a new word, and I felt that I was more consistent in following the established practise (ohm, mho) thanin inventing a new word. I will be the first one to adopt a new word for reluctance, the reciprocal of henry, and for the daraf, the reciprocal for farad, as soon as an international action has been taken.

Presented at the 34th Annual Convention of the American Institute of Electrical Engineers, Atlantic City, N. J., June 28, 1918.

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METHOD OF SYMMETRICAL CO-ORDINATES APPLIED TO THE SOLUTION OF POLYPHASE NETWORKS

BY C. L. FORTESCUE

ABSTRACT OF PAPER

In the introduction a general discussion of unsymmetrical systems of co-planar vectors leads to the conclusion that they may be represented by symmetrical systems of the same number of vectors, the number of symmetrical systems required to define the given system being equal to its degrees of freedom. A few trigonometrical theorems which are to be used in the paper are called to mind. The paper is subdivided into three parts, an abstract of which follows. It is recommended that only that part of Part I up to formula (33) and the portion dealing with star-delta transformations be read before proceeding with Part II.

Part I deals with the resolution of unsymmetrical groups of numbers into symmetrical groups. These numbers may represent rotating vectors of systems of operators. A new operator termed the sequence operator is introduced which simplifies the manipulation. Formulas are derived for three-phase circuits. Star-delta transformations for symmetrical co-ordinates are given and expressions for power deduced. A short discussion of harmonics in three-phase systems is given.

Part II deals with the practical application of this method to symmetrical rotating machines operating on unsymmetrical circuits. General formulas are derived and such special cases, as the single-phase induction motor, synchronous motor-generator, phase converters of various types, are discussed.

INTRODUCTION

IN THE latter part of 1913 the writer had occasion to investigate mathematically the operation of induction motors under unbalanced conditions. The work was first carried out, having particularly in mind the determination of the operating characteristics of phase converters which may be considered as a particular case of unbalanced motor operation, but the scope of the subject broadened out very quickly and the writer undertook this paper in the belief that the subject would be of interest to many.

The most striking thing about the results obtained was their symmetry; the solution always reduced to the sum of two or more symmetrical solutions. The writer was then led to inquire if there were no general principles by which the solution of unbalanced polyphase systems could be reduced to the solution of two or more balanced cases. The present paper is an endeavor to present a general method of solving polyphase network which has peculiar advantages when applied to the type of polyphase networks which include rotating machines.

In physical investigations success depends often on a happy choice of co-ordinates. An electrical network being a dynamic system should also be aided by the selection of a suitable system of co-ordinates. The co-ordinates of a system are quantities which when given, completely define the system. Thus a system of three co-planar concurrent vectors are defined when their magnitude and their angular position with respect to some fixed direction are given. Such a system may be said to have six degrees of freedom, for each vector may vary in magnitude and phase position without regard to the others. If, however, we impose the condition that the vector sum of these vectors shall be zero, we find that with the direction of one vector given, the other two vectors are completely defined when their magnitude alone is given, the system has therefore lost two degrees of freedom by imposing the above condition which in dynamical theory is termed a "constraint". If we impose a further condition that the vectors be symmetrically disposed about their common origin this system will now have but two degrees of freedom.

It is evident from the above definition that a system of n coplanar concurrent vectors may have $2n$ degrees of freedom and that a system of n symmetrically spaced vectors of equal magnitude has but two degrees of freedom. It should be possible then by a simple transformation to define the system of n arbitrary congruent vectors by n other systems of concurrent vectors which are symmetrical and have a common point. The n symmetrical systems so obtained are the symmetrical co-ordinates of the given system of vectors and completely define it.

This method of representing polyphase systems has been employed in the past to a limited extent, but up to the present time there has been as far as the author is aware no systematic presentation of the method. The writer hopes by this paper to interest others in the application of the method, which will be found to be a valuable instrument for the solution of certain classes of polyphase networks.

In dealing with alternating currents in this paper, use is made of the complex variable which in its most general form may be represented as a vector of variable length rotating about a given point at variable angular velocity or better as the resultant of a number of vectors each of constant length rotating at different angular velocities in the same direction about a given point. This vector is represented in the text by $\check{I}$, $\check{E}$, etc., and the conjugate vector which rotates at the same speed in the opposite direction is represented by $\hat{I}$, $\hat{E}$, etc. The effective value of the vector is represented by the symbol without the distinguishing mark as I, E, etc. The impedances Z_a, Z_b, Z_{ab}, etc., are generalized expressions for the self and mutual impedances. For a circuit A the self-impedance operator will be denoted by Z_{aa} or Z_a. In the case of two circuits A and B the self impedance operators would be $Z_{aa}\,Z_{bb}$ and the mutual impedance operator Z_{ab}. The subletters denote the circuits to which the operators apply. These operators are generally functions of the operator, $D = \dfrac{d}{dt}$ and the characteristics of the circuit; these characteristics are constants only when there is no physical motion. It will therefore be necessary to carefully distinguish between $Z_a\,\check{I}_a$ and $\check{I}_a\,Z_a$ when Z_a has the form of a differential operator. In the first case a differential operation is carried out on the time variable $\check{I}_a$ in the second case the differential operator is merely multiplied by $\check{I}_a$.

The most general expression for a simple harmonic quantity e is

$$e = A \cos pt - B \sin pt$$

in exponential form this becomes

$$e = \frac{A + j\,B}{2}\,\epsilon^{j\,pt} + \frac{A - j\,B}{2}\,\epsilon^{-j\,pt}$$

$(A + j\,B)\,\epsilon^{j\,pt}$ represents a vector of length $\sqrt{A^2 + B^2}$ rotating in the positive direction with angular velocity p while $(A - j\,B)$ $\epsilon^{-j\,pt}$ is the conjugate vector rotating at the same angular velocity in the opposite direction. Since $\epsilon^{j\,pt}$ is equal to $\cos pt + j \sin pt$, the positively rotating vector $\check{E} = (A + j\,B)\,\epsilon^{j\,pt}$ will be

$$\check{E} = A \cos pt - B \sin pt + j\,(A \sin pt + B \cos pt)$$

or the real part of $\check{E}$ which is its projection on a given axis is equal to e and therefore $\check{E}$ may be taken to represent e in phase and magnitude. It should be noted that the conjugate vector $\hat{E}$ is equally available, but it is not so convenient since the operation $\frac{d}{dt}\,\epsilon^{-j\,pt}$ gives $-j\,p\,\epsilon^{-j\,pt}$ and the imaginary part of the impedance operator will have a negative sign.

The complex roots of unity will be referred to from time to time in the paper. Thus the complete solution of the equation $x^n - 1 = 0$ requires n different values of x, only one of which is real when n is an odd integer. To obtain the other roots we have the relation

$$\begin{aligned} 1 &= \cos 2\,\pi\,r + j\sin 2\,\pi\,r \\ &= \epsilon^{j\,2\,\pi\,r} \end{aligned}$$

Where r is any integer. We have therefore

$$1^{\frac{1}{n}} = \epsilon^{j\frac{2\,\pi\,r}{n}}$$

and by giving successive integral values to r from 1 to n, all the n roots of $x^n - 1 = 0$ are obtained namely,

$$\begin{aligned} a_1 &= \epsilon^{j\frac{2\,\pi}{n}} = \cos\frac{2\,\pi}{n} + j\sin\frac{2\,\pi}{n} \\ a_2 &= \epsilon^{j\frac{4\,\pi}{n}} = \cos\frac{4\,\pi}{n} + j\sin\frac{4\,\pi}{n} \\ a_3 &= \epsilon^{j\frac{6\,\pi}{n}} = \cos\frac{6\,\pi}{n} + j\sin\frac{6\,\pi}{n} \\ a_n &= \epsilon^{j\,2\,\pi} = 1 \end{aligned}$$

It will be observed that $a_2\,a_3 \ldots . a_n$ are respectively equal to $a_1^2\,a_1^3 \ldots . a_1^{(n-1)}$.

When there is relative motion between the different parts of a circuit as for example in rotating machinery, the mutual inductances enter into the equation as time variables and when the motion is angular the quantities ϵ^{jwt} and ϵ^{-jwt} will appear in the operators. In this case we do not reject the portion of the operator having ϵ^{-jwt} as a factor, because the equations require that each vector shall be operated on by the operator as a whole which when it takes the form of a harmonic time function will contain terms with ϵ^{jwt} and ϵ^{-jwt} in conjugate relation. In some cases as a result of this, solutions will appear with indices of ϵ which are negative time variables; in such cases in the final statement the vectors with negative index should be replaced by their conjugates which rotate in the positive direction.

This paper is subdivided as follows:

Part I.—"The Method of Symmetrical Co-ordinates." Deals with the theory of the method, and its application to simple polyphase circuits.

Part II.—Application to Symmetrical Machines on Unbalanced Polyphase Circuits. Takes up Induction Motors, Generator and Synchronous Motor, Phase Balancers and Phase Convertors.

Part III. Application to Machines having Unsymmetrical Windings.

In the Appendix the mathematical representation of field forms and the derivation of the constants of different forms of networks is taken up.

The portions of Part I dealing with unsymmetrical windings are not required for the applications taken up in Part II and may be deferred to a later reading. The greater part of Part I is taken up in deriving formulas for special cases from the general formulas (**30**) and (**33**), and the reading of the text following these equations may be confined to the special cases of immediate interest.

I wish to express my appreciation of the valuable help and suggestions that have been given me in the preparation of this paper by Prof. Karapetoff who suggested that the subject be presented in a mathematical paper and by Dr. J. Slepian to whom I am indebted for the idea of sequence operators and by others who have been interested in the paper.

PART I

Method of Symmetrical Generalized Co-ordinates

RESOLUTION OF UNBALANCED SYSTEMS OF VECTORS AND OPERATORS

The complex time function $\breve{E}$ may be used instead of the harmonic time function e in any equation algebraic or differential in which it appears linearly. The reason of this is because if any linear operation is performed on $\breve{E}$ the same operation performed on its conjugate $\hat{E}$ will give a result which is conjugate to that obtained from $\breve{E}$, and the sum of the two results obtained is a solution of the same operation performed on $\breve{E} + \hat{E}$, or $2\,e$.

It is customary to interpret $\breve{E}$ and $\hat{E}$ as coplanar vectors, rotating about a common point and e as the projection of either vector on a given line, $\breve{E}$ being a positively rotating vector and

$\hat{E}$ being a negatively rotating vector, and their projection on the given line being

$$e = \frac{\breve{E} + \hat{E}}{2} \tag{1}$$

Obviously if this interpretation is accepted one of the two vectors becomes superfluous and the positively rotating vector $\breve{E}$ may be taken to represent the variable "e" and we may define "e" by saying that "e" is the projection of the vector $\breve{E}$ on a given line or else by saying that "e" is the real part of the complex variable $\breve{E}$.

If (1), $a, a^2 \ldots . a^{n-1}$ are the n roots of the equation $x^n - 1 = 0$ a symmetrical polyphase system of n phases may be represented by

$$\left.\begin{aligned} \breve{E}_{11} &= \breve{E}_{11} \\ \breve{E}_{21} &= a\,\breve{E}_{11} \\ \breve{E}_{31} &= a^2\,\breve{E}_{11} \\ &\ldots\ldots\ldots\ldots\ldots \\ &\ldots\ldots\ldots\ldots\ldots \\ \breve{E}_{n1} &= a^{n-1}\,\breve{E}_{11} \end{aligned}\right\} \tag{2}$$

Another n phase system may be obtained by taking

$$\left.\begin{aligned} \breve{E}_{12} &= \breve{E}_{12} \\ \breve{E}_{22} &= a^2\,\breve{E}_{12} \\ \breve{E}_{32} &= a^4\,\breve{E}_{12} \\ &\ldots\ldots\ldots\ldots\ldots \\ &\ldots\ldots\ldots\ldots\ldots \\ \breve{E}_{n2} &= a^{2\,(n-1)}\breve{E}_{12} \end{aligned}\right\} \tag{3}$$

and this also is symmetrical, although it is entirely different from (**2**).

Since $1 + a + a^2 + a^{n-1} = 0$, the sum of all the vectors of a symmetrical polyphase system is zero.

If $\breve{E}_1\ \breve{E}_2\ \breve{E}_3 \ldots . \breve{E}_n$ be a system of n vectors, the following identities may be proved by inspection:

$$
\left.
\begin{aligned}
\check{E}_1 \equiv {} & \frac{\check{E}_1 + \check{E}_2 + \check{E}_3 + \ldots.\check{E}_n}{n} \\
& + \frac{\check{E}_1 + a\,\check{E}_2 + a^2\,\check{E}_3 + \ldots.a^{n-1}\,\check{E}_n}{n} \\
& + \frac{\check{E}_1 + a^2\,\check{E}_2 + a^4\check{E}_3 + \ldots.a^{2(n-1)}\,\check{E}_n}{n} \\
& + \frac{\check{E}_1 + a^{r-1}\,\check{E}_2 + a^{2\,(r-1)}\,\check{E}_3 + \ldots.a^{(n-1)(r-1)}\,\check{E}_n}{n} \\
& + \ldots.\ \frac{\check{E}_1 + a^{-1}\,\check{E}_2 + a^{-2}\,\check{E}_3 + \ldots.a^{-(n-1)}\,\check{E}_n}{n} \\
\check{E}_2 \equiv {} & \frac{\check{E}_1 + \check{E}_2 + \check{E}_3 + \ldots.\check{E}_n}{n} \\
& + a^{-1}\,\frac{\check{E}_1 + a\,\check{E}_2 + a^2\,\check{E}_3 + \ldots.a^{n-1}\,\check{E}_n}{n} \\
& + a^{-2}\,\frac{\check{E}_1 + a^2\,\check{E}_2 + a^4\,\check{E}_3 + \ldots.a^{2(n-1)}\,\check{E}_n}{n} \\
& + a^{-(r-1)}\,\frac{\check{E}_1 + a^{r-1}\,\check{E}_2 + a^{2(r-1)}\,\check{E}_3 + a^{(n-1)(r-1)}\,\check{E}_n}{n} \\
& + a^{-(n-1)}\,\frac{\check{E}_1 + a^{-1}\,\check{E}_2 + a^{-2}\,\check{E}_3 + \ldots.a^{-(n-1)}\,\check{E}_n}{n} \\
& \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\
& \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\
\check{E}_n \equiv {} & \frac{\check{E}_1 + \check{E}_2 + \check{E}_3 + \ldots.\check{E}_n}{n} \\
& + a^{-(n-1)}\,\frac{\check{E}_1 + a\,\check{E}_2 + a^2\,\check{E}_3 + \ldots.a^{n-1}\,\check{E}_n}{n} \\
& + a^{-2\,(n-1)}\,\frac{\check{E}_1 + a^2\,\check{E}_2 + a^4\,\check{E}_4 + \ldots.a^{2\,(n-1)}\,\check{E}_n}{n} \\
& + a^{-(n-1)(r-1)}\,\frac{\check{E}_1 + a^{r-1}\,\check{E}_2 + \ldots.a^{(n-1)(r-1)}\,\check{E}_n}{n} \\
& + a^{-1}\,\frac{\check{E}_1 + a^{-1}\,\check{E}_2 + a^{-2}\,\check{E}_3 + \ldots.a^{-(n-1)}\,\check{E}_n}{n}
\end{aligned}
\right\} \quad (4)
$$

It will be noted that in the expression for $\check{E}_1$ in the above formulas if the first term of each component is taken the result is $n\,\frac{\check{E}_1}{n}$ or $\check{E}_1$. If the succeeding terms of each component involving

$\check{E}_2 \check{E}_3 \ldots \check{E}_n$ respectively, are taken separately they add up to expressions of the form $\frac{\check{E}_r}{n}$ $(1+a+a^2+\ldots a^{n-1})$ which are all equal to zero since $(1+a+a^2+\ldots a^{n-1})$ is equal to zero. In like manner in the expression for $\check{E}_2 \check{E}_3 \ldots \check{E}_n$ respectively, all the terms of the components involving each of the quantities $\check{E}_1 \check{E}_2 \check{E}_3 \ldots$ etc. excepting the terms involving that one of which the components are to be determined add up to expressions of the form $\frac{\check{E}_r}{n}$ $(1+a+a^2+\ldots a^{n-1})$ all of which are equal to zero, the remaining terms add up to $\check{E}_2$ $\check{E}_3 \ldots E_n$ respectively. It will now be apparent that (**4**), is true whatever may be the nature of $\check{E}_1$ $\check{E}_2$ etc., and therefore it is true of all numbers, real complex or imaginary, whatever they may represent and therefore similar relations may be obtained for current vectors and they may be extended to include not only vectors but also the operators.

In order to simplify the expressions which become unwieldy when applied to the general n-phase system, let us consider a three-phase system of vectors $\check{E}_a$ $\check{E}_b$ $\check{E}_c$. Then we have the following identities:

$$\left.\begin{aligned}
\check{E}_a &\equiv \frac{\check{E}_a+\check{E}_b+\check{E}_c}{3}+\frac{E_a+a\,\check{E}_b+a^2\,E_c}{3} \\
&\qquad +\frac{\check{E}_a+a^2\,\check{E}_b+a\,\check{E}_c}{3} \\
\check{E}_b &\equiv \frac{\check{E}_a+\check{E}_b+\check{E}_c}{3}+a^2\frac{\check{E}_a+a\,\check{E}_b+a^2\,E_c}{3} \\
&\qquad +a\,\frac{\check{E}_a+a^2\,\check{E}_b+a\,\check{E}_c}{3} \\
\check{E}_c &\equiv \frac{\check{E}_a+\check{E}_b+\check{E}_c}{3}+a\,\frac{\check{E}_a+a\,\check{E}_b+a^2\,\check{E}_c}{3} \\
&\qquad +a^2\frac{\check{E}_a+a^2\,\check{E}_b+a\,\check{E}_c}{3}
\end{aligned}\right\} \quad (\mathbf{5})$$

(**4**) states the law that a system of n vectors or quantities may be resolved when n is prime into n different symmetrical groups or systems, one of which consists of n equal vectors and the remaining $(n-1)$ systems consist of n equi-spaced vectors which with the first mentioned groups of equal vectors forms an equal number of symmetrical n-phase systems. When n is not prime some of the n-phase systems degenerate into repetitions of systems having numbers of phases corresponding to the factors of n.

Equation (**5**) states that any three vectors $\check{E}_a$ $\check{E}_b$ $\check{E}_c$ may be resolved into a system of three equal vectors $\check{E}_{a0}$ $\check{E}_{a0}$ $\check{E}_{a0}$ and two symmetrical three-phase systems $\check{E}_{a1}$, $a^2\,\check{E}_{a1}$, $a\,\check{E}_{a1}$, and $\check{E}_{a2}$, $a\,\check{E}_{a2}$, $a^2\,\check{E}_{a2}$, the first of which is of positive phase sequence and the second of negative phase sequence, or

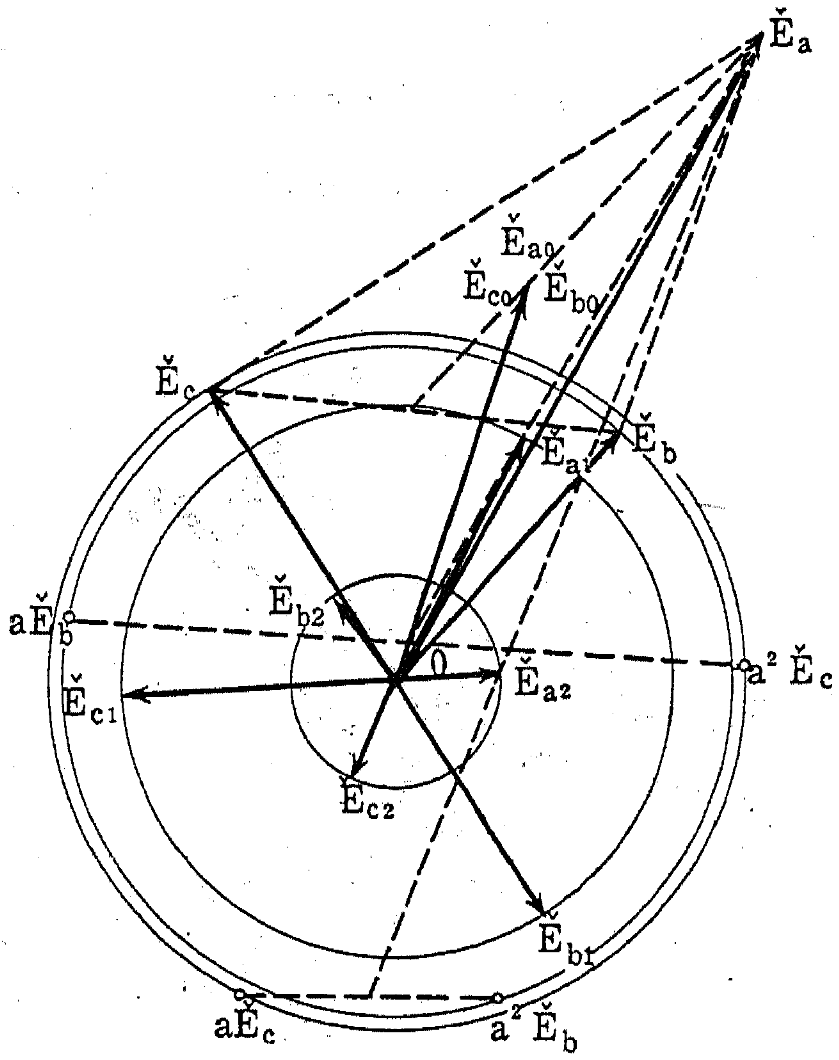

FIG. 1—GRAPHICAL REPRESENTATION OF EQUATION 5.

$$\left.\begin{aligned}\check{E}_a &= \check{E}_{a0} + \check{E}_{a1} + \check{E}_{a2}\\ \check{E}_b &= \check{E}_{a0} + a^2\,\check{E}_{a1} + a\,\check{E}_{a2}\\ \check{E}_c &= \check{E}_{a0} + a\,\check{E}_{a1} + a^2\,\check{E}_{a2}\end{aligned}\right\} \quad (\mathbf{6})$$

Similarly

$$\left.\begin{aligned}\check{I}_a &= \check{I}_{a0} + \check{I}_{a1} + \check{I}_{a2}\\ \check{I}_b &= \check{I}_{a0} + a_2\,\check{I}_{a1} + a\,\check{I}_{a2}\\ \check{I}_c &= \check{I}_{a0} + a\,\check{I}_{a1} + a^2\,\check{I}_{a2}\end{aligned}\right\} \quad (\mathbf{7})$$

Figs. (1) and (2) show a graphical method of resolving three vectors $\check{E}_a$ $\check{E}_b$ and $\check{E}_c$ into their symmetrical three-phase components corresponding to equations (**5**). The construction is as follows:—$\check{E}_{a0}$ $\check{E}_{b0}$, $\check{E}_{c0}$ are obtained by drawing a line from 0 to the centroid of the triangle $\check{E}_a \check{E}_b \check{E}_c$. $\check{E}_{a1} \check{E}_{b1}, \check{E}_{c1}$ are obtained by rotating $\check{E}_b$ positively through an angle $\frac{2\pi}{3}$, and $\check{E}_c$ negatively through the same angle giving the points $a\,\check{E}_b$ and $a^2\,\check{E}_c$ respectively. $\check{E}_{a1}$ is the vector obtained by a line drawn from 0 to the centroid of the triangle $\check{E}_a$, $a\,\check{E}_b$, $a^2\,\check{E}_c$; and $\check{E}_{b1}$ and $\check{E}_{c1}$ *lag* this vector by $\frac{2\pi}{3}$ and $\frac{4\pi}{3}$ respectively. To obtain $\check{E}_{a2} \check{E}_{b2} \check{E}_{c2}$, $\check{E}_b$ is rotated negatively and $\check{E}_c$ positively through the angle $\frac{2\pi}{3}$ giving the points $a_2\,\check{E}_b$ and $a\,\check{E}_c$ respectively; the line drawn from 0 to the centroid of the triangle $\check{E}_a$, $a^2\,\check{E}_b$, $a\,\check{E}_c$ is the

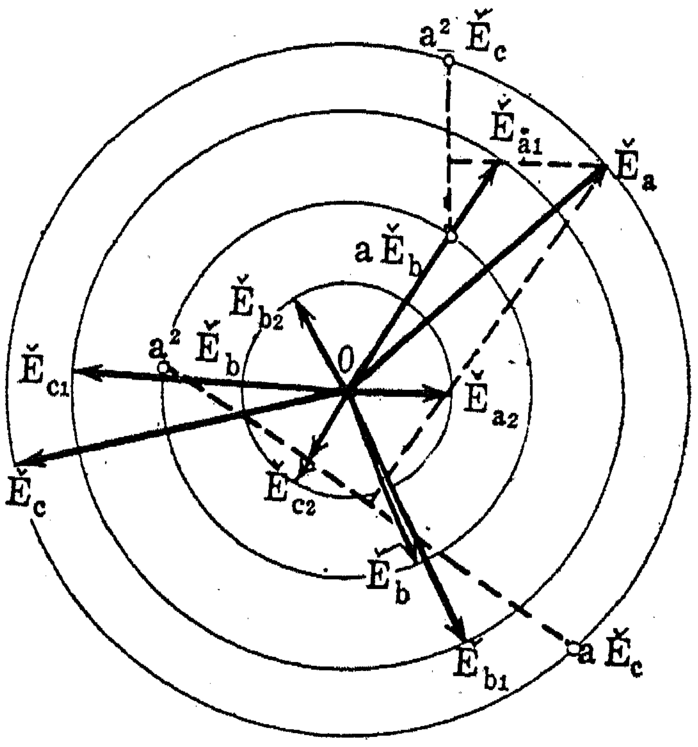

FIG. 2—GRAPHICAL REPRESENTATION OF EQUATION 5.

vector $\check{E}_{a2}$, $\check{E}_{b2}$ and $\check{E}_{c2}$ *lead* this vector by the angles $\frac{2\pi}{3}$ and $\frac{4\pi}{3}$ respectively.

The system of operators $Z_{aa}\, Z_{bb}\, Z_{cc}\, Z_{ab}\, Z_{bc}\, Z_{ca}$ may be resolved in a similar manner into symmetrical groups,

$$\left.\begin{aligned} Z_{aa} &= Z_{aa0} + Z_{aa1} + Z_{aa2} \\ Z_{bb} &= Z_{aa0} + a^2\, Z_{aa1} + a\, Z_{aa2} \\ Z_{cc} &= Z_{aa0} + a\, Z_{aa1} + a^2\, Z_{aa2} \end{aligned}\right\} \qquad (8)$$

$$\left.\begin{aligned} Z_{ab} &= Z_{ab0} + Z_{ab1} + Z_{ab2} \\ Z_{bc} &= Z_{ab0} + a^2\, Z_{ab1} + a\, Z_{ab2} \\ Z_{ca} &= Z_{ab0} + a\, Z_{ab1} + a^2\, Z_{ab2} \end{aligned}\right\} \qquad (9)$$

There are similar relations for n-phase systems.

EXPLANATION OF THEORY AND USE OF SEQUENCE OPERATOR

Let us define the symmetrical sequences of nth roots of unity in the following manner:

$$\left.\begin{array}{l}
S^0 = 1, \quad 1, \quad 1 \ldots . 1 \\
S^1 = 1, \quad a^{-1}, \quad a^{-2} \ldots . a^{-(n-1)} \\
S^2 = 1, \quad a^{-2}, \quad a^{-4} \ldots . a^{-2(n-1)} \\
\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\
S^r = 1, \quad a^{-r}, \quad a^{-2r} \ldots . a^{-(n-1)r} \\
S^{(r+1)} = 1, \; a^{-(r+1)}, \; a^{-2(r+1)} \ldots . a^{-(n-1)(r+1)} \\
\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\
S^{(n-1)} = 1, \; a^{-(n-1)}, \; a^{-2(n-1)} \ldots . a^{-(n-1)^2}
\end{array}\right\} \qquad (10)$$

Consider the sequence obtained by the products of similar terms of S^r and S^1. It will be

$$S^{(r+1)} = 1, \; a^{-(r+1)}, \quad a^{-2(r+1)} \ldots . a^{-(n-1)(r+1)} \qquad (11)$$

Similarly

$$S^k = 1, \quad a^{-k}, \quad a^{-2k} \ldots . a^{-(n-1)k} \qquad (12)$$

and the sequence obtained by products of like terms of this sequence and S^r is

$$S^{(r+k)} = 1, \; a^{-(r+k)}, \quad a^{-2(r+k)} \ldots . a^{-(n-1)(r+k)} \qquad (13)$$

We may therefore apply the law of indices to the products of sequences to obtain the resulting sequence.

In the case of the three-phase system we shall have the following sequences only to consider, viz.:

$$\left.\begin{array}{l}
S^0 = 1, \quad 1, \quad 1 \\
S^1 = 1, \quad a^2, \quad a \\
S^2 = 1, \quad a, \quad a^2
\end{array}\right\} \qquad (14)$$

The complete system of currents $\check{I}_a \; \check{I}_b \; \check{I}_c$ are defined by

$$S\,(\check{I}_a) = S^0\, \check{I}_{a0} + S^1\, \check{I}_{a1} + S^2\, \check{I}_{a2} \qquad (15)$$

Similarly the impedances $Z_{aa}\; Z_{bb}\; Z_{cc}$ may be expressed in symmetrical form

$$S\,(Z_{aa}) \equiv S^0\, Z_{aa0} + S^1\, Z_{aa1} + S^2\, Z_{aa2} \qquad (16)$$

and the mutual impedances Z_{ab}, Z_{bc}, Z_{ca} are expressed by

$$S\,(Z_{ab}) \equiv S^0\, Z_{ab0} + S^1\, Z_{ab1} + S^2\, Z_{ab2} \qquad (17)$$

Attention is called to the importance of preserving the cyclic order of self and mutual impedances, otherwise the rule for the sequence operator will not hold. Thus, Z_{ab}, Z_{bc} and Z_{ca} are in proper sequence as also are Z_{ca}, Z_{ab}, Z_{bc}.

When it is desired to change the first term in the sequence of polyphase vectors the resulting expression will be

$$\left.\begin{aligned} S\,(\check{I}_b) &= S^0\,\check{I}_{a0} + S^1\,a^2\,\check{I}_{a1} + S^2\,a\,\check{I}_{a2} \\ S\,(\check{I}_c) &= S^0\,\check{I}_{a0} + S^1\,a\,\check{I}_{a1} + S^2\,a^2\,\check{I}_{a2} \end{aligned}\right\} \quad (18)$$

Similarly in the case of the operators $S\,(Z_{ab})$ we have

$$\left.\begin{aligned} S\,(Z_{bc}) &= S^0\,Z_{ab0} + S^1\,a^2\,Z_{ab1} + S^2\,a\,Z_{ab2} \\ S\,(Z_{ca}) &= S^0\,Z_{ab0} + S^1\,a\,Z_{ab1} + S^2\,a^2\,Z_{ab2} \end{aligned}\right\} \quad (19)$$

Similar rules apply to the e.m.fs. $E_a\ E_b\ E_c$

$$\left.\begin{aligned} S\,(\check{E}_a) &= S^0\,\check{E}_{a0} + S^1\,\check{E}_{a1} + S^2\,\check{E}_{a2} \\ S\,(\check{E}_b) &= S^0\,\check{E}_{a0} + S^1\,a^2\,\check{E}_{a1} + S^2\,a\,\check{E}_{a2} \\ S\,(\check{E}_c) &= S^0\,\check{E}_{a0} + S^1\,a\,E_{a1} + S^2\,a^2\,\check{E}_{a2} \end{aligned}\right\} \quad (20)$$

It should be kept in mind that any one of the several expressions $S\,(\check{I}_a)$ $S\,(\check{I}_b)$ $S\,(\check{I}_c)$, etc., completely specifies the system, and each of the members of the groups of equations given above is a complete statement of the system of vectors or operators and their relation.

APPLICATION TO SELF AND MUTUAL IMPEDANCE OPERATIONS

We may now proceed with the current systems $S\,(\check{I}_a)$, $S\,(\check{I}_b)$, $S\,(\check{I}_c)$ and the operating groups $S\,(Z_{aa})$ $S\,(Z_{bb})$ $S\,(Z_{cc})$ etc. and the electromotive forces in exactly the same manner as for simple a-c. circuits. Thus,

$$\begin{aligned} S\,(\check{E}_a) &= S\,(Z_{aa})\,S\,(\check{I}_a) + S\,(Z_{ab})\,S\,(\check{I}_b) + S\,(Z_{ca})\,S\,(\check{I}_c) \qquad (21) \\ &= (S^0\,Z_{aa0} + S^1\,Z_{aa1} + S^2\,Z_{aa2})\,(S^0\,\check{I}_{a0} + S^1\,\check{I}_{a1} + S^2\,I_{a2}) \\ &\quad + (S^0\,Z_{ab0} + S^1\,Z_{ab1} + S^2\,Z_{ab2}) \\ &\qquad\qquad (S^0\,\check{I}_{a0} + S^1\,a^2\,\check{I}_a + S^2\,a\,\check{I}_{a2}) \\ &\quad + (S^0\,Z_{ab0} + S^1\,a\,Z_{ab1} + S^2\,a^2\,Z_{ab2}) \\ &\qquad\qquad (S^0\,\check{I}_{a0} + S^1\,a\,\check{I}_{a1} + S^2\,a^2\,\check{I}_{a2}) \\ &= S^0\,(Z_{aa0} + 2\,Z_{ab0})\,\check{I}_{a0} + S^0\,\{Z_{aa2} + (1 + a^2)\,Z_{ab2}\}\,\check{I}_{a1} \end{aligned}$$

$$\begin{aligned}
&+ S^0 \{Z_{aa1} + (1 + a)\, Z_{ab1}\}\, \check{I}_{a2} \\
&+ S^1 \{Z_{aa1} + (1 + a)\, Z_{ab1}\}\, \check{I}_{a0} \\
&+ S^1 \{Z_{aa0} + (a + a^2)\, Z_{ab0}\}\, \check{I}_{a1} \\
&+ S^1 \{Z_{aa2} + 2\, a\, Z_{ab2}\}\, \check{I}_{a2} \\
&+ S^2 \{Z_{aa2} + (1 + a^2)\, Z_{ab2}\}\, \check{I}_{a0} \\
&+ S^2 \{Z_{aa1} + 2\, a^2\, Z_{ab1}\}\, \check{I}_{a1} \\
&+ S^2 \{Z_{aa0} + (a + a^2)\, Z_{ab0}\}\, \check{I}_{a2} \qquad (\mathbf{22})
\end{aligned}$$

Or since $1 + a + a^2 = 0$, $1 + a = - a^2$, $1 + a^2 = - a$ and $a + a^2 = - 1$

$$\begin{aligned}
S\,(\check{E}_a) = {} & S^0\,(Z_{aa0} + 2\, Z_{ab0})\, \check{I}_{a0} + S^0\,(Z_{aa2} - a\, Z_{ab2})\, \check{I}_{a1} \\
&+ S^0\,(Z_{aa1} - a^2\, Z_{ab1})\, \check{I}_{a2} + S^1\,(Z_{aa1} - a^2\, Z_{ab1})\, \check{I}_{a0} \\
&+ S^1\,(Z_{aa0} - Z_{ab0})\, \check{I}_{a1} + S^1\,(Z_{aa2} + 2\, a\, Z_{ab2})\, \check{I}_{a2} \\
&+ S^2\,(Z_{aa2} - a\, Z_{ab2})\, \check{I}_{a0} + S^2\,(Z_{aa1} + 2\, a^2\, Z_{ab1})\, \check{I}_{a1} \\
&+ S^2\,(Z_{aa0} - Z_{ab0})\, \check{I}_{a2} \qquad (\mathbf{23})
\end{aligned}$$

Or since

$$\begin{aligned}
S\,(Z_{bc}) &= S^0\, Z_{bc0} + S^1\, Z_{bc1} + S^2\, Z_{bc2} \\
&= S^0\, Z_{ab0} + S^1\, a^2\, Z_{ab1} + S^2\, a\, Z_{ab2}
\end{aligned}$$

we may write (**23**) in the form

$$\begin{aligned}
S\,(\check{E}_a) = {} & S^0\,(Z_{aa0} + 2\, Z_{bc0})\, \check{I}_{a0} + S^0\,(Z_{aa2} - Z_{bc2})\, \check{I}_{a1} \\
&+ S^0\,(Z_{aa1} - Z_{bc1})\, \check{I}_{a2} + S^1\,(Z_{aa1} - Z_{bc1})\, \check{I}_{a0} \\
&+ S^1\,(Z_{aa0} - Z_{bc0})\, \check{I}_{a1} + S^1\,(Z_{aa2} + 2\, Z_{bc2})\, \check{I}_{a2} \\
&+ S^2\,(Z_{aa2} - Z_{bc2})\, \check{I}_{a0} + S^2\,(Z_{aa1} + 2\, Z_{bc1})\, \check{I}_{a1} \\
&+ S^2\,(Z_{aa0} - Z_{bc0})\, \check{I}_{a2} \qquad (\mathbf{24})
\end{aligned}$$

which is the more symmetrical form. We have therefore from (**24**) by expressing $S\,(\check{E}_a)$ in terms of symmetrical co-ordinates the three symmetrical equations

$$\left.\begin{aligned}
S^0\, \check{E}_{a0} &= S^0\, \{(Z_{aa0} + 2\, Z_{bc0})\, \check{I}_{a0} + (Z_{aa2} - Z_{bc2})\, \check{I}_{a1} \\
&\qquad + (Z_{aa1} - Z_{bc1})\, \check{I}_{a2}\} \\
S^1\, E_{a1} &= S^1\, \{(Z_{aa1} - Z_{bc1})\, \check{I}_{a0} + (Z_{aa0} - Z_{bc0})\, \check{I}_{a1} \\
&\qquad + (Z_{aa2} + 2\, Z_{bc2})\, \check{I}_{a2}\} \\
S^2\, E_{a2} &= S^2\, \{(Z_{aa2} - Z_{bc2})\, \check{I}_{a0} + (Z_{aa1} + 2\, Z_{bc1})\, \check{I}_{a1} \\
&\qquad + (Z_{aa0} - Z_{bc0})\, \check{I}_{a2}\}
\end{aligned}\right\} \qquad (\mathbf{25})$$

An important case to which we must next give consideration is that of mutual inductance between a primary polyphase circuit and a secondary polyphase circuit. The mutual impedances may be arranged in three sets. Let the currents in the secondary windings be I_u I_v and I_w, we may then express the generalized mutual impedances as follows:

$$\left.\begin{array}{r} \text{(I) } Z_{au}\ Z_{bv}\ Z_{cw} \\ \text{(II) } Z_{bw}\ Z_{cu}\ Z_{av} \\ \text{(III) } Z_{cv}\ Z_{aw}\ Z_{bu} \end{array}\right\} \qquad \textbf{(26)}$$

Each set may be resolved into three symmetrical groups, so that

$$\left.\begin{array}{l} S\ (Z_{au}) = S^0\ Z_{au0} + S^1\ Z_{au1} + S^2\ Z_{au2} \\ S\ (Z_{bw}) = S^0\ Z_{bw0} + S^1\ Z_{bw1} + S^2\ Z_{bw2} \\ S\ (Z_{cv}) = S^0\ Z_{cv0} + S^1\ Z_{cv1} + S^2\ Z_{cv2} \end{array}\right\} \qquad \textbf{(27)}$$

and we have for $S\ (\check{E}_a)$ the primary induced e.m.f. due to the secondary currents $S\ (\check{I}_u)$

$$S\ (\check{E}_a) = S\ (Z_{au})\ S\ (\check{I}_u) + S\ (Z_{av})\ S\ (\check{I}_v) + S\ (Z_{aw})\ S\ (\check{I}_w) \qquad \textbf{(28)}$$

Substituting for $S\ (\check{I}_u)$, $S\ (\check{I}_v)$ and $S\ (\check{I}_w)$ and $S\ (Z_{au})$, $S\ (Z_{av})$ $S\ (Z_{aw})$ their symmetrical equivalents we have

$$\begin{aligned} S\ (\check{E}_a) = S^0\ & (Z_{au0} + Z_{bw0} + Z_{cv0})\ I_{uo} \\ & + S^0\ (Z_{au2} + a\ Z_{bw2} + a^2\ Z_{cv2})\ \check{I}_{u1} \\ & + S^0\ (Z_{au1} + a^2\ Z_{bw1} + a\ Z_{cv1})\ \check{I}_{u2} \\ & + S^1\ (Z_{au1} + a\ Z_{bw1} + a^2\ Z_{cv1})\ \check{I}_{u0} \\ & + S^1\ (Z_{au0} + a^2\ Z_{bw0} + a\ Z_{cv0})\ \check{I}_{u1} \\ & + S^1\ (Z_{au2} + Z_{bw2} + Z_{cv2})\ \check{I}_{u2} \\ & + S^2\ (Z_{au2} + a^2\ Z_{bw2} + a\ Z_{v2})\ \check{I}_{u0} \\ & + S^2\ (Z_{au1} + Z_{bw1} + Z_{cv1})\ \check{I}_{u1} \\ & + S^2\ (Z_{au0} + a\ Z_{bw0} + a^2\ Z_{cv0})\ \check{I}_{u2} \qquad \textbf{(29)} \end{aligned}$$

On expressing $S\ (\check{E}_a)$ in symmetrical form we have the following three symmetrical equations

$$\left.\begin{aligned}
S^0\ \check{E}_{a0} = S^0\ \{&(Z_{au0} + Z_{bw0} + Z_{cv0})\ \check{I}_{u0} \\
&+ (Z_{au2} + a\ Z_{bw2} + a^2\ Z_{cv2})\ \check{I}_{u1} \\
&+ (Z_{au1} + a^2\ Z_{bw1} + a\ Z_{cv1})\ \check{I}_{u2}\} \\
S^1\ \check{E}_{a1} = S^1\ \{&(Z_{au1} + a\ Z_{bw1} + a^2\ Z_{cv1})\ \check{I}_{u0} \\
&+ (Z_{au0} + a^2\ Z_{bw0} + a\ Z_{cv0})\ \check{I}_{u1} \\
&+ (Z_{au2} + Z_{bw2} + Z_{cv2})\ \check{I}_{u2}\} \\
S^2\ E_{a2} = S^2\ \{&(Z_{au2} + a^2\ Z_{bw2} + a\ Z_{cv2})\ \check{I}_u{}^0 \\
&+ (Z_{au1} + Z_{bw1} + Z_{cv1})\ \check{I}_{u1} \\
&+ (Z_{au0} + a\ Z_{bw0} + a^2\ Z_{cv0})\ \check{I}_{u2}\}
\end{aligned}\right\} \quad \textbf{(30)}$$

For the e.m.f. $S\ (\check{E}_u)$ induced in the secondary by the primary currents $S\ (\check{I}_a)$ we have

$$S\ (\check{E}_u) = S\ (Z_{au})\ S\ (\check{I}_a) + S\ (Z_{bu})\ S\ (\check{I}_b) + S\ (Z_{cu})\ S\ (\check{I}_c) \quad \textbf{(31)}$$

Since $S\ (Z_{bu})$ bears the same relation to $S\ (Z_{cv})$ as $S\ (Z_{av})$ does to $S\ (Z_{bw})$ and $S\ (Z_{au})$ bears the same relation to $S\ (Z_{bw})$ as $S\ (Z_{aw})$ does to $S\ (Z_{cv})$ to obtain $S\ (\check{E}_u)$ all that will be necessary will be to interchange Z_{bw} and Z_{cv} in (**29**) and change $\check{I}_{u0}\ \check{I}_{u1}\ \check{I}_{u2}$ to $\check{I}_{a0}\ I_{a1}$ and $\check{I}_{a2}$ respectively, this gives

$$\begin{aligned}
S\ (\check{E}_u) = {} & S^0\ (Z_{au0} + Z_{bw0} + Z_{cv0})\ \check{I}_{a0} \\
&+ S^0\ (Z_{au2} + a^2\ Z_{bw2} + a\ Z_{cv2})\ \check{I}_{a1} \\
&+ S^0\ (Z_{au1} + a\ Z_{bw1} + a^2\ Z_{cv1})\ \check{I}_{a2} \\
&+ S^1\ (Z_{au1} + a^2\ Z_{bw1} + a\ Z_{cv1})\ \check{I}_{a0} \\
&+ S^1\ (Z_{au0} + a\ Z_{bw0} + a^2\ Z_{cv0})\ \check{I}_{a1} \\
&+ S^1\ (Z_{au2} + Z_{bw2} + Z_{cv2})\ \check{I}_{a2} \\
&+ S^2\ (Z_{au2} + a\ Z_{bw2} + a^2\ Z_{cv2})\ \check{I}_{a0} \\
&+ S^2\ (Z_{au1} + Z_{bw1} + Z_{cv1})\ \check{I}_{a1} \\
&+ S^2\ (Z_{au0} + Z_{bw0} + Z_{cv0})\ \check{I}_{a2} \qquad \textbf{(32)}
\end{aligned}$$

and the three symmetrical equations will be

$$\left.\begin{aligned}
S^0\,\check{E}_{u0} = S^0\,\{&(Z_{au0} + Z_{bw0} + Z_{cv0})\,\check{I}_{a0}\\
&+ (Z_{au2} + a^2\,Z_{bw2} + a\,Z_{bw2} + a\,Z_{cv2})\,\check{I}_{a1}\\
&+ (Z_{au1} + a\,Z_{bw1} + a^2\,Z_{cv1})\,\check{I}_{a2}\}\\
S^1\,\check{E}_{a1} = S^1\,\{&(Z_{au1} + a^2\,Z_{bw1} + a\,Z_{cv1})\,\check{I}_{a0}\\
&+ (Z_{au0} + a\,Z_{bw0} + a^2\,Z_{cv0})\,\check{I}_{a1}\\
&+ (Z_{au2} + Z_{bw2} + Z_{cv2})\,\check{I}_{a2}\}\\
S^2\,\check{E}_{u2} = S^2\,\{&(Z_{au2} + a\,Z_{bw2} + a^2\,Z_{cv2})\,\check{I}_{a0}\\
&+ (Z_{au1} + Z_{bw1} + Z_{cv1})\,\check{I}_{a1}\\
&+ (Z_{au0} + a^2\,Z_{bw0} + a\,Z_{cv0})\,\check{I}_{a2}\} \; .
\end{aligned}\right\} \quad (33)$$

The same methods may be applied to polyphase systems of any number of phases. When the number of phases is not prime the system may sometimes be dealt with as a number of polyphase systems having mutual inductance between them:—For example, a nine-phase system may be treated as three three-phase systems, a twelve-phase system as three four-phase or four three-phase systems. In certain forms of dissymmetry this method is of great practical value, and its application will be taken up later.

For the present part of the paper we shall confine ourselves to the three-phase system, and dissymmetries of several different kinds.

The operators Z_{au} Z_{aa}, etc., must be interpreted in the broadest sense. They may be simple complex quantities or they may be functions of the differential operator $\frac{d}{dt}$. For if

$$i = \Sigma\,(A_n \cos n\,w\,t + B_n \sin n\,w\,t)$$

it may be expressed in the form

$$\left.\begin{aligned}
i &= \Sigma\left(\frac{A_n - j\,B_n}{2}\,\epsilon^{jnwt} + \frac{A_n + j\,B_n}{2}\,\epsilon^{-jnwt}\right)\\
&= \frac{\check{I}}{2} + \frac{\hat{I}}{2}\\
&= \text{real part of } \check{I}
\end{aligned}\right\} \quad (34)$$

and any linear algebraic operation performed on $\check{I}/2$ will give a result which will be conjugate to that obtained by carrying out the same operation on $\hat{I}/2$ and since the true solution is the sum of these results, it may also be obtained by taking the real part of the result of performing the operation on $\check{I}$.

Modification of the General Case Met With in Practical Networks

Several symmetrical arrangements of the operator Z_{au} etc. are frequently met with in practical networks which result in a much simpler system of equations than those obtained for the general case as in equations (**29**) to (**33**). Thus for example if all the operators in (**26**) are equal, all the operators in (**27**), except $S^0\, Z_{au0}$ $S^0\, Z_{bw0}$ and $S^0\, Z_{cv0}$ are equal to zero, and these three quantities are also equal to one another so that equation (**30**) becomes

$$\left.\begin{aligned} S^0\, \check{E}_{a0} &= S^0\, (Z_{au0} + Z_{bw0} + Z_{cv0})\, \check{I}_{u0} \\ S^1\, \check{E}_{a1} &= 0 \\ S^2\, E_{a2} &= 0 \end{aligned}\right\} \qquad (\mathbf{35})$$

and equation (**33**)

$$\left.\begin{aligned} S^0\, \check{E}_{u0} &= S^0\, (Z_{au0} + Z_{bw0} + Z_{cv0})\, \check{I}_{a0} \\ S^1\, E_{u1} &= 0 \\ S^2\, E_{u2} &= 0 \end{aligned}\right\} \qquad (\mathbf{36})$$

This is the statement in symmetrical co-ordinates that a symmetrically disposed polyphase transmission line will produce no electromagnetic induction in a second similar polyphase system so disposed with respect to the first that mutual inductions between all phases of the two are equal except that due to single-phase currents passing through the conductors.

If in (**26**) the quantities in each group only are equal, equations (**30**) and (**33**) become

$$\left.\begin{aligned} S^0\, \check{E}_{a0} &= S^0\, (Z_{au0} + Z_{bw0} + Z_{cv0})\, \check{I}_{u0} \\ S^1\, \check{E}_{a1} &= S^1\, (Z_{au0} + a^2\, Z_{bw0} + a\, Z_{cv0})\, \check{I}_{u1} \\ S^2\, \check{E}_{a2} &= S^2\, (Z_{au0} + a\, Z_{bw0} + a^2\, Z_{cv0})\, \check{I}_{u2} \end{aligned}\right\} \qquad (\mathbf{37})$$

$$\left.\begin{aligned} S^0\, \check{E}_{u0} &= S^0\, (Z_{au0} + Z_{bw0} + Z_{cv0})\, \check{I}_{a0} \\ S^1\, \check{E}_{a1} &= S^1\, (Z_{au0} + a\, Z_{bw0} + a^2\, Z_{cv0})\, \check{I}_{a1} \\ S^2\, \check{E}_{a2} &= S^1\, (Z_{au0} + a^2\, Z_{bw0} + a\, Z_{cv0})\, \check{I}_{a2} \end{aligned}\right\} \quad (38)$$

SYMMETRICAL FORMS OF COMMON OCCURRENCE

A symmetrical form which is of importance because it is of frequent occurrence in practical polyphase networks has the terms in group (I) equation (**26**) all equal and those in group (II) $\cos \frac{2\pi}{3}$ times those in group (I) and those in group (III) $\cos \frac{4\pi}{3}$ times those in group (I).

Since $\cos \frac{2\pi}{3} = \frac{a + a^2}{2} = \cos \frac{4\pi}{3}$ we have on substituting the values of the impedances in this case,

$$\left.\begin{aligned} S^0\, \check{E}_{a0} &= S^0\, \{Z_{au0}\, (1 + a + a^2)\}\, \check{I}_{u0} = 0 \\ S^1\, \check{E}_{a1} &= S^1\, 1\tfrac{1}{2}\, Z_{au0}\, \check{I}_{u1} \\ S^2\, E_{a2} &= S^2\, 1\tfrac{1}{2}\, Z_{au0}\, \check{I}_{u2} \end{aligned}\right\} \quad (39)$$

$$\left.\begin{aligned} S^0\, E_{u0} &= S^0\, \{Z_{au0}\, (1 + a + a^2)\}\, \check{I}_{a0} = 0 \\ S^1\, \check{E}_{u1} &= S^1\, 1\tfrac{1}{2}\, Z_{au0}\, \check{I}_{a1} \\ S^2\, \check{E}_{u2} &= S^2\, 1\tfrac{1}{2}\, Z_{au0}\, \check{I}_{a2} \end{aligned}\right\} \quad (40)$$

The elements in group I may be unequal but groups II and III may be obtained from group I by multiplying by $\cos \frac{4\pi}{3}$ and $\cos \frac{2\pi}{3}$ respectively.

The members of the three groups will then be related as follows, the same sequence being used as before,

$$\left.\begin{array}{llll} \text{(I)} & Z_{au}, & Z_{bv}, & Z_{cw} \\ \text{(II)} & \frac{a + a^2}{2}\, Z_{cw}, & \frac{a + a^2}{2}\, Z_{au}, & \frac{a + a^2}{2}\, Z_{bv} \\ \text{(III)} & \frac{a + a^2}{2}\, Z_{bv}, & \frac{a + a^2}{2}\, Z_{cw}, & \frac{a + a^2}{2}\, Z_{au} \end{array}\right\} \quad (41)$$

Consequently the following relations are true:

$$\left.\begin{aligned} S^0\, Z_{bw0} &= \frac{a + a^2}{2}\, S^0\, Z_{au0} \\ S^0\, Z_{cv0} &= \frac{a + a^2}{2}\, S^0\, Z_{au0} \\ S^1\, Z_{bw1} &= \frac{1 + a^2}{2}\, S^1\, Z_{au1} \\ S^2\, Z_{bw2} &= \frac{1 + a}{2}\, S^2\, Z_{au2} \\ S^1\, Z_{cv1} &= \frac{1 + a}{2}\, S^1\, Z_{au1} \\ S^2\, Z_{cv2} &= \frac{1 + a^2}{2}\, S^2\, Z_{au2} \end{aligned}\right\} \qquad (\mathbf{42})$$

Substituting these relations in (**30**) and (**33**) we have for this system of mutual impedances

$$\left.\begin{aligned} Z_{au0} + Z_{bw0} + Z_{cv0} &= 0 \\ Z_{au0} + a\, Z_{bw0} + a^2\, Z_{cv0} &= 1\tfrac{1}{2}\, Z_{cv0} \\ Z_{au0} + a^2\, Z_{bw0} + a\, Z_{cv0} &= 1\tfrac{1}{2}\, Z_{aw0} \end{aligned}\right\} \qquad (\mathbf{43})$$

$$\left.\begin{aligned} Z_{au1} + Z_{bw1} + Z_{cv1} &= 1\tfrac{1}{2}\, Z_{au1} \\ Z_{au1} + a\, Z_{bw1} + a^2\, Z_{cv1} &= 1\tfrac{1}{2}\, Z_{au1} \\ Z_{au1} + a^2\, Z_{bw1} + a\, Z_{cv1} &= 0 \end{aligned}\right\} \qquad (\mathbf{44})$$

$$\left.\begin{aligned} Z_{au2} + Z_{bw2} + Z_{cv2} &= 1\tfrac{1}{2}\, Z_{au2} \\ Z_{au2} + a\, Z_{bw2} + a^2\, Z_{cv2} &= 0 \\ Z_{au2} + a^2\, Z_{bw2} + a\, Z_{cv2} &= 1\tfrac{1}{2}\, Z_{au2} \end{aligned}\right\} \qquad (\mathbf{45})$$

which on substitution in (**30**) and (**33**) gives

$$\left.\begin{aligned} S^0\, \check{E}_{a0} &= 0 \\ S^1\, \check{E}_{a1} &= S^1\, \{1\tfrac{1}{2}\, Z_{au1}\, \check{I}_{u0} + 1\tfrac{1}{2}\, Z_{au0}\, \check{I}_{u1} + 1\tfrac{1}{2}\, Z_{au2}\, \check{I}_{u2}\} \\ S^2\, \check{E}_{a2} &= S^2\, \{1\tfrac{1}{2}\, Z_{au2}\, \check{I}_{u0} + 1\tfrac{1}{2}\, Z_{au1}\, \check{I}_{u1} + 1\tfrac{1}{2}\, Z_{au0}\, \check{I}_{u2}\} \end{aligned}\right\} \qquad (\mathbf{46})$$

$$\left.\begin{aligned} S^0\, \check{E}_{u0} &= S^0\, \{1\tfrac{1}{2}\, Z_{au2}\, \check{I}_{a1} + 1\tfrac{1}{2}\, Z_{au1}\, \check{I}_{a2}\} \\ S^1\, \check{E}_{u1} &= S^1\, \{1\tfrac{1}{2}\, Z_{au0}\, \check{I}_{a1} + 1\tfrac{1}{2}\, Z_{au2}\, \check{I}_{a2}\} \\ S^2\, \check{E}_{u2} &= S^2\, \{1\tfrac{1}{2}\, Z_{au1}\, \check{I}_{a1} + 1\tfrac{1}{2}\, Z_{au0}\, \check{I}_{a2}\} \end{aligned}\right\} \qquad (\mathbf{47})$$

The above symmetrical forms in which the factors $\cos \frac{2\pi}{3}$ and $\cos \frac{4\pi}{3}$ occur apply particularly to electromagnetic induction between windings distributed over the surfaces of co-axial cylinders; where if the plane of symmetry of one winding be taken as the datum plane, the mutual impedance between this winding and any other is a harmonic function of the angle between its plane of symmetry and the datum plane. In other words, the mutual impedances are functions of position on the circumference of a circle and may therefore be expanded by Fourier's theorem in a series of integral harmonics of the angle made by the planes of symmetry with the datum plane. Since the same procedure applies to all the terms of the expansion it is necessary only to consider the simple harmonic case. In the partially symmetrical cases of mutual induction, such as that taken up in the preceding discussion, there will be a difference between two possible cases, viz:—Symmetrical primary, unsymmetrical secondary, which is the case just considered, and unsymmetrical primary and symmetrical secondary in which the impedances of (**26**) will have the following values

$$\left.\begin{array}{llll}
\text{(I)} & Z_{au}, & Z_{bc}, & Z_{cw} \\
\text{(II)} & \dfrac{a+a^2}{2} Z_{bc}, & \dfrac{a+a^2}{2} Z_{cw}, & \dfrac{a+a^2}{2} Z_{au} \\
\text{(III)} & \dfrac{a+a^2}{2} Z_{cw}, & \dfrac{a+a^2}{2} Z_{au}, & \dfrac{a+a^2}{2} Z_{bc}
\end{array}\right\} \quad \textbf{(48)}$$

The results may be immediately set down by symmetry from equations (**46**) and (**47**), but the difference between the two cases will be better appreciated by setting down the component symmetrical impedances, thus we have

$$\left.\begin{array}{l}
S^0 Z_{bw0} = \dfrac{a+a^2}{2} S^0 Z_{au0} \\
S^0 Z_{cv0} = \dfrac{a+a^2}{2} S^0 Z_{au0} \\
S^1 Z_{bw1} = \dfrac{1+a}{2} S^1 Z_{au1} \\
S^2 Z_{bw2} = \dfrac{1+a^2}{2} S^2 Z_{au2} \\
S^1 Z_{cv1} = \dfrac{1+a^2}{2} S^1 Z_{cu1} \\
S^2 Z_{cv2} = \dfrac{1+a}{2} S^2 Z_{au2}
\end{array}\right\} \quad \textbf{(49)}$$

Substituting these relations in the impedances used in (**30**) and (**33**) they become

$$\left.\begin{aligned} Z_{au0} + Z_{bw0} + Z_{cv0} &= 0 \\ Z_{au0} + a\, Z_{bw0} + a^2\, Z_{cv0} &= 1\tfrac{1}{2}\, Z_{au0} \\ Z_{au0} + a^2\, Z_{bw0} + a\, Z_{cv0} &= 1\tfrac{1}{2}\, Z_{au0} \end{aligned}\right\} \qquad \textbf{(50)}$$

$$\left.\begin{aligned} Z_{au1} + Z_{bw1} + Z_{cv1} &= 1\tfrac{1}{2}\, Z_{au1} \\ Z_{au1} + a\, Z_{bw1} + a^2\, Z_{cv1} &= 0 \\ Z_{au1} + a^2\, Z_{bw1} + a\, Z_{cv1} &= 1\tfrac{1}{2}\, Z_{au1} \end{aligned}\right\} \qquad \textbf{(51)}$$

$$\left.\begin{aligned} Z_{au2} + Z_{bw2} + Z_{cv2} &= 1\tfrac{1}{2}\, Z_{au2} \\ Z_{au2} + a\, Z_{bw2} + a^2\, Z_{cv2} &= 1\tfrac{1}{2}\, Z_{au2} \\ Z_{au2} + a^2\, Z_{bw2} + a\, Z_{cv2} &= 0 \end{aligned}\right\} \qquad \textbf{(52)}$$

And we have from (**30**) and (**33**), or by symmetry

$$\left.\begin{aligned} S^0\, \check{E}_{a0} &= S^0\, \{1\tfrac{1}{2}\, Z_{au2}\, \check{I}_{u1} + 1\tfrac{1}{2}\, Z_{au1}\, \check{I}_{u2}\} \\ S^1\, \check{E}_{a1} &= S^1\, \{1\tfrac{1}{2}\, Z_{au0}\, \check{I}_{u1} + 1\tfrac{1}{2}\, Z_{au2}\, \check{I}_{u2}\} \\ S^2\, \check{E}_{a2} &= S^2\, \{1\tfrac{1}{2}\, Z_{au1}\, \check{I}_{u1} + 1\tfrac{1}{2}\, Z_{au0}\, \check{I}_{u2} \end{aligned}\right\} \qquad \textbf{(53)}$$

$$\left.\begin{aligned} S^0\, \check{E}_{u0} &= 0 \\ S^1\, \check{E}_{u1} &= S^1\, \{1\tfrac{1}{2}\, Z_{au1}\, \check{I}_{a0} + 1\tfrac{1}{2}\, Z_{au0}\, \check{I}_{u1} + 1\tfrac{1}{2}\, Z_{au2}\, \check{I}_{u2}\} \\ S^2\, \check{E}_{u2} &= S^2\, \{1\tfrac{1}{2}\, Z_{au2}\, \check{I}_{a0} + 1\tfrac{1}{2}\, Z_{au1}\, \check{I}_{u1} + 1\tfrac{1}{2}\, Z_{au0}\, \check{I}_{u2}\} \end{aligned}\right\} \qquad \textbf{(54)}$$

If the angle between the planes of symmetry of the coils and the datum plane are subject to changes, $\cos \frac{2\,\pi}{3}$ and $\cos \frac{4\,\pi}{3}$ in the preceding discussion must be replaced by

$$\left.\begin{aligned} \cos\left(\frac{2\,\pi}{3} + \theta\right) &= \frac{a}{2}\, \epsilon^{j\,\theta} + \frac{a^2}{2}\, \epsilon^{-j\,\theta} \\ \cos\left(\frac{4\,\pi}{3} + \theta\right) &= \frac{a}{2}\, \epsilon^{-j\theta} + \frac{a^2}{2}\, \epsilon^{j\,\theta} \end{aligned}\right\} \qquad \textbf{(55)}$$

where θ is measured from the datum plane

In the strictly symmetrical case of co-axial cylindrical surface windings in which the members of each group of mutual impedances are equal, the result of substituting (**55**) in the equations for induced e.m.f. will be

$$\left.\begin{aligned} S^0\,\check{E}_{a0} &= 0 \\ S^1\,\check{E}_{a1} &= S^1\left(1\tfrac{1}{2}\,Z_{au0}\,\epsilon^{j\theta}\;\check{I}_{u1}\right) \\ S^2\,\check{E}_{a2} &= S^2\left(1\tfrac{1}{2}\,Z_{au0}\,\epsilon^{-j\theta}\,\check{I}_{u2}\right) \end{aligned}\right\} \qquad (\mathbf{56})$$

$$\left.\begin{aligned} S^0\,\check{E}_{u0} &= 0 \\ S^1\,\check{E}_{u1} &= S^1\left(1\tfrac{1}{2}\,Z_{au0}\,\epsilon^{-j\theta}\,\check{I}_{a1}\right) \\ S^2\,\check{E}_{u2} &= S^2\left(1\tfrac{1}{2}\,Z_{au0}\,\epsilon^{j\theta}\;\check{I}_{a2}\right) \end{aligned}\right\} \qquad (\mathbf{57})$$

In the case having symmetrical primary and unsymmetrical secondary in which members of each group are different, but in which there are harmonic relations between corresponding members of the different groups, the impedances are

$$\left.\begin{aligned} &(\mathrm{I})\; Z_{au},\quad Z_{bv},\quad Z_{cw} \\ &(\mathrm{II})\left(\frac{a}{2}\,\epsilon^{j\theta} + \frac{a^2}{2}\,\epsilon^{-j\theta}\right) Z_{cw} \\ &\left(\frac{a}{2}\,\epsilon^{j\theta} + \frac{a^2}{2}\,\epsilon^{-j\theta}\right) Z_{au},\; \left(\frac{a}{2}\,\epsilon^{j\theta} + \frac{a^2}{2}\,\epsilon^{-j\theta}\right) Z_{bv} \\ &(\mathrm{III})\left(\frac{a^2}{2}\,\epsilon^{j\theta} + \frac{a}{2}\,\epsilon^{-j\theta}\right) Z_{bv}, \\ &\left(\frac{a^2}{2}\,\epsilon^{j\theta} + \frac{a}{2}\,\epsilon^{-j\theta}\right) Z_{cw},\; \left(\frac{a^2}{2}\,\epsilon^{j\theta} + \frac{a}{2}\,\epsilon^{-j\theta}\right) Z_{au} \end{aligned}\right\} \qquad (\mathbf{58})$$

The symmetrical component mutual impedances will have the following values in terms of $Z_{au0}\;Z_{au1}\;Z_{au2}$

$$\left.\begin{aligned} S^0\,Z_{bw0} &= \left(\frac{a}{2}\,\epsilon^{j\theta} + \frac{a^2}{2}\,\epsilon^{-j\theta}\right) S^0\,Z_{au0} \\ S^0\,Z_{cv0} &= \left(\frac{a^2}{2}\,\epsilon^{j\theta} + \frac{a}{2}\,\epsilon^{-j\theta}\right) S^0\,Z_{au0} \\ S^1\,Z_{bw1} &= \left(\frac{a^2}{2}\,\epsilon^{j\theta} + \frac{\epsilon^{-j\theta}}{2}\right) S^1\,Z_{au1} \\ S^2\,Z_{bw2} &= \left(\frac{\epsilon^{j\theta}}{2} + \frac{a}{2}\,\epsilon^{-j\theta}\right) S^2\,Z_{au2} \\ S^1\,Z_{cv1} &= \left(\frac{a}{2}\,\epsilon^{j\theta} + \frac{\epsilon^{-j\theta}}{2}\right) S^1\,Z_{au1} \\ S^2\,Z_{cv2} &= \left(\frac{\epsilon^{j\theta}}{2} + \frac{a^2}{2}\,\epsilon^{-j\theta}\right) S^2\,Z_{au2} \end{aligned}\right\} \qquad (\mathbf{59})$$

Substituting these relations in the impedances of equations (**30**) and (**33**) they become

$$\left.\begin{aligned} Z_{au0} + Z_{bw0} + Z_{cv0} &= 0 \\ Z_{au0} + a\, Z_{bw0} + a^2\, Z_{cv0} &= 1\tfrac{1}{2}\, Z_{au0}\, \epsilon^{-j\theta} \\ Z_{au0} + a^2\, Z_{bw0} + a\, Z_{cv0} &= 1\tfrac{1}{2}\, Z_{au0}\, \epsilon^{j\theta} \end{aligned}\right\} \quad (\mathbf{60})$$

$$\left.\begin{aligned} Z_{au1} + Z_{bw1} + Z_{cv1} &= 1\tfrac{1}{2}\, Z_{au1}\, \epsilon^{-j\theta} \\ Z_{au1} + a\, Z_{bw1} + a^2\, Z_{cv1} &= 1\tfrac{1}{2}\, Z_{au1}\, \epsilon^{j\theta} \\ Z_{au1} + a^2\, Z_{bw1} + a\, Z_{cv1} &= 0 \end{aligned}\right\} \quad (\mathbf{61})$$

$$\left.\begin{aligned} Z_{au2} + Z_{bw2} + Z_{cv2} &= 1\tfrac{1}{2}\, Z_{au2}\, \epsilon^{j\theta} \\ Z_{au2} + a\, Z_{bw2} + a^2\, Z_{cv2} &= 0 \\ Z_{au2} + a^2\, Z_{bw2} + a\, Z_{cv2} &= 1\tfrac{1}{2}\, Z_{au}\, \epsilon^{-j\theta} \end{aligned}\right\} \quad (\mathbf{62})$$

which on substitution in (**30**) and (**33**) give

$$\left.\begin{aligned} S^0\, \check{E}_{a0} &= 0 \\ S^1\, \check{E}_{a1} &= S^1\, \{1\tfrac{1}{2}\, Z_{au1}\, \epsilon^{j\theta}\, \check{I}_{u0} + 1\tfrac{1}{2}\, Z_{au0}\, \epsilon^{j\theta}\, \check{I}_{u1} \\ &\qquad + 1\tfrac{1}{2}\, Z_{au2}\, \epsilon^{j\theta}\, \check{I}_{u2}\} \\ S^2\, \check{E}_{a2} &= S^2\, \{1\tfrac{1}{2}\, Z_{au2}\, \epsilon^{-j\theta}\, \check{I}_{u0} + 1\tfrac{1}{2}\, Z_{au1}\, \epsilon^{-j\theta}\, \check{I}_{u1} \\ &\qquad + 1\tfrac{1}{2}\, Z_{au0}\, \epsilon^{-j\theta}\, \check{I}_{u2}\} \end{aligned}\right\} \quad (\mathbf{63})$$

$$\left.\begin{aligned} S^0\, \check{E}_{u0} &= S^0\, \{1\tfrac{1}{2}\, Z_{au2}\, \epsilon^{-j\theta}\, \check{I}_{a1} + 1\tfrac{1}{2}\, Z_{au1}\, \epsilon^{j\theta}\, \check{I}_{a2}\} \\ S^1\, \check{E}_{u1} &= S^1\, \{1\tfrac{1}{2}\, Z_{au0}\, \epsilon^{-j\theta}\, \check{I}_{a1} + 1\tfrac{1}{2}\, Z_{au2}\, \epsilon^{j\theta}\, \check{I}_{a2}\} \\ S^2\, \check{E}_{u2} &= S^2\, \{1\tfrac{1}{2}\, Z_{au1}\, \epsilon^{-j\theta}\, \check{I}_{a1} + 1\tfrac{1}{2}\, Z_{au0}\, \epsilon^{j\theta}\, \check{I}_{a2}\} \end{aligned}\right\} \quad (\mathbf{64})$$

In the case of unsymmetrical primary and symmetrical secondary, we have for the value of the impedance in terms of Z_{au0} Z_{au1} and Z_{au2}

$$\left.\begin{aligned} &(\mathrm{I})\ Z_{au},\quad Z_{bv},\quad Z_{cw} \\ &(\mathrm{II})\ \left(\frac{a}{2}\, \epsilon^{j\theta} + \frac{a^2}{2}\, \epsilon^{-j\theta}\right) Z_{bc}, \\ &\left(\frac{a}{2}\, \epsilon^{j\theta} + \frac{a^2}{2}\, \epsilon^{-j\theta}\right) Z_{cw},\ \left(\frac{a}{2}\, \epsilon^{j\theta} + \frac{a^2}{2}\, \epsilon^{-j\theta}\right) Z_{au} \\ &(\mathrm{III})\ \left(\frac{a^2}{2}\, \epsilon^{j\theta} + \frac{a}{2}\, \epsilon^{-j\theta}\right) Z_{cw}, \\ &\left(\frac{a^2}{2}\, \epsilon^{j\theta} + \frac{a}{2}\, \epsilon^{-j\theta}\right) Z_{au},\ \left(\frac{a^2}{2}\, \epsilon^{j\theta} + \frac{a}{2}\, \epsilon^{-j\theta}\right) Z_{bc} \end{aligned}\right\} \quad (\mathbf{65})$$

The symmetrical component mutual impedances in terms of Z_{au0}, Z_{au1}, Z_{au2} are

$$\left.\begin{aligned}
S^0 Z_{bw0} &= \left(\frac{a}{2}\epsilon^{j\theta} + \frac{a^2}{2}\epsilon^{-j\theta}\right) S^0 Z_{au0} \\
S^0 Z_{cv0} &= \left(\frac{a^2}{2}\epsilon^{j\theta} + \frac{a}{2}\epsilon^{-j\theta}\right) S^0 Z_{au0} \\
S^1 Z_{bw1} &= \left(\frac{\epsilon^{j\theta}}{2} + \frac{a}{2}\epsilon^{-j\theta}\right) S^1 Z_{au1} \\
S^2 Z_{bw2} &= \left(\frac{a^2}{2}\epsilon^{j\theta} + \frac{\epsilon^{-j\theta}}{2}\right) S^2 Z_{au2} \\
S^1 Z_{cv1} &= \left(\frac{\epsilon^{j\theta}}{2} + \frac{a^2}{2}\epsilon^{-j\theta}\right) S^1 Z_{au1} \\
S^2 Z_{cv2} &= \left(\frac{a}{2}\epsilon^{j\theta} + \frac{\epsilon^{-j\theta}}{2}\right) S^2 Z_{au2}
\end{aligned}\right\} \qquad (66)$$

And the impedances of equations (**30**) and (**33**) become

$$\left.\begin{aligned}
Z_{au0} + Z_{bw0} + Z_{cv0} &= 0 \\
Z_{au0} + a\,Z_{bw0} + a^2 Z_{cv0} &= 1\tfrac{1}{2}\, Z_{au0}\,\epsilon^{-j\theta} \\
Z_{au0} + a^2 Z_{bw0} + a\,Z_{cv0} &= 1\tfrac{1}{2}\, Z_{au0}\,\epsilon^{j\theta}
\end{aligned}\right\} \qquad (67)$$

$$\left.\begin{aligned}
Z_{au1} + Z_{bw1} + Z_{cv1} &= 1\tfrac{1}{2}\, Z_{au1}\,\epsilon^{j\theta} \\
Z_{au1} + a\,Z_{bwi} + a^2 Z_{cv1} &= 0 \\
Z_{au1} + a^2 Z_{bw1} + a\,Z_{cv1} &= 1\tfrac{1}{2}\, Z_{au1}\,\epsilon^{-j\theta}
\end{aligned}\right\} \qquad (68)$$

$$\left.\begin{aligned}
Z_{au2} + Z_{bw2} + Z_{cv2} &= 1\tfrac{1}{2}\, Z_{au2}\,\epsilon^{-j\theta} \\
Z_{au2} + a\,Z_{bw2} + a^2 Z_{cv2} &= 1\tfrac{1}{2}\, Z_{au2}\,\epsilon^{j\theta} \\
Z_{au2} + a^2 Z_{bw} + a\,Z_{cv2} &= 0
\end{aligned}\right\} \qquad (69)$$

And on substitution in (**30**) and (**33**), or by symmetry from (**63**) and (**64**), we have

$$\left.\begin{aligned}
S^0 \check{E}_{a0} &= S^0 \left\{1\tfrac{1}{2}\, Z_{au2}\,\epsilon^{j\theta}\ \check{I}_{u1} + 1\tfrac{1}{2}\, Z_{au1}\,\epsilon^{-j\theta}\ \check{I}_{u2}\right\} \\
S^1 \check{E}_{a1} &= S^1 \left\{1\tfrac{1}{2}\, Z_{au0}\,\epsilon^{j\theta}\ \check{I}_{u1} + 1\tfrac{1}{2}\, Z_{au2}\,\epsilon^{-j\theta}\ \check{I}_{u2}\right\} \\
S^2 \check{E}_{a2} &= S^2 \{1\tfrac{1}{2}\, Z_{au1}\,\epsilon^{j\theta}\ \check{I}_{u1} + 1\tfrac{1}{2}\, Z_{au0}\,\epsilon^{-j\theta}\ \check{I}_{u2}
\end{aligned}\right\} \qquad (70)$$

$$\left.\begin{aligned} S^0\, \check{E}_{u0} &= 0 \\ S^1\, \check{E}_{u1} &= S^1\, \{1\tfrac{1}{2}\, Z_{au1}\, \epsilon^{-j\theta}\, \check{I}_{a0} + 1\tfrac{1}{2}\, Z_{au0}\, \epsilon^{-j\theta}\, \check{I}_{u1} \\ &\qquad + 1\tfrac{1}{2}\, Z_{au2}\, \epsilon^{-j\theta}\, \check{I}_{a2}\} \\ S^2\, \check{E}_{u2} &= S^2\, \{1\tfrac{1}{2}\, Z_{au2}\, \epsilon^{j\theta}\, \check{I}_{a0} + 1\tfrac{1}{2}\, Z_{au1}\, \epsilon^{j\theta}\, \check{I}_{a1} \\ &\qquad + 1\tfrac{1}{2}\, Z_{au0}\, \epsilon^{j\theta}\, \check{I}_{a2}\} \end{aligned}\right\} \quad (71)$$

A fuller discussion of self and mutual impedances of co-axial cylindrical windings will be found in the Appendix. It will be sufficient to note here that in the case of self inductance and mutual inductance of stationary windings symmetrically disposed if they are equal

$$\left.\begin{aligned} M_{ab} = M_{bc} = M_{ca} &= \Sigma \left(A_n \cos \frac{2\, n\, \pi}{3} \right) \\ L_{aa} = L_{bb} = L_{cc} = M_{aa} = M_{bb} = M_{cc} &= \Sigma\, A_n \end{aligned}\right\} \quad (72)$$

If the windings are symmetrically disposed but have different number of turns

$$\left.\begin{aligned} L_{aa} &= M_{aa} = \Sigma\, A_n \\ L_{bb} &= M_{bb} = \Sigma\, B_n \\ L_{cc} &= M_{cc} = \Sigma\, C_n \end{aligned}\right\} \quad (73)$$

$$\left.\begin{aligned} M_{ab} &= \Sigma \left(\sqrt{A_n\, B_n} \cos \frac{2\, n\, \pi}{3} \right) \\ M_{bc} &= \Sigma \left(\sqrt{B_n\, C_n} \cos \frac{2\, n\, \pi}{3} \right) \\ M_{ca} &= \Sigma \left(\sqrt{C_n\, A_n} \cos \frac{2\, n\, \pi}{3} \right) \end{aligned}\right\} \quad (74)$$

If the coils are alike but unsymmetrically spaced $L_{aa}\, L_{bb}\, L_{cc}$ have the same values, namely $\Sigma\, A_n$ and

$$\left.\begin{aligned} M_{ab} &= \Sigma \left\{ (A_n \cos n\, \theta_1) \cos \frac{2\, n\, \pi}{3} \right. \\ &\qquad\qquad \left. + (A_n \sin n\, \theta_1) \sin \frac{2\, n\, \pi}{3} \right\} \\ M_{bc} &= \Sigma \left\{ (A_n \cos n\, \theta_2) \cos \frac{2\, n\, \pi}{3} \right. \\ &\qquad\qquad \left. + (A_n \sin n\, \theta_2) \sin \frac{2\, n\, \pi}{3} \right\} \\ M_{ca} &= \Sigma \left\{ (A_n \cos n\, \theta_3) \cos \frac{2\, n\, \pi}{3} \right. \\ &\qquad\qquad \left. + (A_n \sin n\, \theta_3) \sin \frac{2\, n\, \pi}{3} \right\} \end{aligned}\right\} \quad (75)$$

If they are unequal as well as unsymmetrically disposed but are otherwise similar $L_{aa}\ L_{bb}\ L_{cc}$ have values as in (**64**) and

$$\left.\begin{aligned}
M_{ab} &= \Sigma \left\{ (\sqrt{A_n B_n} \cos n\, \theta_1) \cos \frac{2\,n\,\pi}{3} \right.\\
&\qquad \left. + (\sqrt{A_n B_n} \sin n\, \theta_1) \sin \frac{2\,n\,\pi}{3} \right\}\\
M_{bc} &= \Sigma \left\{ (\sqrt{B_n C_n} \cos n\, \theta_2) \cos \frac{2\,n\,\pi}{3} \right.\\
&\qquad \left. + (\sqrt{B_n C_n} \sin n\, \theta_2) \sin \frac{2\,n\,\pi}{3} \right\}\\
M_{ca} &= \Sigma \left\{ (\sqrt{C_n A_n} \cos n\, \theta_1) \cos \frac{2\,n\,\pi}{3} \right.\\
&\qquad \left. + (\sqrt{C_n A_n} \sin n\, \theta_3) \sin \frac{2\,n\,\pi}{3} \right\}
\end{aligned}\right\} \qquad (\mathbf{76})$$

Where the windings are dissimilar in every respect the expressions become more complicated. A short outline of this subject is given in the Appendix.

In the case of mutual inductance between two coaxial cylindrical systems, one of which *A*, *B*, *C* is the primary and the other *U*, *V*, *W* the secondary, the following conventions should be followed:

(a) All angles are measured, taking the primary planes of symmetry as data in a positive direction.

(b) The datum plane for all windings is the plane of symmetry of the primary *A* phase.

(c) All mechanical motions unless otherwise stated shall be considered as positive rotations of the secondary cylinder about its axis.

(d) The conventional disposition of the phases and the direction of rotation of the secondary winding are indicated in Fig. 3.

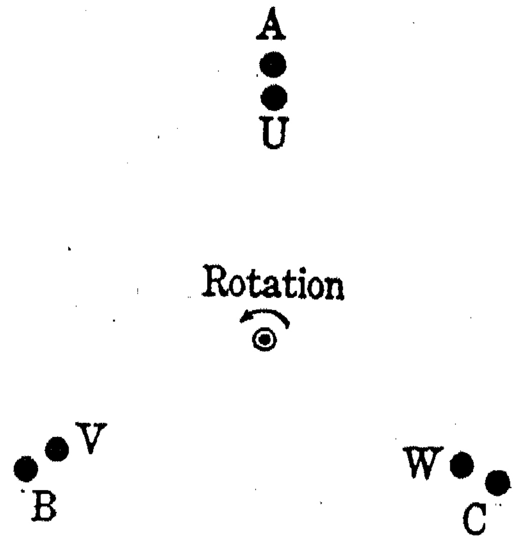

FIG. 3—CONVENTIONAL DISPOSITION OF PHASES AND DIRECTION OF ROTATION.

We shall consider five cases; Case 1 being the completely symmetrical case and the rest being symmetrical in one winding, the other winding being unsymmetrical in magnitude and phase, or both, but all windings having the same form and distribution of coils.

Case I. All Windings Symmetrical.

$$\left.\begin{aligned}
M_{au} &= M_{bv} = M_{cw} = \Sigma A_n \cos n\,\theta \\
M_{bw} &= M_{cu} = M_{av} = \Sigma A_n \cos n\left(\frac{2\,\pi}{3} + \theta\right) \\
M_{cv} &= M_{aw} = M_{bu} = \Sigma A_n \cos n\left(\frac{4\,\pi}{3} + \theta\right)
\end{aligned}\right\}\qquad (77)$$

Case II. Primary Windings equal and Symmetrical, Secondary Windings unequal but otherwise Symmetrical.

$$\left.\begin{aligned}
M_{au} &= \Sigma A_n \cos n\,\theta,\ M_{bv} = \Sigma B_n \cos n\,\theta, \\
&\qquad M_{cw} = \Sigma C_u \cos n\,\theta \\
M_{bw} &= \Sigma C_n \cos n\left(\frac{2\,\pi}{3} + \theta\right), \\
M_{cu} &= \Sigma A_n \cos n\left(\frac{2\,\pi}{3} + \theta\right), \\
M_{av} &= \Sigma B_n \cos n\left(\frac{2\,\pi}{3} + \theta\right) \\
M_{cv} &= \Sigma B_n \cos n\left(\frac{4\,\pi}{3} + \theta\right), \\
M_{aw} &= \Sigma C_n \cos n\left(\frac{4\,\pi}{3} + \theta\right), \\
M_{bu} &= \Sigma A_n \cos n\left(\frac{4\,\pi}{3} + \theta\right)
\end{aligned}\right\}\qquad (78)$$

Case III. Primary Windings Unequal but Otherwise Symmetrical, Secondary Winding Equal and Symmetrical.

$$\left.\begin{aligned}
M_{au} &= \Sigma A_n \cos n\,\theta, M_{bv} = \Sigma B_n \cos n\,\theta,\ M_{cw} = \Sigma C_n \cos n\theta \\
M_{bw} &= \Sigma B_n \cos n\left(\frac{2\,\pi}{3} + \theta\right), \\
M_{cu} &= \Sigma C_n \cos n\left(\frac{2\,\pi}{3} + \theta\right) \\
M_{av} &= \Sigma A_n \cos n\left(\frac{2\,\pi}{3} + \theta\right) \\
M_{cv} &= \Sigma C_n \cos n\left(\frac{4\,\pi}{3} + \theta\right), \\
M_{aw} &= \Sigma A_n \cos n\left(\frac{4\,\pi}{3} + \theta\right), \\
M_{bu} &= \Sigma B_n \cos n\left(\frac{4\,\pi}{3} + \theta\right)
\end{aligned}\right\}\qquad (79)$$

Case IV. Same as Case II except in addition to inequality Secondary Windings are Displaced from Symmetry by angles α_1 α_2 *and* α_3 *whose sum is zero.*

$$\left.\begin{aligned}
M_{au} &= \Sigma\,(A_n \cos\alpha_1 \cos n\,\theta + A_n \sin\alpha_1 \sin n\,\theta)\\
M_{bv} &= \Sigma\,(B_n \cos\alpha_2 \cos n\,\theta + B_n \sin\alpha_2 \sin n\,\theta)\\
M_{cw} &= \Sigma\,(C_n \cos\alpha_3 \cos n\,\theta + C_n \sin\alpha_3 \sin n\,\theta)\\
M_{bw} &= \Sigma\left\{C_n \cos\alpha_3 \cos n\left(\frac{2\,\pi}{3} + \theta\right)\right.\\
&\qquad\left. + C_n \sin\alpha_3 \sin n\left(\frac{2\,\pi}{3} + \theta\right)\right\}\\
M_{cu} &= \Sigma\left\{A_n \cos\alpha_1 \cos n\left(\frac{2\,\pi}{3} + \theta\right)\right.\\
&\qquad\left. + A_n \sin\alpha_1 \sin n\left(\frac{2\,\pi}{3} + \theta\right)\right\}\\
M_{av} &= \Sigma\left\{B_n \cos\alpha_2 \cos n\left(\frac{2\,\pi}{3} + \theta\right)\right.\\
&\qquad\left. + B_n \sin\alpha_2 \sin n\left(\frac{2\,\pi}{3} + \theta\right)\right\}\\
M_{cv} &= \Sigma\left\{B_n \cos\alpha_2 \cos n\left(\frac{4\,\pi}{3} + \theta\right)\right.\\
&\qquad\left. + B_n \sin\alpha_2 \sin n\left(\frac{4\,\pi}{3} + \theta\right)\right\}\\
M_{aw} &= \Sigma\left\{C_n \cos\alpha_3 \cos n\left(\frac{4\,\pi}{3} + \theta\right)\right.\\
&\qquad\left. + C_n \sin\alpha_3 \sin n\left(\frac{4\,\pi}{3} + \theta\right)\right\}\\
M_{bu} &= \Sigma\left\{A_n \cos\alpha_1 \cos n\left(\frac{4\,\pi}{3} + \theta\right)\right.\\
&\qquad\left. + A_n \sin\alpha_1 \sin n\left(\frac{4\,\pi}{3} + \theta\right)\right\}
\end{aligned}\right\}\qquad(80)$$

Case V. Same as Case III except that the Primary Windings are Unsymmetrically disposed with respect to one another as well as being unequal.

$$\left.\begin{aligned}
M_{au} &= \Sigma\,(A_n \cos\alpha_1 \cos n\,\theta + A_n \sin\alpha_1 \sin n\,\theta) \\
M_{bv} &= \Sigma\,(B_n \cos\alpha_2 \cos n\,\theta + B_n \sin\alpha_2 \sin n\,\theta) \\
M_{cw} &= \Sigma\,(C_n \cos\alpha_3 \cos n\,\theta + C_n \sin\alpha_3 \sin n\,\theta) \\
M_{bw} &= \Sigma\,\left\{B_n \cos\alpha_2 \cos n\left(\frac{2\,\pi}{3} + \theta\right)\right. \\
&\qquad \left. + B_n \sin\alpha_2 \sin n\left(\frac{2\,\pi}{3} + \theta\right)\right\} \\
M_{cu} &= \Sigma\,\left\{A_n \cos\alpha_1 \cos n\left(\frac{2\,\pi}{3} + \theta\right)\right. \\
&\qquad \left. + A_n \sin\alpha_1 \sin n\left(\frac{2\,\pi}{3} + \theta\right)\right\} \\
M_{av} &= \Sigma\,\left\{C_n \cos\alpha_3 \cos n\left(\frac{2\,\pi}{3} + \theta\right)\right. \\
&\qquad \left. + C_u \sin\alpha_3 \sin n\left(\frac{2\,\pi}{3} + \theta\right)\right\} \\
M_{cv} &= \Sigma\,\left\{C_n \cos\alpha_3 \cos n\left(\frac{4\,\pi}{3} + \theta\right)\right. \\
&\qquad \left. + C_n \sin\alpha_3 \sin n\left(\frac{4\,\pi}{3} + \theta\right)\right\} \\
M_{aw} &= \Sigma\,\left\{A_n \cos\alpha_1 \cos n\left(\frac{4\,\pi}{3} + \theta\right)\right. \\
&\qquad \left. + A_n \sin\alpha_1 \sin n\left(\frac{4\,\pi}{3} + \theta\right)\right\} \\
M_{bu} &= \Sigma\,\left\{B_n \cos\alpha_2 \cos n\left(\frac{4\,\pi}{3} + \theta\right)\right. \\
&\qquad \left. + B_n \sin\alpha_2 \sin n\left(\frac{4\,\pi}{3} + \theta\right)\right\}
\end{aligned}\right\} \quad (81)$$

The expressions for dissymmetry in both windings and for unsymmetrically wound coils, etc., are more complicated and will be dealt with in the Appendix.

The impedances Z_{aa} Z_{bb}, etc., Z_{au} Z_{bv}, etc., are functions of M_{aa} M_{bb}, etc., M_{au} M_{bv}, etc., and the resistances of the system. The component of e. m. f. proportional to the current due to mutual impedance is so small that it may generally be neglected so that Z_{au} becomes $\frac{d}{dt} M_{au}$, $Z_{bv} = \frac{d}{dt} M_{bv}$ and so forth.

If the secondary winding is rotating at an angular velocity α, θ in equation (**55**) becomes $\alpha\, t$ and the operators Z_{aa}, etc. operate on such products as $\epsilon^{j\alpha t}\, \check{I}_{u1}$ $\epsilon^{j\alpha t}\, \check{I}_{u2}$ where $\check{I}_{u1}$ and $\check{I}_{u2}$ are the variables.

The following relations will be found useful in the application of the method in actual examples.

If D denotes the operator $\frac{d}{d\,x}$ and $\varphi\,(Z)$ is a rational algebraic function of Z

$$\left.\begin{aligned} \varphi\,(D)\, e^{ax} &= \varphi\,(a)\, e^{ax} \\ \varphi\,(D)\,\{e^{ax}\;X\} &= e^{ax}\;\varphi\,(D+a)\,X \\ \varphi\,(D)\,Y &= e^{ax}\,\varphi\,(D+a)\;Y\,e^{-ax} \end{aligned}\right\} \quad (\mathbf{82})$$

Where X and Y may be any function of x.

Star and Delta e.m.fs. and Currents in Terms of Symmetrical Components

It has been shown in the preceding portion of this paper that the e. m. fs. $\check{E}_a$ $\check{E}_b$ and $\check{E}_c$ and the currents $\check{I}_a$ $\check{I}_b$ and $\check{I}_c$ whatever their distortion, may be represented by the sum of symmetrical systems of e. m. fs. or currents so that the two expressions

$$\left.\begin{aligned} S\,(\check{E}_a) &= S^0\,\check{E}_{a0} + S^1\,\check{E}_{a1} + S^2\,\check{E}_{a2} \\ S\,(\check{I}_a) &= S^0\,\check{I}_{a0} + S^1\,\check{I}_{a1} + S^2\,\check{I}_{a2} \end{aligned}\right\} \quad (\mathbf{83})$$

completely define these two systems.

If we take the delta e. m. fs. and currents corresponding to $S^0\,\check{E}_{a0}$, $S^1\,\check{E}_{a1}$ and $S^2\,\check{E}_{a2}$, $S^1\,\check{I}_{a1}$, $S^2\,\check{I}_{a2}$, we have, since $\check{E}_{bc1}$ leads $\check{E}_{a1}$ by $\frac{\pi}{2}$ and $\check{E}_{bc2}$ lags behind $\check{E}_{a2}$ by the same angle

$$\left.\begin{aligned} S^0\,\check{E}_{bc0} &= 0 \\ S^1\,\check{E}_{bc1} &= j\,\sqrt{3}\,S^1\,\check{E}_{a1} \\ S^2\,\check{E}_{bc2} &= -\,j\,\sqrt{3}\,S^2\,\check{E}_{a2} \\ S^0\,\check{I}_{bc0} &= \text{indeterminate from } S\,(\check{I}_a) \\ S^1\,\check{I}_{bc1} &= \;\;j\,\frac{1}{\sqrt{3}}\,S^1\,\check{I}_{a1} \\ S^2\,\check{I}_{bc2} &= -j\,\frac{1}{\sqrt{3}}\,S^2\,\check{I}_{a2} \end{aligned}\right\} \quad (\mathbf{84})$$

And therefore if we take $\check{E}_{ab}$ as the principal vector

$$\left.\begin{aligned} S^0\,\check{E}_{ab0} &= 0 \\ S^1\,E_{ab1} &= j\,a\,s^1\,\sqrt{3}\,\check{E}_{a1} \\ S^2\,\check{E}_{ab2} &= -j\,a^2\,s^2\,\sqrt{3}\,\check{E}_{a2} \\ S\,(\check{E}_{ab}) &= S^1\,\check{E}_{ab1} + S^2\,\check{E}_{ab2} \end{aligned}\right\} \quad \textbf{(85)}$$

The last equation of group (**85**) when expanded gives

$$\left.\begin{aligned} \check{E}_{ab} &= j\,\sqrt{3}\,(a\,\check{E}_{a1} - a^2\,\check{E}_{a2}) \\ \check{E}_{bc} &= j\,\sqrt{3}\,(\check{E}_{a1} - \check{E}_{a2}) \\ \check{E}_{ca} &= j\,\sqrt{3}\,(a^2\,\check{E}_{a1} - a\,\check{E}_{a2}) \end{aligned}\right\} \quad \textbf{(86)}$$

which may also be obtained direct from (**83**) by means of the relations

$$\begin{aligned} \check{E}_{ab} &= \check{E}_b - \check{E}_a \\ \check{E}_{bc} &= \check{E}_c - \check{E}_b \\ \check{E}_{ca} &= \check{E}_a - \check{E}_0 \end{aligned}$$

Similarly

$$\begin{aligned} S^0\,\check{I}_{ab} &= \text{indeterminate from } S\,(\check{I}_a) \\ S^1\,\check{I}_{ab1} &= j\,a\,\frac{1}{\sqrt{3}}\,\check{I}_{a1} \\ S^2\,\check{I}_{ab2} &= j\,a^2\,\frac{1}{\sqrt{3}}\,\check{I}_{a2} \\ S\,(\check{I}_{ab}) &= S^0\,\check{I}_{ab0} + S^1\,I_{ab1} + S^2\,\check{I}_{ab2} \end{aligned} \quad \textbf{(87)}$$

with similar expression for $\check{I}_{ab}$ $\check{I}_{bc}$ and $\check{I}_{ca}$ which may be verified by means of the relations

$$\begin{aligned} \check{I}_a &= \check{I}_{ca} - \check{I}_{ab} + \check{I}_{a0} \\ \check{I}_b &= \check{I}_{ab} - \check{I}_{bc} + \check{I}_{a0} \\ \check{I}_c &= \check{I}_{bc} - \check{I}_{ca} + \check{I}_{a0} \end{aligned}$$

Conversely to (**84**) we have the following relations

$$\left.\begin{aligned}
S^0\,\check{E}_{a0} &= \text{indeterminate from } S\,(\check{E}_{ab})\\
S^1\,\check{E}_{a1} &= -\,j\frac{1}{\sqrt{3}}\,S^1\,\check{E}_{bc1} = -j\,\frac{a^2}{\sqrt{3}}\,S^1\,\check{E}_{ab1}\\
S^2\,\check{E}_{a2} &= \;\;j\,\frac{1}{\sqrt{3}}\,S^2\,E_{bc2} = \;\;j\,\frac{a}{\sqrt{3}}\,S^2\,E_{ab2}\\
S^0\,\check{I}_{a0} &= \text{indeterminate from } S\,(\check{I}_{ab})\\
S^1\,\check{I}_{a1} &= -\,j\,\sqrt{3}\,S^1\,I_{bc1} = -\,j\,a^2\,\sqrt{3}\,S^1\,\check{I}_{ab1}\\
S^2\,\check{I}_{a2} &= \;\;j\,\sqrt{3}\,S^2\,\check{I}_{bc} = \;\;j\,a\,\sqrt{3}\,S^2\,\check{I}_{ab2}
\end{aligned}\right\}\quad(88)$$

It will be sufficient in order to illustrate the application of the principle of symmetrical coordinates to simple circuits to apply it to a few simple cases of transformer connections before proceeding to its application to rotating polyphase systems to which it is particularly adapted.

UNSYMMETRICAL BANK OF DELTA-DELTA TRANSFORMERS OPERATING ON A SYMMETRICAL CIRCUIT SUPPLYING A BALANCED SYSTEM

Let the transformer effective impedances be $Z_{AB}\;Z_{BC}\;Z_{CA}$ and let the secondary load currents be $\check{I}_U\;\check{I}_V$ and $\check{I}_W$ and let the star load impedance be Z. One to one ratio of transformation will be assumed, and the effect of the magnetizing current will be neglected. The symmetrical equations are

$$\left.\begin{aligned}
O &= S^0\,(Z_{AB0}\,\check{I}_{ab0} + Z_{AB2}\,\check{I}_{ab1} + Z_{AB1}\,\check{I}_{ab2})\\
S^1\,\check{E}_{uv1} &= S^1\,\check{E}_{ab1} - S^1\,(Z_{AB1}\,\check{I}_{ab0} + Z_{AB0}\,\check{I}_{ab1} + Z_{AB2}\,\check{I}_{ab2})\\
S^2\,\check{E}_{uv2} &= 0 - S^2\,(Z_{AB2}\,\check{I}_{ab0} + Z_{AB1}\,\check{I}_{ab1} + Z_{AB0}\,\check{I}_{ab2})\\
S^0\,\check{I}_{u0} &= 0\\
S^1\,Z\,\check{I}_{u1} &= \check{E}_{u1}\\
S^2\,Z\,\check{I}_{u2} &= \check{E}_{u2}
\end{aligned}\right\}\quad(89)$$

Since the transformation ratio is unity and the effects of magnetizing currents are negligible $S^1\,\check{I}_{ab1} = S^1\,\check{I}_{UV1}$, $S^2\,\check{I}_{ab2} = S^2\,\check{I}_{UV2}$. And therefore by means of the relations (**85**), the last two equations may be expressed

$$\left.\begin{aligned} S^1\,\check{E}_{uv1} &= S^1\,3\,Z\,\check{I}_{ab1} \\ S^2\,\check{E}_{uv2} &= S^2\,3\,Z\,\check{I}_{ab2} \end{aligned}\right\} \quad (\mathbf{90})$$

in other words, the symmetrical components appear in the secondary as independent systems, 3 Z being the delta load impedance equivalent to the star impedance Z.

Substituting from (**90**) in the second and third equation and eliminating $\check{I}_{ab0}$ by means of the first equation, and we have

$$\left.\begin{aligned} S^1\,\check{E}_{ab1} &= S^1\left\{\left(3\,Z + Z_{AB0} - \frac{Z_{AB1}\,Z_{AB2}}{Z_{AB0}}\right)\check{I}_{ab1}\right. \\ &\qquad \left. + \left(Z_{AB2} - \frac{Z^2_{AB1}}{Z_{AB0}}\right)\check{I}_{ab2}\right\} \\ S^2\,O &= S^2\left\{\left(Z_{AB1} - \frac{Z^2_{AB2}}{Z_{AB0}}\right)\check{I}_{ab1}\right. \\ &\qquad \left. + \left(3\,Z + Z_{AB0} - \frac{Z_{AB1}\,Z_{AB2}}{Z_{AB0}}\right)\check{I}_{ab2}\right\} \end{aligned}\right\} \quad (\mathbf{91})$$

which, when S^1 and S^2 are removed, give two simultaneous equations in $\check{I}_{ab1}$ and $\check{I}_{ab2}$.

A modification of the problem may occur even when the load impedances are symmetrical, as they may have symmetrical but unequal impedances Z_1 and Z_2, to the two components $\check{I}_{v1}$ and $\check{I}_{v2}$ respectively, as in the case of a load consisting of a symmetrical rotating machine. The equations corresponding to (**89**), (**90**) and (**91**) then become

$$\left.\begin{aligned} O &= S^0\,(Z_{AB0}\,\check{I}_{ab0} + Z_{AB2}\,\check{I}_{ab1} + Z_{AB1}\,\check{I}_{ab2}) \\ S^1\,\check{E}_{uv1} &= S^1\,\check{E}_{ab1} - S^1\,(Z_{AB1}\,\check{I}_{ab0} + Z_{AB0}\,\check{I}_{ab1} + Z_{AB2}\,\check{I}_{ab2}) \\ S^2\,E_{uv2} &= O - S^2\,(Z_{AB2}\,\check{I}_{ab0} + Z_{AB1}\,\check{I}_{ab1} + Z_{AB0}\,\check{I}_{ab2}) \\ S^0\,\check{I}_{u0} &= O \\ S^1\,Z_1\,\check{I}_{u1} &= \check{E}_{u1} \\ S^2\,Z_2\,\check{I}_{u2} &= \check{E}_{u2} \end{aligned}\right\} \quad (\mathbf{92})$$

$$\left.\begin{aligned} S^1\,\check{E}_{uv1} &= S^1\,3\,Z_1\,\check{I}_{ab1} \\ S^2\,\check{E}_{uv2} &= S^2\,3\,Z_2\,\check{I}_{ab2} \end{aligned}\right\} \quad (\mathbf{93})$$

$$\left.\begin{aligned} S^1\,\check{E}_{ab1} &= S^1 \left\{ \left(3\,Z_1 + Z_{AB0} - \frac{Z_{AB1}\,Z_{AB2}}{Z_{AB0}} \right) \check{I}_{ab1} + \right. \\ &\qquad \left. \left(Z_{AB2} - \frac{Z^2{}_{AB1}}{Z_{AB0}} \right) \check{I}_{ab2} \right\} \\ S^2\,O &= S^2 \left\{ \left(Z_{AB1} - \frac{Z^2{}_{AB1}}{Z_{AB0}} \right) \check{I}_{ab1} + \right. \\ &\qquad \left. \left(3\,Z_2 + Z_{AB0} - \frac{Z_{AB1}\,Z_{AB2}}{Z_{AB0}} \right) \check{I}_{ab2} \right\} \end{aligned}\right\} \quad (94)$$

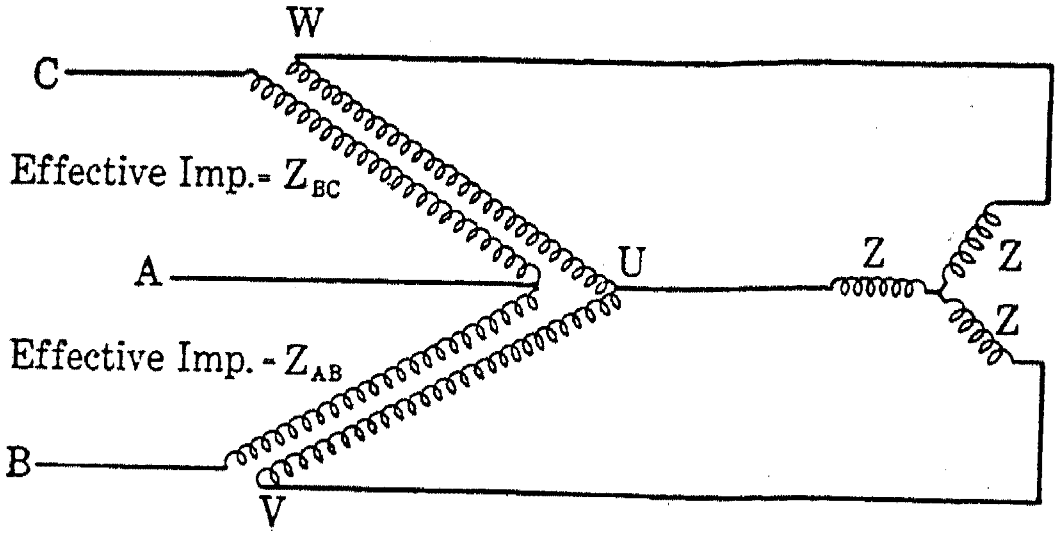

FIG. 4—OPEN DELTA OR V CONNECTION.

In an open delta system $Z_{AB1} = Z_{AB2} = Z_{AB0} - Z_{AB}$ the transformers in this case being both the same. Equation (**91**) becomes in this particular case where Z_{AB0} is infinite

$$\left.\begin{aligned} S^1\,\check{E}_{ab1} &= S^1\,\{(3\,Z + 2\,Z_{AB})\,\check{I}_{ab} + Z_{AB}\,\check{I}_{ab2}\} \\ S^2\,O &= S^2\,\{Z_{AB}\,\check{I}_{ab1} + (3\,Z + 2\,Z_{AB})\,\check{I}_{ab2}\} \end{aligned}\right\} \quad (95)$$

and we have

$$\check{I}_{ab0} = -\check{I}_{ab1} - I_{ab2} \quad (96)$$

Similarly, instead of (**94**) we have

$$\left.\begin{aligned} S^1\,\check{E}_{ab1} &= S\,\{(3\,Z_1 + 2\,Z_{AB})\,\check{I}_{ab1} + Z_{AB}\,\check{I}_{ab2}\} \\ S^2\,O &= S^2\,\{Z_{AB}\,\check{I}_{ab1} + (3\,Z_2 + 2\,Z_{AB})\,\check{I}_{ab2}\} \end{aligned}\right\} \quad (97)$$

The secondary voltages are obtained from (**90**) and (**93**) for this latter case.

The solution of (95) gives

$$\left.\begin{aligned}
\check{I}_{ab1} &= \frac{3\,Z_1 + 2\,Z_{AB}}{(3\,Z_1 + 3\,Z_{AB})\ (3\,Z_1 + Z_{AB})}\ \check{E}_{ab} \\
\check{I}_{ab2} &= -\ \frac{Z_{AB}}{(3\,Z_1 + 3\,Z_{AB})\ (3\,Z_1 + Z_{AB})}\ \check{E}_{ab} \\
\check{I}_{ab0} &= -\ \frac{1}{3\,Z_1 + 3\,Z_{AB}}\ \check{E}_{ab}
\end{aligned}\right\} \qquad (98)$$

And we have

$$\left.\begin{aligned}
S^1\,\check{I}_{a1} &= S^1\ \frac{3\,Z_1 + 2\,Z_{AB}}{3\,(Z_1 + Z_{AB})\left(Z_1 + \dfrac{Z_{AB}}{3}\right)}\ \check{E}_a \\
S^2\,\check{I}_{a2} &= S^2\ \frac{Z_{AB}}{3\,(Z_1 + Z_{AB})\left(Z_1 + \dfrac{Z_{AB}}{3}\right)}\ \check{E}_b
\end{aligned}\right\} \qquad (99)$$

And therefore

$$\left.\begin{aligned}
\check{I}_a &= \frac{\check{E}_a}{Z_1 + \dfrac{Z_{AB}}{3}} + \frac{\frac{1}{3}\,Z_{AB}}{(Z_1 + Z_{AB})\left(Z_1 + \dfrac{Z_{AB}}{3}\right)}\check{E}_{ab} \\
\check{I}_b &= \frac{\check{E}_b}{Z_1 + \dfrac{Z_{AB}}{3}} - \frac{\frac{1}{3}\,Z_{AB}}{(Z_1 + Z_{AB})\left(Z_1 + \dfrac{Z_{AB}}{3}\right)}\check{E}_{ab} \\
\check{I}_c &= \frac{\check{E}_c}{Z_1 + \dfrac{Z_{AB}}{3}}
\end{aligned}\right\} \qquad (100)$$

Three-Phase System with Symmetrical Waves Having Harmonics

We may express $\check{E}_a$ in the following form:

$$\left.\begin{aligned}
\check{E}_a &= E_1\,\epsilon^{jwt} + E_2\,\epsilon^{j2wt} + E_3\,\epsilon^{j\,3wt} + \ .\ . \\
&= \Sigma\,E_n\,\epsilon^{j\,nwt}
\end{aligned}\right\} \qquad (101)$$

where E_n is in general a complex number.

If the system is symmetrical three-phase $\check{E}_b$ is obtained by displacing the complete wave by the angle $-\ \dfrac{2\,\pi}{3}$ or

$$\breve{E}_b = \epsilon^{-j\frac{2\pi}{3}} \cdot E_1 \, \epsilon^{jwt} + \epsilon^{-j\frac{4\pi}{3}} E_2 \, \epsilon^{j2wt} + \epsilon^{-j\frac{6\pi}{3}} E_3 \, \epsilon^{j3wt} + \ldots$$

$$E_c = \epsilon^{j\frac{2\pi}{3}} E_1 \, \epsilon^{jwt} + \epsilon^{j\frac{4\pi}{3}} E_2 \, \epsilon^{j2wt} \quad \epsilon^{j\frac{6\pi}{3}} E_3 \, \epsilon^{j3wt} + \ldots$$

or since $\epsilon^{-j\frac{2\pi}{3}} = a^2$, $\epsilon^{j\frac{2\pi}{3}} = a$ etc.

$$\left.\begin{aligned} \breve{E}_a &= E_1 \, \epsilon^{jwt} + E_2 \, \epsilon^{j2wt} + E_3 \, \epsilon^{j3wt} + \ldots \\ \breve{E}_b &= a^2 E_1 \, \epsilon^{jwt} + a \, E_2 \, \epsilon^{j2wt} + E_3 \, \epsilon^{j3wt} + \ldots \\ \breve{E}_c &= a \, E_1 \, \epsilon^{jwt} + a^2 E_2 \, \epsilon^{j2wt} + E_3 \, \epsilon^{j3wt} + \ldots \end{aligned}\right\} \quad (102)$$

or

$$\left.\begin{aligned} S(\breve{E}_a) = S^0 & \{E_3 \, \epsilon^{j3wt} + E_6 \epsilon^{j6wt} + E_9 \, \epsilon^{j9wt} + \ldots \} \\ + S^1 & \{E_1 \, \epsilon^{jwt} + E_4 \, \epsilon^{j4wt} + E_7 \, \epsilon^{j7wt} + \ldots\} \\ + S^2 & \{E_2 \, \epsilon^{j2wt} + E_5 \, \epsilon^{j5wt} + E_8 \, \epsilon^{j8wt} + \ldots \} \end{aligned}\right\} \quad (103)$$

$$S(\breve{E}_a) = S^0 \, \Sigma \, (E_{3n} \, \epsilon^{j3nwt}) + S^1 \, \Sigma \, (E_{3n-2} \, \epsilon^{j(3n-2)wt}) + S^2 \, \Sigma \, (E_{3n-1} \, \epsilon^{j(3n-1)wt}) \quad (104)$$

This shows that a symmetrical three-phase system having harmonics is made up of positive and negative phase sequence harmonic systems and others of zero phase-sequence, that is to say of the same phase in all windings, which comprise the group of third harmonics. These facts are not generally appreciated though they are factors that may have an appreciable influence in the performance of commercial machines. It should be particularly noted that in three-phase generators provided with dampers the fifth, eleventh, seventeenth, and twenty-third harmonics produce currents in the damper windings.

In dealing with the complex variable it will be convenient to use for the amplitude the root mean square value for each harmonic. When instantaneous values are required, the real part of the complex variable should be multiplied by $\sqrt{2}$. In the remainder of this paper this convention will be adopted.

Power Representation in Symmetrical Co-ordinates

Since the power in an alternating-current system is also a harmonically varying scalar quantity, it may therefore be represented in the same manner as the current or electromotive force, that is to say by a complex variable which we shall denote by $(P + j\,Q) + (P_{\text{H}} + j\,Q_{\text{H}})$, $P + j\,Q$ being the mean value, is the term of the complex variable of zero frequency, P representing the real power and Q the wattless power, $\sqrt{P^2 + Q^2}$ will be the volt-amperes.

The value of the complex variable $(P + j\,Q) + (P_{\text{H}} + j\,Q_{\text{H}})$ may be taken as

$$(P + j\,Q) + (P_{\text{H}} + j\,Q_{\text{H}}) = \check{E}\,\hat{I} + \check{E}\,\check{I} \qquad \textbf{(105)}$$

with the provision that for all terms having negative indices the conjugate terms must be substituted, these terms being present in the product $\hat{E}\,\check{I} + \hat{E}\,\hat{I}$, which is the conjugate of the product (**105**). A similar rule holds good for the symmetrical vector system

$$\left.\begin{aligned} S(\check{E}_a) &= S^0\,\check{E}_{a0} + S^1\,\check{E}_{a1} + \quad . \quad . \quad . \quad + S^{n-1}\,\check{E}_{a(n-1)} \\ S(\check{I}_a) &= S^0\,\check{I}_{a0} + S^1\,\check{I}_{a1} + \quad . \quad . \quad . \quad + S^{n-1}\,\check{I}_{a(n-1)} \end{aligned}\right\} \qquad \textbf{(106)}$$

The conjugate of $S\,\check{I}_a$ is

$$S(\hat{I}_a) = S^0\,\hat{I}_{a0} + S^{(n-1)}\,\hat{I}_{a1} + \quad . \quad . \quad . \quad + S^1\,\hat{I}_{a(n-1)} \qquad \textbf{(107)}$$

and the power is represented by

$$(P + P_b) + j\,(Q + Q_b) = \Sigma\,\{S(\check{E}_a)\,S(\hat{I}_a) + S(\check{E}_a)\,S(\check{I}_a)\} \quad \textbf{(108)}$$

with the same provision for terms having negative indices. The sign Σ signifies that all the products in each sequence are added together.

$$\left.\begin{aligned} \Sigma\,\{S(\hat{I}_a)\,S(\check{E}_a)\} = \Sigma\,S^0\,\{&\hat{I}_{a0}\,\check{E}_{a0} + \hat{I}_{a1}\,\check{E}_{a1} + \quad . \quad . \\ &\qquad + \hat{I}_{a(n-1)}\,\check{E}_{a(n-1)}\} \\ + \Sigma\,S^1\,\{&\hat{I}_{a0}\,\check{E}_{a1} + \hat{I}_{a1}\,\check{E}_{a2} + \hat{I}_{a2}\,\check{E}_{a3} + \quad . \quad . \quad . \\ &\qquad + \hat{I}_{a(n-1)}\,\check{E}_{a0}\} \\ + \Sigma\,S^2\,\{&\hat{I}_{a0}\,\check{E}_{a2} + \hat{I}_{a1}\,\check{E}_{a3} + \hat{I}_{a2}\,\check{E}_{4} + \quad . \quad . \quad . \\ &\qquad + \check{I}_{a(n-1)}\,\check{E}_{a1}\} \\ . \quad . \quad . \quad . \quad . \quad . \quad . \quad &. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \\ + \Sigma\,S^{(n-1)}\,\{&\hat{I}_{a0}\,\check{E}_{a(n-1)} + \hat{I}_{a1}\,\check{E}_{a0} + \quad . \quad . \quad . \\ &\qquad + \hat{I}_{a(n-1)}\,\check{E}_{a(n-2)}\} \end{aligned}\right\} \qquad \textbf{(109)}$$

The terms prefixed by S^1, S^2, S^3 . . . $S^{(n-1)}$ all become zero and since S^0 becomes n

$$\Sigma S(\hat{I}_a) S(\check{E}_a) = n\{\hat{I}_{a0}\check{E}_{a0} + \hat{I}_{a1}\check{E}_{a1} + \ . \ . \ . \ + \hat{I}_{a(n-1)}\check{E}_{a(n-1)}\} \qquad (110)$$

In a similar manner it may be shown that

$$\Sigma S(\check{I}_a) S(\check{E}_a) = n\{\check{I}_{a0}\check{E}_{a0} + \check{I}_{a1}\check{E}_{a(n-1)} + \check{I}_{a2}\check{E}_{a(n-2)} + \ . \ . \ . \ + \check{I}_{a(n-1)}\check{E}_{a1}\} \qquad (111)$$

and therefore

$$\begin{aligned}(P + jQ) + (P_H + jQ_H) = {} & n\{\hat{I}_{a0}\check{E}_{a0} + \hat{I}_{a1}\check{E}_{a1} + \ . \ . \ . \ + \hat{I}_{a(n-1)}\check{E}_{a(n-1)}\} \\ & + n\{\check{I}_{a0}\check{E}_{a0} + \check{I}_{a1}\check{E}_{a(n-1)} + \ . \ . \ . \ + \check{I}_{a(n-1)}\check{E}_{a1}\}\end{aligned} \qquad (112)$$

For a three-phase system the expression reduces to

$$\begin{aligned}(P + jQ) + (P_H + jQ_b) = {} & 3(\hat{I}_{a0}\check{E}_{a0} + \hat{I}_{a1}\check{E}_{a1} + \hat{I}_{a2}\check{E}_{a2}) \\ & + 3(\check{I}_{a0}\check{E}_{a0} + \check{I}_{a1}\check{E}_{a2} + \check{I}_{a2}\check{E}_{a1})\end{aligned} \qquad (113)$$

In the above expression $P + P_H$ is the value of the instantaneous power on the system, P being the mean value and P_H the harmonic portion. When the currents are simple sine waves, Q may be interpreted to be the mean wattless power of the circuit or the sum of the wattless volt-amperes of each circuit. In rotating machinery since the coefficients of mutual induction may be complex harmonic functions of the angular velocity, this is not strictly true for all cases; but if the effective impedances to the various frequencies of the component currents be used, it will be found to be equal to the mean wattless volt-amperes of the system with each harmonic considered independent.

In a balanced polyphase system P_H and Q_H both become zero. The instantaneous power is a quantity of great importance in polyphase systems because the instantaneous torque is proportional to it and this quantity enters into the problem of vibrations which is at times a matter of great importance, especially when caused by unbalanced e. m. fs. A system of currents and e. m. fs. may be transformed to balanced polyphase by means of transformers alone, provided that the value of P_H is zero, while on the other hand polyphase power cannot be supplied from a pulsating power system without means for supplying the necessary storage to make a continuous flow of energy.

PART II

Application of the Method to Rotating Polyphase Networks

The methods of determining the constants Z_a Z_u, M, etc., of co-axial cylindrical networks is taken up in Appendix I of this paper. It will be assumed that the reader has familiarized himself with these quantities and understands their significance. We shall first consider the case of symmetrically wound machines taking up the simple cases first and proceeding to more complex ones.

SYMMETRICALLY WOUND INDUCTION MOTOR OPERATING ON UNSYMMETRICAL POLYPHASE CIRCUIT

Denoting the pole-pitch angle by π let the synchronous angular velocity be ω_0 and let the angular slip velocity be ω_1. And let $S^1\, E_{a1}$ $S^2\, E_{a2}$ be the symmetrical components of impressed polyphase e. m. f. Let R_a be the primary resistance and R_u the secondary resistance. The primary self-inductance being M_{aa}, that of the secondary being M_{uu} and corresponding symbols being used to denote the mutual inductances between the different pairs of windings. Then by means of (**39**), (**40**), (**56**) and (**57**)

$$\left.\begin{aligned}
S^1\, \check{E}_{a1} &= S^1 \Big\{ R_a\, \check{I}_{a1} + 1\tfrac{1}{2}\, M_{aa} \frac{d}{d\,t}\, \check{I}_{a1} \\
&\qquad + 1\tfrac{1}{2}\, M_{au} \frac{d}{d\,t} \epsilon^{j(w_0 - w_1)t}\, \check{I}_{u1} \Big\} \\
S^2\, \check{E}_{a2} &= S^2 \Big\{ R_a\, \check{I}_{a2} + 1\tfrac{1}{2}\, M_{aa} \frac{d}{d\,t}\, \check{I}_{a2} \\
&\qquad + 1\tfrac{1}{2}\, M_{au} \frac{d}{d\,t} \epsilon^{-j(w_0 - w_1)t}\, \check{I}_{u2} \Big\} \\
S^1\, \check{E}_{u1} = O &= S^1 \Big\{ R_u\, \check{I}_{u1} + 1\tfrac{1}{2}\, M_{uu} \frac{d}{d\,t}\, \check{I}_{u1} \\
&\qquad + 1\tfrac{1}{2}\, M_{au} \frac{d}{d\,t} \epsilon^{-j(w_0 - w_1)t}\, \check{I}_{a1} \Big\} \\
S^2\, \check{E}_{u2} = O &= S^2 \Big\{ R_u\, \check{I}_{u2} + 1\tfrac{1}{2}\, M_{uu} \frac{d}{d\,t}\, \check{I}_{u2} \\
&\qquad + 1\tfrac{1}{2}\, M_{au} \frac{d}{d\,t} \epsilon^{j(w_0 - w_1)t}\, \check{I}_{a2} \Big\}
\end{aligned}\right\} \quad (\mathbf{114})$$

denote $1\frac{1}{2}\, M_{aa}$ by L_a and $1\frac{1}{2}\, M_{uu}$ by L_u, $1\frac{1}{2}\, M_{au}$ by M, the equations (**1**) become

$$\left.\begin{aligned}
S^1\, \check{E}_{a1} &= S^1 \left\{ \left(R_a + L_a \frac{d}{d\,t}\right) \check{I}_{a1} \right. \\
&\qquad \left. + M \frac{d}{d\,t}\, \epsilon^{j(w_0 - w_1)t}\, \check{I}_{u1} \right\} \\
S^2\, \check{E}_{a2} &= S^2 \left\{ \left(R_a + L_a \frac{d}{d\,t}\right) \check{I}_{a2} \right. \\
&\qquad \left. + M \frac{d}{d\,t}\, \epsilon^{-j(w_0 - w_1)t}\, \check{I}_{u2} \right\} \\
S^1\, \check{E}_{u1} = O &= S^1 \left\{ \left(R_u + L_u \frac{d}{d\,t}\right) \check{I}_{u1} \right. \\
&\qquad \left. + M \frac{d}{d\,t}\, \epsilon^{-j(w - w_1)t}\, \check{I}_{a1} \right\} \\
S^2\, \check{E}_{u2} = O &= S^2 \left\{ \left(R_u + L_u \frac{d}{d\,t}\right) \check{I}_{u2} \right. \\
&\qquad \left. + M \frac{d}{d\,t}\, \epsilon^{j(w_0 - w_1)t}\, \check{I}_{a2} \right\}
\end{aligned}\right\} \quad (\mathbf{115})$$

From the last two equations we have

$$\check{I}_{u1} = -\frac{M \dfrac{d}{d\,t}}{R_u + L_u \dfrac{d}{d\,t}}\, \epsilon^{-j(w_0 - w_1)t}\, \check{I}_{a1} \qquad (\mathbf{116})$$

$$\check{I}_{u2} = -\frac{M \dfrac{d}{d\,t}}{R_u + L_u \dfrac{d}{d\,t}}\, \epsilon^{j(w_0 - w_1)t}\, \check{I}_{a2} \qquad (\mathbf{117})$$

Substituting these in the first two equations of (**115**) we obtain

$$S'\, \check{E}_{a1} = S^1 \left[\left(R_a + L_a \frac{d}{d\,t}\right) - \frac{M^2 \dfrac{d}{d\,t} \left\{ \dfrac{d}{d\,t} - j\,(w_0 - w_1) \right\}}{R_u + L_u \left\{ \dfrac{d}{d\,t} - j\,(w_0 - w_1) \right\}} \right] \check{I}_{a1} \qquad (\mathbf{118})$$

$$S^2 \check{E}_{a2} = S^2 \left[\left(R_a + L_a \frac{d}{d\,t}\right) - \frac{M^2 \dfrac{d}{d\,t} \left\{ \dfrac{d}{d\,t} + (j\,w_0 - w_1) \right\}}{R_u + L_u \left\{ \dfrac{d}{d\,t} - j\,(w_0 - w_1) \right\}} \right] \check{I}_{a2} \qquad \textbf{(119)}$$

If $\check{E}_{a1} = E_{a1}\,\epsilon^{jwt}$ and $\check{E}_{a2} = E_{a2}\,\epsilon^{jwt}$ the solution for $\check{I}_{a1}$ and $\check{I}_{a2}$ will be

$$\check{I}_{a1} = \frac{\check{E}_{a1}}{Z_1} \qquad \textbf{(120)}$$

$$\check{I}_{a2} = \frac{\check{E}_{a2}}{Z_2} \qquad \textbf{(121)}$$

Where

$$Z_1 = R_a + j\,w_0\,L_a + \frac{w_0\,w_1\,M^2}{R_u^2 + w_1^2\,L_u^2}\,(R_u - j\,w_1\,L_u) \qquad \textbf{(122)}$$

$$Z_2 = R_a + j\,w_0\,L_a + \frac{w_0\,(2\,w_0 - w_1)\,M^2}{R_u^2 + (2\,w_0 - w_1)^2\,L_u^2}\,\{R_u - j\,(2\,w_0 - w_2)\,L_u\} \qquad \textbf{(123)}$$

The impedances Z_1 and Z_2 will be found more convenient to use in the form

$$Z_1 = (R_a + K_1^2\,R_u) + j\,w_0\,(L_a - K_1^2\,L_u) + \frac{w_0 - w_1}{w_1}\,K_1^2\,R_u \qquad \textbf{(124)}$$

$$Z_2 = (R_a + K_2^2\,R_u) + j\,w_0\,(L_a - K_2^2\,L_u) - \frac{w_0 - w_1}{Z\,w_0 - w_1}\,K_2^2\,R_u \qquad \textbf{(125)}$$

Where, as we will see later, K_1^2 and K_2^2 are the squares of the transformation ratios between primary and secondary currents of positive and negative phase sequence.

The last real term in each expression is the virtual resistance due to mechanical rotation and when combined with the mean square current represents mechanical work performed, the positive sign representing work performed and the negative sign work required.

Thus, for example, to enable the currents $S^2\,\check{I}_{a2}$ to flow, the mechanical work $3\,I_{a2}^2\,\dfrac{w_0 - w_1}{2\,w_0 - w_1}\,K_1^2\,R_u$ must be applied to the shaft of the motor, The phase angles of the symmetrical systems $S^1 \check{I}_{a1}$ $S^2 \check{I}_{a2}$ with respect to their impressed e. m. f., $S^1 \check{E}_{a1}$ and $S^2 \check{E}_{a2}$ are given by these impedances so that the complete solution of the primary circuit is thus obtained.

The secondary currents are given by equations (**116**) and (**117**) and are

$$\check{I}_{u1} = - \frac{j\, w_1\, M}{R_u + j\, w_1\, L_u}\, I_{a1}\, \epsilon^{j w_1 t} = \check{K}_1\, I_{a1} \epsilon^{j w_1 t} \qquad \textbf{(126)}$$

$$\check{I}_{u2} = - \frac{j\,(2\, w_0 - w_1)\, M}{R_u + j\,(2\, w_0 - w_1)\, L_u}\, I_{a2}\, \epsilon^{j(2 w_0 - w_1)t} = \check{K}_2\, I_{a2}\, \epsilon^{j(2 w_0 - w_1)t} \qquad \textbf{(127)}$$

In the results just given, M is not the maximum value of mutual inductance between a pair of primary and secondary windings but is equal to the total mutual inductance due to a current passing through the two coils W and V through the coil

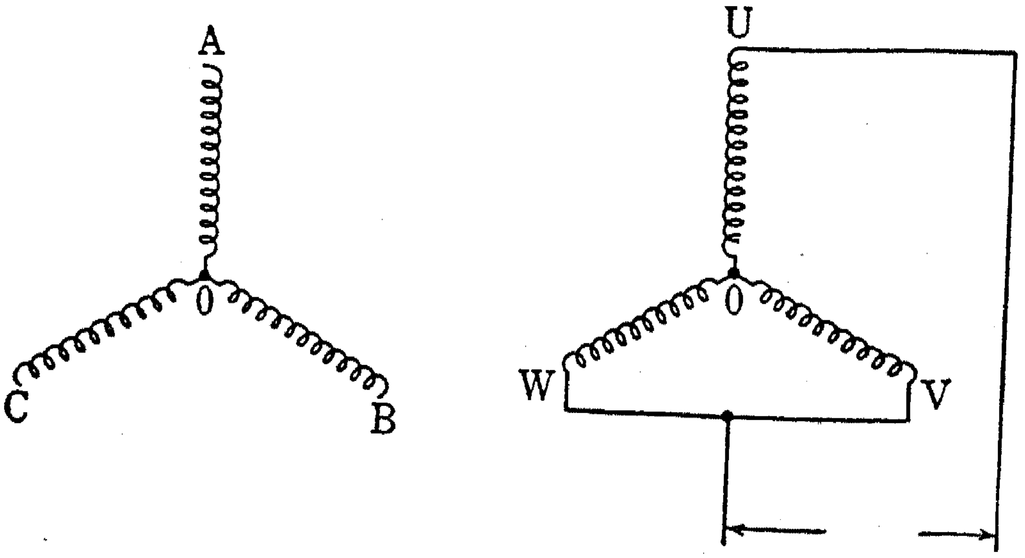

FIG. 5

U as shown in the sketch Fig 5 and the winding "A" when A and U have their planes of symmetry coincident.

Where the windings are symmetrical the induced e. m. f. is independent of the division of current between W and V, but this quantity must not be used in unsymmetrical windings, or with star windings having a neutral point connection so that $\check{I}_{a0}$ is not zero.

The appearance of M in this equation follows from the equation

$$\check{I}_u + \check{I}_v + \check{I}_w = O$$

so that

$$\check{I}_u = - (\check{I}_v + \check{I}_w)$$

The power delivered by the motor is

$$P_Q = 3 \left\{ \frac{w_0 - w_1}{w_1} K_1^2\, I_{a1}^2\, R_u - \frac{w_0 - w_1}{2\, w_0 - w_1} K_2^2\, I_{a2}^2\, R_u \right\} \qquad \textbf{(128)}$$

The copper losses are given by

$$P_L = 3 \{ I_{a1}^2 (R_p + K_1^2 R_u) + I_{a2}^2 (R_p + K_2^2 R_u) \} \qquad (129)$$

The iron loss is independent of the copper loss and power output. The iron loss and windage may be taken as

$$P_F = \text{Iron loss and windage} \qquad (130)$$

The power input as

$$P_I = P_O + P_L + P_F \qquad (131)$$

The mechanical power output is P_O less friction and windage losses.

$$\text{Torque} = 3 \left\{ \frac{1}{w_1} K_1^2 I_{a1}^2 R_u - \frac{1}{2 w_0 - w_1} K_2^2 I_{a2}^2 R_u \right\} \times 10^7 \text{ dyne-cm.} \qquad (132)$$

The kv-a. at the terminals is

$$\sqrt{P_I^2 + Q_I^2} = \text{The effective value of } 3 (E_{a1} I_{a1} + E_{a2} I_{a2}) \qquad (133)$$

This last result may be arrived at in the following way

$$\left.\begin{aligned} S(\breve{E}_a) &= S^1 \breve{E}_{a1} + S^2 \breve{E}_{a2}) \\ S(\hat{I}_a) &= S^2 \hat{I}_{a1} + S^1 \hat{I}_{a2} \end{aligned}\right\} \qquad (134)$$

Since $S^2 \hat{I}_{a1}$ is conjugate to $S^1 \breve{I}_{a1}$, etc.

The product of $\breve{E}_{sa}$ and $\hat{I}_{sa}$ is the power product of the two vectors, $S(\breve{E}_a)$ and $S(\breve{I}_a)$ and omits the harmonic variation as a double frequency quantity, the average wattless appears as an imaginary non-harmonic quantity.

$$P_I + j\, Q_I = \Sigma\, (S^0 \breve{E}_{a1} \hat{I}_{a1} + S^0 \breve{E}_{a2} \hat{I}_{a2} + S^1 \breve{E}_{a2} \hat{I}_{a1} + S^2 \breve{E}_{a1} \hat{I}_{a2}) \qquad (135)$$

The S^1 and S^2 products have zero values, since the sum of the terms of each sequence is zero, hence—

$$P_I + j\, Q_I = 3 (\breve{E}_{a1} \hat{I}_{a1} + \breve{E}_{a2} \hat{I}_{a2}) \qquad (136)$$

$$\sqrt{P_I^2 + Q_I^2} = \text{The effective value of } 3 (\breve{E}_{a1} \hat{I}_{a1} + \breve{E}_{a2} \hat{I}_{a2}) \qquad (137)$$

The solution for the general case of symmetrical motor operating on an unsymmetrical circuit is not of as much interest as certain special cases depending thereon. Some of the most important of these will be taken up in the following paragraphs.

Case I. Single-phase e. m. f. impressed across one phase of three-phase motor.

Assuming the single-phase voltage to be $\breve{E}_{bc}$ impressed across the terminals B C. The known data or constraints are

$$\left.\begin{aligned} \breve{E}_{bc} &= j\,\sqrt{3}\,(\breve{E}_{a1} - \breve{E}_{a2}) \\ \breve{I}_a &= 0,\ \breve{I}_b = -\,\breve{I}_c \end{aligned}\right\} \tag{138}$$

and therefore

$$\breve{I}_{a1} = -\,\breve{I}_{a2} \tag{139}$$

$$\frac{\breve{E}_{a1}}{Z_1} = -\,\frac{\breve{E}_{a2}}{Z_2}$$

$$\breve{E}_{a2} = -\,\frac{Z_2}{Z_1}\,\breve{E}_{a1} \tag{140}$$

Substituting in (**138**)

$$\left.\begin{aligned} \breve{E}_{a1} &= -\,j\,\frac{\breve{E}_{bc}}{\sqrt{3}} \cdot \frac{Z_1}{Z_1 + Z_2} \\ \breve{E}_{a2} &= \,j\,\frac{\breve{E}_{bc}}{\sqrt{3}} \cdot \frac{Z_2}{Z_1 + Z_2} \end{aligned}\right\} \tag{141}$$

and therefore

$$\left.\begin{aligned} \breve{I}_{a1} &= -\,j\,\frac{\breve{E}_{bc}}{\sqrt{3}} \cdot \frac{1}{Z_1 + Z_2} \\ \breve{I}_{a2} &= \,j\,\frac{\breve{E}_{bc}}{\sqrt{3}} \cdot \frac{1}{Z_1 + Z_2} \end{aligned}\right\} \tag{142}$$

Since $\breve{I}_b = \breve{I}_{b1} + \breve{I}_{b2} = a^2\,\breve{I}_{a1} + a\,\breve{I}_{a2}$

$$\breve{I}_b = -\,I_c = -\,\frac{\breve{E}_{bc}}{Z_1 + Z_2} \tag{143}$$

$$P_0 = \left(\frac{w_0 - w_1}{w_1}\,K_2^2\,R_u - \frac{w_0 - w_1}{2\,w_0 - w_1}\,K_2^2\,R_u\right) I_0^2 \tag{144}$$

$$P_1 + j\,Q_1 = I_b^2\,(Z_1 + Z_2) + P_F \tag{145}$$

The power factor is obtained from (**145**) by the formula

$$\cos\alpha = \frac{P_1}{\sqrt{P_1^2 + Q_1^2}} \tag{146}$$

Substituting from (**142**) in equation (**126**) and (**127**) of the general case we obtain for the secondary currents

$$\left.\begin{aligned} \check{I}_{u1} &= -j\,\check{K}_1\,\frac{E_{bc}}{Z_1+Z_2}\,\epsilon^{jw_1 t} \\ \check{I}_{u2} &= j\,\check{K}_2\,\frac{E_{bc}}{Z_1+Z_2}\,\epsilon^{j(2w_0-w_1)t} \end{aligned}\right\} \quad \textbf{(147)}$$

Many unsymmetrical cases may be expressed in terms of the operation of coupled symmetrical motors operating on symmetrical systems. This is invariably the case with symmetrical polyphase motors operating on single-phase circuits. Since the physical interpretations are useful in impressing the facts on ones memory they will be given whenever they appear to be useful.

Equations (**141**) and (**142**) show that single-phase operation is exactly equivalent to operating two duplicate motors in series with a symmetrical polyphase e. m. f. $S^1 E_{ab}$ impressed across one motor, the other being connected in series with the first but with phase sequence reversed, the two motors being directly coupled.

Case II. B and C connected together e. m. f. impressed across A B.

The data given by the conditions of constraint are

$$\left.\begin{aligned} \check{E}_{ab} &= -\,\check{E}_{ca} \\ \check{E}_{bc} &= O = j\,\sqrt{3}\,(\check{E}_{a1} - \check{E}_{a2}) \end{aligned}\right\} \quad \textbf{(148)}$$

We therefore have

$$\check{E}_{a1} = \check{E}_{a2} = -\,\frac{\check{E}_{ab}}{3} \qquad \textbf{(149)}$$

and

$$\left.\begin{aligned} \check{I}_{a1} &= -\,\frac{\check{E}_{ab}}{3\,Z_1} \\ \check{I}_{a2} &= -\,\frac{\check{E}_{ab}}{3\,Z_2} \end{aligned}\right\} \quad \textbf{(150)}$$

The remainder follows from the general solution and need not be repeated here.

(**150**) shows that a motor operated in this manner is the exact equivalent in all respects to two duplicate mechanically coupled polyphase motors, one of which has sequence reversed, operating in parallel on a balanced three-phase circuit of e. m. f. $S^1\,\dfrac{E_{ab}}{\sqrt{3}}$.

The secondary currents follow from substitution of (**150**) in equations (**126**) and (**127**) of the general case.

Case III. B and C connected together by the terminals of a balance coil, the impressed e. m. f. E_{AD} *applied between A and the middle point of the balance coil. Resistance and reactance of balance coil negligible.*

The data furnished by the connection in this case is

$$\check{I}_b = \check{I}_c = -\frac{\check{I}_a}{2} \tag{151}$$

and therefore

$$\check{I}_{a1} = \frac{\check{I}_a - a\dfrac{\check{I}_a}{2} - a^2\dfrac{\check{I}_a}{2}}{3} = \frac{\check{I}_a}{2}$$

$$\check{I}_{a2} = \check{I}_{a1} = \frac{\check{I}_a}{2}$$

We therefore have

$$\left.\begin{aligned} \check{E}_{a1} &= \frac{Z_1\,\check{I}_a}{2} \\ \check{E}_{a2} &= \frac{Z_2\,\check{I}_a}{2} \end{aligned}\right\} \tag{152}$$

we have

$$\begin{aligned} \check{E}_{ab} &= j\sqrt{3}\,(a\,\check{E}_{a1} - a^2\,\check{E}_{a2}) \\ &= j\sqrt{3}\,\frac{\check{I}_a}{2}\,(a\,Z_1 - a^2\,Z_2) \end{aligned}$$

$$\check{E}_{bc} = j\sqrt{3}\,\frac{\check{I}_a}{2}\,(Z_1 - Z_2)$$

$$\left.\begin{aligned} \check{E}_{ad} &= \left(\check{E}_{ab} + \frac{\check{E}_{bc}}{2}\right) \\ &= j\sqrt{3}\,\frac{\check{I}_a}{2}\,\{(a + \tfrac{1}{2})\,Z_1 - (a^2 + \tfrac{1}{2})\,Z_2\} \\ &= -\tfrac{3}{4}\,\check{I}_a\,(Z_1 + Z_2) \end{aligned}\right\} \tag{153}$$

and therefore,

$$\check{I}_a = -1\tfrac{1}{2}\,\frac{\check{E}_{ad}}{Z_1 + Z_2} \tag{154}$$

$$P_0 = \tfrac{3}{4}\left\{\frac{w_0 - w_1}{w_1} K_1^2 - \frac{w_0 - w_1}{2\,w_0 - w_1} K_2^2\right\} I_a^2 R_s \qquad \textbf{(155)}$$

$$P_1 + j\,Q_1 = \tfrac{3}{4}\,I_a^2\,(Z_1 + Z_2) + P_F \qquad \textbf{(156)}$$

$$\cos\alpha = \frac{P_1}{\sqrt{P_1^2 + Q_1^2}} \qquad \textbf{(157)}$$

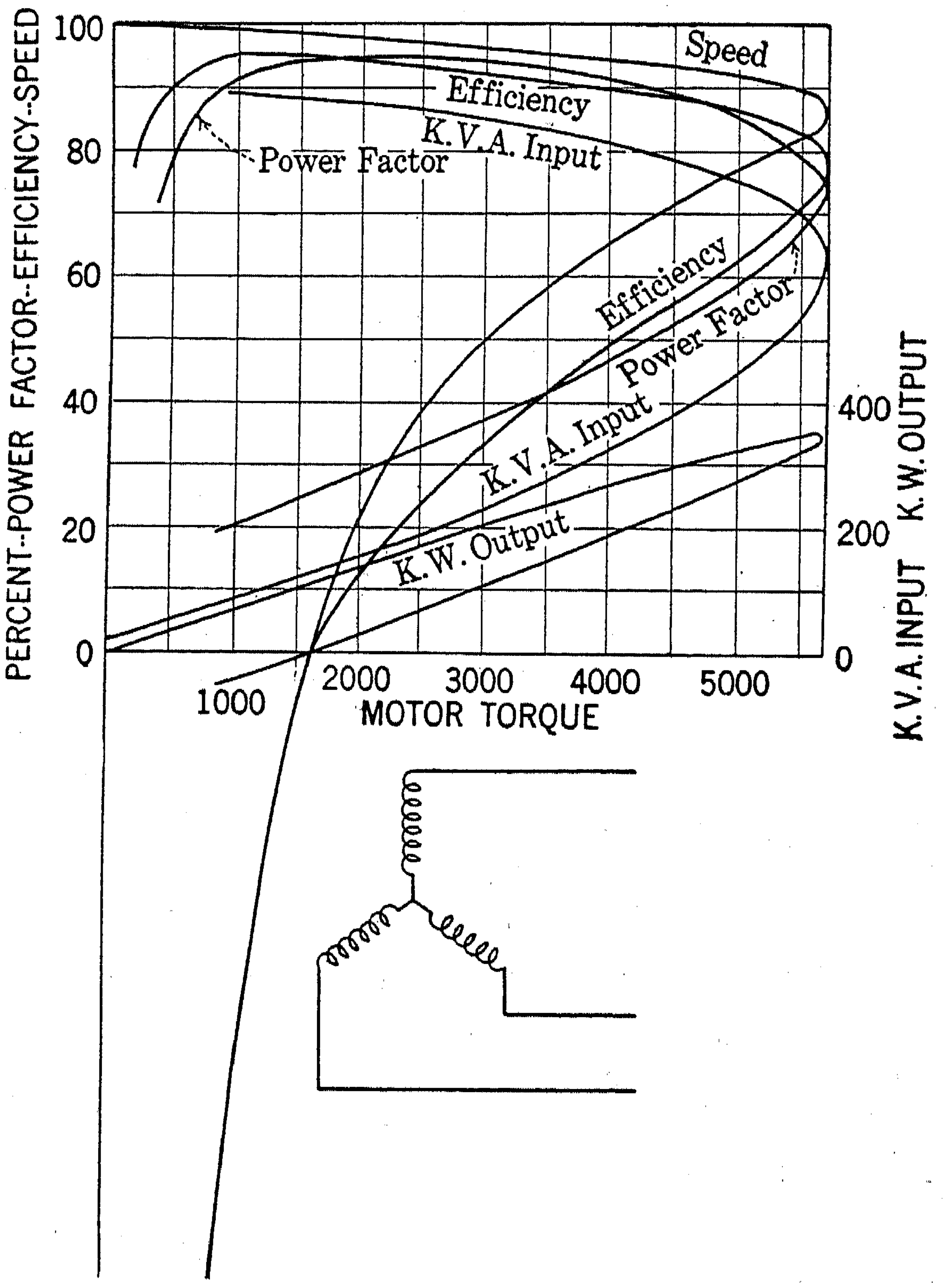

Fig. 6—Characteristics of Three-Phase Induction Motor—Balanced Three-Phase

Evidently (**155**), (**156**) and (**157**) are identical to (**144**), (**145**) and (**146**) if I_a is equal to $I_b \div \frac{\sqrt{3}}{2}$. This will be the case if the value of $E_{ad} = \frac{\sqrt{3}}{2}$ times that of E_{bc}. The total heating of the motors will be the same in each case but the heating in one phase for Case III will be one-third greater than for Case I.

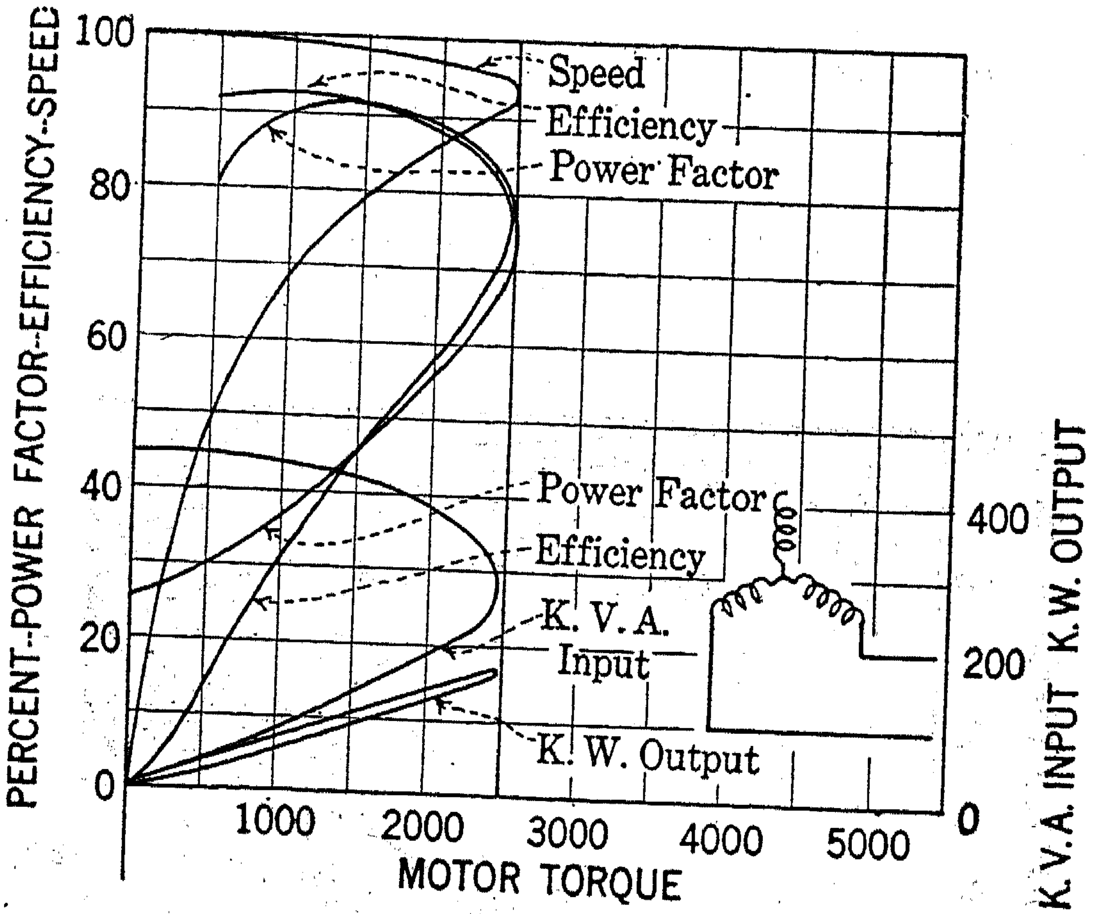

FIG. 7—CHARACTERISTICS OF THREE-PHASE INDUCTION MOTOR—SINGLE-PHASE OPERATION—ONE LEAD OPEN

This method of operation is therefore, as far as total losses, etc. are concerned, the exact counterpart of two polyphase

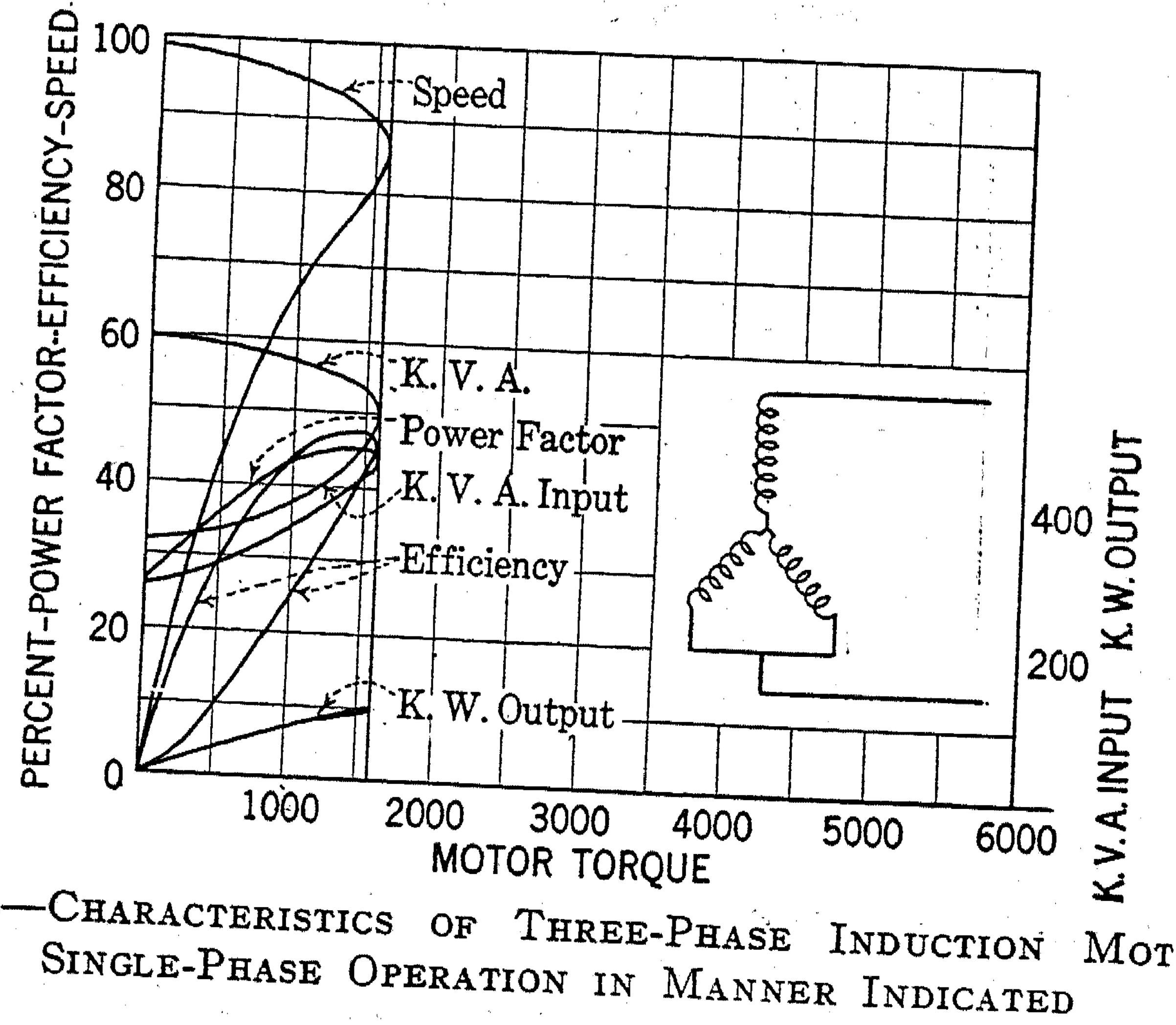

FIG. 8—CHARACTERISTICS OF THREE-PHASE INDUCTION MOTOR—SINGLE-PHASE OPERATION IN MANNER INDICATED

motors connected in series with shafts mechanically connected, one of which has its phase sequence reversed.

Figs. 6, 7 and 8 show characteristic curves of a three-phase induction motor operating respectively on a symmetrical circuit, according to Case I and according to Case II.

Synchronous Machinery

THE SYMMETRICAL THREE-PHASE GENERATOR OPERATING ON UNSYMMETRICALLY LOADED CIRCUIT

The polyphase salient pole generator is not strictly a symmetrical machine, the exciting winding is not a symmetrical polyphase winding and it therefore sets up unsymmetrical trains of harmonics in exactly the same way as they are set up in an induction motor with unsymmetrical secondary winding. These cases will therefore be taken up later on. A three-phase generator may however be wound with a distributed polyphase winding to serve both as exciting and damper winding and if properly connected will be perfectly symmetrical. Such a machine will differ from an induction motor only in respect to the fact that it operates in synchronism and has internally generated symmetrical e. m. fs. which we will denote by $S^1 \check{E}_{a1}$, $S^2 \check{E}_{a2}$ the negative phase sequence component being zero; an e. m. f. $S^0 \check{E}_{a0}$ may exist but since in all the connections that will be considered there will be no neutral connection its value may be ignored. If the load impedances be Z_a', Z_b' and Z_c' they may be expressed by

$$Z_{sa}{}^1 = S^0 Z_{a0}{}^1 + S^1 Z_{a1}' + S^2 Z_{a2}'$$

and the equations of the generator will be

$$\left.\begin{aligned} S^1 \check{E}_{a1} &= S^1 \left\{ \left(R_a + L_a \frac{d}{d\,t} \right) \check{I}_{a1}' + Z_{a0}' \, I_{a1}' \right. \\ &\qquad \left. + Z_{a2}' \, I_{a2}' + M \frac{d}{d\,t} \, \epsilon^{j w_0 t} \, \check{I}_{u1}' \right\} \\ O &= S^2 \left\{ \left(R_a + L_a \frac{d}{d\,t} \right) \check{I}_{a2}' + Z_{a0}' \, I_{a2}' \right. \\ &\qquad \left. + Z_{a1}{}^1 \, \check{I}_{a1}' + M \frac{d}{d\,t} \epsilon^{-j w_0 t} \, \check{I}_{u2}' \right\} \\ O &= \left(R_u + L_u \frac{d}{d\,t} \right) \check{I}_{u1}' + M \frac{d}{d\,t} \, \epsilon^{-j w_0 t} \, \check{I}_{a1}' \\ O &= \left(R_u + L_u \frac{d}{d\,t} \right) \check{I}_{u2}' + M \frac{d}{d\,t} \, \epsilon^{j w_0 t} \, \check{I}_{a2}' \end{aligned}\right\} \quad (158)$$

The last two equations give

$$\left.\begin{aligned} I_{u1}' &= - \frac{M \dfrac{d}{d\,t}}{R_u + L_u \dfrac{d}{d\,t}} \epsilon^{-j w_0 t} \check{I}_{a1}' \\ I_{u2}' &= - \frac{M \dfrac{d}{d\,t}}{R_u + L_u \dfrac{d}{d\,t}} \epsilon^{j w_0 t} I_{a2}' \end{aligned}\right\} \quad (\mathbf{159})$$

which on substitution in the first two equations of (**158**) give the equations

$$\left.\begin{aligned} &\left\{ R_a + L_a \frac{d}{d\,t} - \frac{M^2 \dfrac{d}{d\,t}\left(\dfrac{d}{d\,t} - j\,w_0\right)}{R_u + L_u\left(\dfrac{d}{d\,t} - j\,w_0\right)} \right\} \check{I}_{a1}' \\ &\qquad + Z_{a0}^1 \check{I}_{a1}' + Z_{a2}' \check{I}_{a2}' = \check{E}_{a1} \\ &Z_{a1}^1 \check{I}_{a1}' + \left\{ R_a + L_a \frac{d}{d\,t} \right. \\ &\left. - \frac{M^2 \dfrac{d}{d\,t}\left(\dfrac{d}{d\,t} + j\,w_0\right)}{R_u + L_u\left(\dfrac{d}{d\,t} + j\,w_0\right)} \right\} \check{I}_{a2}' + Z_{a0}^1 \check{I}_{a2}' = 0 \end{aligned}\right\} \quad (\mathbf{160})$$

or if

$$\check{E}_{a1} = E_{a1}\, \epsilon^{j w_0 t} \quad (\mathbf{161})$$

the impedances Z_{a0}, Z_{a1}, Z_{a2} become ordinary impedance for an electrical angular velocity w_0 and equations (**160**) become

$$\left.\begin{aligned} &(R_a + j\,w\,L_a + Z_{a0}^1)\,\check{I}_{a1}' + Z_{a2}'\,\check{I}_{a'2} = \check{E}_{a1} \\ &Z_{a1}'\,\check{I}_{a1}' + \{Z_{a0}' + (R_a + K_2^2\,R_u) + j\,2\,w_0\,(L_a - K_2^2\,L_u) - \\ &\qquad\qquad \tfrac{1}{2}\,K_2^2\,R_u\}\,\check{I}_{a2}' = O \end{aligned}\right\} \quad (\mathbf{162})$$

It is apparent that in the generator the impedances

$$R_a + j\,w_0\,L_a = Z_1'$$

and $\{(R_a + K_2^2\,R_u) + j\,2\,w_0\,(L_a - K_2^2\,L_u) - \frac{1}{2}\,K^2 R_u\} = Z_2'$

take the place of Z_1 and Z_2 in the symmetrical induction motor operating on an unsymmetrical circuit, and we may express equation (**162**)

$$\left.\begin{aligned}(Z_{a0}' + Z_1')\, \check{I}_{a1}' + Z_{a2}'\, \check{I}_{a2}' &= \check{E}_{a1} \\ Z_a'\, \check{I}_{a1}' + (Z_{a0}' + Z_2')\, I_{a2}' &= O\end{aligned}\right\} \quad (\mathbf{163})$$

which gives

$$\check{I}_{a2}' = - \frac{Z_{a1}'}{Z_{a0} + Z_2'}\, \check{I}_{a1}'$$

$$\check{I}_{a1}' = \frac{\check{E}_{a1}}{(Z_{a0}' + Z_1') - \dfrac{Z_{a1}'\, Z_{a2}'}{Z_{a0}' + Z_2'}}$$

Or in more symmetrical form

$$\left.\begin{aligned}\check{I}_{a1}' &= \frac{Z_{a0}' + Z_2'}{(Z_{a0}' + Z_1')\,(Z_{a0}' + Z_2') - Z_{a1}'\, Z_{a2}'}\, \check{E}_{a1} \\ \check{I}_{a2}' &= - \frac{Z_{a1}^1}{(Z_{a0}' + Z_1')\,(Z_{a0}' + Z_2') - Z_{a1}'\, Z_{a2}'}\, \check{E}_{a1}\end{aligned}\right\} \quad (\mathbf{164})$$

From (**159**) we have for the damper currents

$$\left.\begin{aligned}&\check{I}_{u1}' = O \text{ if } R_u > O \\ &\check{I}_{u2}' = - \check{K}_2\, I_{a2}\, \epsilon^{j\,2 w_0 t} \\ &\qquad \text{where } \check{K}_2 = j\, \frac{2\, w_0\, M}{R_u + j\, 2\, w_0\, L_u}\end{aligned}\right\} \quad (\mathbf{165})$$

A particular case of interest is when the load is a Synchronous Motor or Induction Motor with unsymmetrical line impedances in series—Equation (**163**) becomes

$$\left.\begin{aligned}&(Z_{a0}' + Z_1' + Z_1)\, \check{I}_{a1}' + Z_{a2}'\, I_{a2}' = \check{E}_{a2} \\ &\qquad Z_{a1}\, \check{I}_{a1}' + (Z_{a0}' + Z_2' + Z_2)\, I_{a2}' = O \\ &I_{a1}' = \frac{Z_{a0}' + Z_2' + Z_2}{(Z_{a0}' + Z_1' + Z_1)\;(Z_{a0}' + Z_2' + Z_2) - Z_{a1}\, Z_{a2}}\, \check{E}_{a1} \\ &I_{a2}' = \frac{Z_{a1}}{(Z_{a0}' + Z_1' + Z_1)\;(Z_{a0}' + Z_2'\; Z_2) - Z_{a1}\, Z_{a2}}\, \check{E}_{a1}\end{aligned}\right\} \quad (\mathbf{166})$$

An important case is that of a generator feeding into a symmetrical motor and an unsymmetrical load. Let the motor currents be

$\check{I}_a$, $\check{I}_b$, $\check{I}_c$, those of the load I_a', I_b', I_c' and the load impedances Z_a', Z_b', Z_c'. The equations of this system will be

$$\left.\begin{aligned}
S^1\,\check{E}_{a1} &= S^1\{Z_1'(\check{I}_{a1}+\check{I}_{a1}') + Z_{a0}'\,\check{I}_{a1}' + Z_{a2}'\,\check{I}_{a2}'\}\\
S^1\,\check{E}_{a1} &= S^1\{Z_1'(\check{I}_{a1}+\check{I}_{a1}') + Z_1\,\check{I}_{a1}\}\\
S^2\,O &= S^2\{Z_2'(\check{I}_{a2}+\check{I}_{a2}') + Z_{a0}'\,\check{I}_{a2} + Z_{a1}'\,\check{I}_{a1}'\}\\
S^2\,O &= S^2\{Z_2'(\check{I}_{a2}+\check{I}_{a2}') + Z_2\,\check{I}_{a2}\}
\end{aligned}\right\}\qquad(\mathbf{167})$$

Or, omitting the sequence symbols and re-arranging—

$$\left.\begin{aligned}
\check{E}_{a1} &= Z_1'\,\check{I}_{a1} + (Z_1'+Z_{a0}')\,\check{I}_{a1}' + Z_{a2}'\,\check{I}_{a2}'\\
\check{E}_{a1} &= (Z_1'+Z_1)\,\check{I}_{a1} + Z_1'\,\check{I}_{a1}^1\\
O &= Z_2'\,\check{I}_{a2} + Z_{a1}'\,\check{I}_{a1}^1 + (Z_2'+Z_{a0}')\,\check{I}_{a2}'\\
O &= (Z_2'+Z_2)\,\check{I}_{a2} + Z_2'\,\check{I}_{a2}'
\end{aligned}\right\}\qquad(\mathbf{168})$$

These equations can be further simplified as follows:

$$\left.\begin{aligned}
O &= (Z_2'+Z_2)\,\check{I}_{a2} + Z_2'\,\check{I}_{a2}'\\
O &= -Z_2\,\check{I}_{a2} + Z_{a1}'\,\check{I}_{a1}' + Z_{a0}'\,\check{I}_{a2}'\\
O &= -Z_1\,\check{I}_{a1} + Z_{a0}'\,\check{I}_{a0}' + Z_{a2}'\,I_{a2}'\\
\check{E}_{a1} &= (Z_1'+Z_1)\,\check{I}_{a1} + Z_1'\,\check{I}_{a1}'
\end{aligned}\right\}\qquad(\mathbf{169})$$

A set of simultaneous equations which may be easily solved.

The Single-Phase Generator is an Important Case of the Three-Phase Generator Operated on an Unbalanced Load

Let the impedance of the single-phase load be Z and let us suppose it to be made up of three star connected impedances

$$Z_a' = 3\,Z_x + \frac{Z}{2}$$

$$Z_b' = \frac{Z}{2}$$

$$Z_c' = \frac{Z}{2}$$

the value of Z_x in the limit being infinity. Then we have

$$\left.\begin{aligned} Z_{a0}' &= Z_x + \frac{Z}{2} \\ Z_{a1}' &= Z_x \\ Z_{a2}' &= Z_x \end{aligned}\right\} \qquad \mathbf{(170)}$$

Equation (**164**) in the limit when Z_x becomes infinite reduces to

$$\begin{aligned} \check{I}_{a1}' &= \frac{\check{E}_{a1}}{Z + Z_1' + Z_2'} \\ \check{I}'_{a2} &= -\frac{\check{E}_{a1}}{Z + Z_1' + Z_2'} \end{aligned} \qquad \mathbf{(171)}$$

The single-phase load being across the phase $B\ C$, the single-phase current I will therefore be equal to $\check{I}_c$ or

$$\left.\begin{aligned} \check{I} &= \frac{j\sqrt{3}\,\check{E}_{a1}}{Z + Z_1' + Z_2'} \\ &= \frac{\check{E}_{bc}}{Z + Z_1' + Z_2'} \end{aligned}\right\} \qquad \mathbf{(172)}$$

$$\left.\begin{aligned} \check{I}_{u1} &= O \text{ if } R_u > O \\ \check{I}_{u2} &= -j\frac{1}{\sqrt{3}}\check{K}_2\,\check{I}\,\epsilon^{jw_0t} \\ \check{I}_{u2} &= -\frac{j\,\check{K}_2}{\sqrt{3}}\,I\,\epsilon^{j2w_0t} \end{aligned}\right\} \qquad \mathbf{(173)}$$

$\check{I}_{u2}$ is double normal frequency

$$\left.\begin{aligned} P_{\mathrm{I}} + j\,Q_{\mathrm{I}} &= 3\,I^2\,Z \\ P_{\mathrm{L}} + j\,Q_{\mathrm{L}} &= 3\,I^2\,(Z_1' + Z_2') \\ (P + j\,Q) + (P_{\mathrm{H}} + j\,Q_{\mathrm{H}}) &= 3\,\check{E}_{bc}\,(\hat{I} + \check{I}) \end{aligned}\right\} \qquad \mathbf{(174)}$$

In the case of the generally unbalanced three-phase load

$$\left.\begin{aligned} P_1 + j\,Q_1 &= 3\,\{(I_{a1}^2 + I_{a2}^2)\,Z_{a0}' \\ &\qquad + \check{I}_{a1}\,\hat{I}_{a2}\,Z_{a2}' + \hat{I}_{a1}\,\check{I}_{a2}\,Z_{a1}'\} \\ P_{\mathrm{L}} + j\,Q_{\mathrm{L}} &= 3\,\{I_{a1}^2\,Z_1' + I_{a2}^2\,Z_2'\} \\ (P + j\,Q) + (P_{\mathrm{H}} + j\,Q_{\mathrm{H}}) &= 3\,\check{E}_{a1}\,(\check{I}_{a2} + \hat{I}_{a2}) \end{aligned}\right| \qquad \mathbf{(175)}$$

When the generator has harmonics in its wave form equations (**162**) must be written

$$\left.\begin{aligned} &(R_a + j\,w\,L_a + Z_{a0}')\,\breve{I}_{a1}' + Z_{a2}'\,I_{a2}' = \breve{E}_{a1} \\ &\quad Z_{a1}'\,I_{a1}' + \{Z_{a0}' + (R_a + K_2{}^2\,R_u) \\ &\qquad + j\,2\,w\,(L_a - K_2{}^2\,L_u) - \tfrac{1}{2}\,a^2\,R_u\}\,I_{a2}' = \breve{E}_{a2} \end{aligned}\right\} \quad \textbf{(176)}$$

Where $\breve{E}_{a1}$ is finite, $\breve{E}_{a2}$ is zero and vice versa, the frequencies being different in each case, we have therefore a solution for each frequency depending on the phase and amplitude and phase sequence of the e. m. f. of this frequency generated. Of course the values of Z_1' and Z_2' change with each frequency on account of the change in the reactance with frequency, and a value must be taken for w conforming with the frequency of the harmonic under consideration.

Symmetrical Synchronous Motor, Synchronous Condenser, Etc.

As in the case of the generator, the synchronous motor has two impedances, one to the positive phase sequence current of a given frequency and the other to the negative phase sequence current of the same frequency. But, since there is no quantity in the positive phase sequence impedance corresponding to the virtual resistance which indicates mechanical work in an induction motor, its equivalent is furnished by the excitation of the field. Let us denote the e. m. f. due to the field excitation by $S^1\,\breve{E}_{a1}'$ assuming it to be for the present a simple harmonic three-phase system. Let P_0 be the output of the motor which will include the windage and iron losses assumed to be constant. Then for the synchronous motor on a balanced circuit of e. m. f. $S^1\,\breve{E}_{a1}$ we have

$$S^1\,\breve{E}_{a1} = S^1\,\{\breve{I}_{a1}(R_a' + j\,w\,L_a') + \breve{E}_{a1}'\} \qquad \textbf{(177)}$$

$$S^0\,\breve{E}_{a1}\,\hat{I}_{a1} = S^0\left\{\,I_{a1}{}^2\,(R_a' + j\,w\,L_a') + \frac{P_0}{3} - j\,\frac{Q_0}{3}\,\right\} \qquad \textbf{(178)}$$

Where Q_0 is the imaginary part of the product, $\breve{E}_{a1}'\,\hat{I}_{a1}$. (**178**) reduces to

$$E_{a1}\,I_{a1}\cos\alpha = I_{a1}{}^2\,R_a' + \frac{P_0}{3} \qquad \textbf{(179)}$$

Where cos α is the required operating power factor. Solving for I_{a1}

$$I_{a1} = \frac{E_{a1}\cos\alpha}{2 R_a^1}\left\{1 \pm \sqrt{1 - \frac{4 R_a' P_0}{3 E_{a1}^2 \cos^2\alpha}}\right\} \tag{180}$$

$$\check{I}_{a1} = \check{E}_{a1}\frac{\cos\alpha}{2 R_{a1}}\left\{1 \pm \sqrt{1 - \frac{4 R_{a1}' P_0}{3 E_{a1}^2 \cos^2\alpha}}\right\}(\cos\alpha - j\sin\alpha) \tag{181}$$

The apparent impedance of the motor is

$$\frac{2 R_1 \sec\alpha}{1 \pm \sqrt{1 - \frac{4 P_0}{3 E_a^2 \cos^2\alpha}}}(\cos\alpha + j\sin\alpha) \tag{182}$$

and

$$\check{E}_{a1}^1 = \check{E}_{a1}\left[1 - \frac{\cos\alpha}{2 R_a^1}\left\{1 \pm \sqrt{1 - \frac{4 R_a' P_0}{3 E_{a1}^2 \cos^2\alpha}}\right\}(\cos\alpha - j\sin\alpha)\cdot(R_a' + j\,w\,L_a')\right] \tag{183}$$

The same equations apply to the case of the synchronous condenser with the difference that the mechanical work is that required to overcome the iron and windage losses only.

If we take

$$\left.\begin{aligned}\check{E}_{a1} &= E_{a1}(\cos\alpha + j\sin\alpha)\,\epsilon^{j w_0 t} = (A_1 + j B_1)\,\epsilon^{j w_0 t}\\ \check{E}_{a1}' &= (A_1' + j B_1')\,\epsilon^{j w_0 \tau}\end{aligned}\right\} \tag{184}$$

we have

$$\check{I}_{a1} = \frac{A_1}{2 R_a^1}\left(1 \pm \sqrt{1 - \frac{4 R_a' P_0}{3 A_1^2}}\right)\epsilon^{j w_0 t} \tag{185}$$

$$A_1' = \frac{A_1}{2}\left(1 \pm \sqrt{1 - \frac{4 R_a' P_0}{3 A_1^2}}\right)\epsilon^{j w_0 t} \tag{186}$$

$$B_1' = \left\{B_1 - \frac{j\,w\,L_a' A_1}{2 R_a'}\left(1 \pm \sqrt{1 - \frac{4 R_a' P_0}{3 A_1^2}}\right)\right\}\epsilon^{j w_0 t} \tag{187}$$

Since α may be a positive or negative angle, the sine may be positive or negative for a positive cosine, and therefore the power factor will be leading or lagging accordingly as B_1 is negative or positive respectively. The double signs throughout are due to the fact that for any given load and power factor there are always two theoretically possible running conditions. However, since we are concerned only with that one which will give the max. operating efficiency, that is the condition that gives I_{a1} the lesser value, for a given value of P_0 the equations may be written

$$\left.\begin{aligned} \check{I}_{a1} &= \frac{A_1}{2\,R_a^1}\left(1-\sqrt{1-\frac{4\,R_a'\,P_0}{3\,A_1^2}}\right)\epsilon^{j w_0 t} \\ A_1' &= \frac{A_1}{2}\left(1+\sqrt{1-\frac{4\,R_a'\,P_0}{3\,A_1^2}}\right)\epsilon^{j w_0 t} \\ B_1' &= \left\{B_1-\frac{j\,w_0\,L_a'\,A_1}{2\,R_a'}\left(1-\sqrt{1-\frac{4\,R_a'\,P_0}{3\,A_1^2}}\right)\right\} \end{aligned}\right\} \quad (\mathbf{188})$$

And corresponding values for (**180**), (**181**), (**182**) and (**183**) may be obtained by omitting the positive sign in these equations.

Another condition of operation is obtained by inspection of (**180**), due to the fact that I_{a1} must be a real quantity

$$\frac{4\ R_a'\ P_0}{3\ E_{a1}^2\cos^2\alpha}\ \text{must be} > 1 \qquad (\mathbf{189})$$

this is the condition of stability. In terms of (**184**) it becomes

$$\frac{4\,R_a'\,P_0}{3\,A_1^2}\ \text{must be} > 1 \qquad (\mathbf{190})$$

The same conditions apply to the synchronous condenser, the total mechanical load in this case being the iron loss and windage and friction losses.

Proceeding now to operation with unbalanced circuits having sine waves the motor also having a sine wave. In addition to equation (**177**) we shall have

$$S^2\,\check{E}_{a2} = S^2\,Z_2'\,\check{I}_{a2} \qquad (\mathbf{191})$$

The mechanical power delivered through the operation of this negative phase sequence e. m. f. is given by P_N where

$$P_N = -\,3\ I_{a2}^2\,\frac{R_a'}{2} \qquad (\mathbf{192})$$

this quantity must therefore be subtracted from the value of P_0 in all the equations in which P_0 appears when unbalanced circuits are used in connection with equations (**177**) to (**190**) inclusive. These equations, however, give the conditions for maintaining a given mechanical load and a given power factor in the positive phase sequence component, but in practise what is required is the combined power factor of the whole system, or the conditions to give a certain combined factor while delivering a given mechanical load; this may be obtained as follows:

The negative phase sequence component is a perfectly definite impedance and is independent of the load, and therefore the zero frequency part of the product $E_{a2}\, I_{a2}$ may be set down as

$$\check{E}_{a2}\, \hat{I}_{a2} = \frac{P_2}{3} + j\,\frac{Q_2}{3} \qquad \textbf{(193)}$$

we have also for the positive phase sequence power delivered

$$(A_1 + j\,B_1)\ I_{a1} = I_{a1}^2\, R_a' + \frac{P_0}{3} - \frac{P_N}{3} + j\,(w\, I_{a1}\, L_a' + B_1^1)\, I_{a1} \qquad \textbf{(194)}$$

And the power factor is given by cos α, where

$$\tan \alpha = \frac{I_{a1} B_1 + \frac{Q_2}{3}}{I_{a1} A_1 + \frac{P_2}{3}} \qquad \textbf{(195)}$$

From (**194**) we have

$$A_1\, I_{a1} = I_{a1}^2\, R_a^1 + \frac{P_0}{3} - \frac{P_N}{3} \qquad \textbf{(196)}$$

$$B_1 = w\, I_{a1}\, L_a' + B_1' \qquad \textbf{(197)}$$

$$A_1^2 + B_1^2 = E_{a1}^2 \qquad \textbf{(198)}$$

The simplest method of solving these equations is by means of curves. Taking arbitrary values of I_{a1}, B_1 and A_1 are chosen consistent with (**198**) so as to satisfy (**195**), $\frac{P_0}{3}$ A_1' and B_1' are then obtained from (**196**) and (**197**). If there are harmonics in the impressed e. m. f. but there are none in the wave form of the machine, the machine will have a definite impedance to the positive and negative phase sequence components of each harmonic, so that there will be a definite amount of mechanical work contributed by each harmonic which must be subtracted from the total work to be done to give the amount of work contributed by the positive phase sequence fundamental component, the equations will be identical to (**193**), (**194**), (**195**), (**196**), (**197**) and (**198**), if we take P_N to mean the total mechanical work done by the harmonics both positive and negative phase sequence and P_2 and Q_2 to represent the products

$$\Sigma\ ({}_n\check{E}_{a1}\quad {}_n\hat{I}_{a1} + {}_n\check{E}_{a2}\quad {}_n\hat{I}_{a2})$$

the zero frequency part only being taken into account.

When harmonics are present both in the impressed wave and in the generated wave, the problem becomes too complicated to treat generally, but specific cases can be worked out without much difficulty.

Phase Converters and Balancers

The phase converter is a machine to transform energy from single-phase or pulsating form to polyphase or non-pulsating form or vice versa to transform energy from polyphase to single-phase. The transformation may not be complete, that is to say, the polyphase system may not be perfectly balanced when supplied from a single-phase source through the medium of a phase converter. Phase converters may be roughly divided into two classes, namely—shunt type and series type.

Induction Motor or Synchronous Condenser Operating as a Phase Converter of the Shunt Type to Supply a Symmetrical Induction Motor or Synchronous Motor

Let Z_1 and Z_2 be the positive and negative phase sequence impedances of the motor, Z_1', Z_2' those of the phase converter. Let $S^1\,\check{E}_{a1}$ and $S^2\,\check{E}_{a2}$ be the positive and negative phase sequence components of the star e. m. f. impressed on the motor as a result of the operation. The single-phase supply will be one side of the delta e. m. f. $S\,\check{E}_{bc}$ which has positive and negative phase sequence components $S^1\,E_{bc1}$ and $S^2\,E_{bc2}$ the single-phase supply being $\check{E}_{bc} = \check{E}_{bc1} + \check{E}_{bc2}$.

The value of Z_2' may be considered fixed for all practical purposes and since in the induction motor phase converter the speed is practically no-load speed, Z' is practically the no-load impedance plus a real part obtained by increasing the real part of the no-load impedance by the ratio of the normal no-load losses to these same losses plus $\frac{1}{2}$ the secondary losses due to the phase converter currents. The latter may be calculated roughly as even a large error in its value will have an inappreciable effect on the actual results. We have therefore

$$\left.\begin{aligned} S^1\,\check{E}_{a1} &= -\,S^1\,j\,\frac{\check{E}_{bc}}{\sqrt{3}} \\ S^2\,\check{E}_{a2} &= S^2\,j\,\frac{\check{E}_{bc2}}{\sqrt{3}} \end{aligned}\right\} \qquad (\mathbf{199})$$

$$\left.\begin{aligned} S_1^1\,\check{I}_{a1}' &= -\,S^1\,j\,\frac{\check{E}_{bc1}}{\sqrt{3}\,Z_1'} \\ S^1\,\check{I}_{a1} &= -\,S^1\,j\,\frac{\check{E}_{bc1}}{\sqrt{3}\,Z_1} \end{aligned}\right\} \qquad (\mathbf{200})$$

$$\left.\begin{aligned} S^2\,\check{I}_{a2}' &= S^2\,j\,\frac{\check{E}_{bc2}}{\sqrt{3}\,Z_2'} \\ S^2\,\check{I}_{a2} &= S^2\,j\,\frac{\check{E}_{bc2}}{\sqrt{3}\,Z_2} \end{aligned}\right\} \qquad (\mathbf{201})$$

In the common lead of motor and converter we have

$$\check{I}_{a1}' + \check{I}_{a2}' + \check{I}_{a1} + \check{I}_{a2} = O \qquad (\mathbf{202})$$

or, substituting from (**200**) and (**201**)

$$\check{E}_{bc2}\left(\frac{1}{Z_2'} + \frac{1}{Z_2}\right) = \check{E}_{bc1}\left(\frac{1}{Z_1'} + \frac{1}{Z_1}\right) \qquad (\mathbf{203})$$

$$\frac{\check{E}_{bc1}}{\check{E}_{bc2}} = \frac{\dfrac{1}{Z_2'} + \dfrac{1}{Z_2}}{\dfrac{1}{Z_1'} + \dfrac{1}{Z_1}} \qquad (\mathbf{204})$$

$$\check{E}_{bc1} = \frac{\dfrac{1}{Z_2} + \dfrac{1}{Z_2'}}{\left(\dfrac{1}{Z_1} + \dfrac{1}{Z_1'}\right) + \left(\dfrac{1}{Z_2} + \dfrac{1}{Z_2'}\right)}\,\check{E}_{bc} \qquad (\mathbf{205})$$

$$\check{E}_{bc2} = \frac{\dfrac{1}{Z_1} + \dfrac{1}{Z_1'}}{\left(\dfrac{1}{Z_1} + \dfrac{1}{Z_1'}\right) + \left(\dfrac{1}{Z_2} + \dfrac{1}{Z_2'}\right)}\,\check{E}_{bc} \qquad (\mathbf{206})$$

which give the complete solution for all the quantities required with the aid of equations (**200**) and (**201**). For the supply current $\check{I}$

$$\left.\begin{aligned} \check{I} &= \check{I}_{bc1} + \check{I}_{bc2} + I_{bc1}' + I_{bc2}' \\ S\,\check{I}_{bc} &= S^1\,\check{I}_{bc1} + S^2\,\check{I}_{bc2} \\ S\,\check{E}_{bc} &= S^1\,\check{E}_{bc1} + S^2\,E_{bc2} \end{aligned}\right\} \quad (207)$$

$$P_1 + j\,Q_1 = \check{E}_{bc}\,\hat{I} \qquad (208)$$

In order to obtain a perfect balance we may consider the addition of an e. m. f. $S^2\,j\,\frac{E_{x2}}{\sqrt{3}}$ in series with the phase converter whose value must be a function of the load and the phase converter impedances, and therefore equation (**201**) will be replaced by

$$S^2\,\check{I}_{a2}' = S^2\left(j\,\frac{\check{E}_{bc2}}{\sqrt{3}\,Z_2'} + j\,\frac{\check{E}_{x2}}{\sqrt{3}\,Z_2'}\right) \qquad (209)$$

$$S^2\,\check{I}_{a2} = S^2\,j\,\frac{\check{E}_{bc}}{\sqrt{3}\,Z_2}$$

and since the balance is perfect $\check{E}_{bc2}$ is zero, and therefore

$$S^2\,j\,\frac{\check{E}_{x2}}{\sqrt{3}} = S^2\,Z_2'\,\check{I}_{a2}' \qquad (210)$$

An e. m. f. equal and of opposite phase to the negative phase sequence drop through the phase converter is required to produce a perfect balance.

Carrying out the solution in the same manner as in the imperfect converter, we obtain

$$\check{E}_{bc2} = \frac{\frac{1}{Z_1} + \frac{1}{Z_1'}}{\frac{1}{Z_2} + \frac{1}{Z_2'}}\,E_{bc} - \frac{\frac{1}{Z_2'}}{\frac{1}{Z_2} + \frac{1}{Z_2'}}\,\check{E}_{x2} \qquad (211)$$

and since $\check{E}_{bc2}$ is zero and $\check{E}_{bc1} = \check{E}_{bc}$ the single-phase impressed e. m. f., we obtain

$$\check{E}_{x2} = Z_2'\left(\frac{1}{Z_1} + \frac{1}{Z_1^1}\right)\check{E}_{bc} \qquad (212)$$

and therefore from (**210**)

$$S^2\,\check{I}_{a2}' = S^2\,j\left(\frac{1}{Z_1} + \frac{1}{Z_1'}\right)\frac{\check{E}_{bc}}{\sqrt{3}} \qquad (213)$$

$$S^1 \check{I}_{a1}{}^1 = - S^1 j \frac{\check{E}_{bc}}{\sqrt{3} Z_1{}'} \tag{214}$$

$$S^2 \check{I}_{a2} = O \tag{215}$$

$$S^1 \check{I}_{a1} = - S^1 j \frac{\check{E}_{bc}}{\sqrt{3}\ Z_1} \tag{216}$$

Figs. 9, 10, 11 and 12 are vector diagrams of some of the principal compensated shunt-type phase converters. There will be no difficulty in following out these diagrams if the principles of this paper have been grasped.

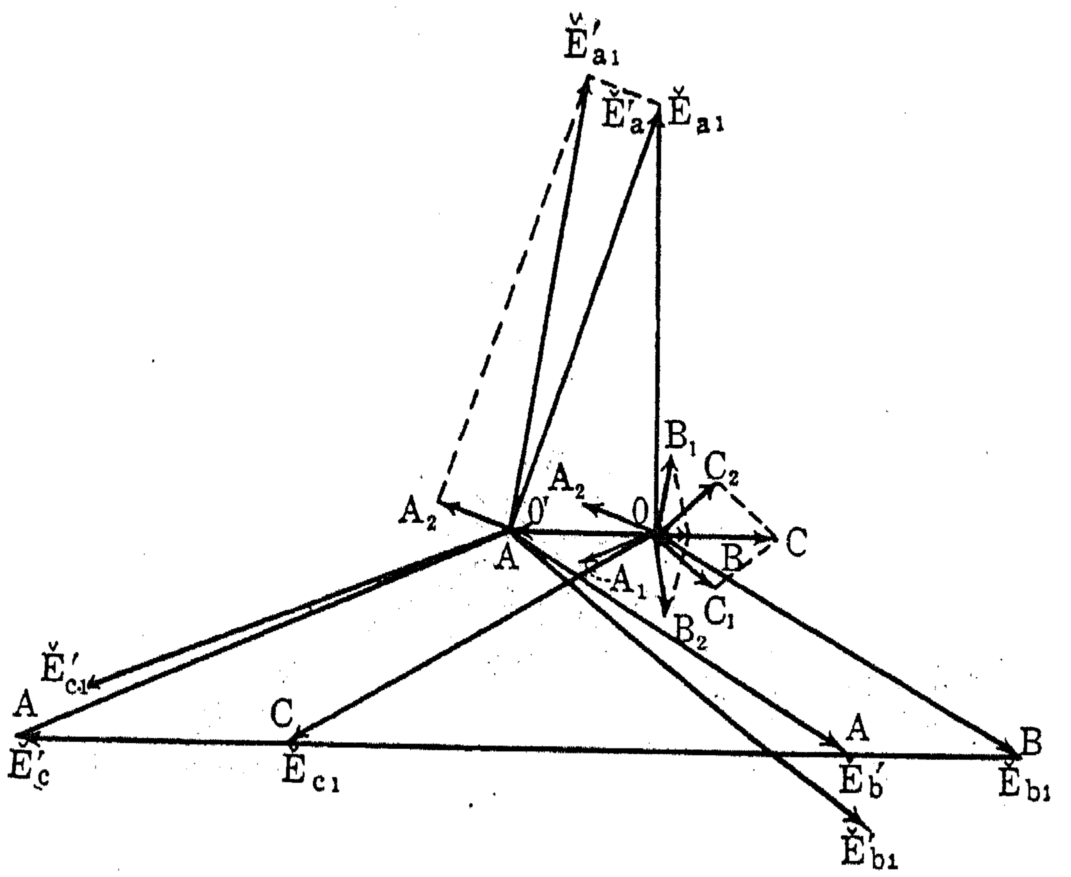

FIG. 9—VECTOR DIAGRAM OF SHUNT-TYPE PHASE CONVERTER OPERATED FROM TRANSFORMER SO AS TO DELIVER BALANCED CURRENTS

Terminal voltages of phase converter $S\check{E}'_a$
Terminal voltages of motor $S^1\check{E}_{a1}$
Negative phase sequence e.m.fs. in phase converter S^2 (OA_2)

The Phase Balancer is a device to maintain symmetry of e. m. fs. at a given point in a polyphase system. It may consist of an induction motor or synchronous condenser with an auxiliary machine connected in series to supply an e. m. f. always proportional to the product of the negative phase sequence current passing through the machine and the negative phase sequence impedance of the balancer. It therefore has the effect of annulling the impedance of the machine to the flow of negative phase sequence current. Thus, in a symmetrical polyphase network, where we have an unbalanced system of currents due to certain conditions

$$S\ \check{I}_a = S^1\ \check{I}_{a1} + S^2\ \check{I}_{a2} \tag{217}$$

If a balancer be placed at the proper point the component $S^2 \check{I}_{a2}$ will circulate between the loads and the phase balancer, the other component $S^1 I_{a1}$ being furnished from the power house. On the other hand, if there be a dissymmetry in the impedance of the system up to the phase balancer, the latter will draw a negative phase sequence current sufficient to counteract the unbalance due to any symmetrical load by causing the proper amount of negative phase sequence current to flow to produce a balance.

The balancer may be made inherently self-balancing by inserting in series with it a machine which is self-exciting and is able to furnish an e. m. f. equal to the negative phase sequence impedance drop. The combination thus has zero impedance to negative phase sequence currents. If in the neighborhood of a phase balancer the loads have impedances

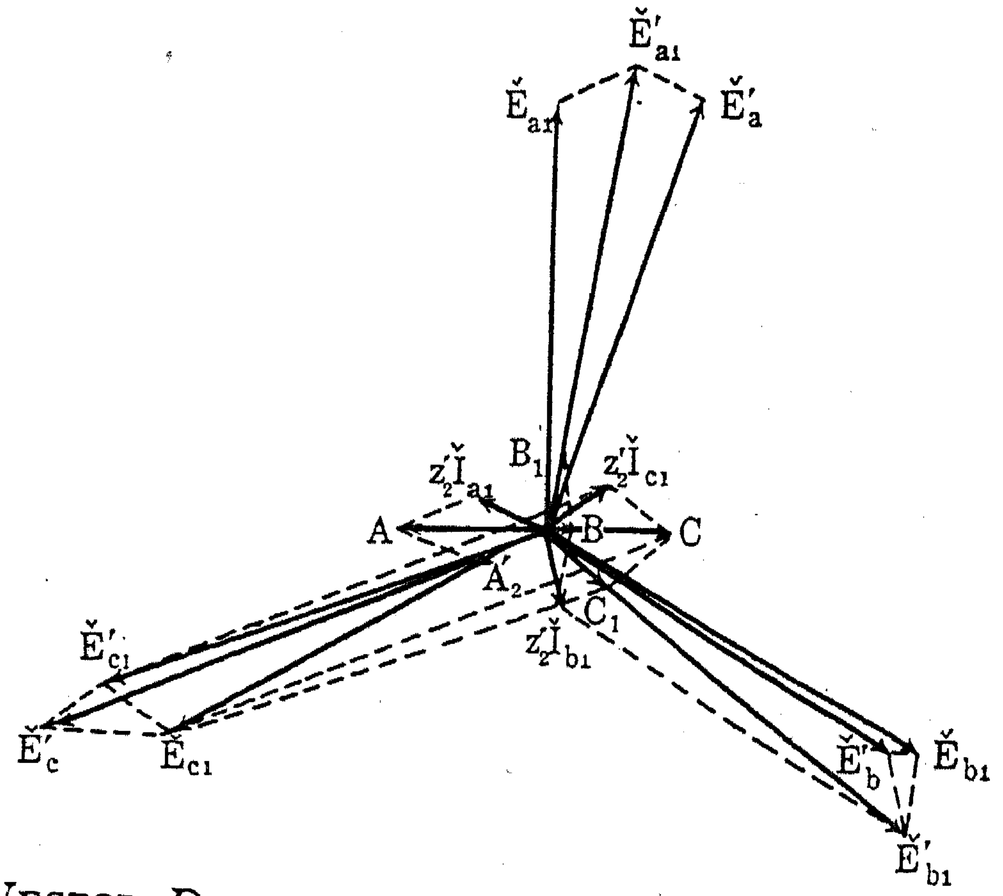

FIG. 10—VECTOR DIAGRAM SHOWING RELATIONS BETWEEN MOTOR TERMINAL E.M.F'S., CONVERTER TERMINAL E.M.FS., AND SYMMETRICAL GENERATED E.M.F'S., SAME CONNECTION AS FOR FIG. 9.

Negative phase sequence drops in phase converter $S^2 Z_2' \check{I}_{a1}$
Conjugate positive phase sequence e.m.fs. $S^1(ABC)$

$$S\, Z_a = S^0\, Z_{a0} + S^1\, Z_{a1} + S^2\, Z_{a2}$$

The equations of the system are

$$\left.\begin{aligned} S^1 \check{E}_{a1} &= S^1 Z_{a0}\, \check{I}_{a1} + S^1 Z_{a2}\, \check{I}_{a2} \\ S^2 E_{a2} = O &= S^2 Z_{a0}\, \check{I}_{a2} + S^2 Z_{a1}\, \check{I}_{a1} \end{aligned}\right\} \quad \textbf{(218)}$$

The currents in the phase balancer are

$$- S^2 \check{I}_{a2} \text{ and } S^1 \frac{\check{E}_{a1}}{Z_1'}$$

The solution of (**218**) gives $S^2 \check{I}_{a2}$ and $S^1 \check{I}_{a1}$, the former of which are the phase balancer currents. The solution is

$$\left.\begin{aligned} \check{I}_{a1} &= \frac{Z_{a0}}{Z_{a0}^2 - Z_{a1} Z_{a2}} \check{E}_{a1} \\ \check{I}_{a2} &= - \frac{Z_{a1}}{Z_{a0}^2 - Z_{a1} Z_{a2}} \check{E}_{a1} \end{aligned}\right\} \quad (\mathbf{219})$$

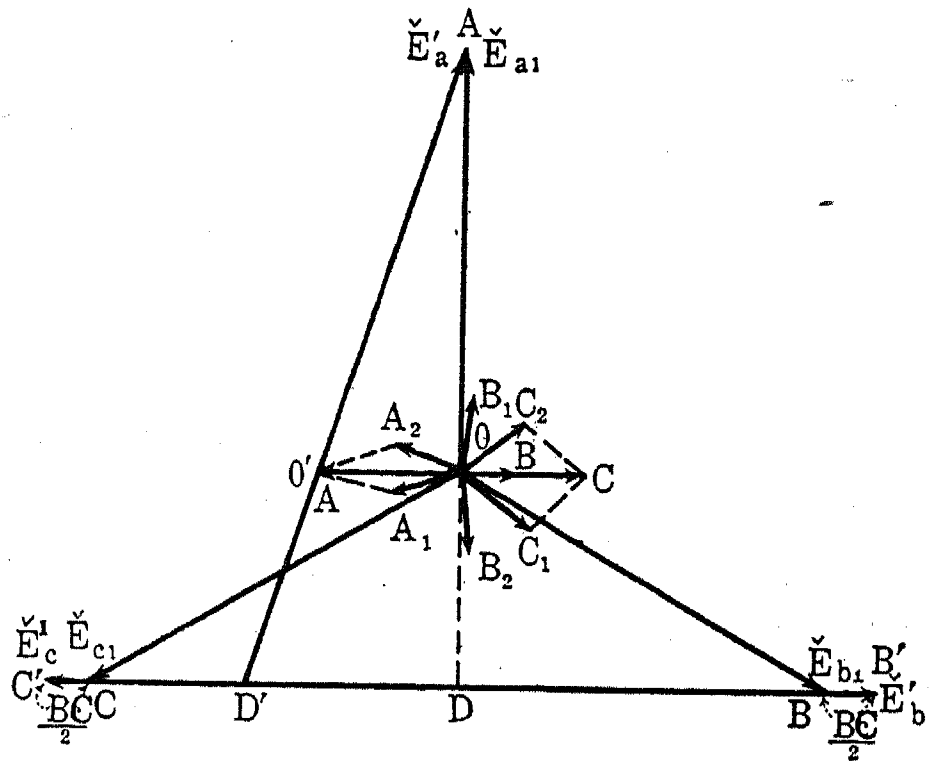

FIG. 11—VECTOR DIAGRAM OF SHUNT-TYPE PHASE CONVERTER SCOTT CONNECTED WITH COMPENSATION BY TRANSFORMER TAPS
Terminal voltages of converter $D'A$ and $B'C^1$
Terminal voltages of motor $S^1\check{E}_{a1}$

The phase balancer is a voltage balancer and will maintain balanced e. m. f. for any condition of impedance, and if the impedance of the mains is unsymmetrical it will draw a sufficient amount of wattless negative phase sequence current through these mains to produce an e. m. f. balance at its terminals. Hence the complete solution requires consideration of all the connections in the network between the supply point and the balancer. Two equations for each mesh and connection are required, one of the positive phase sequence e. m. fs. and the other of the negative phase sequence e. m. f., and these equations may be solved in the usual way.

Series Phase Converter. In discussing the various reactions in rotating machines we have made use of the terms "positive phase sequence impedance" and "negative phase sequence impedance." These terms are definite enough when dealing with relations between machines whose generated e. m. fs. all have the same phase sequence, but require further definition when we are dealing with relations between machines whose e. m. fs. have different phase sequence. We shall retain the symbols Z_1 and Z_2 for the values of the positive and negative phase sequence impedances, depending upon the sequence symbol S to define whether these impedances apply to a negative or positive phase sequence current. Thus, the phase sequence of the currents and

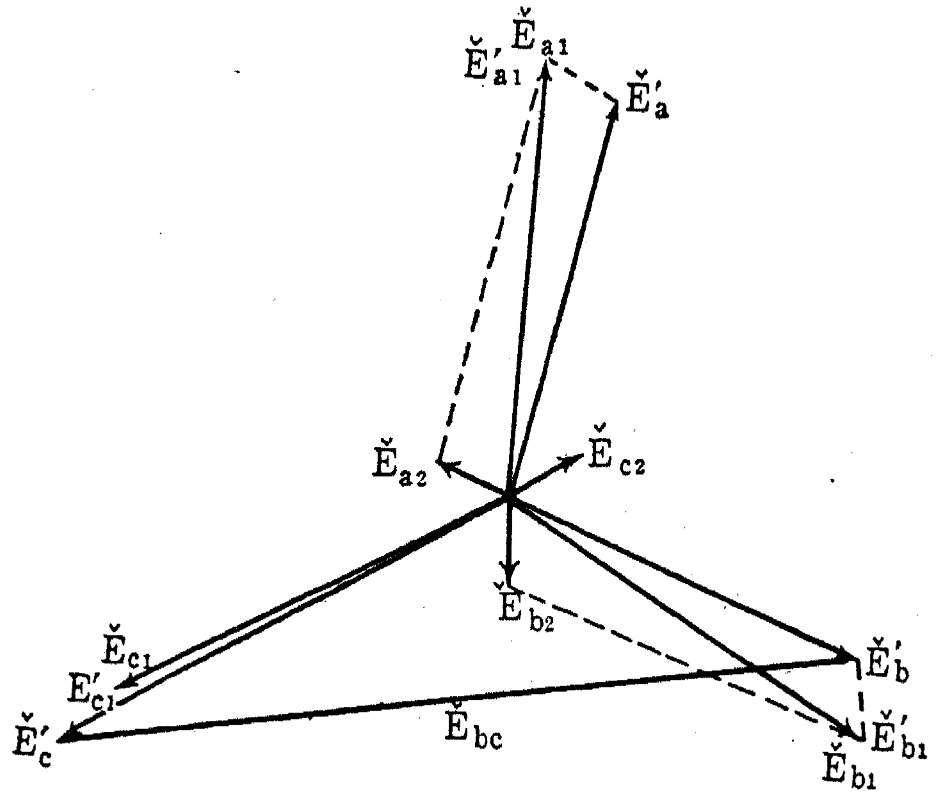

FIG. 12—VECTOR DIAGRAM OF SHUNT-TYPE PHASE CONVERTER WITH AUXILIARY ROTATING COMPENSATOR TO EFFECT A PERFECT BALANCE

Terminal voltages of phase converter $S\,\check{E}_a'$
Terminal voltages of motor $S^1\check{E}_{a1}$
Terminal voltages of compensator $S^2\check{E}_{a2}$

e. m. f. will be defined by the apparatus supplying and receiving power and the impedances of the transmitting apparatus will be defined in relation to these currents. As an example a motor series connected in counter phase sequence relation in a circuit and driven in a positive direction will have impedances

$$\begin{aligned}&\text{positive phase sequence } Z_2\\&\text{negative phase sequence } Z_1\end{aligned}\qquad(220)$$

Where an auxiliary machine is defined as being of negative phase sequence relation to other machines, it will have impedances as given above to the positive and negative phase sequence currents passing through the other machines.

A single-phase transformer winding tapped at the middle point may be regarded as an unbalanced three-phase system where

$$\check{E}_a = 0, \check{E}_b = + \check{E}_s, \check{E}_c = - \check{E}_s$$

2 $\check{E}_s$ being the single-phase e. m. f The system may be represented by the equation

$$S \check{E}_a = S^1 \check{E}_{a1} + S^2 \check{E}_{a2}$$

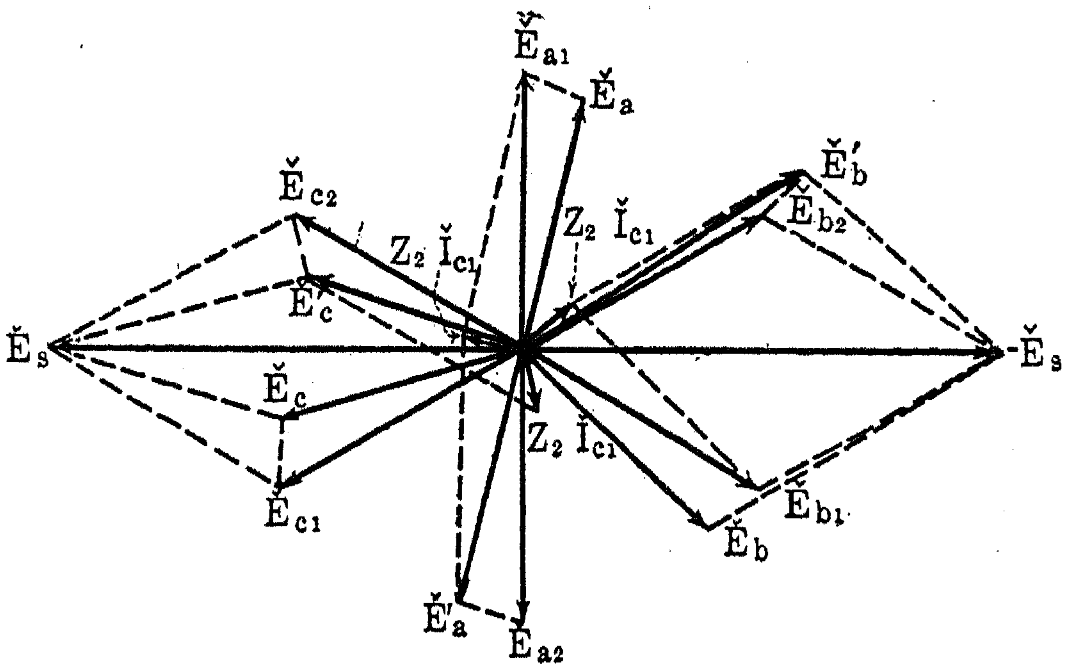

FIG. 13—VECTOR DIAGRAM OF SERIES-TYPE CONVERTER.
NO LOAD E.M.F'S. ACROSS MOTOR TERMINALS $S_1\check{E}_{a1}$
NO LOAD E.M.F'S. ACROSS CONVERTER TERMINALS $S^2\check{E}_{a2}$
SINGLE-PHASE E.M.F'S. $2\check{E}_s$
E.M.F. ACROSS TERMINAL OF MOTOR UNDER LOAD $\check{E}_a\check{E}_b\check{E}_c$
E.M.F. ACROSS TERMINAL OF CONVERTER UNDER LOAD $\check{E}'_a\check{E}'_b\check{E}'_c$

$$\left.\begin{aligned} \text{where } \check{E}_{a1} &= j \frac{\check{E}_s}{\sqrt{3}} \\ \check{E}_{a2} &= -j \frac{\check{E}_s}{\sqrt{3}} \end{aligned}\right\} \quad (221)$$

If, therefore between the single-phase source of power and the load we interpose a polyphase machine with e. m. f. $- S^2 \check{E}_{a2}$, we shall have at the load terminals the e. m. f. $S^1 \check{E}_{a1}$. If we use an induction-type phase converter it will have impedances to motor currents as follows

$$\left.\begin{aligned} &\text{To positive phase sequence } Z_2' \\ &\text{To negative phase sequence } Z_1' \end{aligned}\right\} \quad (222)$$

we therefore have the relations

$$S^1\,\check{E}_{a1} = S^1\,\check{I}_{a1}\,(Z_1 + Z_2') \tag{223}$$

$$S^2\,\check{E}_{a2} = S^2\,\check{I}_{a2}\,(Z_2 + Z_1') \tag{224}$$

If the converter is doing no mechanical work, Z_1' is large compared with Z_2' or Z_2, and therefore the component of negative phase sequence is small in the motor. The value of Z_1' depends upon the slip of the phase converter which will depend on the mechanical load it carries as well as on the load carried by the motors. Approximately the load currents due to the motors produce the equivalent at the phase converter of a mechanical load equal to one-half the rotor loss of the phase converter due to these load currents. Substituting the values given in (**221**) for $S^1\,\check{E}_{a1}$ and $S^2\,\check{E}_{a2}$, we obtain

$$\left.\begin{aligned} S^1\,j\,\frac{\check{E}_s}{\sqrt{3}} &= S^1\,\check{I}_{a1}\,(Z_1 + Z_2') \\ -\,S^2\,j\,\frac{E_s}{\sqrt{3}} &= S^2\,\check{I}_{a2}\,(Z_2 + Z_1') \end{aligned}\right\} \tag{225}$$

$$\begin{aligned} S^1\,\check{I}_{a1} &= \,S^1\,j\,\frac{\check{E}_s}{\sqrt{3}\,(Z_1 + Z_2')} \\ S^2\,\check{I}_{a2} &= -\,S^2\,j\,\frac{\check{E}_s}{\sqrt{3}\,(Z_2 + Z_1')} \end{aligned} \tag{226}$$

If instead of an induction-type phase converter a synchronous phase converter is used an e. m. f. of negative phase sequence $S^2\,\check{E}_{a2}$ *the generated e. m. f. of the phase converter must be introduced in equations* (**224**) *and* (**225**) *and the value and phase of these e. m. fs. will depend upon the load on the phase converter shaft as well as the load carried by the motors.* The equations will be

$$S^1\,\check{E}_{a1} = S^1\,\check{I}_{a1}\,(Z_1 + Z_2') \tag{227}$$

$$S^2\,\check{E}_{a2} = S^2\,\check{I}_{a2}\,(Z_2 + Z_1') + S^2\,\check{E}_{a2}' \tag{228}$$

or

$$\left.\begin{aligned} S^1\,j\,\frac{\check{E}_s}{\sqrt{3}} &= S^1\,\check{I}_{a1}\,(Z_1 + Z_2') \\ -\,S^2\,j\,\frac{\check{E}_s}{\sqrt{3}} &= S^2\,\check{I}_{a2}\,(Z_2 + Z_1') + S^2\,\check{E}_{a2}' \end{aligned}\right\} \tag{229}$$

The last member of equations **(229)** is the equation of a synchronous condenser. Assuming its windage, iron loss and increased losses due to secondary reactions to be P_0, we have by equation **(160)** of the Section on Synchronous Motors

$$\frac{E_s}{\sqrt{3}} I_{a2} \cos \alpha = I_{a2}^2 \, (R_2 + R_1') + \frac{P_0}{3} \qquad \mathbf{(230)}$$

Let

$$\check{I}_{a2} = a_2 + j \, b_2 \qquad \mathbf{(231)}$$

then **(230)** becomes

$$\frac{E_s}{\sqrt{3}} a_2 = (a_2^2 + b_2^2) \, (R_2 + R_1') + \frac{P_0}{3} \qquad \mathbf{(232)}$$

Of the two quantities a_2 and b_2, b_2 alone is arbitrary and depends upon the excitation, a_2 will depend upon the value of b_2 and also upon the losses. Solving therefore for a_2 in terms of b_2, we have

$$a_2 = \frac{E_s}{2 \sqrt{3} \, (R_2 + R_1')} \left\{ 1 - \sqrt{1 - \frac{4 \, (R_2 + R_1') \, \{3 \, b_2^2 \, (R_2 + R_1') + P_0\}}{E_s^2}} \right\} \qquad \mathbf{(233)}$$

Since b_2 is arbitrary we may now determine $\cos \alpha_2 = \frac{a_2}{\sqrt{a_2^2 + b_2^2}}$ and the value of $\check{I}_{a2}$ in terms of the impressed e. m. f. will be by **(181)** of Section on Synchronous Motors

$$S^2 \, \check{I}_{a2} = - S^2 \left[j \frac{\check{E}_s}{\sqrt{3}} \, \frac{\cos \alpha_2}{2 \, (R_2 + R_1')} \left\{ 1 - \sqrt{1 - \frac{4 \, (R_2 + R_1') \, P_0}{E_s^2 \cos^2 \alpha_2}} \right\} \epsilon^{j \alpha} \right] \qquad \mathbf{(234)}$$

The effective value of $\check{I}_{a2}$ in terms of the effective value of $\check{E}_s$ will then be

$$I_{a2} = \frac{E_s}{\sqrt{3}} \, \frac{\cos \alpha_2}{2 \, (R_2 + R_1')} \left\{ 1 - \sqrt{1 - \frac{4 \, (R_2 + R_1') \, P_0}{E_s^2 \cos^2 \alpha_2}} \right\} \qquad \mathbf{(235)}$$

and since the component of the e.m.f. generated in phase with the current is determined only by the magnitude of $\check{I}_{a2}$ and the motor losses, if we define its value by A_2' the quadrature component being B_2' we shall have

$$A_2' = \frac{E_s}{\sqrt{3}} \frac{\cos \alpha_2}{2} \left(1 + \sqrt{1 - \frac{4 (R_2 + R_1') P_0}{E_s^2 \cos^2 \alpha_2}}\right) \tag{236}$$

and

$$B_2' = - \frac{E_s}{\sqrt{3}} \sin \alpha_2 - \frac{w (L_2 + L_2')}{A_2^1} \tag{237}$$

$$= - \frac{E_s}{\sqrt{3}} \Bigg\{ \sin \alpha_2 + \frac{3 w (L_2 + L_1')}{P_0} \cdot \frac{\cos \alpha_2}{2 (R_2 + R_1')} \left(1 - \sqrt{1 - \frac{4 (R_2 + R_1') P_0}{E_s^2 \cos^2 \alpha_2}}\right) \Bigg\} \tag{238}$$

and therefore we have

$$\check{E}_2' = -j \frac{E_s}{\sqrt{3}} \Bigg[\frac{\cos \alpha_2}{2} \left(1 + \sqrt{1 - \frac{4 (R_2 + R_1') P_0}{E_s^2 \cos^2 \alpha_2}}\right) - j \Bigg\{ \sin \alpha_2 + \frac{3 w (L_2 + L_1') \cos \alpha_2}{2 P_0 (R_2 + R_1')} \left(1 - \sqrt{1 - \frac{4 (R_2 + R_1') P_0}{E_s^2 \cos^2 \alpha_2}}\right) \Bigg\} \Bigg] (\epsilon^{jwt}) \tag{239}$$

The impedance of the phase converter to the flow of negative phase sequence current is

$$\frac{2 (R_2 + R_1') \sec \alpha}{1 - \sqrt{1 - \frac{4 (R_2 + R_1') P_0}{E_s^2 \cos^2 \alpha}}} \tag{240}$$

The balance will be at its best when $\check{I}_{a2}$ is a minimum with $\cos \alpha_2$ as the independent variable. This will be the case when $\cos \alpha_2$ is unity; that is to say when b_2 is zero.

Recapitulating the results given above, we have for the general case taking the single-phase e. m. f. $\check{E}_s$ as reference

$$S^1\,\check{I}_{a1} = S^1\,j\,\frac{\check{E}_s}{\sqrt{3}\,(Z_1 + Z_2')} \tag{241}$$

$$S^2\,\check{I}_{a2} = -\,j\,(a_2 + j\,b_2) \tag{242}$$

where b_2 is arbitrary and

$$a_2 = \frac{E_s}{2\sqrt{3}\,(R_2 + R_1')}\left\{1 - \sqrt{1 - \frac{4\,(R_2 + R_1')\,\{3\,b_2^2\,(R_2 + R_1') + P_0\}}{E_s^2}}\right\} \tag{243}$$

Since b_2 is arbitrary $\cos\alpha_2$ is determined by

$$\cos\alpha_2 = \frac{a_2}{\sqrt{a_2^2 + b_2^2}} \tag{244}$$

we may express $\check{I}_{a2}$ in terms of $\check{E}_s$ by

$$S^2\,\check{I}_{a2} = -\,S^2\,j\,\frac{\check{E}_s}{\sqrt{3}}\;\frac{\cos\alpha_2}{2\,(R_2 + R_1')}\left\{1 - \sqrt{1 - \frac{4\,(R_2 + R_1')\,P_0}{E_s^2\cos^2\alpha_2}}\right\}\epsilon^{j\,\alpha_2} \tag{245}$$

The effective value of $\check{I}_{a2}$ will be

$$I_{a2} = \sqrt{a_2^2 + b_2^2} = \frac{E_s}{\sqrt{3}}\;\frac{\cos\alpha_2}{2\,(R_2 + R_1')}\cdot\left\{1 - \sqrt{1 - \frac{4\,(R_2 + R_1')\,P_0}{E_s^2\,\cos^2\alpha_2}}\right\} \tag{246}$$

If A_2' and B_2' are components of $\check{E}_{a2}'$ these being the generated e. m. f. in phase and in quadrature with the current $\check{I}_{a2}$ we shall have

$$\check{E}_{a2}' = -\,j\,(A_2' + j\,B_2') \tag{247}$$

and A_2' and B_2' will have the following values

$$A_2' = \frac{E_s}{\sqrt{3}} \; \frac{\cos \alpha_2}{2} \left(1 + \sqrt{1 - \frac{4\,(R_2 + R_1')\,P_0}{E_s^2 \cos^2 \alpha_2}}\right) \tag{248}$$

$$\begin{aligned} B_2' = - \frac{E_s}{\sqrt{3}} \Biggl\{ \sin \alpha_2 &+ \\ &\frac{3\,w\,(L_2 + L_1')}{P_0} \cdot \frac{\cos \alpha_2}{2\,(R_2 + R_1')} \Biggl(1 - \\ &\sqrt{1 - \frac{4\,(R_2 + Q_1')\,P_0}{E_s^2 \cos \alpha^2}}\Biggr)\Biggr\} \end{aligned} \tag{249}$$

and $\check{E}_{a2}'$ expressed in terms of $\check{E}_s$ becomes

$$\begin{aligned} \check{E}_2' = - j \frac{\check{E}_s}{\sqrt{3}} \Biggl[\frac{\cos \alpha_2}{2} &\left(1 + \sqrt{1 - \frac{4\,(R_2 + R_1')\,P_0}{E_s^2 \cos^2 \alpha_2}}\right) - \\ j \Biggl\{ \sin \alpha_2 &+ \frac{3\,w\,(L_2 + L')\cos \alpha_2}{2\,P_0\,(R_2 + R_1')} \Biggl(1 - \\ &\sqrt{1 - \frac{4\,(R_2 + R_1')\,P_0}{E_s^2 \cos^2 \alpha_2}}\Biggr)\Biggr\}\Biggr] \end{aligned} \tag{250}$$

The effective impedance of the phase converter to the flow of negative phase sequence currents is

$$\frac{2\,(R_2 + R_1')\sec \alpha_2}{1 - \sqrt{1 - \frac{4\,(R_2 + R_1')\,P_0}{E_s^2 \cos^2 \alpha}}} \; (\cos \alpha_2 - j \sin \alpha_2) \tag{251}$$

or

$$\frac{E_s^2}{P_0} \cdot \frac{\cos \alpha_2}{2} \left(1 + \sqrt{1 - \frac{4\,(R_2 + R_1')\,P_0}{E_s^2 \cos^2 \alpha_2}}\right) \epsilon^{-j\,\alpha_2} \tag{252}$$

In the above equations $\cos \alpha_2$ is arbitrary or b_2 may be considered arbitrary and $\cos \alpha_2$ will then be determined.

Minimum Unbalance is obtained when $\cos \alpha_2$ is made unity or when b_2 is made zero in equations (**241**) and (**252**).

Perfect Balance is obtained by driving the phase converter mechanically so as to supply the mechanical power P_0 from a separate or symmetrical source. Under this condition a_2 and b_2 both become zero when $\cos \alpha_2$ is unity. The only equation of the system is then (**241**).

Currents and Power Factor in the Single-Phase Supply Circuit of Series Phase Converter.

The e. m. f. is $2\,\check{E}_s$ and the current supplied is

$$\check{I}_s = \frac{\check{I}_b - \check{I}_c}{2}$$

$$= \frac{\check{I}_{b1} - \check{I}_{c1}}{2} + \frac{\check{I}_{b2} - \check{I}_{c2}}{2} \tag{253}$$

If we take

$$S^1\,\check{I}_{a1} = S^1 j\,(a_1 - j\,b_1) \tag{254}$$

$$\frac{\check{I}_{b1} - \check{I}_{c1}}{2} = \frac{\sqrt{3}}{2}\,(a_1 - j\,b_1) \tag{255}$$

Similarly, since under the same conditions

$$S^2\,\check{I}_{a2} = -\,S^2 j\,(a_2 + j\,b_2) \tag{256}$$

$$\frac{\check{I}_{b2} - \check{I}_{c2}}{2} = \frac{\sqrt{3}}{2}\,(a_2 + j\,b_2) \tag{257}$$

and therefore

$$\check{I}_s = \frac{\sqrt{3}}{2}\,\{(a_1 + a_2) - j\,(b_1 - b_2)\} \tag{258}$$

where a_1, b_1, a_2, b_2 are to be obtained by means of equations (**243**) to (**254**). The single-phase power factor is given by

$$\tan\theta = \frac{b_1 - b_2}{a_1 + a_2} \tag{259}$$

of these quantities a_2 is usually the smallest and its value may be obtained approximately by assigning to b_2 a value which will make the ratio $\frac{b_1 - b_2}{a_1}$ equal to $\tan\theta$, and obtaining the corresponding value of a_2 by (**242**), the value of b_2 may then be recalculated from (**259**) by substituting the tentative value obtained for a_2. This procedure may be repeated until sufficient accuracy has been obtained.

Single-Phase Power Factor in Shunt-Type Phase Converter.

The simplest procedure is to obtain a curve of admittances for varying excitation of the converter and plot the power factor obtained by varying the admittance with a fixed load. The true and wattless power is obtained easily by means of (**208**) whether the system is balanced or unbalanced.

Figs. 14, 15, 16 and 17 are vector diagrams of several methods of using phase converters to supply a balanced 3-phase e. m. f. to a symmetrical load such as an induction motor. The diagrams are all based on a main machine having the same negative phase sequence impedance and the system in each case is delivering the same amount of power at the same voltage and 3-phase power factor without supplying any wattless power. It will be noted that the scheme Fig. 14 has the lowest single-phase power factor, Fig. 16 the highest and the rest arcing alike. It may be remarked, however, that with the shunt-type schemes adjustments can be made for power factor correction which will result also in better regulation.

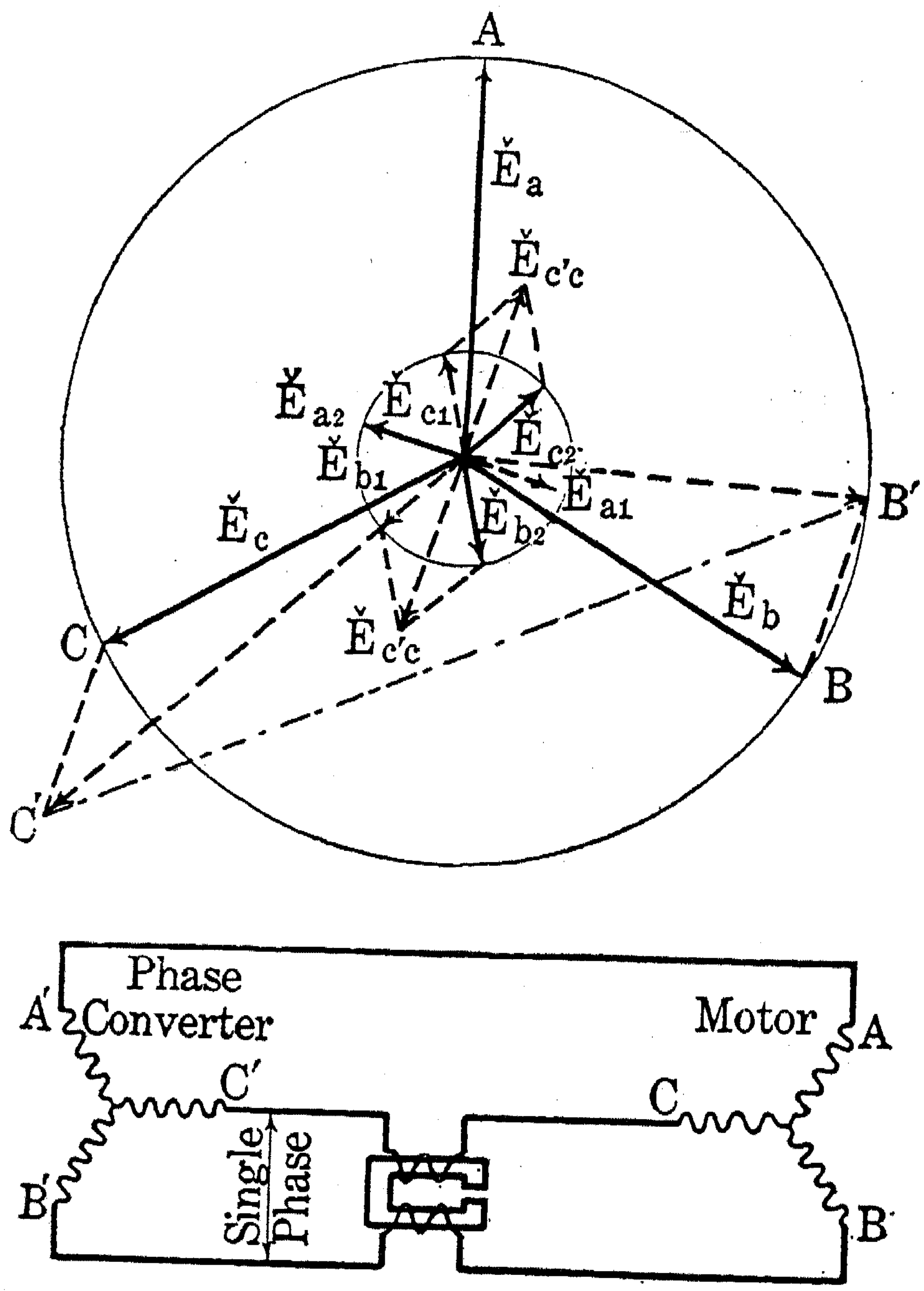

FIG. 14

SINGLE -PHASE IMPRESSED E.M.F. $= B'C'$
MOTOR E.M.F. $= BC$
NEGATIVE PHASE SEQUENCE E.M.FS. $\check{E}_{a2}\check{E}_{b2}\check{E}_{c2}$
CONJUGATE POSITIVE PHASE SEQUENCE E.M.FS. $\check{E}_{a1}\check{E}_{b1}\check{E}_{c1}$
PHASE CONVERTER TERMINAL E.M.F. $AB'C'$

APPENDIX I

Cylindrical Fields in Fourier Harmonics

When we have a diametrical coil around a cylinder concentric with another cylinder which forms the return magnetic path, and the length of the gap is uniform and the coil dimensions are very small, the field across the gap takes the form of a square topped wave, which may be expressed in the form of a Fourier series with the plane of symmetry of the coil as reference plane, and its Fourier expansion is

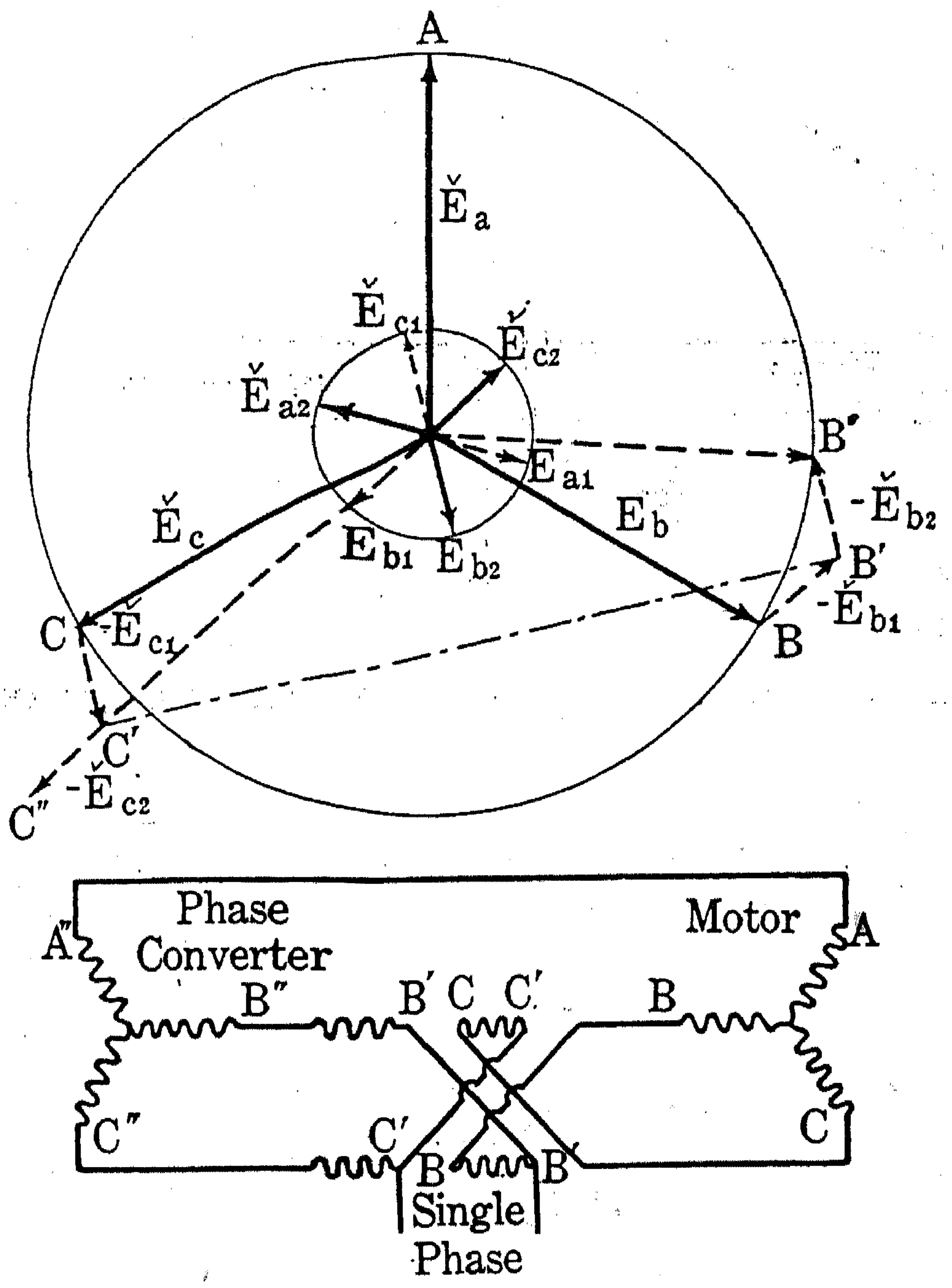

FIG. 15

SINGLE-PHASE IMPRESSED E.M.F. = $B'C'$
MOTOR E.M.F. = BC
PHASE CONVERTER E.M.F. = $B''C''$
NEGATIVE PHASE SEQUENCE E.M.F $\breve{E}_{a2}\breve{E}_{b2}\breve{E}_{c2}$
CONJUGATE POSITIVE PHASE SEQUENCE E.M.F. $\breve{E}_a{}^1E_{b1}E_{c1}$
PHASE CONVERTER TERMINAL E.M.F. $AB''C''$

$$\mathfrak{B} = \frac{4B}{\pi}\left(\cos\theta - \tfrac{1}{3}\cos 3\,\theta + \frac{1}{S}\cos S\,\theta - \,.\,.+\,.\,.\right) \qquad (1)$$

where B is the average induction in the air gap.

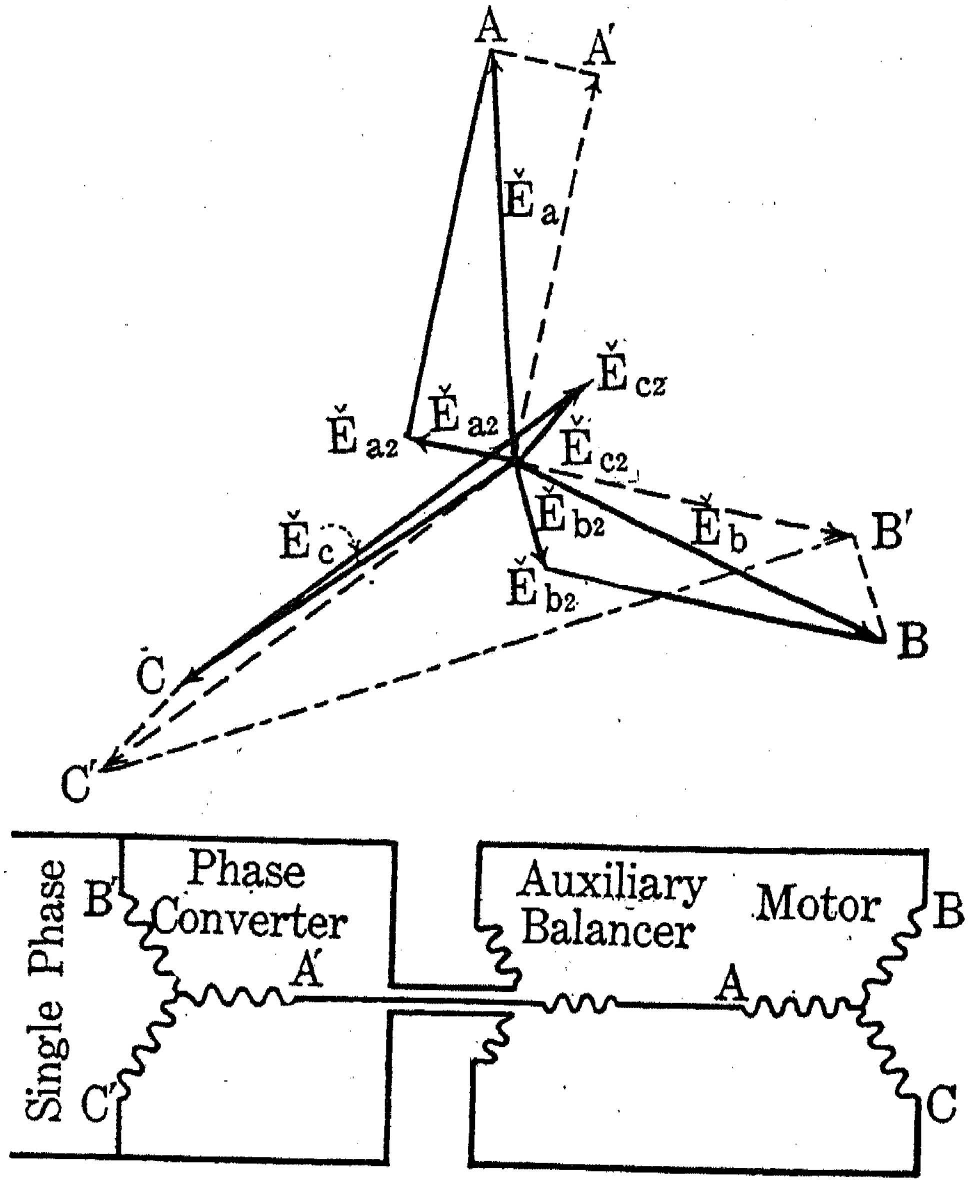

Fig. 16—Phase Converter with Auxiliary Balancer.

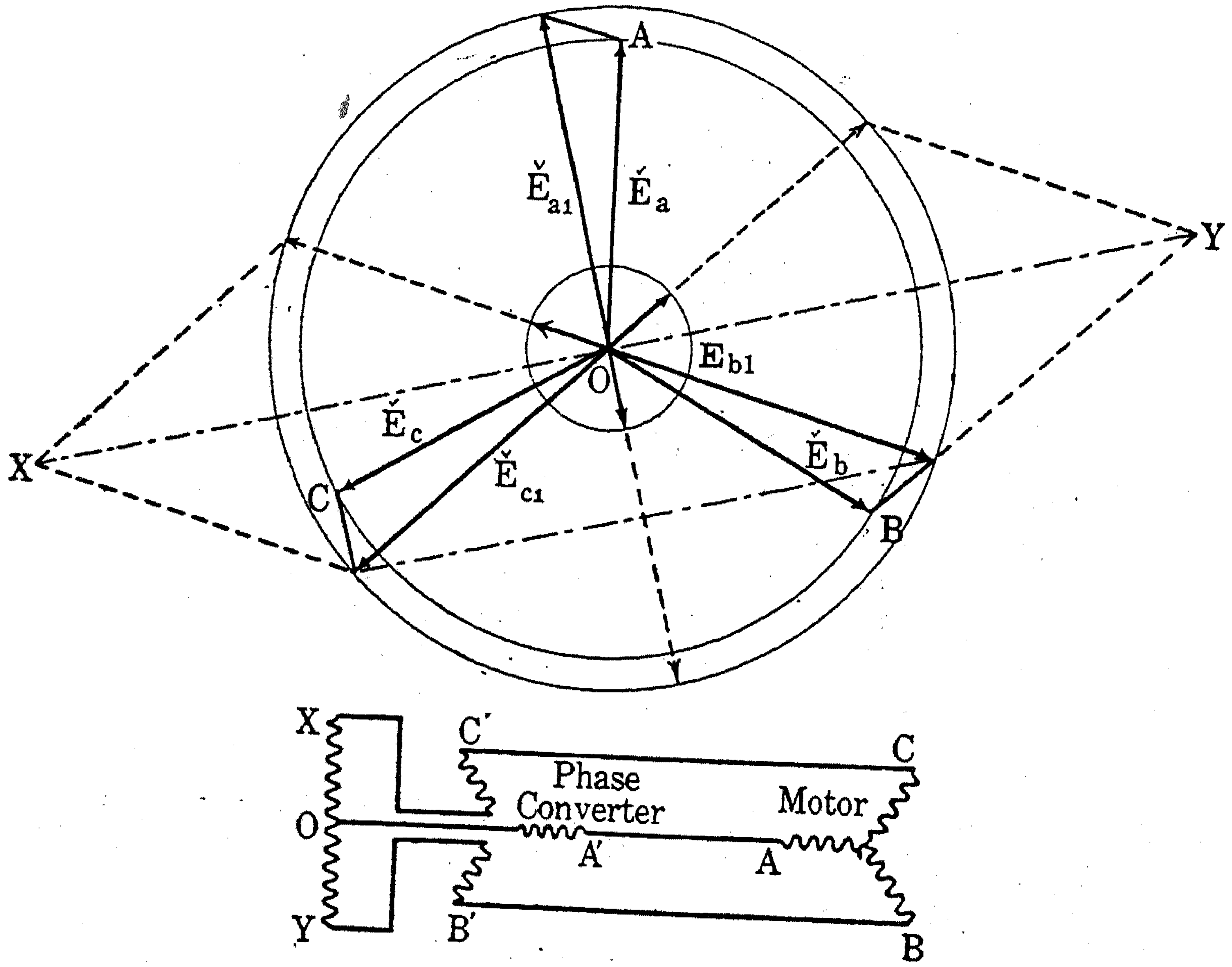

Fig. 17

Single-Phase Impressed e.m.f. = XY

Motor e.m.f. = ABC

There is a 2 to 1 Transformation of e.m.f. from Single-Phase to Three-Phase in This Connection

With pitch less than π the curve will have a different form, the amplitude being greater on one side of the plane of the coils than on the other, the areas of each wave will remain the same and second harmonic terms will appear. Let $2\, m_0\, \pi$ be the new pitch then the average amplitude of the induction will be the same as before, namely B, and the value on one side of the coil will be $2\,(1 - m_0)B$ and on the other side $2\, m_0\, B$ so that the total flux will be the same on either side. To obtain the values of the coefficients we have

$$2\,(1 - m_0 B \int_0^{m_0 \pi} \cos n\,\theta\, d\,\theta + 2\, m_0\, B \int_{m_0 \pi}^{2\,\pi} \cos n\,\theta\, d\,\theta = \frac{\pi}{2} A_n$$

$$2\,(1 - m_0)\, B \left[\frac{1}{n} \sin n\,\theta \right]_0^{m_0 \pi} - 2\, m_0\, B \left[\frac{1}{n} \sin n\,\theta \right]_{m_0 \pi}^{2 \pi} = \frac{\pi}{2} A_n$$

$$A_n = \frac{4\,B}{\pi} \left\{ \frac{(1 - m_0) + m_0}{n} \sin n\, m_0\, \pi \right\}$$

$$A_n = \frac{4\,B}{\pi} \left(\frac{1}{n} \sin n\, m_0\, \pi_1 \right) \tag{2}$$

Let $2\, m_0\, \pi = \frac{2}{3}\, \pi$, then $(1 - m_0)\, \pi = \frac{2}{3}\, \pi$ and

$$\mathcal{B} = \frac{2\,\sqrt{3}\,B}{\pi} \Big(\cos\theta + \frac{1}{2} \cos 2\,\theta - \frac{1}{4} \cos 4\,\theta - \frac{1}{5} \cos 5\,\theta$$

$$+ \frac{1}{7} \cos 7\,\theta + \frac{1}{8} \cos 8\,\theta - \frac{1}{10} \cos 10\,\theta\ .\ .\ . \Big) \tag{3}$$

A general expression for $\mathcal{B}$ where B is the average of the positive and negative, maximum value for any pitch coil would be

$$\mathcal{B} = \frac{4\,B}{\pi}\, \Sigma \left(\frac{1}{n} \sin n\, m_0\, \pi \cos n\,\theta \right) \tag{4}$$

and includes all possible coil pitches. If the number of teeth in a pole pitch be n_T; in addition to the average induction as indicated by (4), there will also be a tooth ripple of flux, the maximum value of which will depend upon the average value of the induction at each point. The value of m_0 must be a fraction having n_T as denominator and an integral numerator. The value of the integral numerator is therefore always $m_0 n_T$. The correct value for the max. induction will therefore be

$$\mathfrak{B}_m = \left\{ \frac{4B}{\pi} \Sigma \left(\frac{1}{n} \sin n\, m_0\, \pi \cos n\, \theta \right) \right\} (1 - (-1)^{m_0 n_T} K_T \cos n_T\, \theta) \qquad (5)$$

where K_T is the ratio of the average to the min. air gap. "m_0" must always be chosen so that $m_0\, n_T$ is an integer.

If the length of the average effective air gap in centimeters be d the value of B is given by

$$B = \frac{4\,\pi}{10} \; \frac{I\,N}{2\,d} \text{ gauss}$$

where I is the maximum value of the current in the coil and N is the number of turns. If d is given in inches we may write

$$B = \frac{4\,\pi}{10} \; \frac{I\,N}{2\,d} \times 2.54 \text{ maxwells per square inch.}$$

If we integrate (5) between the limits $(\theta - m_0\,\pi)$ and $(\theta + m_0\,\pi)$ we shall have the total flux φ through the coil

$$\varphi = \frac{4\,B\,r\,l}{\pi} \int_{\theta - m_0\,\pi}^{\theta + m_0\,\pi} \Sigma \left(\frac{1}{n} \sin n\, m_0\, \pi \cos n\, \theta \right) d\,\theta$$

$$- \frac{4\,B\,r\,l}{\pi} (-1)^{m_0\, n_T} \int_{\theta - m_0\, u}^{\theta + m_0\,\pi} \Sigma \left(\frac{1}{n} \sin n\, m_0 \cos n\, \theta \right) K_T \cos n_T\, \theta\, d\, 0$$

$$= \frac{4\,B\,re}{\pi} \left[\frac{1}{n^2} \sin n\, m_0\, \pi \sin n\, \theta \right]_{\theta - m_0\,\pi}^{\theta + m_0\,\pi}$$

$$- \frac{4\,B\,r\,l}{\pi} (-1)^{m_0\, n_T}\, K_T\, \Sigma \frac{1}{n} \sin m_0\, n\, \pi \left[\frac{\sin (n - n_T)\,\theta}{2\,(n - n_T)} + \frac{\sin (n + n_T\,\theta)}{2\,(n + n_T)} \right]_{\theta - m_0\,\pi}^{\theta + m_0\,\pi} \qquad (6)$$

The second expression is zero for all values of θ which are integral multiples of the tooth pitch angle, so long as $m_0\, n$ is also an integer and therefore it is zero for all mutual inductive relations of similar coils on a symmetrical toothed core we therefore have:

The induction through a coil displaced an angle θ from the axis of a similar coil carrying a current giving a mean induction B both coils being wound on the same symmetrical toothed core is

$$\varphi = \frac{8\,B\,r\,l}{\pi}\,\Sigma\left(\frac{1}{n^2}\sin^2 n\,m_0\,\pi\cos n\,\theta\right) \tag{7}$$

The second term in equation (6) also becomes zero when n_T becomes infinite independent of the value of θ. We may therefore safely make use of an imaginary uniformly distributed winding when considering self and mutual impedances. It will also be shown later on, that with certain groupings of windings the second term may be reduced to zero for every value of θ.

If N_1 be the total number of complete loops in one complete pole pitch, we may take $\frac{N_1}{2\,\pi}$ as the density of winding per unit angle of the complete pole pitch. The mutual induction per turn in a coil angularly displaced an angle θ from another coil of winding density $\frac{N_1}{2\,\pi}$ with an effective total air gap $2\,d$ and with windings subtending an angle $2\,m_1\,\pi$ is given by

$$M_1 = \frac{8\,N_1\,r\,l}{10^9\,\pi\,d}\int_{-m_1\,\pi}^{+m_1\,\pi}\Sigma\left\{\frac{1}{n^2}\sin^2 n\,m_0\,\pi\cos n(\theta+\theta')\right\}d\,\theta' \text{ henrys} \tag{8}$$

$$= \frac{8\,N_1\,r\,l}{10^9\,\pi\,d}\,\Sigma\,\frac{1}{n^3}\sin^2 n\,m_0\,\pi\,[\sin n\,(\theta+\theta)']_{\theta'=-m_1\,\pi}^{\theta'=m_1\,\pi} \text{ henrys}$$

$$M_1 = \frac{16\,N_1\,r\,l}{10^9\,\pi\,d}\,\Sigma\left(\frac{1}{n^3}\sin^2 n\,m_0\,\pi\sin n\,m_1\,\pi\cos n\,\theta\right) \text{ henrys} \tag{9}$$

Next, if the loop of which M_1 is the mutual inductance is part of a winding having distribution density of winding $\frac{N_2}{2\,\pi}$ and subtending an angle $2\,m_2\,\pi$ its mutual inductance with the other winding will be

$$M_{12} = \frac{8\,N_1\,N_2\,r\,l}{10^9\,\pi^2\,d}\int_{-m_2\,\pi}^{m_2\,\pi}\Sigma\frac{1}{n^3}\sin^2 n\,m_0\,\pi\sin n\,m_1\,\pi\cos n\,(\theta+\theta')\,d\,\theta' \text{ henry} \tag{10}$$

$$= \frac{8\,N_1\,N_2\,r\,l}{10^9\,\pi^2\,d}\Sigma\,\frac{1}{n^4}\sin^2 n\,m_0\,\pi\sin n\,m_1\,\pi\,[\sin n\,(\theta+\theta')]_{\theta'=-m_2\,\pi}^{\theta'=m_2\,\pi} \text{ henrys}$$

$$M_{12} = \frac{16\, N_1\, N_2\, r\, l}{10^9\, \pi^2\, d}\, \Sigma \left(\frac{1}{n^4} \sin^2 n\, m_0\, \pi \sin n\, m_1\, \pi \right.$$

$$\left. \sin n\, m_2\, \pi \cos n\, \theta \right) \text{ henrys} \qquad \textbf{(11)}$$

This is the general expression for the mutual inductance between two groups of connected coils of like form on the same cylindrical core. It should be noted how much the harmonics have been reduced due to grouping.

When the coils are not of like design as in the case of a rotor and stator and the pitch of the coils is different in one from the other, sin $n\, m_0\, \pi$ will not appear twice in the equation but one of its values must be replaced by sin $n\, m_x\, \pi$ where $2\, m_x\, \pi$ is the pitch of the new coil. Equation (**11**) then becomes

$$M_{1a} = \frac{16\, N_1\, N_a\, r\, e}{10^9\, \pi^2\, d}\, \Sigma \left(\frac{1}{n^4} \sin n\, m_0\, \pi \sin n\, m_x\, \pi \right.$$

$$\left. \sin n\, m_1\, \pi \sin n\, m_2\, \pi \cos n\, \theta \right) \text{ henrys} \qquad \textbf{(12)}$$

This formula is strictly correct when m_x is an integer and when θ is an integral multiple of the tooth pitch. It is true for all values of θ if either m_0 or m_x or both are unity.

By considering the axes of two similar groups of coils as coincident we obtain the value of $\Delta_1\, L_1$ which is part of the self inductance of the group, thus

$$\Delta_1\, L_1 = \frac{16\, N_1^2\, r\, e}{10^9\, \pi^2\, d}\, \Sigma \left(\frac{1}{n^4} \sin^2 n\, m_0\, \pi \sin^2 n\, m_1\, \pi \right) \qquad \textbf{(13)}$$

The other factor that enters into the self inductance is the slot leakage inductance which depends upon the number of turns in a coil, the number of coils in a group and the width and depth of the slot and the length of the air gap. Since with the value of $\Delta_1\, L_1$ all the field which links the secondary winding has been included, only the portion of the slot leakage which does not link all the turns in the opposed secondary coil should be considered. No hard and fast rule can be made for determining this quantity since it depends upon the shape of the slots, there should be little trouble in making the calculation when the data are given. Denoting this quantity by $\Delta_2\, L_1$ we have

$$L_1 = \Delta_1\, L_1 + \Delta_2\, L_1 \qquad \textbf{(14)}$$

Symmetrically Grouped Windings. The above formulas give the mutual impedance between groups of coils, each group of which may be unsymmetrical. Generally machines are designed so that, although the individual groups of coils due to fractional pitch may be unsymmetrical, the complete winding is symmetrical. When two coils are together in a slot this may be done by connecting one group of coils opposite the north pole in series with the corresponding group opposite the south pole; that is to say, the group displaced electrically by the angle π. If therefore we take equation (**11**) and consider the mutual induction as due to a group having axis at θ = zero and another having its axis at $\theta = \pi$ with a similarly arranged group of coils having its axis at θ, we find that (**11**) becomes

$$M_{12} = \frac{16\, N_1\, N_2\, r\, l}{10^9\, \pi^2\, d}\, \Sigma \left\{ \frac{1}{n^4} \sin^2 n\, m_0\, \pi \sin n\, m_1\, \pi \sin n\, m_2\, \pi\, (1 - \cos n\, \pi)^2 \cos n\, \theta \right\} \text{henrys} \quad (\mathbf{15})$$

Similarly

$$M_{1a} = \frac{16\, N_1\, N_a\, r\, l}{10^9\, \pi^2\, d}\, \Sigma \left\{ \frac{1}{n^4} \sin n\, m_1\, \pi \sin n\, m_x\, \pi \sin n\, m_1\, \pi \sin n\, m_a\, \pi\, (1 - \cos n\, \pi)^2 \cos n\, \theta \right\} \text{henrys} \quad (\mathbf{16})$$

Since $1 - \cos n\pi$ is zero for all even values of n it is evident that (**15**) and (**16**) contain no even harmonics, moreover the above formulas give the mutual induction between two similarly connected groups of windings, but if $(1 - \cos n\, \pi)$ is used only with the first power these formulas give the mutual impedance between one pair of such symmetrically grouped windings and another single group with axis inclined at an angle θ.

The value of self induction is

$$\Delta_1 L_1 = \frac{16\, N_1^2\, r\, l}{10^9\, \pi^2\, d}\, \Sigma \left\{ \frac{1}{n^4} \sin^2 n\, m_0\, \pi \sin^2 n\, m_1\, \pi\, (1 - \cos n\, \pi)^2 \right\} \quad (\mathbf{17})$$

$\Delta_2\, L_1$ is found in the same manner as before

$$L_1 = \Delta_1\, L_1 + \Delta_2\, L_1 \quad (\mathbf{18})$$

It is obvious from (**15**) and (**16**) that the effect of dissymmetry is to introduce more or less double frequency into the wave form of generated e. m. f.

It will be seen from an examination of (**15**) and (**17**) that, for example, a winding of pitch $\frac{2\pi}{3}$ and subtending an angle $\frac{\pi}{3}$ when connected in a symmetrical group of two has the same field form and characteristics as a full pitch winding of the same number of turns subtending an angle $\frac{2\pi}{3}$.

There are many symmetrical forms of winding but all will be found to be covered by the formulas (**15**) and (**16**).

Unsymmetrical Windings. These may take many forms which may be classified:

(1) Dissymmetry of flux form due to even harmonics.

(2) Dissymmetry in axial position of polyphase groups.

(3) Dissymmetry in windings due to incorrect grouping of coils.

(4) Dissymmetry due to unsymmetrical magnetic characteristics of the iron.

Of these various forms of dissymmetry the most common is a combination of (**1**), (**2**) and (**3**). These forms of unsymmetrical windings may all be calculated by the formulas (**11**) to (**16**).

It is to be noted that the mutual inductance between a symmetrical and an unsymmetrical winding is harmonically symmetrical. Hence, if the field of a machine is harmonically symmetrical, the e. m. f. generated will be also harmonically symmetrical whatever may be the form of the windings.

The reciprocal nature of M is fully established by its form, for it is immaterial in obtaining (**16**) whether we start out with the winding whose pitch is m_x or with that whose pitch is m_0, the result will be the same. The effect of saturation will be to tend to alter the values of the coefficients of M but the general form will not vary appreciably. We shall now consider some standard windings of generators and motors.

Three-Phase Symmetrical Full Pitch. Here m_0, m_1 and m_2 are 0.5, 0.1666 and 0.1666 respectively. Using formula (**15**) all the even harmonics disappear and $(1 - \cos n\,\pi)^2$ is equal to 4 or zero.

$$M_{12} = \frac{16\, N_1\, N_2\, r\, l}{10^9\ \pi^2\, d}\left(\cos\theta + \frac{4}{81}\cos 3\,\theta + \frac{1}{625}\cos 5\,\theta + \frac{1}{2401}\cos 7\,\theta + \frac{4}{6561}\cos 9\,\theta + \ldots\right) \qquad \textbf{(19)}$$

Theoretical Symmetrical Three-Phase Winding. Here $m_0 = 0.5, m_1 = m_2 = 0.333$. Using formula (**11**)

$$M_{12} = \frac{3}{4}\,\frac{16\,N_1\,N_2\,r\,l}{10^9\,\pi^2\,d}\left(\cos\theta + \frac{1}{625}\cos 5\,\theta + \frac{1}{2401}\cos 7\,\theta + \frac{1}{14641}\cos 11\,\theta + \ldots\right) \qquad (\mathbf{20})$$

Here the third group of harmonics is entirely eliminated.

Three-Phase Symmetrical $\frac{2\,\pi}{3}$ Pitch Winding. Here $m_0 = 0.333, m_1 = m_2 = 0.166$. Using formula (**15**)

$$M = \frac{3}{4}\,\frac{16\,N_1\,N_2\,r\,l}{10^9\,\pi^2\,a}\left(\cos\theta + \frac{1}{625}\cos 5\,\theta + \frac{1}{2461}\cos 7\,\theta + \frac{1}{14641}\cos 11\,\theta + \ldots.\right) \qquad (\mathbf{21})$$

which gives the same result as (**20**).

FORMULAS FOR SALIENT POLE MACHINES

The formulas given in the preceding discussion are appropriate for distributed winding and non-salient poles. Where salient poles are used the field form due to the poles with a given winding will be arbitrary so that with the polar axis as reference we shall have

$$\mathcal{B} = \frac{2\,\pi\,N_a\,I_a}{d}\,\Sigma\,(A_n\cos n\,\theta) \qquad (\mathbf{22})$$

Where $\mathcal{B}$ is the induction through the armature or stator. When the poles are symmetrical $A_n \cos n\,\theta$ might be chosen at once for this condition and in this case we do not require coefficients of mutual induction between pole windings, since the value of $\mathcal{B}$ is obtained by considering the mutual reaction between pole windings to be such as will produce symmetry. We may however assume $\mathcal{B}$ to be perfectly general in form in which case the flux through a coil of pitch $2\,m_0\,\pi$ is

$$\varphi = \frac{4\,\pi\,N_a\,I_a\,r\,l}{10\,d}\,\Sigma\left(\frac{A_n}{n}\sin n\,m_0\,\pi\cos n\,\theta\right) \qquad (\mathbf{23})$$

We have therefore for the mutual induction between one pole and a group of coils at an angle θ and subtending an angle $2\,m_1\,\pi$

$$M_{a1} = \frac{4\, N_a\, N_1\, r\, l}{10^9\, d}\, \Sigma \left(\frac{A_n}{n^2} \sin n\, m_0\, \pi \sin n\, m_1\, \pi \cos n\, \theta \right) \qquad \textbf{(24)}$$

and where there is symmetry due to grouping of windings, we have

$$M_{a1} = \frac{4\, N_a\, N_1\, r\, l}{10^9\, d}\, \Sigma \left\{ \frac{A_n}{n^2} \sin n\, m_0\, \pi \sin n\, m_1\, \pi \right.$$

$$\left. (1 - \cos n\, \pi)^2 \cos n\, \theta \right\} \qquad \textbf{(25)}$$

where N_a is the number of turns for one pole and (**25**) applies to one pair of poles and the corresponding group of coils. When there are more than one pair of poles in series and the corresponding groups of winding are also in series, if it is desired to consider the mutual inductance of the complete winding, the result given above must be multiplied by the number of pairs of poles.

If in equation (**16**) we take

$$\left. \begin{aligned} &\frac{N_a}{2\,\pi}\, \frac{1}{n} \sin n\, m_a\, \pi = N_a \\ \text{and} \qquad &\frac{1}{\pi\, n} = B_n \end{aligned} \right\} \qquad \textbf{(26)}$$

it becomes

$$M_{1a} = \frac{32\, N_1\, N_a\, r\, l}{10^9\, d}\, \Sigma \left\{ \frac{B_n}{n^2} \sin n\, m_x\, \pi \sin n\, m_0\, \pi \right.$$

$$\left. \sin n\, m_1\, \pi\, (1 - \cos n\, \pi)^2 \cos n\, \theta \right\} \qquad \textbf{(27)}$$

which is the expression corresponding to (**25**) starting with the winding flux form. (**25**) and (**27**) must therefore be identical and we have

$$\frac{32\, N_1\, N_a\, r\, e}{10^9\, d}\, B_n \sin n\, m_x\, \pi - \frac{4\, N_a\, N_1\, r\, e}{10^9\, d}\, A_n$$

or

$$\mathcal{B}_n = \frac{A_u}{8 \sin n\, m_x\, \pi} \qquad \textbf{(28)}$$

and

$$\mathcal{B}_1 = \frac{2\,\pi\, I_1}{10\, d}\, \Sigma\, (B_n \sin n\, m_0\, \pi \cos n\, \theta) \qquad \textbf{(29)}$$

and is the induction wave form for a single turn of the winding.

The expression for the mutual inductance between windings of the same core for salient poles is obtained in terms of the pole flux wave form by substituting in the formulas $\frac{A_n}{8 \sin n\, m_x\, \pi}$ for $\frac{1}{n\, \pi}$. We have therefore the following formulas for salient poles.

General expression considering only one pole and one group of coils.

$$\mathfrak{B}_a = \frac{2\,\pi\, N_a\, I_a}{10\, d}\, \Sigma\, (A_n \cos n\, \theta) \qquad \text{(a)}$$

$$\mathfrak{B}_1 = \frac{\pi\, I_1}{20\, d}\, \Sigma \left(A_n \frac{\sin n\, m_0\, \pi}{\sin n\, m_a\, \pi} \cos n\, \theta\right) \qquad \text{(b)}$$

$$M_{a1} = \frac{4\, N_a N_1\, r\, l}{10^9\, d}\, \Sigma \left(\frac{A_n}{n^2} \sin n\, m_0\, \pi \sin n\, m_1\, \pi \cos n\, \theta\right) \qquad \text{(c)}$$

$$M_{12} = \frac{2\, N_1\, N_2\, r\, l}{10^9\, \pi\, d}\, \Sigma \left(\frac{A_n}{n^3}\, \frac{\sin n\, m_0\, \pi}{\sin n\, m_x\, \pi} \sin n\, m_0\, \pi \sin n\, m_1\, \pi \right.$$
$$\left. \sin n\, m_2\, \pi \cos n\, \theta\right) \qquad \text{(d)}$$

$$\Delta_1\, L_a = \frac{4\,\pi\, N_a^2\, r\, l}{10^9\, d}\, \Sigma \left(\frac{A_n}{n} \sin n\, m_x\, \pi\right) \qquad \text{(e)}$$

$$\Delta_1\, L_1 = \frac{2\, N_1^2\, r\, l}{10^9\, \pi\, d}\, \Sigma \left(\frac{A_n}{n}\, \frac{\sin^2 n\, m_0\, \pi \sin^2 n\, m_1\, \pi}{\sin n\, m_x\, \pi}\right) \qquad \text{(f)}$$

General expressions considering only poles to be symmetrical. Considered on the basis of two poles, N_a being turns on one pole.

$$\mathfrak{B}_a = \frac{2\,\pi\, N_a\, I_a}{10\, d}\, \Sigma\, \{A_n\, (1 - \cos n\, \pi) \cos n\, \theta\} \qquad \text{(a}'\text{)}$$

$$\mathfrak{B}_1 = \frac{\pi\, I_1}{20\, d}\, \Sigma \left\{A_n\, \frac{\sin n\, m_0\, \pi}{\sin n\, m_x\, \pi}\, (1 - \cos n\, \pi) \cos n\, \theta\right\} \qquad \text{(b}'\text{)}$$

$$M_{a1} = \frac{4\, N_a\, N_1\, r\, l}{10^9\, d}\, \Sigma \left\{\frac{A_n}{n^2} \sin n\, m_0\, \pi \sin n\, m_1\, \pi \right.$$
$$\left. (1 - \cos n\, \pi) \cos n\, \theta\right\} \qquad \text{(c}'\text{)}$$

$$M_{12} = \frac{2\,N_1\,N_2\,r\,l}{10^9\,\pi\,d}\,\Sigma\left\{\frac{A_n}{n^3}\;\frac{\sin n\,m_0\,\pi}{\sin n\,m_x\,\pi}\,\sin n\,m_0\,\pi\,\sin n\,m_1\,\pi\right.$$

$$\left.\sin n\,m_2\,\pi\,\cos n\,\theta\right\}\text{henrys} \qquad (\mathbf{d}')$$

$$\Delta_1\,L_a = \frac{4\,\pi\,N_a^2\,r\,l}{10^9\,d}\,\Sigma\left\{\frac{A_n}{n}\,\sin n\,m_x\,\pi\,(1-\cos n\,\pi)\right\} \qquad (\mathbf{e}')$$

$$\Delta_1\,L_1 = \frac{2\,N_1^2\,r\,l}{10^9\,\pi\,d}\,\Sigma\left\{\frac{A_n}{n}\;\frac{\sin^2 n\,m_0\,\pi\,\sin^2 n\,m_1\,\pi}{\sin n\,m_x\,\pi}\right\} \qquad (\mathbf{f}')$$

General expression with both polar and winding symmetry.

$$\mathcal{B}_a = \frac{2\,\pi\,N_a\,I_a}{10\,d}\,\Sigma\,\{A_n\,(1-\cos n\,\pi)\cos n\,\theta) \qquad (\mathbf{a}'')$$

$$\mathcal{B}_1 = \frac{\pi\,I_1}{20\,d}\,\Sigma\left\{A_n\,\frac{\sin n\,m_0\,\pi}{\sin n\,m_x\,\pi}\,(1-\cos n\,\pi)\cos n\,\theta\right\} \qquad (\mathbf{b}'')$$

$$M_{a1} = \frac{4\,N_a\,N_1\,r\,l}{10^9\,d}\,\Sigma\left\{\frac{A_n}{n^2}\,\sin n\,m_0\,\pi\,\sin n\,m_1\,\pi\,(1-\cos n\,\pi)^2\right.$$

$$\left.\cos n\,\theta\right\} \qquad (\mathbf{c}'')$$

$$M_{12} = \frac{2\,N_1\,N_2\,r\,l}{10^9\,\pi\,d}\,\Sigma\left\{\frac{A_n}{n^3}\;\frac{\sin n\,m_0\,\pi}{\sin n\,m_x\,\pi}\,\sin n\,m_0\,\pi\,\sin n\,m_1\,\pi\right.$$

$$\left.\sin n\,m_2\,\pi\;(1-\cos n\,\pi)^2\cos n\,\theta\right\} \qquad (\mathbf{d}'')$$

$$\Delta_1\,L_a = \frac{4\,\pi\,N_a^2\,r\,l}{10^9\,d}\,\Sigma\left\{\frac{A_n}{n}\,\sin n\,m_x\,\pi\,(1-\cos n\,\pi)^2\right\} \qquad (\mathbf{e}'')$$

$$\Delta_1\,L_1 = \frac{2\,N_1^2\,r\,l}{10^9\,\pi\,d}\,\Sigma\left\{\frac{A_n}{n}\;\frac{\sin^2 n\,m_0\,\pi\,\sin^2 n\,m_1\,\pi}{\sin n\,m_x\,\pi}\right.$$

$$\left.(1-\cos n\,\pi)^2\right\} \qquad (\mathbf{f}'')$$

In using any of the formulas given above for machines having more than two poles, it must be divided by the number of pairs of poles and likewise the expression for M or $\Delta_1\,L$ must be multiplied by the number of pairs of poles, which leaves the formula for these quantities unchanged.

Let us next consider the actual induction in the air gap with a distributed winding operating with three-phase currents. Let i_{m1} be the magnetizing current of the first phase i_{m2} and i_{m3} those of the other phases. The induction due to one group of coils of phase 1 is

$$\mathfrak{B}_1 = \frac{8\, N_1\, i_{m1}}{10\, \pi\, d} \Sigma \left\{ \frac{1}{n^2} \sin n\, m_0\, \pi \sin n\, m_1\, \pi \cos n\, \theta \right\} \tag{30}$$

and if the phase displacement of 2 and 3 from 1 be φ_{12} and φ_{13}

$$\mathfrak{B}_2 = \frac{8\, N_2\, i_{m2}}{10\, \pi\, d} \Sigma \left\{ \frac{1}{n^2} \sin n\, m_0\, \pi \sin n\, m_2\, \pi \cos (n\theta - \varphi_{12}) \right\} \tag{31}$$

$$\mathfrak{B}_3 = \frac{8\, N_3\, i_{m3}}{10\, \pi\, d} \Sigma \left\{ \frac{1}{n^2} \sin n\, m_0\, \pi \sin n\, m_2\, \pi \cos (n\, \theta - \varphi_{13}) \right\} \tag{32}$$

For symmetrically grouped coils the formulas become

$$\mathfrak{B}_1 = \frac{8\, N_1\, i_{m1}}{10\, \pi\, d} \Sigma \left\{ \frac{1}{n^2} \sin n\, m_0\, \pi \sin n\, m_1\, \pi\, (1 - \cos n\, \pi) \cos n\, \theta \right\} \tag{33}$$

$$\mathfrak{B}_2 = \frac{8\, N_2\, i_{m2}}{10\, \pi\, d} \Sigma \left\{ \frac{1}{n^2} \sin n\, m_0\, \pi \sin n\, m_2\, \pi (1 - \cos n\, \pi) \cos m\, (\theta - \phi_{12}) \right\} \tag{34}$$

$$\mathfrak{B}_3 = \frac{8\, N_3\, i_{m3}}{10\, \pi\, d} \Sigma \left\{ \frac{1}{n^2} \sin n\, m_0\, \pi \sin n\, m_3\, \pi\, (1 - \cos n\, \pi) \cos m\, (\theta - \varphi_{13}) \right\} \tag{35}$$

For a symmetrical three-phase motor with full pitch coils $m_0 = 0.5$, $m_1 = m_2 = m_3 = 0.166$ (**33**), (**39**) and (**35**) become of the four

$$\mathfrak{B}_1 = \frac{8\, N_1\, i_{m1}}{10\, \pi\, d} \left\{ \cos \theta - \frac{2}{9} \cos 3\, \theta + \frac{1}{25} \cos 5\, \theta + \frac{1}{49} \cos 7\, \theta - \frac{2}{81} \cos 9\, \theta + \frac{1}{121} \cos 11\, \theta + \frac{1}{169} \cos 13\, \theta + \right\} \tag{36}$$

which is the field due to one group of coils alone. The wave is flattened by the third group of harmonics but all the other harmonics are peaking values. There is therefore a decided gain in such a wave form of flux since it permits of high fundamental flux density.

The maximum value of flux is approximately

$$B_{max} = 0.823 \cdot \frac{8\, N_1\, i_m}{10\, \pi\, d} \text{ gaus} \tag{37}$$

where d is given in centimeters.

$$B_{max} = \frac{1.67\, N_1\, i_m'}{\pi\, d} \quad \text{maxwells per square inch,}$$

with d given in inches.

For the total winding the resultant induction will be the sum of B_1, B_2, and B. If we take the symmetrical winding with angles between planes of symmetry

$\varphi_{12} = \frac{2\,\pi}{3}$ and $\varphi_{13} = \frac{4\,\pi}{3}$, we have

$$\left.\begin{aligned} \cos n\,\theta &= \frac{\epsilon^{jn\theta}}{2} + \frac{\epsilon^{-jn\theta}}{2} \\ \cos n\left(\theta - \frac{2\,\pi}{3}\right) &= a^{-n}\,\frac{\epsilon^{jn\theta}}{2} + a^{n}\,\frac{\epsilon^{-jn\theta}}{2} \\ \cos n\left(\theta - \frac{4\,\pi}{3}\right) &= a^{n}\,\frac{\epsilon^{jn\theta}}{2} + a^{-n}\,\frac{\epsilon^{-jn\theta}}{2} \end{aligned}\right\} \tag{38}$$

If we multiply these three quantities successively by $\check{I}_{m1}$, $a^2\check{I}_{m1}$, $a\check{I}_{m1}$ and add, we have

$$\check{I}_{m1}\left\{\frac{\epsilon^{jn\theta}}{2}\,(1 + a^{-(n-2)} + a^{(n+1)})\left(+ \frac{\epsilon^{-jn\theta}}{2} \times (1 + a^{n+2} + a^{-(n-1)})\right)\right\} \tag{39}$$

and giving n successive odd values from 1 up, we find for (**39**) the following values

$n =$ 1 (**39**) becomes $\frac{3}{2}\,\check{I}_{m1}\,\epsilon^{-j\theta}$

$n =$ 3 " " 0

$n = 5$ " " $\frac{3}{2}\, \check{I}_{m1}\, \epsilon^{j\,5\,\theta}$

$n = 7$ " " $\frac{3}{2}\, \check{I}_{m1}\, \epsilon^{-j\,7\,\theta}$

$n = 7$ " " 0

$n = 11$ " " $\frac{3}{2}\, \check{I}_{m1}\, \epsilon^{j\,11\,\theta}$

$n = n$ " " $2\, \check{I}_{m1} \sin^2 \frac{2\,n\,\pi}{3}\, \epsilon^{-j \frac{2}{\sqrt{3}} \sin \frac{2\,n\,\pi}{3}\, n\,\theta}$

We may therefore express $\mathcal{B}$ by
$\mathcal{B}$ = real part of

$$\frac{16\, N_1\, \check{I}_{m1}}{10\,\pi\, d}\, \Sigma \left\{ \frac{1}{n^2} \sin n\, m_0\, \pi \sin n\, m_1\, \pi\, (1 - \cos n\, \pi) \right.$$
$$\left. \times \sin^2 \frac{2\,n\,\pi}{3}\, \epsilon^{-j \frac{2}{\sqrt{3}} \sin \frac{2\,n\,\pi}{3}\, n\,\theta} \right\} \tag{40}$$

It will be obvious that if we proceed around the cylinder in the negative direction of rotation at an angular speed w and $\check{I}_{m1}$ is equal to $I_{m1}\, \epsilon^{jwt}$, for $n = 1$ the value of B_1 will remain constant and real, hence B_1 must be a constant field rotating at angular velocity w in the negative direction. The value of B may be expressed in harmonic form, but in this form it does not illustrate the rotating field theory so aptly. The harmonic form is given below and is simpler in appearance than (**40**).

$$\mathcal{B} = \frac{16\, N_1\, i_{m1}}{10\,\pi\, d}\, \Sigma \left(\frac{1}{n^2} \sin n\, m_0\, \pi \sin n\, m_1\, \pi\, (1 - \cos n\, \pi) \right.$$
$$\left. \sin^2 \frac{2\,n\,\pi}{3} \cos n\,\theta \right) \tag{41}$$

For a symmetrical three-phase motor with full pitch coil ($m_0 = 0.5\; m_1 = 0.166$) $\mathcal{B}$ becomes

$$\mathcal{B} = \frac{12\, N_1\, i_{m1}}{10\,\pi\, d}\, \Sigma \left\{ \cos\theta + \frac{1}{25} \cos 5\,\theta + \frac{1}{49} \cos 7\,\theta \right.$$
$$\left. + \frac{1}{121} \cos 11\,\theta + \frac{1}{169} \cos 13\,\theta + \;.\;.\; \right\} \tag{42}$$

This gives for the maximum induction approximately

$$\mathcal{B}_{max} = \frac{1.075 \times 12\, N_1\, i_{m1}}{10\, \pi\, d} = \frac{1.29\, N_1\, i_m}{\pi\, d} \text{ gauss} \qquad (43)$$

where d is measured in centimeters.

$$\mathcal{B}_{max} = \frac{3.28 \times N_1\, i_{m1}}{\pi\, d} \text{ maxwell per square inch} \qquad (44)$$

where d is measured in inches and N is the total number of turns per pair of poles.

APPENDIX II

Graphical Construction for Obtaining Symmetrical Components

The graphical method for finding the symmetrical components of $S(\check{E}_a)$ given in the text serves as a geometrical interpretation of equation (**5**), but the graphical method shown in Fig. 18 is much simpler and more convenient, the construction is as follows. Find E and F the centroids of the two equilateral triangles with $B\,C$ as base: With 0 the centroid of triangle $A\,B\,C$ as centre describe the two circles passing through E and F then; $E\,O$ extended till it touches the circle through E at the opposite end of the diameter gives $\check{E}_{a1}$; $\check{E}_{b1}$ and $\check{E}_{c1}$ are obtained by laying of points on the circle 60 degrees from E. Similarly if $F\,O$ is extended to meet the circle through F at the opposite end of the diameter we obtain $\check{E}_{a2}$; and $\check{E}_{b2}$ and $\check{E}_{c2}$ are obtained by the same construction as before.

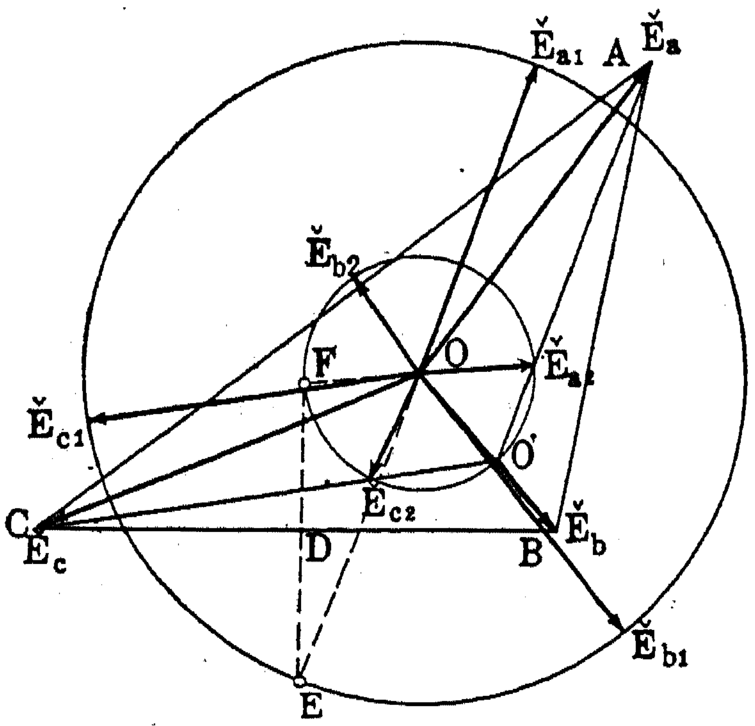

FIG. 18—GRAPHICAL METHOD FOR OBTAINING THE SYMMETRICAL COMPONENTS OF THE THREE-PHASE VECTORS $\check{E}_a$, $\check{E}_b$, $\check{E}_c$

The proof of this construction is as follows: If G and H are the apexes (not shown in figure) of the equilateral triangles having $B\,C$ as base

$$O\,E = \frac{O\,B + O\,C + O\,G}{3} = \frac{-\,O\,A + O\,G}{3} = \frac{-\,\check{E}_a + O\,G}{3}$$

$$O\,F = \frac{O\,B + O\,C + O\,H}{3} = \frac{-\,O\,A + O\,H}{3} = \frac{-\,\check{E}_a + O\,H}{3}$$

$$\begin{aligned}
-\,a^2 : O\,C &= -\,a^2\,(O\,D + D\,C) \\
-\,a\ .\,O\,B &= -\,a\,(O\,D + D\,B) = -\,a\,(O\,D - D\,C) \\
-\,a^2 .\,O\,C - a\,.\,O\,B &= -\,(a + a^2)\,O\,D + (a - a^2)\,.\,D\,C \\
&= O\,D + j\sqrt{3}\,D\,C \\
&= O\,G
\end{aligned}$$

That is

$$O\,G = -\,(a\,\check{E}_b + a^2\,\check{E}_c)$$

Similarly

$$O\,H = -\,(a^2\,\check{E}_b + a\,\check{E}_c)$$

and therefor

$$O\,E = -\,\frac{\check{E}_a + a\,\check{E}_b + a^2\,\check{E}_c}{3} = -\,\check{E}_{a1}$$

and

$$O\,F = -\,\frac{\check{E}_a + a^2\,\check{E}_b + a\,\check{E}_c}{3} = -\,\check{E}_{a2}$$

The construction when $\check{E}_a + \check{E}_b + \check{E}_c$ is not zero is so obvious that it is not necessary to show it here.

If lines be drawn from E_a to $\check{E}_{a2}$, E_b to $\check{E}_{b2}$, E_c to $\check{E}_{c2}$ these lines will be parallel to $\check{E}_a{}'$ $\check{E}_b{}'$ and $\check{E}_c{}'$ respectively and will meet at a point O'. The vectors $O'\,E_a$ $O'\,E_b$ and $O'\,E_c$ give the values of the e. m. f. across each member of a star delta bank of transformers when operated on the three-phase circuit $S\,(\check{E}_a)$ with ratios changed so as to give a balanced secondary triangle of e. m. f's.

DISCUSSION ON "METHOD OF SYMMETRICAL CO-ORDINATES APPLIED TO THE SOLUTION OF POLYPHASE NETWORKS" (FORTESCUE), ATLANTIC CITY, N. J., JUNE 28, 1918.

J. Slepian (by letter): During the past eighteen months it has been my very good fortune to have been in close contact with Mr. Fortescue and to have had many interesting discussions on the ideas embodied in this paper. Since I have had so long to think over and digest these ideas, I think I may be pardoned for going at great length here into the viewpoint I have reached. I feel all the greater need for a long discussion, because on reading the paper I see that the great wealth of material to be presented in a limited space has crowded out much detailed explanation and appeal to analogy, which in discussions with Mr. Fortescue have contributed so much to my clearer conception of the ideas presented here.

The method given here had its origin in considering the operation of balanced induction machines under unbalanced conditions. Using the "coordinates" proposed here, the theory of these machines may be given with beautiful simplicity. I think I am right in saying that the utility of the method is practically entirely limited to the case of rotating induction machines. Purely static apparatus treated in this way does not show any simplification. When one considers, however, that almost every practical alternating-current circuit contains at least one rotating machine, namely the generator, the broad field of application of the method becomes apparent.

The root of the ideas given here is old and was given early in treatments of single-phase motors. The simple constant rotating nature of the flux in a balanced polyphase motor was well known. It was discovered that the more complicated flux in a single-phase motor could be resolved into the sum of two such constant rotating fluxes of opposite rotations, with the magnitude of these fluxes not necessarily equal. It was observed also, that if the balanced polyphase states, which would give independently each of these rotating fields, were superimposed, the resultant state would give correctly the currents. voltages, torques, etc., of the single-phase motor. For a recent discussion of this, see Mr. Lamme's paper, page 627, Volume I, TRANSACTIONS 1918.

Mr. Fortescue has generalized this method of resolution of the flux in a polyphase machine under unbalanced conditions into the sum of fluxes, each corresponding to a balanced condition, to a similar resolution of an unbalanced system of any polyphase quantities whatever.

In my discussion here I shall, for simplicity, confine myself to the three-phase case. To illustrate the resolution of a set of unbalanced three-phase currents into symmetrical or balanced components, consider three arbitrary currents I_a I_b I_c flowing into the terminals of a three-phase, star-connected apparatus. (Fig. 1). Mr. Fortescue shows that this set of currents may be had by adding together the following three sets of currents.

Method of Symmetrical Coördinates. First a current I_{ao} in each phase. These three currents then are all equal and all in phase. They combine to give a current in the neutral. Second: a current I_{ao} in phase a, a current $a^2 I_{a1}$, in phase b, and a current $a I_{a1}$ in phase c, where

$$a = - \tfrac{1}{2} + j \tfrac{1}{2} \sqrt{3}$$

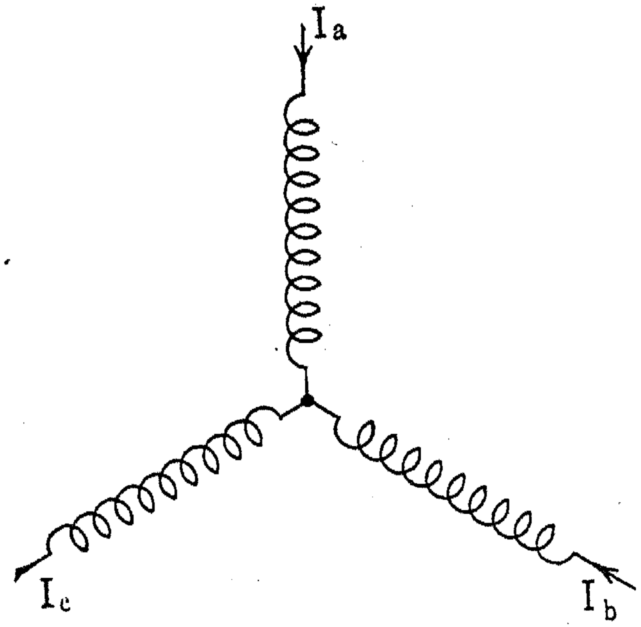

FIG. 1

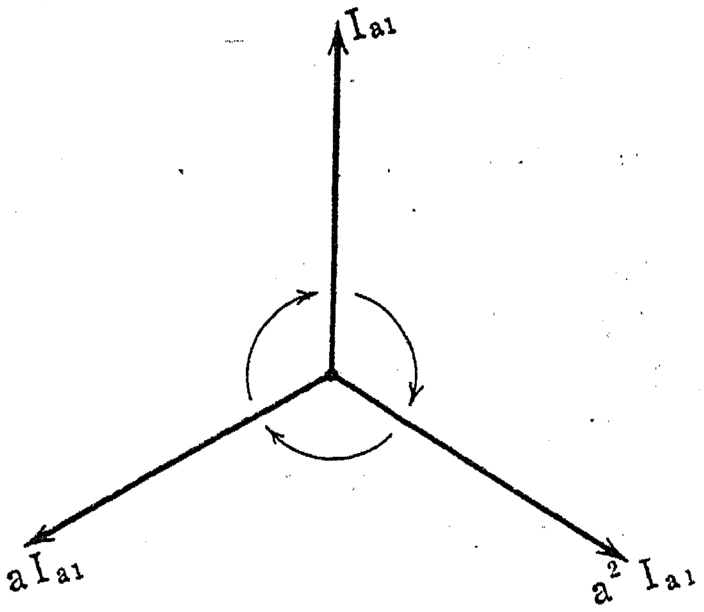

FIG. 2

is a cube root of unity. This set of currents is clearly a balanced polyphase set of what is called positive sequence. (Fig. 2). Third a current I_{a2} in phase a, a current $a\ I_{a2}$ in phase B and a current $a^2\ I_{a2}$ in phase c. (Fig. 3.). This set of currents is clearly a balanced polyphase set of what is called negative phase sequence. If these three sets of currents be made to flow simultaneously in the three-phase apparatus the resultant current in each phase will be respectively I_a, I_b, I_c. The values of I_{a0}, I_{a1}, I_{a2} are obtained from equations (**5**). They are:

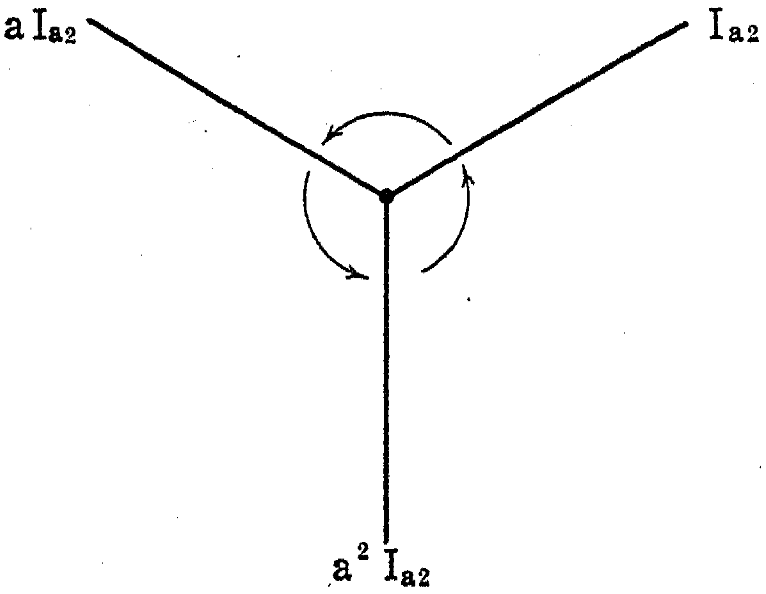

FIG. 3

$$I_{a0} = 1/3\ (I_a + I_b + I_c).$$
$$I_{a1} = 1/3\ (I_a + a I_b + a^2\ I_c).$$
$$I_{a2} = 1/3\ (I_a + a^2 I_b + a I_c).$$

In a similar way, any set of three voltages to neutral acting on the three-phase apparatus can be resolved into the sum of three sets of balanced voltages of zero phase sequence, positive phase sequence, and negative phase sequence, respectively.

The sequence operators, S^0, S^1 and S^2, which Mr. Fortescue introduces, have a close analogy, it seems to me, with the j used in the usual treatment of alternating currents. In the elementary theory, it is found useful to resolve an alternating quantity into two components, one in phase with some reference alternating quantity, and the other component in phase quadrature. If i_1 and i_2 are the magnitudes respectively, of the in-phase and the quadrature components, their resultant I, is denoted by $i = i_1 + ji_2$. Here j may be looked upon as a unit alternating quantity in phase quadrature with a reference alternating quantity. Thus, the equation $i = li_1 + j\,i_2$ states that the quantity i may be obtained by adding together i_1 times a unit in phase quantity, and i_2 times a unit quadrature component.

In the same way, the symbols S°, S^1, S^2 may be regarded as unit polyphase vectors* of zero, positive and negative phase sequence. Thus $S^\circ = (1, 1, 1)$, if referring to currents, represents a unit current in each phase of a three-phase apparatus the three currents being in phase with some reference alternating quantity. Similarly, $S^1 = (1, a^2, a)$ represents a unit balanced, three-phase current, of positive phase sequence, in the three-phase apparatus, the current in the first phase being in phase with the reference alternating quantity. Likewise $S^2 = (1, a, a^2)$ represents a unit balanced, three-phase current of negative phase sequence. Thus the equation (**15**) $S\,(I_a) = S^\circ I_o + S^1 I_{a1} + S^2 I_{a2}$ states that a system of three currents $S\,(I_a)$ is equal to the sum of three sets of currents, the first set being of the type S°, that is of zero phase sequence, the second set of the type S^1, that is of positive phase sequence, and the third set of type S^2, that is of negative phase sequence. The three currents of any set are obtained by multiplying the three currents of the corresponding unit polyphase vector by the complex quantity indicated. Thus the currents in the second set are obtained by multiplying the three currents, 1, a_2, a, respectively, by I_{a1}.

The utility of resolving three-phase currents and voltages in this way, when applied to rotating balanced machines, lies in this fact, that a symmetrical set of voltages of any phase sequence applied to the machine will produce a symmetrical set of currents of the *same phase sequence*, and that a symmetrical set of currents of any phase sequence flowing into the machine will produce a symmetrical set of terminal voltages of the same phase sequence. This fact is well known in the theory usually given of balanced polyphase apparatus, although it is seldom explicitly stated. Other methods of dividing a set of unbalanced three-phase quantities acting on a balanced machine into components would not have this simplicity. For example: suppose the three actual currents in the three phases, and the three actual voltages of the terminals relative to neutral were taken as the components or coordinate of the currents and voltages. Then one component of current alone, that is a current in one phase only, would not produce only the corresponding component of voltage, but would produce voltages in all three phases.

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*This very fortunate term is due to Mr. C. T. Allcutt and expresses the idea of the paragraph most succinctly.

Neglecting saturation, for a given rotor speed, the symmetrical currents produced in a balanced machine by symmetrical applied voltage, are proportional and in a definite phase relation to the symmetrical applied voltage. Thus the currents may be obtained by multiplying the individual members of the symmetrical set of applied voltages by some complex number. Similarly, a symmetrical set of currents flowing into a balanced machine produces a symmetrical set of terminal voltages which may be obtained by multiplying the individual members of the symmetrical set of currents by some complex number. The first complex number could be called the symmetrical admittance of the machine, and the second the symmetrical impedance.

This symmetrical impedance and admittance will be different, of course, for symmetrical components of different phase sequence. Thus for a star-connected, ungrounded neutral machine, the admittance for symmetrical voltage of zero phase sequence will be zero; if the neutral is grounded, the impedance to zero phase sequence current will be principally the leakage reactance between phases; if the rotor is running near synchronism in the sense of positive phase sequence, the impedance Z_1 to positive phase sequence current will be large, while the impedance Z_2 to negative phase sequence will be small. The complete expressions for these impedances Z_1, Z_2, are given in equations (**122**), (**123**). The relations between current and voltage components are given in equations (**120**) and (**121**). In these four equations is concentrated the whole theory of symmetrical machines operating under unbalanced conditions. But so simple are these equations and their physical meaning so clear, that once understood, they enable us to predict qualitatively, without computation, the behavior of rotating balanced machines under any unbalanced condition whatever.

Consider, for example, the simple picture of the action of a phase balancer which the above treatment gives. The balancer is merely a balanced machine across the polyphase line running near synchronism. It offers high impedance for the symmetrical voltage of normal or positive phase sequence, but offers a very low impedance for any negative phase sequence component of voltage. Thus the negative phase sequence voltage is "short-circuited out," and balance on the line is preserved.

The harmful effects of slight unbalance in the terminal voltage of a polyphase machine upon the machine's rating is also clearly shown. Since the negative sequence impedance is very low, a small negative phase sequence voltage will produce large negative phase sequence currents with their attendant heating. This suggests a better quantity to denote the degree of unbalance of a line than the one commonly used. It is the ratio of the negative phase sequence component of the line voltage to the positive phase sequence component.

Balanced stationary apparatus also enjoys the simplicity pointed out in balanced rotating machines, namely that symmetrical voltages of any phase sequence produce symmetrical currents of the same phase sequence, but here the impedances for positive and negative phase sequence are always the same. The zero phase sequence impedance is generally different.

When we pass to unbalanced apparatus, things become more complicated. Symmetrical currents of one phase sequence no longer produce e. m. fs. of that phase sequence alone, but the total e. m. fs. are unsymmetrical and contain components of other phase sequences. Let us consider in detail an unbalanced star impedance, with grounded neutral. Let the phase impedances be Z_a, Z_b, Z_c.

Let us first study the e. m. fs. to neutral produced by a zero phase sequence current. If I_{ao}, I_{ao}, I_{ao}, are the three currents, the three e. m. fs. are Z_aI_{ao}, Z_bI_{ao}, Z_aI_{ao}. Resolving these three e. m. fs. into symmetrical components, we find by equations (**5**) that the zeroth order component is:

$$\tfrac{1}{3}\,(Z_aI_{ao} + Z_bI_{ao} + Z_cI_{ao}) = \tfrac{1}{3}\,(Z_a + Z_b + Z_c)\,I_{ao}$$

The positive phase sequence components will be:

$$\tfrac{1}{3}\,(Z_aI_{ao} + aZ_bI_{ao} + a^2Z_cI_{ao}) = \tfrac{1}{3}\,(Z_a + aZ_b + a^2Z_c)\,I_{ao}$$

The negative phase sequence component will be:

$$\tfrac{1}{3}\,(Z_aI_{ao} + a^2Z_bI_{ao} + aZ_cI_{ao}) = \tfrac{1}{3}\,(Z_a + a^2Z_b + aZ_c)\,I_{ao}$$

Thus the total e. m. f. using the symbols $S^\circ S'$ S^2 would be written:

$$S^\circ\,[\tfrac{1}{3}\,(Z_a + Z_b + Z_c)\,I_{ao}] + S^1\,[\tfrac{1}{3}(Z_a + aZ_b + a^2Z_c)I_{ao}] + S^2\,[\tfrac{1}{3}(Z_a + a^2Z_b + aZ_c]\,I_{ao} = S^\circ\,(Z_{ao}I_{ao}) + S'\,(Z_{a1}I_{ao}) + S^2\,(Z_{a2}I_{ao}).$$

$$\begin{aligned} \text{Where } Z_{ao} &= \tfrac{1}{3}\,(Z_a + Z_b + Z_c) \\ Z_{a1} &= \tfrac{1}{3}\,(Z_a + aZ_b + a^2Z_c) \\ Z_{a2} &= \tfrac{1}{3}\,(Z_a + a^2Z_b + aZ_c) \end{aligned}$$

Now consider the e. m. f's. given by the positive phase sequence currents, I_a, a^2I_{a1}, aI_{a1}. They will be Z_aI_{a1}, $Z_ba^2I_{a1}$ Z_caI_{a1}.

Resolve the system of em. f's. into symmetrical components. We find for the zero phase sequence component:

$$\tfrac{1}{3}\,(Z_aI_{a1} + Z_ba^2I_{a1} + Z_caI_{a1}) = \tfrac{1}{3}\,(Z_a + a^2Z_b + aZ_c)\,I_{a1} = Z_{a2}I_{a1}$$

For the positive phase sequence component we find

$$\tfrac{1}{3}\,(Z_aI_{a1} + aZ_ba^2I_{a1} + a^2Z_caI_{a1}) = \tfrac{1}{3}\,(Z_a + Z_b + Z_c)\,I_{a1} = Z_{a0}I_{a1}$$

For the negative phase sequence component we find

$$\tfrac{1}{3}\,(Z_aI_{a1} + a^2Z_ba^2I_{a1} + aZ_caI_{a1}) = \tfrac{1}{3}\,(Z_a + aZ_b + a^2Z_c)\,I_{a1} = Z_{a1}I_{a1}$$

Lastly, consider the e. m. f.'s produced by negative phase sequence currents, I_{a2}, $a\ I_{a2}$, a^2I_{a2}. The zero phase sequence component will be

$$\tfrac{1}{3}\,(Z_aI_{a2} + Z_baI_{a2} + Z_ca^2I_{a2}) = \tfrac{1}{3}\,(Z_a + aZ_b + a^2Z_c)\,I_{a2} = Z_{a1}I_{a2}.$$

The positive phase sequence e. m. f. component will be

$$\tfrac{1}{3}\,(ZaI_{a2} + aZ_baI_{a2} + a^2Z_ca^2I_{a2}) = \tfrac{1}{3}\,(Z_a + a^2Z_b + aZ_c)\,I_{a2} = Z_{a2}I_{a2}.$$

The negative phase sequence e. m. f. component will be

$$\tfrac{1}{3}\,(Z_aI_{a2} + a^2Z_taI_{a2} + aZ_ca^2I_{a2}) = \tfrac{1}{3}\,(Z_a + Z_t + Z_c)\,I_{a2} = Z_{a0}I_{a2}$$

The components of voltage are then expressed easily in terms of the quantities Z_{a0}, Z_{a1}, Z_{a2}, defined in equations (8).

Let us bring these results together in tabular form where the relations between them can be observed.

Currents	Voltage Components		
	S^0 or zero phase sequence	S^1 or positive phase sequence	S^2 or negative phase sequence
Zero phase sequence, $S^0\ (I_{ao}) = (I_{ao}\ I_{ao}\ I_{ao})$	$Z_{ao}I_{ao}$	$Z_{a1}I_{ao}$	$Z_{a2}I_{ao}$
Positive phase sequence, $S^1\ (I_{a1}) = (I_{a1},\ a^2I_{a1},\ aI_{a1})$	$Z_{a2}I_{a1}$	$Z_{ao}I_{a1}$	$Z_{a1}I_{a1}$
Negative phase sequence $S^2\ (I_{a2}) = (I_{a2},\ aI_{a2},\ a^2I_{a2})$	$Z_{a1}I_{a2}$	$Z_{a2}I_{a2}$	$Z_{ao}I_{ao}$

Studying the table above, we see that each symmetrical component of current gives rise to symmetrical voltages of all three phase sequences. We notice first the e. m. f. component which is of the same phase sequence as the current by which it is produced, is obtained by multiplying the producing currents by Z_{ao}. That is the unbalanced apparatus may be said to have one component of impedance, Z_{ao}, which gives e. m. f. 's of the same phase sequence as the currents. This component of impedance does not change the exponent of the S symbol defining the phase sequence of the currents. Thus Z_{ao} alone corresponding to the currents $S^0\ (I_{ao})$ gives the e. m. fs. $S^0\ (Z_{a1}I_{ao})$; corresponding to the

Currents S^1 (I_{a1}) gives the e. m. fs. S^1 $(Z_{ao}I_{a1})$; and corresponding to the

Currents S^2 (I_{a2}) gives the e. m. fs. S^2 $(Z_2\, I_{a2})$.

Now let us examine how Z_{a1} enters in the above table. Corresponding to the zero phase sequence.

Currents S^0 (I_{ao}) we find the e. m. fs. S^1 $(Z_{a1}I_{ao})$ of positive phase sequence. Corresponding to the positive phase sequence.

Currents S^1 (I_{a1}) we find the e. m. fs. S^2 $(Z_{a1}I_{a1})$ of negative phase sequence. Corresponding to the negative phase sequence.

Currents S^2 (I_{a2}) we find the e. m. fs. S^0 $(Z_{a1}I_{a2})$ of zero phase sequence.

If we agree now to the following definitions of the symbols S when affected with higher exponents:

$$S^3 = S^\circ;\ S^4 = S^1;\ S^5 = S^2;\ S^6 = S^\circ;\ S^7 = S^1;\ \text{etc.}$$

We see that the way in which Z_{a1} terms enter can be summarized in this way:

The unbalanced apparatus has a component of impedance, Z_{a1}, which increases the exponent of the phase sequence symbol by unity.

Lastly, let us see how Z_{a2} enters the above table. We find that:

Currents S° (I_{ao}) give e. m. fs. S^2 $(Z_{a2}I_{ao})$.
Currents S^1 (I_{a1}) give e. m. fs. S° $(Z_{a2}I_{a1})$.
Currents S^2 (I_{a2}) give e. m. fs. S^1 $(Z_{a2}I_{a2})$.

This may be summarized by saying that the unbalanced apparatus has a component of impedance Z_{a2} which increases the exponent of the sequence symbol by two.

All these results will be obtained automatically if we suppose that symbols S°, S^1, S^2, are attached respectively to Z_{ao}, Z_{a1}, Z_{a2}, and when multiplying the current components, S° (I_{ao}), S^1 (I_{a1}), S^2 (I_{a2}), the exponential laws hold. Thus:

$$\begin{aligned}
S^\circ (Z_{ao})\, S^\circ (I_{ao}) &= S^\circ (Z_{ao}\, I_{ao});\ S^1 (Z_{a1})\, S^\circ (I_{ao}) = S^1 (Z_{a1}I_{ao}).\\
S^\circ (Z_{ao})\, S^1 (I_{a1}) &= S^1 (Z_{ao}\, I_{a1});\ S^1 (Z_{a1})\, S^\circ (I_{a1}) = S^2 (Z_{a1}\, I_{a1}).\\
S^\circ (Z_{ao})\, S^2 (I_{a2}) &= S^2 (Z_{ao}\, I_{a2});\ S^1 (Z_{a1})\, S^2 (I_{ao}) = S^3 (Z_{a1}\, I_{a2}).\\
&\qquad\qquad\qquad\qquad\qquad\qquad\quad\ = S^\circ (Z_{a1}\, I_{a2}).
\end{aligned}$$

$$\begin{aligned}
S^2 (Z_{a2})\, S^\circ(I_{ao}) &= S^2 (Z_{a2}I_{ao})\\
S^2 (Z_{a2})\, S^1(I_{a1}) &= S^3 (Z_{a2}I_{a1}) = S^0(Z_{a2}I_{a1})\\
S^2 (Z_{a2})\, S^2 (I_{a2}) &= S^4 (Z_{a2}I_{a2}) = S^1 (Z_{a2}I_{a2})
\end{aligned}$$

The whole result can be expressed by writing the total impedance, $S(Z_a) = S^\circ\, Z_{ao} + S^1\, Z_{a1} + S^2\, Z_{a2}$.

To get the e. m. fs. corresponding to any currents, we multiply together the total impedance by the total current, following merely the rules of algebra, and interpreting higher powers of S as described above. Thus we get:

$$\begin{aligned}
S(E_a) &= S(Z_a)\, S(I_a)\\
&= (S^\circ Z_{ao} + S^1 Z_{a1} + S^2 Z_{a2})\, (S^0 I_{ao} + S^1 I_{a1} + S^2 I_{a2})\\
&= S^\circ (Z_{ao}I_{ao} + Z_{a1}I_{a2} + Z_{a2}I_{a1}) + S^1 (Z_{ao}\, I_{a1}\\
&\qquad + Z_{a1}I_{ao} + Z_{a2}I_{a2}) + S^2 (Z_{ao}I_{a2} + Z_{a1}I_{a1} + Z_{a2}I_{ao})
\end{aligned}$$

We get here the foundation of a complex three-phase algebra, which performs the same functions for three-phase systems as the usual complex quantity does for simple alternating current.

This leads to another interpretation or mode of viewing the sequence symbols, S°, S^1, S^2, and here again we may profit by analogy with the j in the complex algebra of simple alternating quantities. Suppose i is a current in phase with some reference alternating quantity. If this current flows through a resistance r. the resulting e. m. f., ri is also in phase with reference alternating quantity. If the resistance is of unit value the resulting e. m. f. is i. Suppose, however, that the current flows through a reactance whose impedance is x ohms. The resulting e. m. f. will be $x\,I$ in magnitude, but it will be in phase quadrature with the reference alternating quantity. This is taken care of in the usual theory by affecting the expression for the reactance with a j, thus xj, and making j have the property that when multiplying a vector or alternating quantity it does not change the value of the vector but merely advances its phase by ninety degrees This is a different meaning from what was given before. j here no longer represents a unit vector or alternating quantity. It now represents an operator, or a symbol for advancing the phase of any vector which it multiplies. Where j by itself is referred to as a vector, it should be understood that ji is meant; where i is a unit vector in phase with the reference vector.

It is clear that operating twice successively by j upon a vector merely reverses the phase of that vector. Thus $j\,(j.i) = -1$, which leads to the rule of multiplication $J^2 = -1$. Lastly, we see that the e. m. f. induced by a current flowing through an impedance may be obtained correctly both in phase and magnitude by multiplying the current by a number of the form $r + jx$, which represents the complete impedance. Thus the complex algebra of simple alternating quantities is born.

We may attach significance as operators in a similar way to the symbols S°, S^1, S^2. Now they shall no longer represent polyphase vectors, but shall merely be operators which when written next to a polyphase vector, change it into a polyphase vector of another phase sequence. The quantities which the S's affect shall be considered as polyphase vectors. Thus the symbol I shall stand for three currents, each equal to I. Operating on I with S° means multiplying each of these currents by unity. Thus S° is a unity operator and does not change the polyphase vector upon which it operates. A separate symbol for it might have been omitted.

$S^1\,I$ or S^1 operating on I, means that the three currents I, I, I, are to be multiplied by 1, a^2, a, respectively; or that the first current is to be unchanged, the second to be advanced in phase by 120 deg. and the third by 240 deg. S' by itself shall not mean anything unless it is understood to be followed by 1 in which case it stands for the unit polyphase vector of positive, rotation obtained by operating with S^1 on the three currents (1, 1, 1).

Similarly, S^2I or S^2 operating on I, means that the three currents I, I, I, are to be multiplied by I, a, a^2, respectively; or that the first current is to be unchanged, the second advanced in phase by 240 deg. and the third by 120 deg. S^2 by itself shall not mean anything unless it is understood to be I, in which case it stands for the unit polyphase vector of negative phase rotation obtained by operating upon the polyphase quantity (I, I, I).

With the meaning now given to S°, S^1, S^2, it at once follows that they satisfy the law of exponents. For example, S^1 (S^1I) means leave the first current I unchanged; advance the second current I, in phase by 120 deg., and then again by 120 deg; advance the third current, I, by 240 deg. and than again by 240 deg. It is clear that the final results are exactly the same as the results of operating with S^2 on I. Also it is clear that multiplication by an operators and a constant Z is commutative. Thus $ZS^1I = S^1Z_{a1}I$. This serves as the foundation of the complex three-phase algebra.

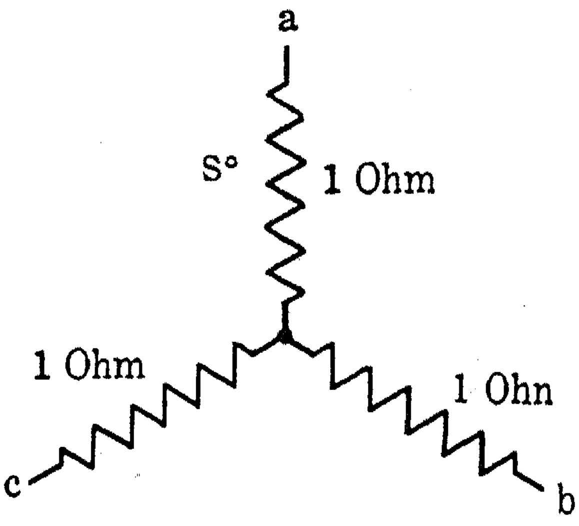

FIG. 4

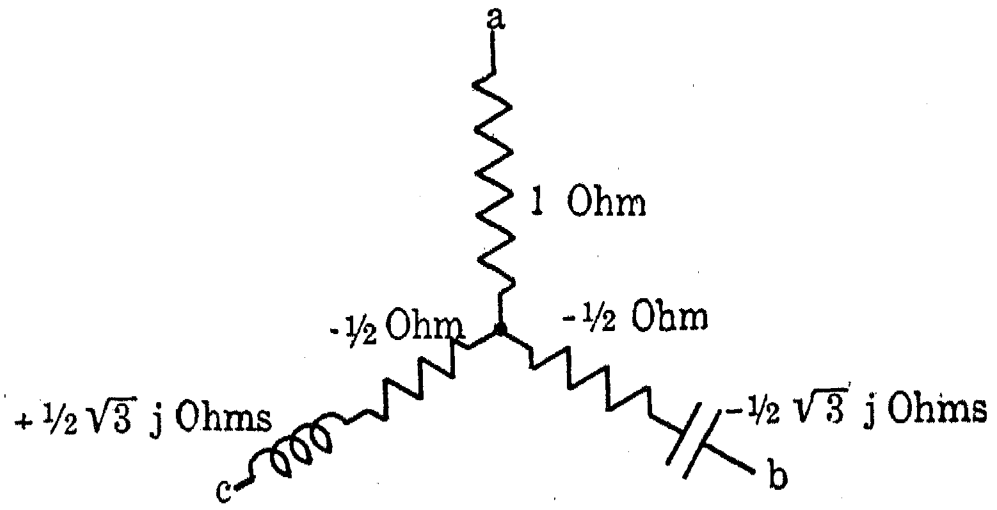

FIG. 5

In the complex algebra of simple alternating quantities, it is easy to give an illustration of the operator J as in impedance. In fact, a reactive impedance of one ohm gives for any current an e. m. f. which may be obtained from the current by multiplying by J. Can we similarly illustrate the three operators S°, S^1, S^2.

S°, of course, is easy. Take a grounded star of resistances, each of one ohm. Then we find for the currents $S^\circ(I) = (I, I, I)$ the voltages, $(I, I, I) = S^\circ I$; for the currents $S^2(I) = (I, aI, a^2I)$ the voltages $(I, aI, a^2I) = S^2I$.

Now for S^1. Take a grounded star in which the impedance of phase a is 1 ohm, of phase b, $(-\frac{1}{2} - \frac{1}{2}\sqrt{3J})$ ohms $= a^2$ ohms, and of phase c, $(-\frac{1}{2} + \frac{1}{2}\sqrt{3J})$ ohms $= a$ ohms. Now we find for the currents $S^\circ(I) = (I, I, I)$, the e. m. fs. $(I, a^2I, aI) = S^1I = S^1(S^\circ I)$ for the currents $S^1(I) = (I, a^2I, aI)$, the e. m. fs. $(I, aI, a^2I) = S^2I = S'(S'I)$; for the currents $S^2(I) = (I, aI, a^2I)$, the e. m. fs. $(I, I, I) = S^\circ I = S^1(S^2I)$. Fig.5.

Lastly, S^2. Take a grounded star in which the impedance of phase a is 1 ohm, of phase b, $(-\frac{1}{2} + \frac{1}{2}\sqrt{3J})$ ohms, and of phase c, $(-\frac{1}{2} - \frac{1}{2}\sqrt{3j})$ ohms. Then we find for the currents $S^\circ\,(I) = (I, I, I)$, the e. m. fs. $(I, aI, a^2I) = S^2I = S^2S^\circ I$; for the currents $S'\,(I) = (I, a^2I, aI)$ the e. m. fs. $(I, I, I) = S^\circ I = S^2S'I$; for the currents $S^2\,(I) = (I, aI, a^2I)$ the e. m. fs. $(I, a^2I, aI) = S^1I = S^2S^2I$. Fig. 6.

In these examples, negative resistances appear. This is not surprising, as it is clear that unbalanced apparatus, when current of one phase sequence flows, may feed energy into impressed e. m. fs. of another phase sequence. The negative resistance may be represented physically by a series commutator a-c. motor, driven at constant speed in the opposite of its motoring direction of rotation.

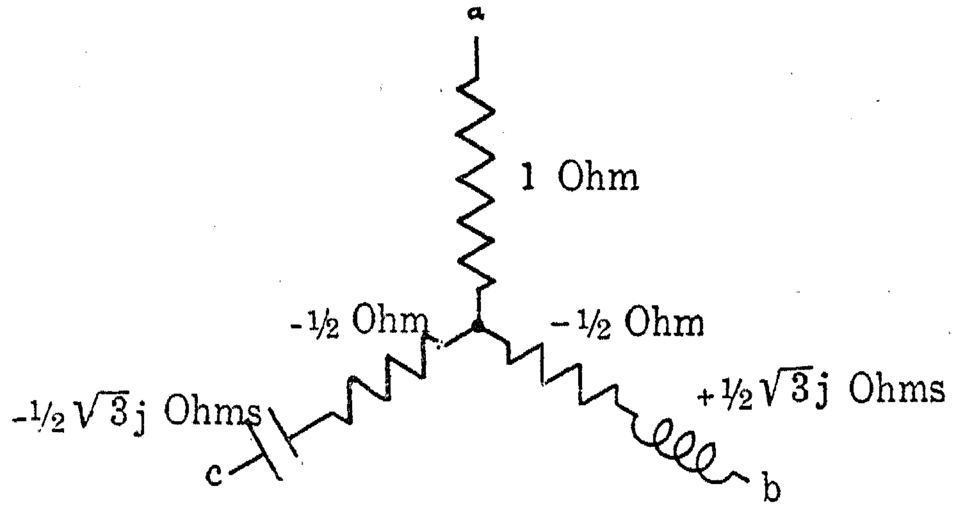

FIG. 6

The resolution of an arbitrary three-phase star impedance into its three-phase components is merely the problem of finding three three-phase impedances of the S°, S^1 and S^2 type, respectively, which put in series as in Fig. 7 will reproduce the given three-phase star.

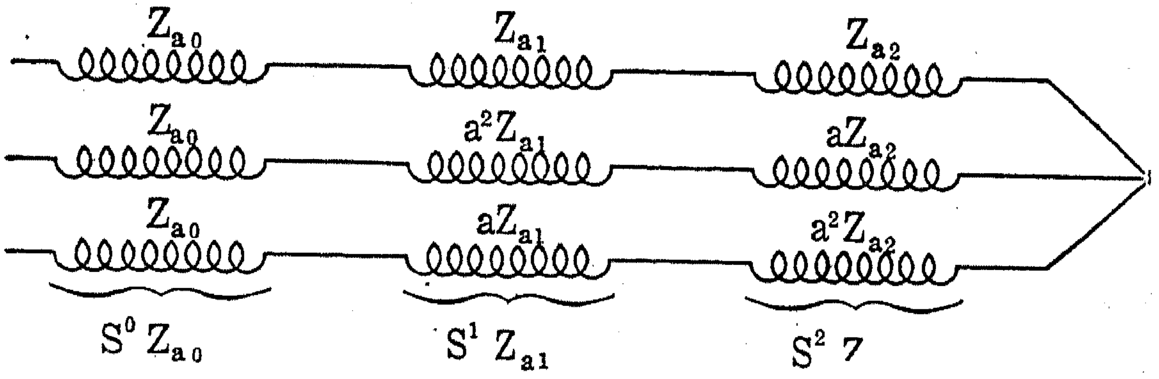

FIG. 7

It is clear that admittances may be treated in the same way as impedances above. Thus the general three-phase admittance may be written in terms of its components, thus:

$$S\,(Y_a) = S^\circ Y_{ao} + S^1 Y_{a1} + S^2 Y_{a2}$$

The currents produced by a general three-phase voltage $S\,(E_a) = S^\circ E_{ao} + S^1E_{a1} + S^2E_{a2}$ will be:

$$
\begin{aligned}
S(I_a) &= (S^\circ Y_{ao} + S^1 Y_{a1} + S^2 Y_{a2})\quad (S^\circ E_{ao} + S^1 E_{a1} + S^2 E_{a2}). \\
&= S^\circ (Y\ E_{ao} + Y_{a1}E_{a2} + Y_{a2}E_{a1}) \\
&\qquad + S^1 (Y_{ao} E_{a1} + Y_{a1}E_{ao} + Y_{a2} E_{a2}) \\
&\qquad + S^2(Y_{ao} E_{a2} + Y_{a1}E_{a1} + Y_{a2}E_{ao}).
\end{aligned}
$$

In the complex algebra of simple alternating currents we pass formally from an impedance $R + jx$ to the corresponding admittance by taking the reciprocal, $\frac{l}{r + jx}$ and multiplying numerator and denominator by $r - jx$ to reduce it to the standard form.

$$\text{Thus } \frac{l}{r + Jx} \times \frac{r-jx}{r-jx} = \frac{r}{r^2 + x^2} - j\frac{x}{r^2 + x^2}$$

Similarly in this three-phase complex algebra we obtain the admittance corresponding to an impedance $S^\circ Z_{ao} + S^1 Z_{a1} + S\ Z_{a2}$ by taking the reciprocal $\frac{1}{S^\circ Z_{ao} + S^1 Z_{a1} + SZ_{a2}}$. But now to reduce to standard form we must multiply numerator and denominator by

$$S^\circ (Z_{ao} + aS^1 Z_{a1} + a^2 S^2 Z_{a2})\ (S^\circ Z_{ao} + a^2 S^1 Z_{a1} + aS^2 Z_{a2}).$$

Thus:

$$
\frac{1}{S^\circ Z_{ao} + S^1 Z_{a1} + S^2 Z_{a2}} \cdot \frac{S^\circ Z_{ao} + aS^1 Z_{a1} + a^2 S^2 Z_{a2}}{S^\circ Z_{ao} + aS^1 Z_{a1} + a^2 S^2 Z_{a2}} \cdot \frac{S^\circ\ Z_{ao} + a^2 S^1 Z_{a1} + aS^2 Z_{a2}}{S^\circ\ Z_{ao} + a^2 S^1 Z_{a1} + aS^2 Z_{a2}}
$$

$$
= \frac{S^\circ\ (Z_{ao}^2 + a^2 Z_{a1} Z_{a2} + aZ_{a1} Z_{a2}) + S^1(a^2 Z_{ao} Z_{a1} + aZ_{ao} Z_{a1} + Z^2_{a2}) + S^2(a^2 Z_{ao} Z_{a2} + Z^2_{a1} + aZ_{ao} Z_{a2})}{D}
$$

$$
= \frac{S^\circ\ (Z^2_{ao} - Z_{a1}\, Z_{a2}) + S^1\ (Z^2_{a2} - Z_{ao}\, Z_{a1}) + S^2\ (Z^2_{a1} - Z_{ao}\, Z_{a2})}{S^\circ (Z^3_{ao} + Z^3_{a1} + Z^3_{a2} - 3\ Z_{ao}\, Z_{a1}\, Z_{a2})}
$$

Remembering that S° is an operator which does not change the quantity upon which it acts, and therefore that a symbol for it might have been omitted, we may leave off the S° in the denominator and get for the final admittance:

$$
\begin{aligned}
S\ (Y_a) = S^\circ & \frac{Z^2_{ao} - Z_{a1}\, Z_{a2}}{Z^3_{ao} + Z^3_{a1} + Z^3_{a2} - 3\ Z_{ao}\, Z_{a1}\, Z_{a2}} \\
&+ S^1 \frac{Z^2_{a2} - Z_{ao}\, Z_{a1}}{Z^3_{ao} + Z^3_{a1} + Z^3_{a2} - 3\ Z_{ao}\, Z_{a1}\, Z_{a2}} \\
&+ S^2 \frac{Z^2_{a1} - Z_{ao}\, Z_{a2}}{Z^3_{ao} + Z^3_{a1} + Z^3_{a2} - 3\ Z_{ao}\, Z_{a1}\, Z_{a2}}
\end{aligned}
$$

The principles just enunciated enable us to put together three-phase apparatus and calculate the resulting three-phase components just as is done in single-phase apparatus. For apparatus in parallel, we add admittances; for apparatus in series we add impedances.

So far, in the static apparatus considered, the mutual inductance between "legs" of the three-phase apparatus has been assumed zero. When these mutual inductances are not zero, things become more complicated. A three-phase apparatus with grounded neutral is, of course, a four terminal network, and as is well known, in the most general case would require $\frac{4 \times 3}{2} = 6$ complex constants to characterize it under steady state conditions. It is clear that a complex three-phase expression $S^o Z_{ao} + S^1 Z_{a1} + S^2 Z_{a2}$ depends on only three complex constants, and hence cannot represent the most general three-phase apparatus.

The relations between the symmetrical components of voltage and current on the general static three-phase network are given in equations (**25**). One way of summarizing these equations would be to say that the three-phase apparatus has three different impedances for the three phase-sequence currents. Thus the impedance to zero phase sequence current as given by equations (**25**) is:

$$S^\circ (Z_{aao} + 2 Z_{bco}) + S^1 (Z_{aa1} - Z_{bc1}) + S^2 (Z_{aa2} - Z_{bc2});$$

the impedance to positive phase sequence currents is

$$S^\circ (Z_{aao} - Z_{bco}) + S^1 (Z_{aa1} + 2 Z_{bc1}) + S^2 (Z_{aa2} - Z_{bc2});$$

and the impedance to negative phase sequence currents is

$$S^\circ (Z_{aao} - Z_{bco}) + S^1 (Z_{aa1} - Z_{bc1}) + S^2 (Z_{aa2} + Z_{bc2}),$$

Another way of summarizing the equations (**25**) would be to say that the actual three-phase apparatus is equivalent to a three-phase network having the three-phase impedance

$$S^\circ(Z_{aao} - Z_{bco}) + S^1 (Z_{aa1} - Z_{bc1}) + S^2 (Z_{aa2} - Z_{bc2}),$$

in series with a network whose impedance to zero, positive and negative phase sequence current are respectively, $S^\circ 3\, Z_{bco}$; $S^1\, 3\, Z_{bc1}$; $S^2\, 3\, Z_{bc2}$.

With the ideas developed in this paper, solutions of problems on symmetrical rotating machines with unbalanced static apparatus may be worked with comparative ease. As an example I shall consider the case of a generator, with ungrounded neutral, acting on a given star load, of leg impedances, Z_a, Z_b, Z_c. Suppose the machine is symmetrical, *i. e.*, non-salient poles, and a damping squirrel cage on the field. In this case the generator has an impedance Z_1 to positive phase sequence current, and a small impedance Z_2 to negative phase sequence currents. The positive phase sequence impedance Z_1 is what has been called the synchronous impedance of the machine such that if E is the no load voltage, $E - Z_1 I$ is the voltage on a positive sequence balanced load I. The problem is to find the currents and voltages on the load, and the voltage between load neutral and generator neutral.

We have given that $I_{ao} = O$.

We calculate the impedance drops of positive phase sequence and put their sum equal to the generated positive sequence voltage. Thus:

$$Z_1 I_{a1} + Z_{ao} I_{a1} + Z_{a2} I_{a2} = E.$$

Similarly, since the generated negative phase sequence voltage is zero,

$$Z_2 I_{a2} + Z_{a1} I_{a1} + Z_{ao} I_{a2} = 0.$$

Solving for I_{a1} and I_{a2}

$$I_{a1} = E \frac{Z_2 + Z_{ao}}{Z_1 Z_2 + (Z_1 + Z_2) Z_{ao} + Z^2_{ao} - Z_{a1} Z_{a2}}$$

$$I_{a2} = E \frac{-Z_{a1}}{Z_1 Z_2 + (Z_1 + Z_2) Z_{ao} + Z^2_{ao} - Z_{a1} Z_{a2}}$$

The terminal voltages will be:

$$E_{a1} = E - Z_1 I_{a1} = Z_{ao} I_{a1} + Z_{a2} I_{a2} = E \,.\, \frac{Z^2_{ao} - Z_{a1} Z_{a2} + Z_2 Z_{ao}}{0}$$

$$E_{a2} = - Z_2 I_{a2} = Z_{a1} I_{a1} + Z_{ao} I_{a2} = E \frac{Z_2 Z_{a1}}{0}$$

The voltage between load neutral and generator neutral will be:

$$E_{ao} = Z_{a1} I_{a2} + Z_{a2} I_{a1} = E \,.\, \frac{-Z^2_{a1} + Z_{ao} Z_{a2} + Z_2 Z_{a2}}{D}$$

A special case of interest is where $Z_b - Z_c$ are zero, that is, a single-phase short circuit. We then have:

$$Z_{ao} = \tfrac{1}{3} Z_a \qquad Z_{a1} = \tfrac{1}{3} Z_{a1} \qquad Z_{a2} = \tfrac{1}{3} Z_a$$

$$I_{a1} = E \frac{(Z_a + 3 Z_2)}{3 Z_1 Z_2 + (Z_1 + Z_2) Z_a}$$

$$I_{a2} = E \frac{-Za}{3 Z_1 Z_2 + (Z_1 + Z_2) Z_a}\,.$$

$$E_{a1} = E \cdot \frac{Z_2 Z_a}{3 Z_1 Z_2 + (Z_1 + Z_2) Z_a}$$

$$E_{a2} = E \cdot \frac{Z_2 Z_a}{3 Z_1 Z_2 + (Z_1 + Z_2) Z_a}$$

$$E_{ao} = E \frac{Z_2 Z_a}{3 Z_1 Z_2 + (Z_1 + Z_2) Z_a}$$

If just previous to the single-phase short circuit the generator was unloaded, Z_a becomes infinite and our final results are:

$$I_{a1} = - I_{a2} = \frac{E}{Z_1 + Z_2}$$

$$E_{a1} = E_{a2} = E_{ao} = E \frac{Z_2}{Z_1 + Z_2}$$

If the generator above did not have a damper winding on the field, it would be an unsymmetrical machine having a single-phase rotor. As we know, the theory of unsymmetrical rotating machines is of considerable complication. Mr. Fortescue has promised a treatment by these methods of the general unsymmetrical rotating machine in a future paper, which I look forward to with great interest.

C. P. Steinmetz: In dealing with calculations or investigations of polyphase systems, or, as usually the case, three-phase systems, the difficulties which we meet are not so much mathematical difficulties, but are what I may call mechanical difficulties. The equations, while mathematically not complicated, lead to expressions which are so complicated and extensive in form as to make any such calculations extremely difficult.

In dealing with the balanced polyphase system, this difficulty has been overcome by the introduction of the equivalent single-phase system, by considering the polyphase system as resolved into a number of single-phase systems, each comprising the circuit from one of the phase wires to the neutral point of the system. This method however, fails in the unbalanced polyphase system—and naturally practically all existing commercial polyphase systems are more or less unbalanced—and the theory of the vector method given to us today in a more extensive description by Mr. Fortescue, gives the solution by showing us in the case of the general three-phase system that it can be resolved into two balanced three-phase systems of opposite phase rotation. We can apply the same plan to other polyphase systems.

V. Karapetoff: Mr. Fortescue deserves the gratitude of the profession for bringing out a new method for numerical computations in unsymmetrical polyphase systems, and also for applying the method to a number of practical cases.

Expressions (**4**) represent the result of solution of certain equations, which equations unfortunately are not given in the paper. The expressions quoted are unnecessarily involved, and it is difficult to see their method of derivation. Moreover, it may be shown that they do not represent the most general case of resolution of an unsymmetrical system of vectors. The following procedure would seem to be preferable.

Definition. A multiple-angle symmetrical polyphase system of vectors is defined as one in which the vectors are numbered not consecutively, but skipping a certain number of vectors. Thus, Fig. 8 represents a triple-angle seven-phase clockwise system, because three angles are comprised between phase 1 and phase 2, and also between phase 2 and 3, etc. Such a generalization of the concept of polyphase systems is useful when the number of phases and the number of angles skipped are prime numbers, so that all the phases may be numbered consecutively, without omitting or repeating any. If the total number of phases n is a prime number, then the total number of possible combinations or multiple systems is $(n - 1)$, the angles between the consecutive phases being $\frac{2\pi}{n}$, $2 \times \frac{2\pi}{n}$, . . . $(n-1)\frac{2\pi}{n}$. It is assumed that the numbering in *all* the systems is either clockwise or counter-clockwise, so that there is no confusion between a positive and a negative phase sequence.

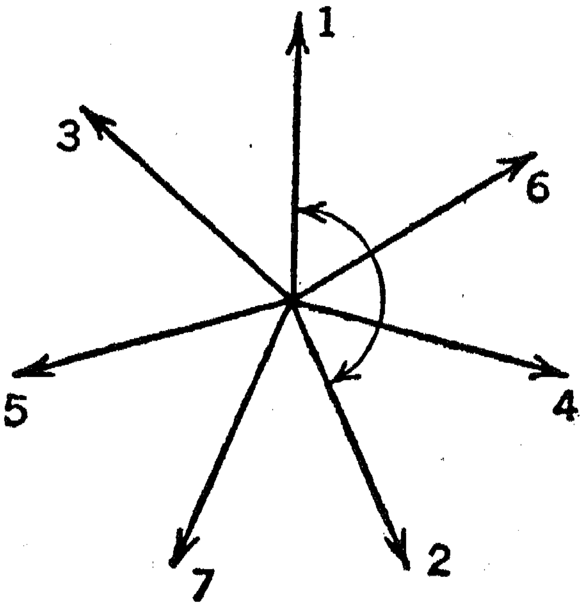

FIG. 8—A TRIPLE-ANGLE SEVEN-PHASE SYMMETRICAL SYSTEM

Theorem 1. An arbitrary system of n unequal and unsymmetrical vectors, *without residue*, may be represented by $(n-1)$ symmetrical multiple n-phase systems. A system without residue is defined as one in which the sum of the vectors is equal to zero. Let the given vectors be $E_1, E_2, \ldots\ldots E_n$, and let the $(n - 1)$ unknown systems be denoted by $A, B, \ldots M$, where A is a single-angle system, B is a double-angle system, etc. It is required to prove that the following equations are consistent and may be solved for $A, B, \ldots\ldots M$:

$$\left.\begin{array}{l} E_1 = A_1 + B_1 + \ldots + M_1 \\ E_2 = A_2 + B_2 + \ldots + M_2 \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ E_n = A_n + B_n + \ldots + M_n \end{array}\right\} \qquad (1)$$

These equations correspond to eqs. (**4**) in Mr. Fortescue's paper when $\Sigma E = 0$. In order to preserve the same conventions as in the original paper, the component systems are assumed to be numbered clockwise (though the vectors are rotating counter-clockwise). Then if a is an operator which rotates a vector by an angle $2\pi/n$ counter-clockwise, the preceding equations become

$$\left.\begin{aligned} E_1 &= A_1 + B_1 + \dots + M_1 \\ E_2 &= a^{-1} A_1 + a^{-2} B_1 + \dots + a^{-(n-1)} M_1 \\ &\dots\dots\dots\dots\dots\dots\dots\dots \\ E_n &= a^{-(n-1)} A_1 + a^{-2(n-1)} B_1 + \dots a^{-(n-1)(n-1)} M_1 \end{aligned}\right\} \quad (2)$$

These n equations contain $(n-1)$ unknown vectors $A_1, B_1, \dots M_1$, but only $(n-1)$ equations are independent of each other, because of the condition $\Sigma E = 0$. To solve for A_1 multiply the second equation by a, the third by a^2, etc. and add them together. The result is

$$n A_1 = E_1 + a E_2 + \dots + a^{n-1} E_n \quad (3)$$

This checks with the second line of Mr. Fortescue's equations (4) By analogy we obtain

$$n B_1 = E_1 + a^2 E_2 + \dots\dots + a^{2(n-1)} E_n \dots\dots \quad (4)$$

and

$$n M_1 = E_1 + a^{n-1} E_2 + \dots\dots a^{(n-1)(n-1)} E_n \quad (5)$$

The theorem is thus proved because equation (**2**) can be solved for the $(n-1)$ unknown vectors in terms of the given E's.

Theorem 2. Referring to theorem 1, if the given system of vectors has a *residue*, say, equal to a vector R, this residue may be split into n arbitrary vectors, $\rho_1, \rho_2, \dots \rho_n$, in phase or out of phase with one another, and equations (**1**) become

$$\left.\begin{aligned} E_1 &= \rho_1 + A_1 + B_1 + \dots\dots + M_1 \\ E_2 &= \rho_2 + A_2 + B_2 + \dots\dots + M_2 \\ &\dots\dots\dots\dots\dots\dots\dots\dots \\ E_n &= \rho_n + A_n + B_n + \dots\dots + M^2 \end{aligned}\right\} \quad (6)$$

This proposition follows at once from the fact that the vectors $(E_1 - \rho_1)$, $(E_2 - \rho_2)$, etc. form a system without residue, so that theorem 1 applies to them. Mr. Fortescue considers only a particular case when

$$\begin{aligned} \rho_1 = \rho_2 = \dots\dots = \rho_n &= R/n \\ &= (E_1 + E_2 + \dots E_n)\ /n \end{aligned} \quad (7)$$

and writes this expression as the first term on the right-hand side in all the formulas of his equations (**4**). This is perfectly justifiable in so far as he is after a symmetrical solution, but there may be practical cases in which a more general representation according to equations (**6**) is preferable. In this case the expressions $(E_1 - \rho_1)$, $(E_2 - \rho_2)$, etc. must be used in equations (**3**) to (**5**), instead of E_1, E_2, etc.

The net result is that only a system without residue may be resolved in a perfectly definite manner, while a system with a residue first must be converted into one without residue by subtracting arbitrary parts of the whole residue from each vector.

With a three-phase system the double-angle component simply becomes a three-phase system of an opposite phase rotation, so that it is not necessary to introduce the question of multiple-angle systems. From a practical point of view the paper would gain much if the author would begin directly with the three-phase system on the basis of his equations (**6**), and leave the general consideration of *n*-phase systems for the end of the paper. For reference purposes one would not have to study the general theory and perhaps get discouraged before obtaining the required bit of specific information.

The introduction of the sequence operator *S* does not seem to be necessary, at least for the three-phase system. A proper cyclic notation should accomplish the same purpose with much less theory and much less writing. As a matter of fact, the index of *S* may be obtained correctly from the subscripts in the expression to which it refers. For example, in Mr. Fortescue's formula (**22**) on the first line the sum of the subscripts of $Z\,I$ is $1 + 2 = 3$, and the order of *S* is 3 or, which is the same, 0. On the second line the sum of the subscripts of $Z\,I$ is $1 + 0 = 1$, and the order of *S* is 1. The same is true for all the fundamental formulas on pp. 1038 to 1042 as well as in applications. Therefore, it would seem that the operator *S* ought to be dropped, and the formulas so rewritten that they would apply to any phase of the system in cyclic rotation.

The fundamental formula (**21**) will then simply become

$$E = Z\,I + W_{+}\,I_{+} + W_{-}\,I_{-} \qquad (\mathbf{8})$$

Here *E*, *Z*, and *I* apply to any of the three phases, I_{+} is the current in the next consecutive phase, and I_{-} is the current in the preceding phase. For example, if *E*, *Z*, and *I* refer to phase *b*, I_{+} refers to phase *c* and I_{-} refers to phase *a*. At the same time the equation is so written that any of the three phases may be taken as the phase under consideration. W_{+} and W_{-} are the corresponding mutual reactances.

When the mutual inductance between a primary and a secondary polyphase circuit must be considered, Fortescue's equation (**28**) applies, and in the simplified cyclic notation it becomes

$$E = X\,J + X_{+}\,J_{+} + X_{-}\,J_{-} \qquad (\mathbf{9})$$

where J, J_{+} and J_{-} are the secondary currents in the three phases. The letter *X* is used instead of *W* to indicate that the mutual inductance is between the phases of the primary and the secondary circuit. *X* without subscript covers the combinations of the "corresponding" phases $a - u$, $b - v$, and $c - w$, while X_{+} refers to the combinations $a - v$, $b - w$, $c - u$ of the phases "tied one forward", and X_{-} refers to the remaining combination of the phases "tied two forward", or "tied one backward".

Each of the quantities in equations (**8**) and (**9**) may be represented by means of symmetrical coordinates in a general manner, without specifying phases *a*, *b* and *c*. Thus,

$$\left.\begin{aligned} I &= I_0 + I_p + I_n \\ I_+ &= I_0 + a\, I_p + a^{-1}\, I_n \\ I_- &= I_0 + a^{-1}\, I_p + a\, I_n \end{aligned}\right\} \qquad \textbf{(10)}$$

Similarly,

$$W_+ = W_0 + a\, W_p + a^{-1}\, W_n \qquad \textbf{(11)}$$

The first equation (**10**) refers to any phase, and states that the current in that phase is a sum of the residue current I_0, plus the vectors I_p and I_n of the symmetrical coordinates *of the same phase*. The subscripts *p* and *n* refer to the systems of positive and negative phase rotation respectively. In an *n*-phase system subscripts 1, 2, 3, etc. should be used in place of $_p$ and $_n$, to indicate single-angle, double-angle, triple-angle, etc. coordinates.

Substituting expressions such as (**10**) and (**11**) into equations (**8**) and (**9**) one obtains comparatively simple polynomials with three groups of terms: those without *a*, those multiplied by *a*, and those multiplied by a^{-1}. These final expressions give the coordinates of *E*, and are the fundamental equations upon which all applications should be based, without any need for the sequence operator *S*.

Incidentally, I believe that the expression "symmetrical components" is a more correct and descriptive term of the method than the term "symmetrical coordinates", and I should like to see it so changed while it is not too late.

I also suggest that the term "mutual reactance" be used throughout the paper in place of the "mutual impedance". The latter term implies a combined resistance and inductance action, while the formulas in the paper seem to refer to the magnetic inductive action exclusively. If the resistance component is intended to be taken into account, a mathematical definition of the mutual impedance becomes necessary, because two coils are presupposed to have different resistances, and it is not clear how these resistances enter into the expression for mutual impedance.

It may also be mentioned here that there are cases of unsymmetrical polyphase connections which can be solved more readily without resolving the given electrical quantities into their symmetrical components. Several such cases are considered in the present writer's work entitled "Ueber Mehrphasige Stromsysteme bei Ungleichmaessiger Belastung (Enke, Stuttgart, 1900). For example, in the solution of any arbitrary *n*-phase star-connected system with given voltages *E* and phase admittances *Y*, it is convenient to take the voltage *e* between the two neutral points as the independent variable. Then, for each phase the current $I = Y\,(E - e)$, and according to the first Kirchhoff law

$$\Sigma\, I = \Sigma\, Y\,(E - e) = 0,$$

so that

$$e = (\Sigma\, Y E)\,/\,\Sigma\, Y.$$

Knowing *e*, the currents in the individual phases are computed from the first equation. In the most general case of a complicated network with a-c. voltages inserted at different places, the Kirchhoff equations of two kinds, written for the vectors of currents and voltages, furnish a complete solution, and one has to consider in individual applications whether or not the resolution into symmetrical components is desirable or not.

A. M. Dudley: Attention has been called to the practical applications of this solution, and one or two may be mentioned here. It has been long known that in the ordinary induction motor in addition to the main rotating working field there may exist several other fields of different amplitude and frequency and phase rotation and that the existence of such fields is probably responsible for the operating freaks sometimes noticed in motors. For example, a motor is sometimes found which will not reverse its mechanical direction of rotation when running light even though the leads be reversed so as to give the opposite direction of phase rotation; or, a two-phase motor will be found which when running light takes power from the line on one phase and returns power to the line on the other, or a, squirrel-cage motor will start from rest with considerably more torque than it is capable of accelerating up to full speed so that it apparently has a so-called sub-synchronous speed. These are actual practical operating conditions often mentioned but never determined quantitatively. Mr. Fortescue's analysis offers the means of such quantitative study.

As a further practical instance may be mentioned the two cases of single-phase connections referred to in the analysis of equations (**141**), (**142**) and (**150**), one of which is equivalent to two coupled polyphase motors connected in parallel and the other to two coupled polyphase motors connected in series. This was fully discussed from another view point in Mr. Lamme's April paper before the Institute and the identity of the two conclusions is a check on Mr. Fortescue's method.

To those who have expressed the idea that the paper is hard to read I would commend Dr. Slepian's discussion also printed herewith as illuminating both to the method and the results. He makes the suggestion that the method as outlined may constitute the basis of a new complex polyphase algebra which will greatly simplify the study of polyphase network problems of all kinds.

Dr. Steinmetz and Prof. Karapetoff have told us that as a mathematical achievement and as a demonstration of theory this paper is a masterpiece. I should like to venture the prediction that as a practical working tool it will eventually come into the greatest usefulness.

Let us then pay our respects to Mr. Fortescue both as a great mathematician and as a scientist and also as the developer of a practical tool which shall make easier the daily task of the engineer in the ranks.

Charles F. Scott: Mathematics and engineering have their varying conditions, and they have their supporters and their critics. To a certain class of engineers the mathematician is an abstract man who starts somewhere and finds something that can lead to consecutive conclusions. On the other hand, a practical man, using that term in the extreme, is apt to rely wholly on experience and to have no use for the mathematics he does not understand. We all know that the really valuable position is the mean between these two, in which the theoretical and the practical do go together.

The reference to papers of a mathematical and explanatory character, and their relations to engineering reminded me somehow of the first Institute paper I came in contact with. It was through the pages of the *Electrical World*. I had been experimenting with alternating currents and was associated with men who knew only slightly more about it than I did. Many things seemed mysterious. Then the paper by Mr. William Stanley on "Alternating-Current Phenomena," appeared, and that mysterious thing which was the stumbling block of about everybody who came up against alternating-current, the simple single-phase phenomenon of current and voltage out of phase, was cleared up. Mr. Stanley drew some triangles and explained a few things. That put me on a new basis. I could see how the mathematical explanation, the physical phenomena, were related, and it gave me a real understanding of the alternating current. A little later came the polyphase, and it was a step from that to the work on the unbalanced polyphase system.

As we advance then, getting into more and more complicated problems, these papers of solution and explanation come to guide the engineer in his work.

What is this paper? When you look at a paper of this sort it looks as if a mathematician had produced it, leading on and on to interesting formulas, one after another. Now, to bring out my point, I want to emphasize and say I do not think this is a product of the mathematician, but of the engineer. Mr. Fortescue did not work these things out from the theoretical, mathematical standpoint, but as an engineer who was confronted with problems in which unbalanced polyphase systems were concerned. It may have been transmission or some kind of particular machinery which involved problems which he did not have the proper tools to solve.

Therefore, in studying the problems and seeing the inadequacy of the mathematical tools at hand, he went to work and constructed tools to perform that work. Mr. Fortescue has done just as some other engineers do, who work first with experiments, then go to mathematics, and then back to the experiments. That combination of first theory and then practise, back and forth, constitutes the work of the engineer who can be constructive and do pioneer work. I think I am right in saying that this is not a paper of a theoretical mathematician, but a paper of a practical engineer, who is developing a new tool for himself and offering it to others.

C. O. Mailloux: This is the second great paper which Mr. Fortescue has presented before this Institute. Much that I had to say in the way of friendly criticism has already been anticipated by Prof. Karapetoff. I sympathize with Prof. Karapetoff's views, and I share them, in regard to the possibilities of simplification of this paper. I thank Mr. Fortescue and many other members of the Institute will also thank him, for having given an introduction to the paper, because that introduction is really the mathematical specifications in accordance with which the paper has been prepared; and to me it was more valuable, as a criterion of the value of the paper, than the paper itself, because the paper itself cannot be read offhand. The bewildering exhibit of subscripts to be found in it is something that will, well, make one pause; but the introduction, to any one who is at all familiar with mathematics, tells exactly the basis on which the paper is written, what it aims to do, and substantially the method by which it accomplished its aim. I am very thankful, indeed, personally, that that was done.

Part I is also very useful, but if it had followed the method of presentation suggested by Prof. Karapetoff, it would have been much simplified. The paper is so valuable, that it would be well worth while to rewrite it for the purpose of making it more easily digestible and more easily accessible to the great body of practical men. It appears to have been written too much from the point of view of the professed mathematician, who seeks first to make a generalization, and then proceeds to experimentation to find practical and special cases. It would have been a better arrangement if that generalization had been put in an appendix, and if the statement had been made that from the general case is derived the particular case which interests us, namely, the case of the three-phase current. I think that to discuss *n-phases* is of academic interest only at present, even though it is possible that there may be some day when it will be of practical importance. Until that seemingly remote day comes, a generalization which includes *n*-phases might as well lie in an appendix, out of harm's way, and especially where it would not encumber the rest of the discussion. If Mr. Fortescue had written the paper by putting into the appendix, the generalization, the thing which makes it complete and comprehensive, he would have simplified it and made it more useful.

It is because I think so highly of the paper, I would like to see it made clearer. It is not a difficult paper, the mathematics are simple, but they look complex, because one is bewildered by a maze of subscripts and by many "operators." The fundamental idea of the operator is a splendid one, but it is just as well to put that explanation in an appendix, in all its generality, and then use it in the simple form that Prof. Karapetoff suggests. Those who deal with the matter practically, would not then have so many equations to analyze and not so many symbols to deal with.

The analysis of the actions and reactions of the various forces present in electrical circuits, and the interpretation of the equations devised for expressing these actions and reactions, under different conditions, whether they occur in balanced or in unbalanced systems, may be facilitated by remembering that all physical forces and all forms of energy are subject to the same general principles of conservation and equilibrium, and, especially, that Newton's principle of dynamic equilibrium,—namely, that under all conditions and at all times, action and reaction are opposed and equal to each other, exactly—, applies to electrical, as well as to mechanical, forces, powers, and energies.

Analogy requires that, in any electrical circuit, at any instant of time whatever, the forces of action and those of reaction should always be opposing and balancing each other exactly, just as they do in mechanical systems; and it is known that they always do balance exactly. Analogy also requires that, for electrical circuits, when dealing with instantaneous values, the equations of electrodynamic equilibrium should have the same general characteristics as the general equation of dynamic equilibrium or balance for mechanical forces. The latter, it is well known, reduces to an algebraical sum of four terms, representing four distinct kinds or amounts of force, or power, or energy, according to the case, whose resultant, when a state of equilibrium occurs, is equal to zero.

The four kinds or amounts of force, power, or energy, in terms of which every dynamic reaction whatever, in mechanics, can be completely expressed, find exact parallels in electrodynamics. Here, also, we find four kinds of force, power, or energy; we have reversible forces of two kinds, corresponding to those which, in mechanics, produce or result from changes in kinetic energy, or changes in potential energy; we have the dissipated force, which is expended in overcoming ohmic resistance, corresponding to the force lost in overcoming friction in mechanics, and we have, for the fourth kind, the force, power, or energy representing the balancing actions or reactions which are necessary to maintain the equilibrium that must exist at every instant of time between action and reaction. The first two kinds represent energy which is not immediately dissipated but is displaced and stored in the system; the third kind represents energy which is immediately consumed in the system; and the fourth kind represents energy which is put into the system from an outside source, or else which is taken out of the system by overflow into another system or circuit, as may be required to maintain equilibrium.

These considerations show that even in an unbalanced electrical system, so-called, there must still be dynamic balance or equilibrium. The balancing force, power, or energy, according to the case, appears in the fourth term, where it represents the compensating force, power, or energy by which equilibrium is maintained.

Charles L. Fortescue: I maintain that the paper is quite simple. The mathematical portion is not difficult to understand, anybody can follow it if he takes the pains. I admit that the appearance of the equations is cumbersome, but that is almost impossible to overcome. The nature of the subject makes the equations cumbersome.

Prof. Karapetoff in his discussion used the word "stimulation". I wish to say that necessarily in a paper of this kind there are many sources of stimulation, as I pointed out in my introduction, many of the ideas are not new. The idea of symmetrical component three-phase systems is being used more or less by others, but the theory has never been presented systematically, and I think the idea of symmetrical operators is new.

Dr. Steinmetz's and Prof. Karapetoff's discussions are what I may term conjugate discussions. Dr. Steinmetz discusses the paper purely from the practical point of view. He thinks that the system is capable of practical application, and will be of great use for that purpose. Dr. Karapetoff points out the possibility, from a theoretical point of view, of the n-phase system. I wish to say nothing would have given me greater pleasure than to go into that purely theoretical matter much more extensively. It is very fascinating and has great promise, but I felt that there was needed, in this paper, some practical justification for presenting it. The theoretical part was so long drawn out, that I felt it was necessary to show by practical illustrations why I presented it.

I felt that the presentation of mathematical solutions did not alone afford a justification for the paper, but that there must also be a good practical reason for it, and I felt if I went into all the theoretical ramifications of this very interesting subject people would ask: "What is he about? What does he mean by driving us through all this painful stuff without giving us a good reason?" So I thought that the presentation of the theory should be as concise and short as possible, and for that reason I left out a good deal of explanation that some people think ought to be there.

Prof. Karapetoff has apparently lost sight of the fact that unless his arbitrary vectors are all equal to $\frac{\breve{E}_1 + \breve{E}_2 + \ldots \breve{E}_n}{n}$ they in turn may be resolved into $(n - 1)$ symmetrical systems and the "residue" will eventually reduce to the same value as that given by me, namely n equal vectors of value

$$\frac{\breve{E}_1 + \breve{E}_2 + \ldots \breve{E}_n}{n}.$$

The form in which the theorem is presented does not preclude the adoption of any artifice such as the subtraction of n arbitrary vectors to make the vector sum equal to zero, when any material advantage is gained thereby. Such an artifice, in fact, is generally adopted in the case of Y-connected circuits where the vector sum is rarely zero on account of the presence of the third harmonic. Furthermore, in any symmetrical system there is a definite impedance to each component symmetrical current including the "residue" as Prof. Karapetoff has termed the constant vector, this is not true if an arbitrary "residue" is adopted.

Prof. Karapetoff brings up another point, the use of sequence symbols. In three-phase systems, once a person has become familiar with the use of this method, it becomes quite a natural system. He thinks quite naturally in terms of symmetrical components, and he usually omits the use of the symbols. In other words, the symbols are not absolutely necessary, but they simply afford a sure method of keeping track of the mechanical part of the mathematical work. If one makes a mistake in carrying out the mathematical work, one can always go back to the sequence symbols and check it up. If a quantity looks unsymmetrical, and a mechanical error is suspected, one can always follow it out with the sequence symbols and be sure of getting the correct result. Furthermore, in dealing with a system of more than three phases, the sequence operator becomes important, because there are so many cross mutual inductances that the formulas become very much involved, and it is necessary to have some reliable mechanical device that will keep one in the right path; the sequence operators are such guides. They are almost indispensable when dealing with power both in three-phase and *n*-phase systems. Those are the principal reasons for their use.

Prof. Karapetoff brings up the question—What is the "mutual impedance?" I have not used the term perhaps quite correctly as I have applied it to cases in which the dissipative forces are not strictly proportional to the velocities or currents. Mutual impedance may be defined in this manner: If we have two terminals of a circuit carrying a given current, and two other terminals of another circuit, the electromotive force produced across the second pair of terminals, given a sine-wave current of a given frequency is, the product of the mutual impedance between the two circuits and the current in the first pair of terminals. In some cases the current follows paths common to both circuits, so that the mutual impedance may have a real component; in other cases the energy component may be due to eddy currents or losses in subsidiary circuits common to both.

Mr. Dudley has pointed out that the practical engineer must always keep track of the theoretical side of his work. The object of all mathematical investigation and theory should be to carry our knowledge of operating conditions further, and to investigate such obscure phenomena as arise from time to time. Very often we find that a new tool has to be devised in order to enable us to carry out our theoretical investigations without becoming involved in too great complication.

I feel that a paper presented before a scientific body that cannot be read by every member is hardly justified. I believe that all can follow this paper if they sincerely try to do so, it may take a little time, but the mathematics is not difficultt.

V. Karapetoff (by letter): In my Theorem 2, ρ_2, ρ_2 etc. are arbitrary vectors such that $\Sigma\ \rho = R$, where R is the total residue of the given system of vectors. Each ρ maybe resolved geometrically into a vector R/n, in phase with total R, and another vector, say ρ, so that $\rho = \rho + (R/n)$. Hence $\Sigma\ \rho + (R/n) = R$, or $\Sigma\ \rho' = 0$. Thus, the system of vectors is without residue and consequently may be resolved into $(n - 1)$ symmetrical systems. The system of n vectors each equal to R/n, in phase with total R, is an *irreducible residue.*

When it is desired to sift out all the symmetrical components from the given vectors, the form R/n for the residue should be used. However, there may be conceivable cases in which there might be some advantage in splitting the residue into certain unequal or unsymmetrical parts, without attempting to extract all of its symmetrical components. For this reason, Theorem 2 is stated in the most general form. I am glad to have my attention called to the fact that only in the case where each ρ is equal to R/n, we have the *true* or the *irreducible* residue.

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SUSTAINED SHORT-CIRCUIT PHENOMENA AND FLUX DISTRIBUTION OF SALIENT-POLE ALTERNATORS

BY N. S. DIAMANT

ABSTRACT OF PAPER

It is shown in Section IV that with the ordinary field forms met with in practise, the resultant flux wave under s. s. c. (sustained short circuits) will be extremely distorted, see Figs. 16, (4th wave) 23, 27, 27A, etc., for the simple reason that with the very low voltages obtained under such conditions, the fundamental of the B-curve of the field is reduced so much by the armature reaction that the higher harmonics assume a very predominant rôle and become several hundred per cent of the s.s.c. fundamental. See Fig 26 and Tables XI, XII, and XIII.

As a corollary to the foregoing it is found that the B-curve under load will not differ radically from the no-load field form since the fundamental will remain large enough to hold its own. See Fig. 22 and also footnote (4). The cross magnetizing effect of the armature reaction is, of course, to make the B-curve unsymmetrical with respect to the mid-pole axis. Compare Figs. 7 and 22.

The magnetic oscillations are studied not only by means of full-pitch stator coils but also by means of rotor coils No. 7 and 8, Fig. 3, and stator coils No. 9, 10, 11, 12, 15, and 16, Figs. 3 and 3A.

Attention is called to the following facts for which explanations and theoretical proofs are offered.

(1) *The ripples at the crest of the e. m. f.* waves of the stator coils are due not so much to the flux pulsations as to (a), the to-and-fro flux swing across the pole, and (b) the cutting of a B-curve similar to wave (b), Fig. 6. See equations 22, and 23; also Table I and Note A, at the end. Compare Figs. 7 and 2nd wave Fig. 4.

(2) *The most important oscillations set up by the armature reaction* are 4 and 6 times machine frequency for two- and three-phase machines respectively. See supplementary Notes B and C; also Figs 8A, 8B wave lengths marked B and D, and Fig. 5, wave length marked B, and Table I.

(3) *Eccentric rotor sets up pulsations* once in every revolution, under load or at no-load, but none under s.s.c. See Fig. 4, 2nd wave, Fig. 5 wave length marked A, and Figs. 8A, 8B, and 8C. Also see Table I, column 4. For a tentative explanation see p 1183.

(4) *The e. m. fs. and fluxes* for coils No. 10 at the top of a tooth are very much larger than the e. m. fs. and fluxes for coil No. 9, at the top of a wedge. See Tables II and III; also Figs. 10, 11, 12.

(5) *The spacing of the ripples of the no-load e. m. fs.* is unequal, contrary to theory. See Fig. 13 and Table IV. Also equations (22) and (23) p. 1195.

(6) *The e. m. fs. and fluxes* for search coil No. 5, at the bottom of the slot, are larger than the e. m. fs. and fluxes for the coil No. 1 at the top of the slot. See Fig. 7 and Tables V, VI, and XV.

(7) *The third harmonic e. m. fs.* of the stator search coils increase with the increasing excitation. See Table IX.

(8) *Under s.s.c. there is present some cross-magnetizing armature reaction* which (a), makes crests 1 and 3 Fig. 27, unequal. See Table XV. (b), it shifts the axis of the flux towards the larger crest, Figs. 23, 27, etc., and thus makes angle B, Fig. 27, less than (1/2) A. See Table XIV.

(9) *The agreement between the armature reaction* obtained by subtracting e_r from e_{fo}, (see Figs. 24, 25, 31 and Figs. 16, 27, etc.) and that obtained by subtracting the equation of the s. s. c. waves as given in Tables XI and XII, from corresponding open-circuit e. m. f. equations as given in Tables VII and VIII, is fairly satisfactory. See last column of Table XIII.

(10) *The direct component of the armature reaction of an imperfect two-phase machine* consists of (a) a regular demagnetizing component, plus (b), a transverse or cross magnetizing component in quadrature to (a). See equations 56, 59, etc.

(11) *The analysis of the no-load field form* gives the 3rd, 5th, and 7th harmonics as 5 to 15 per cent but the $(2q \pm 1)$ ths seem to be small.

(12) *The armature current under s.s.c.* is very nearly sinusoidal. See Figs. 15 and 16. Also Table X. The H-curve of armature reaction set up by such a current will be sinusoidal, see equations 32, 33, etc. and succeeding two paragraphs. The B-curve, however, will not be sinusoidal for lack of constancy and uniformity of the permeability at different points of the magnetic circuit, and on account of saturation. See sections III and IV.

I. INTRODUCTION

IN A paper dealing with the sudden short circuits of alternators, presented at the Panama-Pacific Convention, in passing, the author called attention to the following[1]: First, that the voltages induced in full-pitch stator exploring coils under sudden short circuits "are not of as simple a form as theory would indicate." Second, that the flux distribution within about one or two cycles, after the short circuit takes the general shape which it has under steady or sustained short circuit. The e.m.fs. induced in exploring coils placed at the top, middle and bottom of different slots, all, very shortly after the sudden short circuit, assume a very distorted wave form which persists for the whole length of the film, *i.e.*, about 10 or 15 cycles. The above are clearly seen from Figs. 1 and 2[2] which represent the two extreme cases of symmetrical and totally unsymmetrical armature currents.

The ordinary theories of armature reaction or of short-circuit impedance tests, would hardly lead one to expect results which these oscillograms seem to indicate. A number of valuable investigations on the flux distribution[3] at various loads and power factors have given results quite consistent with those deduced from the usual theory. The flux distribution was in no case sinusoidal, to be sure, but it never deviated greatly from the no-load field form usually met with in practise. This is clearly brought out in a series of interesting oscillograms given by W. J. Foster.[4]

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[1] TRANS., A. I. E. E., 1915. Vol. 34, part II, p. 2259.

[2] See Table III, p. 2255.

[3] *Journal, I. E. E.*, Vol. 37, p. 148. G. W. Worrall and F. T. Wall. See Figs. 2 to 7 inclusive.

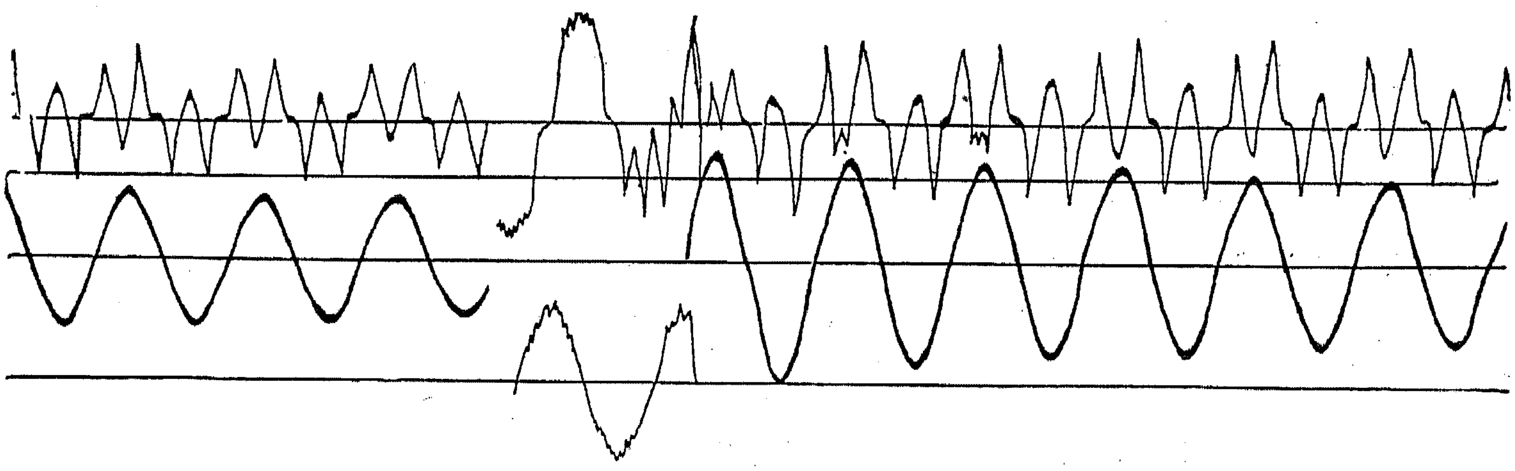

FIG. 1

Oscillogram No. 101. Table III, footnote (2). Machine series Y-connected and suddenly short-circuited at its terminals. Field excitation = 5.1 amperes. Speed = 1200 rev. per min. First wave: E_{1T} = e.m.f. induced in coil No. 1, at top of slot. Second wave: armature current. Third wave: phase voltage. Second straight line: calibration for second wave = 293 amperes.

In view of the above and the entire lack of reliable data, as to the detailed analysis of sustained short-circuit (s.s.c.) phenomena, the writer could make no more definite statement in the paper referred to, than, that "it would seem that a good way to study sudden short circuits was to begin by studying s.s.c. in detail. Although a great deal of work has been done on s.s.c. it is believed that so far little or no attention has been given to the question of flux distribution under such conditions."

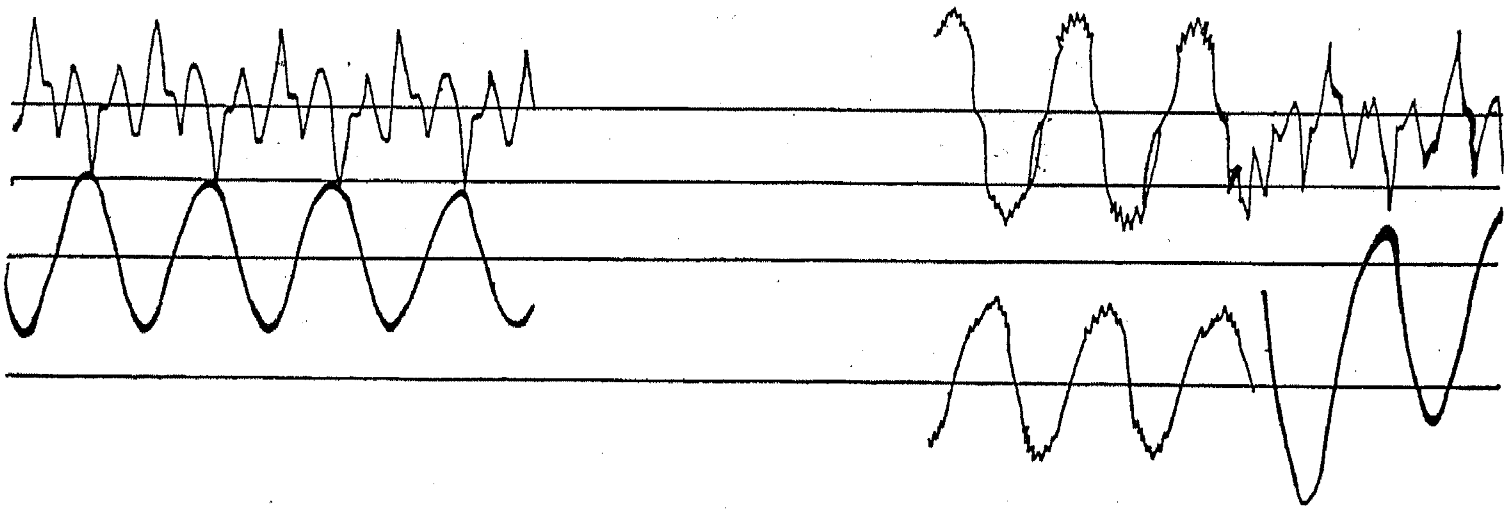

FIG. 2

Oscillogram No. 102. Table III, footnote (2). Machine series Y connected and suddenly short-circuited at its terminals. Field excitation = 5.05 amperes. Normal speed. First wave: E_{3M} =e.m.f. induced in full-pitch coil No. 3 at middle of slot. Second wave: armature current. Third wave: phase voltage. Second straight line: calibration for second wave = 297 amperes.

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[4] TRANS., A. I. E. E., Part I, 1913, p. 749. See Curves 31, 34, 37, 38, 40, 48, etc.

In the following pages an attempt is made to show that all the complicated phenomena of s.s.c. can be explained by taking proper account of certain details and secondary effects. At the end a critical resumé and discussion of results is given which will enable the reader to obtain some of the principal results of the investigation without studying the paper in detail.

II. Data Relating to the Machine, Exploring Coils, etc.

The machine used for the tests is a standard three-phase, six-pole, revolving field alternator rated at 45 kv-a., 480 volts.

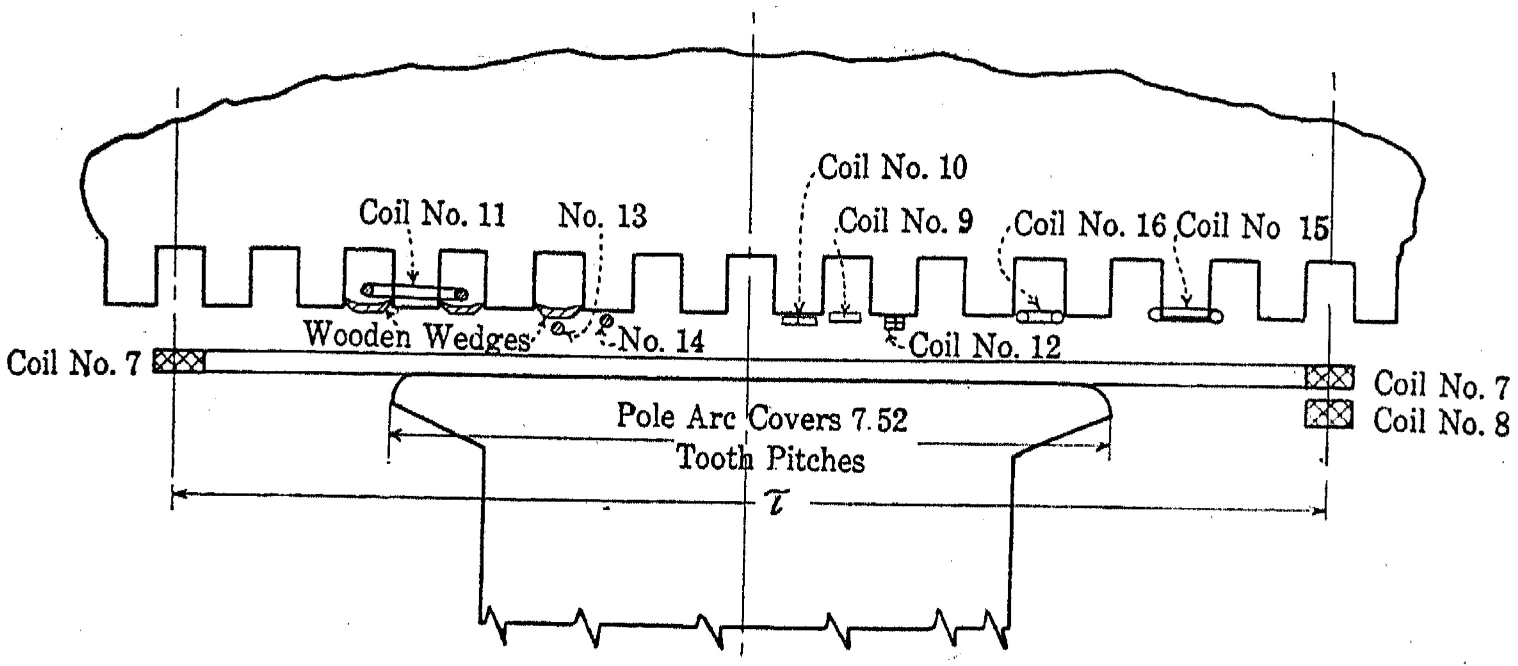

Fig. 3

Coil No. 7: 40 Full pitch concentrated turns.
Coil No. 8: 20 Concentrated turns.
Coil No. 9: 5 Concentrated turns.
Coil No. 10: 5 Concentrated turns.
Coil No. 11: 5 Concentrated turns.
Coil No. 12: 2 Concentrated turns, one on top of other thus having effective pitch of single turn.
Coil No. 13: Single conductor on top of wedge.
Coil No. 14: Single conductor on top of tooth.
Coil No. 15: Single turn around tooth.
Coil No. 16: Single turn around slot.

(278 across phase). It has an amortisseur winding and is equipped with a set of search coils as follows:

Coil No. 1. Full-pitch, 5 concentrated turns at top of slot, right under wooden wedge.
Coil No. 2. Same as No. 1, except that it is $\frac{2}{3}$ pitch.
Coil No. 3. Similar to No. 1, but placed at the middle of the slot.
Coil No. 4. Same as No. 3, except that it is $\frac{2}{3}$ pitch.
Coil No. 5. Similar to No. 1 but placed at the bottom of the slot.
Coil No. 6. Same as No. 5 except that it is $\frac{2}{3}$ pitch.

These coils were placed in different slots. The other search coils are schematically shown and described in Figs. 3 and 3A. The armature winding is of the ordinary double-layer type with a $\frac{2}{3}$ pitch, and it has two circuits which may be connected either in parallel or in series.

In connection with the coils most commonly employed the following notation will be used:

*E_{1T} = e.m.f. induced in coil No. 1. (Subscripts: No. 1 top of slot.)

*E_{3M} = e.m.f. induced in coil No. 3. (Subscripts: No. 3 middle of slot.)

*E_{5B} = e.m.f. induced in coil No. 5. (Subscripts: No. 5 bottom of slot.)

*E_{RF} = e.m.f. induced in coil No. 7, (Subscripts: Rotor full-pitch coil.)

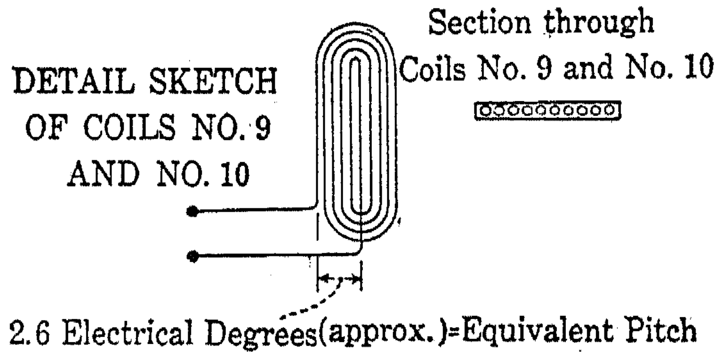
DETAIL SKETCH OF COILS NO. 9 AND NO. 10
Section through Coils No. 9 and No. 10
2.6 Electrical Degrees(approx.)=Equivalent Pitch

FIG. 3A

*E_{RC} = e.m.f. induced in coil No. 8. (Subscripts: Rotor concentrated coil.)

*E_{TW} = e.m.f. induced in coil No. 9. (Subscripts: Coil at top of wedge.)

*E_{TT} = e.m.f. induced in coil No. 10. (Subscripts: Coil at top of tooth.)

*E_{AT} = e.m.f. induced in coil No. 11. (Subscripts: Coil around tooth.)

*E_{TT2} = e.m.f. induced in coil No. 12. (Subscripts: 2 turns on top of tooth.)

*E_{CTS} = e.m.f. induced in coil No. 13. (Subscripts: Conductor on top of slot.)

*E_{CTT} = e.m.f. induced in coil No. 14. (Subscripts: Conductor on top of tooth.)

B = Flux density as further specified by its subscript.

f = (actual) machine-frequency in cycles per second $=$ r.p.s. x p.

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*All under conditions to be specified.

k = Order of space harmonic. See p. 1164 for explanation.
p = Number of pairs of poles.
p_k = Number of pairs of poles of k^{th} space harmonic $= k\,p$
q = Number of slots per pole.
s.s.c. = sustained short circuit.
τ = Pole pitch in centimeters or inches.
τ_k = Pole pitch of k^{th} space harmonic $= \tau/k$.
$e_{rot.st.}$ = Instantaneous value of rotational e.m.f. induced in stator coil.
$e_{pul.st.}$ = Instantaneous value of e.m.f. induced in stator coil due to pulsations of flux. (In magnitude only.)
$e_{sw.st.}$ = Instantaneous value of e.m.f. induced in stator coil due to to-and-fro swing of flux. (Without change in magnitude.)
$e_{pul.r.}$ = Same as $e_{pul.st.}$ except that it refers to coil on rotor.
$e_{sw.r.}$ = Same as $e_{sw.st.}$ except that it refers to coil on rotor.

III. Flux Distribution at Various Loads and Power Factors

The oscillographic study of flux distribution in alternators and other apparatus by means of exploring coils would have been extremely simple, were it not for the fact that the quantity recorded on the film is not the flux wave in which we are interested but a quantity proportional to its *time rate of change.*

A conductor moving across a magnetic field, constant both in time and space, *i.e.*, a field constant in magnitude and stationary with regard to the pole axis, will have an e.m.f. induced in it which may be designated as motional or rotational e.m.f.; this will be maximum when an element $d\,l$, of the conductor under consideration, its velocity v, and the flux density B_m, all three, are mutually perpendicular. However, without any relative motion between flux and conductor, if the flux enclosed by the circuit varies for any cause and in any manner whatsoever, there will be an e.m.f. induced proportional to the rate of change of flux. This may be called static or variational e.m.f.

Thus in the most general case,[5] which is the one met with in practise, the e.m.f. induced in a conductor or coil is:

$$e_{\text{total}} = e_{\text{motional}} + e_{\text{variational}}\,,$$

where e_{total} is the quantity recorded on the film.

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[5] See, in this connection, A. Blondel, *Sur l'énoncé le plus général des lois de l' induction.* Compt. Rend. 1914, Vol. 160, Nos. 20 and 22.

The application of the above to the analysis and interpretation of the complex waves obtained under s.s.c. or even the simpler phenomena of open circuit is difficult. Therefore, it will be well to call attention at first, to some general facts which though not entirely new do not seem to be generally known.

It is clear that the e.m.f. induced in a conductor or a number of concentrated turns (on the armature) will be a true reproduction of the B-curve under the pole, provided the variational e.m.f. were zero—which is seldom the case. The variational or static e.m.f. will depend upon the manner in which B varies with respect to time only. As a result of some valuable researches[6] it has been found that B may vary with respect to time in two ways: First, due to what may be termed the inductor alternator action of the teeth, there may be a *variation in the amount of reluctance* of the magnetic circuit of the machine. This will cause pulsations in the intensity of B and thus an e.m.f. will be induced in the armature coils as well as the pole pieces, yoke, etc. The latter will give rise to parasitic eddy currents which will tend to wipe out the pulsations. Thus, as indicated by theory and as shown by test, the variational e.m.f. due to this cause will be very small (compared to the rotational e.m.f.). Secondly, it has been found that the flux density at any point under the pole may vary due to a *to-and-fro swing* of the flux-wave with respect to the (geometrical) pole-axis. It has been shown by Worrall[6] by trimming the pole-shoes and varying the number of slot-pitches covered by it, that the pulsation is maximum when the pole-arc covers a whole number of slot-pitches, while the flux-swing is maximum when the pole-arc covers (an integer $+ \frac{1}{2}$) slot-pitches. This neglects fringing and in the experiments referred to, it seems to hold remarkably well. However, it seems to the writer that fringing should not be neglected since it is not the geometrical tooth or slot but the manner in which the flux is distributed in these that is responsible for the static e.m.f. Thus, his conclusions seem to be correct, except that it would be more rational to consider the effective teeth and slots (so far as flux is concerned).

The study of the experimental results as well as the quantitative expressions developed in the supplementary notes show that the e.m.f. waves of coils No. 7 and No. 8 give chiefly the

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6. G. W. Worrall, *Jour., Inst. E. E.*, 1907, Vol. 39, p. 206. Also 1908, Vol. 40, p. 413. Guery, *L' Eclairage Electrique*, 1903, Vol. 36, p. 51. Arnold and La Cour, *Samn. Elek. Vort.*, 1901, Vol. 3, p. 58. K. Simon, *E. T. Z.*, 1908, Vol. 27, p. 631. *Electrician*, Vol. 57, p. 581.

e.m.f. due to flux pulsations. See second wave Fig. 4 and Fig. 5. The e.m.f. due to the to-and-fro motion of the flux is given by: (for $x_1 = 0$ see Fig. 6 and note A)

$$e_{sw.r.} = 2\,N\,l\,q\,\omega\,\epsilon^2\,B_1 \sin(4\,q\,\omega\,t) \qquad \textbf{(22)}$$

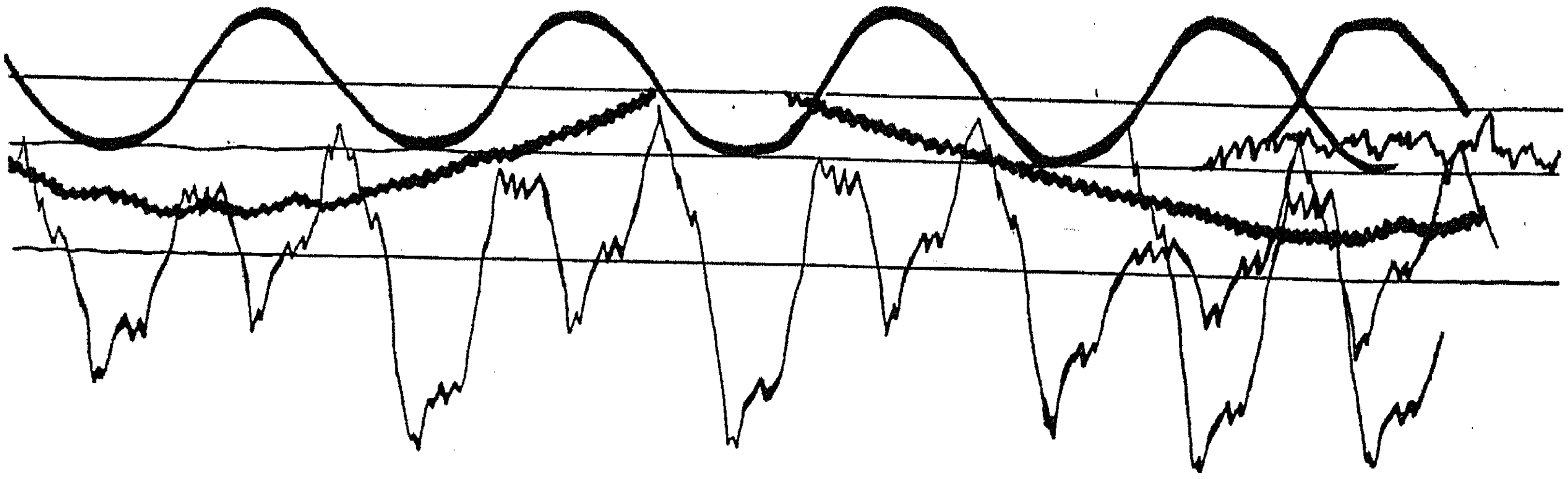

FIG. 4

Oscillogram No. O A 84. Test No. 196. First wave: armature current under s.s.c. at 4.96 amperes field excitation, normal speed and machine connected 2-phase. Third wave: E_{RF} = e.m.f. in full-pitch 40-turn rotor coil No. 7 under same conditions. Second wave: E_{RF} at same excitation and speed at no-load.

and that due to the pulsations of flux by:

$$e_{pul.r.} = 4\,N\,l\,q\,\omega\,\eta\,B_1 \sin(2\,q\,\omega\,t) \qquad \textbf{(23)}$$

where η and ϵ are small fractions denoting the amplitude of the variations of flux, in each case. The former would be expected to be small as it involves ϵ^2 and oscillograms give no indication of the existence of ripples of ($4\,q$ × machine-frequency), as required by (**22**). As seen from Fig. 4, second wave, and Fig. 5, the little kinks in the E_{RF} wave (due to the pulsations of flux) under synchronous motor and no-load operation, are fairly small and have a frequency of $2\,q\,f$, *i.e.*, show $2\,q = 24$ ripples per cycle. See Table I, column 6. This is what would be expected from (**23**).

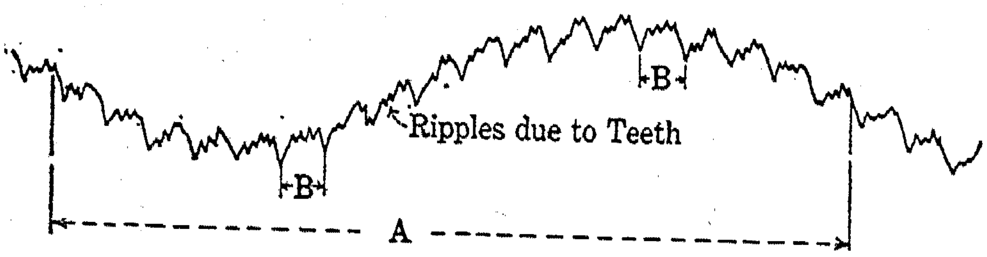

FIG. 5

Oscillogram No. O A 21. Test No. 91. Table I. Machine parallel Y-connected and run as synchronous motor. Field current = 4.4 amperes. Line current = 15.5 amperes. Line voltage = 220. E_{RF} = e.m.f. induced in rotor coil No. 7. Note: A = 1080 deg. B = 60 deg.

While it is clear that the kinks in the E_{RF} wave, Figs. 4 and 5, are due to the teeth, the long flat superimposed wave having a period A, is believed to be due to the rotor being eccentric with respect to the stator. This explanation seems the more plausible when it is considered that A, Fig. 5, is found to be 1080 electrical degrees, as indicated in Table I column 4; *i.e.*, the superimposed wave is found to have under load or at no-load a frequency $(\frac{1}{3})\ f = (1/p)\ f =$ one cycle per revolution.

TABLE I.

Data relating to the e.m.fs. induced in rotor exploring coils under conditions given below.

No. of oscill. / No. of test	E_{RF} / E_{RC}	Field current in amps.	A in degrees see Fig. 5	B in degrees see Fig. 8	No. of ripples per cycle	Freq. in cycles per sec.	Amplitude in max. volts see Figs. 8a, b, c pos.	Amplitude in max. volts see Figs. 8a, b, c neg.	Line current in amps.	Approx. power factor
1	2	3	4	5	6	7	8	9	10	11
Synchronous Motor Operation										
O A 21/91	E_{RF}	4.4	1080	54.8	24	59.5	0.69	0.82	15.5	1
O A 22/92	E_{AP}*	4.4	1080	60		59.5	0.66		15.2	1
O A 39/115	E_{AP}*	1.0	1080		24	59.5	1.12	mean	79 †	0.27 lg.
O A 40/116	E_{RC}	8.3	1080	61.3		59.5	1.12	mean	76	0.28 ld.
O A 36/118	E_{RC}			61	24	59.5	0.037	0.053	85.5	0.25 lg.
O A 37/109	E_{RC}	8		60.6	24	59.5	0.037	0.053	71.2	0.24 ld.
Sustained Short Circuit. Series Y Connection										
–/164	E_{RF}	5			24	54	0.97 mean			
–/165	E_{RC}	5			24	50	0.03 mean			
–/166	E_{RC}	3.16			24	52	0.027 mean			
–/167	E_{RF}	3.16			24	52	0.71 mean			
226/–	E_{RF}	5		59.2	24	60				
Open-Circuit Operation										
–/163	E_{RF}	5				60	2.46			
–/162	E_{RF}	3.16				60	1.94			
–/161	E_{RF}	5				54	2.31			
–/160	E_{RF}	3.16				54	1.98			
O A 24/94	E_{RF}	5	1080	60	24					

*E_{AP} = e.m.f. induced in search coil around pole, right under pole-shoe; this gave practically same results as full-pitch coil No. 7 and therefore it was disconnected from slip-ring and not used.

†lg. = lagging. ld. = leading.

For a conductor placed at the middle of the pole-arc, *i.e.*, at $x_1 = \tau/2$, Fig. 6, $e_{sw.r.}$ would be maximum and $e_{pul.r.}$[7] would be zero; *i.e.*, such a conductor would give ripples due to the flux-swing only. Attempts were made to take advantage of this but owing to the high centrifugal force it was found difficult to fasten down securely wires at the mid-pole position.

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[7] See equations (**21**) and (**20**) respectively in supplementary notes.

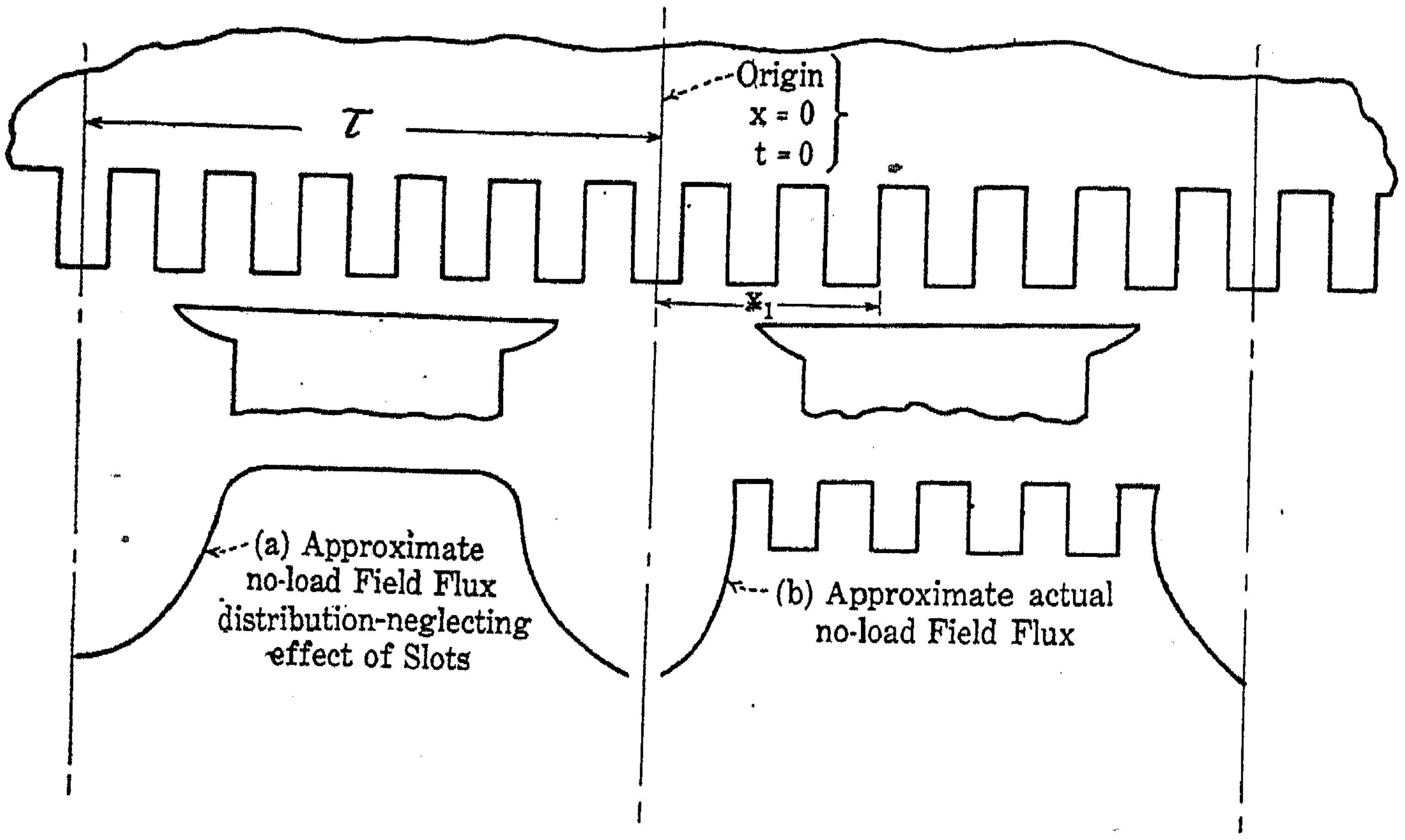
τ
Origin
x = 0
t = 0
x_1
(a) Approximate no-load Field Flux distribution-neglecting effect of Slots
(b) Approximate actual no-load Field Flux

Fig. 6

However, a very good idea of the relative magnitude of the e.m.f. due to flux-swing, can be obtained by studying the e.m.f. waves induced in the stator coils. These will consist of (e_{rot} + e_{pul} + e_{sw}). As indicated by equations (**7**), (**13**) and (**18**) Note A, e_{pul} will be negligible for a rotor coil placed at the middle of the pole-shoe, *i.e.*, at $x = \tau/2$, or for a stator coil

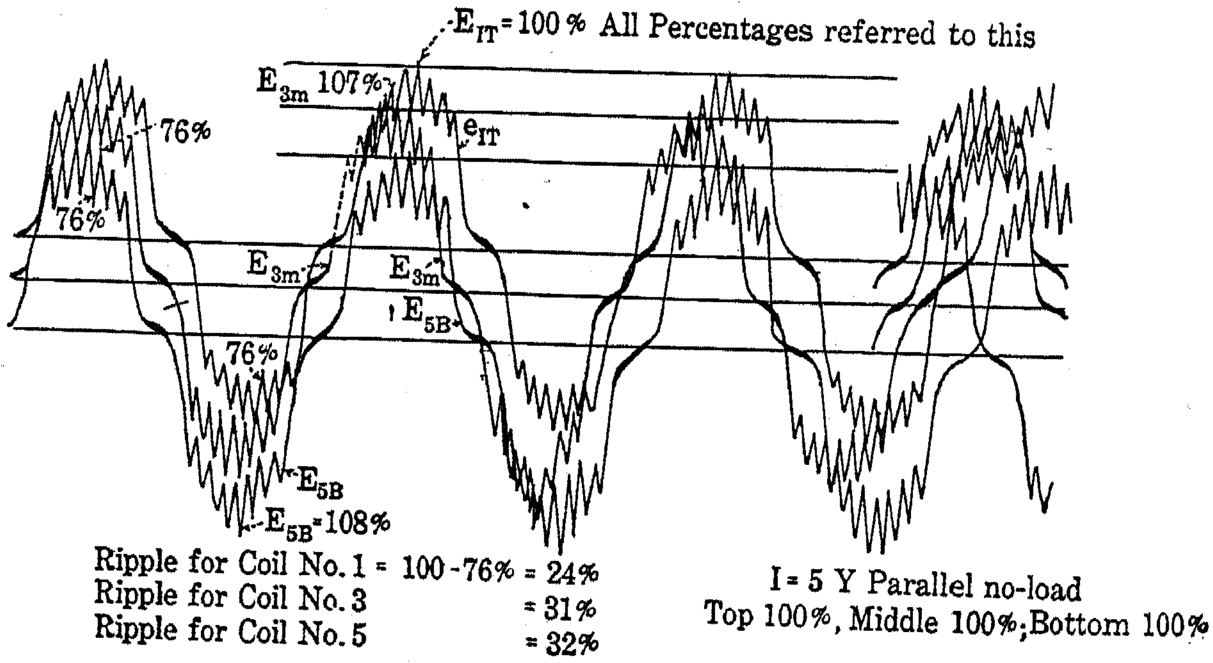

Fig. 7

Oscillogram No. O A 38. Test No. 110. Tables IV, V and VI. Machine run on open-circuit. First wave: E_{1T} = e.m.f. induced in exploring coil No. 1 at top of slot. Second wave: E_{3M} = e.m.f. induced in exploring coil No. 3, at middle of slot. E_{5B} = e.m.f. induced in exploring coil No. 5, at bottom of slot. Speed normal, *i.e.*, corresponding to 60. Excitation = 5 amperes. Same vibrator and resistance in vibrator circuit used for all waves. Compare to Fig. 22.

whenever $x = \tau/2$, see Fig. 6. In other words, e_{pul} will be negligibly small *whenever* $e_{rot.st.}$ is maximum. Thus the ripples at the crest of the e.m.f. wave $(x = \tau/2)$ of the stator coils 1, 3 and 5 are due mainly to flux-swing and a comparison of the ripples in them, Fig. 7, with the kinks of the E_{RF} wave, second wave Fig. 4 and Fig. 5, and E_{RC}, Fig. 8a, will show at once that the kinks (due to pulsation) are quite negligible compared to the ripples (due to flux-swing)—especially when it is remembered that coils No. 8 and 7 have many more turns than the stator coils and the volts per centimeter for the calibration of the former is about ten per cent of that for the latter.

The subject of tooth ripples is of great practical importance, not only in case of accurate tests and measurements but also in connection with the operation of large alternators, induction motors, etc. Though effective methods have been developed for their elimination there seems to be lack of general concensus of opinion[8] as to their origin and detailed analysis. In this connection it seems pertinent to ask, whether the ripples are entirely variational in their origin or are they partly motional. The actual flux density in the air-gap has the general form indicated by curve (*b*) rather than curve (*a*) Fig. 6. Since the motional

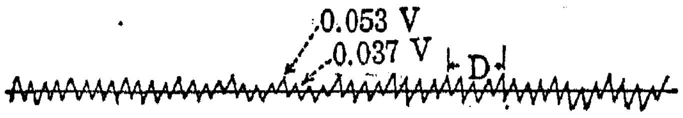

FIG. 8A

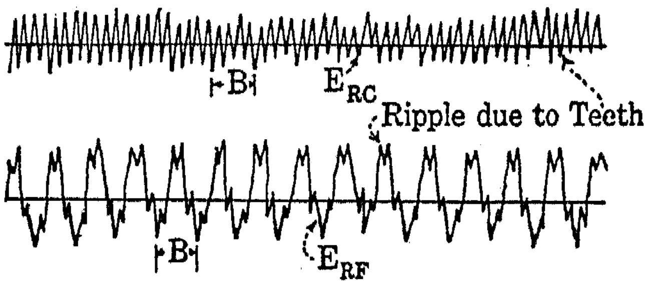

FIG. 8B

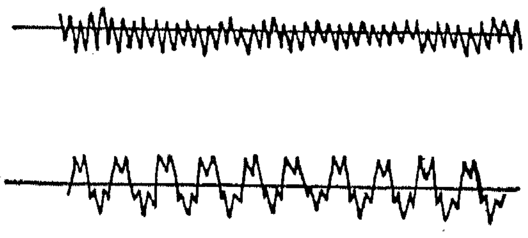

FIG. 8C

Fig. 8a Oscillogram No. O A 36. Table I. Machine run as synchronous motor. Line current 85.5 amperes lag. Line voltage = 200. E_{RC} = e.m.f. induced in coil No. 8. Exactly same wave shape and amplitude obtained in oscillogram No. O A 37 with 71 amperes of leading line current and 238 line voltage.

Fig. 8b Oscillogram No. O A 49. First wave E_{RC} = e.m.f. induced in coil No. 8. B = 60 deg. Second wave: E_{RF} = e.m.f. induced in coil No. 7. B = 60 deg. Machine series Y-connected and short-circuited at its terminals. Field current = 5 amperes. Frequency = 54 cycles per second. Resistance in vibrator circuit = 0 ohms and gold-leaf fuses short-circuited.

Fig. 8c Oscillogram No. O A 50. Similar to O A 49 in every respect, except field excitation = 3.16 amperes. Same vibrators used.

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[8] S. P. Smith and R. H. Boulding, *Jour., I. E. E.*, 1915, Vol. 53, p. 205. Discussion p. 238. Also see correspondence *Electrician*, June, 1914. Further, see references given under foot-note (6).

e.m.f. will be duplicate of the B-curve, it is clear that the tooth ripples will partly have motional origin, *i.e.*, will be partly due to the cutting of a B-curve like that shown in (*b*) Fig. 6.

A general idea of the relative importance of the *tooth kinks in the B-curve* may be gained from Fig. 9 which is self-explanatory. The flux measurements were obtained by pulling out of the air gap an exploring coil fastened on a piece of cardboard and having a length approximately equal to the axial thickness of the armature core, and a width about 30 per cent of that of a tooth. In the test recorded in Fig. 9, the machine was stationary; in the next one, the rotor was excited and slowly rotated and the following maximum throws of the Grassot fluxmeter were observed.

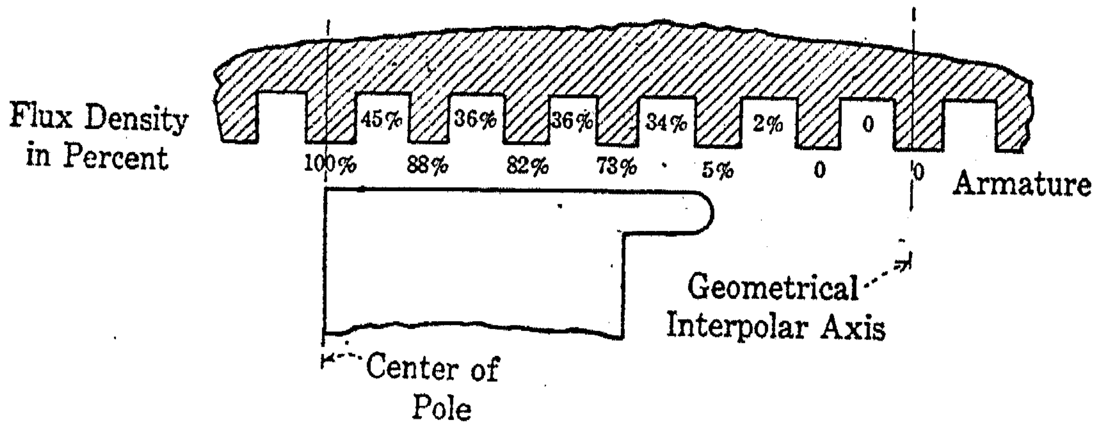

FIG. 9

Flux distribution with the machine stationary as measured by Grassot fluxmeter.

Coil No. 10—coil at top of tooth—maximum deflection = 100 per cent.

Coil No. 9—coil at top of wedge—maximum deflection = 41 per cent.

Coil No. 8—concentrated coil—maximum deflection = 8 per cent

The actual flux density under running conditions was further investigated by means of the exploring coils No. 9, 10 and 11, Figs. 3 and 3A, by taking oscillographic records of the e.m.fs. induced in them under no-load, s.s.c. and synchronous motor operation. The data obtained in this series of tests are brought together in Table II. As seen from Figs. 10, 11 and 12 the E_{AT}, E_{T1} and E_{TW} waves have two prominent crests above, and two below, the zero line, per cycle. These are quite characteristic and are designated as 1st and 2nd positive and 1st and 2nd negative. See Table II, columns 8 and 11. The angular distance between successive + or − crests is designated by α and that between successive positive and negative ones by β. See Fig. 10 and Table II columns 6 and 7.

TABLE II.—Data relating to the e.m.fs. induced in Exploring Coils No. 9, No. 10 and No. 11 (Fig. 3) under open-, sustained short-circuit and synchronous motor operation.

No. of oscillograms / No. of test	$E_{A.T.}$ $E_{T.T.}$ $E_{T.W.}$	No load, s.s.c. or synch. mot. / field amperes	Line current in amperes	Approximate power factor leading or lagging	β in degrees See Figs. 10, 12, etc.	α	Consecutive crest values in max. volts per turn			
							Positive crests		Negative crests	
							1st	2nd	1st	2nd
1	2	3	4	5	6	7	8	9	10	11
O A 25/95	$E_{A.T.}$	N. L./5	0		118°	61°	1.47	1.39	1.49	1.27
O A 61/172	$E_{T.T.}$	N. L./4.95	0		120.5°	59.5°	0.508	0.422	0.466	0.416
O A 20/90	$E_{A.T.}$	S. M./4.49	15.5	0.95 lg.	117°	59.9°	1.18	1.24	1.02	1.23
O A 27/97	$E_{T.T.}$	S. M./4.24	15.5	1.0	116.5°	62.3°	0.33	0.33	0.38	0.34
O A 27/97	$T_{T.W.}$	S. M./4.24	15.5	1.0	119°	59.4°	0.15	0.16	0.15	0.15 +
O A 23/93	$E_{A.T.}$	S. M./3	18.5	0.81 lg.	113°	65.1°	1.19	1.07	1.05	1.19
O A 28/98	$E_{T.T.}$	S. M./2.9	39.5	0.56 lg.	119°	62.2°	0.24	0.28	0.24	0.29
O A 28/98	$E_{T.W.}$	S. M./2.9	39.5	0.56 lg.	115°	60.9°	0.12	0.14	0.12 +	0.14 +
O A 26/96	$E_{A.T.}$	S. M./	40.7	0.4 ld.	124.8°	58.5°	1.41		1.44	1.47
O A 29/99	$E_{T.T.}$	S. M./5.4	39.5	0.45 ld.	118.4°	62.15°	0.47	0.48	0.48 +	0.48 −
O A 29/99	$E_{T,W.}$	S. M./6.4	39.5	0.45 ld.	119.5°	61.7°	0.19	0.196	0.194	0.194
O A 37/109	$E_{A.T.}$	S. M./8	71.2	0.24 ld.	121.5°	57.3°	1.47	1.47	1.51	1.51
O A 36/108	$E_{T.T.}$]	S. M./−	85.5	0.25 lg.	117.5°	63.3°	0.71 +	0.94	0.71 +	1.0 −
O A 41/117	E_{TT2}	N. L./ −	0		117°	62.4°	0.017	0.014		
O A 41/117	E_{TT2}	S. M./ −	24.7	0.93 lg.	116°	61.4°	0.041	0.034		
O A 40/116	E_{TT2}	S. M./−	76	0.26 ld.	120°	61.5°	0.045	0.047	0.045	0.047
O A 39/115	E_{TT2}	S. M./−	79	0.27 lg.	111.5°	62.6°	0.022	0.033	0.022	0.033
		Field amperes	Kind of s.s.c.	Frequency						
O A 61/172	E_{TT}	4.95	Y series	60	118.4°	61.6°	0.23	0.187	0.25	0.187
O A 59/180	T_{TW}	5.0	Y series	60	119°	60.1°	0.18	0.12	0.16	0.12

NOTE: α and β are in terms of machine-frequency. Thus if one cycle at machine-frequency occupies m cms. on the film and α is n cms., then $\alpha = \frac{n}{m} \times 360$ deg.

The peculiar shape of these curves may be explained as follows: As a concentrated coil like No. 9 or No. 10, moves past a (stationary) field pole, the amount of change in the enclosed flux will depend upon the tooth kinks of the B-curve, (b) Fig. 6. If the B-curve were flat topped, see (a) Fig. 6, there would be no change of flux through coils No. 9 or 10 while under the pole-arc. Thus the small ripples in the portion of the B-curve marked β, Fig. 10, are due partly to the bunching of the flux at the teeth and partly due to the flux variations (= pulsations in magnitude, and to-and-fro swing), as already explained. As the coil emerges from under the horn of, say, a north pole and begins to enter the interpolar region and then the tip of the south pole, there will be, two, comparatively large changes in the flux enclosed; these flux changes will give rise to two peaks whose angular spacing α, as seen from Table II is about 60 elec. deg. *i.e.*, α corresponds to the angular distance between two consecutive pole-horns. Evidently these crests should be on the same side of the zero line since one is due to a decreasing flux issuing from a north pole, while the other is due to an increasing flux of opposite polarity.[9]

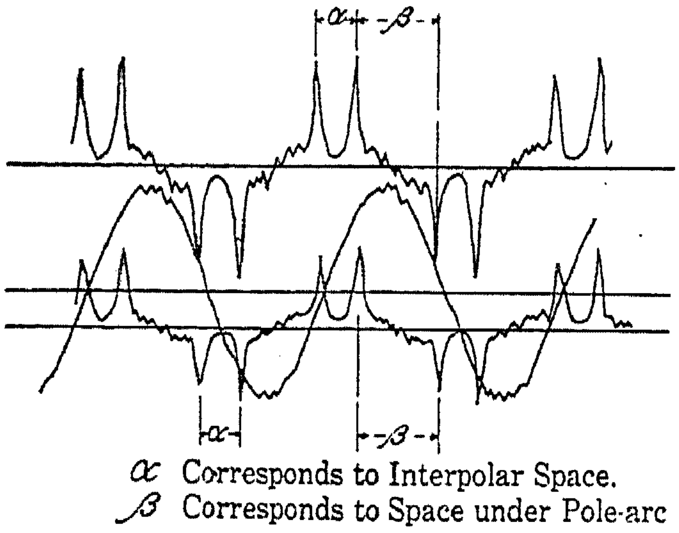

FIG. 10

Oscillogram No. O A 28. Test No. 98, Table II. First wave: E_{AT} = e.m.f. induced in coil No. 11 around tooth. Second wave: volts across the terminals of the machine. Third wave: E_{TW} = e.m.f. induced in coil No. 9 on top of wedge. Machine parallel Y-connected and operated as a synchronous motor. Line current = 39.5 amperes. Line voltage = 212 volts. Power factor = 56 deg. lag. α corresponds to interpolar space. β corresponds to space under pole-arc.

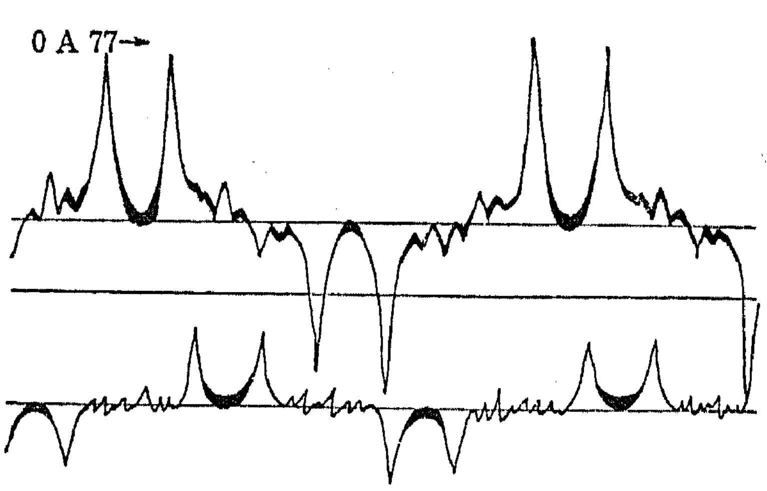

FIG. 11

Oscillogram No. O A 77. Test No. 189. First wave: E_{TT} = e.m.f. induced in search coil No. 10 at top of tooth. Second wave: E_{TW} = e.m.f. induced in exploring coil No. 9 at top of wedge. Machine run at open-circuit and same vibrator and same resistance in vibrator circuit for both waves.

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[9] See curve 32 given by W. J. Foster, TRANS., A. I. E. E., 1913, Vol. 32, p. 754.

At first sight it might be expected that the e.m.fs. induced in coils No. 9 and 10 should be similar in shape and equal in amount since the only difference between them is that one is placed on top of a tooth, while the other is on top of a wedge. However, as seen from Table II this is by no means the case; further as seen from Table III the area enclosed by E_{TT} for the coil at the top of a tooth, is very much larger than E_{TW} for the coil at the top of a wedge. This can be explained on the assumption that a given peak is due to the change of the enclosed flux to nearly zero, from the value it had just before the coil left the horn of the pole. This change will be much larger for a tooth than for a slot. The above, in connection with Tables II and III, give a fairly good idea of the flux distribution and flux pulsations under actual running conditions, in contrast to the phenomena which obtain with the machine stationary and which are easily investigated, as indicated in Fig. 9.

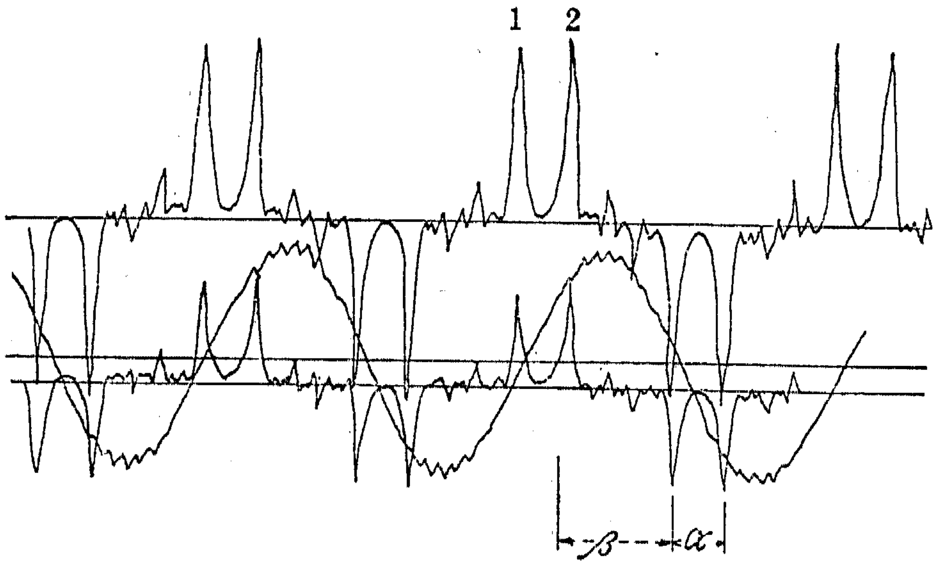

FIG. 12

Oscillogram No. O A 29. Test No. 99. Table II. Machine Y parallel connected and run as a synchronous motor. Field excitation, 6.4 amperes. Line voltage = 237 volts Line current = 39.5. Approximate power factor = 0.45 leading. First wave: E_{TT} = e.m.f. induced in coil No. 10 at top of tooth. Second wave: Line voltage. Third wave: E_{TW} = e.m.f. induced in coil No. 9 at top of wedge.

TABLE III.

Data relating to synchronous motor operation of machine and giving the area per one-half cycle per turn enclosed by the e.m.f. waves induced in coils No. 9 and No. 10.

No. of oscill.	Field amps.	Line amps.	Line volts	Power factor	Maxwells or volts per turn per ½ cycle
O A 29	6.4	39.5	237	0.45 ld.	45,000, coil No. 10
O A 29	6.4	39.5	237	0.45 ld.	12,036, coil No. 9
O A 27	4.24	15.5	218	1.00	59,400, coil No. 10
O A 27	4.24	15.5	218	1.00	29,800, coil No. 9

Study of Table II and the oscillograms given in connection therewith reveals several interesting facts. A comparison of Figs. 10 and 12 will show that the kinks between a + crest and a − crest, *i.e.*, under the pole-arc, are larger when the armature current is leading or demagnetizing than when it is lagging or magnetizing. Both E_{TT} and E_{TW} are larger for a given armature current when it is leading than when it is lagging; for example at 39.5 amperes of line current (under synchronous motor operation) $E_{TT} = 0.24$ and $E_{TW} = 0.12$ volts per turn. At 39.5 amperes of condensive current the volts per turn increase to 0.47 and 0.19 respectively. The field excitation in the former case was 2.9 amperes and the terminal voltage 212 (effective volts per phase); the corresponding quantities for the latter case being 6.4 amperes and 237 volts. In the light of the above, (see Table II) the explanation seems to be that in spite of the demagnetizing action of the leading currents the flux density in the gap increases on account of the higher excitation and voltage.

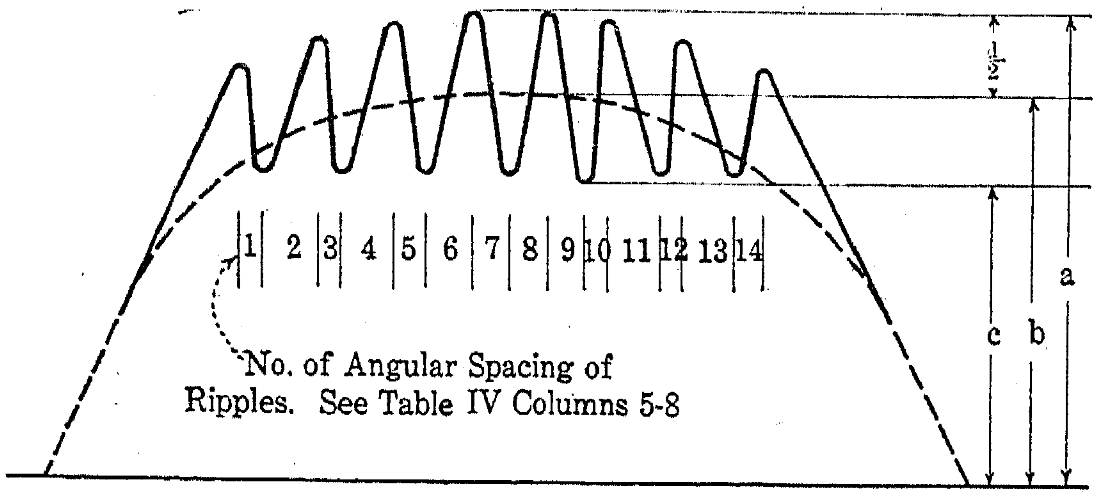

FIG. 13

Exaggerated figure showing e.m.f. induced in exploring coils at no-load and unequal spacing of tooth ripples.

Under s.s.c. the crest voltages E_{TT} and E_{TW} do not differ as widely which is no doubt due to the very peculiar and distorted flux distribution as will be explained in the next section.

In connection with the subject of tooth ripples, attention may be called to several other peculiarities. As seen from Table IV and indicated in Fig. 13, the angular spacing of the tooth ripples from crest to crest is not uniform as ordinary theory would indicate. (See equations **13** and **14**). In the first place, the angular spacing (1 + 2), (3 + 4), etc. Fig. 13, is slightly less than π/q, or in this case 15 electrical deg. Secondly the angular spaces 2, 4, 6, 11, and 13 are larger than 1, 3, 5, etc.; this is probably due to the hysteresis effect in the teeth. Thirdly, the amplitude *d*, Fig. 13 is distinctly smaller for E_{1T} than for E_{5B}

this is easily seen from Fig. 7 in which the waves were taken under the same operating conditions, using the same vibrator and the same vibrator resistance. A quantitative idea of the variations may be gained from Table V.

TABLE IV.

Angular spacing in degrees (electrical) of the maximum and minimum crests of the tooth ripples of the no-load search coil e.m.fs.

No. of oscill.	Search coil	Field amperes	Mean angular spacing of (see Fig. 13)			
			1, 3, 5 & 7	2, 4 & 6	(1 + 2), (3 + 4), etc.	(2 + 3), (4 + 5), etc.
1	2	3	4	5	6	7
272	No. 1, top of slot	8.8	6	8.5		
O A 35	No 1, top of slot		7.15	7.35	14.62	13.29
O A 35	No.2, midd. of slot		6.46	7.81	13.9	14
O A 35	No. 5, bott. of slot		6.94	7.21	14	14.2
O A 38	No. 1, top of slot	5	6.64	7.45	14.18	14.4
O A 38	No.3,midd. of slot	5	6.64	7.08	13.77	14
O A 38	No. 5, bott. of slot	5	6.84	8.03		
260	No. 1, top of slot	5	6.41	9.13		
263	No. 3,midd.of slot	4	6.03	8.43		
268	No.5, bott. of slot	5	6.37	8		
Mean:			6.55	7.89	14.09	13.18

In Table VI are given the areas enclosed per ½ cycle by E_{1T}, E_{3M} and E_{5B}, and it is seen that for the bottom coil No. 5 the flux is 107.5 per cent of that enclosed by the top coil. Under synchronous motor operation the percentage is smaller, about

TABLE V.

No. of oscillograms	E.m.f. of search coil	$\frac{d}{2} \times \frac{1}{b} \times 100$ See Fig. 13	Field amperes	Frequency cycles per second
272	E_{1T}	7.5%	8.8 appr.	50
272	E_{5B}	8.3%	8.8	50
O A 38	E_{1T}	7.2%	5	60
O A 38	E_{3M}	7.9%	5	60
O A 38	E_{5B}	9.2%	5	60

103 per cent for leading, *i.e.* demagnetizing, and larger, about 113 per cent for lagging currents.

These phenomena may be explained on the basis of the following assumptions. Consider two surfaces, one passing through the top and the other through the bottom of the armature slots.

TABLE VI.

Mean areas of consecutive positive and negative half-waves of e.m.f. induced in full-pitch, 5-turn exploring coils at top, middle, and bottom of slots = E_{1T}, E_{3M} and E_{5B} respectively.

No. of oscill. test No.	E_{1T} E_{3M} or E_{5B}	Frequency in cycles per sec.	Synchr. motor, short-circuit or no load operation	Approx. power factor	Line current V	$(\phi_1-\phi_2)$ in mega-maxwells or volts per turn per cycle	$(\phi_1-\phi_2)$ in per cent	Field amps.
1	2	3	4	5	6	7 8	9	10
O A 32/104	E_{3M}	59.5	S. M.	0.42 ld.	45.2 ld.	2.31	100	
O A 32/104	E_{5B}	59.5	S. M.	0.42 ld	45.2 ld.	2.40	104	
O A 34/106	E_{1T}	59.5	S. M.	0.41 lg.	52.6 lg.	1.95	100	
O A 34/106	E_{3M}	59.5	S. M.	0.41 lg.	52.6 lg.	2.05	105	
O A 35/107	E_{1T}	59.5	S. M.	0.37 ld.	52.6 ld.	2.22	100	
O A 35/107	E_{3M}	59.5	S. M.	0.37 ld.	52.6 ld.	2.235	101.5	
O A 35/107	E_{5B}	59.5	S. M.	0.37 ld.	52.6 ld.	2.281	103	
O A 33/105	E_{1T}	59.5	S. M.	lag.	60 appro	1.93	100	
O A 35/105	E_{3M}	59.5	S. M.	lag.	60 appro	2.0	103.7	
O A 35/105	E_{5B}	59.5	S. M.	lag.	60 appro	2.18	113	
O A 38/110	E_{1T}	60	no load			2.316	100	5
O A 38/110	E_{3M}	60	no load			2.38	103.0	5
O A 44	E_{1T}	54	s.s.c. connection Y series			0.415	100	5
O A 72	E_{5B}	54	s.s.c. connection Y series			0.282	70.3	5
O A 51	E_{1T}	52	s.s.c. connection Y series			0.3095	100	3.16
O A 54	E_{3M}	52	s.s.c. connection Y series			0.245	79.2	3.16
O A 67	E_{5B}	52	s.s.c. connection Y series			0.0969	34.4	3.16
O A 81	E_{1T}	60	s.s.c. connection 2-phase			0.1348	100	4.96
O A 82	E_{5B}	60	s.s.c. connection 2-phase			0.1056	78.3	4.96

Assume that the flux density in the slots is smaller and in the teeth is larger and more nearly perpendicular[10] to the direction of rotation of the armature conductors, for the second surface than for the first. Further, if the armature were revolving, the

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[10] Compare equations (4) and (4a) in supplementary notes.

TABLE VII.

Order of harmonics	Wave analysis No.: ($I-2$) No. of oscillogram: 206 Field current, amperes: 6.5 Frequency, cycles per sec.: 60 Exploring coil: No. 1, top of slot			($I-4$) 201 6 60 No. 1, top of slot			($I-5$) 206 5 60 No. 1, top of slot		
	A, volts B, volts	C, volts (C/C_1) 100	$\frac{A}{C_1} \times 100$ $\frac{B}{C_1} \times 100$	A, volts B, volts	C, volts $\left(\frac{C}{C_1} \times 100\right)$	$\frac{A}{C_1} \times 100$ $\frac{B}{C_1} \times 100$	A, volts B, volts	C, volts $\left(\frac{C}{C_1} \times 100\right)$	$\frac{A}{C_1} \times 100$ $\frac{B}{C_1} \times 100$
1	28.82	28.9	99.7%	27.93	27.99	99.85%	25.3	25.3	100%
	−0.62	100%	−2.14%	1.39	100%	4.97%	0.17	100%	0.66%
3	−0.73	1.08	−2.53%	−1.03	1.05	−3.68%	−0.88	0.95	−3.44%
	0.805	3.77%	2.78%	−0.21	3.75%	−0.75%	0.35	3.75%	1.36%
5	−2.22	2.24	−7.65%	−2.41	2.42	−8.61%	−2.18	2.19	−8.63%
	0.33	7.76%	1.12%	−0.24	8.65%	−0.86%	−0.14	8.65%	0.56%
7	−2.28	2.28	−7.89%	−2.09	2.55	−0.75%	−2.02	2.05	−7.9%
	0.12	7.89%	0.403%	−1.47	0.91%	−5.26%	−0.33	8.09%	−1.3%
9	−0.81	0.94	−2.79%	0.02	0.09	0.07%	2.68	2.7	10.64%
	−0.48	3.25%	−1.67%	−0.09	0.03%	−0.03%	−0.00	10.65%	−0.02%
11	0.35	0.39	1.22%	0.14	0.64	0.05%	0.62	0.64	2.42%
	−0.17	1.35%	−0.58%	0.63	2.29%	2.25%	0.15	2.5%	0.59%
13	1.15	1.16	3.98%	0.44	0.72	1.57%	0.34	0.38	1.31%
	−0.15	4.02%	−0.52%	0.58	2.57%	2.07%	0.20	1.5%	0.78%
15	0.30	0.56	1.04%				0.13	0.26	0.50%
	0.47	1.93%	1.63%				0.22	1.02%	0.89%
17	−0.26	0.26	−0.88%				0.40	0.52	1.60%
	−0.03	0.9%	−0.12%				0.32	2.04%	1.26%
23	1.14	1.17	3.95%	−0.29	0.87	−1.04%	0.79	1.12	3.14%
	−0.27	4.05%	−0.9%	0.81	3.1%	2.93%	0.78	4.43%	3.09%
25	−0.87	0.88	−3%	0.49	1.09	1.75%	−0.65	0.77	−2.57%
	0.14	3.04%	4.95%	−0.97	3.89%	1.39%	−0.42	3.04%	−1.66%

NOTE: Wave analysis, $I -$ etc. refers to open-circuit waves. First column gives the coefficients of the series: $A_1 \sin \theta + A_3 \sin 3\theta +$ etc. $+ B_1 \cos \theta + B_3 \cos 3\theta +$ etc. The second column gives the coefficients of the same wave when expressed as: $C_1 \sin \overline{\theta + \varphi_1} + C_3 \sin \overline{3\theta + 3\varphi_3} +$ etc. A, B and C are also given in per cent of C_1 in second and third columns.

TABLE VIII.

Wave analysis No.:	$(I-10)$		$(I-13)$		$(I-15)$		$(I-21)$		$(I-25)$	
No. of oscillogram:	206		263		263		201		228	
Field amperes:	2.75		4		3		6		3	
Frequency:	60		60		60		60		60	
Exploring coil,	No. 1, top of slot		No. 3, middle of slot		No. 3, middle of slot		No. 5, bottom of slot		No. 5, bottom of slot	
Order of harmonics	A, volts B, volts	C, volts (C/C_1) 100	A, volts B, volts	C, volts $\frac{C}{C_1} \times 100$	A, volts B, volts	C, volts $\frac{C}{C_1} \times 100$	A, volts B, volts	C, volts $\frac{C}{C_1} \times 100$	A, volts B, volts	C, volts $\frac{C}{C_1} \times 100$
1	16.75	16.8	22.2	2.22	17.2	17.2	28.9	28.9	17.4	17.43
	0.97	100%	−0.15	100%	0.05	100%	0.72	100%	0.002	100%
3	−0.66	0.74	−1.0	1.09	−0.9	−0.97	−1.89	−1.89	−0.62	0.79
	0.35	4.4%	0.28	4.91%	0.36	5.64%	0.05	6.54%	0.5	4.55%
5	−1.58	1.78	−2.4	2.41	−2.12	2.12	−3.66	3.75	−0.66	0.83
	−0.8	10.59%	−0.11	10.85%	−0.14	1.23%	−0.8	12.96%	−0.5	4.76%
7	−0.89	1.25	−1.08	1.12	−1.27	1.29	−1.67	1.67	−1.93	2.83
	−0.88	7.44%	−0.28	5.04%	−0.26	7.50%	−0.35	5.78%	−2.07	16.23%
9	0.05	0.09	1.01	1.01	0.06	0.15	0.39	0.68	−1.31	1.43
	−0.08	0.56%	0.03	4.55%	−0.13	8.55%	−0.56	2.35%	−0.58	8.20%
11	0.11	0.58	0.96	0.80	−0.68	0.68	0.92	1.08	0.14	1.65
	0.57	3.45%	0.26	3.60%	0.097	3.95%	0.57	3.74%	1.64	9.47%
13	−0.1	0.48					0.35	0.5		
	0.47	2.85%					0.36	1.73%		
15	0.75	0.17					−0.24	0.26		
	−0.16	1.01%					0.1	0.9%		
17	−0.44	1.13					−0.36	3.98		
	1.04	6.71%					−3.96	13.76%		
23	−0.34	−0.34					0.67	1.02		
	−0.00	2.02					1.77	3.53%		
25	0.47	0.47					−0.72	1.25	0.48	0.55
	0.067	2.8					−1.02	4.33%	0.26%	3.17%

NOTE: Wave analysis $I -$ etc. refers to open-circuit waves. First column gives the coefficients of the series: $A_1 \sin \theta + A_3 \sin 3\,\theta$ etc. $+ B_1 \cos \theta + B_3 \cos 3\,\theta$ etc. Second column gives coefficients of the same wave when transformed into: $C_1 \sin \overline{\theta + \varphi_1} + C_3 \sin \overline{3\theta + \varphi_3}$ etc. Coefficients C in per cent of fundamentals C_1 are also given in column (2).

peripheral velocity would have been larger for the second surface. These assumptions will partly at least explain the above and they seem reasonable as may be seen by examining the approximate flux distribution[11] in slotted armatures under different load conditions.

Tables VII and VIII give wave analyses of the e.m.fs. induced at no-load in different exploring coils. It will be seen that for a given field current, for example 6 amperes, wave analysis

TABLE IX.

Showing increase of 3rd harmonic of e.m.f. induced in stator exploring coils, with increasing excitation.

Excitation in amps. $= I_f$	Third harmonic e.m.fs. induced in stator full pitch exploring coils			Per cent values of columns				No. of oscillogram from which columns		
								No. 2	No. 3	No. 4
	No. 1 E_{1T}	No. 3 E_{3M}	No. 5 E_{5B}	No. 1 I_f	No. 2 E_{1T}	No. 3 E_{3M}	No. 4 E_{5B}	were calculated		
1	2	3	4	5	6	7	8	9	10	11
				No-load operation						
8.8	2.21		3.94	320	298		532	272		272
6.5	1.08			236	146			206		
6.0	1.05		1.89	218	142		255	201		201
5.0	0.95	1.67	1.28	182	128	225	173	206	261	266
4.0		1.09		145.5		147			263	
3.0	0.75	0.97	0.79	109	101	131	107	228	263	228
2.75	0.74			100	100			206		
				Y series sustained short circuit						
5	3.99		7.52	182	532		1000	202		286
4			5.12	145.5			684			286
3	2.46		4.04	109	328		538	229		286

NOTE: The sustained s.c. percentages are in terms of 0.75 volts, induced in top search coil No. 1 at no-load and 3 amperes excitation.

$I - 4$, the amplitudes in maximum volts for the 1st, 3rd and 5th harmonics are respectively, 27.99, 1.05 and 2.42 for coil No. 1. The corresponding values from wave analysis $I - 21$, Table VIII, for coil No. 5 at the bottom of the slot, are: 28.9, 1.89

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[11] See for example, Figs. 110 and 111 p. 282 and p. 283, Principles of Electrical Design, by A. Still. *Jour., I. E. E.*, Magnetic flux distribution in toothed core armatures. H. S. Hele-Shaw, A. Hay and P. H. Powell. Figs. 183, 223 etc. pp. 165 and 220, in S. P. Thompson's Polyphase Electric Currents, Second Edition.

and 3.75. As the third harmonic is of special interest in three-phase work Table IX has also been prepared; inspection of this table shows that taking the e.m.f. induced in coil No. 1, at the top of the slot, at 2.75 amperes of field excitation, as 100 per cent, the third harmonic of e.m.f. increases for coil No. 1 with increasing excitation until it reaches 298 per cent at 8.8 amperes of field current. Under s.s.c. we have a similar condition; the third harmonic for coil No. 5 reaches 1000 per cent while the corresponding value for coil No. 2 is 532 per cent at 5 amperes.[12]

In Fig. 14 the e.m.fs. E_{CTT} and E_{CTS} induced in a single conductor placed on top of a tooth and on top of a slot respectively (see coils No. 13 and 14, Fig. 3) show certain peculiar differences in that E_{CTT} has nine distinct kinks while E_{CTS} has eight. This is easily explained by following step by step the variational e.m.f. induced in a conductor on top of a tooth and one on top of a wedge, as the armature slots move with respect to the field poles.

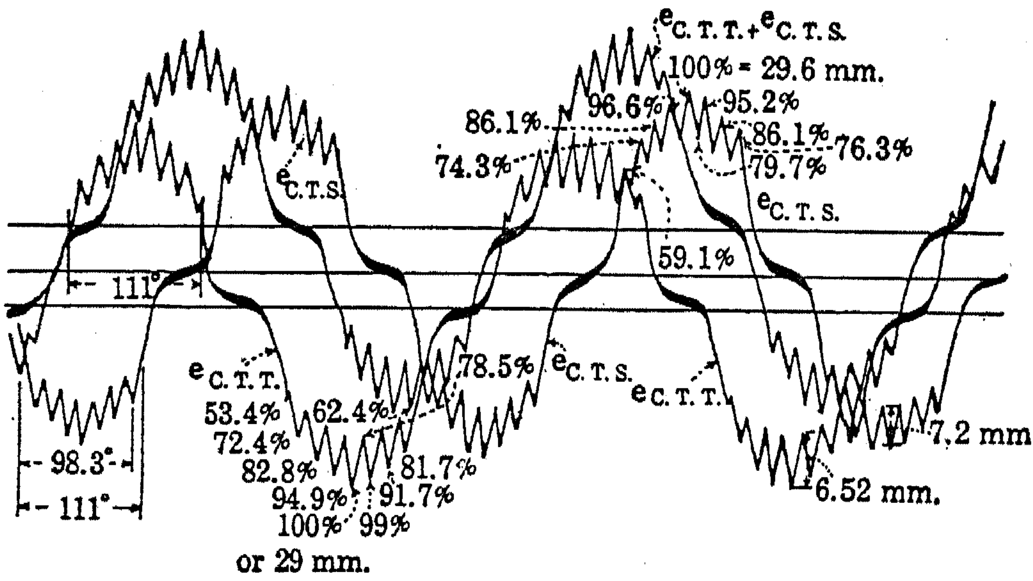

Fig. 14

Oscillogram No. O A 75. Test No. 187. Open-circuit test at 5 amperes field excitation and normal speed. First wave: (e.m.f. induced in single conductor No. 14 at top of tooth) + (e.m.f. induced in single conductor No. 13 at top of immediately succeeding wedge) = $E_{CCT} + E_{CTS}$. Second wave: E_{CTS}. Third wave: E_{CTT}. Same vibrator used for all exposures. 30 ohms resistance in vibrator for $E_{CTT} + E_{CTS}$; 10 ohms for other two waves.

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[12] For a tentative explanation of the fact observed, namely that the third harmonic of e.m.f. for a given exploring coil will increase with excitation see, P. Janet, *Lecons d' électrotechnique générale*, T. II, p. 191. However, note further that not only the 3rd harmonic increases for a given coil with increase of field current but it seems to increase faster for a search coil at the bottom of slot than one at the top of a slot.

IV. Experimental Investigation of the Direct Component of Armature Reaction and the Resultant Flux Distribution Under Sustained Short-Circuit Conditions

The armature current under s.s.c. has been found to be very closely sinusoidal, as can be seen by inspecting first two waves in Fig. 15 and fifth wave in Fig. 16, which are for Y and delta connections. A quantitative idea of the approach of these waves to a sine wave may be gained from Table X.

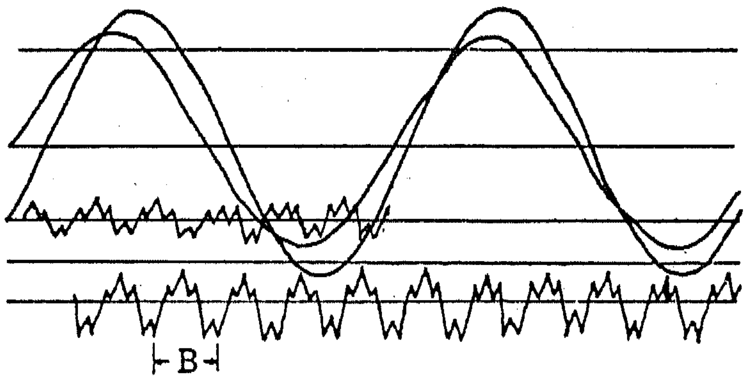

Fig. 15

Oscillogram No. 281. Table X. Sustained short circuit with machine series delta connected. Second straight line; zero line for first two waves representing armature phase current at 3.27 and 4.1 amperes field excitation respectively. Third and fourth waves give e.m.f. induced in 29-turn full-pitch rotor exploring coil No. 7, under same conditions.

Throughout this paper the armature current under s.s.c. *will be assumed* to be sinusoidal and if the armature winding were sinusoidally distributed, see Fig. 17, the armature m.m.f. would be sinusoidal. However, it can be shown (see supplementary notes, equation **30**) than an ordinary armature winding with N conductors per slot and one slot per pole per phase, see Fig.

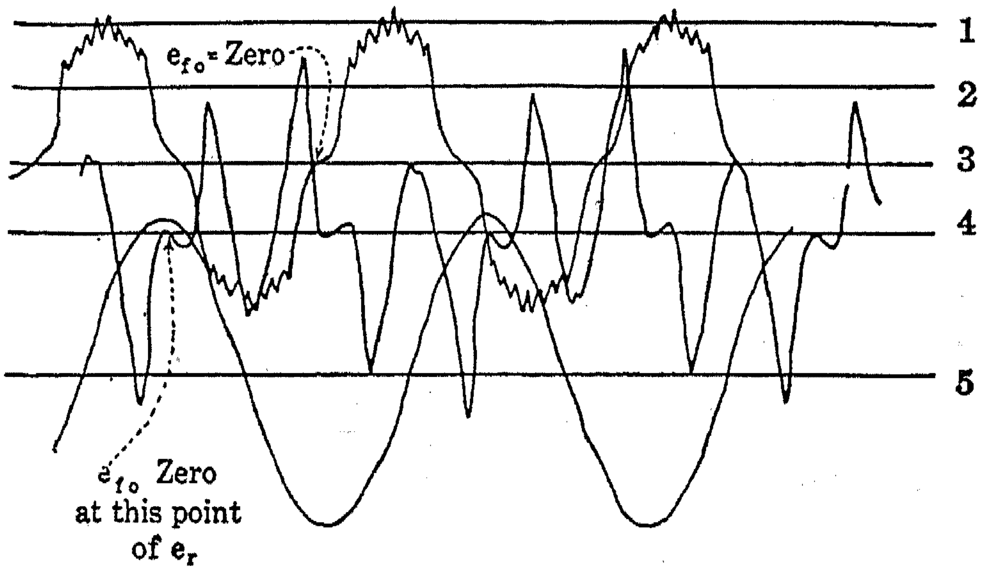

Fig. 16

Oscillogram No. O A 44. Test No. 148, Table XV. *Exposure No.* 1. Open-circuit test at 5 amperes field excitation and 54 cycles per second. First wave: contactor. Third wave: E_{1T} = e.m.f. induced in coil No. 1. *Exposure No.* 2. Sustained short-circuit test with machine series Y connected. Same excitation and speed as before. Fourth wave: E_{1T}. Fifth wave: line current. Second wave: contactor.

18, supplied with a sinusoidal current will be exactly equivalent to: (1) a winding sinusoidally distributed (in space) having a pitch of τ cm. or inches. (2) a winding sinusoidally distributed having a pitch of $\tau/3$ and $\frac{1}{3}$ as many turns as the first, etc., etc. In other words, imagine a set of sinusoidally distributed windings, of pitch τ/k, where k is the order of the space harmonic, all placed in series and carrying the same sinusoidal current (with respect to time): These will produce the same H-curve as the actual non-sinusoidal winding carrying the sine-wave current of fundamental (with respect to time) frequency.

TABLE X.

Per cent amplitude of armature current, i_a, at every 30 deg., under s.s.c. and the machine Y-series and delta-series connected.

θ in degrees	0	30	60	90	120	150	180
sin θ × 100	0	50	86.6	100	86.6	50	0
i_a, Y conn. (oscill. O A 44)	0	51.1	84.5	100	85.4	50	0
i_a, Δ conn. (oscill.281)	0	48.7	81.2	100	85.1	48.7	0

Without going further into the analysis of this, suffice to state[13] that for a polyphase alternator, with a sine wave (with respect to time) armature current and any ordinary type (non-

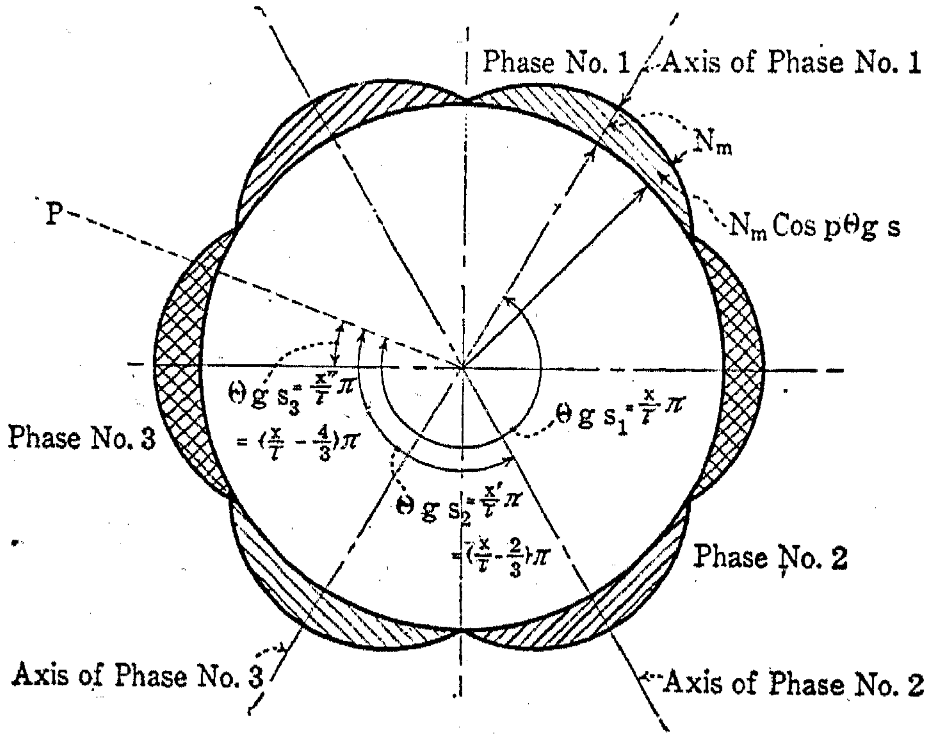
Phase No. 1
Axis of Phase No. 1
N_m
$N_m \cos p\theta g s$
P
$\theta g s_3 = \frac{x''}{\tau}\pi$
$= (\frac{x}{\tau} - \frac{4}{3})\pi$
$\theta g s_1 = \frac{x}{\tau}\pi$
Phase No. 3
$\theta g s_2 = \frac{x'}{\tau}\pi$
$= (\frac{x}{\tau} - \frac{2}{3})\pi$
Phase No. 2
Axis of Phase No. 3
Axis of Phase No. 2

FIG. 17

Schematic representation of three-phase two-pole armature winding which is sinusoidally distributed (in space).

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[13] See A. Potier, *Journal de Physique*, 1897, 3^{me} serie, T. VI, p.341 and 483. A. Blondel, *L' Eclairage Electrique*, 1895, T. IV, pp. 241, 308 et 358. T. V., p. 268. See also P. Janet, *Lecons d' Electrotechnique générale*, T. III, Chp. V. Alex. Russell, Altern. Currents Vol. II, Chp. XIV.

sinusoidally distributed, in space) winding, the resultant m.m.f. curve can be considered as consisting of a series of H-curves, all of fundamental frequency (in time) but having a pole pitch τ/k and an amplitude equal to:

$$H_a = \frac{N_{ph}}{2} \cdot \frac{H_m}{k} S \cdot D_f \cdot S_{p \cdot f.} \tag{31}$$

where N_{ph} = number of phases.

H_m = $(4/\pi)\ Y_m$. See Fig. 18.

S = slots per pole per phase.

D_f = distribution factor = $\dfrac{\sin \frac{1}{2} S \frac{\lambda}{\tau} \pi}{S \sin \frac{1}{2} \cdot \frac{\lambda}{\tau} \pi}$

λ = slot pitch.

$S_{p \cdot f.}$ = short-pitch factor = $\cos \epsilon\, k\, \pi$, where $(1 - \epsilon)\, \tau$ = winding pitch.

For a three-phase alternator the first, seventh, thirteenth etc., or $(6\, n + 1)$ $t\, h$ space harmonics, ($n = 0, 1, 2, 3$ etc.), will rotate in the same direction at a speed of $2\, \pi f/k$ or at an angular velocity $\omega/p\, k$. The $(6\, n - 1)$ $t\, h$ harmonics will rotate in the opposite direction with the same velocity. In mathematical short-hand the above may be expressed as follows:

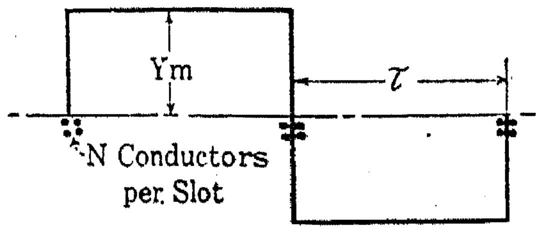

FIG. 18

$$H_1 = H_a \cos\left(\omega\, t - \frac{x}{\tau}\, \pi\right) \tag{32}$$

$$H_3 = 0 \tag{33}$$

$$H_5 = H_a \cos\left(\omega\, t + \frac{5\, x}{\tau}\, \pi\right) \tag{34}$$

$$H_7 = H_a \cos\left(\omega\, t - \frac{7\, x}{\tau}\, \pi\right), \text{ etc.} \tag{35}$$

where H_a is given by **(31)**.

In case of two-phase machines the first, fifth etc., or $(4\, n + 1)$ $t\, h$ space harmonics will travel in one direction while the third, seventh etc. or $(4\, n - 1)$ $t\, h$ space harmonics will travel in the opposite direction. The expressions for these will be similar to the ones given above and their amplitude will be given by **(31)**

The m.m.f. per centimeter at any point x of the stator periphery will be $(4\ \pi .\ 10^{-1}\ H)$ where,

$$H = H_1 + H_5 + H_7 \text{ etc.}$$

= sum of series of sinusoidal H-waves, all of fundamental frequency and having phase angles, $(-\ 6\ x_1\ \pi/\tau)$, $(+\ 6\ x_1\ \pi/\tau)$, $(-\ 12\ x_1\ \pi/\tau)$, etc., with respect to the fundamental whose phase angle $(-\ x_1\ \pi/\tau)$ is assumed to be zero. See Notes B and C.

This leads to the following important proposition: With polyphase balanced alternating currents varying according to a sine law (in time) and flowing through ordinary non-sinusoidally distributed windings, (in space) the H-curve will be sinusoidal (*i.e.*, will be the sum of a series of sine waves). Thus, under load or s.s.c., in case of induction motors or alternators, the H-curve due to sinusoidal (in time) armature currents will be sinusoidal. Consequently if the permeability be constant, and uniform across the pole-face, the B-curve due to armature reaction will also be sinusoidal. However, if we find the e.m.fs. induced in exploring coils due to armature reaction to be distorted this must be charged to the permeability μ, of the circuit. The importance of this result will be appreciated when we attempt to explain certain phenomena to be considered presently.

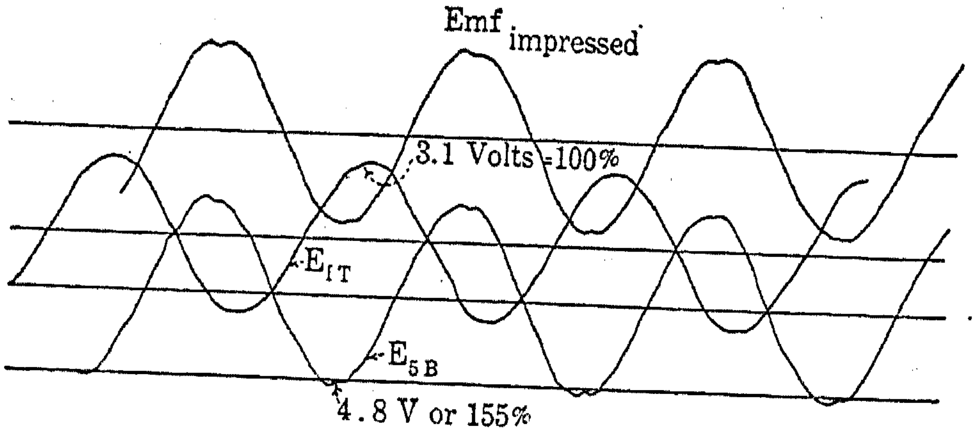

FIG. 19

Oscillogram No. O A 13. Test No. 32. Rotor removed and three-phase 85.8 amperes (158 per cent normal current) introduced into the stator from outside source. First wave: e.m.f. impressed on stator. Second wave: E_{1T} = induced in full pitch search coil No. 1 Third wave: E_{5B} = e.m.f. induced in full pitch exploring coil No. 5. Second and fou.th lines: calibrations for E_{1T} and E_{5B} respectively. Note E_{1T} and E_{5B} are due to the rotating field set up by three-phase currents.

Fig. 19 gives E_{1T} and E_{5B} when the stator was supplied with 3-phase currents from an outside source and the rotor removed. It is seen that the impressed e.m.f. does not deviate greatly from a sine wave. But the e.m.f. E_{5B}, induced in the coil at the bottom of the slot is peaked and over 150 per cent of E_{1T} = e.m.f. induced in coil on top of slot. The waves given in Fig. 20 were taken with the rotor in place and running without excitation at a speed corresponding exactly to that of the gliding field produced by the three-phase stator currents. In this case the current supplied was only 28 per cent of full-load current. As in Fig. 19, E_{5B} is more peaked than E_{1T}; $E_{5B} = 1.17\ E_{1T}$.

Assuming the m.m.f. per centimeter to be the same at the top and bottom of the slot, it is clear that the inequality between E_{1T} and E_{5B} must be charged, at least partly, to the difference in permeance between the points considered. Generalizing the above it can be stated that although the H-curve due to armature currents may be sinusoidal the B-curve will differ from the former for two reasons: First, the lack of constancy and uniformity in μ across the pole face; further the B-curve at the bottom of the slot may differ from that at the top owing to the difference in the permeance per unit area at these two points.

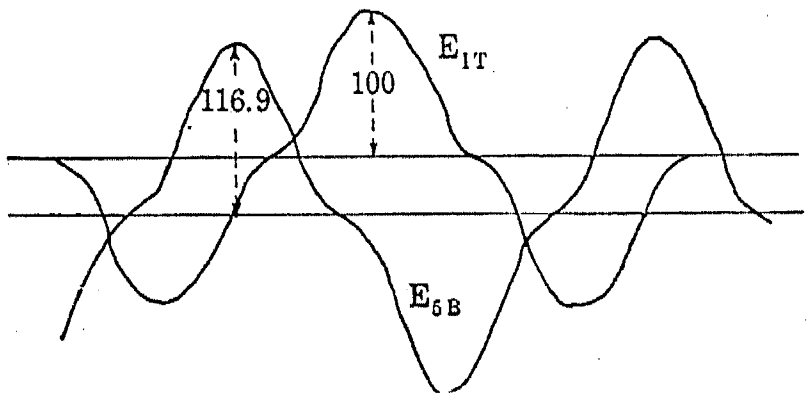

FIG. 20

Oscillogram O A 66. Test 177. First wave: E_{1T} = e.m.f. induced in full-pitch search coil at top of slot. Second wave: E_{5B} = e.m.f. induced in full-pitch search coil at bottom of slot. A-c. supplied from outside source = 15.1 amperes. Two exposures taken under identical conditions. Rotor not excited and run at synchronous speed. Same vibrator used for both exposures.

Second, according to the degree of saturation the B-curve will be more or less flat topped; this of course is well known especially in connection with induction motor design.

It is possible for the B-curve due to armature reaction alone, to undergo certain oscillations, similar to those described in Section III. However, a little study of Table I, columns 8 and 9 and of Figs. 19, 20 and of others to be given later will show that these secondary effects may safely be neglected.

Summarizing then we can state that the experimental and theoretical studies given herein and by others show that the no-load flux may and does deviate from a sine wave to the extent of containing the 3rd, 5th and 7th harmonics to the amount of 10 per cent or thereabouts. The field-form of most modern machines, at no load or full load approaches reasonably well the general shape of a sine wave[14] and is symmetrical with respect to the $\pi/2$ or the (geometrical) pole-axis except in case of loads having considerable cross-magnetizing effect.

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[14] W. J. Foster, TRANS., A. I. E. E., 1913, Vol. 32, p. 749. L. A. Herdt, TRANS., A. I. E. E., 1902, Vol. 19, p. 1093.

As explained before the H-curve due to armature reaction will be sinusoidal while the B-curve will show no *extreme* deviations from a sine wave. This, rather theoretical result, is further corroborated by the oscillograms given in Figs. 19 and 20. According to these, either with the rotor removed or in place and running at synchronous speed, the e.m.fs. induced in stator search coils, due to stator currents alone, are fairly smooth. Furthermore the flux pulsations as indicated by the rotor coil e.m.fs., E_{RF}, and E_{RC} (under s.s.c.) were found to be extremely small.

TABLE XI

Wave analysis No.: No. of oscillogram: Field excit. amperes: Expl. coil,	$II-1$ 202 4.95 No. 1, top of slot		$II-2$ 229 3 No. 1, top of slot		$II-3$ 286 5.02 No. 5, bottom of slot		$II-4$ 286 3 No. 5, bottom of slot	
Order of harmonics	A, volts B, volts	C, volts $\frac{C}{C_1} \times 100$	A, volts B, volts	C, volts $\left(\frac{C}{C_1}\right) 100$	A, volts B, volts	C, volts $\left(\frac{C}{C_1}\right) 100$	A, volts B, volts	C, volts $\left(\frac{C}{C_1}\right) 100$
1	1.24 − 0.004	1.24 100%	1.16 − 0.03	1.16 100%	− 1.05 − 0.92	1.39 100%	0.85 0.13	0.86 100%
3	1.96 3.32	3.99 323%	0.98 2.26	2.46 213%	− 3.64 − 6.47	7.42 532%	1.97 3.74	4.24 490.5%
5	1.06 − 1.86	2.14 172.5%	0.17 − 1.16	1.77 152.5%	1.77 − 1.37	2.24 160.5%	− 1.15 − 0.96	1.49 173%
7	0.91 − 1.12	1.45 116.8%	0.67 − 0.78	1.1 95%	− 1.05 − 2.47	2.7 193%	1.25 − 1.61	2.04 236%
9	0.45 0.027	0.45 36.5%	0.03 − 0.001	0.03 2.8%	− 1.18 0.72	1.38 98.9%	1.51 0.87	1.75 203%
11	− 0.31 − 0.097	0.32 25.86%	− 0.24 − 0.01	0.03 2.24%	− 0.11 − 0.02	0.11 81%	− 0.18 − 0.34	0.38 44.4%
13	− 0.193 − 0.34	0.39 31.7%	− 0.01 − 0.02	0.27 2.29%	− 0.88 0.95	1.29 92%	− 0.33 0.63	0.71 82.5%
15	0.128 0.086	0.15 12.5%	0.004 0.003	0.004 0.4%				
17	− 0.056 0.10	0.12 9.5%						
23	0.005 0.12	0.12 9.5%						
25	0.004 0.004	0.006 0.5%						

NOTE: Wave analysis, II-etc. refers to analysis of waves obtained under s.c.c. with the machine series Y connected. See also note, Table VIII, as to meaning of A, B, C.

The resultant of two or more sine waves is, of course, a sine wave, and therefore it might be expected, at first sight, that the resultant of the *B*-curve due to the field and that due to the armature reaction will not deviate greatly from a sine wave, since neither of them is very distorted to begin with. However, turning to oscillograms obtained under s.s.c. we find indications of an extremely distorted flux distribution. A glance at Figs. 23, (fourth wave) 16, (fourth wave) etc. and Table XI which gives the coefficients of the harmonics of the waves obtained under s.s.c., show that the 3rd harmonic of e.m.f. induced in the stator search coils is several hundred per cent of the fundamental and that a number of other harmonics are quite comparable to it.

In order to avoid any false deductions and detect any peculiar defects in the machine under test to which the distorted s.s.c. waves might be due it was considered advisable to investigate briefly the *B*-curves obtained under different loads. For this purpose the machine was run as a synchronous motor and a number of oscillograms taken, a few of which have been included in the paper.

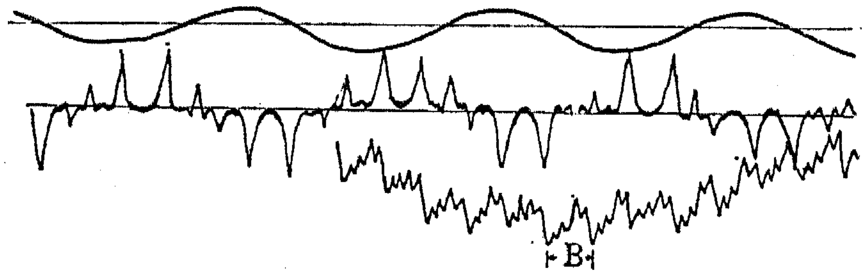

FIG. 21

Oscillogram No. O A 40. Test No. 116. Tables I and II. First wave: line current Second wave: E_{TT_2} = e.m.f. induced in coil No. 12. Third wave: E_{RF} = e.m.f induced in full-pitch rotor coil No. 7. Synchronous motor operation. Line current = 76 amperes lead. Line voltage = 241 volts. Power factor = 0.28 ld.

As seen from Figs. 21, 22 etc., the waves E_{1T}, E_{3M}, E_{RF}, E_{RC} etc. are not unlike those obtained at no-load with small variations which might have been expected. It will be observed by comparing Fig. 22 and Fig. 7 that the tooth ripples are smaller under synchronous motor operation. One of the important results of this series of tests was that even with heavy magnetizing or demagnetizing armature reactions, (no field excitation, or 150 per cent or more excitation) the flux distribution did not in any way approach the *B*-curves obtained under s.s.c.

Other points of interest have already been considered in the previous section and as the subject is not new it will not be considered any further. However, attention may be called to the e.m.f. induced in the rotor search coils No. 7 and 8. The e.m.f. waves of these coils show $2q = 24$ small indentations per cycle which are due to the teeth. The e.m.f. of full-pitch coil No. 7, shows a large superimposed oscillation which recurs every B deg., see Fig. 21, 3rd wave.

As shown clearly in the supplementary notes this is due to the fifth and seventh space harmonics of armature reaction and as indicated in Table I column 5, $B = 60$ elec. deg., Fig. 21, *i.e.*, the periodicity of the superimposed wave is $360/60 = 6$ times machine-frequency, as we would expect. The long flat oscillations which repeat themselves every $A = 1080$ elec. deg., see Fig. 5, and Table I column 4, are due to eccentric rotor, as explained before.

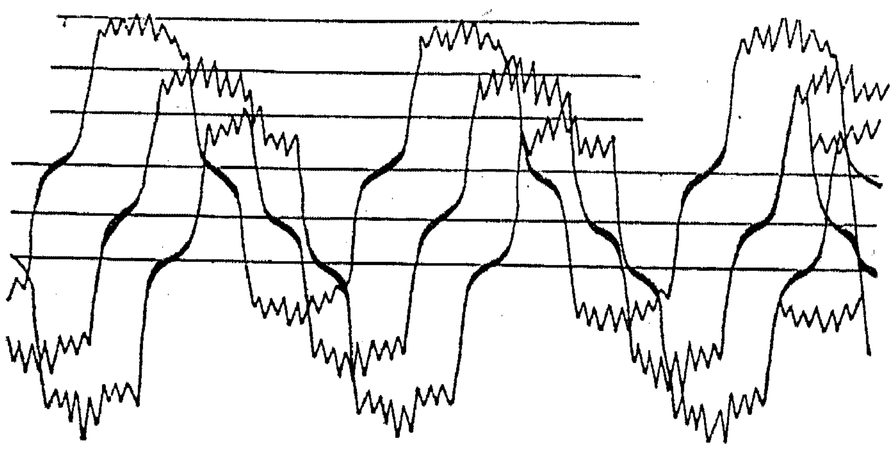

FIG. 22

Oscillogram No. O A 35. Test No. 106. Table VI. Machine Y parallel connected and run as synchronous motor. First wave: E_{1T} = e.m.f. induced in exploring coil No. 1 at top of slot. Second wave: E_{3M} = e.m.f. induced in exploring coil No. 3 at middle of slot. Third wave: E_{5B} = e.m.f. induced in exploring coil No. 5 at bottom of slot. Line current = 52.6 lead. Power factor = 0.37 approx. Compare to Fig. 7.

To determine the armature reaction under s.s.c., *experimentally*, in terms of the voltages induced in the stator coils the writer resorted to the following method. A contactor[15] having a fixed space relation with respect to the poles was placed on the machine. This closed a direct-current circuit of very little inductance so that the rise of current was quite abrupt and the instant of make well defined. See Fig. 23. An exposure was first made giving the contactor wave and the open circuit e.m.f. induced in one of the search coils. The zero lines of both waves were then moved, in the same direction, and a second exposure was made giving the contactor and search coil e.m.f. under the same operating conditions except that the machine was short-circuited at its terminals.

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[15] The contactor used is similar to the one described in TRANS., A. I. E. E., Vol. 34, p. 2254, except for slight changes in the design of the brush rigging.

Obviously it is thus possible to determine accurately the point on the s.s.c. wave corresponding to any point on the open-circuit wave. If we designate by e_{fo} the e.m.f. induced in one of the stator coils No. 1, 3, or 5 at no-load and given field current and speed, and by e_r the e.m.f. induced in the same coil under the same conditions of speed and excitation but with the machine short-circuited, we can, as a fair approximation assume that e_{fo} plus a certain e.m.f. e_a, which we can charge to armature reaction, give us the resultant e.m.f. e_r.

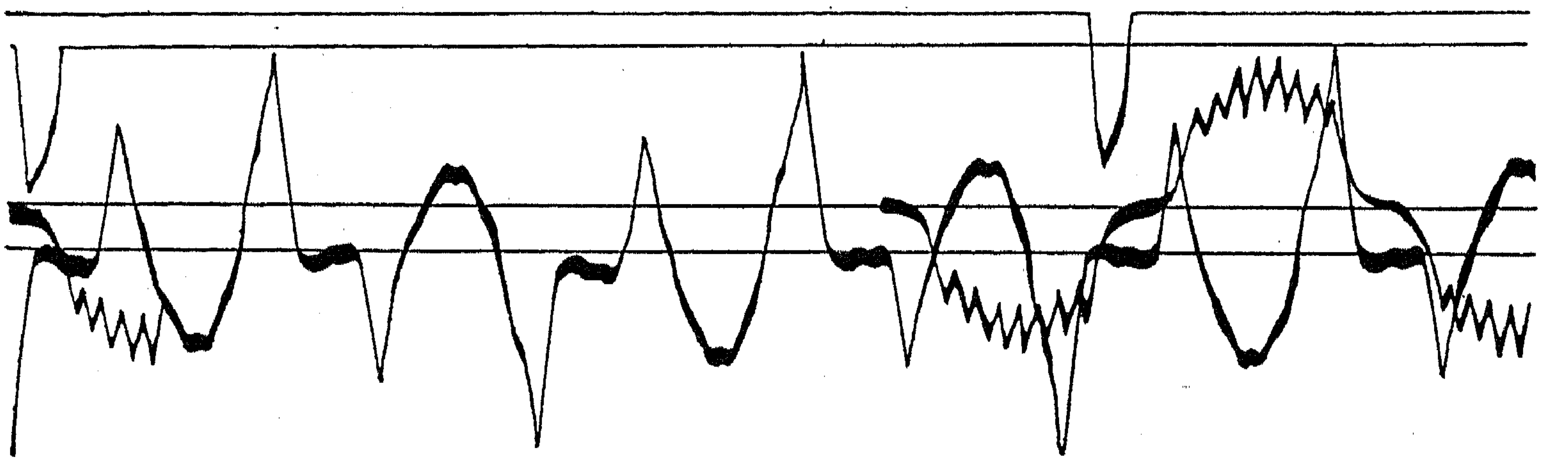


Fig. 23

Oscillogram No. O A 72. Test No. 184. Table XV. *Exposure No.* 1. Open-circuit run at 5 amperes field current and 54 cycles per second. First wave: contactor. Third wave: E_{5B} = e.m.f. induced in exploring coil No. 5, at bottom of slot. *Exposure No.* 2 Sustained short circuit; series Y connection and under same conditions. Second wave Contactor. Fourth wave: E_{5B}.

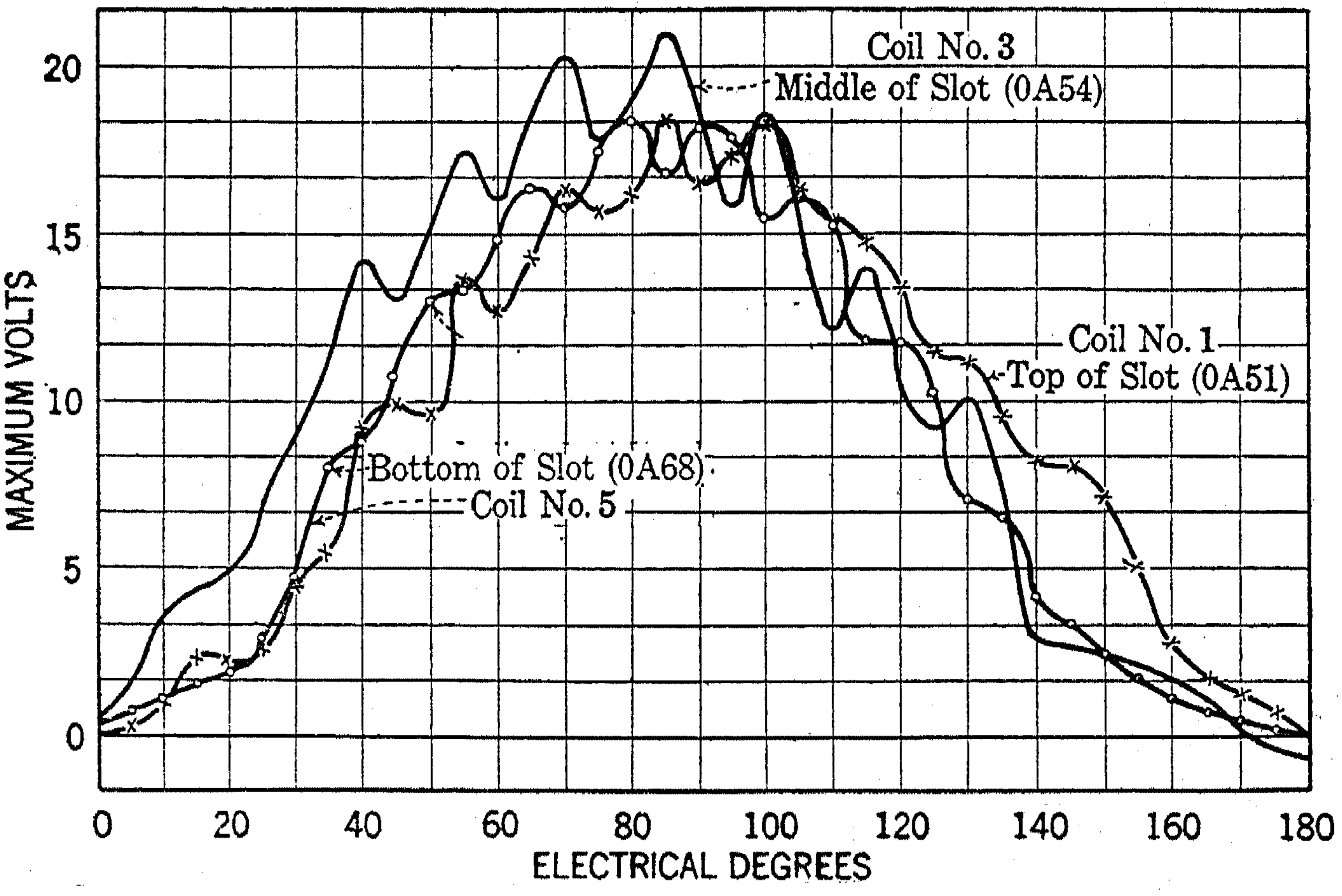


Fig. 24

Ea curves = difference between no-load and s.s.c. e.m.fs. induced in top, bottom and middle of slot full-pitch exploring coils, Nos. 1, 3 and 5 = $(E_{fo} - E_r)$—field current = 3.16 amperes — periodicity = 52 cycles per second — Y series connection.

Fig. 24 shows $e_a = (e_r - e_{fo})$ for the three full-pitch stator exploring coils taken at a field excitation of 3.16 amperes and a

TABLE XII.

	Wave analysis: VI – 1 No. of oscillogram: O A 44 Field excitation: 5 Frequency: 54 Exploring coil: No. 1, top of slot			(VI – 2) O A 51 3.16 52 No. 1, top of slot			(VI – 3) O A 68 3.1 52 No. 5, bottom of slot		
Order of harmonics	A, volts B, volts	(A/C_1) 100 (B/C_1) 100	C, volts (C/C_1) 100	A, volts B, volts	(A/C_1) 100 (B/C_1) 100	C, volts (C/C_1) 100	A, volts B, volts	(A/C_1) 100 (B/C_1) 100	C, volts (C/C_1) 100
1	– 23.02	– 99.8%	23.07	– 15.22	– 99.4%	15.31	14.75	100%	14.75
	1.16	5.03%	100%	1.64	10.73%	100%	0.2	1.35%	100%
3	4.87	21.0%	5.06	2.09	13.62%	2.14	0.85	5.76%	0.92
	– 1.38	– 6.01%	21.05%	– 0.65	– 4.27%	14%	0.35	2.37%	6.24%
5	– 0.87	– 3.77%	0.9	0.73			– 0.6	– 4.1%	0.62
	– 0.25	– 1.10%	3.93%	– 0.34			– 0.12	– 0.8%	4.18%
7	0.56	2.44%	0.68	– 0.08			0.22	1.47%	0.35
	0.38	1.67%	2.94%	– 0.52			0.27	1.8%	2.35%

NOTE: Wave analysis VI – etc. refers to C_a curves of armature reaction See Figs. 25 and 24. First column gives coefficients of series: $A_1 \sin \theta + A_3 \sin 3\theta$ etc. $+ B_3 \cos \theta + B \cos 3\theta$ etc. Second column gives A and B in per cent of C_1. Third column gives coefficients of series $C_1 \sin \overline{\theta + \varphi_1} + C_3 \sin \overline{3\theta + \varphi_3}$ etc. where $C^2 = A^2 + B^2$; also it gives C in per cent of C_1

frequency of 52 cycles per second; 3.16 amperes being the field excitation necessary to send full-load current under s.s.c. Fig. 25 gives e_a for a frequency of 54 cycles per second and a field current of five amperes.

Table XII gives the wave analysis of e_a, Fig. 25 coil No. 1, and Fig. 24 coils No. 1 and 5. It is seen that the 3rd harmonic is 21 per cent for five amperes field excitation and drops to about 5 and 6 per cent at 3.1 amperes of excitation. A general idea of the shape of these e_a curves of armature reaction may be obtained from Fig. 24A which is an average "smoothed out" curve representing the waves given in Fig. 24 for coils No. 1 and 5.

The last two columns of Table XIII give the wave analysis of e_a as obtained by subtracting (1 − 5) Table IV, from (*II* − 1) Table XI. (*I* − 5) gives the wave analysis of the e.m.f. induced in coil No. 1 at no-load and 5 amperes of excitation, while (*II* − 1) gives the analysis of the corresponding wave of the same coil with the machine Y connected and short-circuited. Comparing (*II* − 1) − (*I* − 5) with the wave analysis of e_a, Table XII, it will be seen that the agreement is satisfactory.

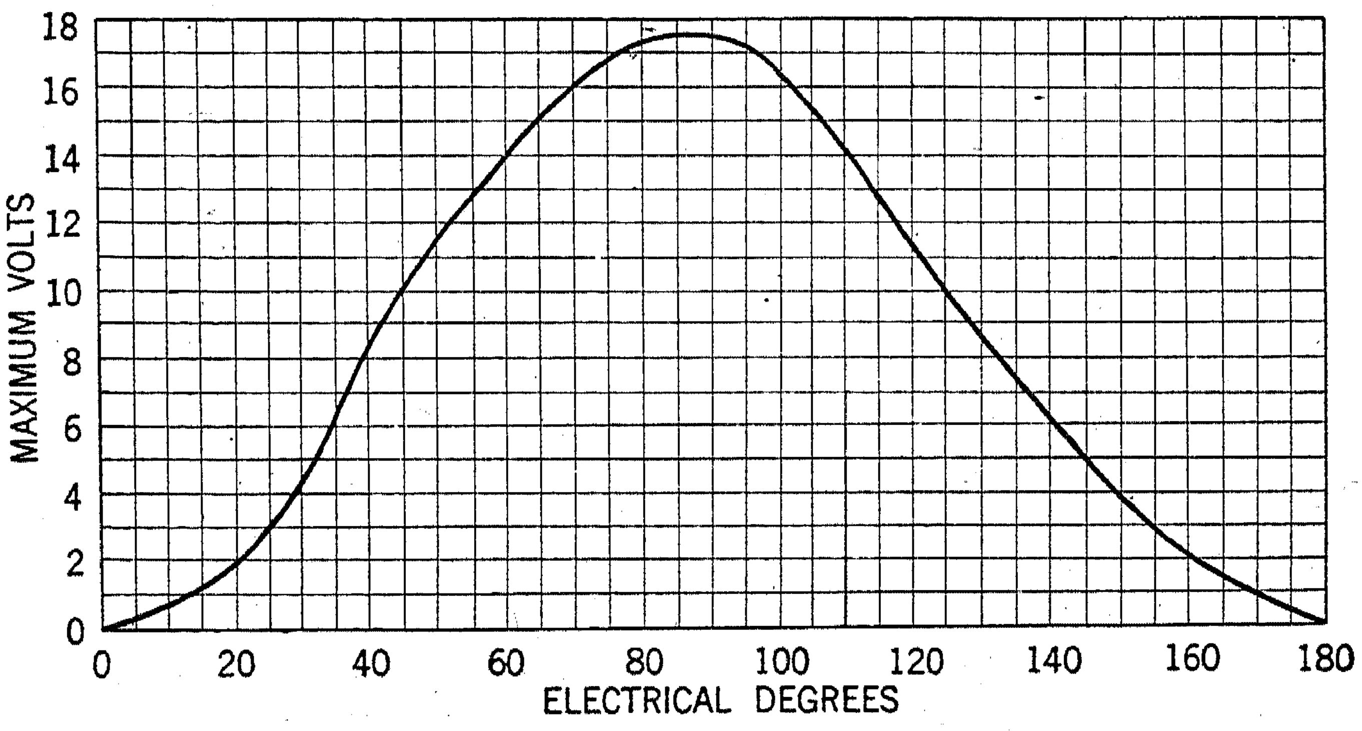


FIG. 24A

Average "Smoothed Out" curve representing e.m.f., E_a induced in coils No. 1 and No. 5 Fig. 24.

The important result derived from the above is that the s.s.c. flux distribution is badly distorted primarily on account of the fact that the fundamental of the resultant flux is so greatly reduced that the higher harmonics become quite predominant. For example, from Table VII, wave analysis (*I*−5), the 3rd is seen to be 0.95 volts which is 3.75 per cent of the no-load fundamental, which is 25.3 volts; but it is (0.95/1.24) × 100 = 67 per cent

TABLE XIII.

	Wave analysis No. III – 2 No. of oscillogram: 225 Field excitation amperes: 5.05 Exploring coil, No. 1, top of slot		(III – 4) 227 3 No. 1, top of slot		(V – 4) O A 81 5 No. 1, top of slot		(V – 5) O A 82 5 No. 5, bottom of slot		(II – 1) – (I – 5) 5 No. 1, top of slot	
Order of harmonic	*A*, volts *B*, volts	*C*, volts (C/C_1) 100	*A*, volts *B*, volts	*C* volts (C/C_1) 100	*A*, volts *B*, volts	*C*, volts (C/C_1) 100	*A*, volts *B*, volts	*C*, volts (C/C_1) 100	*A*, volts *B*, volts	$(A/25.3)$ 100 $(B/25.3)$ 100
1	– 0.75	1.21	0.525	1.00	– 2.65	2.65	– 2.19	2.64	– 24.06	95.2
	0.96	100%	0.86	100%	0.05	100%	1.48	100%	– 0.18	0.71
3	1.74	3.54	1.17	2.13	– 3.77	3.78	– 4.9	4.96	2.84	– 11.21
	3.09	291.5%	1.78	213%	0.28	144%	0.82	188	2.97	– 11.74
5	– 1.4	1.76	– 0.94	1.0	1.93	1.99	1.6	2.15	3.24	– 12.8
	– 1.07	144.8%	0.38	100%	– 0.47	75%	– 1.44	81.5	– 1.72	6.8
7	0.97	0.99	0.38	0.64	1.22	1.375	1.29	1.9	2.93	– 11.58
	– 0.20	81.6%	0.51	63.7%	– 0.63	51.8	– 1.4	71.9	– 0.79	3.12
9	0.89	0.94	0.19	0.39					– 2.23	8.82
	– 0.28	77.1%	– 0.35	39.3%					0.03	
11	– 0.47	0.94	0.048	0.99					– 0.93	3.7
	0.82	77.4%	– 0.08	9.57%					– 0.25	0.99
13	– 0.25	0.40	0.15	0.32					– 0.53	2
	– 0.31	33.1%	– 0.28	0.32%					– 0.54	2.13
15	0.25	0.29							0	
	0.01	23.8%							– 0.31	1.23
17			0.07	0.25					– 0.46	1.82
			– 0.24	25.1%					– 0.22	0.87

NOTE: Wave analysis III etc. refers to analysis of waves obtained under s.s.c. with the machine series Y connected and neutral in. V etc. refers to waves obtained under s.s.c. with the machine connected two-phase. See Fig. 31. For II – 1 see Table XI; for I – 5 see Table VII. As to meaning of *A*, *B* and *C* see Table XII.

of the s.s.c. fundamental. From wave analysis (*II* – 1) Table XI, the third harmonic under s.s.c. is 3.99 volts *i.e.*, (3.99/25.3) × 100 = 16 per cent of the no-load fundamental, but (3.94/1.24) × 100 = 323 per cent of the s.s.c. fundamental. The increase from 0.95 volts to 3.99, on the other hand, must be charged mainly to the *B*-curve of armature reaction which will be distorted, unlike the *H*-curve producing it, which will be sinusoidal

Turning to Table XII, consider wave analysis No. (*V I* – 1) which gives the coefficients for E_{IT} at five amperes excitation and 54 cycles per second. The fundamental is seen to be 23

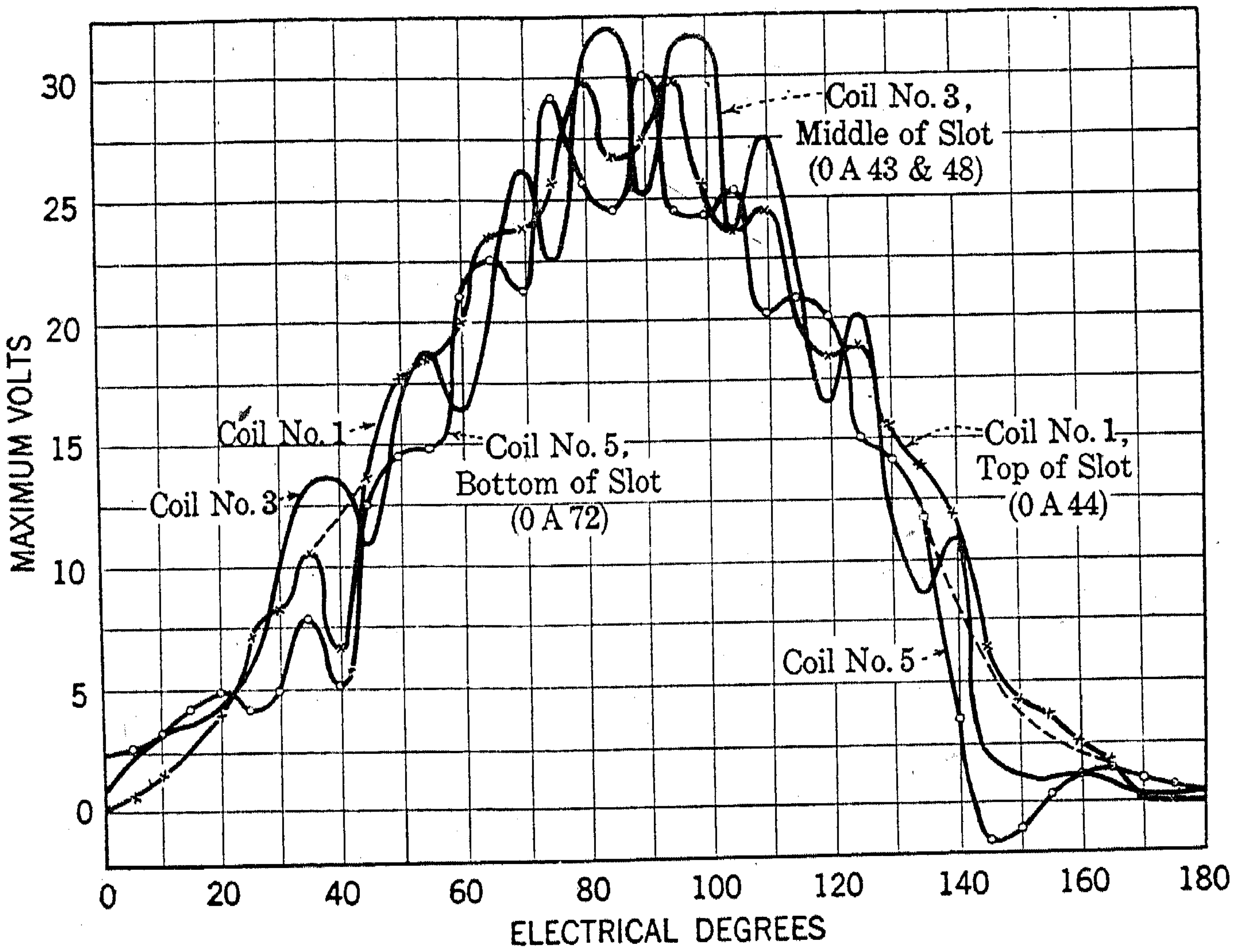


FIG. 25

E_a curves $= E_{fo} - E_r$ = difference between open-circuit and s.s.c. e.m.fs. induced in full-pitch exploring coils Nos. 1, 3 and 5 – field current = 5 amperes—periodicity = 54 cycles per second—Y series connection.

volts; the 3rd is 21 per cent of e_a but (60/54) (5.05/1.24) ×100 equals 450 per cent of the s.s.c. fundamental.

In general, then, if both the flux distribution at no-load and the flux wave due to armature reaction are distorted to the extent of containing 10 to 20 per cent 3rd and 5th harmonics, the resultant s.s.c. flux distortion will be badly distorted, because the above harmonics will amount to several hundred per cent of the fundamental, of the s.s.c. wave. Assume, for example, that e_{fo} and e_a consist of:

	e_{fo}	e_a
1st harmonic	100 per cent	– 95 per cent
3rd "	10 " "	10 " "
5th "	5 " "	5 " "

These values are quite conservative as inspection of Tables VII, VIII, and XII will show, and neither of the waves can be said to be very distorted. The resultant s.s.c. flux distribution will be equal to $e_r = (e_{fo} + e_a)$ and the amplitudes of its 1st, 3rd, and 5th harmonics will be 5, 20 and 10 or 100, 400 and 200 per cent respectively, in terms of the s.s.c. fundamental. These

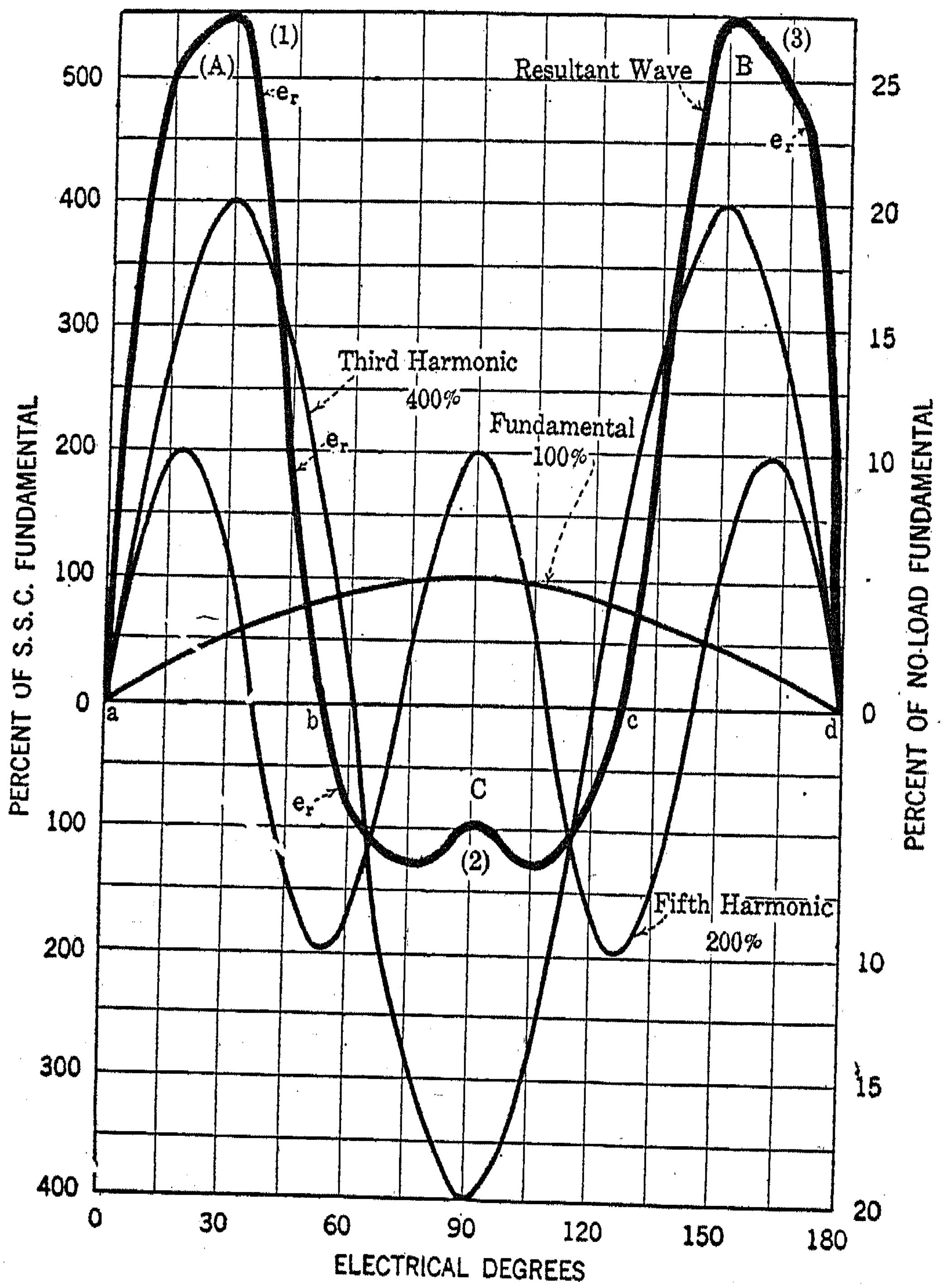


FIG. 26

Illustrating how the extremely distorted resultant wave forms are produced under sustained short circuit.

are drawn in Fig. 26 and their sum, e_r is seen to be extremely distorted and not unlike the waves obtained experimentally under s.s.c.

It is clear now why the flux distribution waves under synchronous motor operation deviate comparatively little from the no-load curve. Obviously at very low voltages such as are obtained under s.s.c., the harmonics of the field and armature reaction will assume a predominant rôle (unless they happen to be out of phase by $\frac{1}{2}$ of their own period and thus neutralize each other).

In Fig. 27 the angular spacings between different crests are designated, *A*, *B*, *D*, *E* and *F* and their values for different kinds of s.s.c. are given in Table XIV. From this it is seen that *A* is approximately 105 deg., or nearly equal to the pole-arc, as it would be expected—see Fig. 26; *E*, Fig. 27, is about 75 deg.

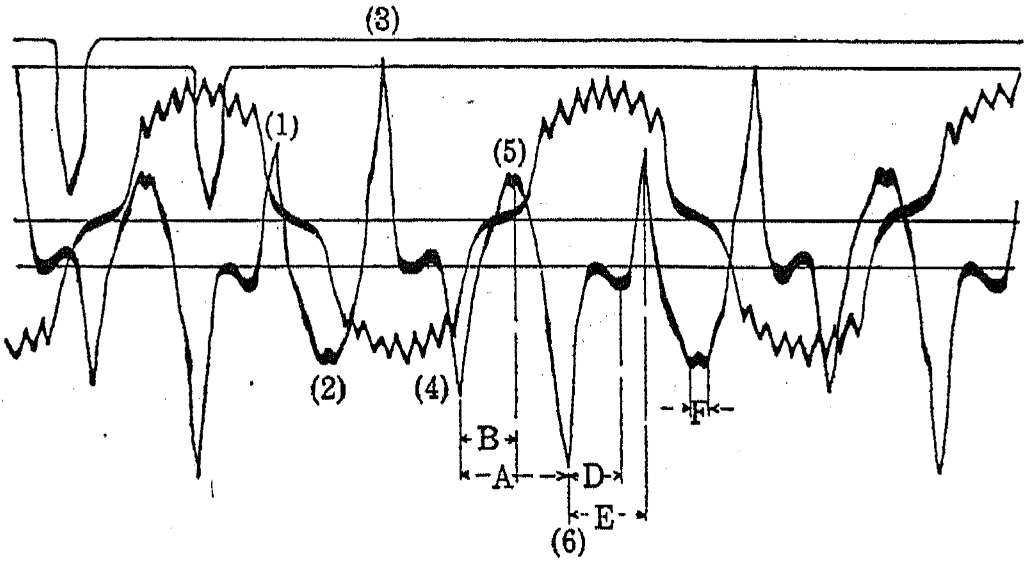


FIG. 27

Oscillogram No. O A 68. *Exposure No.* 1. First wave: contactor. Third wave e.m.f. induced in coil No. 5 at no load, 3.1 amperes field excitation, 52 cycles per second. *Exposure No.* 2: Machine series Y connected and short-circuited at same excitation and speed. Second wave: contactor. Fourth wave: E_{5B}. See Fig. 24.

TABLE XIV.

Data relating to the e.m.fs. induced in the coils at the top and bottom of different slots under s.s.c. and the machine connected, series Y, series △ and two-phase. Frequency = 60 cycles per second.

No. of oscill.	Field amps.	E_{1T} E_{5B}	Kinds of short-circuit	*A*	*B*	*D*	*E*	*F*
						in degrees see Figs. 27, 34, etc.		
202	4.95	E_{1T}	Y	105	51.3	40	75.5	19
O A 88	5	E_{1T}	Y.	106.3	48	35.6	65.2	10
226	5	E_{1T}	Y neutral in	109	51	37	70	
O A 89	5	E_{5B}	Y	103.2	50.4	37.1	73.6	10
O A 88	5	E_{1T}	△	104.5	49.7	41.8	75.4	10
275	4.9	E_{5B}	△	106.2	50.8	39.2	73.5	
O A 89	5	E_{5B}	△	104.5	49.3	36.8	72.0	
O A 81	4.96	E_{1T}	two-phase	105	51.1	36.8	72.3	17.4
O A 86	4.96	E_{1T}	two-phase	105.3	47.8	48.2	76.1	15
O A 82	4.96	E_{5B}	two-phase	104.9	52.4	42.9	79.2	10.1

so that $(A + E) = 180$ deg. *D* is nearly $(1/2\, E)$ but *B* is less than $(1/2\, A)$; referring to the theoretical assumed curve e_r of Fig. 26, it will be easily seen that if the transverse armature reaction (due to the active or in-phase component of armature current with respect to the geometrical pole-axis) is not entirely negligible, the peaks (1) and (3) will be unequal, as it is the case in actual oscillograms Figs. 23 and 27, etc. A little consideration will show that the cross-magnetizing armature reaction will not only reduce, say crest (3), Fig. 26, but will shift crest (2) towards (3). Thus the angle B, between (2) and (3), Fig. 26, will be less than $(1/2\,A)$ = angle between crests (1) and (3), Fig. 27.

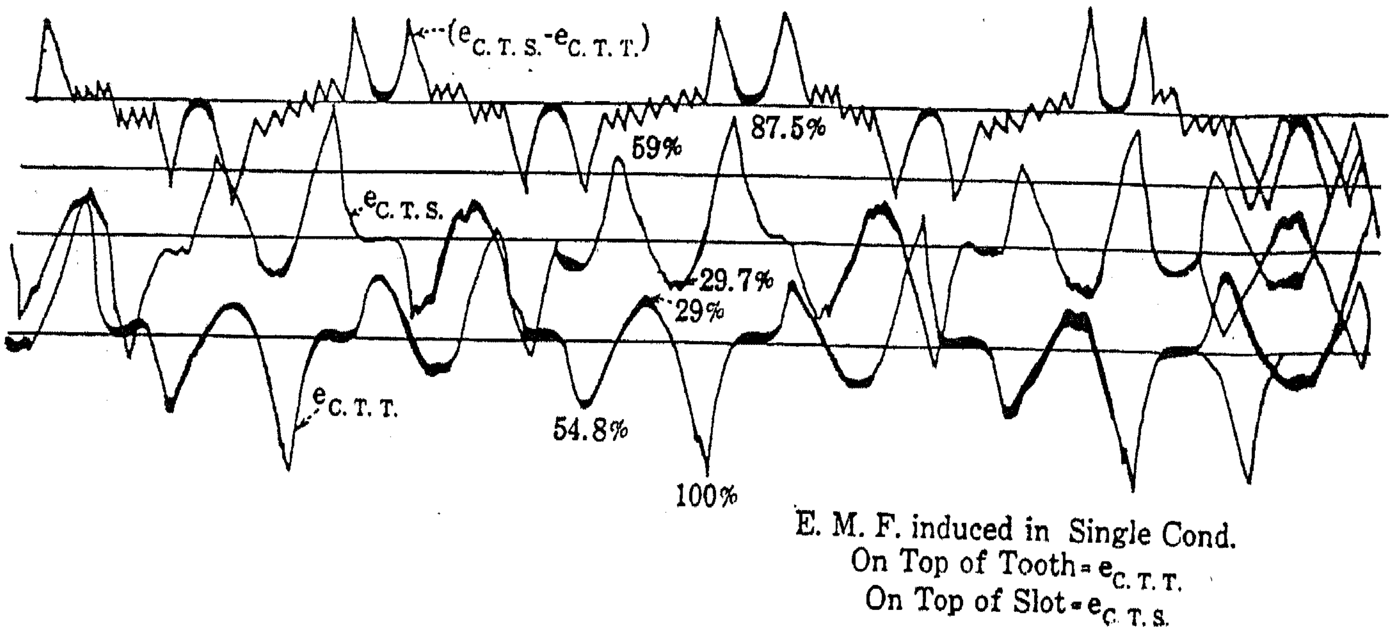


FIG. 27A

Oscillogram No. O A 76. Test No. 188. *Exposure No.* 1. Open circuit run at 5 amperes field excitation and normal speed. First wave = ($E_{CTT} - E_{CTS}$) = (e.m.f. induced in single conductor at center of tooth, No. 14, Fig. 3)−(e.m.f. induced in single conductor at top of succeeding wedge, No. 13, Fig. 3). This should be compared with Fig. 14. *Exposure No.* 2. s.s.c. run under same conditions. Second wave: E_{CTS}. Third wave: E_{CTT}. These two waves should be compared with Figs. 23, 27, 28a, etc., which give the e.m.fs. induced in full-pitch exploring coils.

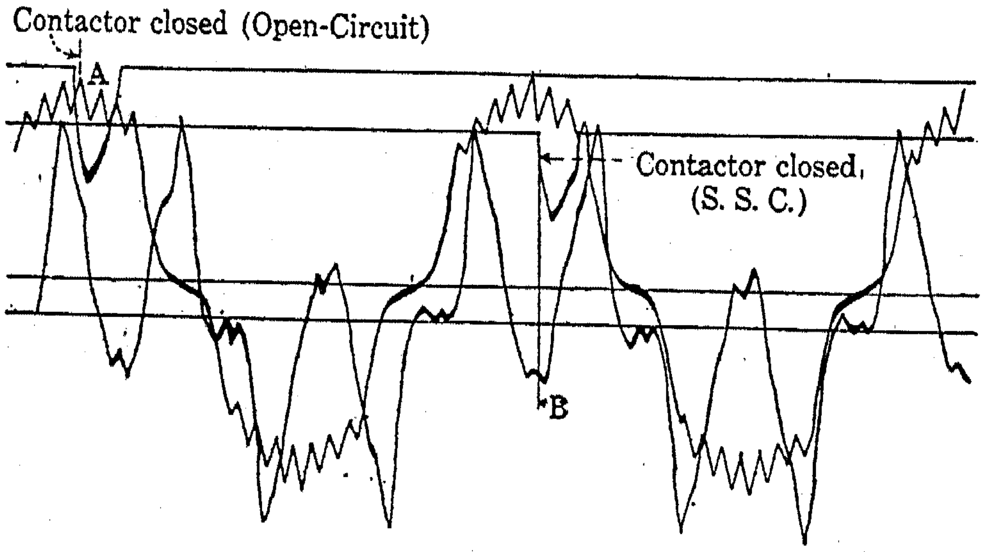


FIG. 28

Oscillogram No. O A 81. Test No. 193. Tables XIV and XV. First wave: Contactor Third wave: E_{1T} = e.m.f. of coil No. 1 at 4.96 amperes of field current, 60 cycles per second and no-load. Second wave: contactor. Fourth wave: E_{1T} under same conditions except machine short-circuited and connected two-phase.

In Fig. 27 the different crests are designated 1, 2, 3, etc. and their values in maximum volts for different kinds of s.s.c. are given in Table XV. Study of this table shows that for three-phase Y or delta s.s.c. the peaks on the same side of the zero line, *i.e.*, (1), (3) or (4), (6) are quite unequal which might have been expected according to the preceding paragraph. Compare also Fig. 27A. In case of two-phase s.s.c., however, the corresponding crests (1), (3) or (4), (6) are very nearly equal. See Fig. 28.

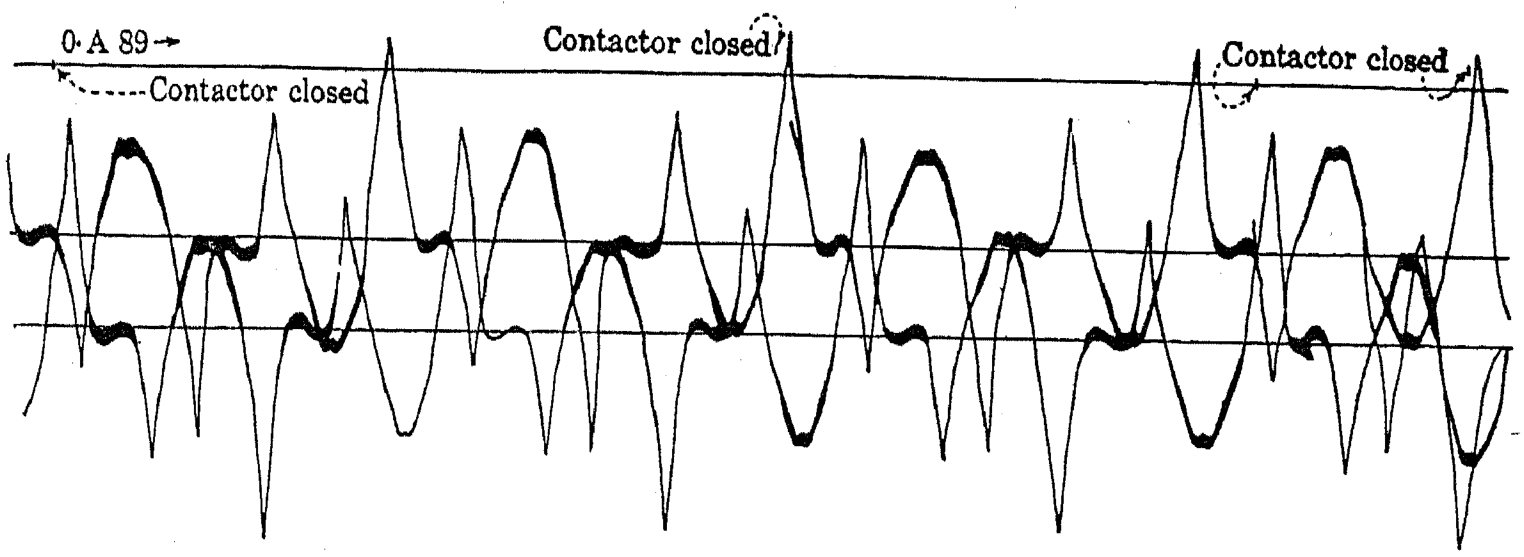
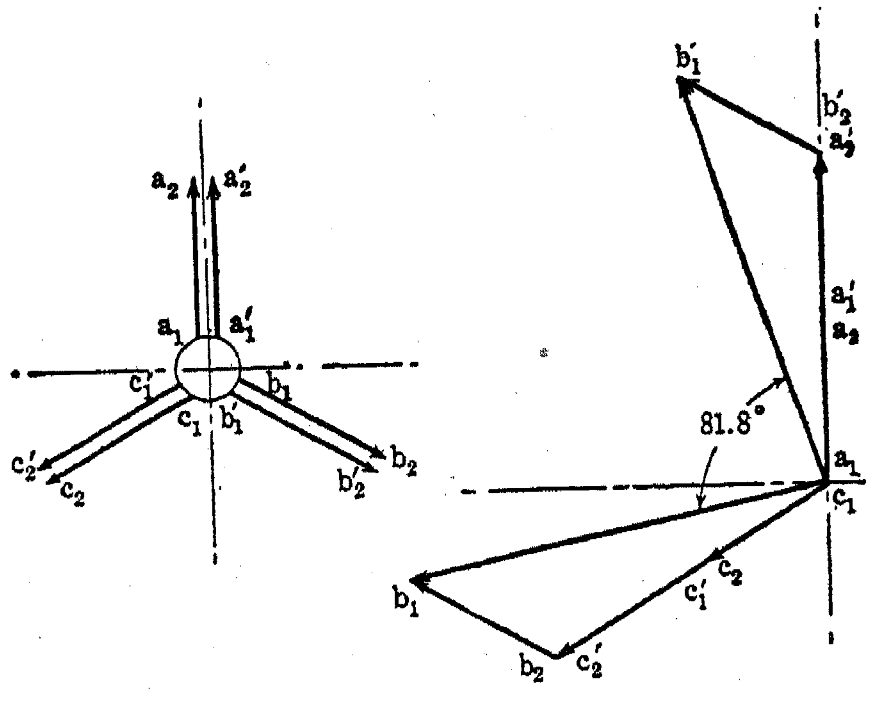


Fig. 28A

Oscillogram No. O A 89. Test No. 201. Table XIV and XV. First wave: Contactor. Third wave: E_{5B} = e.m.f. induced in search coil No. 5 under sustained short circuit at 5 amperes field excitation, 60 cycles per second and series delta connection. Second wave Contactor. Fourth wave: E_{5B} under same conditions, using same vibrator, etc., except machine series Y connected.

Before comparing further the s.s.c. phenomena with the alternator connected Y and delta, see Fig. 28A, also Y with the neutral in, and two-phase, attention should be called to the fact that the two-phase connection was only an imperfect one, the angle



Three-phase two -circuit armature·

Two-phase connection giving 81.8 degrees between phases

Fig. 29

between the phases being 81.8 deg. instead of 90. This is clearly shown in Fig. 29 which is self-explanatory. Further as stated in the beginning of this section the armature current under three-phase s.s.c. is sinusoidal, which is not true in this case as may be seen from Fig. 4, first wave.

In view of these two discrepancies it seems rather remarkable that the general wave shapes of the e.m.fs. induced in the search coils under two-phase short circuit, Fig. 28, are so similar to those obtained with three-phase connection, see Figs. 23, 27 etc., except for some small differences which will be considered presently.

In all the tests with two- or three-phase connection E_{5B} was always found to be considerably larger than E_{1T} so that in many cases it was necessary to insert extra resistance into the vibrator circuit. It will be recalled that this difference in the e.m.fs. induced in coils No. 1 and 5 has already been pointed out in connection with Figs. 7, 20, 22 and Table VII and IX, and it has been ascribed to the fact that the flux density at a surface bounding the bottom of the slots may be larger and more nearly perpendicular to the search coil conductors, than the flux density at the top of the slots. To illustrate this point quantitatively, assume Fig. 30A and 30B to represent the flux distribution at two surfaces bounded by the top and bottom of the slots respectively. For different values of β, the ratio of the 1st and 3rd harmonic of wave 30B to the 1st and 3rd harmonic of Fig. 30A, if $B_T = B_B$, is as follows; (B_B = flux density at bottom of slot. B_T = flux density at top of slot).

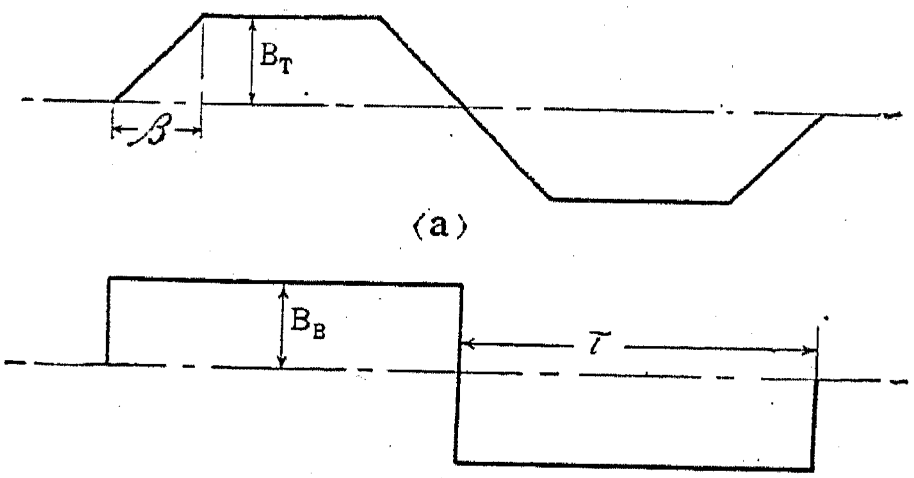


FIG. 30

Assumed flux distribution at top and bottom of slot.

β in degrees	1	5	10	20	30
Ratio of firsts	1.022	1.005	1	1.02	1.11
Ratio of thirds	1.005	1.01	1.005	1.205	1.575

If $B_B = (1.09\ B_T)$ which is a fair assumption (see Fig. 20) the ratios are as follows:

β in degrees	1	5	10	20	30
Ratio of firsts	1.115	1.09	1.09	1.1	1.21
Ratio of thirds	1.092	1.103	1.37	1.31	1.91

Further, with all connections the crests (2) and (5) columns (7) and (10) Table XV, Figs. 23 and 27 are distinctly larger for E_{5B} than for E_{1T}. Again, the amplitudes of these crests in volts do not differ greatly for delta or for Y connection but are smaller for two-phase connection. For example, crest (2) for E_{1T} is about 3.7 for delta and it is 4 volts for Y, but it is only about 1.8 or 2.6 for two-phase connection. The reason for this will be seen from Fig. 26 which shows that crest (2) is due to the amplitude of the armature reaction which is the same for delta or Y but it is smaller for two-phase connection (imperfect) on account of the distribution factor of the winding.

TABLE XV.

Data relating to the crest values of the e.m.fs. induced in the stator search coils No. 1, No. 3 and No. 5 under s.s.c. with the machine connected series Y, series △ and two-phase.

No. of oscill.	Field amps.	Kind of s.s.c.	Frequency	Coil e.m.f.	Amplitudes in max. volts of:					
					Crest (1)	Crest (2)	Crest (3)	Crest (4)	Crest (5)	Crest (6)
					See Figs. 27, 28, etc.					
1	2	3	4	5	6	7	8	9	10	11
202	4.95	Y	60	E_{1T}	6.62	4.01	7.88	6.4	3.7	7.56
O A 44	5	Y	54	E_{1T}	5.45	3.25	7.12	5.6	3.25	7.49
O A 88	5	Y	60	E_{1T}	7.21	6.12	11.24	7.17	6.21	11.24
O A 43	5	Y	54	E_{3M}	4.24	4.24	9.48	4.24	4.69	9.48
O A 72	5	Y	54	E_{5B}	6.43	5.01	10.62	6.3	5.28	10.43
O A 89	5	Y	60	E_{5B}	7.47	5.66	11.75	6.89	5.89	11.64
275	4.64	△	60	E_{1T}	7.86	3.73	9.59	8.06	3.73	9.45
275	4.9	△	60	E_{5B}	8.34	6.12	12.32	7.92	6.53	12.67
O A 88	5	△	60	E_{1T}	6.20	3.77	8.05	6.2	3.69	8.05
O A 89	5	△	60	E_{5B}	7.37	5.98	11.32	6.83	5.98	11.54
205	5 +	two-phase	60	E_{1T}	8.23	2.62	8.11	8.11	3.08	8.11
O A 81	4.96	two-phase	60	E_{1T}	7.89	1.85	8.32	7.68	1.85	7.89
O A 86	4.96	two-phase	60	E_{1T}	9.42	1.95	9.74	9 42	1.95	9.74
O A 52	3.16	Y	52	E_{1T}	3.1	1.94	4.08	3.1	1.75	4.37
O A 54	3.16	Y	52	E_{3M}	2.52	2.81	5.91	2.71	2.47	5.91

Taking everything into consideration, it is found that the e.m.fs. induced in rotor coils with two-phase connection are fairly regular; this is further illustrated by wave analysis $V-4$ and $V-5$. Table XIII and also the e_a curves given in Fig. 31.

However, leaving aside, for a moment, all secondary effects it may be well to refer to a graphical explanation given by A.

E. Clayton.[16] He considers an ideal alternator with no magnetic oscillations or secondary effects, no fringing, etc.; the no-load field form may then be assumed to be rectangular and the armature reaction to be portion of a sine wave as indicated in Figs. 32A and 32B. The resultant curve is given in Fig. 32C which resembles in general the waves obtained under s.s.c. conditions.

The rotor exploring coils under s.s.c. show $2q = 24$ ripples per cycle due to the teeth. See Table I, column 6 and Figs. 8B and 8C. They also indicate oscillations which repeat themselves every 60 deg. See column 5. These as already explained, are due to armature reaction and their jagged unsymmetrical shape may be accounted for as suggested in Fig. 33 where A and B are due to armature reaction and the teeth, and have a frequency of $6f$ and $24f = 2qf$, respectively. Their resultant is seen to be similar to the E_{RF} waves Figs. 8B and 8C. In this connection it may be noted that the long flat oscillations which have been

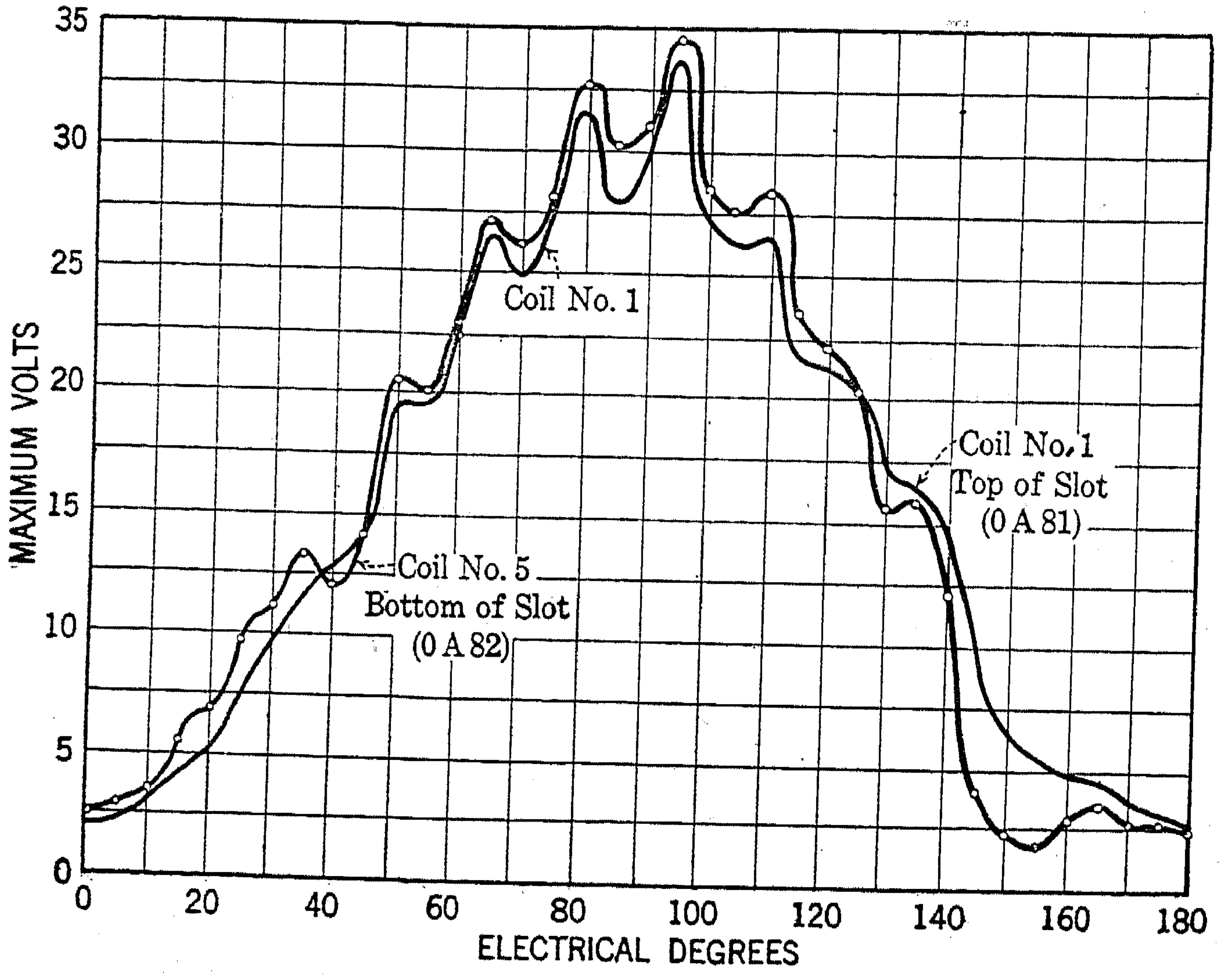


FIG. 31

E_a curves = $E_{fo} - E_r$ = difference between open circuit and s.s.c. e.m.fs. induced in full-pitch top and bottom search coils—field excitation = 5 amperes—periodicity = 60 cycles per second—connection = two-phase.

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[16] The wave shapes obtained with a-c. generators working under steady short-circuit conditions. *Journal, Inst. E. E.*, (London), 1916, Vol. 54, p. 34. This is the first paper, so far as I am aware, to throw any light on this complex subject.

ascribed to the eccentricity of the rotor disappear from the E_{fr} wave under s.s.c. Compare Figs. 4, 5 and 8. This may be explained if the peculiar flux distribution under s.s.c. be considered; taking this to be approximately as represented in Fig. 26 e_r wave, assume that at the position of minimum reluctance the area designated by (A), under crest (1) between $a\ b$, $= 83 = B$ and $C = 66$ $=$ area under crest (2) between $b\ c$; thus the total net flux per pole under s.s.c. will be $166 - 66 = 100$ per cent. Next, at the position of minimum permeance, assume $A = 75 = B$ and $C = 50$; the net flux per pole is again $150 - 50 = 100$ per cent. Obviously under these conditions the e.m.f. due to the eccentricity of the rotor would disappear. However, the areas A, B and C when actually measured were not in the proportion assumed; but it is believed that some such explanation as the one just given is responsible for the disappearance of these oscillations which occur once in every revolution under no-load or synchronous motor operation.

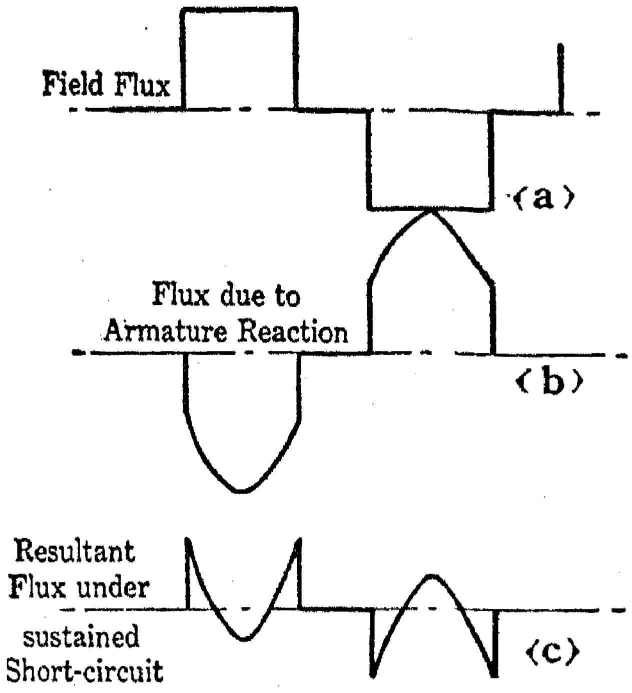
Field Flux
(a)
Flux due to Armature Reaction
(b)
Resultant Flux under sustained Short-circuit
(c)

FIG. 32

Flux distribution curves of ideal salient-pole alternator under sustained short circuit

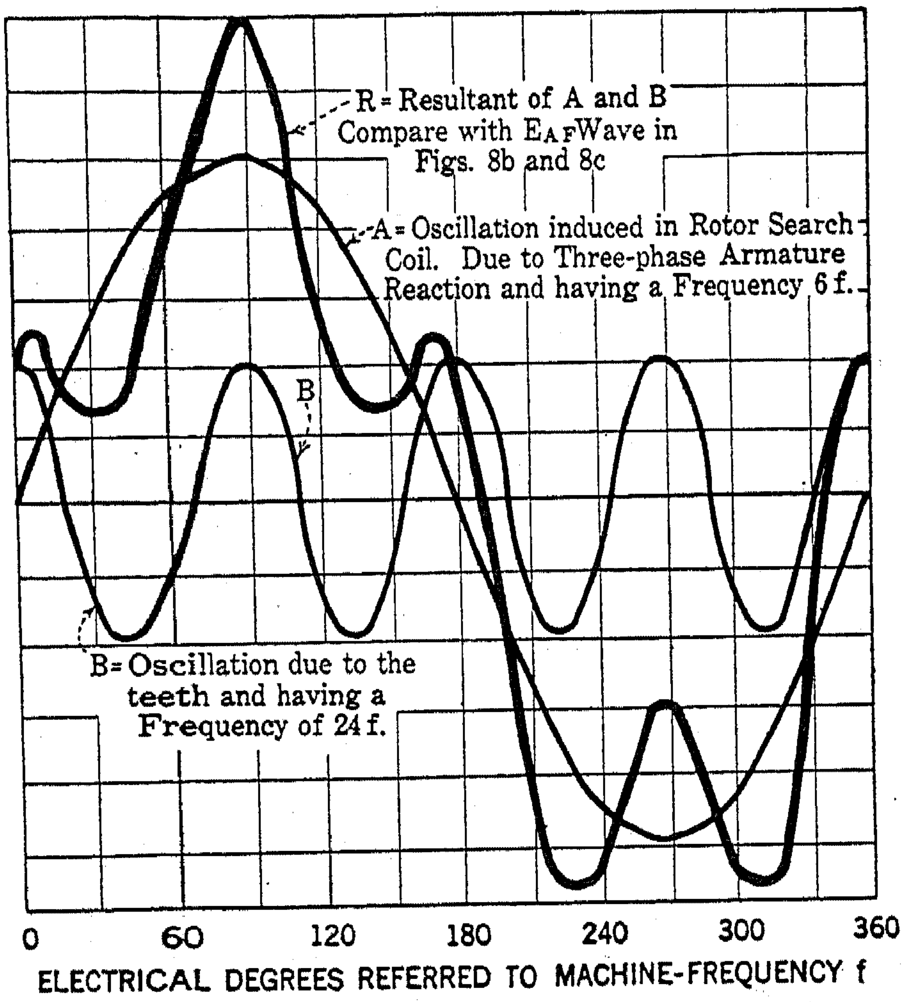

FIG. 33

Theoretical explanation of the production of the peculiar e.m.f. waves shown in Figs. 8B and 8C.

The e.m.fs. induced in rotor coils No. 8 and 7 with two-phase connection are given in Fig. 4 (third wave) and Fig. 34 (first wave) respectively. E_{RF} = e.m.f. induced in coil No. 7, is nearly 3 times (2.7) as large under two-phase s.s.c. as it is with Y connection. See Table I, columns 8 and 9. As to E_{RC} = e.m.f. induced in coil No. 8, see Fig. 34, it is found to be nearly 6 times (5.7) as large with two-phase s.s.c. as with three-phase s.s.c. Under synchronous motor operation Fig. 8A, E_{RC} is about 0.053 at 59.5 cycles against 0.03 at 50 cycles with three-phase s.s.c. Table I, Test No. 165.

In short, the flux oscillations with two-phase connection are very much larger than with three-phase connection. This is believed to be due mainly to the fact that the two phases were not exactly in quadrature.

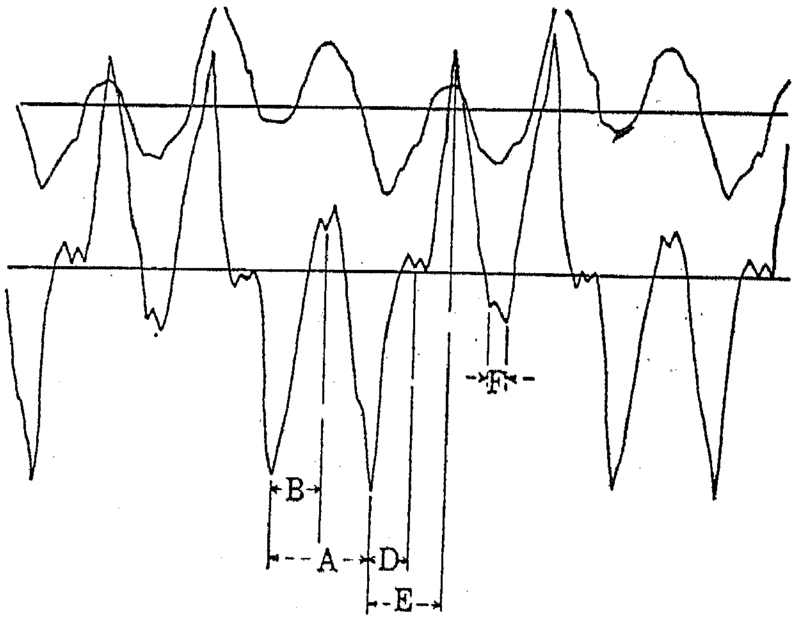

FIG. 34

Oscillogram No. O A 86. Test No. 198. Tables XIV and XV. Sustained short circuit at 4.96 amperes field excitation, and normal speed, two-phase connection. First wave: E_{RC} = e.m.f. induced in rotor coil No. 8. Second wave: E_{1T} = e.m.f. induced in coil No. 1.

It can be shown, (see Note C), that if the angle between the two phases is $(90 - \alpha)$ deg., (= 81.8 deg. in our case, see Fig. 29), the resultant armature reaction will consist approximately of (*a*) a regular travelling wave whose amplitude is approximately equal to the value obtained if $\alpha = 0$. See equation (**56**). (*b*), it will consist of a wave in *quadrature* to the first and travelling in the opposite direction and at the same speed but having an amplitude $(\sin 2\alpha)/2$ of (*a*). Hence it will be seen, that neglecting other effects, this second rotating field, which is *in quadrature to the first and maximum at* $x_1 = 0$, *i.e., at the coil-side of coils No.* 7 *and* 8, will be responsible for the e.m.fs. induced in them under two-phase s.s.c. Further it will account for the peculiarly distorted shape of the E_{RF} wave. This is illustrated in Fig. 35 where the e.m.f. induced in the rotor coil No. 7 is assumed to be due to H_{3R}, H_3^α and H_{5R}, *i.e.*, regular 3rd space harmonic travelling wave, 3rd space harmonic due to lack of quadrature and regular 5th space harmonic travelling wave. See equations (**59**) and (**62**). The amplitude of these waves is assumed to be 100, 27 and 66 per cent respectively, *i.e.*, $H_3^\alpha = 0.27\,H_{3R} = (\frac{1}{2}\sin 4\,\alpha)$, approximately. $H_{5R} = H_{3R}\,(4f/6f) = 0.66\,H_{3R}$, where $(6:4)$ = ratio of their relative velocities with respect to the rotor. Thus it is seen that, Fig. 35 explains

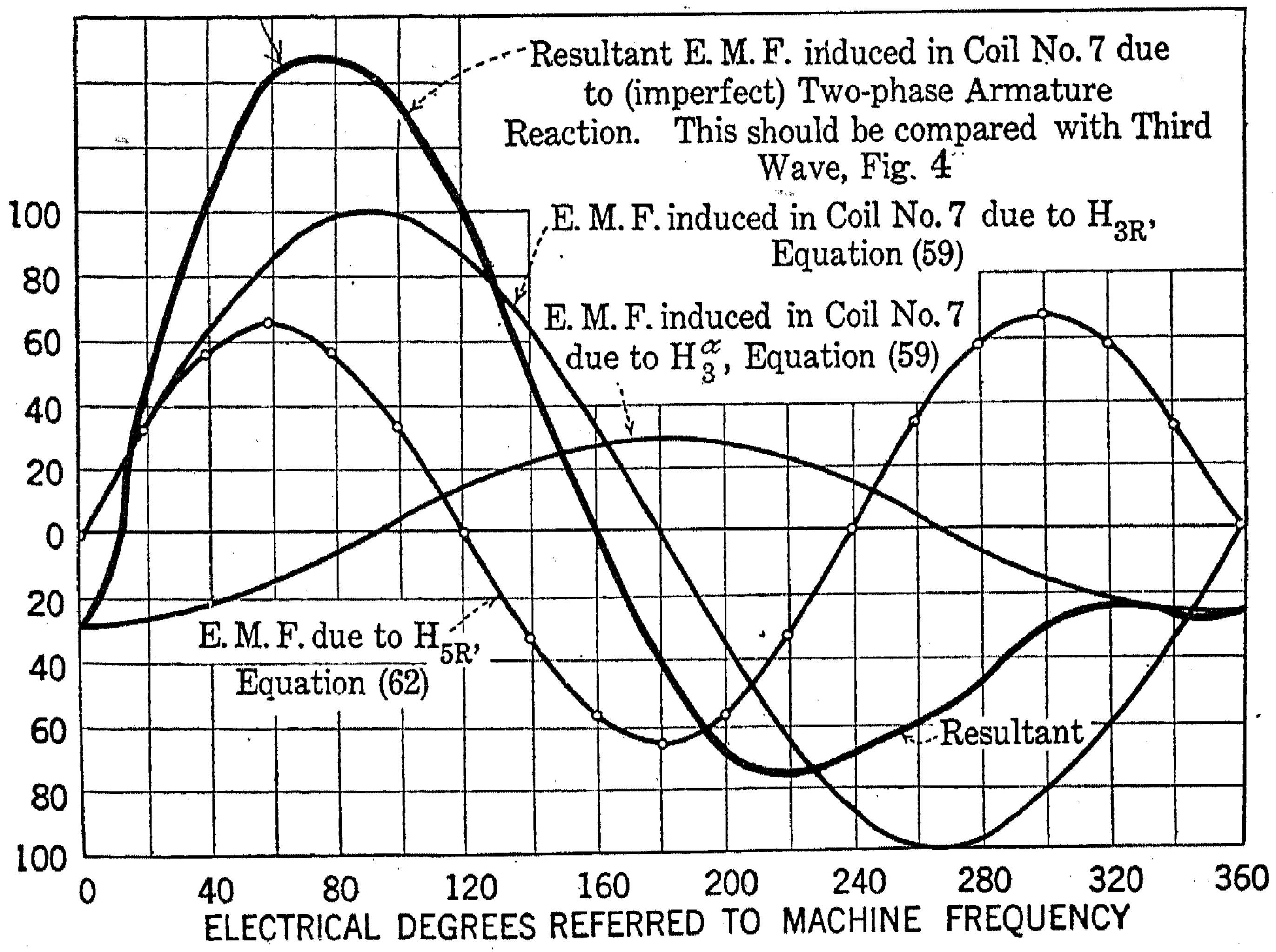

FIG. 35

Theoretical explanation of production of the peculiar E_{RF} wave under 2-phase s. s. c. See third wave Fig. 4.

approximately the peculiar shape of the E_{rf} wave given in Figs. 4 and 34.

Finally considering the e.m.fs. induced under s.s.c. with Y connection, in coils No. 9 and 10, Figs 3 and 3A, we find these to conform to the general wave shape we would expect, according to the explanation given in Section III. In Fig. 36, E_{TT} at no-load and at s.s.c. is given and it is seen that both are characterized by two prominent peaks, and they are quite similar to each other except for the decided difference in shape between a + crest and a − crest. The reason for this will be clear if it be recalled, as explained in detail in Section III, that the portion of the wave between a + and a − crest is produced while the coil is under the pole-arc; on open-circuit the portion of the *B*-curve under the pole is more or less toothed and it gives rise to little kinks see curve (*b*), Fig. 6. Under s.s.c., however, the *B*-curve is very small or even negative at the middle of the pole as indicated by crest (2) Figs. 26, 28, 37 etc. Consequently, as the coil passes from one pole tip to the next its enclosed flux undergoes considerable change, which produces the portion of the curve between a + and a – crest, as indicated in Figs. 36 and 37.

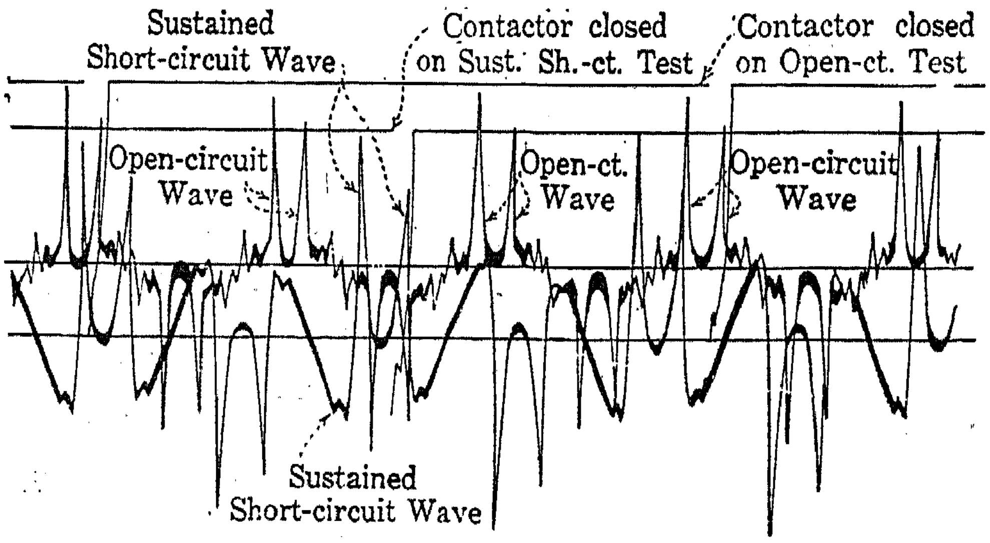

Fig. 36

Oscillogram No. O A 61. Test No. 172. Table II. *Exposure No.* 1. Open-circuit run at 60 cycles per second and 4.95 amperes field excitation. First wave: contactor Third wave: E_{TT} = e.m.f. induced in concentrated exploring coil No. 10, on top of tooth. *Exposure No.* 2. Sustained short circuit with series Y connected and same field current and speed as before. Second wave: contactor. Fourth wave: E_{TT}: This shows that there is no displacement in E_{TT} wave under short-circuit condition.

Space forbids considering single-phase s.s.c. phenomena; however, for the sake of completeness it may be stated that from a series of tests with the machine delta- or Y-connected and short-circuited between two terminals, or one terminal and the neutral, (in case of Y) the exploring coil e.m.fs. were found to be fairly similar to those already described. The similarity is rather striking when the presence of complex secondary effects and pulsations of single-phase alternators is considered.

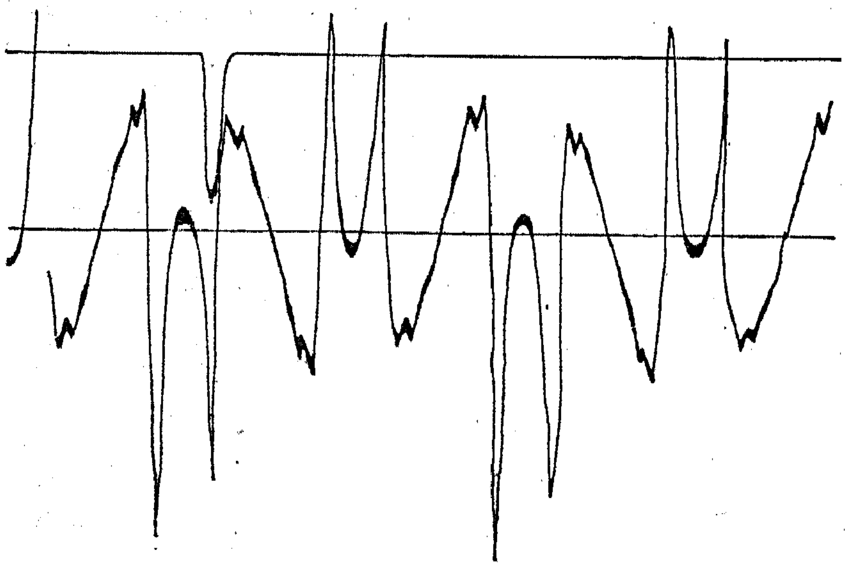

Fig. 37

Oscillogram O A 69. Test No. 180. Sustained short circuit at 5 amperes field excitation, and frequency of 60 cycles per second. Y series connection. First wave: E_{TT} = e.m.f. induced in coil No. 10 on top of tooth.

V. CRITICAL RESUME AND CONCLUSIONS

It seems fair to conclude from the above that the sustained short-circuit (s.s.c.) phenomena of alternators in spite of their complexity and the extremely distorted wave-forms to which they give rise, are amenable to the ordinary theories of synchronous generators as given by Blondel, Potier and others. It is only necessary to restrict the usual sweeping simplifying assumptions of so-called text-books in order to arrive at some of the general results given in the preceding pages. It has been shown that unless the field and armature flux waves are sinusoidal the resultant field under s.s.c. will be badly distorted, see for example Figs. 23, 27 etc., for the simple reason that with the very low voltages which obtain under s.s.c. conditions the fundamental is so greatly reduced that the higher harmonics assume a very predominant role. Starting with a field flux-wave consisting of, 1st 100 per cent, 3rd + 10 per cent, 5th + 5 per cent, and a corresponding armature flux curve containing the same harmonics with relative amplitudes of − 95 per cent, + 10 per cent and + 5 per cent, the resultant flux distribution will be given by e_r, Fig. 26; this is not unlike the actual s.s.c. oscillograms given in Figs. 23, 28, etc.

Furthermore it has been shown in Section IV, that with sinusoidal (in time) armature currents the *H*-curve of armature reaction will be sinusoidal, but not so with the *B*-curve; the latter, as is well known will be flat-topped depending upon the degree of saturation, and will further differ from the *H*-curve in that it will be toothed and have different values, and directions with respect to the armature conductors, at a surface bounded by the top of the slots than at a surface bounded by the bottom of the slots. Thus it is clear that from the view-point of s.s.c. phenomena from among the different schemes used for obtaining "sine-wave alternators" the best is the one which attempts to get to the root of the evil and obtain a sinusoidal flux distribution, under the poles to begin with. However, even then harmonics may and will be introduced by armature reaction. It is well in this connection to emphasize the fact that the *B*-curve of armature reaction will differ more or less from the *H*-curve producing it for the reasons just given. Consequently the elimination of the higher harmonics from the *B*-curve (of armature reaction) is very difficult, if not impossible, since even such devices as fractional slots per pole will avail little unless the permeance per unit area across the pole face is made uniform, constant and free from extreme effects of saturation. However, with a thorough understanding of the subject it should be possible to reduce the effects of the B-curve to a negligible quantity.

The magnetic oscillations under load, no-load and s.s.c. conditions are studied not only by means of full-pitch exploring coils placed on the stator and the rotor but also concentrated coils placed on top of a wedge or a tooth and having a pitch equal to a slot-pitch or fraction thereof. See Figs. 3 and 3A.

The principal oscillations observed are:

(1) *Those due to the teeth*; taking the amplitude (a), Fig. 13 as 100 per cent for the coil No. 1 on top of the slot, we find (a) to be 107 per cent for coil No. 3 and 108 per cent for coil No. 5. On the same basis of percentages, (d), Fig. 13, is 24, 31 and 32 per cent respectively for the same coils. See Fig. 7. These oscillations (due to the teeth) decrease under synchronous motor operation. Compare Figs. 7 and 22.

(2) *Oscillations due to the eccentricity of the rotor.* These have a frequency of ($1/p$) machine-frequency, or one cycle per revolution.

(3) *Oscillations due to armature reaction* and having a frequency of 6 times and 4 times machine-periodicity in case of three-phase and two-phase connection. It seems rather remarkable that oscillations (2) disappears under s.s.c.; an explanation of this is given on p. 1183.

(4) *Oscillations due to the pulsations of magnetic reluctance* caused by the salient-pole construction of the machine. These are best illustrated by means of exploring coils placed on top of a slot and on top of a tooth and having a pitch equal to that of a tooth. See Figs. 10, 11, etc. These oscillations are analyzed in detail in Table II and it is seen that the e.m.f. induced in a concentrated coil on top of a tooth is about twice as large as the e.m.f. induced in an exactly similar coil placed on top of a wedge: These oscillations are studied in detail and they give a clear idea of the flux distribution under *actual running conditions*, which are, of course very important from the point of view of iron losses etc. In this connection it is well to call attention to the methods of analysis and investigation followed in this research; guided by a simplified general mathematical theory the flux pulsations and oscillations were separated and analyzed by taking oscillograms of the e. m. f.'s induced in different stator and rotor exploring coils as explained in detail in Section I.

Certain pecularities and differences between the e.m.fs. induced in search coils at the top and bottom of slots are considered and, at least, tentative explanations are given. For these the reader can consult sections III and IV.

In regard to different kinds of short circuits it is shown that there is practically no difference between s.s.c. phenomena with the machine connected delta, Y, and Y with the neutral in. This might have been expected considering the elimination of triple frequency circulating currents due to the $\frac{2}{3}$ pitch armature winding. In case of two-phase connection, however, the armature current is not as sinusoidal as with three-phase connection; the variational e.m.fs. induced in the rotor coils under s.s.c. show very peculiar wave distortion as seen from Fig. 34, first wave, etc. An explanation of this is indicated in Fig. 35.

Before concluding these remarks it may be well to call attention to the bearing of the results given in the paper upon what are considered standard methods of testing large alternators. The thorough understanding of the underlying facts and the collection of quantitative data on the subject for salient as well as non-salient pole generators will throw a great deal of light on efficiency tests, cyclic heat run tests, short-circuit load loss tests, phase-shifting or feeding back method of load tests, etc. In such tests the flux distribution and the flux density at different points of the magnetic circuit is of considerable importance. It seems to the writer that, in such cases, even if it be extremely difficult to devise tests and means of predetermining by calculation certain characteristics of the machine, without actually loading it, it is very desirable to know and understand, as well as possible, the internal reactions so that proper rational allowances may be made. In any case, the detailed analysis of these phenomena seems to be a step in the right direction, and although the application of such information to design and test problems does not fall within the scope of this paper, it may be well to give a simple quantitative illustration of it.

Assume that the open-circuit core loss of an alternator is 2.8 per cent of its rating and that half of this can be charged to hysteresis and the other half to eddy current loss; let the s.s.c. test loss, excluding friction and windage, be 1.5 per cent. According to the ordinary assumption that the flux distribution under s.s.c. is sinusoidal and 10 per cent of the open-circuit value the hysteresis loss, P_h and the eddy current loss, P_e will be:

$$P_h = \left(\frac{2.8}{2}\right) \cdot \left(\frac{0.10\,B}{B}\right)^{1\cdot 6} \cdot \left(\frac{f}{f}\right)$$
$$= 0.035 \text{ per cent}$$
$$P_e = \left(\frac{2.8}{2}\right) \cdot \left(\frac{0.10\,Bf}{Bf}\right)^{2}$$
$$= 0.014 \text{ per cent}$$

Consequently, $P_h + P_e = 0.049$ per cent.

Next assume that the fundamental is 5 per cent, the third harmonic is 15 per cent and the fifth 10 per cent; these are fair and very conservative values according to the results of this investigation. Then,

$$P_h = \left(\frac{2.8}{2}\right) \cdot \left(\frac{0.05\,B}{B}\right)^{1\cdot 6} \cdot \left(\frac{f}{f}\right) + \left(\frac{0.15\,B}{B}\right)^{1\cdot 6} \cdot \left(\frac{3f}{f}\right) + \left(\frac{0.10\,B}{B}\right)^{1\cdot 6} \cdot \left(\frac{5f}{f}\right)$$
$$= 0.114 \text{ per cent}$$

and,

$$P_e = \left(\frac{2.8}{2}\right) \cdot \left(\frac{0.05\,Bf}{B.f}\right)^{2} + \left(\frac{0.15\,B.\,3f}{B.f}\right)^{2} + \left(\frac{0.10\,B.\,5f}{B.f}\right)^{2}$$
$$= 0.45 \text{ per cent}$$

hence, $(P_h + P_e) = 0.564$ per cent.

Thus according to the usual assumption $(P_h + P_e)$ under s.s.c. tests is negligible, being (0.049/1.5). 100 = 3.3 per cent, nearly, in this example. But if we take the actual flux distribution under a s.s.c. test into consideration, and assume the very conservative values taken above for the 3rd and 5th harmonics of the flux-wave, we find that the eddy current and hysteresis losses amount to, (0.564/1.5). 100 = 38 per cent, nearly, of the total s.s.c. test losses, *i.e.*, the iron losses are not necessarily negligible.

Finally it seems almost unnecessary to call attention to the necessity of standardizing our terminology. In specifications or technical papers and discussions it is only fair that in case of least ambiguity some qualifying term be used to specify the kind of short circuit implied. The words "sudden" and "sustained" seem suitable. However, the question of importance is that we should appreciate the great difference between these two important and complex phenomena of sudden and sustained short circuits and never leave any doubt as to which one is meant.

In conclusion it is a pleasure to the author to acknowledge his indebtedness to the authorities of the Rice Institute for the facilities afforded and particularly the moral support they have lent him. He is also glad to take this opportunity and acknowledge the conscientious and very able assistance of Mr. J. S. Waters, Jr., of the Class of 1917.

SUPPLEMENTARY NOTES

NOTE A. GENERAL E.M.F. EQUATIONS

1. *E.M.F. Induced in Stator Search Coils.* As explained in the e.m.f. equations.

$$e = -N \frac{d\phi}{dt} \tag{1}$$

ϕ may be and is usually, a function of the time t, as well as the position x of the coil with respect to the pole. Thus,

$$\frac{d}{dt}(\phi_{x,\,t}) = \frac{\partial\phi}{\partial x}\cdot\frac{dx}{dt} + \frac{\partial\phi}{\partial t}\cdot\frac{dt}{dt}$$

$$= v\frac{\partial\phi}{\partial x} + \frac{\partial\phi}{\partial t} \tag{2}$$

But,

$$\phi_{(x,\,t)} = \int B_{(x,\,t)}\cdot l dx \tag{3}$$

whence,

$$e = -N\frac{d\phi}{dt} = -N\,l\left[v\frac{\partial}{\partial x}\left(\int B\,dx\right) + \frac{\partial}{\partial t}(B\,dx)\right] \tag{4}$$

$$= e_{\text{motional}} + e_{\text{variational}}$$

Strictly speaking, $\phi = \int B\,da.\cos(B,\,da)$, where $da = ldx$

= element of area at which the flux density is B, and $\cos(B, da)$ $= \cos\theta$, is the angle between the vector B and the normal to the elementary surface $d\,a$; $\cos\theta$ is, of course, ordinarily neglected, except where methods of vector analysis are used. In our attempt to explain certain phenomena, we shall find it necessary to take account of this—which at first may seem an unnecessary refinement. Thus strictly (**4**) should be written:

$$e = - N l v \frac{\partial}{\partial x} \left[\int B \, dx \cos \theta \right] - N l \frac{\partial}{\partial t} \left[\int B \, dx \cos \theta \right] \qquad \textbf{(4a)}$$

$$= e_{\text{rotational}} + e_{\text{variational}}{}^{17}$$

By means of (**4**) or (**4a**), which are quite general, it is easy to obtain expressions for the rotational and variational e.m.fs. induced in any of the stator or rotor coils. Assuming the flux under the pole to be symmetrical with respect to the interpolar axis, we can write,

$$B = \left[B_{1m} \sin \left(\pi \frac{x}{\tau} \right) + B_{3m} \sin \left(3 \pi \frac{x}{\tau} \right) + \text{etc.} \right.$$

$$\left. + B_{1m} \cos \left(\pi \frac{x}{\tau} \right) + B_{3m} \cos \left(3 \pi \frac{x}{\tau} \right) + \text{etc.} \right] \qquad \textbf{(5)}$$

The rotational e.m.f. induced in a stator coil of N concentrated turns and having an active length of l cm. will be:

$$e_{rot.st.} = - N l v \frac{d}{dx} \int_{x_1}^{x_1 + \tau} (B \, dx) \qquad \textbf{(6)}$$

$$= 2 N l v B = 2 N l v \left(B_{1m} \cdot \sin \left(\frac{x}{\tau} \pi \right) + \text{etc.} \right) \qquad \textbf{(7)}$$

since according to the assumption made in (**5**) even harmonics are absent, and in case of a full-pitch coil B at $x_1 = - B$ at $(x_1 + \tau)$. For a short-pitch coil (**7**) must be multiplied by the pitch factor.

In order to find expressions for the variational e.m.fs. let us make the assumption that the *flux variations are sinusoidal.* This is a very convenient assumption, since our equations already involve sine functions, and it is reasonably correct, as indicated by experience, because we are dealing with a *correction term* to be *added* to the comparatively large rotational e.m.f. Thus if $2 q$ = number of slots per pair of poles, p, and (f/p) = speed of the machine in rev. per sec., obviously, $2 q \, (f/p)$ will be the number of slots per second passing a given point x_1; this will set

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[17] S. J. Barnett, Electromagnetic Theory, pp. 333 to 340.

up an oscillation of small amplitude and of frequency $2\,q\,f$, which will be expressed in mathematical symbolism by:

$$B_{pul} = B\ (\eta \cos\ (2\,q\ \omega\ t)\ ) \tag{8}$$

Neglecting the higher harmonics of the B curve, see (5), we get,

$$B_{pul} = \eta\ B_1 \left(\sin \left(\frac{x}{\tau}\ \pi \right) \right) (\cos\ (2\,q\ \omega\ t)\ ) \tag{9}$$

whence,

$$\frac{\partial}{\partial t}\ (B_{pul}) = -\ \eta\ B_1\, 2\,q\ \omega \left(\sin \left(\frac{x}{\tau}\ \pi \right) \right) (\ \sin\ (2\,q\ \omega\ t)\ ) \tag{10}$$

and according to the second term of the right hand member of (4) the variational e.m.f. is proportional to:

$$\int_x^{x+\tau} \frac{\partial B_{pul}}{\partial t}\ dx = -\ \eta\ B_1\, 4\,q\ \omega\ \frac{\tau}{\pi} \left(\cos \frac{x}{\tau}\ \pi \right) (\sin\ 2\,q\ \omega\ t) \tag{11}$$

To find the e.m.f. induced in any armature conductor it must be remembered that owing to the relative movement between armature and field,

$$\begin{aligned} x &= v\ t = (2\ \pi\ r)\ (\text{r.p.s.})\ t \\ &= 2 \left(\frac{2\ \pi\ r}{2\ p} \right) (\text{r.p.s.})\ t = (2\,\tau\,f)\ t = \left(\omega\ \frac{\tau}{\pi} \right) t \end{aligned} \tag{12}$$

where r = radius of rotor, so that,

$$\begin{aligned} e_{pul.st.} &= -\ N\ l\ \frac{\partial}{\partial t} \int B\ dx \\ &= 4\ N\ l\ \frac{\tau}{\pi}\ q\ \omega \cdot \eta\ B_1\ (\cos\ \omega\ t)\ (\sin\ 2\,q\ \omega\ t) \end{aligned} \tag{13}$$

$$\begin{aligned} &= 4\ N\ l\ q\ \omega\ \frac{\tau}{\pi}\ \eta\ B_1\ [\sin\ (2\,q\ \omega + 1)\ \omega\ t \\ &\qquad + \sin\ (2\,q - 1)\ \omega\ t] \end{aligned} \tag{14}$$

To find analytical expressions for the static or variational e.m.f. in an armature conductor due to the to-and-fro swing of the flux, we shall assume that the B-curve is constant in magnitude, but it moves across the pole-face according to simple harmonic motion, at a frequency of $2\,q\,f$. Hence neglecting the higher harmonics of (5) we may write,

$$B_{sw} = B_1 \left[\sin \frac{\pi}{\tau}\ (x - \epsilon \sin 2\,q\ \omega\ t) \right] \tag{15}$$

where $2\,q\,\omega$ is the angular velocity and ϵ the amplitude of the variations.

Differentiating (15) with respect to t and making the reasonable assumption that,

$$\cos\left(\frac{\pi}{\tau}\ \epsilon \sin 2\,q\,\omega\,t\right) = 1, \text{ and}$$

$$\sin\left(\frac{\pi}{\tau}\ \epsilon \sin 2\,q\,\omega\,t\right) \cong \frac{\pi}{\tau}\ \epsilon\ (\sin 2\,q\,\omega\,t)$$

we obtain:

$$\frac{\partial B_{sw}}{\partial t} = -\,B_1\left[\cos \frac{\pi}{\tau}\ (x - \epsilon \sin 2\,q\,\omega\,t)\right]$$

$$\left[2\,\epsilon\,q\,\omega\ \frac{\pi}{\tau}\ \cos 2\,q\,\omega\,t\right] \tag{16}$$

and

$$\int_x^{x+\tau} \frac{\partial B_{sw}}{\partial t}\ d\,x = [4\,\epsilon\,B_1\,q\,\omega \cos\,(2\,q\,\omega\,t)]$$

$$\left[\sin \frac{\pi}{\tau}\ (x - \epsilon \sin 2\,q\,\omega\,t) \cos \frac{x}{\tau}\ \pi\right] \tag{17}$$

As before, to find the variational e.m.f. in a given armature conductor due to the flux-swing, substitute,

$$x = (2\,f\,\tau\,t) = \left(\omega\ \frac{\tau}{\pi}\ t\right)$$

Thus,

$$e_{sw.st.} = -\,4\,N\,l\,q\,\omega\ (\epsilon\,B_1)\ (\cos 2\,q\,\omega\,t)$$

$$[\sin\,(\omega\,t - \epsilon \sin 2\,q\,\omega\,t) \cos \omega\,t]$$

$$= -\,4\,N\,l\,q\,\omega\ (\epsilon\,B_1)\left[\{\sin\,(2\,q + l)\,\omega\,t - \sin\,(2\,q - l)\,\omega\,t\}\right.$$

$$\left. - \left\{\frac{\epsilon}{2}\ \sin\,(4\,q + 1)\,\omega\,t + \sin\,(4\,q - 1)\,\omega\,t\right\}\right] \tag{18}$$

2. *E.M.Fs. Induced in Field Exploring Coils.* Equation (7) gives the rotational e.m.f. induced in a coil of N concentrated turns having a relative speed v with respect to B. Evidently this is zero for a coil placed on the rotor, and therefore,

$$e_{mot.r.} = 0 \tag{19}$$

To find the variational e.m.f. due to flux pulsations in a field (rotor) search coil whose coil-side is at any point x_1, simply substitute $x = x_1$ in (**13**). Thus,

$$e_{\overline{pul.r}} = 4\,N\,l\,q\,\omega\,(n\,B_1)\cos\left(\frac{\pi}{\tau}\,x_1\right)(\sin 2\,q\,\omega\,t) \qquad \textbf{(20)}$$

Similarly from (**17**) and (**4**) we get,

$$\begin{aligned} e_{sw.r.} &= -\,N\,l\,\frac{\partial}{\partial t}\int B\cdot d\,x \\ &= 4\,N\,l\,q\,\omega\,(\epsilon\,B_1)\,(\cos 2\,q\,\omega\,t) \\ &\qquad \left[\left\{\sin\left(\frac{\pi}{\tau}\,x_1-(\epsilon\sin 2\,q\,\omega\,t)\right)\right\}\cos\frac{\pi}{\tau}\,x_1\right] \qquad \textbf{(21)} \end{aligned}$$

For coils No. 7 and 8, $x_1 = 0$, whence we obtain directly (**22**) and (**23**) from (**20**) and (**21**) respectively, as given in the first part of the paper:

$$e_{sw.r.} = 2\,N\,l\,q\,\omega\,\epsilon^2\,B_1\sin(4\,q\,\omega\,t) \qquad \textbf{(22)}$$

and,

$$e_{pul.r.} = 4\,N\,l\,q\,\omega\,\eta\,B_1\sin(2\,q\,\omega\,t) \qquad \textbf{(23)}$$

NOTE B

1. *Theoretical Investigation of Armature Reaction.* Make the reasonable assumption (as shown on p. 1163) that the armature current under s.s.c. is sinusoidal (with respect to time). Further assume that the armature winding of say, a three-phase alternator is sinusoidally distributed (in space) as schematically represented in Fig. 17. If one of the phase windings were supplied with direct-current the magnetic field intensity H, at any point θ_{gsr}, geometrical space radians or x centimeters, from the axis of reference, see Fig. 17, would be,

$$H_1' = \gamma\,I_{d.c.}\,(N_{max}\cos p\,\theta_{gsr}) = \gamma\,I_{d.c.}\,N_{max}\cos\left(\frac{x}{\tau}\pi\right) \qquad \textbf{(24)}$$

where, γ = constant, and $\left(\frac{x}{\tau}\,\pi\right) = p\,\theta_{gsr}$ = electrical space radians. Similarly for the other phases:

$$\left.\begin{aligned} H_1'' &= \gamma\,I_{d.c.}\,N_m\cos\left(\frac{x'}{\tau}\,\pi\right) = \gamma\,I_{d.c.}\,N_m\cos\left(\frac{x}{\tau}-\frac{2}{3}\right)\pi \\ H_1''' &= \gamma\,I_{d.c.}\,N_m\cos\left(\frac{x''}{\tau}\,\pi\right) = \gamma\,I_{d.c.}\,N_m\cos\left(\frac{x}{\tau}-\frac{4}{3}\right)\pi \end{aligned}\right\} \qquad \textbf{(25)}$$

When these windings are supplied with balanced three-phase currents the resultant field intensity due to armature reaction will be:

$$H_1' = \gamma\,[I_m \cos \omega\, t] \left[N_m \cos \frac{x}{\tau}\,\pi\right] \tag{26}$$

$$H_1'' = \gamma \left[I_m \cos \left(\omega\, t - \frac{2}{3}\,\pi\right)\right] \left[N_m \cos \left(\frac{x}{\tau} - \frac{2}{3}\,\pi\right)\right] \tag{27}$$

$$H_1''' = \gamma \left[I_m \cos \left(\omega\, t - \frac{4}{3}\,\pi\right)\right] \left[N_m \cos \left(\frac{x}{\tau} - \frac{4}{3}\,\pi\right)\right] \tag{28}$$

whence by trigonometry,

$$\begin{aligned} H = H_1' + H_1'' + H_1''' &= \frac{3}{2}\, H_m \cos \left(\omega\, t - \frac{x}{\tau}\pi\right) \\ &= \frac{3}{2}\, \gamma\, I_m \cos \left(\omega\, t - \frac{x}{\tau}\,\pi\right) \end{aligned} \tag{29}$$

This gives the resultant field intensity due to armature reaction and it will be recognized to be the well known equation of a travelling wave which is a function of the time t, as well as the space position x, and whose maximum, H_m, occurs when $\omega\, t = \left(\frac{x}{\tau}\,\pi\right)$; in other words, whose maximum H_m travels at a speed $(x/t) = (\omega\,\tau/\pi) = (2 f \tau)$, which is the peripheral speed of the rotor. See (**12**). Next, consider the case of N conductors per slot and one slot per pole per phase carrying $I_{dc.}$ amperes. Evidently, as indicated in Fig. 18 the field intensity H, will be rectangular in shape and may be represented by the series:

$$H = \frac{4}{\pi}\, Y_m \left[\sin \left(\frac{x}{\tau}\,\pi\right) + \frac{1}{3} \sin \left(3\,\frac{x}{\tau}\,\pi\right) + \dots\right] \tag{30}$$

Equation (**30**) takes the place of (**24**) and similar expressions can be written out for the other phases, corresponding to (**25**) and (**26**). However, without going any further into this, suffice to call attention to the references given in foot-note (**13**) and state that if the non-sinusoidally distributed winding represented in Fig. 18, be supplied with balanced alternating-current instead of direct-current, expressions exactly analogous to (**26**), (**27**) etc. will be obtained. Combining these trigonometrically, we get (**32**), (**33**) etc. which were given in the first part, in section IV.

2. *E.M.F. Induced in Rotor and Stator Exploring Coils by the Armature Reaction.* It is a simple matter, now to obtain general expressions for the e.m.fs. induced in the different search coils, if we assume that the B-curve is sinusoidal; the rotational e.m.f. of a stator coil whose coil-side is at a fixed distance x_1 from the origin, see Fig. 6 will be according to (**4**):

$$e_{rot.st} = -N\,l\,v\,\frac{\partial}{\partial x}\int B\,d\,x \tag{36}$$

$$= -N\,l\,(2f\tau)\,\frac{\partial}{\partial x}\int \frac{3}{2}B_a\cos\left(\omega\,t-\frac{x_1}{\tau}\,\pi\right)\cdot d\,x \tag{37}$$

$$= -N\,l\,(2f\tau)\left(\frac{3}{2}\,B_a\right)\cos\left(\omega\,t-\frac{x_1}{\tau}\,\pi\right) \tag{38}$$

This is a sinusoidal e.m.f. of fundamental frequency and having a phase angle $\left(\frac{x_1}{\tau}\,\pi\right)$ with respect to the origin.

The fifth space harmonic of armature reaction will give:

$$e_{rot.st.} = -N\,l\,\frac{2\,f\,\tau}{5}\cdot\frac{\partial}{\partial x}\int\frac{3}{2}\,B_a\cos\left(\omega\,t+\frac{5\,x_1}{\tau}\,\pi\right) \tag{39}$$

$$= -N\,l\,\frac{2\,f\,\tau}{5}\cdot\frac{3}{2}\,B_a\cos\left(\omega\,t+\frac{5\,x_1}{\tau}\,\pi\right) \tag{40}$$

i.e., an e.m.f. of fundamental machine frequency and having a phase angle $(6\,x_1/\tau)\,\pi$ with respect to the origin. Similar expressions may be derived for all other harmonics.

However, as already explained, the sinusoidal H-curve produced by the armature reaction will give rise to a distorted B-curve. This will produce in the stator coils a motional e.m.f. which will contain higher *time harmonics*. To find analytical expressions for these e.m.fs. (due to a distorted gliding or rotating B-curve) it will be convenient to assume the B-curve as stationary and the exploring coils revolving at the same speed in the opposite direction. In this manner the problem is reduced to the one already considered in Note A, and it seems hardly necessary to derive equations for them here.

As to the e.m.fs. induced in the rotor search coils (due to armature reaction). it is clear that the fundamental travelling wave given by (**32**), will induce no e.m.f. The 3rd harmonic will, of course, be zero for a three-phase machine. See (**33**).

To find the motional e.m.f. induced in a rotor coil due to the fifth space harmonic B_5,

$$B_5 = (1/2)\, B_a \cos\,(\omega\, t + 5\, x\, \pi/\tau) \tag{41}$$

it must be remembered that x increases at the rate of $2 f \tau$ with respect to the rotor. Thus substituting in (**41**),

$$\omega\, t = \pi\, x/\tau = (\pi/\tau)\, 2 f \tau\, t \tag{42}$$

we obtain,

$$B_5 = (3/2)\, B_a\, (\cos 6\, \omega\, t) \tag{43}$$

Similarly,

$$B_7 = (3/2)\, B_a\, (\cos 6\, \omega\, t) \tag{44}$$

That is, the 5th and 7th space harmonics though of fundamental frequency with respect to the stator are of sextuple frequency with respect to the rotor. This will be clear when it is considered that the 5th space harmonic glides backward at a speed $(2 f \tau/5)$ while the seventh travels forward at a speed $(2 f \tau/7)$. Assuming for convenience the rotor as stationary, it is evident, that the relative speed between the rotor and these gliding fields will be:

$$(2 f \tau) - \left(- \frac{2 f \tau}{5}\right) = 6 \left(\frac{2 f \tau}{5}\right) \tag{45}$$

and

$$2 f \tau - \left(+ \frac{2 f \tau}{7}\right) = 6 \left(\frac{2 f \tau}{7}\right) \tag{46}$$

respectively. The pole pitches τ_5 and τ_7 of the fifth and seventh fields are $(\tau/5)$ and $(\tau/7)$, and the corresponding number of pairs of poles are $5\, p$ and $7\, p$ respectively. Consequenty the frequency of the e.m.fs. induced in a rotor coil by them will be:

$$f_5 = (\text{r.p.s.})_5\, (p_5) = (\text{speed})_5 \left(\frac{1}{2 \tau p}\right) (5\, p) \tag{47}$$

$$= 6 \left(\frac{2 \tau f}{5}\right) \cdot \left(\frac{1}{2 \tau p}\right) (5\, p) = 6 f \tag{48}$$

Similarly for the 7th space harmonic,

$$f_7 = 6 \left(\frac{2 \tau f}{7}\right) \left(\frac{1}{2 \tau p}\right) 7\, p = 6 f \tag{49}$$

In general for the $(6\, n - 1)$ *th* space harmonic which travels backward with respect to the fundamental,

$$f_{(6n-1)} = 6\,n\Big(2\tau f/(6\,n-1)\Big)\Big((6\,n-1)\,p/2\tau\,p\Big) = 6\,n f \qquad (50)$$

and for the $(6\,n+1)\,t\,h$ harmonic,

$$f_{(6n+1)} = 6\,n\Big(2\tau f/(6\,n+1)\Big)\Big((6\,n+1)\,p/2\tau\,p\Big) = 6\,n f \qquad (51)$$

Thus the 11th and 13th will give rise to rotor e.m.fs. having periodicities of $12f$, and so on for the others.

In case of two-phase alternators the 3rd and 5th will give rise to e.m.fs. of (4. machine frequency). The seventh and ninth will produce e.m.fs. of (8. machine frequency), etc.

However, as shown by the oscillograms given before, the rotor search coils indicate only pulsations having periodicities of $6f$ in case of three-phase connection and $4f$ in case of two-phase connection. In concluding this note attention may be called to the fact that the above analysis of armature reaction and the e.m.fs. induced by it in the stator and rotor coils is applicable to induction motors; as a matter of fact it is a little difficult for the writer to understand why standard texts do not treat armature reaction in alternators etc. in exactly the same way as the rotating fields of induction motors, as it has been done above, because it is more convenient and lends itself better to the derivation of quantitative expressions for the e.m.fs. induced in the stator and rotor by armature reaction alone.

NOTE C. DIRECT COMPONENT OF ARMATURE REACTION OF AN IMPERFECT TWO-PHASE ALTERNATOR

As already explained in detail the resultant magnetic field intensity due to balanced two-phase currents flowing through sinusoidally distributed two-phase windings will be given by:

$$H_1' = H_1' + H_1'' = H_m\Big[\Big\{\cos\omega t\Big\}\Big\{\cos\Big(\frac{x}{\tau}\,\pi\Big)\Big\} + \Big\{\cos\Big(\omega t + \frac{\pi}{2}\Big)\Big\}\Big\{\cos\Big(\frac{x}{\tau}\,\pi + \frac{\pi}{2}\Big)\Big\}\Big] \qquad (52)$$

Now in case of an imperfect machine which has a displacement between phases of $\left(\frac{\pi}{2} - \alpha\right)$ deg. instead of $\frac{\pi}{2}$, the sum of H_1' and H_1'' will be:

$$H_1 = H_m \Big[\{\cos \omega t\} \Big\{ \cos \Big(\frac{x}{\tau}\, \pi \Big) \Big\}$$

$$+ \Big\{ \cos \Big(\omega t + \frac{\pi}{2} - \alpha \Big) \Big\} \Big\{ \cos \Big(\frac{x}{\tau}\, \pi + \frac{\pi}{2} - \alpha \Big) \Big\} \Big] \quad \textbf{(53)}$$

$$= H_m \Big[\frac{1}{2} \cos \Big(\omega t + \frac{x}{\tau}\, \pi \Big) + \frac{1}{2} \cos \Big(\omega t - \frac{x}{\tau}\, \pi \Big) -$$

$$\frac{1}{2} \cos \Big(\omega t + \frac{x}{\tau}\, \pi - 2\alpha \Big) + \frac{1}{2} \cos \Big(\omega t - \frac{x}{\tau}\, \pi \Big) \Big] \quad \textbf{(54)}$$

$$= H_m \Big[\Big\{ \cos \Big(\omega t + \frac{x}{\tau}\, \pi \Big) \Big\} \Big\{ \frac{1}{2} - \frac{\cos 2\alpha}{2} \Big\}$$

$$- \Big\{ \frac{\sin 2\alpha}{2} \Big\} \Big\{ \sin \Big(\omega t + \frac{x}{\tau}\, \pi \Big) \Big\} + \Big\{ \cos \Big(\omega t - \frac{x}{\tau}\, \pi \Big) \Big\} \Big] \quad \textbf{(55)}$$

or approximately,

$$H_1 = H_m \Big[\cos \Big(\omega t - \frac{x}{\tau}\, \pi \Big) - \Big(\frac{\sin 2\alpha}{2} \Big) \Big(\sin \Big(\omega t + \frac{x}{\tau}\, \pi \Big) \Big) \Big] \quad \textbf{(56)}$$

= regular fundamental (space) wave gliding forward + a second wave due to lack of quadrature, and gliding backwards.

Further it can be shown, as it was explained in Note B, that with ordinary non-sinusoidally distributed windings, supplied with alternating current of fundamental frequency, (with respect to time) there will be a 3rd space harmonic of the form, $H_3 = H_3' + H_3''$,

$$H_3 = H_a \Big[\{\cos (\omega t)\} \Big\{ \cos \Big(\frac{3x}{\tau}\, \pi \Big) \Big]$$

$$+ \Big\{ \cos \Big(\omega t + \frac{\pi}{2} - \alpha \Big) \Big\} \Big\{ \cos \Big(\frac{3x}{\tau}\, \pi + \frac{3\pi}{2} - 3\alpha \Big) \Big\} \Big] \quad \textbf{(57)}$$

Translating (**57**) into words we shall find that it simply represents the field intensity due to a current of fundamental pulsation ω flowing through the 3rd harmonic of a non-sinusoidal winding. Simplifying this equation, we get,

$$H_3 = H_a \Bigg[\left\{\cos\left(\omega\, t + \frac{3\,x}{x}\ \pi\right)\right\} \left\{\left(\frac{1}{2} + \frac{\cos 4\,\alpha}{2}\right)\right\}$$

$$+ \left\{\cos\left(\omega\, t - \frac{3\,x}{\tau}\ \pi\right)\right\} \left\{\left(\frac{1}{2} - \frac{\cos 2\,\alpha}{2}\right)\right\}$$

$$+ \left\{\sin\left(\omega\, t + \frac{3\,x}{\tau}\ \pi\right)\right\} \left\{\left(\frac{\sin 4\,\alpha}{2}\right)\right\}$$

$$- \left\{\sin\left(\omega\, t - \frac{3\,x}{\tau}\ \pi\right)\right\} \left\{\frac{\sin 2\,\alpha}{2}\right\} \Bigg] \qquad \textbf{(58)}$$

or approximately,

$$H_3 = H_a \Bigg[\cos\left(\omega\, t + \frac{3\,x}{\tau}\ \pi\right)$$

$$+ \left(\sin\left(\omega\, t + \frac{3\,x}{\tau}\ \pi\right)\right)\left(\frac{1}{2}\sin 4\,\alpha\right)$$

$$+ \left(\sin\left(\omega\, t - \frac{3\,x}{\tau}\ \pi\right)\right)\left(\frac{1}{2}\sin 2\,\alpha\right)\Bigg] \qquad \textbf{(59)}$$

$= H_{3R} + H_3{}^{\alpha} + H_{31}{}^{\alpha}$

$=$ regular third space harmonic travelling backward at a speed $2\,f\,\tau/3$. $+$ a second wave due to lack of quadrature and in quadrature to the first and travelling in the same direction and at the same speed as the first. $+$ a third wave due to lack of quadrature and in phase with the second but travelling forwards, at the same speed as the first two.

If desired the third wave in (**59**), $H_{31}{}^{\alpha}$, may be neglected, and H_3 be taken equal to H_{3R} and $H_3{}^{\alpha}$.

Exactly as before for the 5th space harmonic, due to a sine wave of current of fundamental pulsation ($\omega = 2\,\pi\,f$), we have,

$$H_5 = H_a \Bigg[\{\cos\,(\omega\, t)\} \left\{\cos\left(\frac{5\,x}{\tau}\ \pi\right)\right\}$$

$$+ \left\{\cos\left(\omega\, t + \frac{\pi}{2} - \alpha\right)\right\} \left\{\cos\left(\frac{5\,x}{\tau}\ \pi + \frac{5\,\pi}{2} - 5\,\alpha\right)\right\}\Bigg] \qquad \textbf{(60)}$$

Simplifying this, we find,

$$\begin{aligned} H_5 = H_a \Bigg[& \left\{ \cos\left(\omega t + \frac{5x}{\tau}\pi\right)\right\} \left\{ \frac{1}{2} - \frac{\cos 6\alpha}{2} \right\} \\ & + \left\{ \cos\left(\omega t - \frac{5x}{\tau}\pi\right)\right\} \left\{ \frac{1}{2} + \frac{\cos 4\alpha}{2} \right\} \\ & - \left\{ \sin\left(\omega t + \frac{5x}{\tau}\pi\right)\right\} \left\{ \frac{1}{2} \cdot \sin 6\alpha \right\} \\ & - \left\{ \sin\left(\omega t - \frac{5x}{\tau}\pi\right)\right\} \left\{ \frac{1}{2} \cdot \sin 4\alpha \right\} \Bigg] \end{aligned} \tag{61}$$

or approximately,

$$\begin{aligned} H_5 = H_a \Bigg[& \cos\left(\omega t - \frac{5x}{\tau}\pi\right) \\ & - \left(\sin\left(\omega t + \frac{5x}{\tau}\pi\right)\right) \frac{1}{2} \cdot \sin 6\alpha \\ & - \left(\sin\left(\omega t - \frac{5x}{\tau}\pi\right)\right) \frac{1}{2} \cdot \sin 4\alpha \Bigg] \end{aligned} \tag{62}$$

$= H_{5R} + H_5{}^\alpha \; H_{51}{}^\alpha$

= a regular fifth space harmonic, travelling forward at a speed $2f\tau/5$. + a second wave in quadrature to the first and travelling backward at the same speed. + a third wave in quadrature to the first and gliding forward at the same speed.

As in (**59**) H_5 may be taken equal to $(H_{5R} + H_5{}^\alpha)$ and the third wave be neglected. Similar expressions for other space harmonics may be derived by exactly the same procedure. Furthermore, in all the above we have assumed the current to be of the form $I_m \cos \omega t$. If the current contains higher time harmonics the resultant armature reaction may be found in a similar manner.

DISCUSSION ON "SUSTAINED SHORT-CIRCUIT PHENOMENA AND FLUX DISTRIBUTION OF SALIENT-POLE ALTERNATORS" (DIAMANT), ATLANTIC CITY, N. J., JUNE 28, 1918.

F. D. Newbury: To a considerable extent, this paper appears to cover the same ground as the paper by A. E. Clayton on "Wave Shapes of A-C. Generators with Steady Short-Circuit Conditions," presented before the British Institution in 1915. The latter paper has an important advantage in that it shows the current waves and flux distributions of a number of different generators having a considerable range in size and of commercial design. This English paper treats this complex subject in a very clear and simple manner by showing the close correspondence between theoretical curves derived by graphical methods and the oscillograms.

The greater part of Prof. Diamant's paper is concerned with the results from a single 45-kv-a. generator, whose design proportions are not given. In considering these results, it cannot be too strongly emphasized that the significance of the various curves depends entirely upon the detail design proportions of the generator, and the location and spread of the search coils. For example, the double peaks in the curves of Fig. 11 are caused, as the author points out, simply by the reduction of flux density in the space between poles, and thus indicate a very familiar fact by an unfamiliar method.

It is dangerous to draw general conclusions from the results obtained from a single generator. I have in mind the results obtained from a 1500-kv-a., 12,000-volt synchronous motor, having very wide armature slots in comparison with the air gap. The ratio between armature slot opening and air gap was something over 3. There was a very pronounced ripple in both the field form and in the no-voltage e. m. f. wave form. A search coil of one tooth pitch indicated curves very similar to those shown by the author in Fig. 11. While there were only six slots per pole, the wave forms indicated a much higher frequency disturbance and analysis of the e. m. f. wave showed a large twenty-third harmonic. Further investigation of the design developed the fact that partially closed damper slots in the rotor had a pitch only slightly greater than half the slot pitch of the stator, and this conjunction of stator and rotor slots was the cause of the voltage pulsation. This conclusion was confirmed later by building another motor using the same stator punchings, but with a larger tooth pitch for the damper winding, in which the wave form had a relatively small ripple, in spite of the large ratio between stator slot opening and air gap. While this example illustrates a well-known cause of wave-shape distortion, it nevertheless emphasizes the point that the higher harmonics sometimes found in commercial generators are due to a variety of obscure causes, and the search coil e. m. f. waves shown in the paper may or may not have general significance.

There are a number of specific points in the paper to which attention should be called.

It is stated that the subject of tooth ripples is of great practical importance. Tooth ripples are important in synchronous generators and motors only in connection with questions of telephone interference, and then only in connection with salient-pole alternators, in which air gaps are inherently small. One is apt to obtain an exaggerated idea of the importance of these variations in air gap reduction from this and other statements made by the author. For example, in Fig. 9 the variations in air gap flux were obtained with the rotor at rest, or moving slowly. Under these conditions, much greater variations are obtained than are obtained during normal operation when eddy currents very largely reduce the flux variations that tend to exist. In Fig. 9, the flux opposite a slot is 45 per cent of the flux opposite a tooth. In Fig. 7, obtained presumably from the same generator, this ratio for coil No. 1 is 76 per cent. In generators of commercial design, the ripples rarely, if ever, produce greater variations than those shown in Fig. 7.

Throughout the discussion of armature reaction, it is assumed that the space distribution of m. m. f. in a single concentrated armature coil is sinusoidal. At any instant of time, the space distribution of m. m. f. and flux, assuming uniform reluctance, is rectangular, as illustrated in Fig. 18. At equal intervals of time, the m. m. f. and flux may be represented by a series of rectangles, such as Fig. 18, the ordinates of which vary sinusoidally. This is important, because it means that the space distribution of the armature reaction flux depends on the space distribution of the armature coils, and the latter should be taken into account in all calculations concerning armature reaction.

This assumption of sine-shaped armature reaction flux, and the assumption recorded that the field form of most modern machines approaches the sine shape, leads to the statement that the resultant of the main field and armature field under sustained short-circuit conditions must also be approximately sine shaped. These assumptions are immediately shown to be false, by the statement that "we find indications of an extremely distorted flux distribution." The reason for this distortion is explained by Fig. 32, taken from Clayton's British paper, in which the main flux is assumed to be rectangular, and the armature flux a segment of a sine curve. It seems hardly worth while to build up a theory upon assumptions that are immediately shown to be wide of the fact.

In general, harmonics in the distribution curve of flux have a considerably reduced effect on the voltage generated in the armature winding, because the voltages generated by any particular harmonic of the flux distribution curve are in most cases (except the third and multiples) opposed in different conductors connected in series. Furthermore, the current on short circuit is still closer to a sine wave on account of the increasing reactance of the winding for the higher harmonics. Therefore, while the flux distribution on short circuit, as shown in the paper, varies widely from the flux distribution at normal voltage and no load, the generated voltage on short circuit and the current resulting therefrom, depart less and less from the sine form.

One matter of practical importance in connection with the sustained short-circuit operation is this distortion of the flux, due to the relatively weak field excitation under this condition. This matter has been quite completely treated in Clayton's paper previously referred to, and is moreover quite simply treated graphically, even when actual flux distributions are taken into account. One important result of this distorted flux is the increase in losses under short-circuit conditions in some generators. The short-circuit losses of large two and four-pole turbo-generators, when operated on short circuit, are very much larger than can be accounted for by the losses in the armature winding and the core loss corresponding to the internal voltage. The departure of the flux distribution from the sine increases rapidly as the ratio of field to armature m. m. f. decreases, and with the ratio of unity or less, now commonly used, it is quite probable that the large short-circuit losses in these machines are largely due to the flux distortion produced by the armature reaction. This is a matter of considerable practical importance, as the present Institute Standardization Rules require that the total short-circuit losses be included in the loss existing under normal operating conditions. There is opportunity here for experimental and analytical work that will enable us to more completely determine the source of loss under the condition of sustained short circuit.

R. E. Doherty: I wish to agree with practically all that Mr. Newbury has said in regard to this paper, but there is one point which I think he has put a little too strong. I think it is not Mr. Diamant's intention to give quantitative results to be used as a guide in designing the machines, but rather to give a qualitative explanation of some of the curious wave irregularities that we all recognize as existing in sustained short-circuits. Of this he has given the very clear explanation that while under sustained short-circuit the fundamental is practically annihilated, the harmonics are not very much affected and, therefore, the relative importance of the harmonics as compared to the fundamental has been increased hundreds of times in some cases. This produces those curious waves shown on the oscillogram of coil voltage at sustained short circuit.

William F. Dawson: In the average modern turbo-alternator the field form is trapezoidal until saturation is reached (in the field teeth nearest the pole) when the corners of the trapesoid are rounded off. The trapezoid is a close approach to a sine wave and is more so when rounded at the corners by saturation.

With a satisfactory flux wave as the foundation, conditions are further improved by a generous number of slots per phase and pole, and by the practical elimination of the third harmonic by making the armature coil pitch approximately two thirds, the resulting e. m. f. and current waves at the armature terminal are very close to true sine waves at all loads. Considerations of ventilation, regulation and the minimizing of core loss insure such large air gaps as compared with slot widths that while tooth ripples still exist their amplitude is too small to create any objectionable disturbance even under sustained short circuit.

C. M. Davis: I wish to emphasize what Mr. Doherty has said, that this paper presents the results only of tests upon a particular machine, and it is not the author's intention in any way to imply that the case mentioned is a general one. He is merely seeking a way of explaining, to his own satisfaction, some of the actions which are not evident on first examination.

N. S. Diamant: In answer to Mr. Newbury's remarks, may I say that I am rather disappointed in that I was expecting to hear something worth while and profitable considering his experience which I am sure is very wide. I was expecting to have him tell us something about the more complex aspects of the subject, which are carefully taken up in my paper.

As Mr. Newbury does not seem to be sure to what extent my paper and that of A. E. Clayton cover the same ground, I may add that reference is made to A. E. Clayton's 1916 paper on 1181 and 1182. There, in a general way Clayton's explanation is given, and a further note made that his paper; namely, Clayton's, is the first one, as far as I am aware, to throw any light on this complex subject. As explained in my introduction, I had started this work before Clayton's paper appeared. The explanation and theory of Clayton, which is graphical, is very good; so good in fact that I have included it in my paper. My explanation of the same phenomenon is a little different as may be seen by referring to Figs. 24, 24A, 25 and 31, also oscillograms Figs. 23, 27 etc., and Tables VII, VIII and XI.

Furthermore, I have taken questions of flux distribution under load and no-load conditions as well as under sustained short circuit. It is disappointing that Mr. Newbury, with his wide experience, doesn't tell us more definitely in regard to several points, at least twelve of which are very specifically referred to in the abstract and very carefully treated, in the paper itself. For example, Mr. Newbury states that the "paper is concerned with results from a single 45-kv-a. generator", and that it is dangerous to draw general conclusions from the results obtained from a single machine. This may be true enough but it would be very much more useful to me and to the reader for Mr. Newbury, instead of making these general remarks which are of no practical value, to tell us more specifically just what points in the paper he thinks could be generalized and what points should not. Frankly, it is not quite clear to me just what is the relation between the example he gives about a 1500-kv-a. machine and the danger of generalizing, unless it be to show that it is a rather extravagant way of investigating flux pulsations, "by building another motor using the same stator punchings but with a larger tooth-pitch for the damper winding." To be more specific, may I ask Mr. Newbury to tell us what he thinks is the origin and cause of to-and-fro flux oscillations and pulsations. From his experience is the general theory given by Worall and referred to in my paper acceptable to him. What does Mr. Newbury think about:

1. My method of determining experimentally the armature reaction by subtracting no-load and sustained short-circuit exploring coil voltages, as given in Figs. 23, 27, etc.
2. What does he think is the explanation of the fact that the third harmonic seems to increase with increasing excitation as given in Table IX.
3. What does Mr. Newbury think about my experimental and mathematical analysis of armature reaction of imperfect two-phase alternator as given in Fig. 4, Fig. 35 and the supplementary note C page 1199.

(A tentative explanation of this is referred to in foot note 12.)

I don't quite agree with his statement that "tooth ripples are important only in connection with questions of telephone interference, and then only in connection with salient-pole alternators." Is not the question of tooth ripples and higher harmonics important in connection with iron losses and sometimes dielectric losses also. However, I may add that my paper did not attempt to state just how important higher harmonics may become in certain cases, or of how little importance they may be in other cases. The object is more to consider the fundamental theory of the origin and production of ripples in m. m. f. curves, in flux waves and induced voltages, and I, for one, will appreciate it if I could have Mr. Newbury's ideas on the subject.

Mr. Newbury states that the assumption of sine-shaped armation reaction flux is made in the paper. This is not quite so. As stated on p. 1163, the armature current under short-circuit conditions is assumed to be sinusoidal and then very clear distinction is made as to whether the m. m. f. will be sinusoidal or not and whether the flux, due to the m. m. f. will be sine-shaped or not. If I am not mistaken, in most discussions this distinction, namely, whether a sinusoidal m. m. f. may or may not give a sinusoidal flux, is not quite as clearly adhered to as it is in the paper.

Although outside of the scope of the paper, in connection with the "critical resume conclusions" attention is called to the practical bearing of the results of the paper on standard methods of testing alternators by means of cyclic heat run tests, short-circuit load loss tests, phase-shifting load tests etc. It will be interesting to hear something in regard to this.

There is little to add to Mr. Doherty's discussion except that as in Mr. Newbury's case, considering his wide experience covering a great many types and sizes of machines, it will be of considerable interest to the profession to have his views on the many different specific points in connection with flux distribution that are taken up and at least tentatively explained in the paper.

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REACTANCE OF SYNCHRONOUS MACHINES AND ITS APPLICATIONS

BY R. E. DOHERTY AND O. E. SHIRLEY

ABSTRACT OF PAPER

Part I treats of the calculation and application of the armature self-inductive reactance of synchronous machines. A short, reliable method is given in the form of curves, Figs. 20A, B, C, making the calculation from design sheet data a matter of a few minutes. Table I shows a comparison of calculated and test values (obtained from saturation and synchronous impedance curves) for 138 machines, ranging from high-speed turbine generators to the low-speed engine type.

Three points were brought out during the investigation:

(1) That in polyphase machines, the armature self-inductive reactance, just as the armature reaction, is a polyphase, not a single-phase phenomenon, and therefore the mutual induction of phases in a three-phase machine increases the effective self-induction of each phase by approximately 50 per cent over the single phase value, while in two-phase machines, in which the mutual induction of phases is zero, the effective self-induction of the phase is the same for two-phase or single-phase operation.

(2) That the variation of armature reactance during the cycle, due to salient-pole construction, is practically eliminated in Y-connected, three-phase machines for the reason that the variation, consisting almost entirely of a third harmonic, is cancelled in such machines. This leaves, in effect, a uniform reluctance for the leakage flux emanating from the tooth tips.

(3) That in the familiar method of obtaining the armature self-induction from the saturation and synchronous impedance curves (*i.e.*, by subtracting the armature reaction, that is, the demagnetizing ampere turns of normal current under sustained short circuit, from the corresponding field ampere turns), a very large error in this test value of self-induction may occur, if, as is usually done, the armature reaction is calculated on sine wave assumptions. A set of curves shown in Fig. 20, which are plotted from results derived in Appendices A and B, give values of the correction factor which applies to calculations based on sine wave.

An approximate, but convenient, method of applying the armature reactance in the calculation of field excitation under load is given in Appendix C.

In Part II it is shown that the initial short-circuit current of synchronous machines is determined not only by the armature self-inductive reactance, as is often assumed, but also by the field self-inductive reactance. Neglecting the field reactance in calculation may give a calculated short-circuit current 50 per cent or more, too high. A formula is derived for calculating the field reactance which, added to the armature reactance, gives the total which determines the initial short-circuit current and which it is proposed should be called, as previously recommended by

other authors, *transient reactance*. Table IV shows a comparison on eleven machines of the actual short-circuit currents as determined by oscillograph, and as calculated by methods proposed in the paper.

The calculations of *transient reactance* apply strictly to salient laminated pole alternators without amortisseur winding. However, such experience as is recorded in Table IV, if analyzed in connection with theoretical considerations, affords a basis for estimating the *transient reactance* of turbine generators with massive steel rotors and of salient-pole machines with amortisseur windings. These points are summed up in "Summary and Conclusions".

An attempt is made to describe the apparently complicated physical phenomena of sudden short circuits in terms as free as possible from mathematics. One interesting and important point which the authors establish from the physical interpretation of the problem is that there is a very significant rise in flux at the bottom of the pole at short circuit. This may be (if the rotor is entirely laminated) 30 or 40 per cent of normal flux, and apparently explains why, in machines with solid steel rotor rims, the field attenuation factor, α_f, changes radically after the first few cycles after short circuit.

INTRODUCTION

As the march of time obliges designers to become ever more efficient in the use of materials, it becomes more and more necessary that they should extend design calculations to further completeness. The self-inductive reactance of synchronous machines stands in very important relation to this problem. In the predetermination of field excitation, of voltage regulation, and of the initial short-circuit current, the reactance is, of course a very essential factor.

Three years ago the authors undertook a survey of the work which had been done on the calculation of armature reactance in the hope of finding, if not in any particular existing formula, at least in a combination of different ones, an expression for reactance that would give reliable calculations—reliable, we mean when applied to the entire range of machines as built by one of the large manufacturing companies. The result was probably very much the same as that which others have obtained who may have made a similar attempt. Particular expressions applied well to particular classes of machines, but none was found, and no combination of existing expressions was found, that would apply generally. Moreover, on account of the many factors in the problem, the equation for reactance is a very clumsy thing to handle—too clumsy, indeed, to make practical the calculation of reactance on each new, proposed design. It is nevertheless desirable to have this calculation. By methods described later an expression was finally arrived at which does apply generally to all classes of machines from two-pole turbine-generators to 136-pole water-wheel driven generators, of any voltage and frequency, three-phase, two-phase or single-phase, and the results are put in convenient shape for the designer's use, thereby making the calculation for each machine a practical matter. A tabulated comparison of calculated and test results (taken from ordinary commercial tests for open and short-circuit characteristics) is shown in Table I.

The importance of predetermining the initial short-circuit current of synchronous machines need not be dwelled upon. Mechanical stress involved, the rupturing capacity of circuit breakers, heating, etc., all make it a matter of supreme importance. The theory underlying short circuits has been thoroughly worked out by Steinmetz, Berg and Bouchert; and Diamant[1] has extended the mathematics involved and investigated the possibility of determining experimentally the attenuation factors, r/L, which determine the rate of decrease of the transients. That is, the complete equation for the instantaneous values of short-circuit current has been available for some time. The difficulty in its use has been that the factors involved could not be predetermined, and, to some extent, there has been uncertainty regarding what constitutes the reactance which determines the short-circuit current. The authors will show that this reactance includes the field self-induction as well as that of the armature, and will derive a formula by which the total reactance may be calculated. To the author's knowledge this has not been previously worked out. Table IV gives a comparison of calculated and actual values of short-circuit current.

In the process of obtaining the above results, another closely related problem was incidentally solved, namely, the effect of the harmonics in the no-load flux wave upon the field excitation required for normal voltage and also upon the armature reaction. This is given in Appendices A and B.

The object of the paper, therefore, is (1) to present a reliable, general formula for the calculation of self-inductive armature reactance of synchronous machines. (Given in Part I).

(2) To give a simplification of the general equation, permitted by standard construction, accompanied by working curves making it possible to calculate the per cent armature reactance from the design sheet in less than five minutes. (Given in Part I).

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[1] TRANS., A. I. E. E. 1915, Vol. XXXIV, p. 2237.

TABLE I
COMPARISON OF CALCULATED AND TEST VALUES AT ARMATURE SELF-INDUCTIVE REACTANCE

SALIENT POLE MACHINES

	Kv-a.	ϕ	Freq. ~	R.p.m.	Voltage	x_{pa}	
						Calc.	Test
1	33	3	60	1200	440	13.4	8.06
2	50	3	60	300	2300	16.0	13.9
3	125	3	62½	750	2300	13.5	11.0
4	180	3	60	900	220	10.9	12.3
5	240	3	60	150	440	17.1	15.2
6	265	3	25	150	400	28.1	34.8
7	300	3	60	1200	4000/2300	15.6	13.0
8	320	3	50	200	5000	12.5	15.5
9	345	3	60	720	4000	13.1	10.7
10	370	3	50	187½	500	14.5	20.0
11	375	3	60	150	2300	14.7	13.2
12	375	3	60	150	3100	14.4	11.4
13	400	3	25	300	2300	15.2	21.0
14	400	3	60	300	4000	27.5	30.1
15	400	3	60	200	600	18.9	19.3
16	400	3	60	150	6600	24.6	21.4
17	450	3	25	750	13200	8.8	3.1
18	475	3	50	214	440	16.4	15.3
19	500	3	60	600	2300	7.6	7.5
20	500	3	60	150	2300	16.0	13.7
21	500	3	60	100	240	14.4	13.9
22	540	3	60	164	6600	31.0	27.3
23	560	3	25	750	480	6.6	5.8
24	560	3	60	720	13200	15.1	12.0
25	560	2	60	720	2400	18.6	23.2
26	570	3	25	750	13200	9.0	8.9
27	580	3	60	720	2300	13.4	15.6
28	585	3	60	90	2000	14.4	15.6
29	605	3	60	150	2200	16.8	18.1
30	625	3	60	150	2300	10.7	10.0
31	625	3	60	100	480	21.1	15.8
32	675	3	60	90	6600	36.8	33.5
33	700	3	25	750	2200	10.1	6.00
34	700	3	25	750	13200	13.2	16
35	700	3	25	500	6600	11.4	7.3
36	700	3	60	900	4000	13.8	12.7
37	700	3	40	300	600	10.5	10.2
38	750	3	60	300	600	22.5	23.6
39	750	3	60	150	2300	18.4	19.5
40	780	3	60	120	2300	21.7	21.0
41	800	3	60	90	2300	41.9	44.0
42	850	3	25	500	6600	5.6	8.4
43	900	3	60	360	600	20.5	18.6
44	900	3	60	150	2300	22.1	18.2
45	1000	3	60	450	2300	22.5	19.8
46	1000	3	60	200	600	21.2	16.8
47	1200	3	25	500	4400	16.3	16.9
48	1250	3	25	750	13200	10.5	10.5
49	1250	3	50	500	5500	12.3	12.8
50	1250	3	60	300	2300	12.0	10.5
51	1250	2	62.5	375	5400	12.1	10.2

TABLE I. (*Continued*)

COMPARISON OF CALCULATED AND TEST VALUES AT ARMATURE SELF-INDUCTIVE REACTANCE

SALIENT POLE MACHINES

	Kv-a.	ϕ	Freq. ~	R.p.m.	Voltage	x_{pa}	
						Calc.	Test
52	1350	3	25	375	6600	11.3	12.8
53	1400	3	25	500	2200	10.0	8.9
54	1400	3	25	375	13200	11.5	13.8
55	1400	3	60	514	13200	10.8	10.9
56	1400	3	60	514	13200	12.9	11.9
57	1420	3	25	300	2300	11.0	5.8
58	1425	3	60	240	2200	21.1	22.0
59	1450	3	60	720	2300	11.3	8.6
60	1500	3	25	500	2300	7.9	10.2
61	1500	3	60	514	2200	14.5	15.0
62	1500	2	62.5	750	2300	14.3	20.0
63	1625	3	25	750	3300	10.5	10.5
64	1800	3	60	300	2300	10.6	12.6
65	2000	3	60	720	2200	13.8	14.0
66	2000	3	60	100	2300	8.4	8.0
67	2000	2	60	360	2400	9.6	7.2
68	2100	3	60	514	4600	15.7	15.7
69	2125	3	40	200	600	15.3	14.1
70	2200	3	25	750	3300	9.1	15.0
71	2200	3	25	375	2300	9.1	9.7
72	2250	3	25	214	2300	5.9	4.4
73	2250	3	60	514	6000	15.3	16.9
74	2250	3	60	450	3450	17.8	17.1
75	2500	3	25	250	2200	5.8	4.0
76	2800	3	25	300	2300	5.5	4.9
77	2000	3	60	514	13200	18.9	19.2
78	2800	3	60	450	4000	15.0	18.0
79	3000	3	50	500	6600	13.1	17.4
80	3000	3	60	450	2300	8.25	11.0
81	3125	3	25	750	14000	16.1	15.8
82	3500	3	50	500	11500	19.2	20.0
83	4000	3	60	600	2300	13.8	11.2
84	5000	3	25	300	2300	8.9	10.2
85	5000	3	25	88.3	6600	13.9	13.8
86	5650	3	60	360	6600	19.8	21.3
87	6000	3	50	500	3450	7.65	6.1
88	6000	3	50	500	16500	21.6	26.5
89	6600	3	25	375	14000	9.5	11.0
90	6250	3	60	600	6600	16.8	13.9
91	6750	3	50	600	11000	19.3	25.6
92	7050	3	40	400	600	6.6	8.5
93	7050	3	50	375	6600	13.9	13.0
94	7500	3	60	600	13800	13.3	16.9
95	8750	3	50	500	600	15.1	19.4
96	8750	3	50	500	6600	6.9	7.1
97	9000	3	25	57.7	11000	13.9	16.4
98	10000	3	60	514	6600	6.81	7.9
99	10000	3	62.5	55.6	6600	20.4	19.4
100	12000	3	25	375	6600	13.2	12.6
101	15000	3	50	375	6600	10.2	12.3

TABLE I. (*Continued*)

TURBINE ALTERNATORS

	Kv-a.	ϕ	Freq. ~	R.p.m.	Voltage	x_{pa}	
						Calc.	Test
1	3750	3	60	3600	2300	9	14.5
2	4375	3	60	1800	2400	5.9	2.4
3	4375	3	60	1800	4500	6.7	3.9
4	4375	3	60	1800	2300	7.7	8.2
5	4375	2	60	1800	2300	6.25	5.1
6	5000	3	25	1500	5500	10.6	10.2
7	5000	3	25	1500	6600	6.4	6.8
8	5000	3	25	1500	6600	7.3	5.5
9	5000	3	60	3600	2300	9.0	8.9
10	5625	3	25	1500	2300	6.1	3.3
11	6250	3	25	1500	6600	10.1	10.6
12	6250	3	60	3600	2300	8.6	12.2
13	6250	3	60	1800	6600	8.6	7.8
14	7500	3	25	1500	2300	7.1	6.9
15	7500	3	25	1500	6600	10.5	7.1
16	7500	3	25	1500	6600	10.5	11.8
17	7500	3	60	1800	4000/2300	9.4	5.0
18	7500	3	60	1800	4160/2300	9.1	8.2
19	7500	3	60	1800	2300	7.6	9.1
20	7812	3	25	1500	2300	7.7	7.8
21	7812	3	60	1800	2300	10.1	9.5
22	7812	3	60	1800	4000/2300	10.6	8.6
23	7812	3	60	1800	4000/2300	11.8	10.9
24	9375	3	25	1500	13200	8.9	9.3
25	9375	3	60	1800	4000/2300	8.0	8.6
26	9375	3	60	1800	5000	11.0	9.2
27	10000	3	25	1500	6600	9.9	9.8
28	10000	3	40	2400	10000	12.5	13.8
29	10000	3	60	1800	6600	14.2	9.2
30	12500	3	25	1500	11000	8.4	8.2
31	12500	3	60	1800	4000/2300	9.1	7.4
32	12500	3	60	1800	11000	13.0	17.1
33	12500	3	60	1800	13200	19.2	13.5
34	12500	3	60	1800	2300	10.2	8.6
35	18333	2	37.3	2240	4469	6.3	7.4
36	18750	3	60	1800	2300	13.8	12.0
37	25000	3	60	1800	13200	14.4	15.4

(3) To derive a formula for calculating the field self-induction which combines with the armature self-induction to determine the initial short-circuit current; and further to describe in part the physical phenomena which exist in the machine at short circuit. (Given in Part II).

(4) To derive a formula for calculating armature reaction, taking into account the effect of field distribution. (It is necessary to determine the armature reaction accurately, before the armature self-induction can be determined from standard open and short-circuit tests). (Given in Appendix B).

(5) To show the effect of flux distribution (*i.e.* the effect of the harmonics in the no-load flux wave) on the field excitation. (Given in Appendix A).

(6) To give a simplified method of calculating field excitation which has worked out well in connection with the *per cent* armature reactance. (Given in Appendix C).

PART I.

ARMATURE REACTANCE

Of all the work which had been done on armature reactance, that of Gray[2] was, in our opinion, the most complete. While it involved certain familiar factors, such as that for the slot reactance and the leakage from tooth tip to tooth tip which have appeared in other reactance formulas, it yet took into account for the first time, we believe, the variation of tooth-tip reactance in salient-pole machines for different positions of the pole, which is very important. Fechheimer introduced a factor to account for the effect of fractional pitch armature coils. The results given in this paper have been obtained by a modification and extension of the work done by Gray, Fechheimer[3] and others. A bibliography which may be of historical interest is included at the end of the paper.

There were three difficulties of fundamental importance which it was necessary to overcome before consistent results were obtained. The first was the influence of the harmonics in the no-load flux wave upon the effective armature magnetomotive force or armature reaction. The armature self-inductive reactance, as obtained from the saturation and synchronous impedance curves (the open-circuit and short-circuit characteristics) involves the difference between the actual field m.m.f. on short circuit and the effective armature m.m.f. for the corresponding current. Since the armature reaction is calculated, this difference must also, to that extent, be considered as calculated. The familiar, approximate formula usually employed in determining effective armature m.m.f. per pole,

$$A = \frac{2.12\, N_q\, I}{K_p\, K_d} \quad \text{(three-phase)}$$

is based upon sinusoidal distribution of flux and armature m.m.f.

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[2] Gray's Elec. Machine Design, 1st Edition, page 215.

[3] TRANS. A. I. E. E., Vol. 31, p. 578.

N_q = series turns per pole per phase
I = amperes per turn
K_p and K_d = coefficients, greater than unity, which take account of the effect of fractional pitch coils and the distribution of coils respectively.

Often the flux distribution is very different from a sine wave. In such cases the actual effective armature m.m.f. may be different by 8 to 10 per cent from the value based on sine-wave assumptions—*i.e.*, as calculated from the above formula. This error in armature m.m.f., of course causes a much larger percentage error in the difference between field and armature m.m.f's. on short circuit, which difference, correctly determined, represents the armature self-induction. It is therefore obvious that the effect of flux distribution must be taken into account. This has been done by a single factor, K_ϕ, which is derived in Appendix A, and is given in Fig. 20.

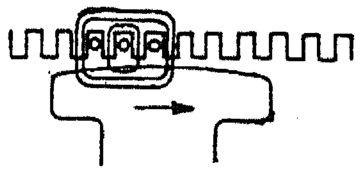

FIG. 1—TOOTH-TIP LEAKAGE FLUX

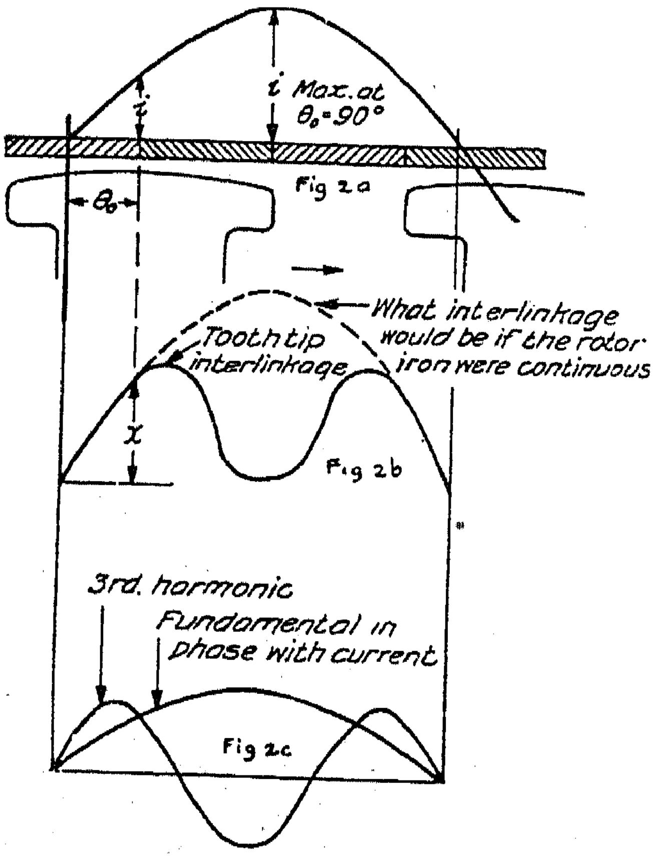
i Max. at $\theta_0 = 90°$
i
θ_0
Fig 2a
What interlinkage would be if the rotor iron were continuous
Tooth tip interlinkage
x
Fig 2b
3rd. harmonic
Fundamental in phase with current
Fig 2c

FIG. 2—TOOTH-TIP INTERLINKAGES AT ZERO POWER FACTOR

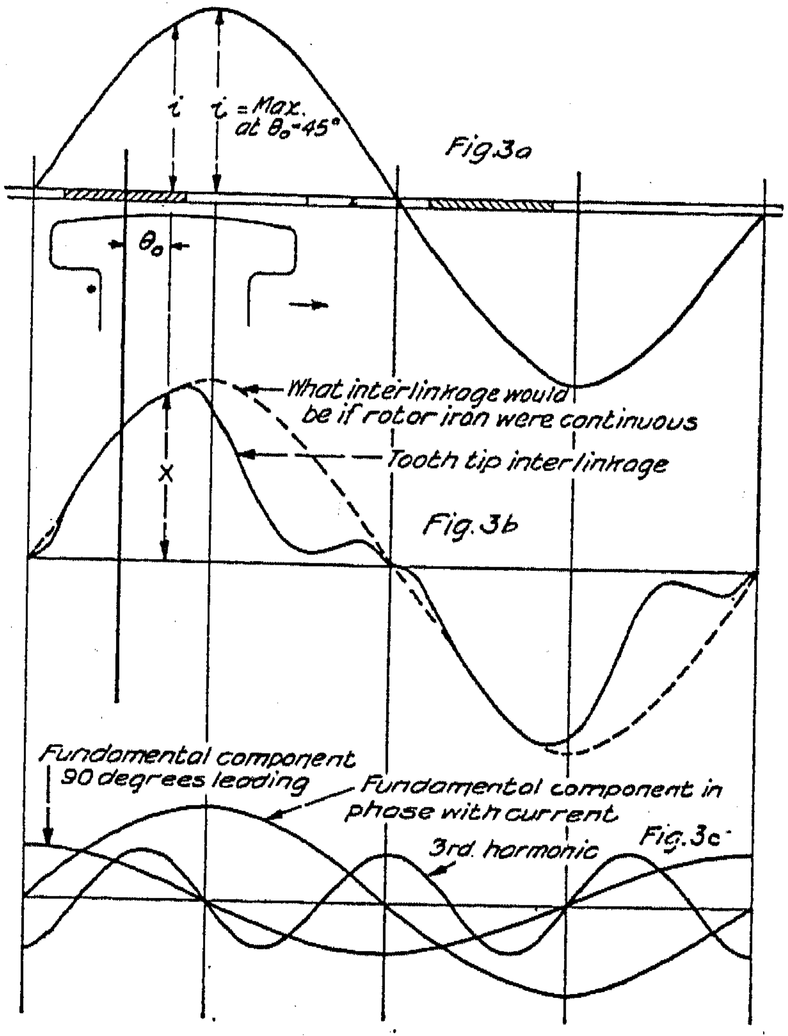
i
i = Max. at $\theta_0 = 45°$
Fig. 3a
θ_0
What interlinkage would be if rotor iron were continuous
Tooth tip interlinkage
X
Fig. 3b
Fundamental component 90 degrees leading
Fundamental component in phase with current
3rd. harmonic
Fig. 3c

FIG. 3—TOOTH-TIP INTERLINKAGES AT 80 PERCENT POWER FACTOR

The second difficulty was met in accounting correctly for the, variation in the permeance of the "tooth-tip" leakage path for different positions of the pole. Gray's method of accounting for this neglected certain serious factors, as will be shown later. Fig. 1 shows *tooth-tip* leakage of one-phase belt at a time when the pole-face iron forms part of the leakage path. It is obvious from Fig 1, that as the pole moves on, a part, and finally all, of the path between tooth-tips will be air. At zero power factor the current in any particular phase group is a maximum when that phase group is half way between poles, when the angle, θ_0, between the center of the phase group and the center of the pole Fig. 2A, is 90 deg.; and is zero when the group is over the middle of the pole, when θ_0 is zero. This produces a curve of leakage interlinkages shown as x in Fig. 2B. The dotted curve indicates what the leakage curve would be if the iron of the rotor were continuous. Now for any other power factor, say $\cos \theta = 0.7$, if the distortion of the magnetic field is neglected, the current will be a maximum when

$$\theta_0 = \theta = 45 \text{ deg.}$$

and zero when $\theta_0 = -45$ deg. But distortion can not be neglected. By shifting the flux toward the trailing tip (in a generator) the position of the pole is relatively advanced, thereby increasing the total angle between the center line of the pole and the position of the phase group when the current is a maximum, to

$$\theta_0 = \theta + \theta_d$$

where θ_d is the distortion angle, shown in Fig. 18. For illustration let the current be a maximum when θ_0 equals 45 deg. Fig. 3A. This would correspond in certain machines to a power factor of about 80 per cent and would produce a curve of leakage interlinkages, x, shown in Fig. 3B. The dotted curve indicates the leakage which would exist if the rotor iron were continuous.

An analysis of the curves of interlinkages shown in Figs. 2B and 3B discloses an interesting and very important fact: that these curves as shown in Figs. 2C and 3C, are made up principally of a fundamental and a large third harmonic, all other harmonics being negligible; and that since in a Y-connected 3-phase machine the third is eliminated, the fundamental alone remains for consideration in such machines—which probably comprise 95 per cent of all that are now being manufactured. And even in two-phase or Δ-connected three-phase machines the effective results are much the same. Experience has been that the short-circuit characteristic and saturation curves show practically the same value of reactance in the same machine whether it is connected Y or Δ. The probable explanation of this is that in Δ-connected or two-phase machines the current, especially under short circuit, is not a sine wave, as it is for Y-connected machines, but instead, contains a negative third harmonic, as shown in Fig. 4. This operates to lower the peaks in Fig. 2B, that is, to decrease the third harmonic in the reactance. And although, of course, the effect of the remaining, decreased third harmonic in the reactance voltage is to restrict the flow of current at certain points of the cycle, and therefore produce a current of smaller average value than would exist if the third harmonic were eliminated, nevertheless the form factor of the resulting peaked wave shown in Fig. 4, is correspondingly higher, that is, the ammeter will read correspondingly higher, and therefore show in this case about the same reading for sustained short-circuit current as would be shown in the case of a Y-connected machine. In general, then, it is necessary to consider only the fundamental in the tooth-tip leakage.

The salient pole construction, which is the cause of the large third harmonic mentioned above, is therefore, in effect replaced by a continuous rotor of uniform reluctance, which is greater than that of iron but lower than that of air, and which is different for different power factors, being lower at high power factor. This is obvious from a comparison of Figs. 2B and 3B.

At high power factor, an interesting circumstance arises. In addition to that component of the fundamental of the tooth-tip leakage which is in phase with the current and which therefore produces a reactive voltage lagging behind the current by 90 deg., there is also a cosine term, leading the current by 90 deg., which produces a reactive voltage in phase with the current. This has important significance in the matter of field excitation required under load, as pointed out later.

Gray's value for the tooth-tip factor is based on a root-mean-square of instantaneous values of a curve somewhat similar to that shown in Fig. 2B., the principal difference in the curves being that Fig. 2B., takes account of the distribution of the coils in the phase belt. Being based on the root-mean-square and neglecting the spread of the phase belt, Gray's value for this factor is approximately 100 per cent higher than that given in this paper, and does not take account of the in-phase component of reactive voltage, or of the effect of the distortion of the magnetic field—both of which occur under watt load.

The third difficulty involved the conception that on a polyphase machine, the self-induction of the armature is a polyphase, not a single-phase phenomenon; and that therefore the reactance per phase of the same machine, connected three phase in one case, two phase in another, would be approximately 1.5 times the single-phase value in the former, 1.0 times the single-phase value in the latter. In other words the magnetic field of self-induction is a rotating polyphase field just as is the field of armature reaction, and the resulting overlapping of phase produces greater leakage interlinkages in any particular phase of a three-phase machine than would exist by the m.m.f. of the phase alone. For instance, in a three-phase machine, when the

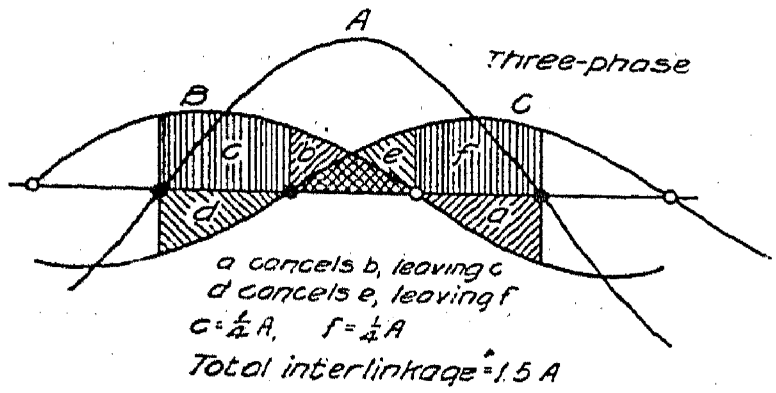

FIG. 5A

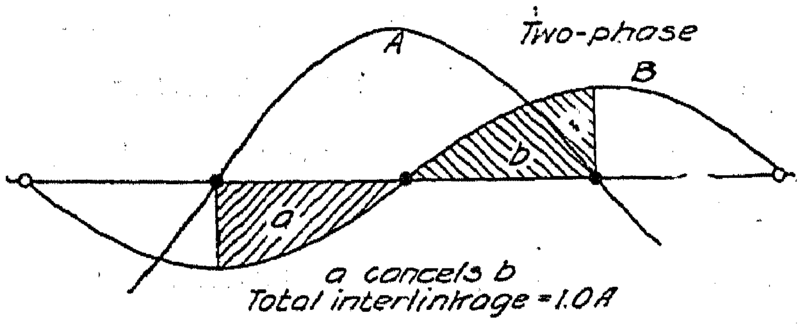

FIG. 5B
MUTUAL INTERLINKAGES—POLYPHASE MACHINE

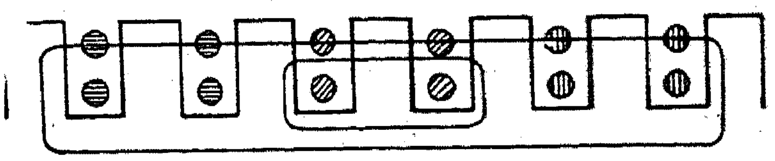

FIG. 6 A

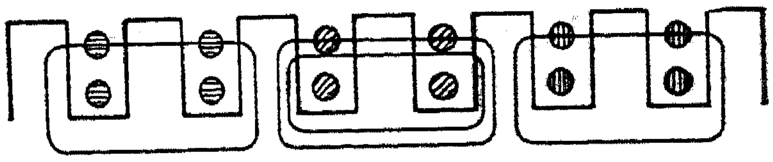

FIG. 6B
SLOT LEAKAGE IN THREE-PHASE MACHINE

current is maximum in phase A, it is one-half maximum in B and C. The increase of interlinkages in A, due to B and C, is shown in Fig. 5A. For two-phase, the current is zero in B when a maximum in A, and moreover they are displaced in space so that the mutual interlinkages must be zero at all times as shown in Fig. 5B.

Unless this fact is taken into account when considering three-phase machines, the calculated value, which is the reactance per leg, will be only two-thirds as large as the test value, unless some of the factors in the reactance equation are overburdened to make up the difference. Then, obviously, the formula would not apply to two-phase machines; it would give calculated results 50 per cent too high. This is just what had happened during this investigation. By using Gray's formula modified by Fechheimer's "Short Pitch Factor", calculations checked fairly well on certain classes of three-phase machines, but not on two-phase. The factor of tooth-tip leakage being 100 per cent too high and the other factors being approximately right, made up for neglecting the increase of 50 per cent on the total, and naturally made two-phase calculations too high. By using the proper value of tooth-tip leakage, and allowing for the polyphase action, the equation applies equally well to three-phase, two-phase and single-phase machines.

There is, however, one term of the reactance equation which is not affected by the mutual induction of phases, namely, the slot leakage. Fig. 6B shows the three different phases of a two-slot per pole per phase machine, and the slot leakage which exists when the current is a maximum in the middle group, one-half maximum in the other two. Fig. 6A is equivalent to Fig. 6B, and it is clear that the interlinkages of the middle group have not been increased by the m.m.f. of the other two groups. The same reasoning can be applied to other instants, when the currents have different values, to show that there is no mutual induction. This operates to reduce the ratio of three-phase to single-phase (terminal to neutral) reactance to a value slightly lower than 1.5.

It is not obvious at first that this overlapping of phases occurs in the case of tooth-tip leakage. Consider, however, that the elimination of the third harmonic in this term, in effect produces a path of uniform reluctance (similar to a path of air only having lower reluctance) for the tooth-tip leakage. In other words, the case is similar to that of the end winding whose leakage path is air, or to that of the induction motor rotating main field. And, as in the two latter cases, the polyphase field is greater than that produced by one phase alone by about 50 per cent, as shown in Fig. 5A.

Turning to the question of applying the reactance in the calculation of field excitation required under load, experience has demonstrated that the use of the zero power factor value of reactance (that is, the value determined from open-circuit and short-circuit test) gives reasonably reliable calculation of field current for loads of any power factor. Why this could be true when the reactance at 80 per cent power factor is, by reason of the different pole position when the current is a maximum, about 60 per cent higher than that at zero power factor, was a perplexing question. It is answered by the appearance of the in-phase component of reactive voltage in the tooth-tip factor. The $r\,I$ drop is in phase opposition to the current; and the voltage consumed by it is in phase with the current. The $x_1\,I$ drop, produced by leakage interlinkages in phase with the current, is 90 deg. behind the current; and the voltage consumed by it is 90 deg. ahead of the current. The $x_2\,I$ drop produced by leakage interlinkages 90 deg. ahead of the current, as shown in Fig. 3c, is in phase with the current; and the voltage which it consumes is in phase opposition to the current. Fig. 7 shows these relations for approximately 80 per cent power factor, and makes clear why the internal voltage E_1', determined by the use of the zero power factor reactance, is practically the same as E_1 which is actually required by the much higher reactance, corresponding to 80 per cent power factor, the effect of the latter being mitigated by the in-phase component $x_2\,I$. There is a difference in phase between E_1' and E_1 which, by using the zero power factor value of reactance, will give a calculated excitation slightly too low, but for practical purposes the difference is considered negligible.

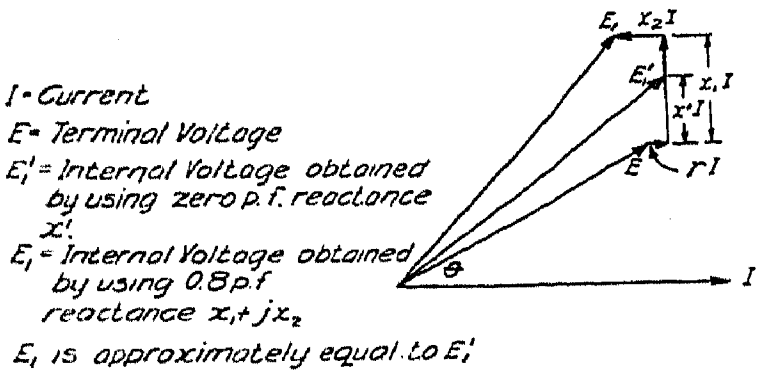
E_1
$x_2 I$
E_1'
$x_1 I$
$x' I$
E
$r I$
θ
I
I = Current
E = Terminal Voltage
E_1' = Internal Voltage obtained by using zero p. f. reactance x'.
E_1 = Internal Voltage obtained by using 0.8 p. f reactance $x_1 + j x_2$
E_1 is approximately equal to E_1'

FIG. 7—VOLTAGE DIAGRAM AT 80 PERCENT POWER FACTOR

In the interest of simplicity and convenience in calculation, therefore, it is proposed that for the present at least, the zero power factor value of reactance be used. The method of calculating field excitation which is given in this paper follows that assumption. In the future it may become desirable to make correction for the distortion angle and power factor; hence the problem remains open to this further refinement.

CALCULATION OF ARMATURE REACTANCE

A = effective armature reaction in ampere turns per pole

a = conductors per slot; for a double layer winding a is twice the number of turns per coil.

C = air-gap coefficient based on Carter's fringing coefficient.

c = number of circuits in multiple in armature winding.

C_a = reduction factor for Φ_a at zero power factor, depending on ratio of pole arc to pole pitch.

C_i = reduction factor for Φ_i at zero power factor, depending on ratio of pole arc to pole pitch.

C_1 = 1.5 C_i for three-phase
= C_i for two-phase.
C_2 = 1.5 C_a for three-phase
= C_a for two-phase.
d_1 = distance from top of copper to bottom of copper in the armature winding Fig. 8.
d_2 = distance from top of copper to armature face Fig. 8.
e = voltage per phase
f = frequency in cycles per second.
g = average air gap.
I_n = normal current per phase in amperes
i = instantaneous value of current in conductor
K = reduction factor for Φ_s Φ_i and Φ_a depending on pitch of armature coils.
K_d = factor for effect of distribution of armature coils in calculation of induced voltage, depending on slots per pole and phase.

TABLE II—VALUES OF K_d

s		1	2	3	4	5	6	7	8up
K_d	Three-phase	1.0	1.035	1.043	1.044	1.045	1.046	1.047	1.047
K_d	Two-phase	1.0	1.083	1.098	1.103	1.106	1.108	1.109	1.11

K_i = factor for Φ_i depending on slots per pole and phase.
K_p = factor for effect of short pitch of armature coils in calculation of induced voltage.

$$K_p = \frac{1}{\text{Sin } (90^\circ \times p)}$$

where p is pitch of coil in per cent of full pole pitch.

K_ϕ = factor for effect of form of flux wave shape in calculation of induced voltage. Values of K_ϕ are given in Fig. 20.
L_e = length of one phase belt of end connections.
l = gross stacked length of armature core (iron + ducts)
N_s = total number of stator coils
n = number of phases.
p = per cent pitch of armature winding (= 0.8 for 80 per cent pitch)
q = number of poles
s = slots per pole and phase.
w_s = width of slot

w_t = width of tooth at armature face.

For w_s and w_t see Fig. 8.

x_a = reactance in ohms

x_{pa} = armature reactance in per cent.

α = ratio of pole arc to pole pitch

τ = pole pitch at armature core surface.

Φ = flux per pole in maxwells.

Φ_a = effective tooth-tip interlinkages per phase belt per ampere conductor for unit length of armature, when the entire phase belt is between the poles.

ϕ_a = effective instantaneous value of Φ_a.

Φ_e = effective end connection interlinkages per phase belt per ampere conductor for unit length of phase belt of end connections.

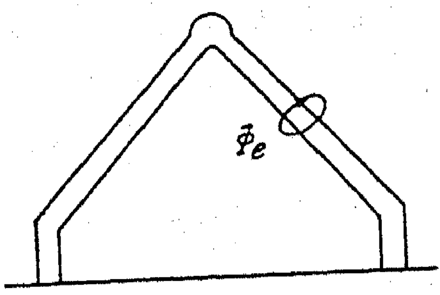

Fig 9 a

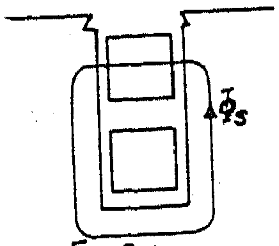

Fig 9 b

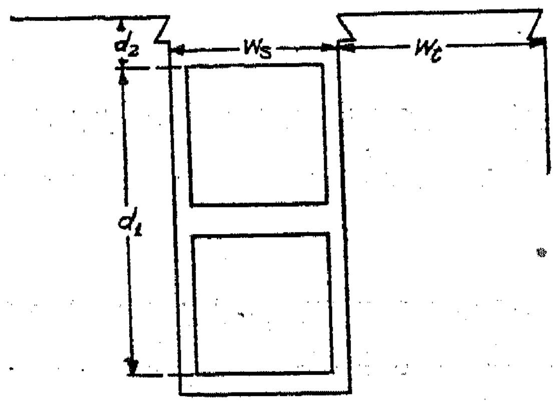

FIG. 8—SLOT DIMENSIONS

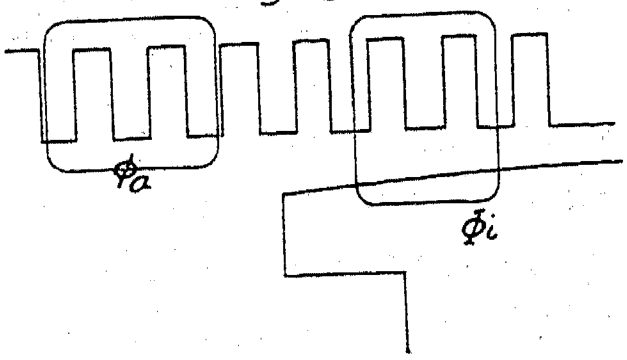

FIG. 9C
ARMATURE LEAKAGE FLUX DIAGRAMS

Φ_i = effective tooth-tip interlinkages per phase belt per ampere conductor for unit length of armature, when the entire phase belt is under a pole.

ϕ_i = effective instantaneous value of Φ_i.

Φ_s = effective slot interlinkages per phase belt per ampere conductor for unit length of armature.

ψ = electrical angle by which the armature coil span is less than 180 deg.

Theory. The armature leakage flux in an alternating-current generator may be divided into three factors.

(1) End connection leakage flux, represented by $\Phi_e L_e$, the effective flux per ampere conductor interlinking one phase belt of end connections. (Fig. 9 A).

(2) Slot leakage flux represented by Φ_s, the effective flux per ampere conductor through the slot interlinking unit length of one phase belt. (Fig. 9 B).

(3) Tooth-tip leakage flux represented by

(*a*) Φ_i, the effective flux per ampere conductor, crossing the air gap into the pole face and returning across the gap and interlinking unit length of one phase belt, when the entire phase belt is under the pole. (Fig. 9C.)

(b) Φ_a, the effective flux per ampere conductor emerging from the tooth tips and interlinking unit length of one phase belt when the entire phase belt is between the poles. (Fig. 9c.)

The expression for reactance has been derived by a number of writers, and may be expressed in our notation by the following equation:

$$x = \frac{2 \pi f s^2 a^2 q l}{10^8 c^2} L_0 \tag{1}$$

Where L_0 is the total interlinkages per ampere conductor in the phase per unit length of the armature[4].

The factor $\frac{2 \pi f s^2 a^2 q l}{10^8 c^2}$ will be replaced by M in the following derivation.

End Connection Leakage. The end connection leakage can only be approximated, but the value of $\Phi_e L_e$ is obviously between the following two limits.

The first limit assumes that the length of the path around the phase belt varies directly as the width of the belt. See Fig. 9A.

Then

$$\Phi_e = K_1 \frac{n}{\tau}$$

where K_1 is a constant.

The second limit assumes that the length of the path is independent of the width of the belt.

Then Φ_e would be constant.

Obviously Φ_e can not change directly as $\frac{n}{\tau}$ because the depth of coil is a factor of the length of path; on the other hand, the length of path *must* increase some as τ increases, that is, it can not be independent of τ. Hence the variation of Φ_e must be between the first power of $\frac{n}{\tau}$ and the zero power of $\frac{n}{\tau}$. The analysis of tests for entire armature reactance on over 100 machines, as well as exploring coil tests for end leakage reactance on a 125-kv-a. generator indicates that Φ_e varies approximately as $\sqrt{\frac{n}{\tau}}$ that is

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[4] Electrical Machine Design, Gray Edition 1913, p. 21.

$$\Phi_e = K_2 \sqrt{\frac{n}{\tau}}$$

The length of the phase belt $L_e = p\,\tau\,K_3$.

Then $$\Phi_e\,L_e = K_4 \sqrt{n}\; p\, \sqrt{\tau}$$

For a three-phase double-layer winding, and dimensions in centimeters, $K_4\sqrt{n} = 2.8$ was found to give satisfactory results.

The factor for a two-phase two-layer winding will be reduced in the ratio of $\frac{\sqrt{2}}{\sqrt{3}}$ and also by the factor 0.667 on account of zero mutual induction between phases in this case, and will be 1.6 in round numbers.

The end connection reactance for a double-layer winding will be

$$x = \frac{2.8\, M\, p\, \sqrt{\tau}}{l} \qquad \text{Three-phase} \quad (2)$$

$$= \frac{1.6\, M\, p\, \sqrt{\tau}}{l} \qquad \text{Two-phase} \quad (3)$$

For a single-layer winding the value of $\Phi_e\,L_e$ will be doubled as proposed by Gray.

Slot Leakage. Three-phase coil pitch between 66.66 and 100 per cent. The expression for slot reactance with short-pitch armature winding was derived by Fechheimer[5], and is used with a correction in the number of *A* and *B* coils, which is $\frac{N_s\,\psi}{180}$ instead of $\frac{N_s\,\psi}{2 \times 180}$ as given in his paper.

Omitting the factors for the top of the partially closed slot and modifying for our notation, the expression for inductance of open slots becomes

$$L = 0.4\,\pi\,a^2 \left[\left(\frac{19}{96}\,\frac{d_1}{w_s} + \frac{3}{8}\,\frac{d_2}{w_s}\right)\frac{N_s\,\psi}{3 \times 60} + \right.$$

$$\left.\left(\frac{7}{96}\,\frac{d_1}{w_s} + \frac{3}{8}\,\frac{d_2}{w_s}\right)\frac{N_s\,\psi}{3 \times 60} + \frac{N_s}{3}\left(1 - \frac{\psi}{60}\right)\left(\frac{d_1}{3\,w_s} + \frac{d_2}{w_s}\right)\right] \qquad (4)$$

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[5] Fechheimer "Synchronous Motors" A. I. E. E., Vol. XXXI, 1912, p. 580.

Simplifying

$$L = 0.4\,\pi\,a^2 N_s \left[\left(1 - \frac{\psi}{320}\right)\frac{d_1}{3\,w_s} + \left(1 - \frac{\psi}{240}\right)\frac{d_2}{w_s}\right] \tag{5}$$

$$x = M\,\frac{0.4\,\pi}{s}\left[\left(1 - \frac{\psi}{320}\right)\frac{d_1}{3\,w_s} + \left(1 - \frac{\psi}{240}\right)\frac{d_2}{w_s}\right] \tag{6}$$

The derivation of this expression in the above paper omits the allowance for the out of phase component of the reactive voltage induced in one phase by the half coils of another phase in the same slot. The final expression however is correct as it can readily be seen by reference to Fig. 10, the reactance component of E_2, the voltage induced in phase one by phase two, will be exactly balanced by the reactive component of E_3 the voltage induced in phase one by phase three.

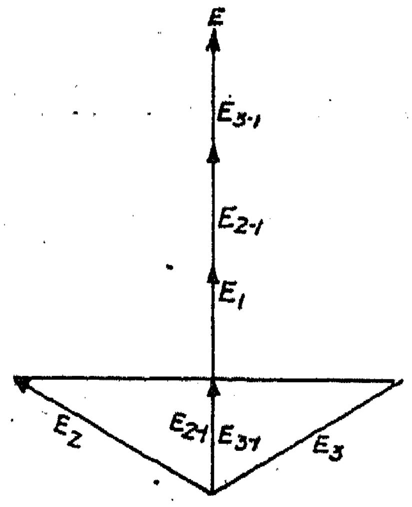

FIG. 10—MUTUAL INDUCTION IN SHORT-PITCH ARMATURE WINDINGS

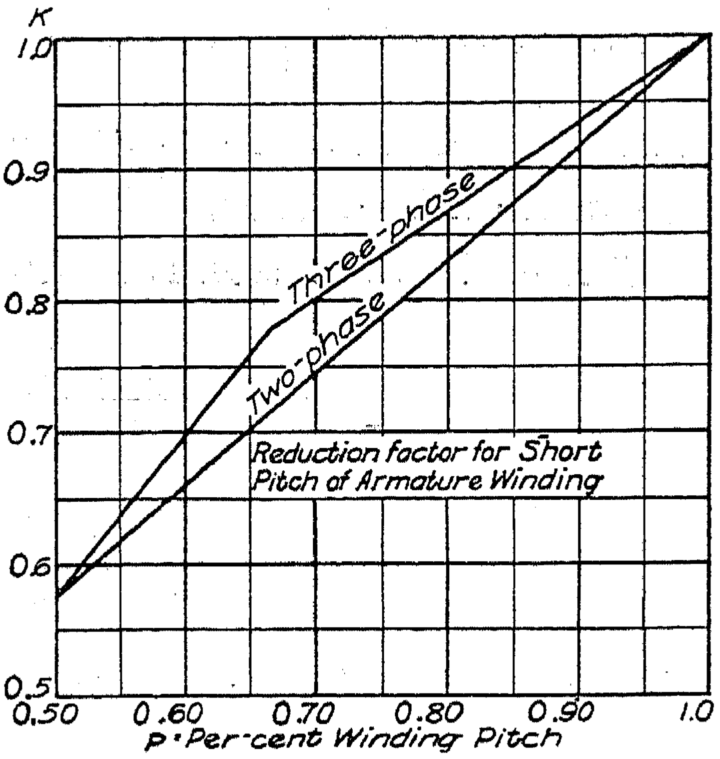

FIG. 11

For simplicity and with very little decrease in accuracy, we may replace the two short-pitch factors in equation (6) by an average value K from Fig. 11.

Then
$$x = M\,\frac{0.4\,\pi}{s}\,K\left(\frac{d_1}{3\,w_s} + \frac{d_2}{w_s}\right) \tag{7}$$

Pitch below $66\frac{2}{3}$ per cent.

Following Fechheimer's method with the correction for number of half coils, the expression for reactance where the pitch is less than $66\frac{2}{3}$ per cent will be

$$x = M\,\frac{0.4\,\pi}{s}\left[\left(1.187 - \frac{\psi}{160}\right)\frac{d_1}{3\,w_s} + \left(1.25 - \frac{\psi}{120}\right)\frac{d_2}{w_s}\right] \tag{8}$$

Replacing the two factors by an average value K from Fig. 11

$$x = M \frac{0.4\,\pi}{s} K \left(\frac{d_1}{3\,w_s} + \frac{d_2}{w_s}\right) \tag{9}$$

Two phase.

Pitch between 50 and 100 per cent.

The expression for two-phase slot reactance with short pitch derived by a similar process is

$$x = M \frac{0.4\,\pi}{s} \left[\left(1 - \frac{\psi}{240}\right)\frac{d_1}{3\,w_s} + \left(1 - \frac{\psi}{180}\right)\frac{d_2}{w_s}\right] \tag{10}$$

Using an average value of reduction factor from Fig. 11

$$x = M \frac{0.4\,\pi}{s} K \left(\frac{d_1}{3\,w_s} + \frac{d_2}{w_s}\right) \tag{11}$$

Tooth-Tip Leakage. The tooth-tip leakage may be most conveniently calculated by dividing it into two components; the effective leakage flux around the part of the phase belt opposite the iron of the pole face (maximum value Φ_i); and the effective leakage flux around the part of the phase belt between the poles (maximum value Φ_a).

The values of Φ_i and Φ_a for a complete phase belt were determined by Gray.[6] His method has been followed for Φ_i except in determining the area of the path across the air gap. The leakage flux will fringe from the teeth in almost exactly the same way as does the no-load flux, hence the area of the path across the air gap is the length of armature times the tooth pitch divided by the air gap coefficient, Fig. 12B.

For one and two slots per pole and phase,

$$\Phi_i = 0.4\,\pi \frac{w_s + w_t}{2\,g\,C} \tag{12}$$

Following Gray's derivation, it will be found that for higher numbers of slots per pole and phase

$$\Phi_i = 0.4\,\pi\,K_i \left(\frac{w_s + w_t}{2\,g\,C}\right) \tag{13}$$

A curve for K_i plotted against slots per pole and phase is given in Fig. 12A.

Gray's expression for Φ_a has been adopted without change, and simplified by assuming $w_s = w_t$, which gives a definite value of Φ_a for any given value of slots per pole and phase.

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[6] Electrical Machine Design, Gray Edition, 1913, p. 220.

Φ_a plotted against slot per pole and phase is shown in Fig. 12.

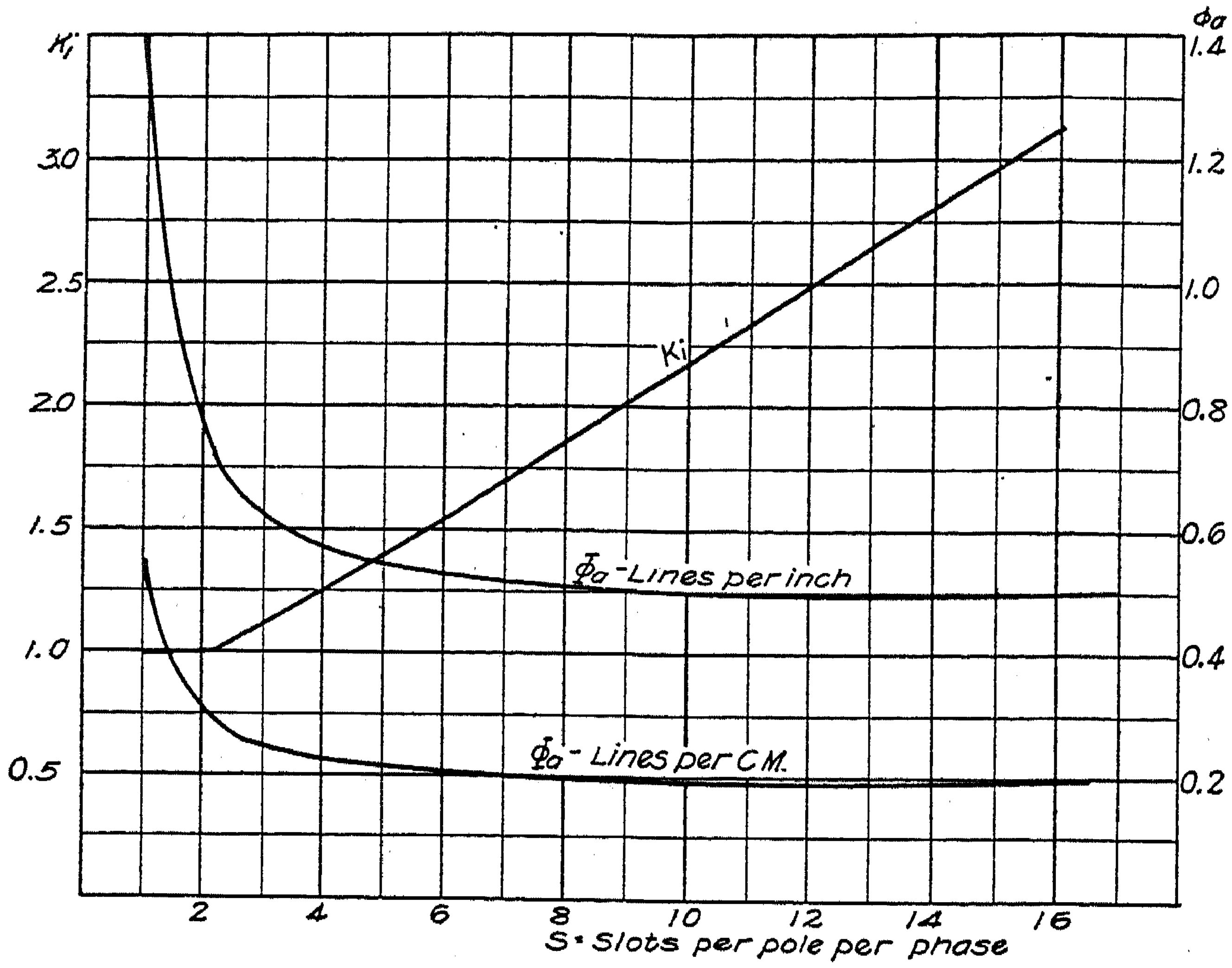

FIG. 12A

Modification of Φ_i and Φ_a by Definite Pole Construction. It should be noted that Φ_i, would be the leakage flux per ampere conductor in the phase belt, if the phase belt were opposite the

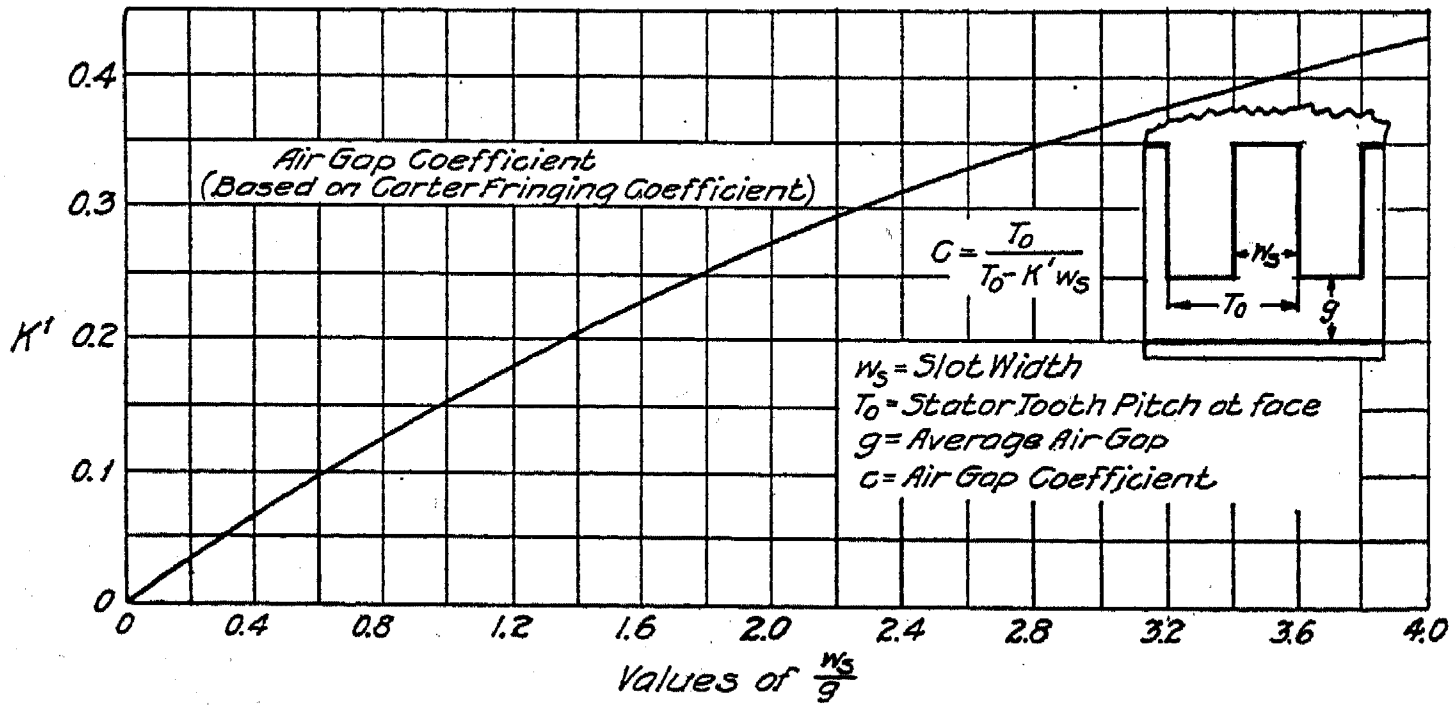

FIG. 12B

pole face iron throughout the cycle. Also Φ_a would be this leakage flux if there were no iron in the tooth-tip leakage path. The actual condition is a path partly iron and partly air and varying in proportion of air and iron during the cycle. This condition may be approximated by calculating the interlinkages in the iron part of the path, and also the interlinkages in the air part of the path. The ratio of the former to Φ_i, and of the latter to Φ_a will be reduction factors to be applied to Φ_i and Φ_a in calculating the reactances due to tooth-tip leakage. The actual tooth-tip leakage will be obtained by adding the two parts as calculated in this way.

The general theory of the variation of Φ_i and Φ_a at zero power factor has been discussed in the first part of this paper, and it will now be applied to the actual calculation.

The instantaneous values of leakage flux $\phi_i\ i$ vary from zero when the current is zero (Fig. 13A position a) to a maximum approximately where the pole begins to leave the phase belt (Fig. 13A position b) and then decreases to zero where the phase belt is entirely between the poles. (Fig. 13A position c). The instantaneous value of leakage flux $\phi_a\ i$ is zero when the current is zero, and remains zero until the pole begins to leave the phase belt, after which it increases gradually to a maximum where the current is a maximum at 90 deg. Obviously the curves for $\phi_i\ i$ and $\phi_a\ i$ are symmetrical about the 90 deg. axis. Fig. 14.

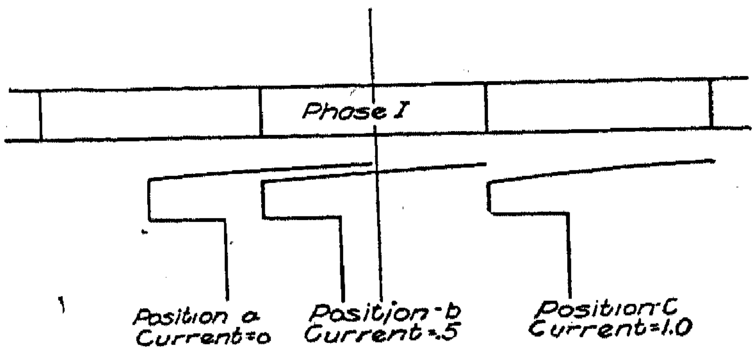

FIG. 13A—DIAGRAM OF POLE POSITIONS

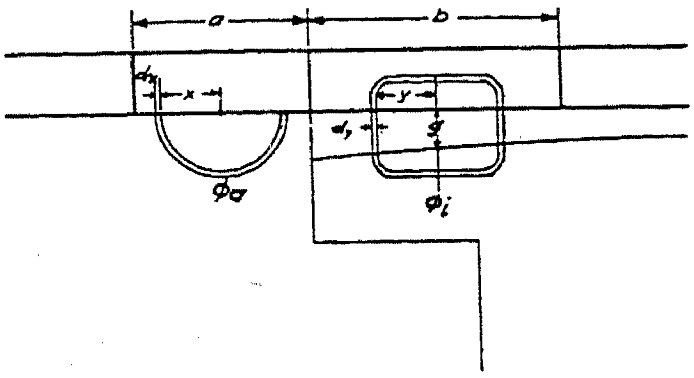

FIG. 13B—TOOTH-TIP LEAKAGE

The values of $\phi_i\ i$ and $\phi_a\ i$ for complete distribution of the armature winding were calculated by the following methods.

Tooth-tip leakage $\phi_a\ i$

For one ampere in the phase belt and unit length of phase belt the m. m. f. at x (See Fig. 13B) is

$$\text{m. m. f.} = \frac{2\,x}{\dfrac{\tau}{n}} = \frac{2\,n\,x}{\tau} \tag{14}$$

where n = number of phases

and τ = pole pitch

$$d\,\phi = 0.4\,\pi\,\frac{2\,n\,x}{\tau}\;\frac{d\,x}{\pi\,x} \tag{15}$$

$$= 0.4\,\pi\,\frac{2\,n}{\pi\,\tau}\,d\,x \tag{16}$$

Let effective interlinkages due to $d\,\phi = d\,\phi_a$

Then $$d\,\phi_a = \frac{2\,x}{\frac{\tau}{n}}\,d\,\phi \tag{17}$$

$$= \frac{2\,n\,x}{\tau}\,0.4\,\pi\,\frac{2\,n}{\pi\,\tau}\,d\,x \tag{18}$$

$$\phi_a = \int_0^{a/2} d\,\phi_a = 0.4\,\pi \times \frac{2\,n^2}{\pi}\;\frac{x}{\tau^2} + C\Big]_0^{a/2} \tag{19}$$

$$= 0.4\,\pi\;\frac{n^2}{2\,\pi}\left(\frac{a}{\tau}\right)^2 + C \tag{20}$$

For $a = 0$ $\phi_a = 0$ Hence $C = 0$

$$\phi_a = 0.4\,\pi\;\frac{n^2}{2\,\pi}\left(\frac{a}{\tau}\right)^2 \tag{21}$$

for one centimeter of phase belt.

$$\phi_a = 3.2\;\frac{n^2}{2\,\pi}\left(\frac{a}{\tau}\right)^2 \tag{22}$$

for one inch of phase belt.

For the special case of three-phase and where a is expressed as a per cent. of τ.

$$\phi_a = 4.6\,a^2 \text{ for one inch of phase belt.} \tag{23}$$

Tooth-tip Leakage $\phi_i\,i$

Referring to Fig. 13B

$$\text{m. m. f.} = \frac{2\,y\,n}{\tau} \tag{24}$$

Let g_b = average air gap over b.

$$d_\phi = 0.4\,\pi\,\frac{2\,y\,n}{\tau}\;\frac{d\,x}{2\,g_b} \tag{25}$$

Let effective interlinkages due to $d\,\phi = d\,\phi_i$

$$d\,\phi_i = \frac{2\,y\,n}{\tau}\,d\,\phi \tag{26}$$

$$= \frac{0.4\,\pi\,2\,n^2\,y^2}{g_b\,\tau^2}\,d\,y \tag{27}$$

$$\phi_i = 0.4\,\pi\,\frac{2\,n^2}{g_b\,\tau^2}\int_0^{b/2} y^2\,d\,y \tag{28}$$

$$= 0.4\,\pi\,\frac{2\,n^2}{3\,g_b\tau^2}\,y^3 + c\Big]_0^{b/2} \tag{29}$$

At $y = 0$ $\phi_i = 0$ hence $c = 0$

$$\phi_i = 0.4\,\pi\,\frac{n^2\,b^3}{12\,g_b\tau^2} \tag{30}$$

Where b and g_b are expressed as a per cent. of τ

$$\phi_i = 0.4\,\pi\,\frac{n^2\,b^3}{12\,g_b} \tag{31}$$

per centimeter of phase belt.

$$= 3.2\,\frac{n^2\,b^3}{12\,g_b} \tag{32}$$

per inch of phase belt.

For three-phase and one-inch length of phase belt.

$$\phi_i = 2.4\,\frac{b^3}{g_b} \tag{33}$$

The area of leakage path across the air gap in actual machines will be reduced due to the effect of armature slots and this will be allowed for by the introduction of the air gap coefficient, C, given in Fig. 12B.

$$\phi_i = 2.4\,\frac{b^3}{C\,g_b} \tag{34}$$

The derivation of the actual values of $\phi_i\,i$ and $\phi_a\,i$ will be illustrated by an example for three-phase where α is $66\frac{2}{3}$ per cent.; the minimum gap is 1.5 per cent of the pole pitch; and the ratio of maximum to minimum gap is 2 to 1, giving an average gap of approximately 2 per cent of the pole pitch. Average values of air gap coefficient will be assumed. The armature winding will be assumed to be completely distributed.

Table III gives the calculation of $\phi_i\, i$ and $\phi_a\, i$ as the current varies through 90 deg. calculated from the formulas.

$$\phi_i = 2.4 \frac{b^3}{C\, g_b} \qquad \text{from} \quad \mathbf{(33)}$$

$$\phi_a = 4.6\, a^2 \qquad \text{from} \quad \mathbf{(23)}$$

TABLE III.

θ	b	g_b	C	b^3	$C g_b$	ϕ_i	a	a^2	ϕ_a	i	$\phi_i\, i$	$\phi_a\, i$
0	0.333	0.0165	1.09	0.036	0.018	4.76	0	0	0	0	0	0
15	0.333	0.0177	1.09	0.036	0.0193	4.43	0	0	0	0.26	1.15	0
30	0.333	0.0188	1.08	0.036	0.204	4.2	0	0	0	0.50	2.1	0
45	0.25	0.0196	1.08	0.0156	0.0212	1.75	0.083	0.007	0.032	0.71	1.24	0.023
60	0.167	0.020	1.08	0.0047	0.0216	0.52	0.167	0.028	0.128	0.87	0.45	0.111
75	0.083	0.023	1.07	0.006	0.0246	0.06	0.250	0.063	0.29	0.97	0.06	0.28
90	0	0.027	1.07	0	..	0	0.333	0.110	0.505	1.0	..	0.505

Dimensions are expressed as per cent of pole pitch.
Average gap = 0.02

$$\Phi_i = 2.4 \frac{0.333^3}{1.08 \times 0.02} = 4.08$$

$$\Phi_a = 4.6 \times 0.333^2 = 0.505$$

Fig. 14 shows the above values of $\phi_i i$ and $\phi_a i$, and the total tooth-tip leakage over 180 deg. These waves were analyzed for harmonics and the equation is given in the figure. The fundamentals only are plotted.

It will be noted that the third is the most important harmonic, and since the third harmonics cancel in the terminal voltages of star connected three-phase machines, the effect of only the fundamental needs to be considered. It is obvious from the equation in Fig. 14 that the higher harmonics may be neglected without introducing any appreciable error.

C_i is the ratio of the maximum value of the fundamental of $\phi_i\, i$ Fig. 14 to the value of Φ_i that is,

$$C_i = \frac{0.89}{4.08} = 0.218$$

Similarly

$$C_a = \frac{0.22}{0.505} = 0.435$$

In the derivation of C_i and C_a a sine wave of armature current has been assumed. This is very nearly the case with a star-connected machine, and, as has been explained, in a Δ-connection the short-circuit current as read on an ammeter is practically equal to that with star-connection for the same field current. The values of C_i and C_a as derived for star connection may therefore be used for delta connection, even though the current is not a sine wave in the latter case.

The values of C_i and C_a were calculated for complete distribution of the armature winding with various ratios of pole arc to pole pitch for three phase and two phase by the method shown in Table III, and are given in Fig. 15.

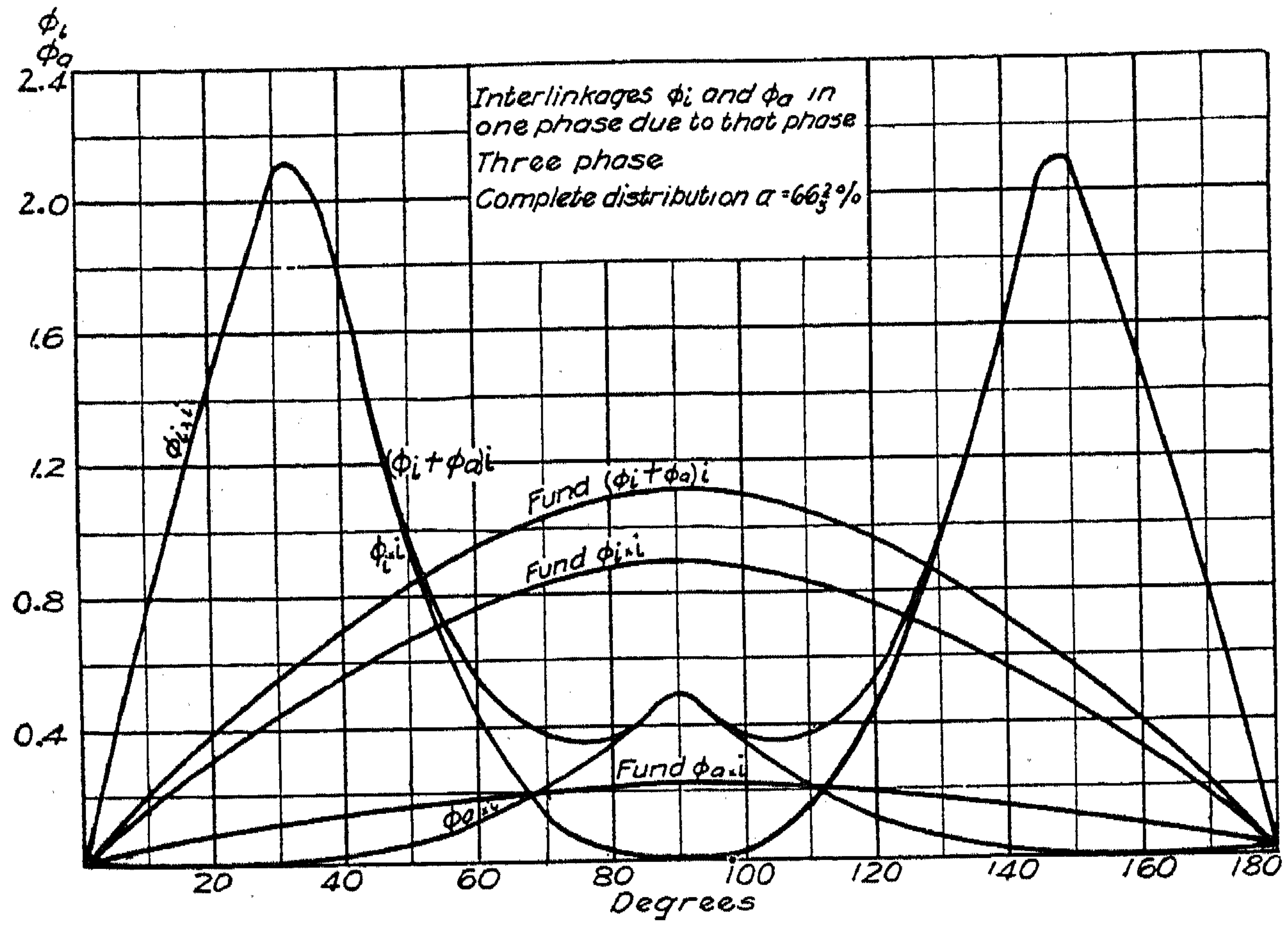

FIG. 14

$$\phi_i = .89 \sin \theta + 1.22 \sin 3\,\theta + .29 \sin 5\,\theta - .15 \sin 7\,\theta - .18 \sin 9\,\theta - .06 \sin 11\,\theta$$

$$\phi_a = .22 \sin \theta - .13 \sin 3\,\theta + .06 \sin 5\,\theta - .04 \sin 7\,\theta + .04 \sin 9\,\theta - .01 \sin 11\,\theta$$

$$\phi_i + \phi_a = 1.11 \sin \theta + 1.09 \sin 3\,\theta + .35 \sin 5\,\theta - .19 \sin 7\,\theta - .14 \sin 9\,\theta - .07 \sin 11\,\theta$$

The variation of C_i and C_a with distribution of the armature winding was investigated and it was found the results obtained by the application of these factors C_i and C_a to Φ_i and Φ_a would give practically correct results when applied to the case of one slot per pole and phase. It is evident that there is just one leakage path around the coil in the case of one slot per pole and phase, and this path will be partly through the pole face iron and partly through air, but the length of path in each will vary as the pole moves along. The total interlinkages were calculated over 90 deg. for this condition and the waves analyzed. It was found that the ratio of the fundamental of the total interlinkage wave in this case to the value of $\Phi_i + \Phi_a$ was practically the same as the ratio of $C_i \Phi_i + C_a \Phi_a$ to $\Phi_i + \Phi_a$, that is the values of C_i and C_a derived for complete distribution apply also to one slot per pole and phase.

It is very important to note that while the values of C_i and C_a do not change with distribution, Φ_i and Φ_a do show a large variation.

The polyphase effect in tooth-tip reactance has been discussed in the first part of this paper and is taken into account by increasing the values of C_i and C_a by 50 per cent for three phase. For two phase the values of C_i and C_a are used as calculated for one phase of the winding.

Let C_1 be the factor by which Φ_i is multiplied to obtain the effective interlinkages per ampere, and C_2 be the factor for Φ_a Then

$$C_1 = 1.5\, C_i \text{ for three phase}$$
$$= C_i \quad \text{for two phase}$$
$$C_2 = 1.5\, C_a \text{ for three phase}$$
$$= C_a \text{ for two phase}$$

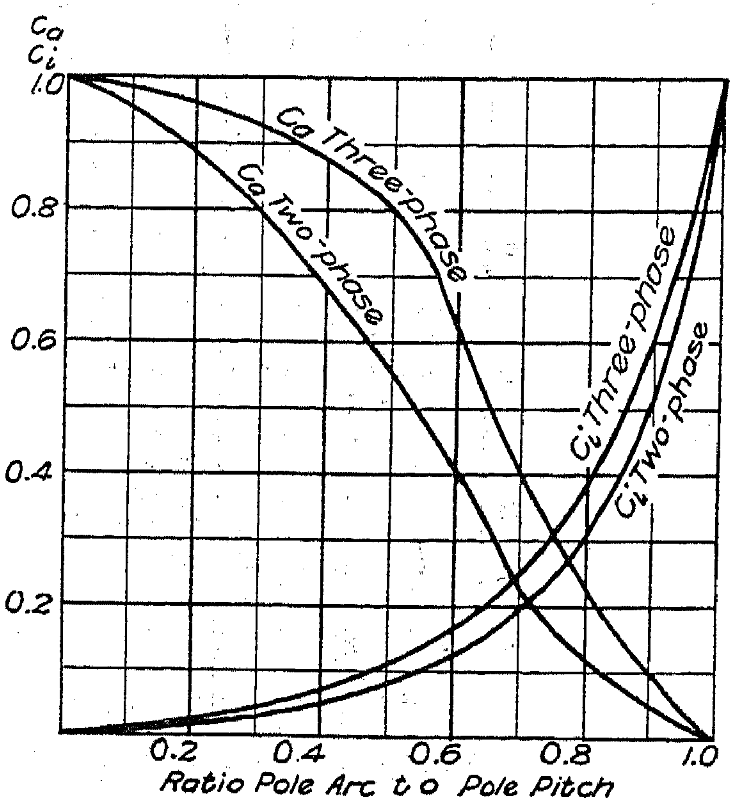

FIG. 15—VALUES OF C_i AND C_a

It will be noted that the values of C_i and C_a are not the same for two phase as for three phase.

The factors C_1 and C_2 for both three phase and two phase are plotted against ratio of pole arc to pole pitch in Fig. 16.

The reduction factor for tooth-tip reactance accounting for short pitch coils may be taken as the average factor K from Fig. 11.

Reactance Formulas. The various factors of the reactance formula may be combined to give the following equation for a double-layer armature winding.

For dimension in centimeters.

Three Phase.

$$x_a = \frac{2\pi f\, s^2 a^2\, q l}{10^8 c^2} \left\{ \overset{\text{End}}{\frac{2.8\, p \sqrt{\tau}}{l}} + \right.$$

$$\left. K \left[\overset{\text{Slot}}{\frac{0.4\pi}{s} \left(\frac{d_1}{3\, w_s} + \frac{d_2}{w_s} \right)} + \overset{\text{Tooth Tip}}{0.4\pi\, C_1\, K_i \frac{w_s + w_t}{2gC}} + C_2 \Phi_a \right] \right\} \qquad (35)$$

Two Phase.

$$x_a = \frac{2\ \pi f\, s^2\, a^2\ q\, 1}{10^2\, c^2} \left\{ \overset{\text{End}}{\frac{1.6\ p\ \sqrt{\tau}}{1}} + \right.$$

$$\left. K \left[\overset{\text{Slot}}{\frac{0.4\ \pi}{s} \left(\frac{d_1}{3\, w_s} + \frac{d_2}{w_s} \right)} + \overset{\text{Tooth Tip}}{0.4\ \pi\, C_1\, K_i\, \frac{w_s + w_t}{2\, g\, C} + C_2 \Phi_a} \right] \right\} \quad \textbf{(36)}$$

For single layer armature winding, multiply end connection factor by 2.0 and use $K = 1.0$.

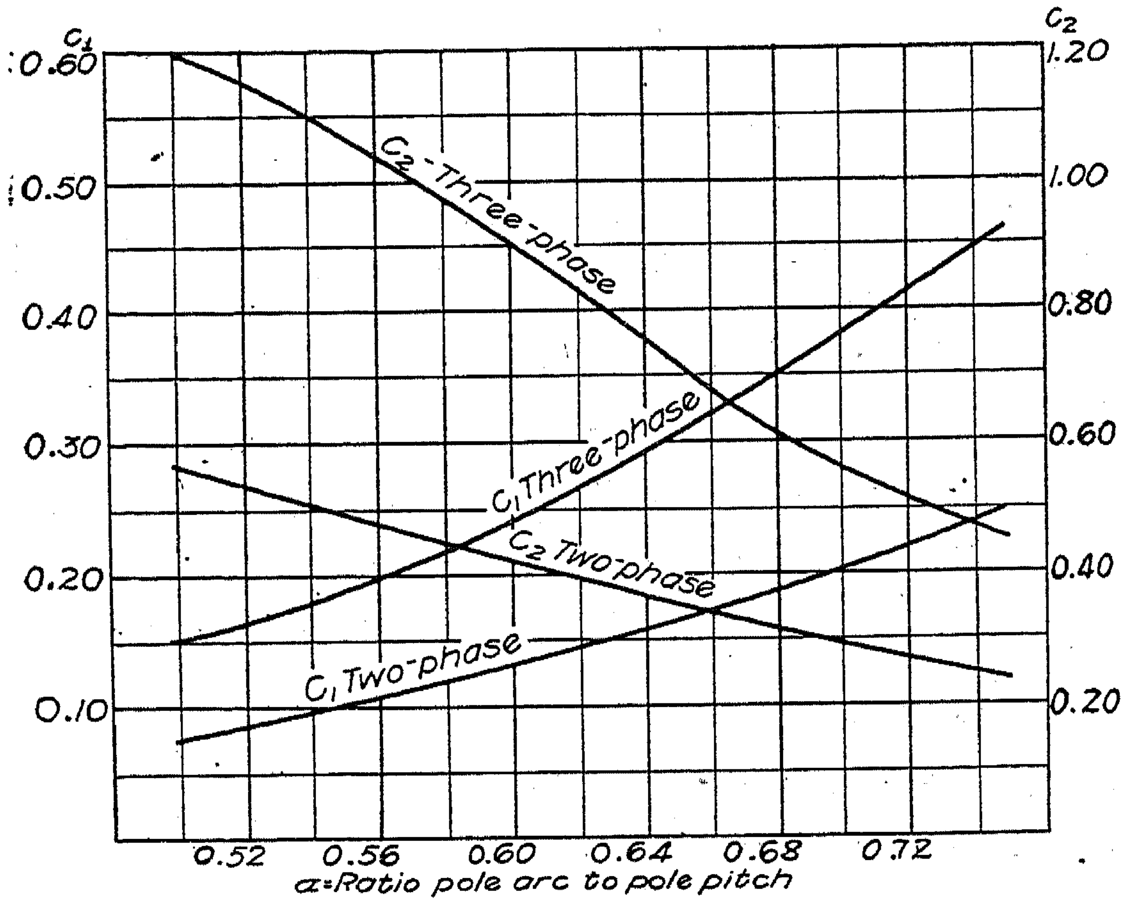

FIG. 16—VALUES OF C_1 AND C_2

REDUCTION FACTORS FOR TOOTH-TIP LEAKAGE

$C_1 = 1.5\ C_i$ for three phase

$= C_i$ for two phase.

$C_2 = 1.5\ C_\alpha$ for three phase

$= C_\alpha$ for two phase.

For dimensions in inches.

Substitute 4.5 for 2.8 in factor for three-phase end leakage; 2.5 for 1.6 in case of two-phase: and 3.2 for 0.4π in slot leakage and first factor of tooth-tip leakage. Use Φ_a in lines per inch.

Where x_a = armature reactance per phase in ohms.

f = frequency in cycles per second

s = slots per pole and phase

a = conductors per slot; for a double layer winding a is twice the number of turns per coil.

q = number of poles

l = gross stacked length of armature core (iron + ducts)

c = number of circuits in multiple in armature winding
p = per cent pitch of armature winding (= 0.8 for 80 per cent pitch)
τ = pole pitch at armature core face
K = reduction factor depending on pitch of armature coils
d_1 = distance from top of copper to bottom of copper in the armature winding. Fig. 8
d_2 = distance from top of copper to armature face Fig. 8
C_1 = 1.5 C_i for three phase
= C_i for two phase
C_2 = 1.5 C_a for three phase
= C_a for two phase
See Fig. 16 for C_1 and C_2
K_i = factor for Φ_i depending on slots per pole and phase
w_s = width of slot Fig. 8
w_t = width of tooth at armature face Fig. 8
g = average air gap
C = air gap coefficient based on Carter's fringing coefficient
Φ_a = effective tooth interlinkages from Fig. 12

Reactance in Per Cent. It is very convenient to have the reactance expressed in per cent and in terms of quantities easily available from the design of the machine.

Three Phase

Per cent reactance is given by $$x_{pa} = \frac{I_n\, x_a}{e} \qquad \textbf{(37)}$$

(x_{pa} = 0.20 means 20 per cent reactance)
Where I_n = normal current per phase
x_a = reactance per phase in ohms
e = voltage per phase

$$\text{Armature reaction } A = \frac{2.12\ s\ a\ I_n}{2\ K_p\ K_d\ K_\phi\ c} \qquad \textbf{(38)}$$

$$\text{Flux per pole } \Phi = \frac{e \times 10^8\ K_p\ K_d\ K_\phi\ c}{2.22\ s\ a\ q\ f} \qquad \textbf{(39)}$$

Let L_0 = bracket expression from reactance formulas (**35**) or (**36**)

$$x_a = \frac{2\,\pi\, f\ \ s^2\, a^2\, q}{10^8\ \ c^2}\ L_0 \qquad \textbf{(40)}$$

From (38)

$$I = \frac{2\ K_p\, K_d\, K_\phi\, A}{2.12\ s^1\ a} \tag{41}$$

and from (39)

$$e = \frac{2.22\ s\ a\ q\ f}{10^8\ K_p\, K_d\, K_\phi\ c}\,\Phi \tag{42}$$

$$x_{pa} = \frac{I\ x_a}{e} = \frac{2.67\ K_p^2\, K_d^2\, K_\phi^2 A\ l}{\Phi}\ L_0 \tag{43}$$

Two Phase

Replacing the factor 2.12 in the armature reaction formula by 1.41 the formula for two phase becomes

$$x_{pa} = \frac{4.\ K_p^2\, K_d^2\, K_\phi^2\ A\ l}{\Phi}\ L_0 \tag{44}$$

Where K_d, and L_0 are based on the two-phase curves and formulas. See reactance formula (36).

Working Formulas. The expression for

$$\Phi_i = 0.4\,\pi\ K_i\ \frac{w_s + w_t}{2\ g\ C}$$

for dimensions in centimeters and

$$3.2\ K_i\ \frac{w_s + w_t}{2\ g\ C}$$

for dimensions in inches may be simplified by assuming $w_s = w_t$ and then a series of curves for Φ_i plotted against the values of $\frac{w_s}{g}$ may be derived for different values of slots per pole and phase. The curves are given for dimensions in inches in Fig. 17. The working formulas for reactance of double layer windings with lengths expressed in centimeters are as follows:

Three Phase

$$x_{pa} = \frac{2.67\ (K_p\, K_d\, K_\phi)^2\, A\ l}{\Phi} \left\{ \overset{\text{End}}{\frac{2.8\ p\ \sqrt{\tau}}{l}} + \right.$$

$$\left. K \left[\overset{\text{Slot}}{\frac{0.4\pi}{s}\left(\frac{d_1}{3\ w_s} + \frac{d_2}{w_s}\right)} + \overset{\text{Tooth Tip}}{0.39\,(C_1\,\Phi_i + C_2\ \Phi_a)} \right] \right\} \tag{45}$$

Two Phase

$$x_{pa} = \frac{4.\,(K_p\,K_d\,K_\phi)^2\,A\,l}{\Phi}\left\{\overset{\text{End}}{\frac{1.6\,p\,\sqrt{\tau}}{l}} + K\left[\overset{\text{Slot}}{\frac{0.4\pi}{s}\left(\frac{d_1}{3\,w_s}+\frac{d_2}{w_s}\right)} + \overset{\text{Tooth Tip}}{0.39\,(C_1\,\Phi_i + C_2\,\phi_a)}\right]\right\} \tag{46}$$

Note that factor 0.39 is introduced in the tooth-tip leakage because Φ_i and Φ_a in the curves are expressed in leakage lines per inch length of core.

For dimensions in inches.

Three Phase.

$$x_{pa} = \frac{2.67\,(K_p\,K_d\,K_\phi)^2\,A\,l}{\Phi}\left\{\overset{\text{End}}{\frac{4.5\,p\,\sqrt{\tau}}{l}} + K\left[\overset{\text{Slot}}{\frac{3.2}{s}\left(\frac{d_1}{3w_s}+\frac{d_2}{w_s}\right)} + \overset{\text{Tooth Tip}}{C_1\,\Phi_i + C_2\,\Phi_a}\right]\right\} \tag{47}$$

Two Phase

$$x_{pa} = \frac{4.0\,(K_p\,K_d\,K_\phi)^2\,A\,l}{\Phi}\left\{\overset{\text{End}}{\frac{2.5\,p\,\sqrt{\tau}}{l}} + K\left[\overset{\text{Slot}}{\frac{3.2}{s}\left(\frac{d_1}{3\,w_s}+\frac{d_2}{w_s}\right)} + \overset{\text{Tooth Tip}}{C_1\,\Phi_i + C_2\,\Phi_a}\right]\right\} \tag{48}$$

See Fig. 11 for K, Fig. 17 for Φ_i, Fig. 12 for ϕ_a and Fig. 16 for C_1 and C_2. ($x_{pa}=0.2$ means 20 per cent reactance.)

Reactance Curves. The formula may be still further simplified by assuming average values of d_1, d_2 and w_s for a certain narrow range of slot pitches. A series of five curves covering the ordinary ranges of slot pitches, and giving values of end, slot and tooth-tip factors as a function of the slots per pole and phase are given in Figs. 20A, 20B, etc. The curves will give the accuracy necessary for most calculations of reactances and make the work quite simple.

Reactance of Non-Salient Pole Generators. The reactance of turbine generators, the most common type of non-salient pole machines, was calculated by assuming the ratio of pole arc to pole pitch to be the ratio of the unslotted pole centre to pole pitch. Since practically all machines of this type now have five or more slots per pole and phase, the expression for Φ_i was simplified by substituting in equations **(35)** and **(36)** the value of K_i expressed as a function of the slots per pole and phase.

That is $$K_i = 0.625 + 0.156\, s \text{ (above } s=4) \tag{49}$$

With a ratio of pole arc to pole pitch of 33 per cent, an approximate average for turbine generators, the value of C_1 is 0.075 for three phase and 0.035 for two phase, and C_2 is 1.32 for three phase and 0.60 for two phase.

Three Phase

$$C_1 \Phi_i = 0.075\,(0.625 + 0.156\, s)\; 0.4\, \pi\, \frac{w_s + w_t}{2\, g\, C} \tag{50}$$

in lines per centimeter.

$$C_1\, \Phi_i = 0.075\,(0.625 + 0.156\, s)\; 3.2\, \frac{w_s + w_t}{2\, g\, C} \tag{51}$$

in lines per ampere cond'r per inch.

Allowing the approximation $w_s = w_t$ and simplifying we have

$$C_1\, \Phi_i = (0.059 + 0.015\, s)\, \frac{w_s}{g\, C} \tag{52}$$

in lines per centimeter.

$$C_1\, \Phi_i = (0.15 + 0.038\, s)\, \frac{w_s}{g\, C} \tag{53}$$

in lines per inch.

$$C_2\, \phi_a = 1.32 \times 0.198 = 0.26 \text{ line per centimeter}$$
$$= 1.32 \times 0.50 = 0.66 \text{ line per inch.}$$

Two Phase

Similarly

$$C_1\, \Phi_i = (0.0275 + 0.007\, s)\, \frac{w_s}{g\, C} \tag{54}$$

lines per centimeter.

$$C_1 \Phi_i = (0.07 + 0.018\, s)\, \frac{w_s}{g\, C} \tag{55}$$

lines per inch.

$$C_2 \Phi_a = 0.12 \text{ line per centimeter.}$$
$$= 0.3 \text{ line per inch}$$

Substituting these values in equations (**45**), (**46**), (**47**), and (**48**) will give the formulas for turbine generator reactances.

Application of Formula. The use of the reactance formulas will be illustrated by the application of equation (**47**) to a 6600-kv-a. three-phase 25-cycle 14,000-volt generator.

Example:

$s = 5.25$ $p = 82.5\% = 0.825$ $A = 11{,}000$ $g = 0.925$

$\tau = 36.5$ $\alpha = 0.698$ $w_s = 1.125$

$d_1 = 3.2$ $d_2 = 1.05$ $\Phi = 47.8 \times 10^6$

$K_p = 1.04$ $K_d = 1.046$ $K_\phi = 1.016$

$$\frac{2.67\,(1.04 \times 1.046 \times 1.016)^2\; 11{,}000 \times 38}{47.8 \times 10^6} = 0.0283$$

$$\frac{4.5\, p\, \sqrt{\tau}}{l} = \frac{4.5 \times 0.825\ \sqrt{36.5}}{38} = 0.59$$

$$\frac{3.2}{s}\left(\frac{d_1}{3\, w_s} + \frac{d_2}{w_s}\right) = \frac{3.2}{5.25}\left(\frac{3.2}{3.375} + \frac{1.05}{1.125}\right) = 1.15$$

$$K = 0.885 \qquad K \times 1.15 = 1.02$$

$$\frac{w_s}{g} = \frac{1.125}{0.925} = 1.22 \qquad \Phi_i = 5.1 \qquad C_1 = 0.37$$

$$K\, \Phi_i\, C_1 = 5.1 \times 0.37 \times 0.885 = 1.67$$

$$\Phi_a = 0.54 \qquad C_2 = 0.57$$

$$K\, \Phi_a\, C_2 = 0.54 \times 0.57 \times 0.885 = \underline{0.27}$$

$$3.55$$

$$x_{pa} = 0.0283 \times 3.55 = 0.10$$
$$= 10 \text{ per cent}$$

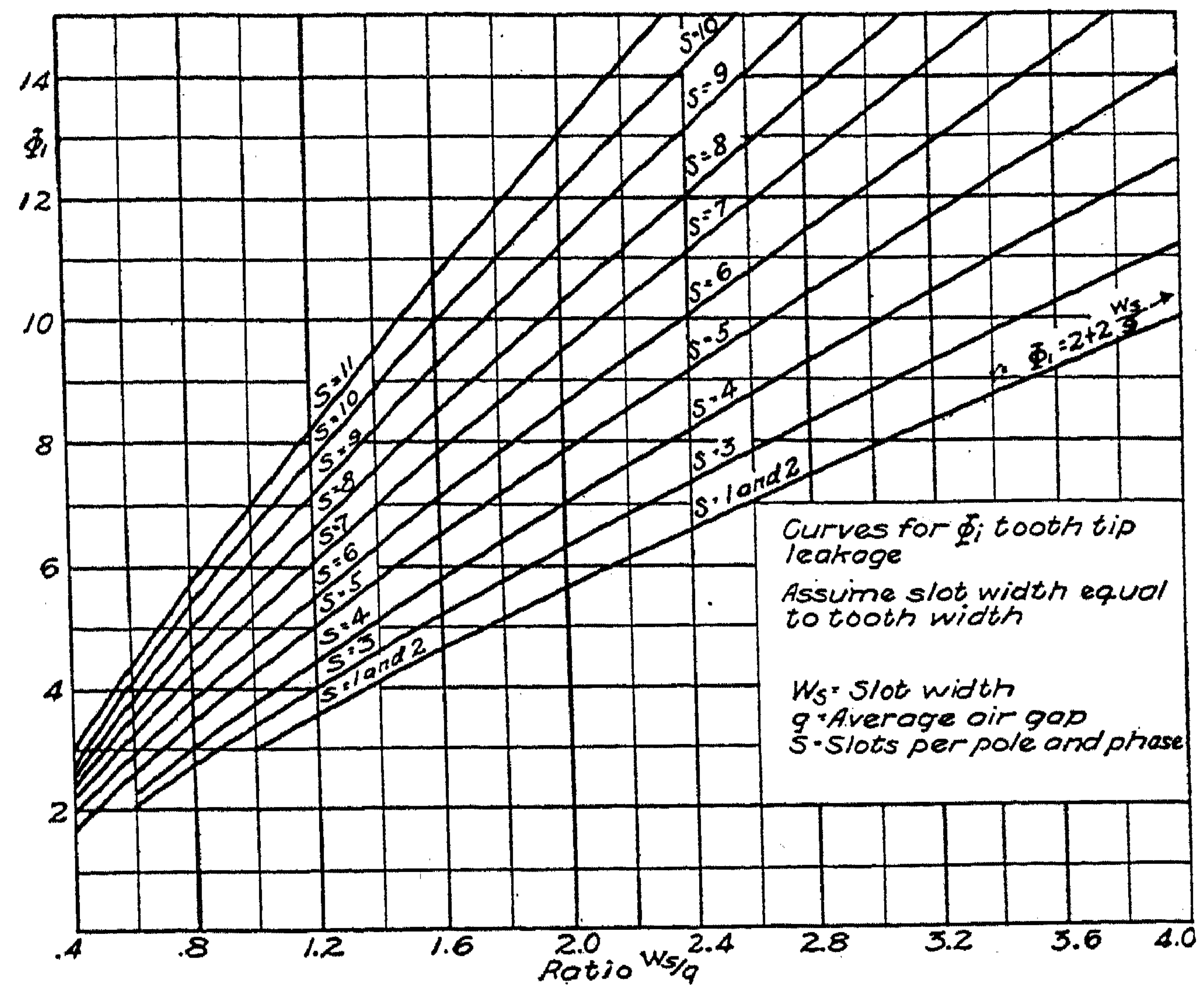

FIG. 17

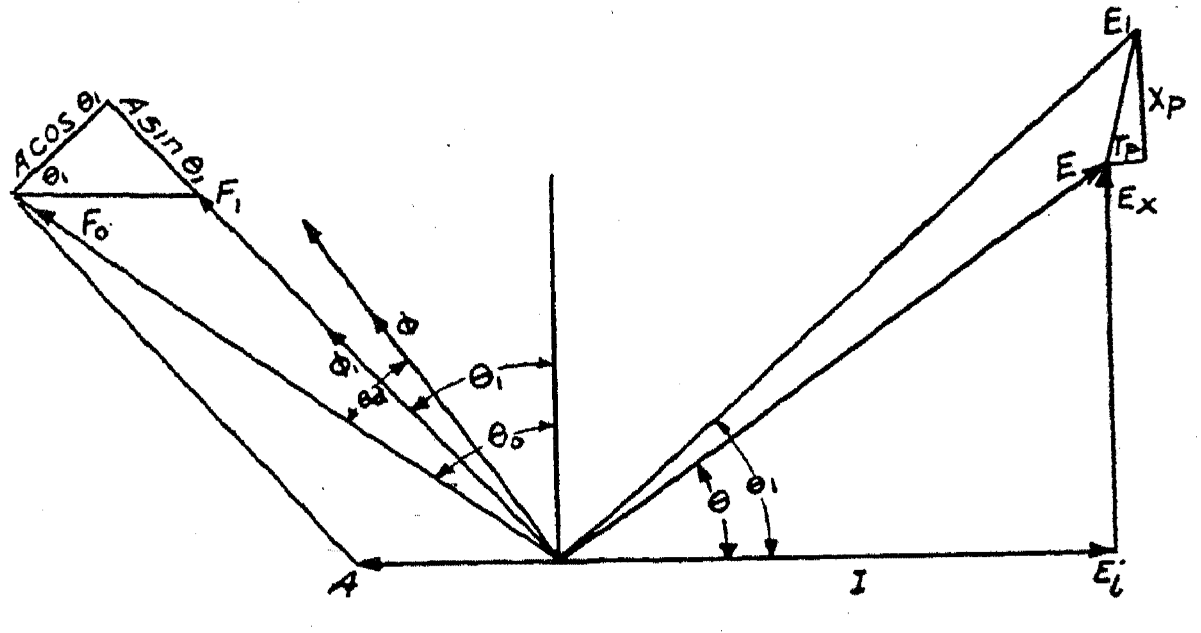

FIG. 18—EXCITATION DIAGRAM

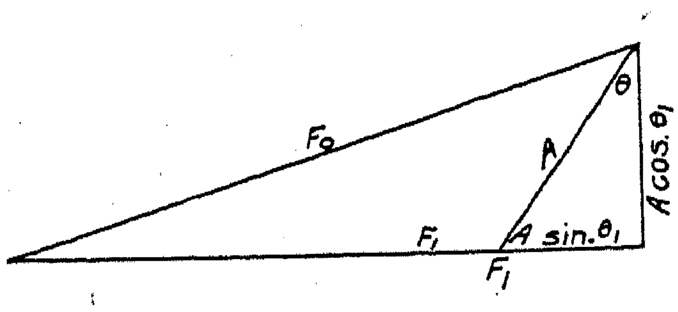

FIG. 19A—M. M. F. DIAGRAM FOR OVER-EXCITED MACHINE

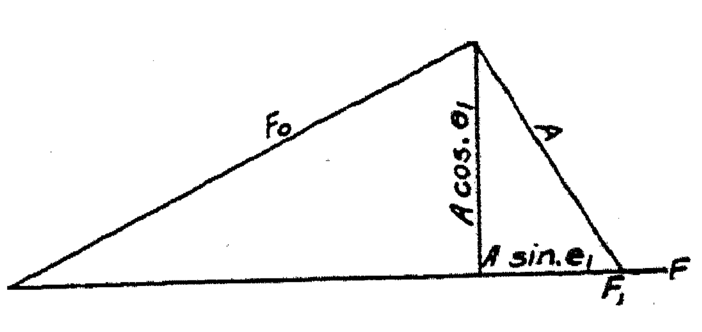

FIG. 19B—M. M. F. DIAGRAM FOR UNDER-EXCITED MACHINE

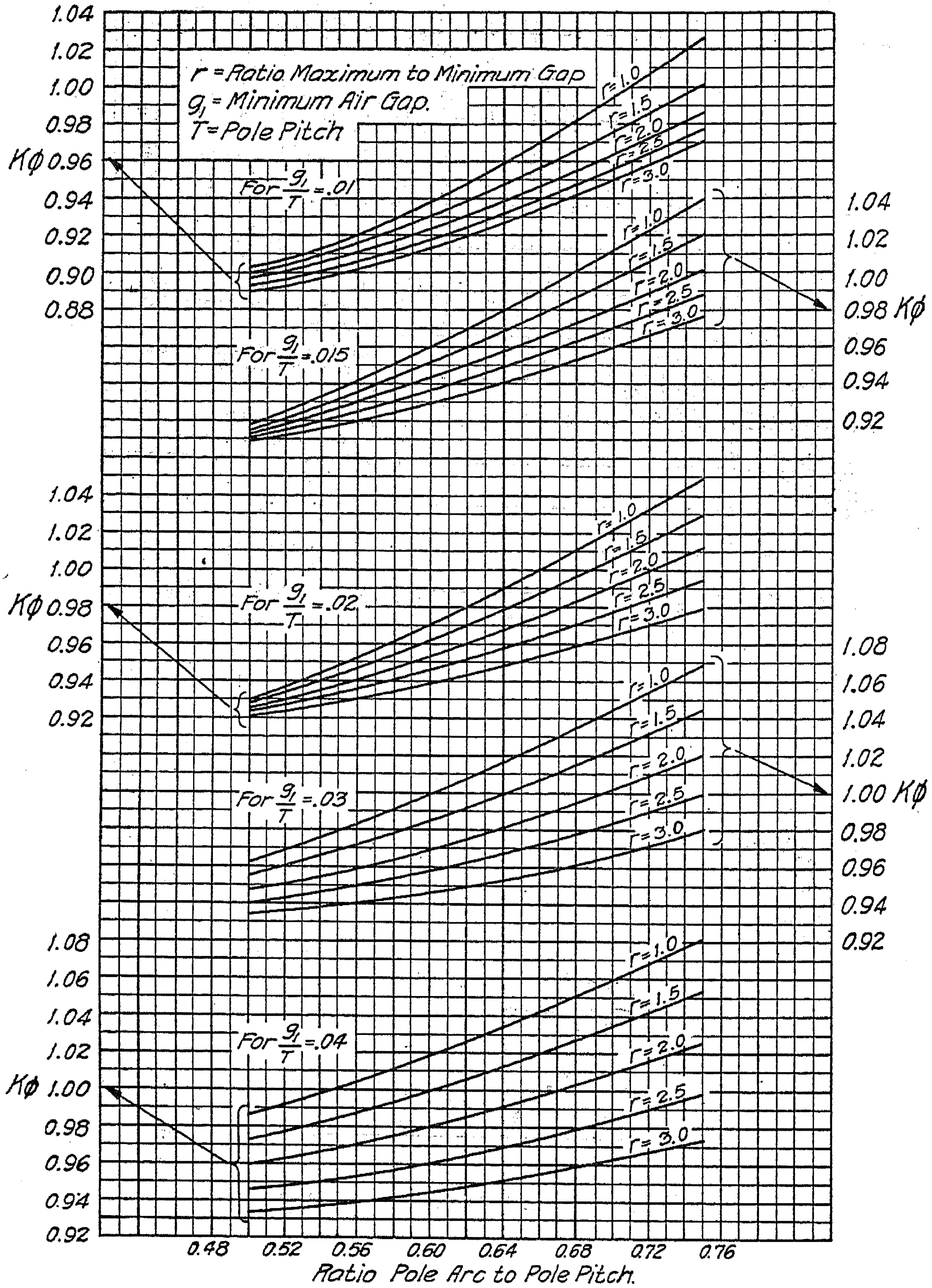

FIG. 20

FLUX DISTRIBUTION COEFFICIENT, K_ϕ

Note: Curves apply to poles having a constant radius of pole arc for all points of the arc except the extreme tip, which is rounded off by small radius. The maximum air gap is taken as the radial intercept, at the pole edge, between the armature face and the pole arc extended. That is, the rounded tip of small radius is neglected in determining the maximum gap.

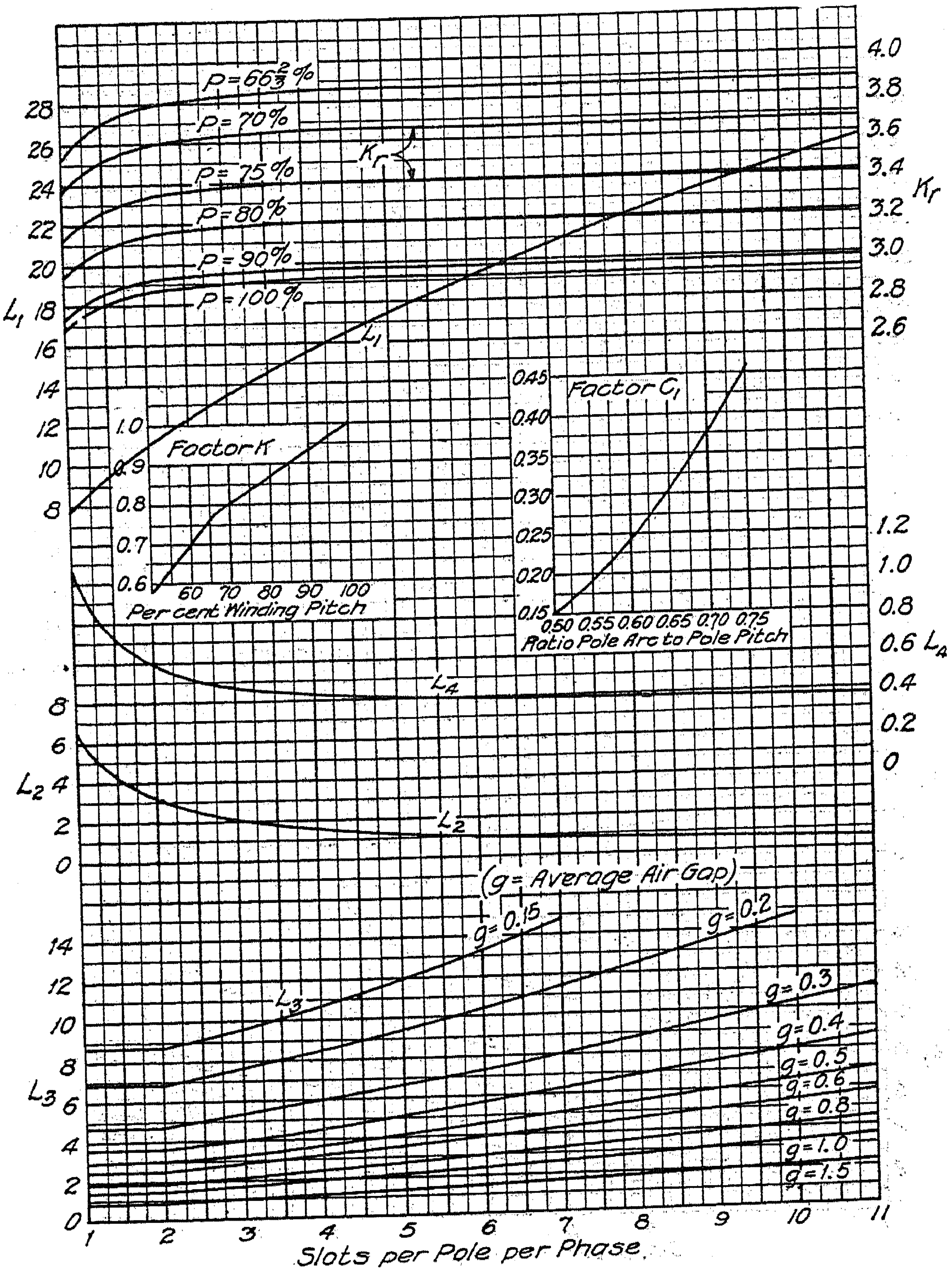

FIG. 20 A

ARMATURE REACTANCE OF THREE-PHASE ALTERNATORS

$$\text{Armature Reactance } x_{pa} = \frac{K_r\, K_\phi^2\, A\, l}{\Phi}\left[\frac{p\, L_1}{l} + K\,(L_2 + C_1 L_3 + L_4)\right]$$

(For 20% Reactance $x_{pa} = 0.20$)

K_r, K, L_1, L_2, L_3, L_4 and C_1 are given by curves.

A = $A\ T$ armature reaction per pole.

Φ = magnetic flux per pole in maxwells.

l = gross length of armature in inches (iron + ducts).

p = arm. winding pitch (0.8 for 80 per cent)

K_ϕ = flux distribution factor (Fig. 20)

Curves for Slot Pitch 0.9 to 1.1. Two-Layer Windings.

For Single-Layer Windings multiply L_1 by 2.0 and use $K = 1.0$.

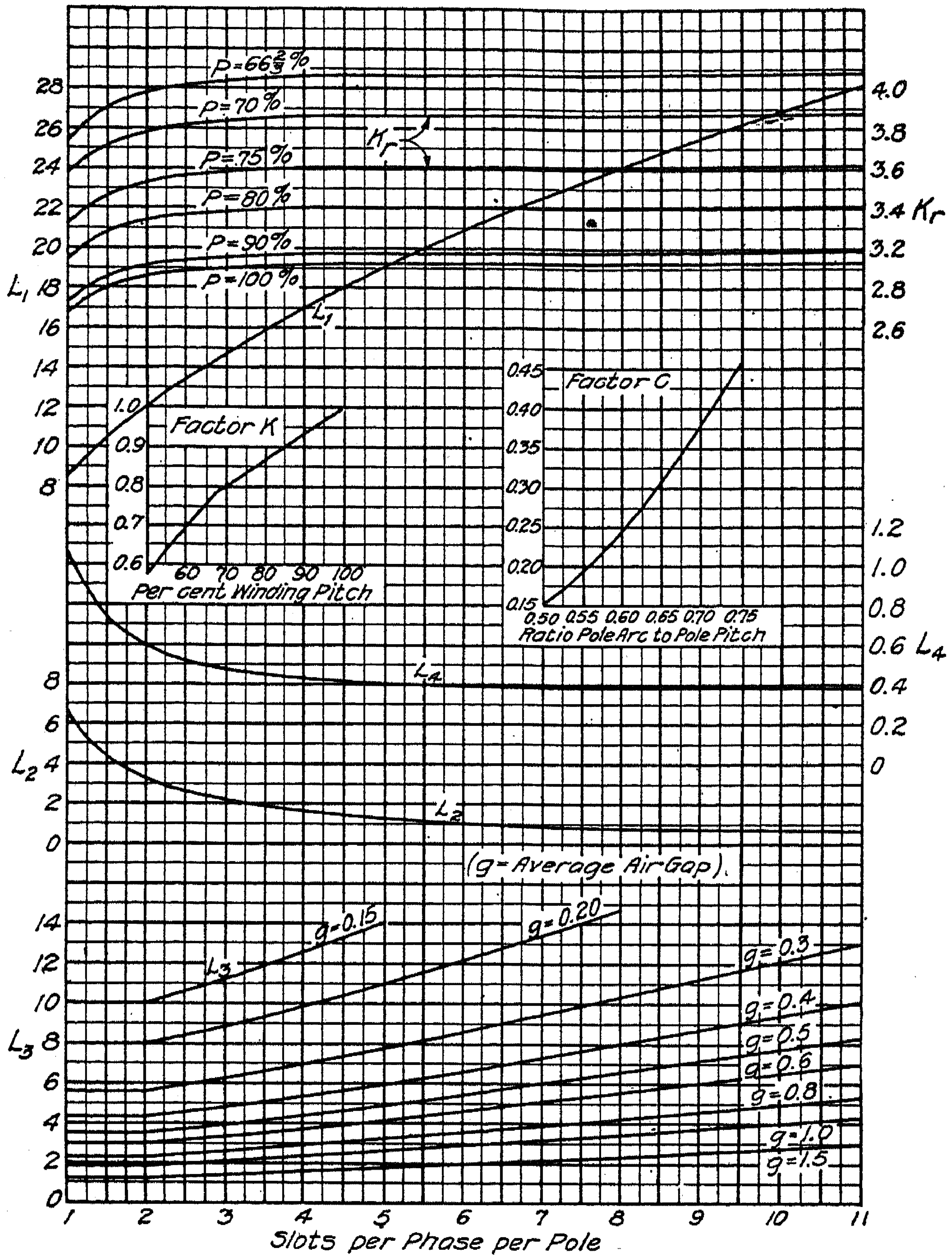

Fig. 20 b

Armature Reactance of Three-Phase Alternators

$$\text{Armature Reactance } x_{pa} = \frac{K_r\, K_\phi^2\, A\, l}{\Phi}\left[\frac{p\, L_1}{l} + K\,(L_2 + C_1\, L_3 + L_4)\right]$$

(For 20% Reactance $x_{pa} = 0.20$)

K_r, K, L_1, L_2, L_4 and C_1 are given by curves.

A = $A\ T$ armature reaction per pole.

Φ = magnetic flux per pole in maxwells.

l = gross length of armature in inches (iron + ducts).

p = arm. winding pitch (0.8 for 80 per cent)

K_ϕ = flux distribution factor (Fig. 20)

Curves for Slot Pitch 1.1 to 1.3. Two-Layer Windings.

For Single-Layer Windings multiply L_1 by 2.0 and use $K = 1.0$.

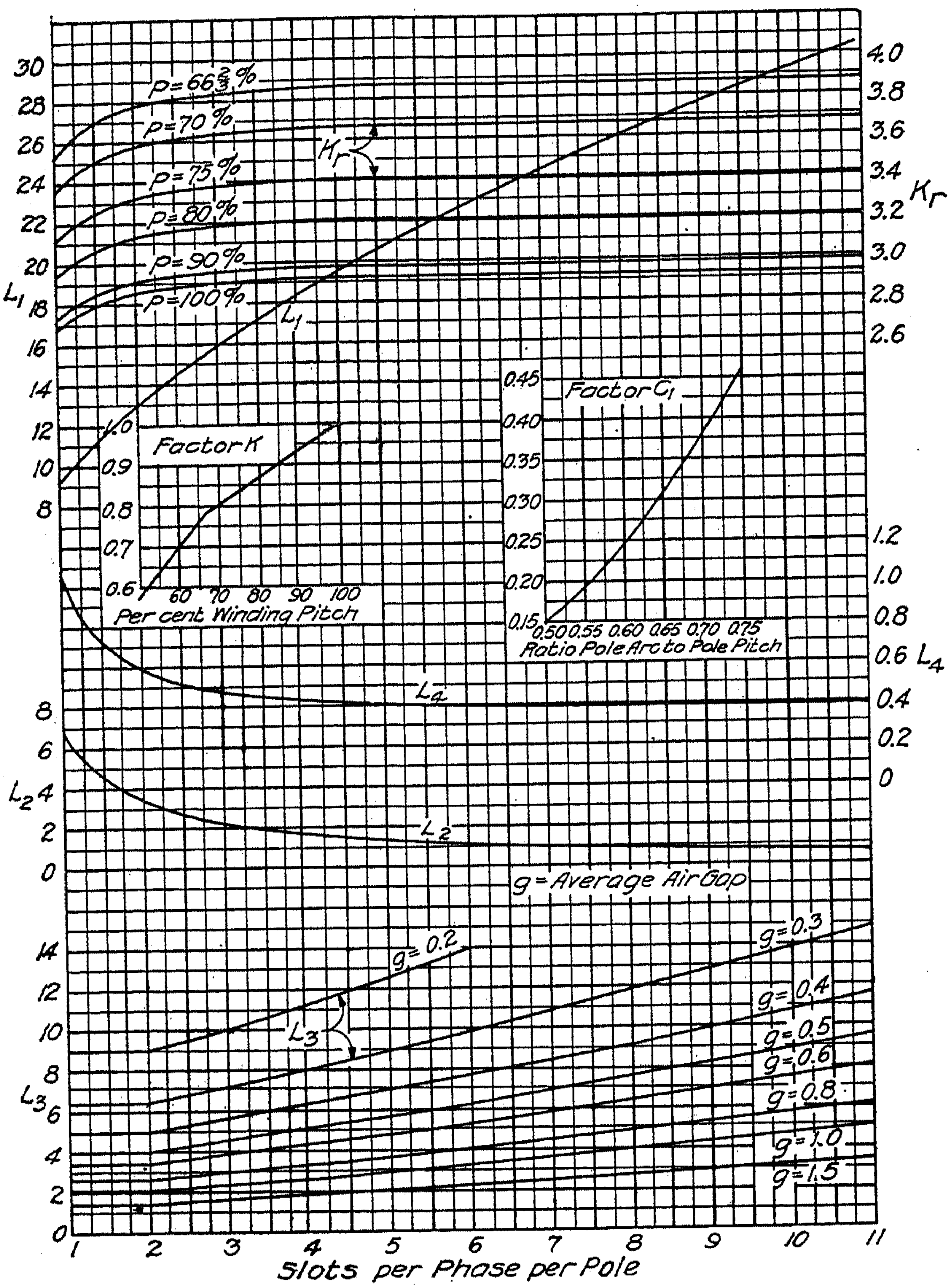

FIG. 20c

ARMATURE REACTANCE OF THREE-PHASE ALTERNATORS

Armature Reactance $$x_{pa} = \frac{K_r\, K_\phi^2\, A\, l}{\Phi}\left[\frac{p\, L_1}{l} + K\,(L_2 + C_1\, L_3 + L_4)\right]$$

(For 20% Reactance $x_{pa} = 0.20$)

K_r, K, L_1, L_2, L_3, L_4 and C_1 are given by curves.

A = $A\ T$ armature reaction per pole.

Φ = magnetic flux per pole in maxwells.

l = gross length of armature in inches (iron + ducts).

p = arm. winding pitch (0.8 for 80 per cent)

K_ϕ = flux distribution factor (Fig. 20)

Curves for Slot Pitch 1.3 to 1.5. Two-Layer Windings.

For Single-Layer Windings multiply L_1 by 2.0 and use $K = 1.0$.

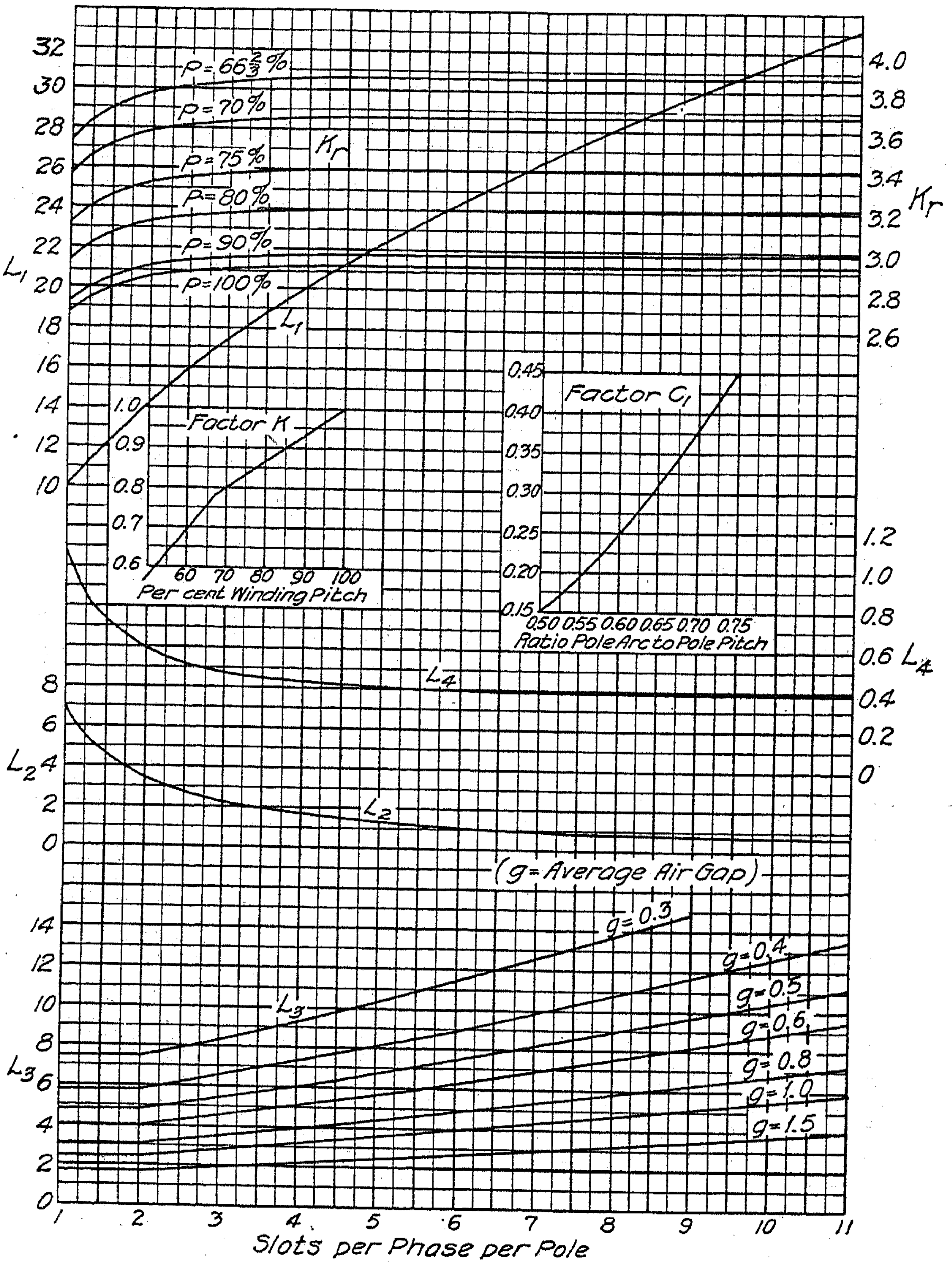

FIG. 20D

ARMATURE REACTANCE OF THREE-PHASE ALTERNATORS

Armature Reactance $x_{pa} = \dfrac{K_r\, K\phi^2\, A\, l}{\Phi}\left[\dfrac{p\, L_1}{l} + K\,(L_2 + C_1\, L_3 + L_4)\right]$

(For 20% Reactance $x_{pa} = 0.20$)

K_r, K, L_1, L_2, L_3, L_4 and C_1 are given by curves.

A = $A\ T$ armature reaction per pole.

Φ = magnetic flux per pole in maxwells.

l = gross length of armature in inches (iron + ducts).

p = arm. winding pitch (0.8 for 80 per cent)

K = flux distribution factor (Fig. 20)

Curves for Slot Pitch 1.5 to 1.9. Two-Layer Windings.

For Single-Layer Windings multiply L_1 by 2.0 and use $K = 1.0$.

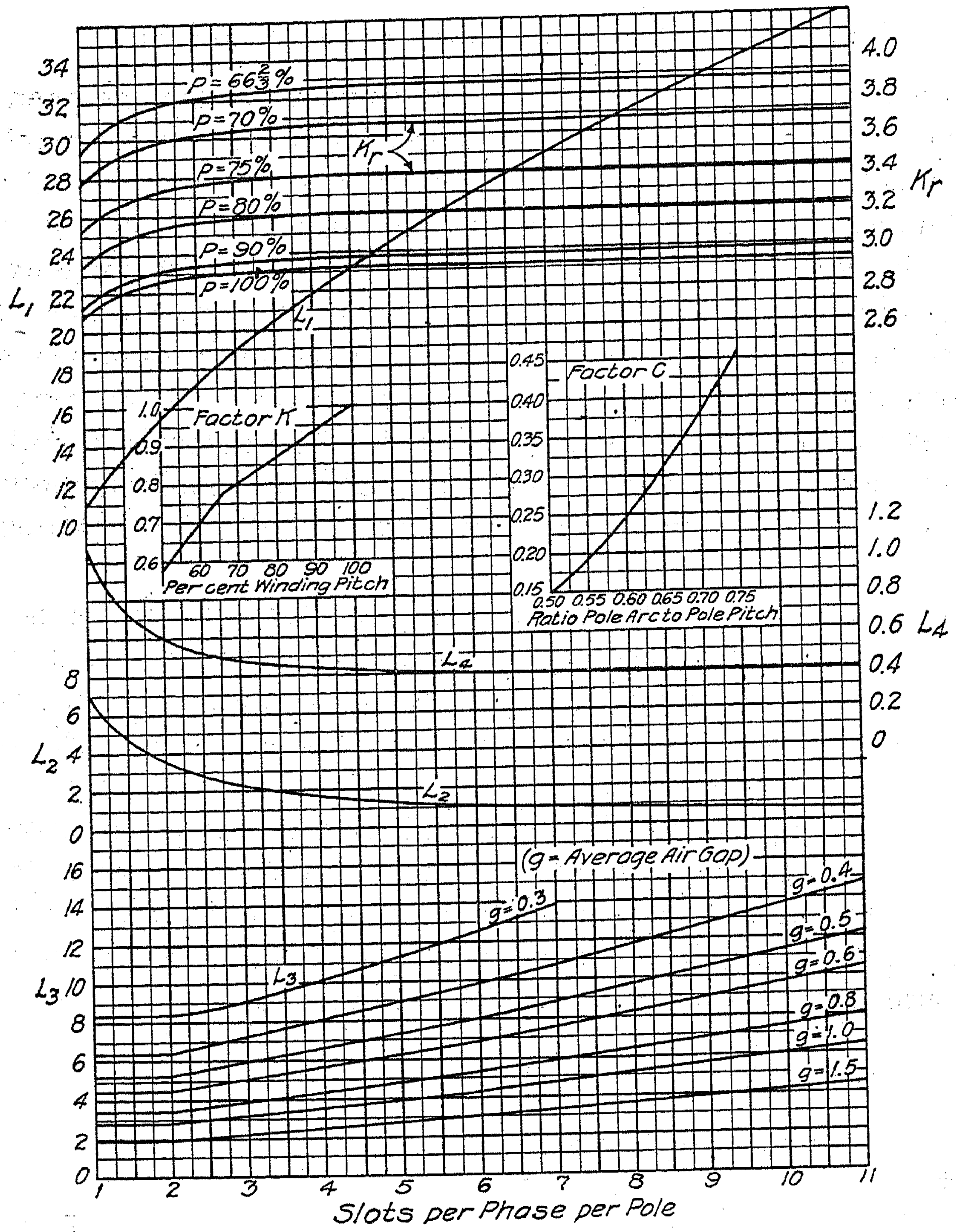

FIG. 20E

ARMATURE REACTANCE OF THREE-PHASE ALTERNATORS

$$\text{Armature Reactance } x_{pa} = \frac{K_r\, K_\phi^2\, A\, l}{\Phi}\left[\frac{p\, L_1}{l} + K\,(L_2 + C_1\, L_3 + L_4)\right]$$

(For 20% Reactance $x_{pa} = 0.20$)

K_r, K, L_1, L_2, L_3, L_4 and C_1 are given by curves.

A = $A\ T$ armature reaction per pole.

Φ = magnetic flux per pole in maxwells.

l = gross length of armature in inches (iron + ducts).

p = arm. winding pitch (0.8 for 80%)

K_ϕ = flux distribution factor (Fig. 20)

Curves for Slot Pitch 1.9 to 2.1. Two-Layer Windings.

For Single-Layer Windings multiply L_1 by 2.0 and use $K = 1.0$.

PART II

TRANSIENT REACTANCE AND SHORT CIRCUITS

A great deal of work has been done toward placing the phenomena of short circuits upon a calculable basis, but there is much yet to be done. Diamant's paper[7] reviewed historically the development of the theory underlying short circuits giving in detail the equations of Berg and Boucherot, and with some extensions in the mathematics, gave an expression from which, if all the factors were known, the entire wave of instantaneous current, from the instant of short circuit until the sustained value is approached, might be predicted. The potential usefulness of such an expression is obvious; but it is equally obvious to what extent its usefulness is restricted until all of the factors are known.

This paper offers methods by which the factors may be calculated with practical accuracy on certain classes of machines, (those with salient, laminated poles without amortisseur winding) but there are refinements yet to be made, and extensions to be worked out for other types of machines.

Diamant's equation **(26)**[7], which gives the curve of positive crests, is perhaps the most practical form for the engineer's use. In our notation and simplified for the condition of short circuit occurring at zero voltage (*i.e.* wave completely offset, which must be assumed in most engineering calculations) is,

$$i_c = \sqrt{2}\,(I_0 - I_s)\,\epsilon^{-\alpha_f t} + \sqrt{2}\,I_0\,\epsilon^{-\alpha_a t} + \sqrt{2}\,I_s$$

$$= i_1 + i_2 + \sqrt{2}\,I_s$$

where

I_0 = *alternating component* of short-circuit current $= \dfrac{E}{x_0}$

E = voltage per leg.

x_0 = total reactance effective on sudden short circuit. This should be called *transient reactance*, as suggested by Durgin and Whitehead[9] and by F. D. Newbury[10].

I_s = value of sustained short-circuit current.

α_f = field attenuation factor, defined later.

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[7] TRANS. A. I. E. E., 1915, Vol. 34, p. 2237.

[8] Term proposed by Hewlett, Mahoney and Burnham, TRANS. A. I. E. E., 1918, Vol. XXXVII, p. 128.

[9] TRANS. A. I. E. E., Vol 31, 1912, p. 1662.

[10] *Elec. Journal*, April 1914, p. 196.

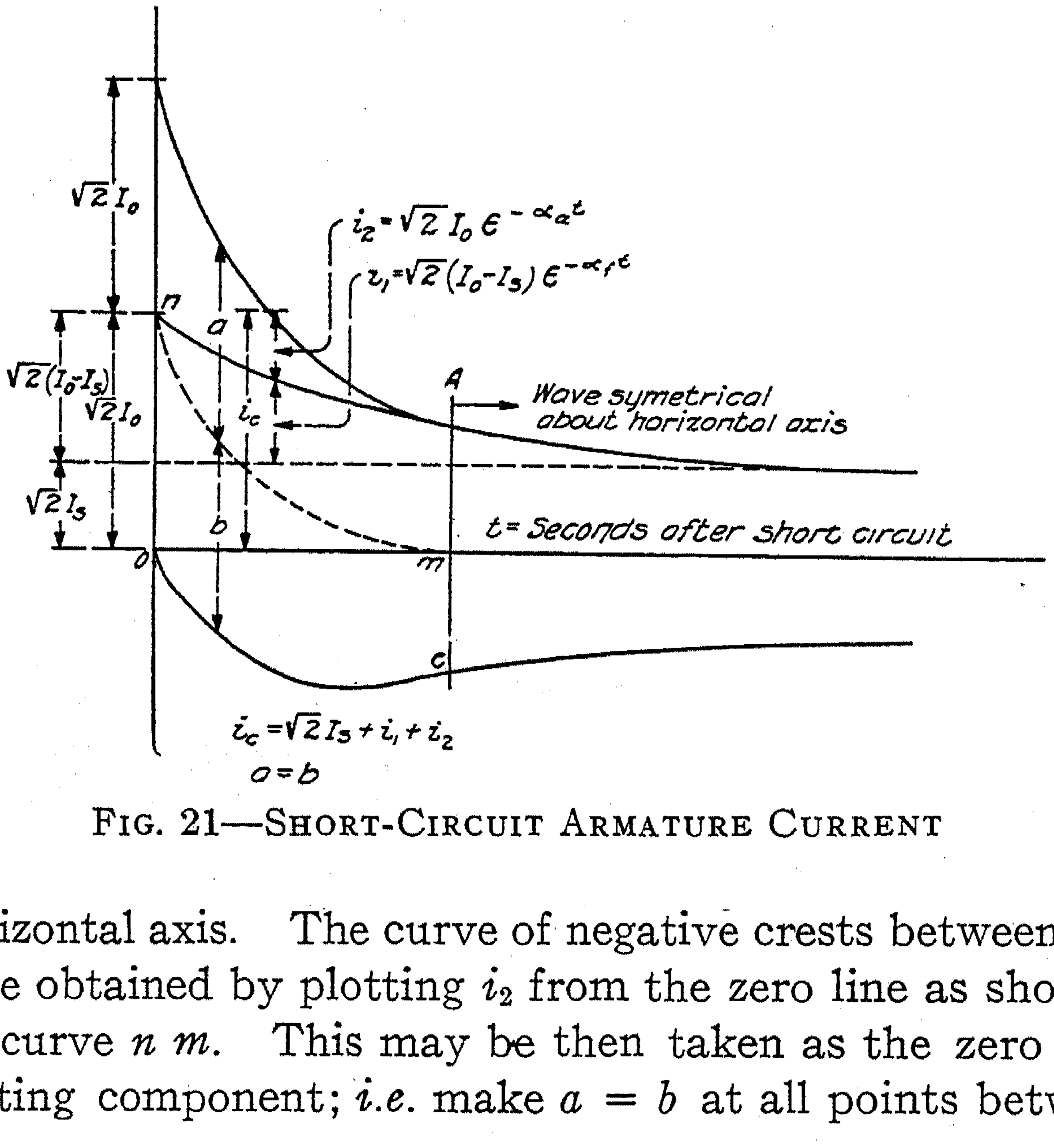
α_a = armature attenuation factor, defined later.
t = seconds after short circuit.

For calculation, a convenient form is:

1st term,
$$i_1 = \sqrt{2}\,(I_0 - I_s)\,\epsilon^{-\alpha_f t}$$
$$\log_e \frac{\sqrt{2}\,(I_0 - I_s)}{i_1} = \alpha_f\, t$$

2d term,
$$i_2 = \sqrt{2}\, I_0\, \epsilon^{-\alpha_a t}$$
$$\log_e \frac{\sqrt{2}\, I_0}{i_2} = \alpha_a\, t$$

Fig. 21 shows the skeleton thus derived. Beyond the ordinate A, when i_2 is practically zero, the wave is symmetrical about the horizontal axis. The curve of negative crests between o and c can be obtained by plotting i_2 from the zero line as shown by dotted curve $n\ m$. This may be then taken as the zero of the alternating component; *i.e.* make $a = b$ at all points between o and m.



$\sqrt{2} I_0$
$\sqrt{2}(I_0-I_s)$
$\sqrt{2} I_0$
$\sqrt{2} I_s$
$i_2 = \sqrt{2} I_0 \epsilon^{-\alpha_a t}$
$i_1 = \sqrt{2}(I_0 - I_s) \epsilon^{-\alpha_f t}$
n
a
i_c
A
Wave symetrical about horizontal axis
b
o
m
c
t = Seconds after short circuit
$i_c = \sqrt{2} I_s + i_1 + i_2$
a = b

FIG. 21—SHORT-CIRCUIT ARMATURE CURRENT

With the curve shown in Fig. 21, the three questions which the designing engineer is asked, can be more accurately answered than in the past; namely, what is the first rush of current; what is the current at, say, ¼ or ½ second after short circuit, this interval depending usually upon the operating time of the circuit breaker; and what is the sustained value.

The sustained value is easily taken from the design sheet by the well known method. The value of I_0 is determined by the transient reactance, x_0. And it will be shown later that α_f and α_a depend upon x_0 and the resistance of field and armature circuits.

In the past, the rate of decrease of the current has been predetermined by empirical curves, based upon oscillographic data, such as given by Hewlett, Mahoney and Burnham[11]. After all, for commercial engineering work, such curves will still be used; but in their preparation or in the investigation of special cases, the above equation will be very useful. But before those curves can be used, the generator transient reactance which adds directly to the external reactance, must be known.

The authors would make it clear at this point that they are not in agreement with conception given in Diamant's paper, of the factors which determine the first rush of current, and of what constitutes the factors α_f and α_a. He states that the initial value of the alternating component (crest value), which he calls $(A+B)$, "is equal to the maximum phase voltage before short circuit divided by the armature impedance, with good approximation." As brought out in the following paragraph, this assumption would lead to serious error in many cases. The matter of α_f and α_a will be discussed in subsequent pages.

Turning now to the problem in hand, the reactance which limits the initial short-circuit current in most synchronous machines is not the armature self-inductive reactance only; it is rather the combined self-inductive reactance of both the armature and field circuits. The calculated short-circuit current, based on the armature self-induction, is therefore higher, in some cases 40 to 50 per cent higher, than the actual short-circuit current. In salient, laminated pole alternators, in which the field leakage flux, that is, the self-induction, is large and in which rotor eddy currents are practically negligible, this difference is more pronounced than in the continuous rotor construction of turbine generators, in which the effective field self-induction is largely reduced by the distribution of the field winding and also by the solid steel rotor,[12] or in salient pole machines with low resistance amortisseur winding. Hence in the latter two cases, as might be expected and as experience indicates, the armature self-inductive reactance is a reasonably accurate measure of the short-circuit current; and it is proposed, for the present at least, that this basis be used for these cases. This is not the point of view taken by Durgin and Whitehead[13] in 1912. Their conclusion, based on tests of a 12,000-kw., 9000-volt, 25-cycle turbo-generator in the Fisk Street Station of the Commonwealth Edison Company, was that "the short-circuit currents of alternators are limited by reactance much more complex and much higher than the self-inductive reactance of the armature." While the authors agree, as stated above, that this is true for alternators with salient, laminated poles, they submit that it is not true for many turbo-generators, as now ordinarily constructed, or in the case of salient pole machines with low resistance amortisseur windings. Tests in Table IV, confirm this.

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[11] TRANS. A. I. E. E., 1918, Vol. XXXVII, p. 130.

[12] A. B. Field, TRANS. A. I. E. E., Vol. 31, Part 2, p. 1651.

[13] TRANS. A. I. E. E., Vol. 31, Part 2, p. 1657.

In order to establish a conception of the effect of armature and field self-induction, consider in part the phenomena which occur in an alternator under short circuit, and the fundamental principle involved. *A closed electric circuit, without resistance, must persist magnetically in the same condition as at the instant of closing; that is, must contain, so long as it is closed, the same number of magnetic interlinkages.* If the flux tends to increase or decrease, this will produce a current of sufficient magnitude to maintain the interlinkages which existed when the circuit was closed. This law determines, in a large measure, what happens under short circuit; because, although both the armature and field circuits actually contain some resistance, and the currents therefore die down in the familiar transients, Fig. 22, yet its effect during the first cycle or so after short circuit is, for qualitative considerations at least, negligible.

It follows that the flux linked with the field winding before short circuit must persist after short circuit; that likewise whatever flux is caught in the armature circuit at the instant of short circuit, must there persist; and that the flux from the rotating field poles can not now enter the armature, *i. e.*, can not change the magnetic interlinkages of the armature circuit.

Consider a simple case of closed circuit in Fig. 23. The closed-ring conductor, without resistance, is moved from position *a* to the center of the magnetic field, ϕ, in *b*. This causes a current I_0 in the ring—a current sufficient to produce around the leakage paths of the ring a flux equal to $-\phi$. This is illustrated hypothetically in *c*. But I_0 in the ring in *b* tends to demagnetize the field. This requires a corresponding increase in m.m.f. of the field coil, to maintain the flux ϕ linked with the coil. The result, as shown on *b*, is to force the flux around the ring.

Suppose the ring is closed around the flux as shown in *d*. Then if the ring is moved to position in *f*, the result will be as shown: the magnetic flux ϕ will remain linked with the field

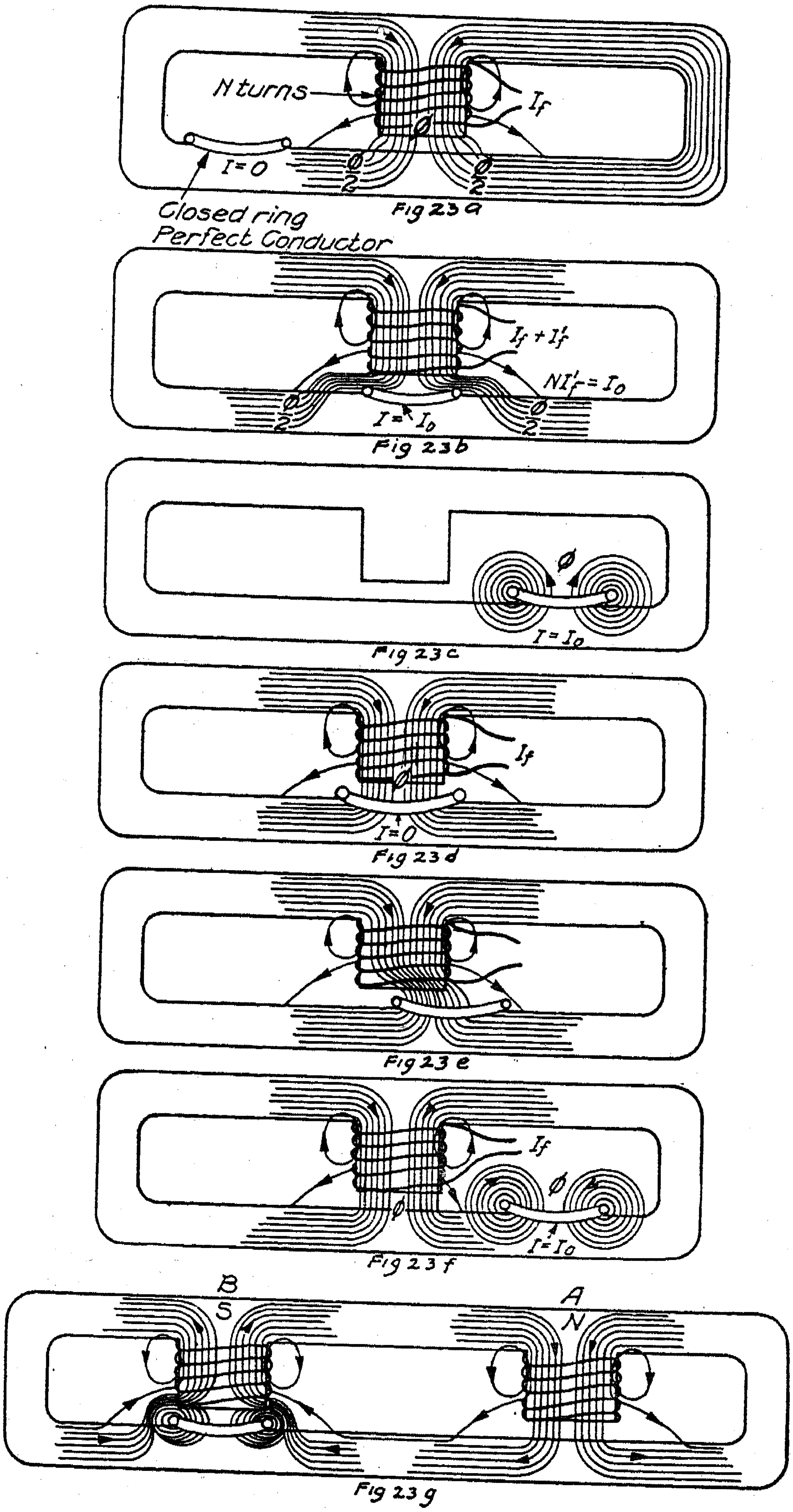


Fig. 23—Diagrammatic Representation of Short-Circuit Fluxes

coil, as in *d*, and in addition, another magnetic field of equal value will be maintained around the ring by a current I_0, the same value, (assuming equal reluctance of leakage path),

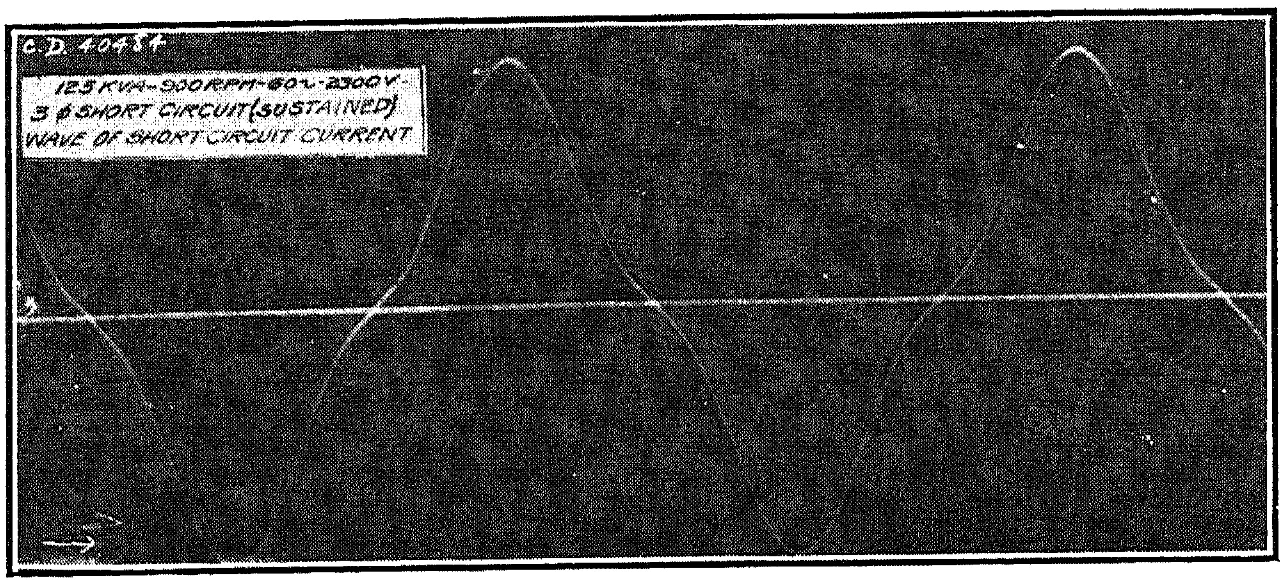


FIG. 4

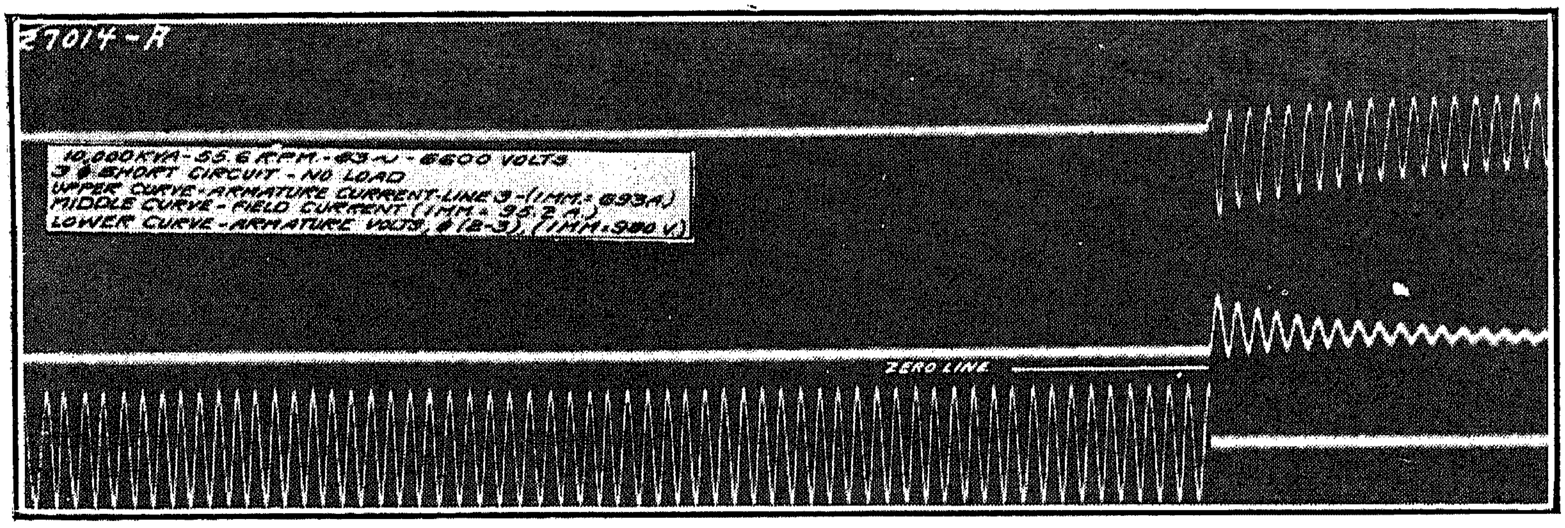
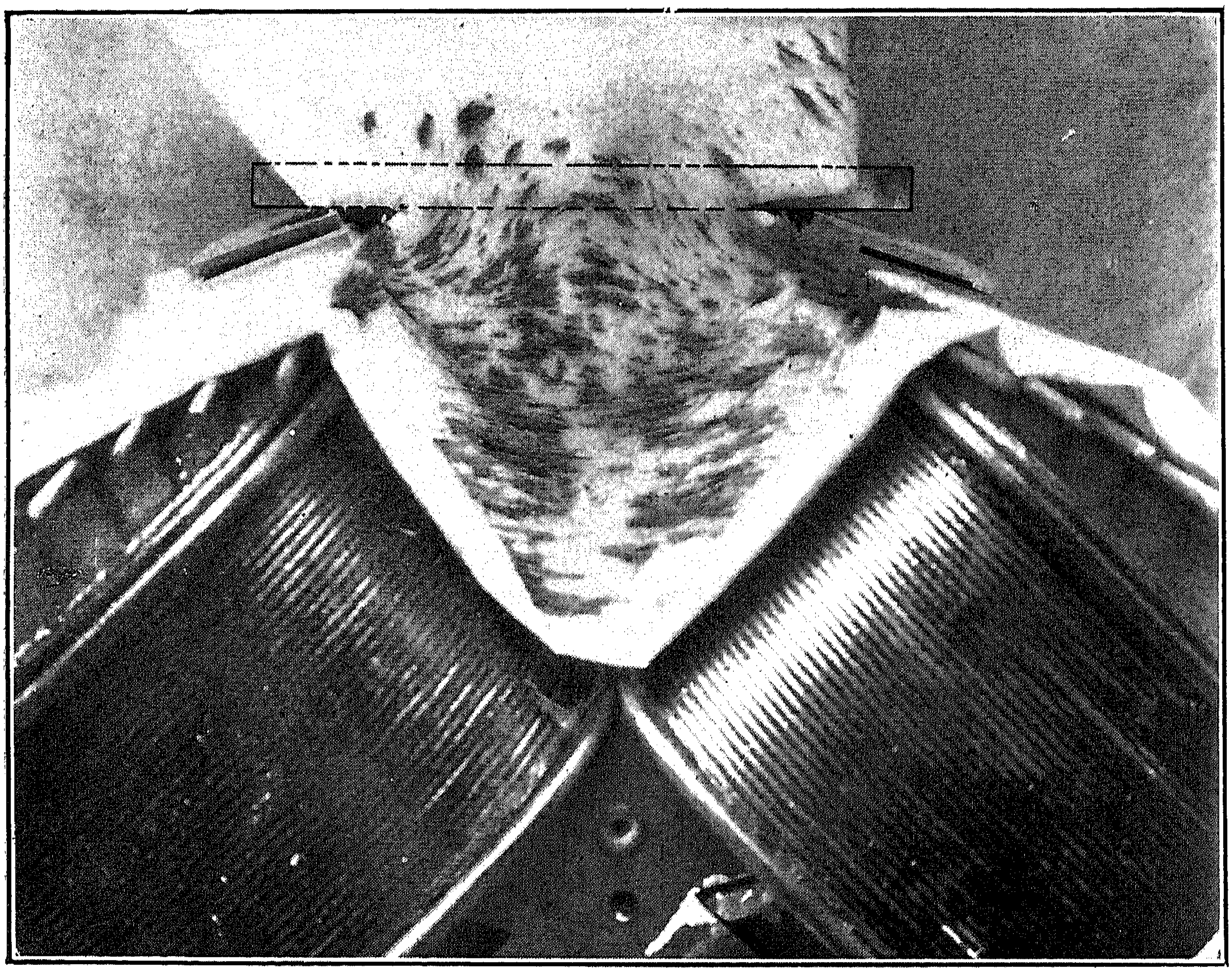


FIG. 22



[DOHERTY AND SHIRLEY]

FIG. 25B.—ACTUAL MAGNETIC DISTRIBUTION OF A FOUR-POLE FIELD—DERIVED DISTRIBUTION SHOWN IN FIG. 25A

which was necessary in b. Force is required to move the ring from d to f, as shown by the distortion in e. The work done in moving the ring is the energy stored in the new magnetic field around the ring in f. If the ring containing zero flux, as in a, were moved along in g, the current would rise to I_0 when under A, decrease to zero between A and B, and rise to $-I_0$ under B; in other words, an alternating current would be produced. If, however, the ring, enclosing the flux ϕ and containing the direct current I_0, as in f, were moved as above, the current would decrease to zero under A, rise again to I_0 between A and B, and reach $2I_0$ under B. In this case, the resultant current, varying from zero to $2I_0$ is made up of an *alternating component* and a *direct component*, the latter being equal to the maximum of the former.

The events occurring in an alternator immediately after a no-load single-phase short circuit have thus been illustrated. The short circuit at maximum voltage is represented by the ring enclosing zero flux, the short circuit at zero voltage, by the ring enclosing the flux ϕ. Fig. 23e illustrates how a large alternating torque is produced when the short circuit occurs at zero voltage.

A three-phase short circuit at no load is different from the single-phase short circuit in this important respect, that it is impossible, three-phase, to have the short circuit occur when zero flux is enclosed in the short-circuited armature winding. The entire flux is at all times enclosed by some parts of the armature winding, and therefore in whatever position the field structure happens to place the flux when the short circuit occurs, there it must remain, a series of polar regions, stationary in space, while each field pole with its equal value of flux moves on These stationary polar regions of normal intensity, of course generate in the rotating field winding an alternating current of normal frequency, and of a maximum value in m.m.f. equal to the direct current which maintains the stator polar regions. That is, the flux, once intact through the entire normal magnetic path is now literally torn apart at the air gap, and at 180 electrical degrees from the instant of short circuit, forms two separate magnetic circuits, closing through the leakage paths between armature and field windings. At the end of one cycle the flux is again intact through the normal path. All the work which, during the first half cycle, was done on the field in tearing it apart, in actually establishing normal interlinkages in circuits of much lower permeance than that of the normal circuit, is returned to the rotor during the last half cycle. The torque which accompanies this transfer of energy has been calculated by S. H. Weaver, *G. E. Review*, Nov., 1915.

Return to the question of the self-inductive reactance existing at sudden short circuit. It has been generally understood how the armature self-induction limits the initial short-circuit current; that is, how, if a machine had 10 per cent armature self-induction, normal armature current would cause flux through the armature leakage paths equal to 10 per cent of normal flux, and that therefore on sudden short circuit, when (neglecting field self-induction) the total normal flux must be absorbed in the leakage paths, the armature current would have to rise to

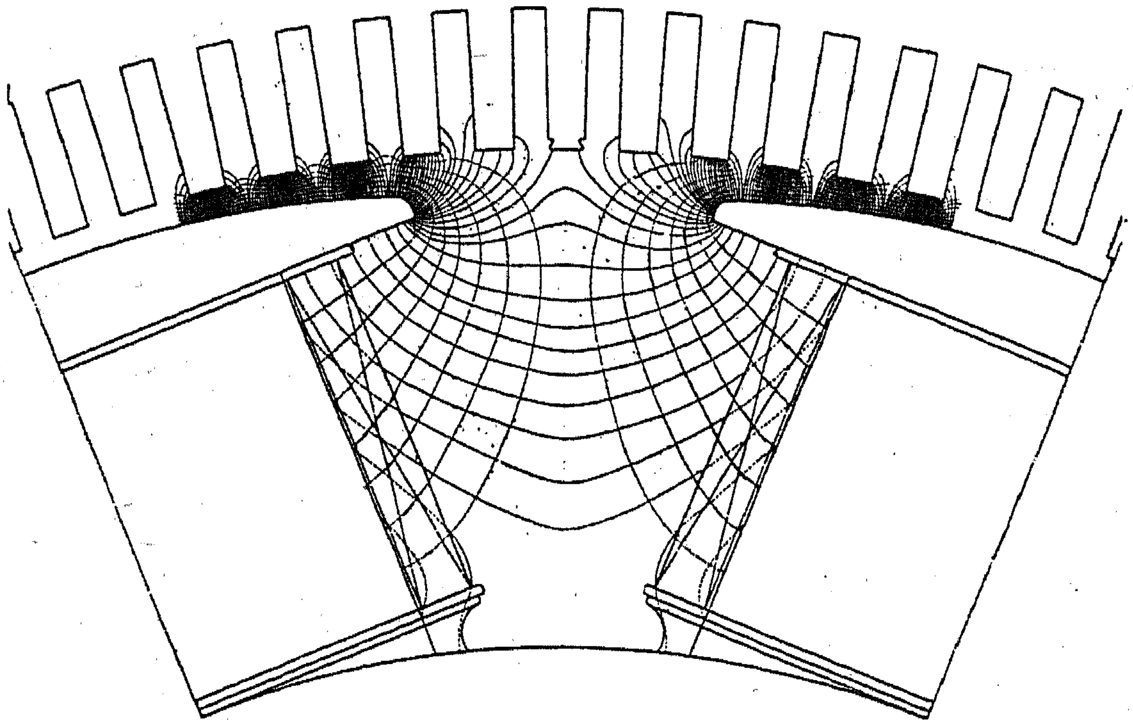


FIG. 24A—DERIVED DIAGRAM OF MAGNETIC DISTRIBUTION OF SALIENT-POLE MACHINE

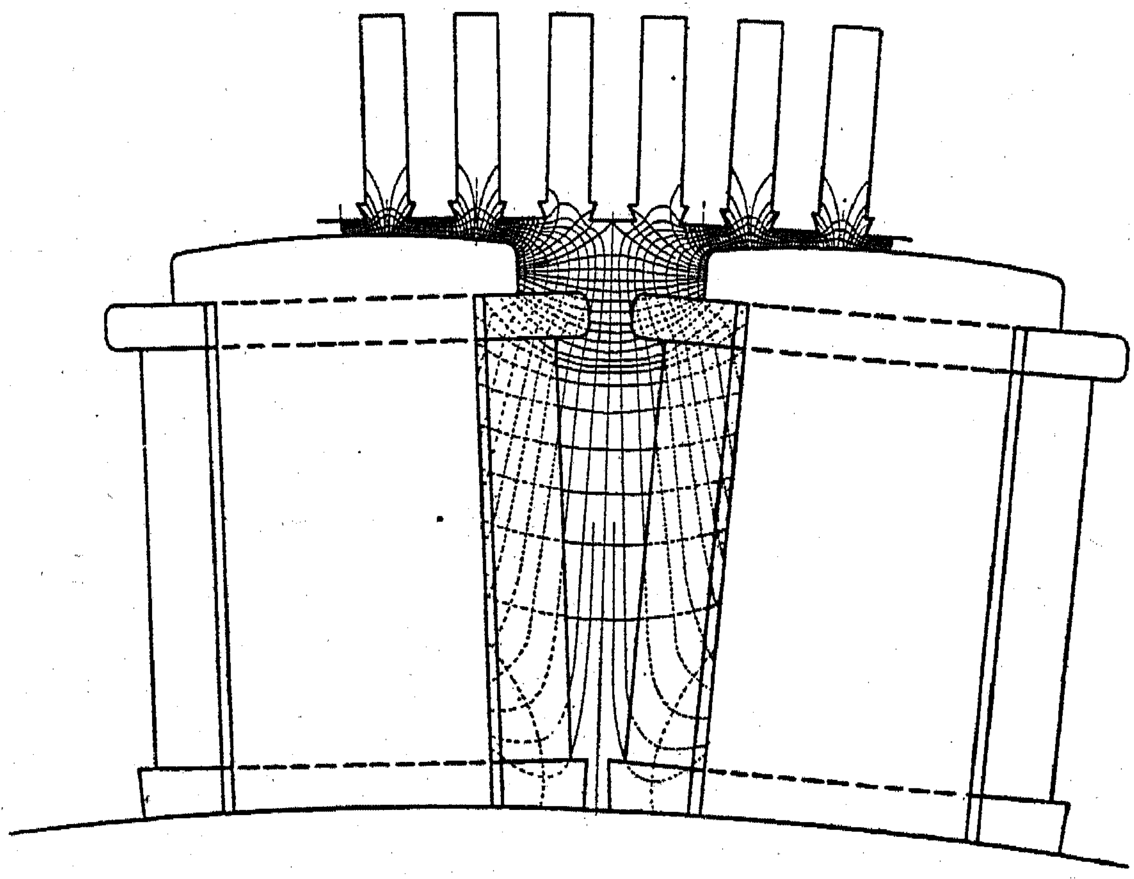


FIG. 24B—DERIVED DIAGRAM OF MAGNETIC DISTRIBUTION OF SALIENT-POLE MACHINE

ten times normal. It may not have been so clear how and to what extent the field self-induction affects the short-circuit current. From foregoing considerations and from an approximate knowledge of the field leakage paths, it is possible to determine this point. The leakage paths have been approximately determined by plotting equi-magnetic potential lines as shown in Figs. 24A and 24B. The flux must be at right angles to these lines. The entire flux emanating from field poles of different proportions is thus illustrated in these diagrams. Figs. 25A and 25B show the graphically determined and actual magnetic distribution of a 4-pole rotor with an iron plate bridging the poles, wooden blocks supporting the ends and forming an air gap. It will be noted that a large portion of the leakage flux emanates from the under side of the pole tip, which, we believe has not generally been considered in leakage calculations. In Fig. 25B the paths as indicated by the iron filings can not be traced to the pole body and to all of the under surface of the pole tip, on account of the field winding. But it is clear from the direction of the paths at the edge of the field winding that they will terminate approximately as shown in Fig. 25A. These figures of course do not show quite what the distribution would be if the rotor were placed in the stator. Then, both the derived and actual distribution would be different, but not greatly different. This experiment was made to confirm the method. Fig. 26 shows the distribution on a machine with a large number of poles.

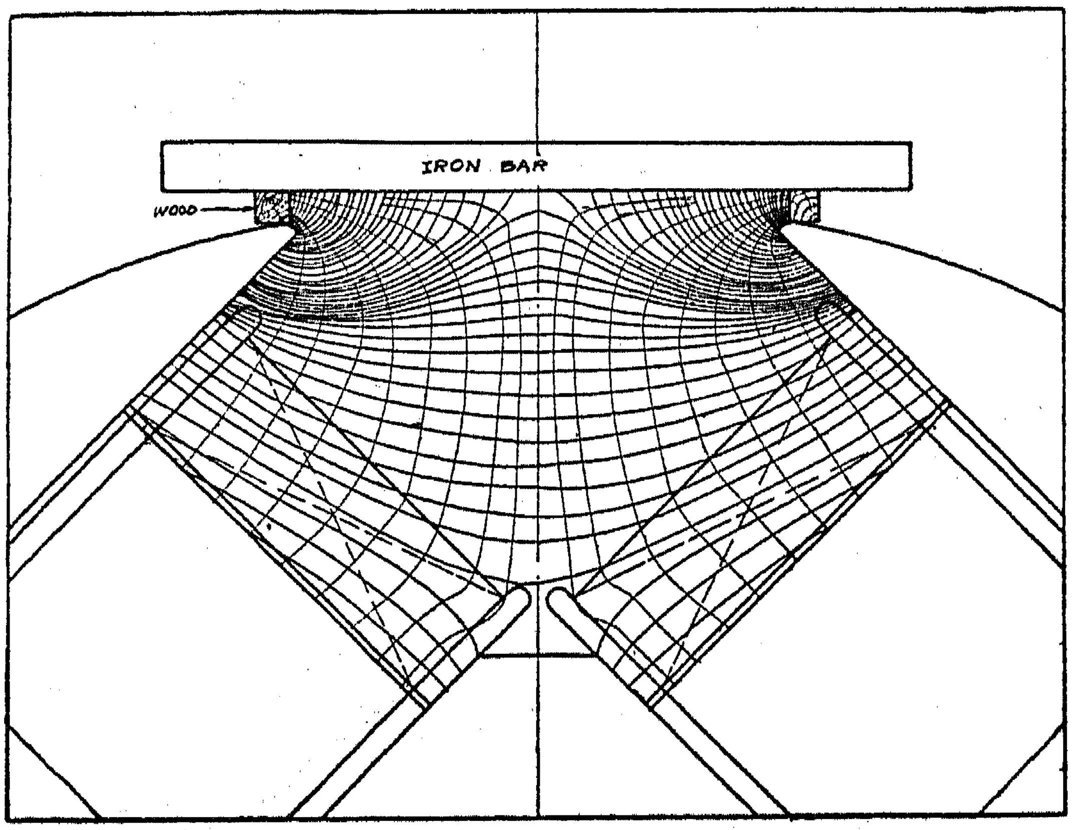


FIG. 25A—DERIVED DIAGRAMS OF MAGNETIC DISTRIBUTION OF THE FOUR-POLE FIELD SHOWN IN FIG. 25B

Fig. 27A represents the distribution of flux at no load on a three-phase generator. *A*, *B* and *C* represent the different phase belts. Fig. 27B represents the same machine with open field circuit, the field poles rotating synchronously in position shown, and with a three-phase current in the armature. The entire magnetic flux shown in Fig. 27B represents the synchronous reactance. All magnetic lines in Fig. 27B which do not link with the field winding represent the armature self-induction, or armature reactance. Likewise in Fig. 27A, that part of the field flux which does not link with the armature winding represents field self-induction, or field reactance. Just

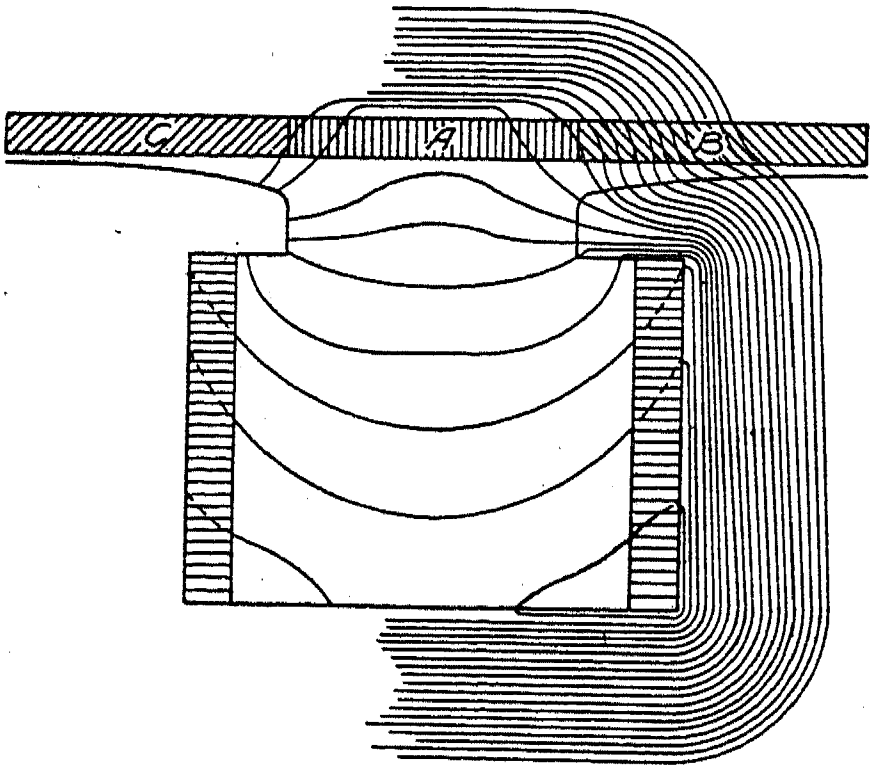


Fig. 27A—Diagram of Magnetic Field at No-Load Excitation on Field

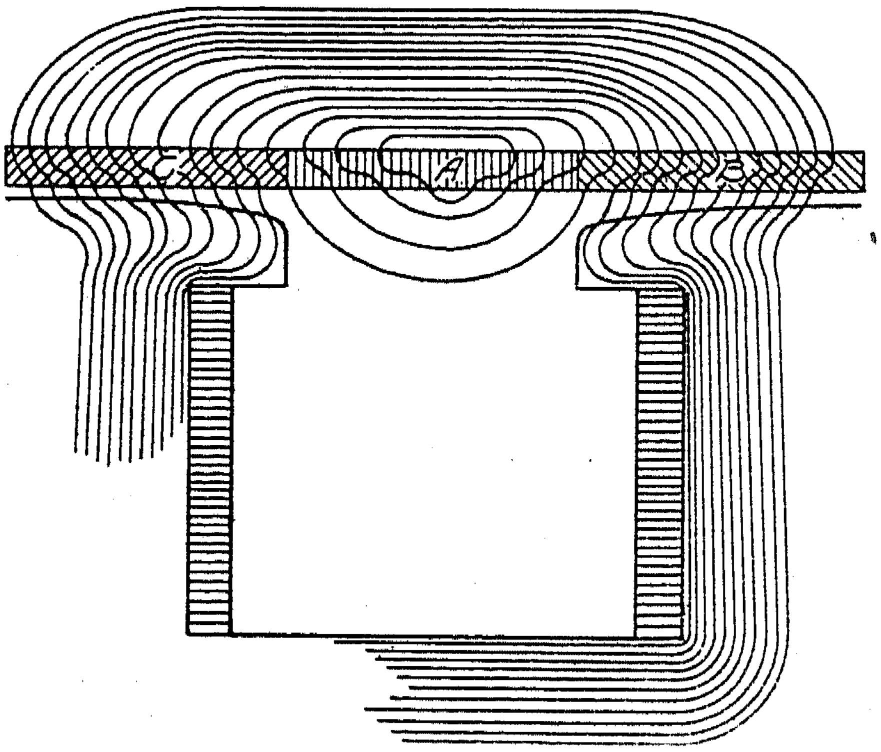
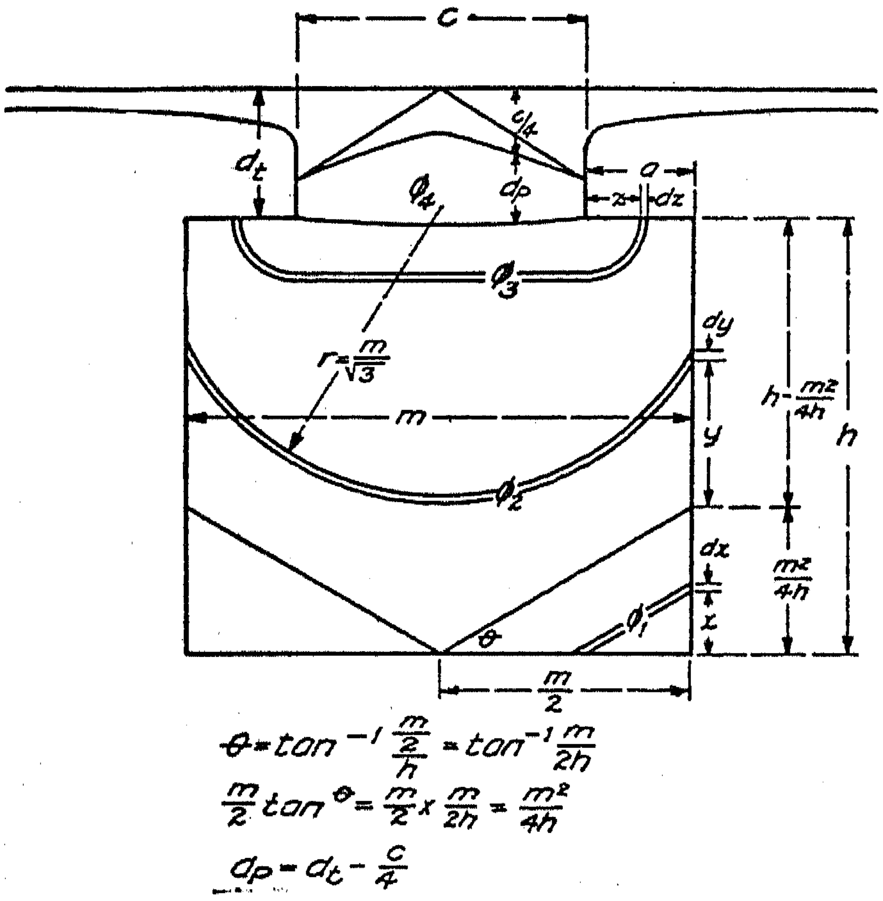


Fig. 27B—Diagram of Magnetic Field at No-Load—Three-Phase Excitation on Armature Field Revolving at Synchronous Speed

as on sustained short circuit, the field current causes the armature current to flow, and must therefore be greater in m.m.f. than the armature current, by the armature self-induction, that is greater in the ratio,

$$\frac{\text{synchronous reactance } AT}{\text{effective armature reaction}} = \frac{\text{total flux linked with armature, Fig. 27B}}{\text{mutual flux, Fig. 27B}}$$

so also on sudden short circuit, when the armature current causes the rise in field current, the armature m.m.f. must be



c
$c/4$
d_t
d_p
a
z
dz
ϕ_4
ϕ_3
dy
$r = \frac{m}{\sqrt{3}}$
m
$h - \frac{m^2}{4h}$
h
y
ϕ_2
dx
$\frac{m^2}{4h}$
x
ϕ_1
θ
$\frac{m}{2}$
$\theta = \tan^{-1}\frac{\frac{m}{2}}{h} = \tan^{-1}\frac{m}{2h}$
$\frac{m}{2}\tan\theta = \frac{m}{2} \times \frac{m}{2h} = \frac{m^2}{4h}$
$d_p = d_t - \frac{c}{4}$

FIG. 28—DIAGRAM FOR CALCULATING FIELD LEAKAGE

greater than the rise in field m.m.f. by the field self-induction, that is greater in the ratio,

$$\frac{\text{total field winding interlinkages, Fig. 27A}}{\text{mutual flux interlinkages, Fig. 27A}}$$

For convenience in developing the point, assume that all of the field leakage is concentrated between pole tips shown as ϕ_4 in Fig. 28. On sudden short circuit, since the normal flux which is linked with the field winding and which before short circuit entered the armature, can now neither enter the armature nor die out through the field winding, it must be forced through the leakage paths. At the instant at which the armature current in its rise at short circuit passes through normal value, the accompanying rise in field m.m.f. will be approximately equal (actually less) to the effective armature reaction, A, corresponding to normal current. This increase in field m.m.f. must cause a corresponding increase in ϕ_4, Fig. 28; but since ϕ_4 links with the field circuit, and the field interlinkages can not increase, it follows that the increase in ϕ_4 represents the portion of the normal flux Φ, which is absorbed on short circuit by the field self-induction when the field and armature m.m.f's. have risen to A. As an illustration, let the flux ϕ_4 corresponding to a field m.m.f., A, be 4 per cent, and let the armature self-induction be 10 per cent. Then as the armature current passes through normal value, 14 per cent of normal flux Φ, will have been absorbed in the leakage paths; and the armature current will rise to 7.15 times normal value.

But ϕ_4 does not, as above assumed, represent all of the field leakage. Other paths of ϕ_1, ϕ_2, ϕ_3, etc., Fig. 28, exist. It is not so obvious how this additional leakage from the sides and ends of the pole, behaves, and how it operates to limit the short-circuit current. It would appear that none of the normal flux, Φ, could sink into leakage paths of ϕ_1, and ϕ_2, since in doing so it would have to cut through the closed field winding; and also, and for the same reason, that the leakage ϕ_1 and ϕ_2 could not increase. Actually, both happen. The condition which must be fulfilled is that *the total effective interlinkages of the field circuit shall not change.* Two lines linking one half of the field turns is equivalent to one line linking all of the turns; four lines linking one fourth of the turns is equivalent to one line linking all, etc., etc. What actually happens, therefore, is that as the short-circuit current rises, some of the normal flux, Φ, sinks into the paths of ϕ_1 and ϕ_2; *and to just the extent to which interlinkages are decreased in this process at the top of the winding, new flux, compensating for the loss, appears at the bottom of the winding.* In machines which have rotors entirely laminated, this increase in flux may be 30 to 40 per cent. So that the total field leakage flux, Φ_l, existing under sudden short circuit, is the same (or nearly so) as would exist on open circuit with the same field m.m.f.; and the effective interlinkages which Φ_l represents is the portion of the normal flux Φ, which would be absorbed in the total field self-induction on short circuit.

Let ϕ'_{ln} be the effective field leakage interlinkages per unit length of machine, corresponding to a m.m.f. on the field equal to normal armature reaction, A. Then, *if* a rise to normal current in the armature would cause a rise in field m.m.f. equal to A, the ratio

$$x'_{pf} = \frac{l\phi'_{ln}}{\Phi} \tag{1}$$

would represent the per cent field self-induction. Actually the rise in field m.m.f. corresponding to normal armature current is less than A, as already pointed out, by the ratio

$$\frac{\text{mutual flux interlinkages,} \quad \text{Fig. 27A}}{\text{total field winding interlinkages,} \quad \text{Fig. 27A}}$$

$$= \frac{\Phi}{\Phi + \frac{F}{A}\, l\, \phi'_{ln}} = \frac{1}{1 + \frac{F}{A}\,\frac{l\,\phi'_{ln}}{\Phi}} = \frac{1}{1 + \frac{F}{A} x'_{p_f}} \tag{2}$$

where

F = no-load field m.m.f.

l = length of machine

It is still further reduced by the field self-induction in another way: the portion of normal flux which at short circuit is absorbed in the field leakage paths no longer requires, as it did before short circuit, a corresponding portion of the no-load field m.m.f. F to maintain it across the air gap. But all of F remains at short circuit. Hence the difference between F and the portion of F which is actually needed for the flux which is not absorbed in field leakage paths, is balanced by a correspondingly decreased rise in field m.m.f. Hence, a rise in armature m.m.f. equal to A, produces a rise in field m.m.f. equal to

$$F_a = \frac{A}{1 + \frac{F}{A}\, x'_{p_f}} - x_{p_f} F \tag{3}$$

Where X_{pf} represents the per cent actual field self-induction or per cent field reactance. This is

$$x_{pf} = \frac{F_a}{A}\, x'_{p_f} \tag{4}$$

But

$$x'_{p_f} = \frac{l\,\phi'_{ln}}{\Phi}$$

and

$$\phi'_{ln} = A\,L \quad \text{(from equation 35)}$$

Hence

$$x'_{p_f} = \frac{A\,l\,L}{\Phi} \tag{5}$$

Substituting (**3**) and (**5**) in (**4**) and simplifying

$$x_{p_f} = \frac{A}{F} \frac{\frac{F\,l\,L}{\Phi}}{\left(1 + \frac{F\,l\,L}{\Phi}\right)^2} \tag{6}$$

Where

A = effective polyphase armature reaction AT corresponding to normal current I_n

F = no-load field AT

Φ = normal flux per pole (in maxwells) entering the armature

$$L = \left[1.42 \frac{h}{m} + 0.16 \left(\frac{m}{h}\right)^5 + 1.6 \log \left(1 + \pi \frac{a}{c}\right) + 5.0 \frac{d_p}{c} \left\{ 1 + 0.62 \frac{c}{l} \log \left(1 + \frac{\pi\, b}{4\, c}\right) \right\} + 0.55 \frac{d}{l} \right]$$

If length is in inches, the expression for L becomes

$$L = \left[3.6 \frac{h}{m} + 0.04 \left(\frac{m}{h}\right)^5 + 4 \log \left(1 + \pi \frac{a}{c}\right) + 12.8 \frac{d_p}{c} \left\{ 1 + 0.62 \frac{c}{l} \log \left(1 + \frac{\pi}{4} \frac{b}{c}\right) \right\} + 1.4 \frac{d}{l} \right]$$

a, b, c, d, h, m, d_p, see Fig. 28 and Fig. 34

m = distance between pole cores at $h \div 2$

l = gross stacked length of machine (core + ducts)

d = width of pole core

L = applies for all values of $\frac{h}{m}$ above 0.5, and is the effective leakage lines per unit length of machine, linking the entire field winding when the field m.m.f. is one ampere turn. The expression for L is derived in the following pages.

The terms $4 \log \left(1 + \pi \frac{a}{c}\right)$ and $0.62 \log \left(1 + \frac{\pi}{4} \frac{b}{c}\right)$ are plotted in Fig. 29.

The total per cent reactance therefore which determines the initial short-circuit current (that is, the transient reactance) is given by

$$x_{p_0} = x_{p_a} + x_{p_f} \tag{7}$$

where x_{p_a} = per cent armature self-inductive reactance (or armature reactance) calculated by equation (**47**) Part I.

x_{p_f} = per cent field self-inductive reactance (or field reactance) calculated by equation (**6**) Part II

(x_{p_0} = 0.2, means 20 per cent reactance)

Hence the effective value of the alternating component of the initial short-circuit current is

$$I_0 \quad \frac{I_n}{x_{p_0}} \tag{8}$$

where I_m = normal current.

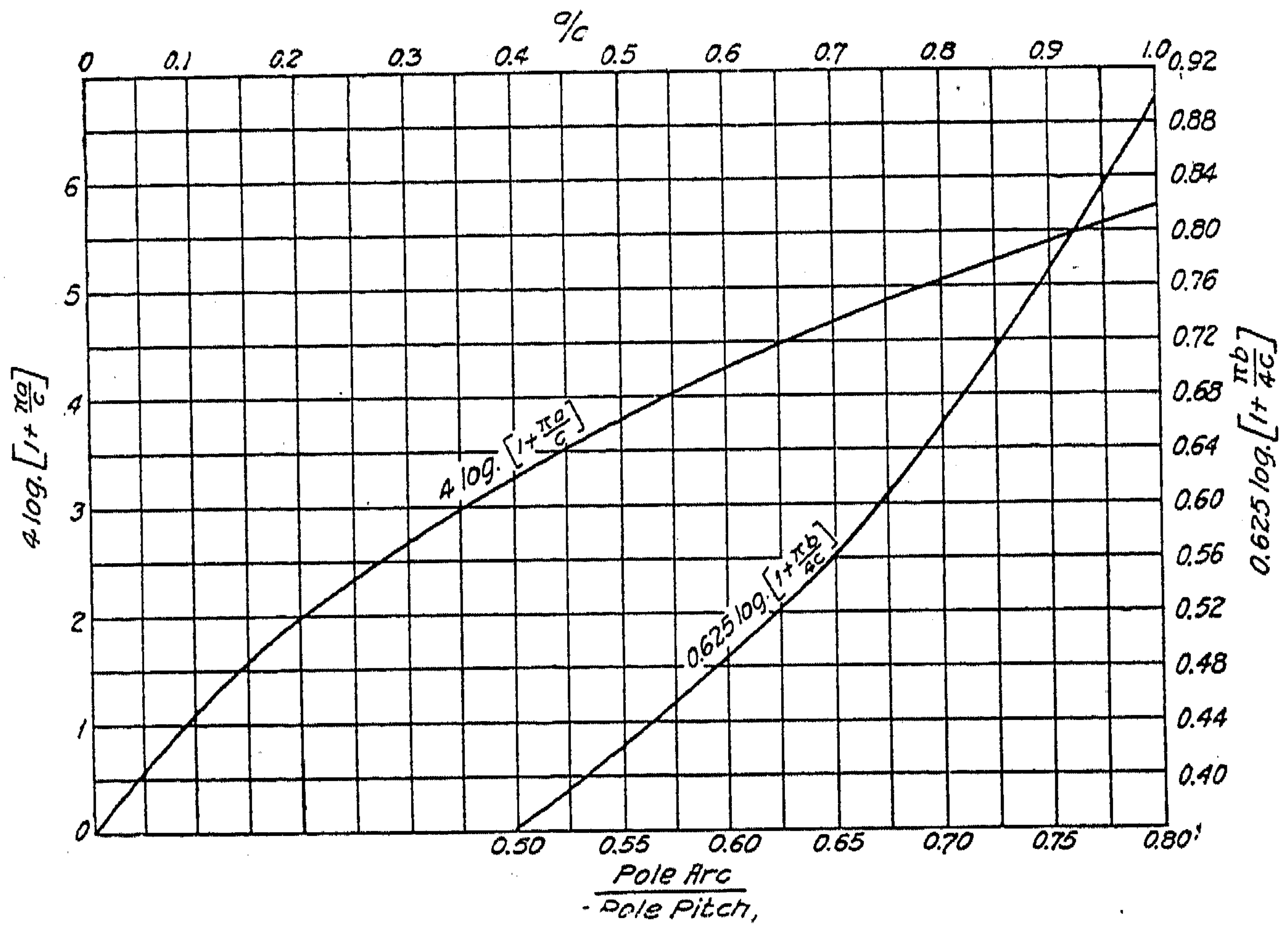


FIG. 29

The above discussion has considered only the alternating component of the initial short-circuit current. Actually a direct component of armature current, approximately equal in m.m.f. to the alternating component of armature current must always exist on a three-phase short circuit, and *may* exist on a single-phase short circuit, as already mentioned. That is at 180 degrees after short circuit, it is necessary, in such a case, for twice normal flux to exist across the leakage paths of the armature and field. It is obvious that this additional flux of normal value would be absorbed in the leakage paths in the same manner as above described.

Assuming a sine distribution of both the field flux Fig. 27A and of the armature flux Fig. 27B, the direct component of current would be equal to the crest of the alternating component; that is

$$I_{dc} = \sqrt{2}\, I_0 \tag{9}$$

The distribution is, however, usually not sinusoidal. Since the effect of the important harmonics on short-circuit current (alternating component) is practically eliminated, as discussed in Part I, it follows that the magnitude of the alternating component is determined by the fundamental of the flux wave. The direct component, however, is determined by the total flux, that is by the entire flux which the armature encloses, and may therefore be greater or less than $\sqrt{2}\, I_0$ by a small amount. Fig. 30 and Fig. 31 show both cases. In Fig. 30 the direct component is greater than the maximum of the alternating component; in Fig. 31, less.

Fig. 30 also shows the third harmonic voltage, terminal to neutral, existing on short circuit.

The formula for x_{p_0} applies to polyphase short circuits on salient laminated pole machines without amortisseur windings. (The effect of amortisseur windings is discussed later). It will apply to single-phase short circuit, terminal to neutral, if: (1) A, in the formula for x_{pf} is taken as the maximum value of the pulsating armature m.m.f. of normal current, *i.e.*

$$A_1 = \sqrt{2}\, K'\, N_q\, I_n \tag{10}$$

where

N_q = series turns per phase (*i.e.* per leg) per pole.
I_n = normal current per turn
K' = factor accounting for short coil pitch, coil distribution, and flux distribution. (2) the armature reactance is taken equal to two thirds of the three-phase value: since as pointed out in Part I, three-phase reactance is approximately 50 per cent higher than the single-phase. That is, for single-phase short circuit, terminal to neutral,

$$x_{p_{01}} = \frac{2}{3}\, x_{p_a} + x_{p_{f1}} \tag{11}$$

where $x_{p_{f1}}$ is field reactance for single-phase short circuit. But since the three-phase armature reaction is

$$A_3 = 1.5\, \sqrt{2}\, N_q\, I_n\, K' \tag{12}$$

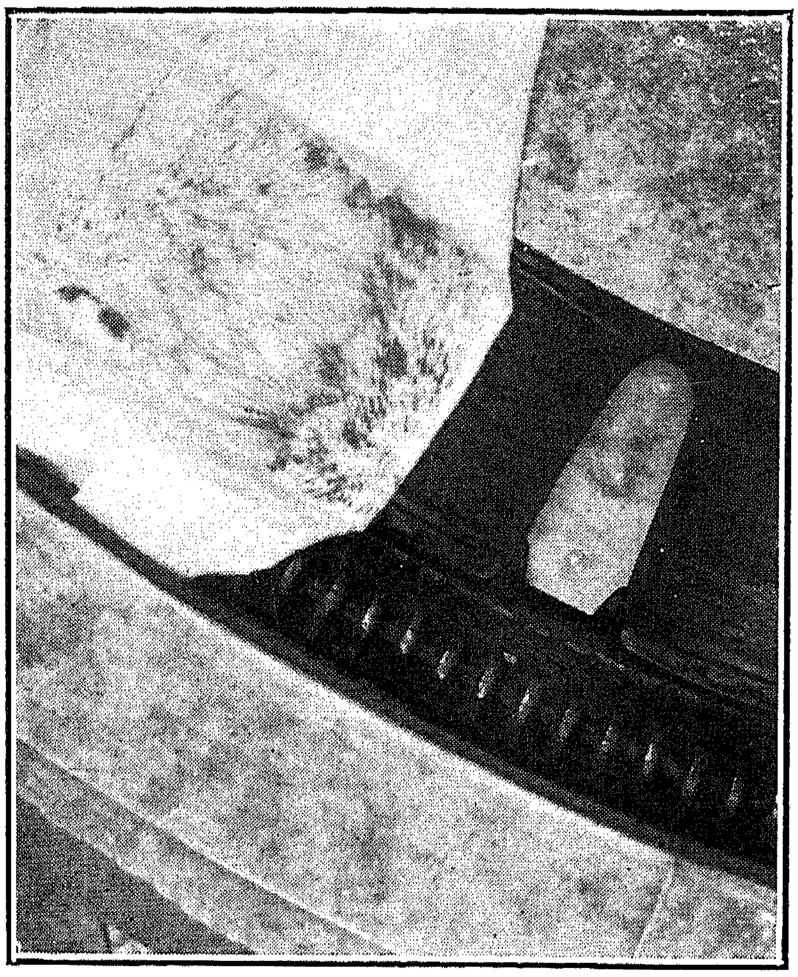


FIG. 26—ACTUAL MAGNETIC DISTRIBUTION

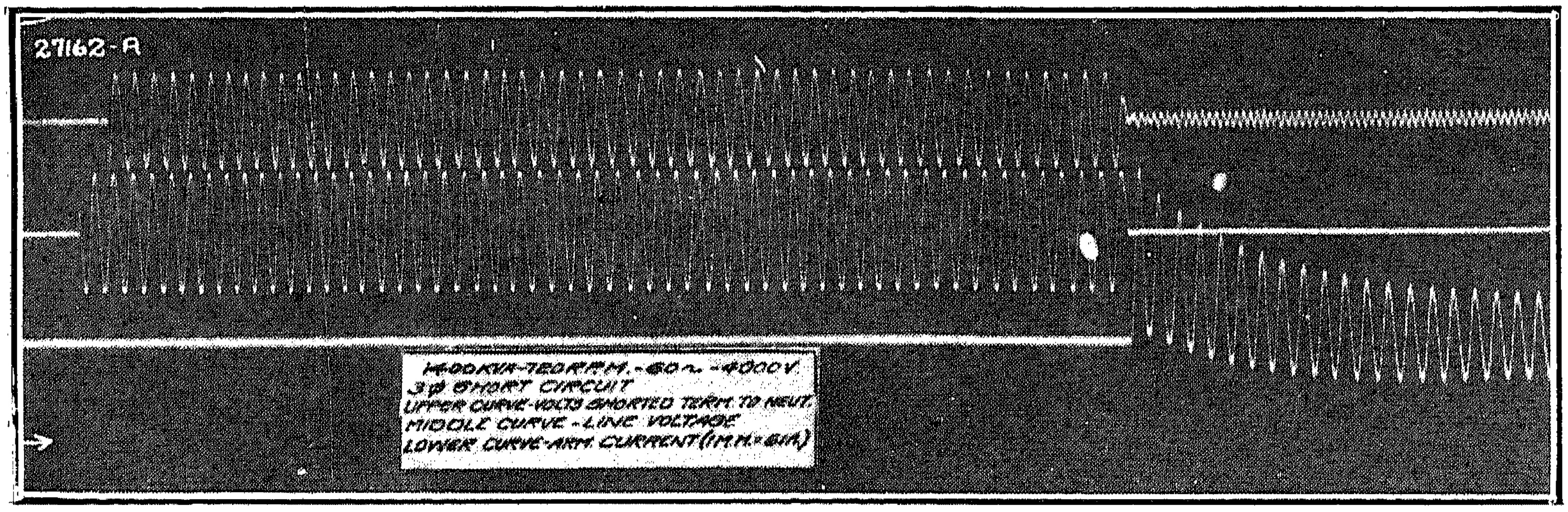


FIG. 30

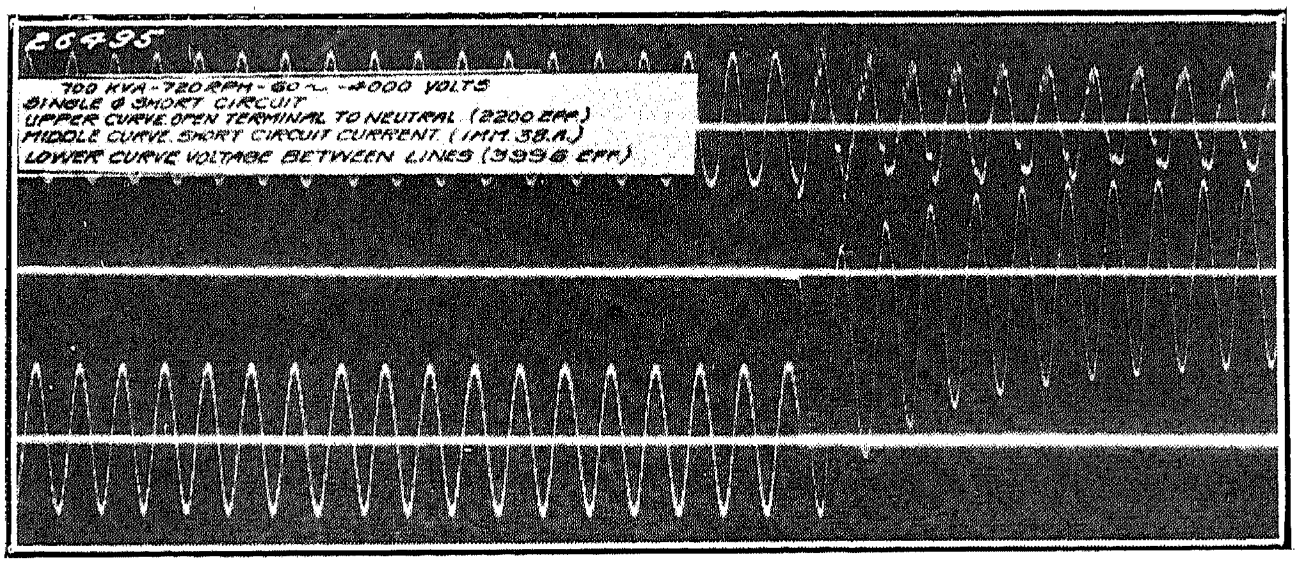


FIG. 31

[DOHERTY AND SHIRLEY]

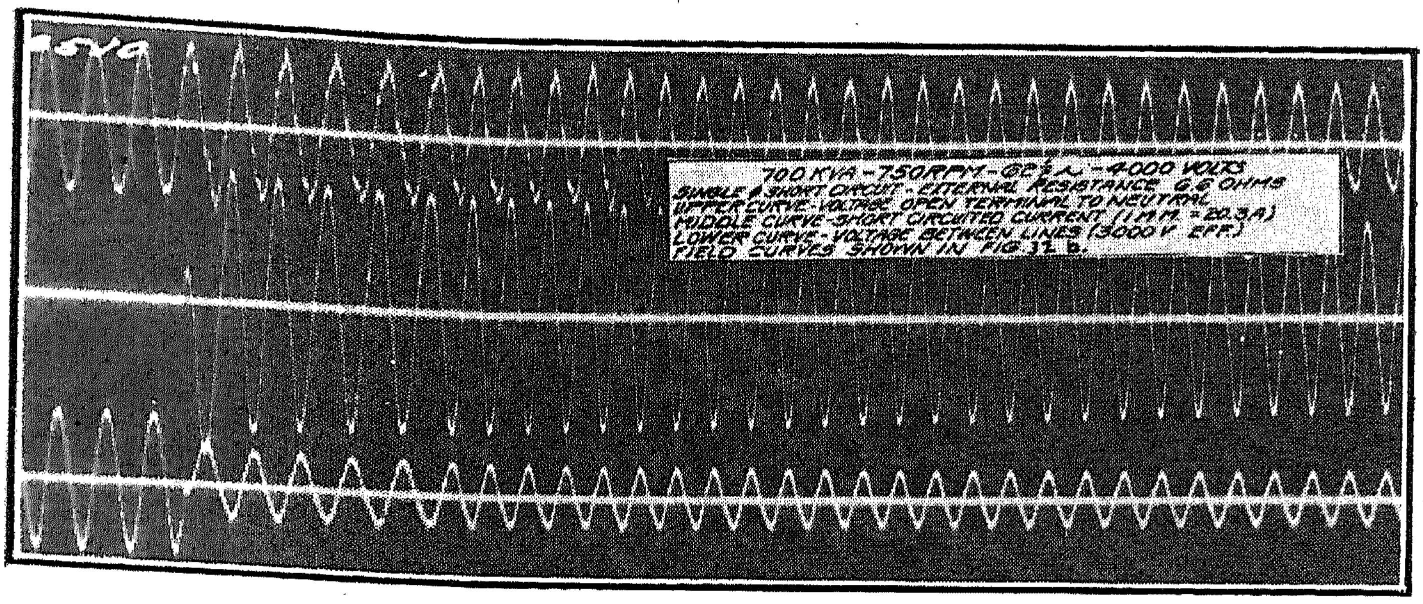


FIG. 32A

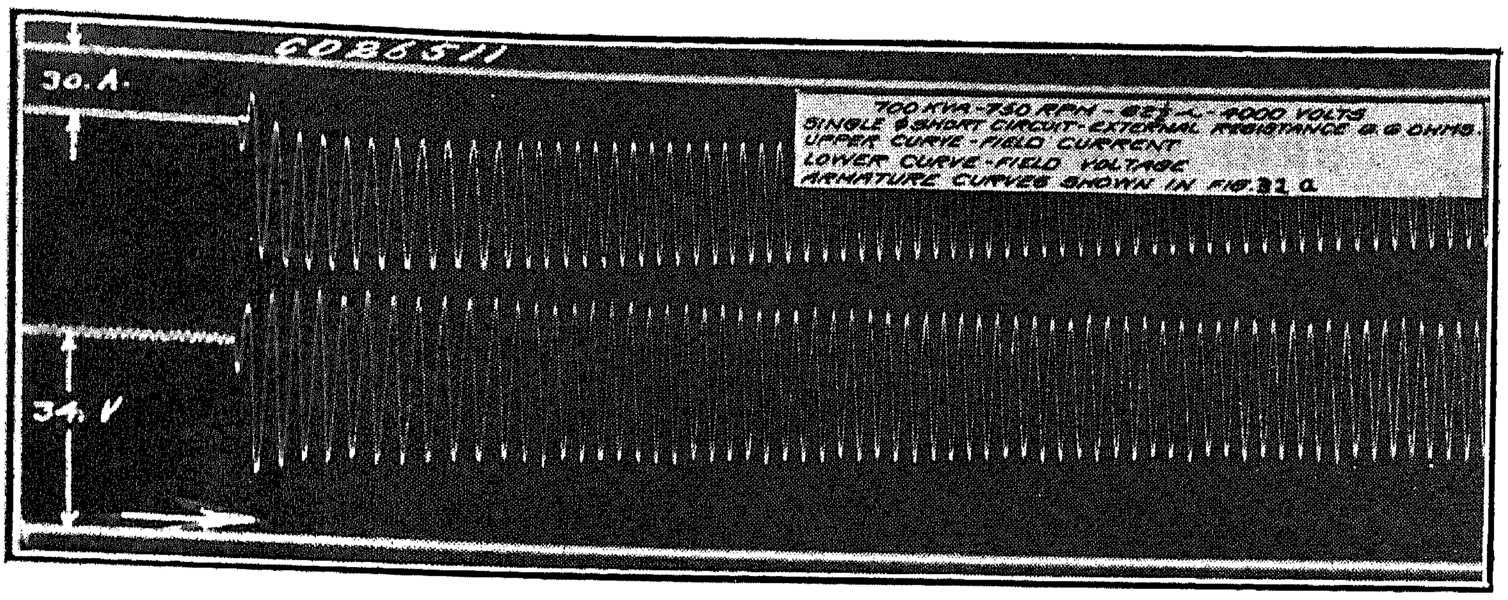
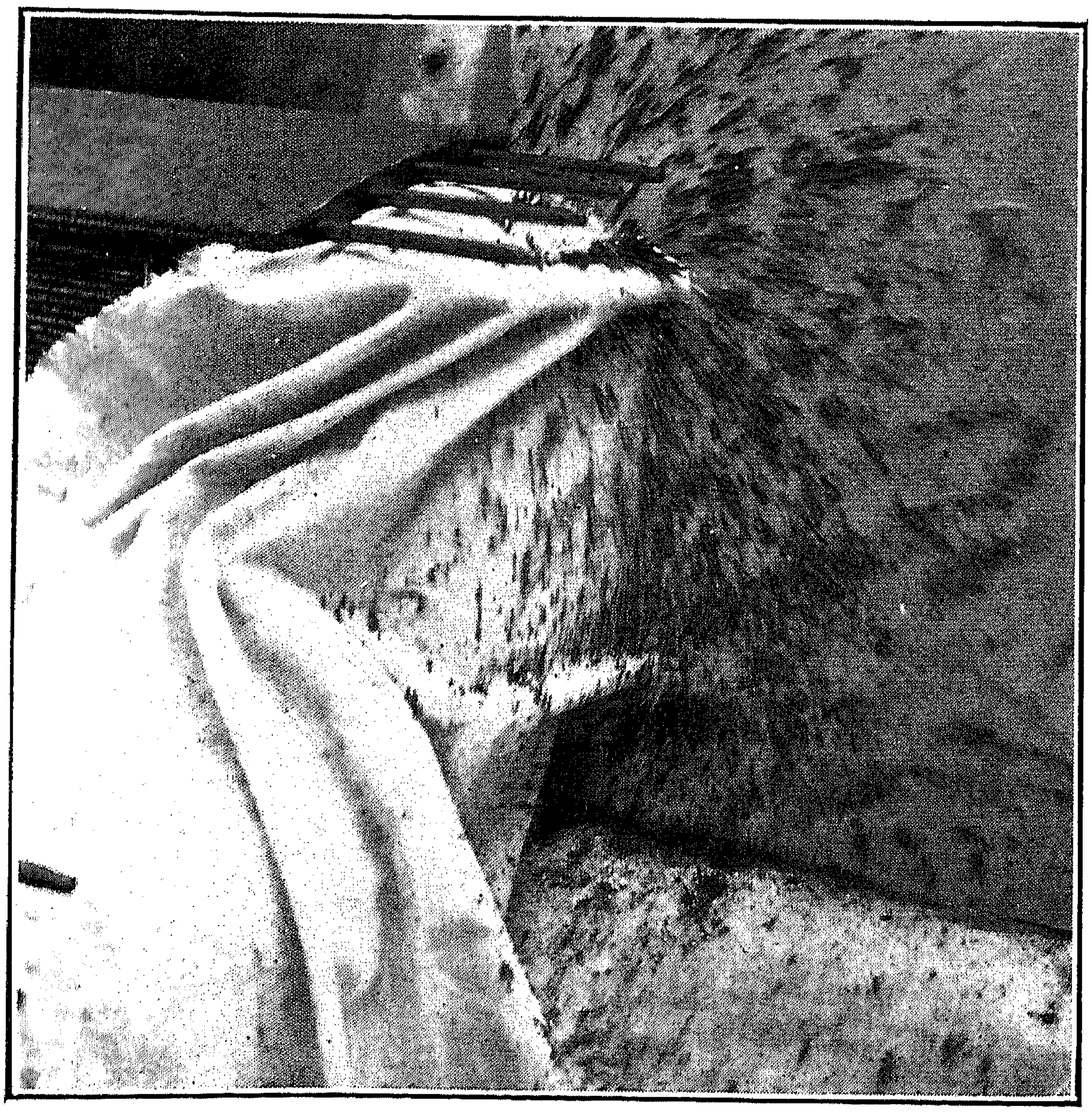


FIG. 32B



[DOHERTY AND SHIRLEY]

FIG. 35B—FIELD LEAKAGE PATH AT END OF POLE

the equation for x_{p01} can be written

$$x_{p_{01}} = \frac{2}{3}\left(x_{p_a} + x_{p_f}\right) = \frac{2}{3}\, x_{p_0} \qquad (13)$$

That is, on a single-phase short circuit, terminal to neutral, the current should be about 50 per cent larger than on a three-phase short circuit. The authors have not made tests to confirm this point.

The action of the field and armature on single-phase short circuit is as follows: if the short circuit occurs when the armature encloses zero flux, that is, at maximum voltage, Figs. 32A and 32B, the pulsating armature reaction produces in the field, for well known reasons, a direct m.m.f. equal to about one-half (actually less) of the maximum armature m.m.f., and a double frequency m.m.f. whose maximum value equals the direct m.m.f. The double frequency m.m.f. is of course at negative maximum at instant of short circuit, so that at the end of the first half cycle the total field m.m.f. is about equal to the maximum armature m.m.f.; that is, the armature and field absorb in their leakage paths at this instant, the total normal flux in proportion to their respective self-inductions.

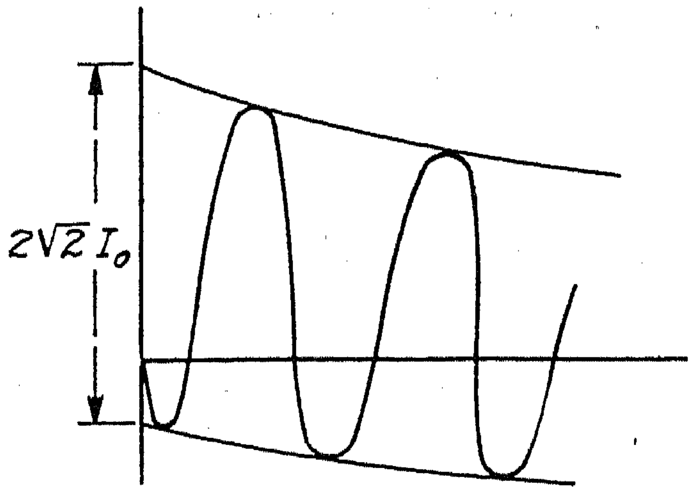


FIG. 33—SHORT CIRCUIT ARMATURE CURRENT

A single-phase short circuit between terminals gives, in most cases, practically the same current as a three-phase short circuit, as experience has shown, and may therefore be calculated by the formula for three-phase until a more accurate method is worked out.

The total self-inductive reactance effective at short circuit expressed in ohms, that is, the transient reactance, is

$$x_0 = \frac{x_{p_0} \times \text{normal voltage per leg}}{\text{normal current per leg}} \qquad (14)$$

Table IV shows a comparison of calculated and actual values of the alternating component of short-circuit current in terms of normal current. The actual values were determined by projecting to the axis of zero time the curve of positive crests and also the curve of negative crests, Fig. 33. The intercept on the vertical axis represents $2\sqrt{2}\, I_0$, *i.e.* two times the crest value of the alternating component.

TABLE IV—SALIENT-POLE MACHINES

Machine					Conditions of short circuit			Type of rotor	Amortisseur winding	Ratio—Normal cur. to R. M. S. of alternating component of short-circuit current†	
Kv-a.	Phase	Cyc.	R. P. M.	Voltage	Phase	Load	Voltage			Calc.	Test
10000	3	63	55 6	6600	3	None	6600	Laminated salient pole	None	0.27	0.25
"	"	"	"	"	3	None	"	"	"	"	0.25‡
"	"	"	"	"	3	None	"	"	"	"	0.26‡
"	"	"	"	"	3	None	"	"	"	"	0.25
"	"	"	"	"	3	7500kw	"	"	"	"	0.22
"	"	"	"	"	3	7500kw	"	"	"		0.24
9000	3	25	57.7	11000	3	None	11000	"	None	0.22	0.26
"	3	"	"	"	3	"	11000	"	"	"	0.26
"	"	"	"	"	3	"	11000	"	"	"	0.25
"	"	"	"	"	3	"	8250	"	"	"	0.25
"	"	"	"	"	1	"	8250	"	"	"	0.26
"	"	"	"	"	1		8250	"	"	"	0.29
6600	3	25	375	14000	3	None	16000	"	None	0.15	0.12
"	"	"	"	"	3	"	14000	"	"	"	0.13
"	"	"	"	"	3	"	14000	"	"	"	0.13
"	"	"	"	"	3	"	8000	"	"	"	0 14‡
4300	1	25	300	11000	1	None	11000	"	Low-Res.	0.19*	0.20
"	"	"	"	"	1	"	11000	"	"	"	0.22
"	"	"	"	"	1	"	11000	"	"	"	0.19
"	"	"	"	"	1	"	8250	"	"	"	0.20
"	"	"	"	"	1	"	8250	"	"	"	0.19
"	"	"	"	"	1	"	8250	"	"	"	9.23
"	"	"	"	"	1	"	5500	"	"	"	0.23
					1	"	5500	"	"	"	0.23
1500	3	50	500	6600	3	None	6600	"	None	0.16	0.12
1500	3	50	500	6600	3	"	4950	"	"	"	0.13
1500	3	50	500	6600	3	"	3300	"	"	"	0.14
1500	3	50	500	6600	3	"	1650	"	"	"	0.15
1400	3	60	720	2300/4000	3	None	4000	"	High-Res.	0.18	0.21
"	"	"	"	"	3	"	"	"	"	"	0.16
"	"	"	"	"	3	"	"	"	"	"	0.21
"	"	"	"	"	3	"	"	"	"	"	0.21
"	"	"	"	"	1	"	"	"	"	"	0.21
"	"	"	"	"	1	"	"	"	"	"	0.18
"	"	"	"	"	1	"	"	"	"	"	0.21
"	"	"	"	"	1	"	"	"	"	"	0.22
"	"	"	"	"	1	"	"	"	"	"	0.20
"	"	"	"	"	1	"	3000	"	"	"	0.21

SALIENT-POLE MACHINES (Continued)

Machine					Condition of short circuit			Type of rotor	Amortissuer Winding	Ratio—Normal cur. to R. M. S. of alternating component of short-circuit current†	
Kv-a	Phase	Cyc.	R. P. M.	Voltage	Phase	Load	Voltage			Calc.	Test
700	3	60	720	2300/4000	1	None	3996	Laminated salient pole	High Res	0.18	0.20
"	"	"	"	"	1	"	3996	"	"	"	0.23
"	"	"	"	"	1	"	3996	"	"	"	0.16
"	"	"	"	"	1	"	3036	"	"	"	0.18
"	"	"	"	"	1	"	3036	"	"	"	0.20
"	"	"	"	"	1	"	2016	"	"	"	0.17
"	"	"	"	"	1	"	2016	"	"	"	0.19
"	"	"	"	"	3	"	3996	"	"	"	0.15
"	"	"	"	"	3	"	3996	"	"	"	0.14
"	"	"	"	"	3	"	3036	"	"	"	0.15
"	"	"	"	"	3	"	2016	"	"	"	0.17
"	"	"	"	"	3	"	2016	"	"	"	0.15
125	3	60	900	2300	3	None	2300	"	High-Res	0.18	0.19
"	"	"	"	"	3	"	2300	"	High-Res.	"	0.19
"	"	"	"	"	3	"	2300	"	None	"	0.18
"	"	"	"	"	3	"	2300	"	None	"	0.18
"	"	"	"	"	3	"	2300	"	None	"	0.17
"	"	"	"	"	3	"	1150	"	None	"	0.20
"	"	"	"	"	3	"	1150	"	None	"	0.21
TURBINE ALTERNATORS											
10000	3	40	2400	10000	3	None	12000	Continuous solid steel	None	0.12*	0.12‡
"	"	"	"	"	"	"	10000	"	"	"	0.13‡
"	"	"	"	"	"	"	5000	"	"	"	0.16‡
"	"	"	"	"	"	Full Load 10 P. F.	10000	"	"	"	0.15‡
"	"	"	"	"	"	"	10000	"	"	"	0.15‡
"	"	"	"	"	"	Full Load O P. F.	10000	"	"	"	0.12‡
6250	3	60	3600	2300	3	None	2300	"	"	0.10*	0.09
3750	"	60	3600	2300/4000	3	None	4000	"	None	0.09*	0.10
"	"	"	"	"	"	"	4000	"	"	"	0.08
"	"	"	"	"	"	"	3000	"	"	"	0.11
"	"	"	"	"	"	"	3000	"	"	"	0.09
"	"	"	"	"	"	"	2000	"	"	"	0.12

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†In order to have ratios on same basis, normal current is decreased or increased according as voltage at short circuit is less or greater than normal.

*Transient reactance taken equal to armature reactance.

‡Average from three waves on one film.

Example: 6600-kv-a., three-phase, 25-cycle, 14,000-volt generator.

$$l = 38 \quad A = 10970 \quad \Phi = 47.7 \times 10^6$$

$$F = 15600 \quad \frac{h}{m} = 0.687 \qquad \frac{a}{c} = 0.413$$

$d_t = 2.0$ $\qquad d_p = 2.0 - \dfrac{11.5}{4}$ (neglect d_p if it is zero or negative) $\qquad \dfrac{d}{l} = 0.407$

Ratio $\dfrac{\text{pole arc}}{\text{pole pitch}} = 0.675 \qquad \dfrac{b}{c} = 2.08$

$$L = \left[3.6 \times 0.687 + 0.04 \left(\frac{1}{0.687}\right)^5 + 4 \log\,(1 + \pi \times 0.413) + 1.4 \times 0.407 \right]$$

$$= [2.47 + 0.26 + 3.3 + 0.57] = 6.6$$

$$x_{p_f} = \frac{10970}{15600} \frac{\dfrac{38 \times 15600 \times 6.6}{47.7 \times 10^6}}{\left(1 + \dfrac{38 \times 15600 \times 6.6}{47.7 \times 10^6}\right)^2} = 0.049$$

Armature reactance $= x_{pa} = 0.10$ (see example, Part I)

$$x_{p0} = x_{pa} + x_{pf} = 0.10 + 0.049 = 0.149$$

That is, the three-phase short-circuit current (*a-c.* component) would be

$$I_0 = \frac{1}{0.149} = 6.7 \text{ times normal}$$

Consider now the question of attenuation factors, α_f and α_a. The attenuation factor of a circuit is the ratio of the effective resistance to the (interlinkages per ampere), that is, the inductance. In the field attenuation factor, α_f, the inductance is not as Diamant gives[14] the exciting inductance of the machine, (using his notation)

$$\frac{\Phi N_f}{I_f \, 10^{-1}}$$

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[14] Trans. A. I. E. E., Vol. 34, p. 2247.

where Φ is the flux which the normal field excitation, I_f, would produce in the normal magnetic circuit, as given by the design sheet. It is rather

$$\frac{\Phi' N_f}{I_f 10^{-1}}$$

where Φ' is the flux, linking the field winding, which I_f would produce in the *leakage paths of both field and armature.* The correct attenuation factor of the field to be used in determining the rate of decrease of field flux on short circuit is the ratio of the effective resistance of the circuit to the (transient field interlinkages maintained through the leakage paths of both armature and field on short circuit, per ampere rise in the field circuit). Therefore the question is, what is the value of fleld interlinkages thus maintained per ampere *rise* in field current on short circuit. It is,

$$\frac{\Phi_t N_f}{I_{f0}} \quad \text{per pole}$$

or

$$\frac{q \, \Phi_t N_f}{I_{f0}} \quad \text{total}$$

That is,

$$\frac{q \, \Phi_t N_f}{10^8 I_{f0}} \quad \text{henrys} \tag{15}$$

Where

Φ_t = transient flux = $\Phi - \Phi_s$
Φ = normal flux per pole
Φ_s = flux per pole, absorbed in armature self-induction on sustained short circuit

$$\Phi_s = \frac{F \Phi}{\dfrac{A}{x_{p_a}} + F_g}$$

x_{p_a} = armature reactance
($x_{pa} = 0.2$ means 20 per cent)
F_g = m.m.f. consumed in air gap by normal flux
N_f = turns per pole
q = number of poles

I_{f0} = *d-c.* component of *rise* in field current at short circuit. This corresponds to the reaction of the *alternating component* in the armature, and for reasons previously stated is less than the armature m.m.f. as shown by equation (**3**).

$$I_{f_0} = \frac{F_a}{x_{p_0}\, N_f} = \frac{F}{x_{p_0}\, N_f} \left[\frac{\dfrac{A}{F}}{1 + \dfrac{F\,l\,L}{\Phi}} - x_{p_f} \right] \qquad \textbf{(16)}$$

Hence the field inductance during transient is,

$$L_{f_0} = \frac{q\, x_{p_0}\, \Phi_t\, N_f^2}{10^8\, F} \; \frac{1}{\left[\dfrac{\dfrac{A}{F}}{1 + \dfrac{F\,l\,L}{\Phi}} - x_{p_f} \right]} =$$

$$L_{f_0} = \frac{q\, x_{p_0} \Phi\, N_f^2}{10^8\, F} \; \frac{\left(1 - \dfrac{F}{\dfrac{A}{x_{p_a}} + F_g} \right)}{\left(\dfrac{\dfrac{A}{F}}{1 + \dfrac{F\,l\,L}{\Phi}} - x_{p_f} \right)} \text{ henrys} \qquad \textbf{(17)}$$

The field attenuation factor is therefore

$$\alpha_f = \frac{r_f}{L_{f_0}} \qquad \textbf{(18)}$$

where r_f = effective resistance of the field circuit. In laminated pole machines, this may be taken in approximate calculations as the ohmic resistance of the field circuit.

The inductance in the armature attenuation factor α_a, which determines the rate of decay of the *direct component*, is determined as follows: the interlinkages of the armature per pole per phase, maintained through the leakage paths of both the armature and the field, per effective ampere turn rise in the armature m.m.f. on short circuit, is

$$\frac{(K'\, N_q)\, \Phi}{\dfrac{A}{x_{p_0}}} \qquad \textbf{(19)}$$

or, for all poles

$$\frac{q\, x_{p_0}\, \Phi\, (K'\, N_q)}{A} \qquad \textbf{(20)}$$

But A, for three-phase, is the product

$$\sqrt{2}\, I_n \times 1.5\, (K'\, N_q) \tag{21}$$

where K' is a constant depending upon the pitch and distribution of coils, and upon the flux distribution; I_n = normal rated current; and N_q = series turns per pole per phase. Hence, the inductance, that is the interlinkages with $(K'\, N_q)$, is

$$L_{ao} = \frac{1.5\, q\, x_{p0}\, \Phi\, (K'\, N_q)^2}{10^8} \text{ henrys} \tag{22}$$

The factor 1.5 accounts for the mutual induction of phases. For single phase or two phase this factor would be unity. L_{a0} can be arrived at in another and more convenient way.
It is equal to

$$L_{ao} = \frac{x_0}{2\, \pi f} \tag{23}$$

Where x_0 = transient reactance in ohms
f = frequency in cycles per second

$$x_o = \frac{x_{p0}\, E}{I_n}$$

Hence

$$L_{ao} = \frac{x_{p0}\, E}{2\, \pi\, f\, I_n} \tag{24}$$

The armature attenuation factor is

$$\alpha_a = \frac{r_a}{L_{ao}} \tag{25}$$

where r_a = effective resistance per phase.

It may seem at first thought that since the sum of the inductances of both the field and armature are involved in the attenuation factors, the sum of resistance of both circuits also should enter. However, this is not the case. The armature and field transients involve magnetic energy storage under unstable conditions. In the case of the field transient, immediately after short circuit the energy stored in the field is the product of normal field flux times the large field m.m.f. which now maintains it. The flux can die down only if its energy is dissipated in the circuit with which it links, because, by the very way it is constituted, the energy can decrease only by a decrease in interlinkages, and this can not happen except by resistance loss in the linking circuit. From a slightly different viewpoint, suppose the field circuit has no resistance. Then the field flux and current would never die out, regardless of the armature resistance. The armature m.m.f. operates only to increase the reluctance[15] of the path through which the flux must pass on short circuit. The armature resistance operates only to decrease the initial rise in current, that is to decrease the amount of flux which on short circuit must be sent through the paths of increased reluctance, and therefore to increase the sustained value of flux, that is, to decrease the magnitude of the transient, but not its time. The same reasoning will apply to the armature transient.

The relative values of α_f and α_a can be illustrated by taking some rough average figures for salient-pole machines. The ratio, armature reactance to armature resistance is, say, 20; field reactance to field resistance, 80; field reactance to armature reactance, 0.5.

Hence

$$\text{Armature} - \frac{\text{resistance}}{\text{transient reactance}} = \frac{1}{30}$$

$$\alpha_a = \frac{2\pi f}{30}$$

$$\text{field} - \frac{\text{res stance}}{\text{transient reactance}} = \frac{1}{240}.$$

$$\alpha_f = \frac{2\pi f}{240}$$

That is, the armature transient will die out eight times as fast as the field, in this case.

Diamant found from oscillographic tests that the field attenuation factor was larger during the first few cycles than during the rest of the transient. The authors have found this to be the case; and agree with Diamant that, until this difference, due probably to saturation and eddy currents, can be further studied, the lower value (which corresponds to our calculation) should be used. This is on the safe side, since by this assumption, at any time after short circuit the current will be lower than the calculated value.

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[15] An equivalent reluctance could theoretically be produecd, with the same effect on the field transient, by a sudden increase, at open circuit, in full reluctance of the armature iron.

Saturation of the leakage paths at short circuit and the consequent reduction of reactance may cause an increase in α_f, and probably does so in machines which have restricted leakage paths—turbine generators, for instance. In salient-pole machines, however, where tests show the short-circuit current to be practically proportional to the voltage at short circuit, it is probable that the increase is largely due to eddy currents. The increase in flux at the bottom of the field pole—*i. e.* next to the rotor rim—at sudden short circuit, as explained in preceding pages, operates in two ways to increase α_f. If the rotor rim is solid, as is usually the case, this change in flux is largely restricted by eddies in the solid rim. To whatever extent it is restricted, the effective field self-induction is reduced, which would increase α_f. But a much larger increase in α_f results from loss of energy due to eddies in the rim, increasing the effective resistance. Since the alternating flux in the rim is caused by the direct component of the armature current, *i. e.* the armature transient, the increased α_f would by this theory, exist only during the armature transient, which lasts about ¼ second. That is what actually happens, as shown in Fig. 33A. The initial value of α_f is 1.7 times the constant value which exists after the termination of the armature transient.

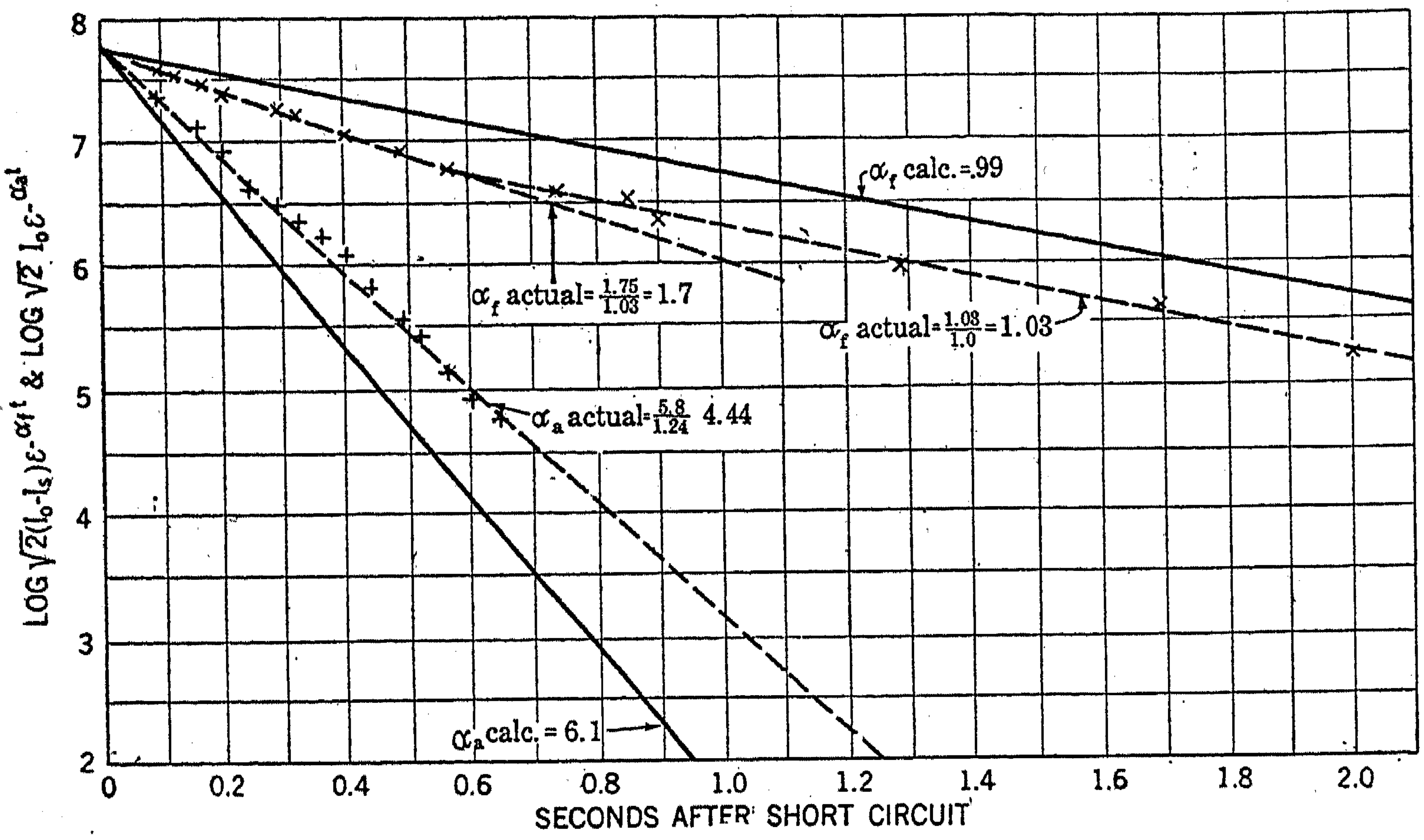

FIG. 33A—ATTENUATION FACTORS—6600 KV-A.—375 REV. PER MIN.—14000-VOLT ALTERNATOR

There is further evidence that this theory is correct. If the entire rotor magnetic circuit were laminated the change in flux at the bottom of the pole could occur without producing serious eddies, and therefore, without appreciable change in α_f. Actual tests on such a machine shows this to be the case. The results are shown in Fig. 33 B. There is no appreciable change in slope of the curve.

The question arises, how much will calculations of reactance be thrown out by saturation? From the data available it would seem that in salient-pole machines without amortisseur windings, saturation is not a very serious factor. Although at 180 deg. after a three-phase short circuit there must be two times normal flux through the leakage paths, these paths are widely distributed, and therefore very serious concentration is prevented. Yet there is some. The smaller the section of the principal leakage paths (the pole tip and teeth) the greater the saturation. It is still further increased by the action of an amortisseur winding, and by reason of the slots that contain it. But we believe it is safe to assume that in the case of most laminated, salient-pole machines, without amortisseur windings, the short-circuit current is practically proportional to the voltage at which short circuit occurs. When thin pole tips and shallow armature slots are used, the short-circuit current may be 10 to 20 per cent greater than indicated by calculations which neglect saturation. In the case of low resistance amortisseur windings, solid steel poles, or in turbine generators, where the leakage paths are much more restricted,

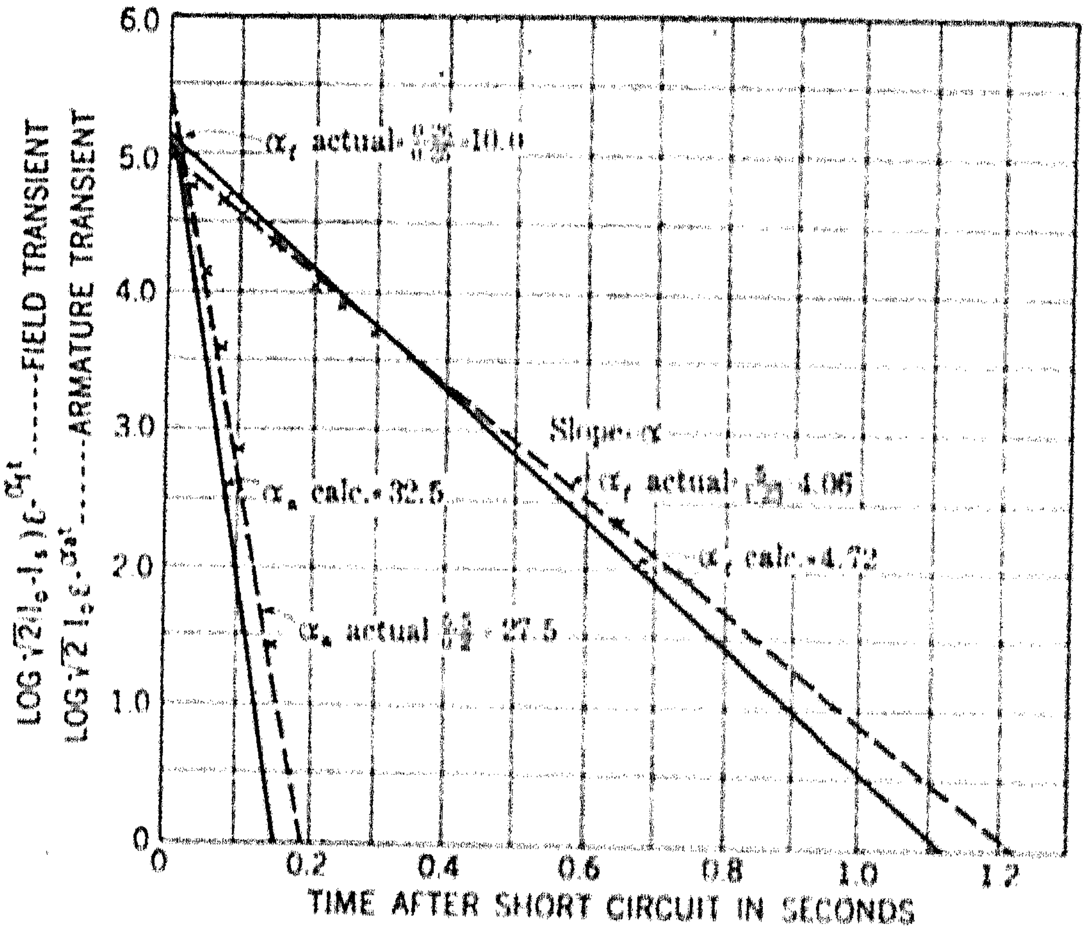

FIG. 33B—ATTENUATION FACTORS—125-KV-A.—900 REV. PER MIN.—60-CYCLE—2300-VOLT ALTERNATOR

saturation makes a greater difference. Hence, for this, as well as another reason given in the following paragraph, the calculation of such machines should be based upon the armature self-induction only. The matter of saturation of leakage paths is one which should be much further investigated.

Another matter of importance is the effect of amortisseur windings, short-circuiting collars around the pole, solid steel poles, etc. Obviously if the amortisseur winding were without resistance, the flux at short circuit could not change through it and consequently the effect of field self-induction would be nil, since the field m.m.f. would not rise. Moreover, the armature leakage paths would be more restricted. That is the transient reactance would be less than the armature self-induction. But, of course, amortisseur windings and collars are never perfect conductors. On the other hand, many of them are poor conductors; hence in the case of laminated field poles they permit a large part of the field self-induction to exert itself—the more, naturally the higher the resistance of the amortisseur winding. However, in single-phase generators, or such machines, in which heavy copper amortisseur windings are used, and also in the case of solid steel poles, the resulting increase in short-circuit current can not be neglected. For safety, therefore, it is best in such cases, as proposed above, to neglect the field self-induction in calculating the short-circuit current; that is, to assume that the transient reactance is equal to the armature reactance.

External, as well as internal, field reactance will decrease the short-circuit current. The rate at which the field current rises at short circuit will not be impeded by the external reactance, because the new magnetic interlinkages established in the external reactance represent a corresponding decrease in the interlinkages of the field winding itself, since the net total of the circuit can not change appreciably during the first cycle or so. But, by reason of this decrease in interlinkages of the field winding itself, or what is the same thing, the increase in interlinkages in the external reactance, there remains in the field less flux to be forced across the leakage paths in the machine; hence the short-circuit current will be less by just that amount.

With zero external field reactance the rise in field m.m.f. on short circuit is less than the armature m.m.f. in the ratio

$$G = \frac{1}{1 + \frac{F\,l\,L}{\Phi}} \text{ by equations } (\mathbf{2}) \text{ and } (\mathbf{5})$$

Or, in terms of total interlinkages in the machine,

$$G = \frac{q\,\Phi\,N_f}{q\,\Phi\,N_f + \dfrac{F}{N_f}\,L_f\,10^8} = \frac{1}{1 + \dfrac{F\,L_f\,10^8}{N_f^2\,\Phi\,q}}$$

where L_f = coefficient of self-induction of the field winding.

$$L_f = \frac{q\,l\,L\,N_f^2}{10^8} \text{ henrys}$$

F = no-load field m.m.f.
l = gross length of armature core (iron + ducts)
L = bracket quantity in equation (**33**) or (**33a**)
Φ = normal flux per pole (maxwells)
q = number of poles
N_f = turns per pole on field.

If an external reactance, x_{fx}, (coefficient of self-induction, $= L_{fx}$), is placed in the field circuit, the ratio becomes,

$$G = \frac{1}{1 + \dfrac{F\,L_f\,10^8}{N_f^2\,\Phi\,q} + \dfrac{F\,L_{fx}\,10^8}{N_f^2\,\Phi\,q}}$$

$$G = \frac{1}{1 + \dfrac{F\,10^8}{q\,N_f^2\,\Phi}\,(L_f + L_{fx})}$$

Let x_{pf}'' represent the per cent reactance of the field circuit, assuming the rise in field m.m.f. equals the armature m.m.f. on short circuit. Then

$$x_{pf}'' = \frac{\dfrac{A}{N_f}L_f\,10^8}{\Phi\,N_f} + \frac{\dfrac{A}{N_f}L_{fx}\,10^8}{\Phi\,N_f}$$

$$x_{pf}'' = \frac{A\,10^8}{N_f^2\,\Phi}\,(L_f + L_{fx})$$

The reactance of the field circuit is

$$x_{pfx} = G\,x''_{pf}$$

$$= \left[\frac{\dfrac{A\,10^8}{N_f^2\,\Phi}\,(L_f + L_{fx})}{1 + \dfrac{F\,10^8}{N_f^2\,q\,\Phi}\,(L_f + L_{fx})}\right]$$

$$x_{pfx} = \frac{A}{F}\left[\frac{1}{1 + \dfrac{1}{\dfrac{F\,10^8}{N_f^2\,\Phi\,q}\,(L_f + L_{fx})}}\right] \qquad \textbf{(25a)}$$

The total transient reactance of the machine with external field reactance is

$$x_{p0} = x_{pa} + x_{pf_x}$$

where x_{pa} = armature reactance

(x_{po} = 0.2 means 20 per cent reactance)

The principal objection[16] to the use of reactance in the field circuit is the high voltage which occurs across its terminals at short circuit. With an external reactance in the field circuit equal to the internal field reactance, this voltage may be of the order of 50,000 volts in the case of large generators.

Under load the short-circuit current is slightly higher than at no-load. The alternating component, which depends upon the flux per pole, is greater for short circuit under load than at no-load, by the increase in flux which is required under load to overcome the true impedance of the armature. Hence for the same load current, the alternating component of short-circuit current will be greater, the lower the power factor, until at zero power factor, it will be increased by the per cent armature reactance of the machine—10 per cent reactance, 10 per cent increase in the alternating component. At unity power factor, the 10 per cent being added in quadrature, would not cause appreciable increase. The direct component, depending upon the flux linked with the armature circuit, *i.e.* upon the terminal voltage, is the same for any load or power factor, providing there is no external reactance in the armature circuit. External reactance decreases the direct component because it decreases the net magnetic interlinkages of the armature circuit. With 10 per cent external reactance, zero power factor load, the direct component would be 0.9 as large as if the short circuit occurred at no load, the same terminal voltage. The reverse would be true of leading current. For unity power factor, the difference would be negligible, for reasons mentioned above.

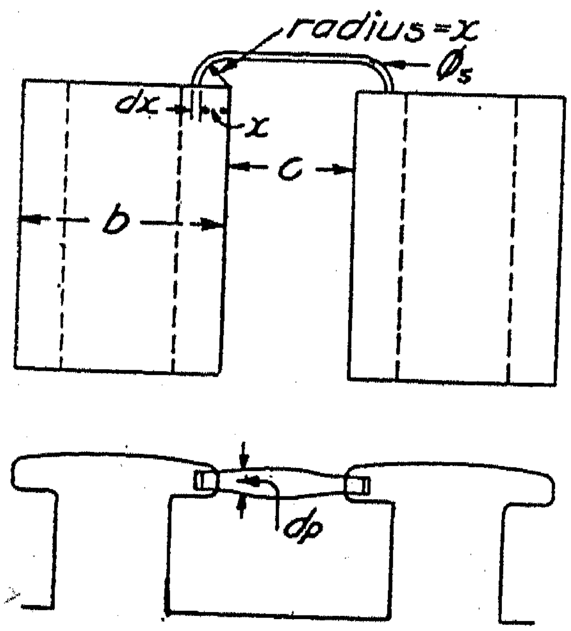

FIG. 34

DERIVATION OF FORMULA FOR CALCULATING FIELD SELF-INDUCTION

The field leakage paths shown in Figs. 24A and 24B are approximated by the assumed paths shown in Figs. 28, 34 and 35A.

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[16] This point has been observed by K. Ito, *Journal Elec. Society* (Japanese), Dec. 1917, p. 891.

With a m.m.f. equal to NI per pole the effective leakage lines, linking the entire field winding, per unit length of machine will be as follows: the flux element $d\,\phi_1$ is

$$d\,\phi_1 = 2 \times 0.4\,\pi \frac{x}{h} NI \frac{d\,x}{Kx}$$

where $$K = \frac{1}{\tan\theta} \text{ (approximately) } = \frac{2\,h}{m}$$

$$d\,\phi_1 = 0.4\,\pi\,NI \frac{m}{h^2}\,dx$$

Effective flux due to ϕ_1 linking entire winding is

$$\phi_{n1} = \int_0^{\frac{m^2}{4\,h}} \frac{x}{h}\,d\phi_1$$

$$\phi_{n1} = 0.039\,NI \left(\frac{m}{h}\right)^5 \qquad (26)$$

$$d\,\phi_2 = 2 \times 0.4\,\pi \left(\frac{\frac{m^2}{4\,h} + y}{h}\right) 2\,NI \frac{dy}{\frac{2\,\pi}{3}\;\frac{m}{\sqrt{3}}}$$

$$d\,\phi_2 = 1.05\,\frac{NI}{h^2\,m}(m^2 + 4\,h\,y)\,d\,y$$

It is here assumed that the length of path is one third of the arc of a circle whose radius is $\frac{m}{\sqrt{3}}$, that is, the pole sides intercept $\frac{1}{3}$ of the arc.

Effective flux of ϕ_2 linking the entire field winding is

$$\phi_{n2} = \int_0^{h-\frac{m^2}{4h}} \left(\frac{\frac{m^2}{4\,h} + y}{h}\right) d\,\phi_2$$

$$= 0.27\,\frac{N\,I}{h^4\,m} \int_0^{h-\frac{m^2}{4\,h}} (m^2 + 4\,h\,y)^2\,d\,y$$

$$\phi_{n2} = 0.27\,N\,I\left[\frac{16}{3}\,\frac{h}{m} - \frac{1}{12}\left(\frac{m}{h}\right)^5\right] \qquad (27)$$

$$d\,\phi_3 = 2 \times 0.4\,\pi \times 2\,N\,I \frac{d\,z}{c + \pi\,z}$$

$$d\,\phi_3 = 5\,N\,I\,\frac{d\,z}{c + \pi\,z}$$

All of ϕ_3 is assumed to be linked with the winding. Hence

$$\phi_{n3} = \phi_3 = 5\,NI \int_0^a \frac{d\,z}{c + \pi\,z}$$

$$\phi_{n3} = 1.6\,N\,I \log\left(1 + \pi\,\frac{a}{c}\right) \qquad (28)$$

In determining ϕ_4, it is necessary to find the portion of the pole tip from which ϕ_4 emanates, Fig. 28, and further to find what depth of path in the air between tips should be used. By plotting a large number of diagrams, such as Figs. 24A and 24B for different pole proportions, it was found that this average depth of path could be approximated by the expression

$$d_p = d_t - \frac{c}{4} \qquad (29)$$

That is the factor may entirely disappear, which happens in the case of thin pole tips and low pole arc ratio. When $\frac{c}{4}$ is greater than d_t, ϕ_{n4} should be neglected.

$$\phi_{n4} = \phi_4 = 2 \times 0.4\,\pi \times 2\,N\,I\,\frac{d_p}{c}$$

$$\phi_{n4} = 5\,N\,I\,\frac{d_p}{c} \qquad (30)$$

$$d\,\phi'_5 = 4 \times 0.4\,\pi \times 2\,N\,I\,\frac{d_p\,d\,x}{\pi\,x + c} \quad \text{(See Fig. 11)}$$

$$d\,\phi'_5 = 10\,N\,I\,d_p \int_0^{b/4} \frac{d\,x}{\pi\,x + c}$$

$d\,\phi'_5$ is integrated between 0 and $\frac{b}{4}$ because toward the middle of the pole the leakage from the end of pole shoe no longer goes from pole to pole, but instead bends upward and enters the stator iron, cutting the armature winding. The approximation is therefore made that the dividing line falls at $b/4$.

$$\phi_5{}' = 3.2\, N\, I\, d_p \log \left(1 + \frac{\pi}{4} \frac{b}{c} \right)$$

$$\phi_{ns} = \frac{\phi_s{}'}{l} = 3.2\, N I \frac{d_p}{l} \log \left(1 + \frac{\pi}{4} \frac{b}{c} \right) \qquad \textbf{(31)}$$

where l = gross length of armature core (iron + ducts).

The leakage at the end of the pole is in paths practically straight and downward to the field spider rim as shown in Fig. 35A and 35B. At about three-fourths of the way up the pole the leakage bends up and enters the stator iron, part of it, certainly, cutting the armature winding. Of course, depending upon design, the division line may fall at one-half or may be at the top of the pole. But in the authors' judgment, three-fourths represents the average case. On this and the further assumption that the length of path at any height ν, Fig. 35, is 1.3 ν,

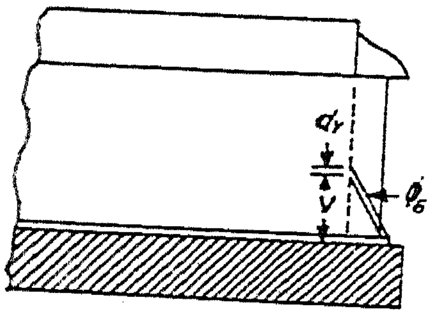

FIG. 35A

$$d\, \phi'_6 = 2 \times 0.4\, \pi \frac{\nu}{h} NI \frac{d\, d\nu}{1.3\, \nu}$$

where h = height of pole core

$$d\, \phi'_6 = 1.95 \frac{d}{h} N I\, d\, \nu$$

$$\phi'_{n6} = \int_0^{\frac{3}{4}h} \frac{\nu}{h^2}\, d\, \phi_6{}'$$

$$= 1.95\, N\, I \frac{d}{h^2} \int_0^{\frac{3}{4}h} \nu\, d\, \nu$$

$$\phi_{n6} = \frac{\phi'_{n6}}{l} = 0.55\, N\, I \frac{d}{l} \qquad \textbf{(32)}$$

While one or two rather arbitrary assumptions were made in deriving the above equations, they were made in the light of an experimental and graphical study of the leakage paths, and with the object of avoiding complicated expressions; and it is believed that the error in the final result will not be serious, one reason being that the assumptions involving the most uncertainties are in the smallest factors, such as ϕ_{n5} and ϕ_{n6}.

Adding equations (**26**), (**27**), (**28**), (**30**), (**31**) and (**32**), the total self-induction of the field is

$$\phi_{ln} = \phi_{n1} + \phi_{n2} + \phi_{n3} + \phi_{n4} + \phi_{n5} + \phi_{n6}$$

For lengths in centimeters,

$$\phi_{ln} = N\,I\Big[\,0.039\left(\frac{m}{h}\right)^{5} + 0.27\left\{\frac{16}{3}\,\frac{h}{m} - \frac{1}{12}\left(\frac{m}{h}\right)^{5}\right\}$$

$$+\,1.6\log\left(1 + \pi\,\frac{a}{c}\right) + 5\,\frac{d_p}{c}$$

$$+\,3.2\,\frac{d_p}{l}\,\log\left(1 + \frac{\pi}{4}\,\frac{b}{c}\right) + 0.55\,\frac{d}{l}\,\Big]$$

$$\phi_{ln} = N\,I\Big[\,1.42\,\frac{h}{m} + 0.16\left(\frac{m}{h}\right)^{5} + 1.6\log\left(1 + \pi\,\frac{a}{c}\right)$$

$$+\,5\,\frac{d_p}{c}\left\{1 + 0.62\,\frac{c}{l}\,\log\left(1 + \frac{\pi}{4}\,\frac{b}{c}\right)\right\} + 0.55\,\frac{d}{l}\Big] \qquad (\mathbf{33})$$

If inches are used, the equation becomes

$$\phi_{ln} = N\,I\Big[3.6\,\frac{h}{m} + 0.04\left(\frac{m}{h}\right)^{5} + 4\log\left(1 + \pi\,\frac{a}{c}\right)$$

$$+\,12.8\,\frac{d_p}{c}\left\{1 + 0.62\,\frac{c}{l}\log\left(1 + \frac{\pi\,b}{4\,c}\right)\right\} + 1.4\,\frac{d}{l}\Big] \qquad (\mathbf{33a})$$

ϕ_{ln} is the effective leakage lines per pole, linking the entire field winding, per unit length of machine, produced by a m.m.f. of NI ampere turns.

The coefficient of self-induction of the field is therefore

$$L_f = \frac{q\,N_f^2\,l\,L}{10^8}\ \text{henrys} \qquad (\mathbf{34})$$

where q = number of poles
N_f = turns per pole
l = gross stacked length of core (iron and ducts)
L = bracket quantity in equation (**33**) or (**33a**)

For the value of leakage, ϕ'_{ln}, per unit length, corresponding to a m.m.f. on the field equal to normal armature reaction, A, substitute A for $N\,I$ in equation (**33**) and (**33a**). That is

$$\phi'_{ln} = A\,L \qquad (\mathbf{35})$$

SUMMARY AND CONCLUSIONS

(1) A reliable prediction of the armature self-inductive reactance of synchronous machines can be made by equations (**45**) to (**48**) inclusive, given in Part I. Moreover, working curves for three-phase, based on those equations and given in Figs. 20A 20B, etc., make the complete calculation from the design sheet a matter of four or five minutes. It is proposed that this reactance be called, for brevity, *armature reactance.*

(2) The variation of the effective armature m.m.f. or armature reaction, (under zero power factor conditions) with the no-load flux distribution has been determined. The armature reaction of three-phase machines, as calculated by the familiar formula based on sinusoidal distribution, is the basis of the new calculation. This result is modified by a single factor, K_ϕ given in Fig. 20, and developed in appendices A and B. The correct calculation of armature reaction is very important in its relation to the armature self-inductive reactance, since the latter is determined from test by the difference between the field and armature m.m.f.'s on sustained short circuit.

(3) As a corollary of (2), the effect of the harmonics in the no-load flux distribution upon the generated voltage is accounted for also by K_ϕ. See appendix A.

(4) A simplified method of calculating field excitation that has worked out well in conjunction with the reactance given by Figs. 20A, 20B, etc., is shown in appendix C.

(5) The reactance which determines the initial short-circuit current of most synchronous machines is not the armature self-induction only; it is rather the combined self-induction of both the armature and field circuits. The internal field self-induction (which, on short circuit is transient) is often 50 per cent of the armature self-induction; in which case, the calculated short-circuit current, based on the armature self-induction alone, would, of course, be 50 per cent too high.

(6) A method of calculating the transient field self-induction is given in Part II, equation (**6**). It is proposed that this reactance be called, for brevity, *field reactance.*

(7) Equation (**7**), Part II, gives a value, x_{p0} of the total per cent reactance, which determines the alternating component of short-circuit current. It is proposed that this reactance be called *transient reactance.*

(8) The attenuation factors α_f and α_a, which determine the

rate of decrease of the field and armature transients, respectively are given in equations (**18**) and (**25**).

(9) External field reactance reduces the short-circuit current. Its effect can be calculated by equation (**25a**). The high voltage across the field collector rings (which may be 50,000 volts on large generators, assuming external field reactance equal to internal) precludes its use in most cases.

(10) Table IV confirms the following points, which are brought out in the theory given in Part II:

(a) The short-circuit current of synchronous machines can be calculated with practical accuracy.

(b) In the case of turbine generators with solid steel rotors, and also of salient-pole machines with low resistance amortisseur windings, the transient reactance should be taken equal to the armature reactance.

(c) In the case of high resistance amortisseur windings, the same value of transient reactance should be used as for laminated salient-pole machines without amortisseur windings. That is, the value given by equation (**7**) Part II.

(d) While strictly, the complete calculations apply only to salient, laminated pole machines without amortisseur windings, and the proposals in (b) and (c) are only approximations, nevertheless the results shown in Table IV justify the use of these approximations until the method is further extended.

(e) Saturation at short circuit does not practically affect value of short-circuit current of salient, laminated pole machines without amortisseur windings, unless the magnetic densities in the pole tips and teeth are high at normal voltage, as in the case of the 1500-kv-a. generator in Table IV. It does affect the short-circuit current of turbine generators, and, probably, of salient-pole machines with low resistance amortisseur windings.

(f) The short-circuit current is practically the same whether the short circuit on three-phase machines occurs between two terminals or three.

(g) A single-phase short circuit-terminal to neutral, produces a current 50 per cent greater than that of a three-phase short circuit.

In conclusion the authors wish to express their thanks to Dr. C. P. Steinmetz and W. J. Foster for their interest in reviewing the results of the paper, and to Messrs. E.J. Burnham, E. S. Henningsen, H. K. Humphrey, and J. J. Thalheimer for heir assistance.

APPENDIX A

EFFECT OF HARMONICS IN NO-LOAD FLUX WAVE

The shape of the wave of flux distribution is determined largely by the shape of the pole face and the ratio of the pole arc to pole pitch. The armature teeth of course add ripples of high order, but usually of small magnitude. The latter are neglected in the following derivation because, even if they are of appreciable magnitude to cause trouble in telephone circuits, etc., they have little effect upon the effective armature m.m.f. or upon the value of terminal voltage. The third, fifth, seventh, and possibly the ninth, all of whose magnitude are determined by the design of the pole, are the terms which introduce errors into calculations based upon fundamental sine functions. To determine the effect of these harmonics, the following equations have been developed. They show that the effect of the harmonics upon the terminal voltage, for a given value of flux per pole, can be taken into account by a single factor, K_ϕ.

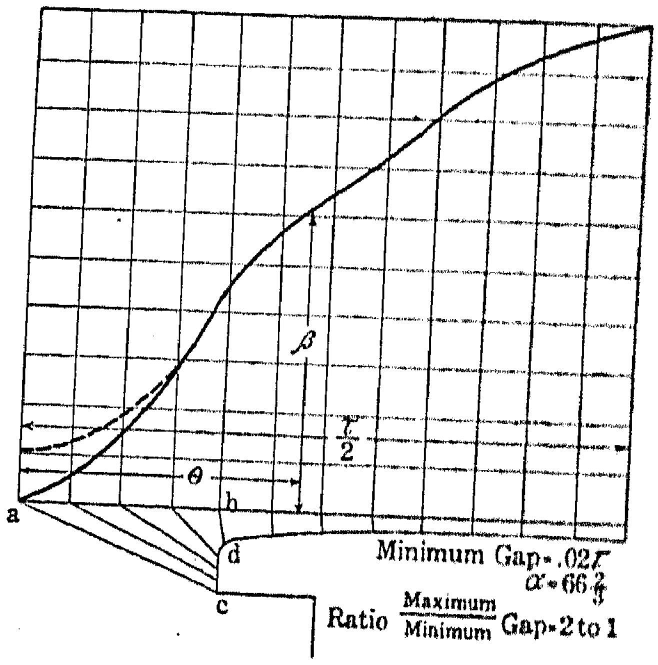

FIG. 36—FLUX DISTRIBUTION CURVE

The flux waves of a number of representative machines were plotted by the following method. Refer to Fig. 36.

The flux at the middle of the pole was assumed to be 10, and the flux at any point toward the edge of the pole was taken inversely as the air gap. To obtain the flux beyond the edge of the pole, the distance *a b* was divided into the same number of spaces as the distance *c d* and lines drawn joining corresponding points on these lines. Assuming the flux to be inversely proportional to the length of air gap, the broken curve was derived. Since the effect of the adjacent poles would be to make the flux zero at the point midway between the poles, the solid line was drawn gradually deviating from the broken curve to pass through zero at the middle point.

The calculated flux waves were compared with test flux waves taken by exploring coils on three machines and found to agree very closely. The effect of saturation of the teeth in the middle of the pole was found to make a slight difference in the flux wave shapes on the same machines at different voltages.

At ordinary tooth densities, however, the variation will make only one or two per cent difference in the value of K_ϕ, for values of voltage from one half to normal.

Flux waves were obtained by the method shown in Fig. 36 for ratios of pole arc to pole pitch of 50, $66\frac{2}{3}$ and 75 per cent, for ratios of maximum to minimum air gap from unity to three to one, and for different minimum air gaps varying from one to four per cent of the pole pitch. The flux waves were then analyzed for the harmonics up to the eleventh, and the values of K_ϕ were calculated. These values of K_ϕ are given in Fig. 20.

The factor K_ϕ gives the ratio of the total flux in the irregular wave to the total flux in a sine wave which would give the same effective voltage at the terminals of the machine.

This is exactly true only when the terminal voltage has a form factor equal to that of a sine wave. This is approximately the case with practically all commercial machines as now built, as the distribution of armature winding, short pitch of armature coils and star connection of the machines all operate to eliminate the effect of harmonics in the flux wave.

Let Fig. 36 represent the flux distribution at no load.

$$\beta = (\beta_1 \sin \theta + \beta_3 \sin 3\,\theta + \beta_5 \sin 5\,\theta + \ldots.) \qquad (1)$$

Instantaneous voltage generated in unit length of one conductor is

$$e' = \frac{\nu \beta}{10^8} = \frac{\nu}{10^8} (\beta_1 \sin \theta + \beta_3 \sin 3\,\theta + \beta_5 \sin 5\,\theta + \ldots.) \quad (2)$$

Where

ν = velocity of flux relative to conductor, in centimeters per second.

β = flux density, lines per sq. cm.

Effective value of voltage is

$$e_{eff.} = \sqrt{\frac{2}{\pi} \int_0^{\pi/2} e'^2 d\,\theta}$$

$$= \frac{\nu}{10^8} \sqrt{\frac{2}{\pi} \int_0^{\pi/2} \beta^2\, d\,\theta} \qquad (3)$$

$$\beta^2 = (\beta_1 \sin \theta + \beta_3 \sin 3\,\theta + \beta_5 \sin 5\,\theta + \ldots.)^2$$
$$= \beta_1^2 \sin^2 \theta + \beta_3^2 \sin^2 3\,\theta + \beta_5^2 \sin^2 5\,\theta + \ldots.$$
$$+ 2\,(\beta_1\,\beta_3 \sin \theta \sin 3\,\theta + \beta_1\,\beta_5 \sin \theta \sin 5\,\theta + \beta_3\,\beta_5 \sin 3\,\theta \sin 5\,\theta + \ldots.)$$

Since $2\int_0^{\pi/2} (\beta_1\,\beta_3 \sin \theta \sin 3\,\theta + \beta_1\,\beta_5 \sin \theta \sin 5\,\theta + \beta_3\,\beta_5 \sin 3\,\theta \sin 5\,\theta + \ldots.)$ is zero.

$$e_{eff.} = \frac{\nu}{10^8} \sqrt{\frac{2}{\pi}\int_0^{\pi/2} (\beta_1^2 \sin^2 \theta + \beta_3^2 \sin^2 3\,\theta + \beta_5^2 \sin^2 5\,\theta + \ldots)\, d\,\theta}$$

$$e_{eff.} = \frac{\nu}{10^8\sqrt{2}} \sqrt{(\beta_1^2 + \beta_3^2 + \beta_5^2 + \ldots.)} \tag{4}$$

$$e_{ff.} = \frac{\nu\,\beta_1}{10^8\sqrt{2}} \sqrt{1 + k_3^2 + k_5^2 + \ldots.} \tag{5}$$

Where

$$k_3 = \beta_3 \div \beta_1$$
$$k_5 = \beta_5 \div \beta_1 \quad \text{etc.}$$
$$\nu = 2\,\tau f \tag{6}$$

where

τ = pole pitch

f = cycles per second.

$$e_{ff.} = \frac{2\,\tau f\,\beta_1}{10^8\,\sqrt{2}} \sqrt{1 + k_3^2 + k_5^2 + \ldots.} \tag{7}$$

The effective voltage of a complete, full-pitch coil of N_q series turns, (that is, concentrated winding) is

$$E_{eff.} = 2\,N_q\,l\,e_{eff.} \tag{8}$$

where

l = gross stacked length of core (iron + ducts)

$$E_{eff.} = \frac{4\,\tau\,l\,f\,N_q\,\beta_1}{10^8\,\sqrt{2}} \sqrt{1 + k_3^2 + k_5^2 + \ldots.} \tag{9}$$

If the coil does not span the full pitch τ, the quantity under the radical is reduced. The voltage of the two coil sides are out of phase by angle $(1-p)\ \pi$, for fundamental and by angle

$$(1 - p)\ n\ \pi$$

for n *th* harmonic,
where,

$$p = \frac{\text{coil pitch}}{\text{pole pitch}}$$

Hence the reduction factor will be the cosine of half the angle, that is,

$$k_p = \cos (1-p) \frac{\pi}{2} \qquad (10)$$

for fundamental and

$$k_{p_n} = \cos (1-p) \frac{n\ \pi}{2} \qquad (11)$$

for n th harmonic.

[17]Adams has given curves for k_{pn}

If the N_q series turns per pole per phase are distributed in two or more slots, there is an additional reduction. This factor k_{d_n} is given in table prepared also by [17]Adams.

If the three-phase machine is connected Y, there is a still further reduction, but since the reduction for fundamental is same as for all harmonics, namely 0.866, except the 3rd and 9th which are entirely eliminated, it is not necessary to take this into account separately, as is done with k_{p_n} and k_{d_n}. [17]Adams gives also a table showing the product

$$k_{p_n} k_{d_n} = k_{r_n} \qquad (12)$$

for different conditions.

Hence the effective voltage per phase (per leg) is

$$E_{eff.} = \frac{4\,\tau\, l\, f\, N_p\, \beta_1}{10^8 \sqrt{2}} \sqrt{k_r^2 + k_{r3}^2 k_3^2 + k_{rs} k^2_5 + \dots} \qquad (13)$$

Where

N_p = series turns per phase.

To find value of β_1, refer to equation (**1**)

$$\beta_1 = \beta_1 \sin \theta + \beta_3 \sin 3\ \theta + \beta_5 \sin 5\ \theta + \dots \qquad (1)$$

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17. Trans. A. I. E. E., Vol 28, Part 2, p. 798, 799, 805.

$$\beta_{ave.} = \frac{2}{\pi}\int_0^{\pi/2} \beta \, d\,\theta$$

$$\beta_{ave.} = \frac{2}{\pi}\left[\beta_1 + \frac{\beta_3}{3} + \frac{\beta_5}{5} + \ldots.\right]$$

$$\beta_{ave.} = \frac{2}{\pi}\,\beta_1\left(1 + \frac{k_3}{3} + \frac{k_5}{5} + \ldots.\right) \qquad (14)$$

Total flux per pole is

$$\Phi = \tau\, l\ \beta_{ave.}$$

$$\Phi = \frac{2}{\pi}\,\tau\, l\, \beta_1\left(1 + \frac{k_3}{3} + \frac{k_5}{5} + \ldots.\right) \qquad (15)$$

Let

$$K_\phi = \left(1 + \frac{k_3}{3} + \frac{k_5}{5} + \ldots.\right) \qquad (16)$$

$$\Phi = \frac{2}{\pi}\,\tau\, l\, \beta_1\, K_\phi$$

$$\beta_1 = \frac{\Phi}{\frac{2}{\pi}\,\tau\, l\, K_\phi} \qquad (17)$$

Substituting in (13)

$$E_{eff.} = \frac{4.44\, f\, N_p\, \Phi}{10^8\, K_\phi}\sqrt{k_r^2 + k_{r3}^2\, k_3^2 + k_{r5}^2\, k_5^2 + \ldots.} \qquad (18)$$

It is obvious from (**18**) that if fractional pitch is used and the coils are distributed, k_{rn} becomes practically negligible for the higher harmonics. If the machine is Y-connected, the third and its multiples are eliminated, leaving the value of the radical very near to

which is k_r

$k_p\, k_d$

In most cases the radical may be taken as $k_r = k_p\, k_d$ with very little error in $E_{eff.}$

In concentrated windings, full pitch, Δ-connected, the radical cannot be neglected. It can be neglected with little error for machines with two or more slots per pole per phase, or fractional slots per pole per phase, Y-connected armature. In such cases,

$$E_{eff.} = \frac{4.44\, f\, N_p\, \Phi\, k_p\, k_d}{10^8\, K_\phi} \qquad (19)$$

This means that the flux of the fundamental only

$$\Phi_1 = \frac{\Phi}{K_\phi}$$

is useful. That is, the useful flux is

$$\Phi_1 = \int_0^\pi \beta_1 \sin\theta\, d\,\theta \qquad (20)$$

This value Φ_1 is what design calculations have usually been based on, *i.e.* calculations which assumed sine distributions of flux.

Therefore, such calculations should be modified only by the factor

$$K_\phi$$

Thus

$$\Phi = K_\phi\, \Phi_1$$

$$\Phi = \frac{E_{eff.} \times K_p \times K_d \times K_\phi \times 10^8}{4.44\, f\, N_p} \qquad (21)$$

where

$E_{eff.}$ = effective voltage per phase. *i.e.* per leg

N_p = series turns per phase.

f = frequency.

$K_p = \frac{1}{k_p}$ = fractional pitch coefficient ordinarily used in design

$K_d = \frac{1}{k_d}$ = distributed coefficient ordinarily used in design, and given in Table II.

$K_\phi = \left(1 + \frac{k_3}{3} + \frac{k_5}{5} + \ldots.\right)$, and is given in Fig. 20.

Where

k_3 = percent third harmonic in flux wave
k_5 = " fifth " " " "
k_7 = " seventh " " " "
etc.

If the radical in equation (**18**) is not negligible, (**21**) becomes

$$\Phi = \frac{E_{ff} \cdot \times K_\phi \times 10^8}{4.44 f N_p} \sqrt{k_r^2 + k_{r3}^2 k_3^2 + k_{r5}^2 k_5^2 + \ldots} \qquad (22)$$

APPENDIX B

ARMATURE REACTION

Much has been written about armature reaction, particularly in Europe and during the last few years[18]. Practically without exception the method of attacking the problem has been to obtain an expression for the wave of armature m.m.f. and an expression for the wave of field m.m.f. and combine the two for the resultant which determines the flux. But at this point difficulty is encountered, especially on salient-pole machines. One must know the reluctance at all points, as well as the m.m.f. before the flux can be obtained. On turbine generators where reluctance is practically uniform over the whole pole pitch, the flux can be assumed to be proportional to the m.m.f., and the problem works out well. However, such a method has not succeeded in solving, without questionable assumptions, the case of salient poles.

The method given in the following pages for solving the case of salient poles is based on the principle that *the effectiveness of an armature ampere turn at any instant and at zero power factor, as compared with a field ampere turn, in establishing flux in the mutual magnetic path, depends upon the percentage of the total useful no-load flux that the armature turn would enclose at that instant on open circuit.* The method was worked out during the investigation of armature self-induction covered in the main body of the paper. The authors were met in this investigation by the necessity of determining more accurately the relative strength of armature and field, since the armature self-induction is tested by taking the difference between these quantities on sustained short circuit. This, as in the case given in Appendix A, is one of the few instances where an apparently hopeless case has worked out to the simplest result: the armature reaction is calculated by the familiar formula based on sine wave assumption, and this is modified by a single factor which may be taken directly from the curve shown in Fig. 20

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[18] See bibliography at end of Appendix B.

It was found by methods described in Appendix A that the no-load flux wave could be very closely determined from the dimensions of the machine. The flux-wave shape was thus plotted for the variety of conditions within the ranges of normal design. These waves were expressed as a Fourier's series. With the equation of the wave given, the solution easily followed.

Let Fig. 37A represent the no load flux wave. If one armature ampere turn is placed symmetrically about the field magnetic circuit (*i.e.* directly opposite it, when $\theta = O$ in Fig. 37A), the flux wave will be a replica of that produced by one field ampere turn. If the coil does not span the entire pole pitch, then within the portion which is spanned the flux wave will be a replica of that same portion of a complete wave of a full-pitch turn, as in Fig. 37B.

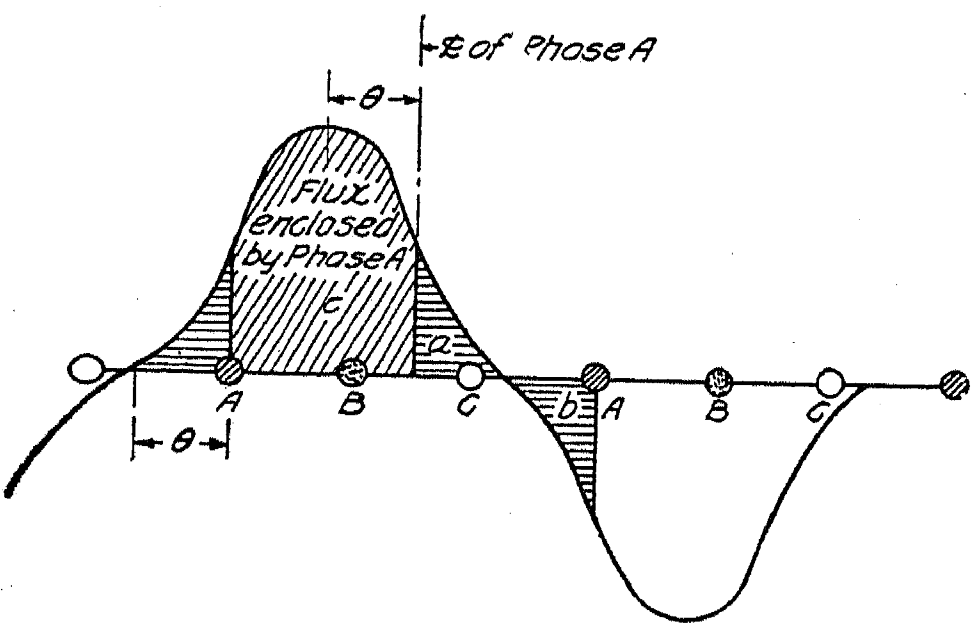

FIG. 37A

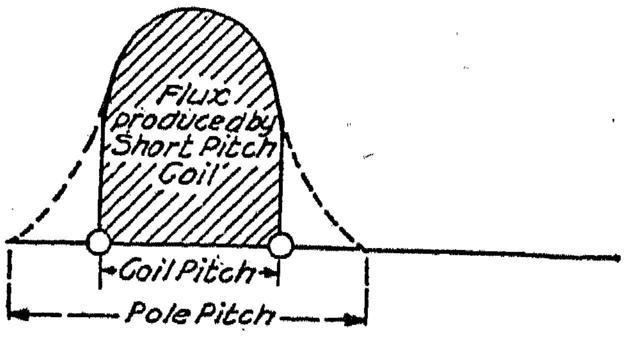

FIG. 37B

On sustained short circuit, an armature current is maintained which is just sufficient to create an opposing m.m.f. acting on the mutual magnetic circuit, equal to that of the field; or in other words, sufficient to annihilate the mutual flux. Hence, as mentioned above, the measure of the effectiveness of any turn at any instant is the percentage of the total mutual flux it would enclose on open circuit.

Refer to Fig. 37A. A full-pitch turn, A, encloses all of the flux when in position $\theta = 0$. At any angle θ, observe that the flux enclosed by A is the same as if the two sides of the coil were moved toward the center of the flux wave by the same angle, θ;

because the positive flux, a, is now neutralized by the negative flux b.

The effectiveness of the turns in phase A at the angle θ is

$$\lambda_\theta = \frac{\text{area } c}{\text{area total wave}} = \frac{c}{c + 2a} \qquad (1)$$

As in Appendix A, the equation of the wave is

$$y = \sin\theta + k_3 \sin 3\,\theta + k_5 \sin 5\,\theta + \ldots. \qquad (2)$$

$$a = \int_0^\theta y\,d\,\theta \qquad (3)$$

From equations (14) and (16), Appendix A, it follows that

$$c + 2a = \pi \times \text{average ordinate}$$

$$= \pi \times 0.636\left(1 + \frac{k_3}{3} + \frac{k_5}{5} + \ldots.\right)$$

$$c + 2a = 2\,K_\phi \qquad (4)$$

From (3)

$$a = \int_0^\theta y\,d\,\theta = \int_0^\theta (\sin\theta + k_3 \sin 3\,\theta + k_5 \sin 5\,\theta + ..)\,d\,\theta$$

$$a = (1 - \cos\theta) + \frac{k_3}{3}(1 - \cos 3\,\theta) + \frac{k_5}{5}(1 - \cos 5\,\theta) + \ldots.$$

$$a = K_\phi - \left(\cos\theta + \frac{k_3}{3}\cos 3\,\theta + \frac{k_5}{5}\cos 5\,\theta + \ldots.\right) \qquad (5)$$

From (4),

$$c = 2\,(K_\phi - a)$$

Hence, from (1)

$$\lambda_\theta = \frac{2\,(K_\phi - a)}{2\,K_\phi}$$

$$\lambda_\theta = \left(1 - \frac{a}{K_\phi}\right) \qquad (6)$$

$$= 1 - \frac{K_\phi - \left(\cos\theta + \frac{k_3}{3}\cos 3\,\theta + \frac{k_5}{5}\cos 5\,\theta + \ldots.\right)}{K_\phi}$$

$$\lambda_\theta = \frac{1}{K_\phi}\left(\cos\theta + \frac{k_3}{3}\cos 3\,\theta + \frac{k_5}{5}\cos 5\,\theta + \ldots\right) \tag{7}$$

For the case of a three-phase machine with 60-deg. phase belt, and concentrated winding, and assuming a sine wave armature current (which practically exists on sustained short circuit), the effective m.m.f. of phase A, Fig. 37A is

$$\lambda_\theta \sqrt{2}\, N_q\, I \cos\theta \tag{8}$$

of phase B,

$$\lambda_{(\theta+60)} \sqrt{2}\, N_q\, I \cos(\theta + 60) \tag{9}$$

of phase C,

$$\lambda_{(\theta+120)} N_q\, I \cos(\theta + 120) \tag{10}$$

where,

N_q = series armature turns per pole per phase.
I = r.m.s. amperes per turn.

$$\lambda_{(\theta+60)} = \frac{1}{K_\phi}\left[\cos(\theta + 60) + \frac{k_3}{3}\cos(3\,\theta + 180) + \frac{k_5}{5}\cos(5\,\theta + 300) + \ldots\right]$$

$$\lambda_{(\theta+60)} = \frac{1}{K_\phi}\left[(0.5\cos\theta - 0.866\sin\theta) - \frac{k_3}{3}\cos 3\,\theta + \frac{k_5}{5}(0.5\cos 5\,\theta + 0.866\sin 5\,\theta) + \frac{k_7}{7}(0.5\cos 7\,\theta - 0.866\sin 7\,\theta)\right] \tag{11}$$

This neglects harmonics above 7th.
By the same reasoning,

$$\lambda_{(\theta+120)} = \frac{1}{K_\phi}\left[(-\,0.5\cos\theta - 0.866\sin\theta) + \frac{k_3}{3}\cos 3\,\theta + \frac{k_5}{5}(-\,0.5\cos 5\,\theta + 0.866\sin 5\,\theta) + \frac{k_7}{7}(-\,0.5\cos 7\,\theta - 0.866\sin 7\,\theta)\right] \tag{12}$$

The total m.m.f. per pole (*i.e.* the armature reaction) is

$$\begin{aligned} A = {} & \lambda_\theta \sqrt{2}\, N_q\, I \cos\theta \\ & + \lambda_{(\theta+60)} \sqrt{2}\, N_q\, I\, (0.5 \cos\theta - 0.866 \sin\theta) \\ & - \lambda_{(\theta+120)} \sqrt{2}\, N_q\, I\, (0.5 \cos\theta + 0.866 \sin\theta) \end{aligned}$$

$$\begin{aligned} A = {} & \sqrt{2}\, N_q\, I\, [\lambda_\theta \cos\theta + \lambda_{(\theta+60)} (0.5 \cos\theta - 0.866 \sin\theta) \\ & - \lambda_{(\theta+120)} (0.5 \cos\theta + 0.866 \sin\theta)] \end{aligned} \qquad (\mathbf{13})$$

Substituting equations (**7**), (**11**), and (**12**),

$$\begin{aligned} A = \sqrt{2}\, N_q\, I \Bigg[& \frac{\cos\theta}{K_\phi} \left\{ \cos\theta + \frac{k_3}{3} \cos 3\,\theta + \frac{k_5}{5} \cos 5\theta + \frac{k_7}{7} \cos 7\,\theta \right\} \\ & + \frac{(0.5 \cos\theta - 0.866 \sin\theta)}{K_\phi} \Big\{ (0.5 \cos\theta - 0.866 \sin\theta) - \frac{k_3}{3} \cos 3\,\theta \\ & + \frac{k_7}{7} (0.5 \cos 7\,\theta - 0.866 \sin 7\,\theta) \Big\} \\ & - \frac{(0.5 \cos\theta + 0.866 \sin\theta)}{k_\phi} \Big\{ (-0.5 \cos\theta - 0.866 \sin\theta) \\ & + \frac{k_3}{3} \cos 3\,\theta + \frac{k_5}{5} (-0.5 \cos 5\,\theta + 0.866 \sin 5\,\theta) \\ & + \frac{k_7}{7} (-0.5 \cos 7\,\theta - 0.866 \sin 7\,\theta) \Big\} \Bigg] \end{aligned}$$

Simplifying, this becomes

$$\begin{aligned} A = \frac{1.5 \sqrt{2}\, N_q\, I}{K_\phi} \Bigg[& 1 + \cos\theta \left(\frac{k_5}{5} \cos 5\,\theta + \frac{k_7}{7} \cos 7\,\theta \right) \\ & + \sin\theta \left(- \frac{k_5}{5} \sin 5\,\theta - \frac{k_7}{7} \sin 7\,\theta \right) \Bigg] \end{aligned} \qquad (\mathbf{14})$$

If the armature turns are distributed over the phase belt in a number of coils instead of being concentrated in one coil, and if the coils do not span the full pole pitch, the bracket quantity of equation (**14**) is modified as follows:

1 becomes $k_p\ k_d$

k_5 and k_7 are multiplied by k_{rn}

Adams[19] has given tables for these reduction factors.

However, since the 5th and 7th are usually not large, and since, as shown by (**14**), their effect is decreased by the order of the harmonic, it may be safely assumed that the further decrease by k_{rn} renders negligible all of the harmonics, in cases where the winding is distributed in more than one slot per pole per phase.

Hence for such cases equation (**14**) becomes,

$$A = \frac{1.5\sqrt{2}\,N_q\,I}{K_\phi\ K_d\ K_p} = \frac{2.12\,N_q\,I}{K_\phi\ K_d\,K_p}\ \text{(three-phase)} \qquad (\mathbf{15})$$

where

$$K_p = \frac{1}{k_p} = \frac{1}{\sin\ (p\ 90)}$$

$$K_d = \frac{1}{k_d}\ \text{(See Table II, Part I, for value of } K_d.)$$

N_q = series turns per pole per phase

I = current per turn

K_ϕ = flux distribution coefficient given in Fig. 20.

APPENDIX C

CALCULATION OF EXCITATION

The reactance as calculated by the method developed in this paper may be used in calculating excitation of alternating-current machines. The method used is an application of the generally accepted principles of excitation calculation, which have been developed by a number of writers in slightly different forms.

The diagram, Fig. 18, illustrates the factors used in the calculation for a generator with lagging current output.

I_n = normal armature current.

E = terminal voltage (is taken as unity for calculations on the per cent basis.)

E_i = in-phase component of terminal voltage
= $\cos\ \theta$

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[19] TRANS. A. I. E. E., Vol. 28, Part 2, pp. 798, 799, 850.

E_x = quadrature component of terminal voltage
= sin θ
cos θ = power factor
r_{pa} = voltage required to overcome resistance drop in per cent of E. ($r_{p_a} = 0.01$ means 1 per cent)
x_{pa} = voltage required to overcome reactance drop in per cent of E. ($x_{p_a} = 0.2$ means 20 per cent)
E_1 = internal induced voltage under load in per cent of E.
$= (\cos\theta \pm r_{pa}) + j(\sin\theta \pm x_{pa})$

From the diagram it can be seen that $\cos\theta = \frac{E_i + r_{pa}}{E_1}$

Note that for a motor the sign of r_{pa} will be minus.

Also $\sin\theta = \frac{E_x + x_{p_a}}{E}$.

This expression holds for the over-excited condition, that is, for a generator with lagging current or a motor with leading current, but the sign of x_{p_a} will be minus for the under-excited conditions.

A = ampere turns per pole required to overcome the effective armature reaction.

$$A = \frac{2.12\, N_q\, I_n}{K_p\, K_d\, K_\phi} \text{ for three phase}$$

$$= \frac{1.41\, N_q\, I_n}{K_p\, K_d\, K_\phi} \text{ for two phase}$$

N_q = series armature turns per pole per phase
K_p = fractional pitch coefficient
K_d = armature winding distribution coefficient.
K_ϕ = flux distribution coefficient.
Φ = no load flux per pole for normal voltage E.
Φ_1 = full-load flux per pole for internal induced voltage E_1
Φ_2 = flux for impedance drop.
F_1 = ampere turns from no-load saturation curve required to produce Φ_1.
F_0 = full-load ampere turns per pole.

The m.m.f. diagram is all that is required for the calculation of the field current and this diagram taken from Fig. 18 for the over-excited condition is given in Fig. 19A, and for under-excited condition in 19B. The following relation may be derived from this diagram by inspection.

$$F_0 = (F_1 + A \sin \theta_1) + j A \cos \theta_1$$

For the under-excited condition the expression

$$F_0 = (F - A \sin \theta_1) + j A \cos \theta_1$$

The application of this theory can best be illustrated by an example. The formulas necessary for the calculations are given below

$$\cos \theta_1 = \frac{\cos \theta \pm r_{pa}}{E_1}$$

Sign is + for generator
Sign is – for motor

$$\sin \theta_1 = \frac{\sin \theta \pm x_{pa}}{E_1}$$

$$F_0 = (F_1 \pm A \sin \theta_1) + j A \cos \theta_1$$

Sign is + for generator lagging current or motor leading current.

Sign is – for generator leading current or motor lagging current.

$$E_1 = (\cos \theta \pm r_{pa}) + j (\sin \theta \pm X_{pa})$$

Example:

Generator—Lagging current.

Assume $r_{pa} = 0.01 = 1$ per cent
$x_{pa} = 0.15 = 15$ per cent
Power factor $\cos \theta = 0.80$
$\sin \theta = 0.60$
$F = 3800$ ampere turns (not used in calculations)
$F_1 = 4500$ ampere turns.
$A = 3000$ ampere turns.
Then $E_1 = (\cos \theta + r_{pa}) + j (\sin \theta + x_{pa})$
$= 0.81 + j\, 0.75 = 1.10$

The value of F_1 is obtained from the no-load saturation curve (test or calculated) at 110 per cent normal terminal voltage and is 4500 ampere turns.

$$\sin \theta_1 = \frac{0.75}{1.10} = 0.68$$

$$\cos \theta_1 = \frac{0.81}{1.10} = 0.735$$

$$\begin{aligned} F_0 &= (4500 + 0.68 \times 3000) + j\,0.735 \times 3000 \\ &= 6900 \text{ ampere turns.} \end{aligned}$$

Motor—Lagging current.
Same data as above except $F_1 = 3400$ ampere turns.

$$\begin{aligned} E &= (\cos \theta - r_{pa}) + j\,(\sin \theta - x_{pa}) \\ &= 0.79 + j\,0.45 = 0.91 \end{aligned}$$

F_1 corresponds to 91 per cent of normal voltage and is 3400 ampere turns.

$$\cos \theta_1 = \frac{0.79}{0.91} = 0.87$$

$$\sin \theta_1 = \frac{0.45}{0.91} = 0.495$$

$$\begin{aligned} F_0 &= (3400 - 0.495 \times 3000) + j\,0.87 \times 3000 \\ &= 3230 \text{ ampere turns.} \end{aligned}$$

The calculation of the above generator may be represented by the following simple calculation.

$$E \left\{ \begin{array}{c|} 0.81 \\ 0.75 \end{array} \right. = 1.10$$

$$F_0 \left\{ \begin{array}{lcc} & & \begin{array}{l} 3000 = A \\ \underline{4500} = F \end{array} \\ \dfrac{3000}{1.10} \times 0.75 = & \left. \begin{array}{r} \underline{2040} \\ 6540 \end{array} \right| & \\ & & = 6900 = F_0 \\ \dfrac{3000}{1.10} \times 0.81 = & \left. 2210 \right| & \end{array} \right.$$

Note that $\left. \begin{array}{r} 0.81 \\ 0.75 \end{array} \right| = 1.10$ and $\left. \begin{array}{r} 6540 \\ 2210 \end{array} \right| = 6900$

indicate combination of vectors at 90 deg.

This method of calculation of field current is an approximation, and does not allow for the increased field leakage under load. This is probably compensated for by including the added no-load saturation in the armature magnetic circuit which does not exist under load. The method has been found to give a very close check on test results on a very large number of commercial machines.

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DISCUSSION ON "REACTANCE OF SYNCHRONOUS MACHINES AND ITS APPLICATIONS" (DOHERTY AND SHIRLEY), ATLANTIC CITY N. J., JUNE 28, 1918.

Carl J. Fechheimer: All who have been identified with the design of a-c. synchronous machinery have appreciated the great importance of internal reactance of the stator. All of us have grappled with the problem of estimating the value of the reactance, and probably there are no two who will agree either on a method of calculating its value or even on its magnitude after the machine is tested. Certainly, we cannot agree upon the value of reactance until we establish what reactance means, and are in accord upon its valuation from test data. Thus, as pointed out by the authors, the reactance determined from instantaneous short circuits may be different than that which obtains during steady operation.

We shall enumerate the most important ways by which the stator reactance can be measured.

(1) With the use of the open-circuit saturation and short-circuit curves, and a knowledge of the proportions of the machine, the ampere turns required to neutralize the armature reaction on sustained short circuit may be deducted from the total ampere turns on short circuit, thus leaving the ampere turns which are needed to drive the flux through the magnetic circuit. From this latter, and with the open-circuit saturation curve, the reactance can be estimated.

(2) From no-load saturation curve, a corrected curve which assumes full-load leakage may be drawn. From this latter curve, and with the use of the load saturation curve at nearly zero power factor, the drop may be broken up into armature reaction and armature reactance by assuming that these are constant along the curve.

(3) The rotor may be removed and single-phase current at normal frequency may be circulated in the stator windings between one terminal and neutral for three-phase star connection. Inasmuch as the resistance drop is usually negligible, we may take the reactance to be the volts from terminal to neutral, divided by the current.

For three-phase delta, circulate current in one leg: for two-phase, circulate current in one phase.

(4) Same as 3, except that instead of single-phase current between neutral and terminal, polyphase current may be circulated between terminals.

(5) Same as 3, except with the rotor in position. (This gives different values for different positions of the rotor.)

(6) Same as 4, except with the rotor in position. (This gives different values for different positions of the rotor.)

(7) A condenser may be connected across the terminals of the machine, the value of the capacity of which may be varied, so that approximate resonance can be obtained. It is then a simple matter to calculate the inductance.

(8) The reactance may be estimated from oscillograms obtained on instantaneous short circuits. In that case, the peaks of the current should be projected back to the ordinates drawn at the instant of short circuit.

These eight methods of measurement give various values of reactance, although two or more methods may be found to check in some machines. What then shall we say is the correct method? The subject is well worthy of the attention of the Standardization Committee of the Institute. We shall attempt to discuss these methods briefly.

No. 1 (determination from short-circuit curve) has two decided objections: (a) It is difficult to estimate with sufficient accuracy the armature reaction, and a relatively small error in this estimate will be magnified considerably in the value of the reactance; thus 5 per cent error in estimate of armature reaction may result in 40 per cent error in reactance. (b) The voltage which is induced in the individual conductors on sustained short circuit (and which is entirely consumed by impedance) is in general of badly distorted wave form.

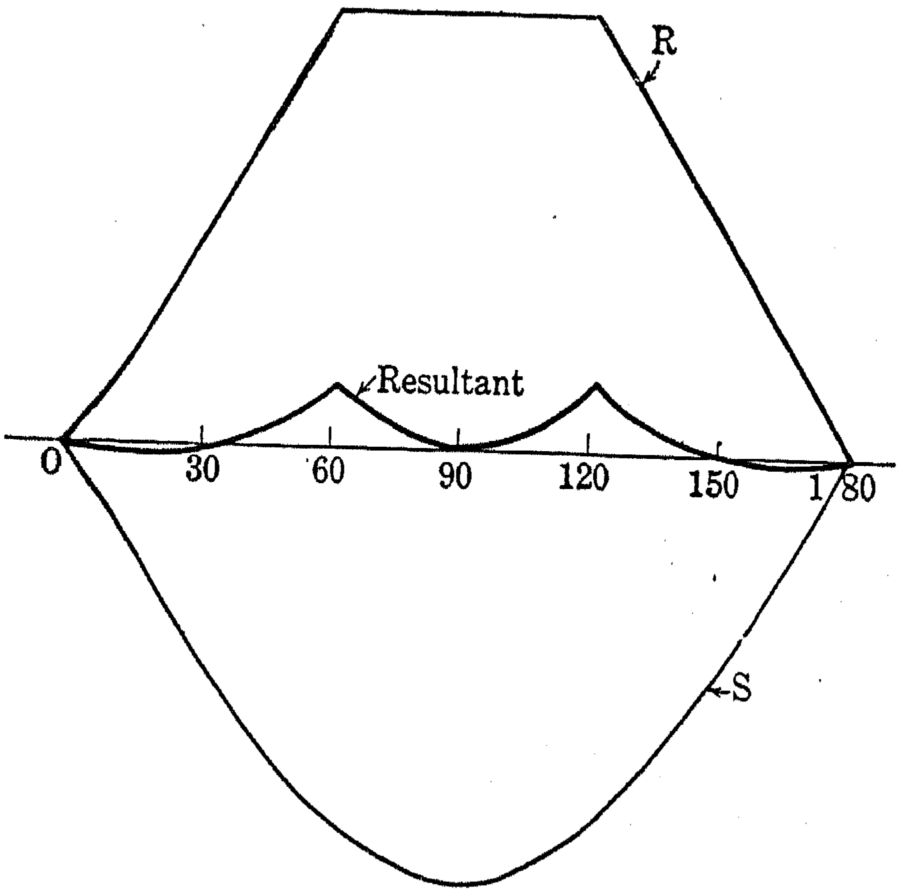

FIG. 1

For instance, we are showing in Fig. 1 the ideal field form of the magnetomotive force of the rotor of a turbo-generator *R*, and an assumed sine wave space distribution of the m. m. f. of the stator current *S* (ideal conditions). The resultant of these two gives the m. m. f. which drives the flux through the magnetic circuit. Although the higher harmonics which would result from the distortion are pretty well damped out in the current wave form, they still appear in the wave of induced e. m. f. in the individual conductors. It will therefore readily be seen that reactance computed in this way from tests is liable to be considerably in error. That the distortion is very much reduced at full load, zero power factor, is quite evident.

No. 2 (determination from no-load and zero power factor load curves) has the objection that it involves calculation of the full-load leakage no-load saturation curve, which is subject to error. The proportions of reaction and reactance as finally determined by this method, even if we were able to estimate the full-load leakage curve, are also liable to be in error. Furthermore, saturation in the magnetic circuit may cause the reactance to change in magnitude along the curve.

Nos. 3 to 6, inclusive, (pertaining to circulating current in windings) all give different results, but our observations lead us to believe that No. 3 gives results which are more nearly correct than 4, 5 or 6. It is certain that with the rotor in position the reactance measured is considerably higher than that which actually obtains. With three-phase currents, there is sufficient mutual inductance to give about 15 per cent higher inductance than with single-phase.

No. 7 has the disadvantage that a condenser has not lent itself readily to use in our test rooms. Furthermore, we have had too little experience with this method to compare it with others, but believe there are possibilities and we therefore suggest that it be further investigated.

No. 8 (pertaining to instantaneous short circuit) has the drawback that it subjects the machine to severe strains, and is in general expensive and too elaborate for general commercial use.

Discussing first the sustained reactance, it is our understanding that the authors of the paper used Method No. 1 (short-circuit, and no-load saturation curves) for checking their formula for sustained reactance against test. All of us will agree that an analytical determination of reactance is next to impossible, and we must have recourse to test data, and thereby obtain empirical constants; especially is this true of end connection leakage, and we think that it is essential for tooth-tip leakage also. Inasmuch as the authors used a test method which is subject to an error of the order of 50 per cent, we must be cautious in adopting their formula, and rely upon the accuracy of reactance estimated thereby. Aside from the error pointed out in determining reactance from test, we find in Table I, that 40 per cent of the machines show a difference between calculated and test values of more than 20 per cent, and the discrepancy in one case is 145 per cent, and in another 185 per cent.

We believe that the authors have allowed too much for increase due to mutual inductance with three-phase over single-phase. Tests which we have made removing the rotor and circulating current through windings, indicate only 16 per cent higher reactance three-phase (star-connection) than single-phase terminal to neutral.

Turning now to the reactance which obtains in instantaneous short circuit, we are in agreement with the authors that the leakage flux of the rotor plays a considerable part in reducing the rush of current, although we do not view the phenomenon from the same point of view. For instance, we believe that saturation of the rotor poles in conjunction with rotor leakage flux, may have considerable influence in reducing the pole leakage. For example, let us assume that the combined transient reactance is such that the stator current can rise to ten times normal r. m. s. value, and that the total ampere turns in the rotor also rise to ten times the no-load value. Then the difference in magnetic potential between poles, being ten times, the leakage flux will rise to ten times. If the no-load field leakage were 12 per cent, then with ten times field current, there will be $12 \times 10 = 120$ per cent leakage, so that, if the density in the poles at no load were 90,000 lines per square inch, the density would rise to $\left(\frac{1 + 1.2}{1 + 0.12}\right) 90{,}000 = 177{,}000$ lines per sq. in. That the density in the poles does not reach such high value will be quite evident from a glance at the B - H curve for the steel, and the result is that some of the flux which had previously been useful as generating e. m. f. in the stator conductors, "sinks" into leakage flux, so as to maintain substantially constant interlinkages with the conductors in the rotor (these may be field turns, rivets, or any paths for the currents). Saturation of the stator teeth may also play some part, for with the reduction in main flux, more leakage flux may flow, and this may, in some teeth, be of sufficient magnitude to reduce the percentage leakage. With smooth, solid rotors, such as those in turbo-generators, the phenomenon as regards pole saturation is quite different from that which obtains with definite poles, because most of the counter m. m. f. in the rotor is produced by eddy-currents near the rotor surface, and they are not, in turn, productive, of a great percentage increase in leakage on instantaneous short circuit. Although it is very difficult to check with the sustained short circuit and no-load and zero power factor saturation curves, it seems that the transient and sustained reactances in turbo-generators are not in very great disagreement, and we note that the authors are of the same opinion.

Other factors of not negligible importance as affecting the transient reactance might be cited. We shall mention only one: In salient-pole alternators, most of the spiders are solid, and as is well known, the flux (pulsates whether polyphase or single-phase short circuit) and eddy-currents are induced in the spider, which in turn influence the rotor leakage.

As we look over the authors' paper we note that they believe "saturation is not a very serious factor," and they have not considered eddy currents in the spider. We are very much of the opinion that such factors cannot be ignored if even reasonably approximate accurate results are desired.

Again, eddy-currents and saturation play their parts in estimating the attenuation factors—as both affect resistance and inductance. Owing to the fact that the saturation of the poles decreases with time, due to reduction in m. m. f. and flux, and therefore (1) the distribution of leakage fluxes from pole to pole are altered (assuming salient pole-construction) which in turn affects the interlinkages with the field winding, (thereby changing the coefficient of self-induction of the field circuit); and (2) the percentage of the total flux which enters the stator is automatically increased. Inasmuch as eddy currents are in parallel with the field currents, insofar as net m. m. f. is concerned, the resistance and inductance of their paths are necessarily coupled magnetically with that of the field circuit. It will, therefore, be seen that an analytical determination of the attenuation factors is well nigh impossible, and that these factors may not be constant, but change with time.

The whole problem involves so many difficulties and complications that we believe that we must rely upon empirical data. Our first problem is to find a way of measuring sustained reactance, and then modify the constants in an empirical equation to enable prediction of reactance within a maximum error of, say, 15 to 20 per cent. Then we can multiply this value of reactance by some factor or factors determined from test so as to enable agreement with transient reactance, the factors being of different magnitude for various types of machines. The determination of the field and armature attenuation factors which are probably not constants, must be determined from oscillograms of short circuits, and should be some function of the time.

In regard to the measurement of sustained reactance, we are planning to conduct some tests in line with method No. 7 (involving a condenser) and take oscillograms to determine the magnitude of the fundamental as well as of the harmonics. This method will not, we believe, be suitable for general application, especially not on large machines, because of difficulty in obtaining large condensers, but we hope thereby to establish some constants for modifying measured sustained reactance (say by circulating current in stator with rotor removed) which we can adopt as reliable figures.

I have noted the authors' reference to an error in the writer's paper, and wish to thank them for calling attention to it. I previously noted this mistake about four years ago, but have not called it to the attention of the Institute members.

I believe the authors deserve a great deal of credit for the great amount of work which they have done, for more work of this character would be helpful in cleaning up some of the problems with which we are confronted.

V. Karapetoff: It seems to me that the terms reactance and inductance are used somewhat indiscriminately in this paper. After formula (3) in Part II. the authors say: "The effective field self-induction or field reactance." Reactance is a different concept from self-inductance, and in a paper which is intended for reference purposes it would seem rather unfortunate not to have these terms more definitely discriminated because one has to find out from the context whether the reactance or the inductance is meant. The same criticism applies to other places in the paper.

I should like to see the next administration of the Institute make a strong effort towards improving the "readibility" of the Institute papers. In a mathematical paper like the one under discussion a very careful list of symbols should be demanded of the author, and not left to the author's discretion. It is true that in this particular paper partial lists of symbols are found in two places, but they do not include, by any means, all the symbols, nor do they show in which units these symbols are expressed. I think that this is an important matter, and should not be left to the discretion of the author, but should be made uniform by definite rules of the Editing Committee of the Institute.

Now, as to the method of computation of inductance. A number of years ago our new honorary member, Mr. Oliver Heaviside, showed that inductance and magnetic permeance are almost identical concepts, that the inductance of a coil is equal to the permeance of the magnetic path, times the square of the number of turns. Since the number of turns is a numeric, the physical dimension of inductance is the same as that of magnetic permeance.

The expression for a magnetic permeance is similar to that of the electrical conductance. The permeance of a uniform path is equal to the permeabilty of the medium times the cross-section of the path, and inversely as the length of the path. If the path is not uniform a differential expression for permeance must be properly integrated to cover the whole field. What I am driving at is that computations of inductance should be performed in the same way as computations of the permeance or conductance of an irregular path, that is, by adding the permeances of the individual component paths. Most authors including these authors, compute inductances by taking ratios of magnetomotive forces to the fluxes. That is an unnecessary complication.

Suppose you had an irregular resistance to figure out, you would not assume certain e. m. fs. and currents, and divide one by the other, you would use a formula for an element of resistance, and properly make a summation of such terms for an irregular shape. Why, then, is it necessary in a paper on inductance always to use the ratio of ampere-turns to fluxe ? If you will glance at the formulas in the paper you will find everywhere a ratio of flux to ampere-turns. Why not call this ratio the permeance, and deal with permeances entirely? This procedure has been followed consistently in the chapters on inductance in my book entitled "The Magnetic Circuit."

I notice from the list of references that the authors are more familiar with the English and German literature, and have apparently neglected the French contributions to the subject. While they give credit to Rezelman, through a translation in *The Electrician*, his greater work on leakage reactance in alternators in *La Lumiere Electrique* is not mentioned. Also there is no reference to the latest contributions by André Blondel on the armature reaction in synchronous machines. I believe the paper would gain if more use had been made of the work of the French and Belgian investigators on the subject.

I do not wish to detract from the value of the work done by the authors and the remarkable frankness with which the results are presented, including the test data. I fully appreciate the importance of such a paper for practical designers and students, and my only plea is for a better and clearer presentation of the subject. We are swamped with new reading matter, and yet a day still has only twenty-four hours, so that the skill in presentation almost determines the fate of a paper, that is, whether it will be used or not. If it is not properly presented so that it takes too long to get the meat out of it, many people will not take the trouble to refer to it.

F. D. Newbury: Before we can harmonize various methods of calculating reactance, we must agree upon a method of determining reactance by test; we must define the quantity we are attempting to calculate.

In the alternator, there are two fluxes that are set up by the armature current and the difficulty and confusion in this matter arises from the difficulty in separating one of these fluxes from the other. One produces armature *reaction* and the other produces armature *reactance.*

When the alternator is operated at synchronous speed, on short circuit, and rated current circulates through the armature, the value of field ampere turns is a measure of the m. m. f. necessary to produce both of these fluxes.

The flux of armature *reaction* is that part of the flux set up by the armature current that combines with the field winding flux to produce the effective or resultant exciting flux; the flux of armature *reactance* is that part of the armature flux that is more or less completely localized about the armature winding, and that, by its self-inducing effect, produces a voltage of self-induction in the armature winding that combines with the main induced voltage to produce the terminal voltage. Thus armature reaction may be conveniently treated as a flux phenomenon, as its action does not progress beyond the flux stage; following the same thought, the armature reactance may be considered as a voltage phenomenon. The effect of armature reaction on the total m. m. f. required for a given voltage and current condition is the same at zero voltage, normal voltage, or any other terminal voltage; the effect of armature reactance on the total m. m. f., on the contrary, varies considerably with the terminal voltage because it appears as a voltage and the m. m. f. to generate a certain additional voltage at low terminal voltages is very much less than the m. m. f. required to generate the same additional voltage at a high terminal voltage, because of magnetic saturation.

This distinction is important because it leads to a fairly satisfactory method of separating these two parts of the armature flux. I refer to the graphical method based on the no-load saturation curve and saturation curve at zero-power factor and full-load current. This method, I believe, was first called to the attention of the Institute some fifteen years ago by Mr. B. A. Behrend. In my experience, it gives consistent results for a wide range of machines. It has the advantage that it is obtained by test under flux and voltage conditions very nearly approaching those existing during normal operation. This cannot be said when the separation is based entirely on the short-circuit current at zero terminal voltage which is the method used by the authors. The authors calculate the armature reaction, which is the larger of the two parts, and subtract it from a test value, representing the total armature flux. This test method is in reality a calculated method, because an error of, say five per cent in the calculation of the armature reaction flux, may result in an error of 20 to 40 per cent in the reactance voltage. The graphical method referred to above also involves a calculated correction but a given error in this has a much smaller effect on the result than does the same percentage error in the calculated part in the authors' method.

Mr. Fechheimer referred, in his communication, to various methods of obtaining the reactance by test. Each of these methods involves different conditions, and no two methods really give the same quantity. The first difficulty in this problem, and the main difficulty, I feel, is that we have not agreed on a method of test for reactance. We have not agreed on the fundamental definition of the quantity we propose to calculate. If we can take that step, I think it is a fairly easy matter to obtain methods of calculating reactance that will check with such test results. It may not be possible to obtain a general formula such as the authors have tried to do, that will hold for all machines of all proportions, but it is not a difficult matter to make up a more or less empirical formula for a given line of machines which will check with test results.

As an example of results from the use of some of the methods cited by Mr. Fechheimer, I give the test results of a 4500-kv-a., 11,000-volts, single-phase, 25-cycles, 500 rev. per min. generator.

Tests were made at 4500 kv-a., single-phase, and at 7800 kv-a., three-phase:

	At 4500-kv-a. 1-phase.	At 7800-kv-a. 3-phase.
From instantaneous short-circuit test, (by oscillograph)	13%	10%
From no-load and zero power factor saturation curves	13	11
From locked saturation test (rotor at rest)	15	12.5
From test without rotor in position	21	18

This machine had a low resistance damper winding on the rotor. From the locked saturation test, that is, with the rotor at rest, the reactance is somewhat higher than during synchronous operation, but still quite close to it. This is reasonable, because although the rotor is at rest, the low resistance damper winding practically wipes out that part of the flux corresponding to armature reaction in synchronous operation, so that there is left practically the same flux that, in synchronous operation, is responsible for armature reactance. The same agreement would not exist if the generator did not have the damper winding. A test without the rotor in position showed the reactance to be about 50 per cent greater, or 21 per cent. This, again, is reasonable, because the entire flux set up by the armature winding is effective in producing reactance voltage.

The second part of the paper, referring to sudden short-circuit phenomena, is extremely important, in a practical way. Reactance of alternator windings is of interest, quantitatively, only to designers, except as applied to sudden short-circuit conditions. In this matter, users generally are interested in the value of reactance since it largely determines the value of transient currents during short circuit.

The authors state that the total reactance effective during sudden short circuit is considerably greater than armature reactance, this difference amounting in some cases to 50 per cent of the armature reactance. The authors' method of obtaining armature reactance from tests is so open to question that this generalization is not warranted. In my own experience the armature reactance during synchronous operation (obtained from no-load and full-load saturation curves) is not materially different in value from the total reactance effective during sudden short circuit. This does not mean that the two reactances are identical; it only means that in the total reactance (effective during short circuit) the increase in reactance due to the field has been neutralized by some other action. What this complete action is, we don't know. The fact remains that in short-circuit tests made on generators varying from 4000 to 20,000 kv-a. in rating, with a wide range in speed, and with laminated and solid rotors, with and without damper windings, the tested sustained armature reactance has practically the same value as the total reactance obtained from short-circuit oscillograms. Here again, the difference in test results and conclusions may be due to differences and errors in determining the true armature reactance and agreement can hardly be expected until this question of test method is settled.

W. F. Dawson: Mr. Newbury raised the point of the discrepancies between the figures shown by Messrs. Doherty and Shirley in their method, and his own method, and I am moved to ask if he has considered fully the question of end coil support, the nature of the material that supports the end coil, particularly in turbo-alternators, and the material that clamps these coils? I know personally there is a slightly different method right within the General Electric Company's works. The Schenectady engineers employ a different material for coil support, and rather more of it that I do myself, and I believe the Westinghouse support is different again, and the method of clamping the coils is different. There certainly must be a difference in reactance, whether it is transient reactance or the steady reactance, that effects the excitation, due to these several different methods of support. I simply offer this as a suggestion which may explain the discrepancy.

V. Karapetoff: I should like to ask the authors regarding Fig. 25A. The orthogonal rulings shown there indicate magnetic lines of force and equipotential surfaces. This ruling extends into the space occupied by the coils. As far as I am aware, the magnetic flux has no potential in the space occupied by the current, and I do not quite see how equipotential surfaces could consistently be continued into the space occupied by the coils.

In the simplest case of a circular conductor the magnetic lines of force are concentric circles surrounding the conductor, and the flux inside the conductor also consists of concentric circles. The equipotential surfaces outside the conductor are radial planes, but these planes do not extend inside the conductor, because there the difference of magnetic potential between two points depends upon the path selected. I therefore feel that it might be better to omit the orthogonal ruling or at least the equipotential lines, within the space occupied by the coils, so as to avoid a possible criticism on this point.

Charles L. Fortescue: Is not the uncertainty of this method due, as Mr. Newbury says, to its empirical nature? If we consider the dynamic equations of a system like this, we find that we have a set of three or more linear differential equations of the first order. The solution of such a system of equations depends upon another differential equation, which is of the third or fourth order according to the number of equations. That would mean that the actual transient solution would involve three or more arbitrary constants and, instead of one reactance three or four different reactances would enter into the transient solution in a manner depending on the conditions of the initial short-circuit. It seems to me therefore that it would be very difficult, to express the results in terms of a single compound reactance. We would find possibly that the value of the effective reactance would depend upon at what point on the voltage wave the short circuit took place and that the solution for other conditions, would require different values for the effective reactance, or, putting it another way, one would have to have a different effective reactance for each short circuit. The problem is quite complicated, and as I said before, it is possible that the empirical nature of this reactance accounts for the fact that it does not check up under all conditions.

H. R. Summerhayes: This question of reactance is exceedingly important, not only from the standpoint of the designing engineer to ascertain what the machine will do but for the engineer working on the system who wants to know what amount of energy is released when a short circuit occurs. His calculations on the rupturing capacity of switches, the strength of transformers the installation of protective reactance and setting of relays all depend on the short-circuit values of the generator for different time intervals. In order to properly design the system we must know the characteristics of the generator and this work of the authors is a pains taking attempt to outline methods of obtaining these data.

I hope that the paper will lead to a discussion among the engineers and manufacturing companies which will, if it does not bring to standardization all methods of calculation, at least cause them to come to some agreement on the nomenclature, so that, when the reactance of synchronous machines is discussed engineers will talk the same language and will know what kind of reactance is meant and what factors are included. This will go a long way toward the correct determination of the actual and comparative values for different machines.

N. S. Diamant: In order to make the discussion of this paper as brief and as definite as possible let me start by making the following statements, even at the risk of seeming to be a little dogmatic:

A. The writers do not in any way contradict or disprove any theoretical or experimental results given in my 1915 paper on sudden short circuits.

B. It is very gratifying that they corroborate a number of points brought out in the paper referred to and even those which they think they contradict.

C. The writers are laboring under the serious and entirely false impression that *they* for the first time are calling attention to a fundamental fact which is the a, b, c, of sudden short circuits or of any transient phenomenon for that matter. It was clearly explained by Boucherot in 1911; it was referred to by Berg and experimentally illustrated in my paper.

D. The writers are using in their paper, as published at present, a terminology entirely different from that of other writers and mine, and for some reason which I cannot quite understand, they have failed to ascribe to the terms self-inductance the proper or at least a possible meaning and thus were led first to believe that they disagree with me in certain points, when they actually do not, and secondly were led to the false impression stated under C.

E. The writers make several valuable contributions: (1) They give an expression for calculating the armature leakage reactance. (2) An expression for calculating the leakage reactance of the field. This so far as I am aware is new, although the leakage of the field in "per cent" is derived and used by designers. (3) They have shown how unpractical and inefficient it is to use terms in a loose and unscientific fashion. (4) They have shown that the bibliography they give at the end of their paper, (to which it would be well to add Boucherot's memoirs and Douglas' paper) if properly studied and used, would have shortened their paper considerably and made it more useful. However, the mental attitude of the writers towards the work of others does not seem to be critical and aggressive enough if one is to judge from their statement—"A bibliography which may be of historical interest is included".

Now I shall proceed to prove the above statements, one by one. I regret that with reference to C and D I shall have to introduce here some elementary text-book material, as follows:

DEFINITIONS OF FOUR KINDS OF INDUCTANCES OR REACTANCES

A straight thin wire carrying ten amperes and having Φ lines set up around it will have a self-inductance L equal to Φ abhenries. In case of a solenoid, etc. exactly the same definition holds, except that the calculation of L becomes more complicated. (See Bulletin Bureau of Standards, Vol. 8, No. 1, where formulas and tables are given for the calculation of L. For experimental measurement of L see Laws, Eric Gerard and other works on electrical measurements.) Now instead of a single isolated coil or system as above we have a second coil, mutually inductively related to the first, we introduce a second concept—that of the mutual inductance M. M is defined in any text in physics or electrical engineering as $\Phi_{12c}\, n_2$; where, Φ_{12c} = flux per abampere (flowing in the primary) which is common to both coils or links both coils, and n_2 = number of turns in secondary. Similarly if we reverse the role of the coils and send 10 amperes through coil No. 2, instead of No. 1 we get $M = \Phi_{21c}\, n_1$ ($= \Phi_{12c}\, n_2$ if μ is constant). For calculation and measurement of M see references given for L.

The above concepts of self- and mutual inductance are fundamental and absolutely necessary both to the physicist and engineer. However, the former seems to find these sufficient while the engineer is forced to introduce two more concepts, namely that of "leakage inductance" of primary, or secondary or of armature or of field etc. and that of "total leakage inductance reduced to" primary or to secondary or to armature, as the case may be. Let us denote these by α and λ respectively. Then $L =$ leakage inductance of coil No. $1 = n_1 \Phi_{1l}$ where Φ_{1l} is the leakage flux of coil No. 1, *i. e.* the flux that interlinks with coil No. 1 only. Thus if $\Phi_1 =$ total flux set up by 10 amperes flowing through coil No. $1 = (\Phi_{12c} + \Phi_{1l})$, we have,

$L_1 = n_1 \Phi_1 =$ self-inductance of coil No. 1
$\alpha_1 = n_1 \Phi_{1l} =$ leakage inductance of coil No. 1
$M = n_2 \Phi_{12c} =$ mutual inductance of coil No. 1

Similarly if we reverse the role of the two coils we get,

$L_2 = n_2 \Phi_2 =$ self-inductance of coil No. 2
$\alpha_2 = n_2 \Phi_{2l} =$ leakage inductance of coil No. 2
$M = n_2 \Phi_{21c} =$ mutual inductance of coil No. 2

From the above it follows directly that,

$$M^2 = (L_1 - \alpha_1)(L_2 - \alpha_2)$$

and if we imagine the leakage of No. 2 to be zero and charge all the leakage to coil No. 1 we get

$$M^2 = (L_1 - \lambda_1) L_2$$

or $\lambda_1 = \left(L_1 - \dfrac{M^2}{L_2}\right) =$ total leakage inductance reduced to coil No. 1.

Similarly for λ_2.

Simple Application of Above. Consider the case of a single coil or in general the case where the mutually inductive effect of coil No. 2 is nil. Then, if we impress an alternating e. m. f. of frequency f across coil No. 1, $I_1 =$ current in coil No. 1 $= E_1/Z_1$ where $Z_1^2 = \overline{r_1^2 + 2\pi f L_1^2}$. This of course refers to permanent conditions. Under transient conditions, with either alternating or direct current we will find that the rise or decay of current depends upon $\alpha_1 =$ attenuation factor of coil No. 1 $= r_1/L_1$.

Next let us take the case where $M \pm O$, *i. e.* there is a closed coil No. 2, mutually inductively related to No. 1. If we impress the same e. m. f. as before, we shall find that $I_1 =$ current in coil No. $1 = E_1/Z_1$ where now Z_1^2 is equal to $(\overline{r_1^2 + 2\pi f \lambda_1^2})$ and not $(\overline{r_1^2 + 2\pi f L_1^2})$ and the latter may be 5 or 10 or 20 times as large as the former. Furthermore $\alpha_1 =$ attenuation factor of coil No. $1 = r_1/\lambda_1$ and not r_1/L_1 as before. Thus as we all know, from our study of a choke coil and transformer, it makes all the difference in the world whether there is a secondary coil affecting the primary coil or not. At this point let me call attention to: First, that the matter is simplified like this, if the secondary resistance is negligible; and secondly, that though I have used the general terms coil No. 1 and No. 2 the above apply to any system be it primary and secondary or field and armature or rotor and stator, etc.

In my papers I have used the above terminology and I propose to do the same in this discussion, and for sake of consistency in quoting from Doherty's and Shirley's paper I shall call the *leakage inductance*, leakage inductance and not self-inductance. It is true that with our typical engineering looseness and crude superficiality some authors use the two terms as synonymous, just like the writers do; but I find it extremely difficult to explain or understand why the writers failed or refused to ascribe to the term self-inductance, as is used in my paper, its usual meaning of self-inductance, at least as a possible interpretation. Instead no matter how clearly I expressed myself they have taken self-inductance to mean leakage inductance, or something akin to it.

With reference to the reactance which affects sudden short circuits, they state that "the authors will show that this reactance includes the field leakage inductance (writers use term, self-induction) as well as that of the armature." Unfortunately for the authors this has been shown by Boucherot in 1911, who discussed the matter, in a very much more thorough and masterly way than the authors have done. It may be well to call attention here to the fact that my discussion of L, α, M and λ is not quite as thorough as Boucherot's who gives seven and one-half pages to this question, from a purely theoretical point of view. Those interested further on this may consult his memoir. However, even if the writers did not consult Boucherot, it is difficult to understand why they overlooked the following statements in my paper in 1915 TRANS., page 2244: ". . . x_f is not the self-inductive reactance of the field spools but the total leakage reactance of the field and armature. Similarly α_a is a complicated quantity . . . The complex nature of these quantities has been recognized by several investigators such as Miles Walker, and Berg who without attempting to define them rigorously (from a mathematical point of view as Boucherot has done) has called them the equivalent resistance and reactance. . . Others, however, have apparently failed to understand the real meaning of these and have attempted either to simplify them too much . . . or complicate them unnecessarily. . ." Following these remarks I described experimental methods of determining α_a and α_f from the rise of direct current in the armature and field respectively. I showed that the attenuation factor α_f obtained with the secondary or armature open-circuited and at rest is quite different from the attenuation obtained with the armature dead short-short-circuited and the machine running at full speed. I went a step further and showed that α_f obtained from the rise of direct current in the field with the armature running at full speed and short-cricuited through a high resistance is practically the same as α_f obtained with the armature open-circuited and at rest. Thus I illustrated fully the effect of mutual inductance which may be negligible even with the secondary closed. Now it is clear that in the 1st and 3rd cases, above $\alpha_f = r_f/L_f$; while in the 2nd case $\alpha_f = r_f/\lambda_f$. I called attention to these facts at the top and bottom of page 2246 TRANS. 1915.

To quote from page 2246 this is what I said: in this case we have "the important difference that x_f is no longer the true self-inductive reactance but the total leakage reactance reduced to the field circuit" I fail to see how one could have expressed the facts more clearly. Again on page 2247 it is stated that "for this particular machine, . . . the ratio of the self inductive reactance to the total leakage reactance reduced to the field is about three." Immediately following this, I state that ordinary methods of calculating x_f from equation (**14**):

$$\frac{N_f \Phi_f}{I_f 10^{-1}} \text{ abhenry, or more accurately}$$

$$\frac{N_f \Phi_f}{I_f 10^{-1}} (1 + \text{per cent leakage of poles}) \qquad (14)$$

"utterly fail to give correct results" since "it is the attenuation factor involving the leakage reactance that enters the problem." All the above quotations occur within four pages of each other. However in spite of all this, the writers give the foregoing equation and triumphantly state that in the field attenuation factor, the inductance is not as Diamant gives the exciting inductance of the machine . . . but rather $\Phi^1 N_f / I_1 10^{-1}$ where Φ^1 is the flux, linking the field winding, which I_f would produce in the leakage paths of both field and armature." It may be well for the writers to compare this and other long clumsy expressions which they use—see for example (**5**) in their statement of conclusions, etc.—to the standard brief and clear statement: $\Phi^1 N_f / I_f 10^{-1} =$ total leakage inductance reduced to the field.

On page 1250 the writers make the following statement: "The authors would make it clear at this point that they are not in agreement with conception given in Diamant's paper . . ." I trust that it is clear to any one by this time that the authors really do not disagree with any of my conceptions—at least I agree with theirs although I am inclined to think that they are not practical and scientific enough either in expressing their ideas or using the proper terms. I shall take up these points again later, but right here, let me illustrate the truth of my statements: They begin by stating clearly that they do not agree with the *conception* given in my paper and to prove this they quote from my paper that the alternating component "is equal to the maximum phase voltage divided by the armature impedance, with good approximation." Now I fail to see why the writers use such poor logic. In engineering problems we frame a more or less ideal theory and base our conceptions on it; but in practical calculations we are invariably forced to make approximations and the good engineer is the one who can make good approximations. Thus my *approximations* do not necessarily affect my *conceptions*.

However, if we go a step further we shall find the extremely gratifying fact that the writers are entirely in agreement with my "good approximation" which I found to be true for 1 or 2 machines and theoretically sound; while, they have found them to be true from extensive tests on a great many machines. Within one-half page from where they say that they do not agree with me, they state that in case of machines with solid steel rotors or alternators equipped with low resistance amortisseur windings "the armature leakage inductance is a reasonably accurate measure of the sudden short-circuit current." Furthermore they not only thus corroborate my approximation but "propose its use for at least the present." See also (b) and (c) page 1281. In connection with this the writers deserve credit for determing that this approximation is allowable for machines with low resistance amortisseur windings only, as was the case with the machine I used, as stated in my paper.

In 1915 I called attention to the fact that single-phase sudden short circuits are quite complicated and that in case of a short circuit between neutral and line the armature current, for a given set of conditions, is about 140 or 150 per cent higher than in case of three-phase or line to line short circuit. Further I urged care in such cases and state clearly whether by single-phase short circuit reference is made to a short circuit between line and line, or line and neutral.

It is very gratifying that from extensive tests the writers corroborate both of my results, namely, that for a given set of conditions, three-phase or line to line short circuits give about the same rush of current; but short circuit between line and neutral is more serious. They offer an explanation for the latter on the basis that with three-phase currents flowing in the armature the leakage inductance is increased 50 per cent on account of the mutual inductance between phases. (See their paper bottom page 1262 and top of 1263.) Reprints of the paper were available so late that I have been unable to check this explanation. However, I would ask the authors to explain on the same basis why there seems to be practically no difference between a three-phase and line to line sudden short circuit.

I called attention in 1915 to the fact that α_f, when determined, either directly from oscillographic records according to methods developed on pages 2260-2264 or when obtained experimentally from the rise of direct current in the field with the armature dead short-circuited and running at full speed, comes out larger for the first cycle or so of the transient than for subsequent ones. I gave an explanation of this on page 2262 and it is gratifying that the writers not only corroborate this fact but agree with me that until we know more about the subject it is well to use the lower and safer value.

On page 1255 they state that "it may not have been so clear how and to what extent the field leakage inductance affects the short-circuit current." As I have already shown the writers are simply laboring under the wrong impression, in making a statement like this and I regret to say that their attempt, to show that the quantity to be used is the total leakage inductance reduced either to field or armature is rather crude and superficial compared to the explanation of Boucherot. They devote several pages to this (pages 1251 to 1259) and therefore I may be justified in calling attention to the following. On page 1251 they give the important law that "an electric circuit, without resistance, must so long as it is closed contain the same number of magnetic interlinkages. Now it is really unnecessary to make the ideal assumption of a circuit without resistance." Let me state the law, which as they say *is* very important, in a clear general way as follows. Let the flux in the field, or any electric circuit in general, change from Φ_1 to Φ_2 during the transient interval t; then the e. m. f. induced by this change will be $\frac{\Phi_1 - \Phi_2}{t}$. This transient e. m. f. will produce a quantity of electricity $Q = i \times t$ where i is a decaying direct current. As I pointed out in a recent article in the *Electrical World* this is a well known fundamental law used sometimes in electrical measurements (see *Electrical World*, May 18, 1918 page 1030, foot note 15). Let us apply this principle to the alternator. During sudden short circuit the flux in the field dies down from Φ_f to Φ_{fssc}. This produces a decaying direct current in the field which in turn produces the decaying alternating component of the armature current. Again if for any given armature phase there be a change of flux from Φ_a to Φ_{assc}; this will produce a decaying direct current in the armature etc.

For further details the reader may consult Boucherot or the 1915 paper of mine referred to by the writers or the recent articles in the *Electrical World* of May 18 and June 1. Before leaving this subject which has been well thrashed out in the above references let me call attention to the fact that the writers should be careful not to confuse the explanation of sudden short circuits based on the flux per phase or m. m. f. per phase and another possible explanation based on the total polyphase flux or m. m. f. of the armature circuit. The latter explanation is given in a short article, illustrated by oscillograms, in the *Electrical World*, Dec. 29, 1917, page 1251 and it is not necessary to go into it here.

Another point that I must mention in this connection is this. The statement of the writers on page 1275 that "the alternating component, ... depends upon the flux per pole, ..." is entirely wrong. So is the next statement, that the "direct component depends upon the flux linked with the armature circuit." As I have just explained, and as required by the fundamental law given on page 1251 by the writers, and which I have endeavored to generalize, the alternating and direct components depend upon the *difference* between normal and sustained short-circuit fluxes $(\Phi_f - \Phi_{fssc})$ and $(\Phi_a - \Phi_{assc})$ and not upon the flux per pole or the armature circuit as the writers state. This is a very fundamental point and there is little use for the writers to take several pages to explain things in a crude fashion, compared to Boucherot's explanation or the one given by me in the references already mentioned, and then come back and state that the direct component depends upon the flux linked with the armature circuit. No, first it depends *not* upon the *flux* but the *difference* in flux at the instant of the sudden short circuit and at the end of same; secondly, it depends upon the difference in flux at the start and end of sudden short circuit linking with the phase we are considering and not linking with the armature circuit.

Furthermore it seems to me that their statement that the direct component depends upon the terminal voltage is wrong. I think, it should be internal voltage due to the resultant flux.

There are other points in connection with appendix A and the factor $K\phi$, etc., that I would like to discuss, but as I have taken enough time I shall leave those to others. However, as I am anxious to see some good come out of this discussion, I will make the following definite proposals.

(1) That the qualifying adjectives sudden and sustained or permanent, etc., be used to qualify the kind of short circuit implied whenever there is room for ambiguity. Otherwise the shorter term short-circuit be used.

(2) That the terms leakage inductance and leakage reactance be exclusively used in referring to these quantities.

(3) That the term self-inductance or self-inductive reactance never be used to mean leakage inductance or leakage reactance. This is unscientific and impractical, since no term is left to be used to refer to "self-induction." Furthermore the word leakage is 50 per cent shorter than self-inductive and tells us the whole story.

(4) The term self-inductance be reserved to refer to what is sometimes called coefficient of self-induction. This is the universal term used consistently by physicists, writers on electrical measurements etc. etc.

(5) In conformity with transformer practise the sum of leakage reactances of two systems, be referred to total leakage inductance reduced to primary, or armature etc, as the case may be.

(6) If desired, the term inductance or reactance be referred to any of the above when there is no danger of ambiguity.

(7) According to the above it is the only common sense and practical scientific procedure not to introduce any terms like "transient reactance," "exciting reactance" etc. The x_{pa} of the author's is simply the leakage reactance of the armature; the quantity that one would use in calculating regulation etc. x'_{pf} is simply the leakage reactance of the field. x_{pf} (see equation (4) page 1259) is simply the leakage reactance of the field reduced to the armature, and it is extremely interesting to note the way the

writers express this on page 1259. They say, "the effective field self-induction or field reactance, x_{pf}, expressed as per cent reactance in the same terms as the per cent armature self-inductive reactance." Why not say x_{pf} is the "leakage reactance of the field reduced to the armature?" This is very much more simple and familiar, since we have the same thing in connection with the transformer. The writers, however, do not stop here, they take the sum of the leakage reactance of the armature and leakage reactance of the field $= x_{pa} + x_{pf} = x_o$ and they give to this a new impressive, misleading name of "transient reactance." I say misleading because I wonder how many engineers suspected that this wonderful "transient reactance" is nothing but the "total leakage reactance reduced to the armature."

It is well to note here that the writers in introducing this short term did not even attempt to explain its real meaning or give any reasons for introducing such a term. I am afraid they are thus preparing the ground for further unnecessary complications and misunderstandings; these are very common and annoying in practise and there are cases in which they cost quite a good deal of money and trouble. The point I wish to make is that the loose unscientific terminology of the writers or any engineer is not an academic question but an extremely practical one.

Thus why not call x_o, the total leakage reactance reduced to armature—because that is what it is. The length of the term will not cause half as much trouble as misunderstandings in discussions, letters, contracts and specifications etc., etc. Furthermore if it is necessary to have a short term why not call it "equivalent reactance" this I believe is used in transformer practise in referring to ($x_{\text{primary}} + \alpha^2 x_{\text{sec.}}$) and we ought to try and keep our electrical engineering theory, practise, terminology etc. as a single unit rather than narrow so much that those who specialize in alternators, transformers, electrical measurements etc. etc. should develop theories and terminologies without due regard to the needs of their fellow workers. Personally I care very little what the outcome of the above may be, but I am inclined to think that it is to the best interests of the profession to standardize things in some such way as given above and not introduce any unnecessary weird sounding impressive terms.

In conclusion I would like to ask the opinion of the authors as to the results of a paper which was presented at the Pittsburgh meeting of the A. I. E. E. by Douglas. This paper, represented what seemed to be fairly exhaustive work carried on at Cornell on the question of reluctance and leakage reactance of machines. It gave flux distribution figures similar to 24a, 24b, etc. and checked these experimentally. It stated clearly that present day formulas for calculation of field leakage, etc. are very inaccurate. I hope this paper was of more than "historical interest" and I am interested to know what the writers think about it.

C. M. Laffoon (communicated after adjournment): Additional refinements in the methods of calculating the reactance of synchronous machines, as suggested by the authors, are important on account of the inherent relation of the reactance to the still unsettled questions of field excitation, voltage regulation, and generator protection under short-circuit conditions. Before such refinements, however, can be applied, and the results generally accepted without serious confusion, the following two fundamental conditions should be universally agreed upon and adopted;

(a) A concise interpretation of what is meant by, and included in the term armature reactance under different stable and transient conditions; and,

(b) A standardized reliable test method for checking the calculated values of armature reactance and the determination of constants of reactance formulas.

In extending the refinements to the calculation of the reactance of polyphase synchronous machines, the authors are justified in raising the question whether or not a portion of the armature reactance flux of each phase interlinks part of the conductors of the other phases. Obviously, this is true to a limited extent, with regard to the end connections, in which the reactance is produced by the total flux interlinking this part of the winding; but for the leakage flux, which crosses the slots and leaves the tooth-tips, it is not so apparent that there is mutual reactance effect between the different phases. On polyphase synchronous machines the magnitude of the mutual inductance effect, if any exists, of the different leakage reactance fluxes, can be determined analytically only by actually laying out the flux diagrams produced by all of the electric circuits acting simultaneously (taking into consideration the difference in permeability and the damping currents in the iron portions of the magnetic circuit) and combining the different fluxes in the proper space relations at different instants of time. This is an extremely difficult task to accomplish on account of the varying saturation in the iron, and irregularity or indefiniteness of the flux paths. At best it can only be very roughly approximated, even when wide sweeping simplifying assumptions are made. Since the authors apparently have not followed this procedure in arriving at their conclusions, the writer feels that they have neither established nor justified their major premise that the leakage reactance in three-phase synchronous machines is increased approximately 50 per cent, due to a mutual leakage inductance effect, while in two-phase machines it is negligible.

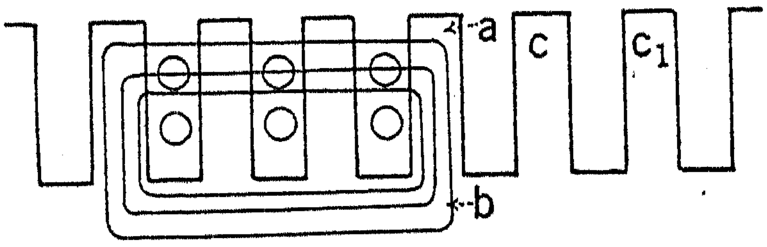

FIG. 2

If the slot leakage flux is assumed, to act independent of other flux interlinking the coils, it is evident from Fig. 2 that the flux produced by the current in a given phase belt will not cross the slots of the other phases from *a* to *c*, c_1, etc., unless there is an appreciable m. m. f. drop from *a* to *b*. Then, on the basis of the usual assumptions, wherein the saturation of the teeth is totally neglected, there would be no m. m. f. drop from *a* to *b*, and hence no flux from *a* to *c*, c_1, etc. But in actuality there must be more or less of an m. m. f. drop from *a* to *b*, due to the other flux, which is present, and hence some mutual inductive effect must exist. However, the increase in slot reactance, due to this effect, would in all probability be more than compensated for by considering the total m. m. f. of each phase belt acting across its own slots.

The extent to which the tooth-tip leakage flux produces a mutual inductance effect upon the adjacent phases depends upon the path that it is constrained to take. If the leakage paths were confined to an air circuit, the density of the tooth-tip leakage flux would diminish very rapidly, (assuming as before that this flux is free to act independent of all other fluxes) as the distance from the magnetic center of the phase group around the armature periphery increases, and consequently the mutual leakage flux would be small, if not inappreciable. However, when iron is in the presence of the teeth, the path of the tooth-tip flux is dependent on the action of the damping currents in the iron. It is the writer's opinion that the authors have idealized the laminated salient-pole rotor, and have not given sufficient consideration to the damping action of the eddy currents which unquestionably circulate in the lamination of the pole faces, rivets, bolts, etc. The cross-section of the tooth-tip flux path must be more or less restricted and confined to the air gap on account of the damping action of these currents. This in turn tends to minimize the mutual effect of this part of the leakage flux. The authors have not made it clear in their presentation how the armature current can force leakage flux across the air gap from the pole face to the teeth of other phases when the same m. m. f. has been used on the same path as a counter m. m. f. to the field m. m. f.

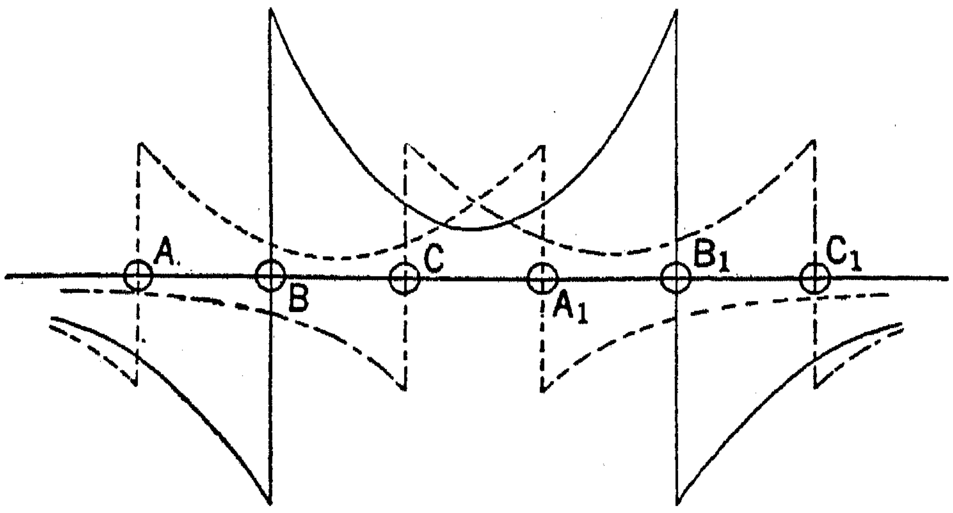

FIG. 3

In the case of the end connections, where there is no question concerning the existence of a mutual inductance effect, the flux density diagram produced by each phase will be as in Fig. 3. if the following radical simplifying assumptions are made.

(a) The adjacent iron parts of the machine do not affect the flux interlinking the end connections;

(b) The end connections are rectangular in shape and very long;

(c) The machine has only one slot per phase per pole; and

(d) The equivalent conductors are circular in section.

Under these conditions, it can readily be shown, by combining the respective flux forms, that the mutual inductance effect could at the outset amount to only a few per cent. Moreover, in actual machines the percentage of mutual inductance effect will be still further reduced on account of the fact that in most machines with V-shaped end connections, half of the end connectors of each phase are at approximately right angles to one-half of the end connectors of the other phases, and the mutual flux interlinking the end turns is thereby very appreciably reduced.

At all events, it is certainly evident from the above that the space distribution of the leakage flux of each phase is not sinusoidal and, consequently, in three-phase machines there is not a simple rotating field of leakage flux, as in the case of the work flux of an induction motor, and the mutual inductance effect in no wise amounts to 50 per cent. These conclusions are practically confirmed by test results, when single-phase and polyphase currents are circulated in the stator with the rotor in and removed, as indicated in Mr. Newbury's discussion.

The argument, advanced by the authors, that the mutual inductance effect must be 50 per cent because certain formulas which gave good results on three-phase machines gave results approximately 50 per cent too high on two-phase machines, is not necessarily conclusive, for the fault might have rested elsewhere in the formula. The writer has used certain formulas that did not consider any mutual leakage inductance effect, but which checked test values of reactance equally well on both two and three-phase machines.

Even if it is assumed that the damping currents are negligible, and the leakage flux is free to enter or leave the pole face, the authors' method of calculating the tooth-tip leakage flux is not fundamentally sound, for it assumes that all of the tooth-tip flux passes from teeth to pole face, independent of the ratio of the slot width to the air-gap radial length. It is evident that if the slot is wide, compared to the air gap, as shown in Fig. 4 a most of the tooth-tip leakage flux would cross the gap and then return to the next tooth. However, if the slot width is narrow, compared to air-gap width, as shown in Fig. 4b, the most of the tooth-tip leakage flux will not enter the pole face at all.* Since in actual machines the ratio of slot width to air gap varies through fairly wide limits, the method of calculating tooth-tip leakage must consider the total flux that leaves the tooth tip. That is, it should include the flux that crosses the equipotential section *ab-bc* (Fig. 5). The part of the flux that enters *ab* can be found by using Carter's pole fringing constant for different ratios of slot width to air gap; the part that crosses *bc* is more difficult to determine. In determining this flux, the author has apparently used Carter's coefficient for slots and vent ducts, which is based on the assumption that the tips of the teeth are at the same magnetic potential and that there is no tendency for the flux to pass from tooth to tooth. This is evidently incorrect, for it does not include the flux crossing section *bc*. Table I shows the values of reactance as calculated by the authors' formula, and the formulas used by the writer for machines with different ratios of slot width to air gap. These results indicate that in the case of machines having small gap and wide slots, the calculated values of reactance obtained by the two formulas are in reasonably close agreement, but as the ratio of slot width to air gap decreases, and a smaller percentage of the tooth-tip flux crosses the gap to the pole face, the calculated values of reactance by the two formulas diverge very rapidly.

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*This argument applies particularly to machines which have few slots per pole per phase.

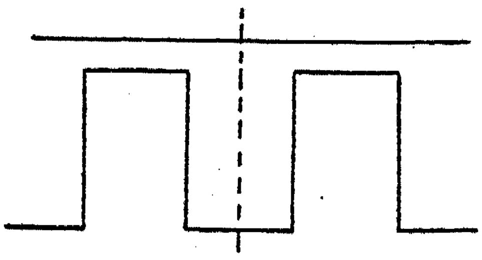

FIG. 4A

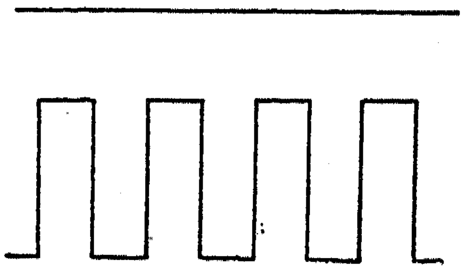

FIG. 4B

In this connection, the writer also questions the advisability of using curves which are based on the ratio of the maximum to the minimum air gap, to determine constants for the tooth-tip leakage formula. In the first place, such curves could only be applicable to machines having a given pole-face shape, for the maximum and minimum gaps have little or no relation to the variation of the air gap in the intervening space. In the second place, the maximum gap is not a definite quantity and is subject to considerable variation in interpretation as to where it should be measured. At least, this was the writer's experience in attempting to calculate the reactances of several machines by the authors' formulas. The most natural and logical place to measure the maximum air gap would be along the path that the

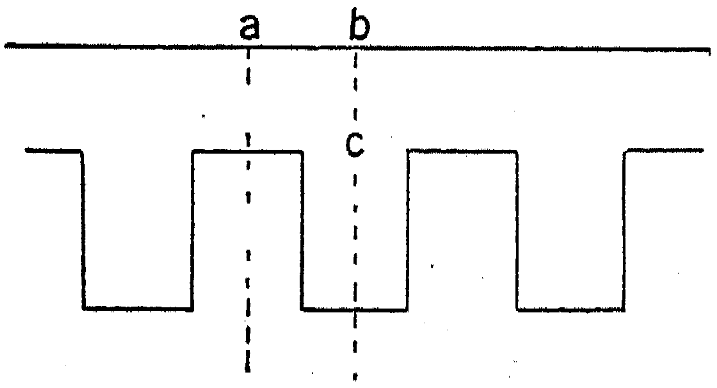

FIG. 5

no-load flux is assumed to take. It is true that in laying out the no-load flux form, it does not make a great deal of difference how the lines are assumed to pass, so long as a consistent policy is followed. But in the determination of the authors' constants, it makes an appreciable difference, for a small per cent increase in the maximum air gap may mean quite a large variation in the ratio of maximum to minimum gap.

Much of the confusion and difference of opinion concerning the reactance of synchronous machines is due, as has been previously suggested, to the fact that there is no standard and accepted method for determining it experimentally. The test method used by the author is in reality a calculated method, and, as such, is subject to serious limitations, for the armature reaction forms the major portion of the measured field ampere-turns, and a small percentage error in its calculation causes a large percentage error in the test value of the reactance.

In the writer's opinion, the most satisfactory experimental method, at present, of determining the reactance is from the no-load saturation and full-load zero power factor saturation test curves, used in connection with the calculated curve, which shows the increase of ampere turns on the no-load saturation curve due to the full-load leakage. This method also more nearly approaches the actual load conditions than any of the others. It is true that it is subject to limitations in that (a) the increase due to full-load leakage curve is calculated and subject to error, and (b) the reactance is probably not constant for all points on the saturation curve. The latter is not felt to be a serious disturbing limitation, for it is probably of the same order of magnitude as the errors in ordinary commercial tests. Granting the same degree of accuracy in the calculated terms by the two methods, the error in the test reactance obtained from the above method would be considerably less than that obtained by the method used by the authors, (as has been pointed out in Mr. Newbury's discussion). However, in most cases of salient-pole machines, in which the saturation of the pole core and yoke is not carried too high and the percentage field leakage flux is not too large, the increase in the no-load saturation curve, due to full-load leakage, can be calculated with a reasonable degree of accuracy. However, the method of obtaining it is apt to be rather long and tedious, and require painstaking effort. The accuracy of the calculated curve can be verified to a certain extent by checking the no-load and full-load zero power factor saturation test curves by the same method.

In columns 5, 6, and 7 of Table I are tabulated the per cent reactance as obtained by the authors' formula, the formula used by the writer, and by the above mentioned experimental method for a number of two and three-phase laminated salient-pole synchronous machines of quite a wide range in rating. This comparison indicates that there is a very close agreement between the results obtained by the method used by the writer and the test method for both two and three-phase machines, whereas, as was previously stated, the values obtained by the authors' method check the test results reasonably close for machines which have a large ratio of slot width to air gap, but for machines with a small ratio of slot width to air gap, it gives results from 30 to 50 per cent too low.

It is becoming more generally recognized by different engineers that the maximum value of the initial current delivered by an a-c. generator under short circuit is limited by a reactance more complex than the armature reactance under normal balanced load conditions. For, under short-circuit conditions, the armature reaction pulsates in magnitude, and shifts in position with respect to the field poles, and alternating currents are induced in the rotor and field circuits. The magnetic leakage of these circuits, consequently, produces a self-induction which must (after multiplying by the proper ratio of transformation) be added to the armature self-induction in order to give the short-circuit or transient reactance, just as in a transformer.

TABLE I.

Machine	Kv-a.	Phase	Slot width / Air gap	Per cent reactance		Test
				†D. & S.	‡*L*	
1	500	3	4.94	20.6	22.7	23.1
2	175	2	4.67	..	21.5	22.75
3	1132	2	3.56	..	18.5	19.0
*4	1250	3	3.4	9.6	10.9	..
5	875	3	2.78	13.6	16.7	16.5
*6	700	3	2.4	13.6	22.85	..
7	600	3	2.34	10.2	13.2	..
8	1132	2	1.92	..	15.0	14.1
9	1132	2	1.92	..	17.7	18.1
10	1250	3	1.62	11.7	15.7	..
11	3750	3	1.6	9.36	13.63	13.25
*12	560	3	1.067	17.6	26.0	..
13	12500	3	0.94	13.2	20.2	19.3

*The design data on machines No. 4, 6 & 12 were submitted to writer by Messrs. Doherty & Shirley.

†The values of reactance in column 5 were obtained by Doherty & Shirley formula.

‡The values of reactance in column 6 were obtained by formula used by writer.

The authors, in treating this problem, apparently have assumed that the armature component of the transient reactance is the same as the armature reactance under normal load conditions, and have derived their reactance formula to meet this condition. It would be more logical to consider the armature reactance under stable and transient conditions as different phenomena, and treat each case separately.

It has been the writer's experience that the calculated value of the armature reactance does not happen to differ widely from the value of the transient reactance determined from short-circuit oscillograms. Table II shows the calculated value of the armature reactance and the transient reactance as determined from oscillographic records for several different types of synchronous machines. There does not seem to be any very definite relation between the stator reactance under load and the transient reactance for different classes of machines, and it is our opinion that it is more or less of a coincidence. However, the coincidence can be partially explained on the basis as suggested above, that the armature reactance under short circuit is not the same as under load; that is, the value of the armature reactance under short circuit is decreased by an amount approximately equal to the added equivalent field reactance. This reduction of the armature reactance may be due to (a) the saturation of the iron part of the magnetic circuit of the leakage paths, and (b) the reduction in cross-section of the leakage flux paths by the damping currents in the field poles and adjacent iron parts. The writer is also of the opinion that this damping action is not confined to solid types of rotors, but exists to an appreciable extent in laminated salient-pole machines. However, definite quantitative conclusions, regarding the effect of saturation and the damping currents on the armature reactance under short-circuit conditions, cannot be given until more complete experimental data are available.

TABLE II.

No.	Machine.	Kv-a.	Phase.	Number phases short-circuited.	Ohms reactance	
					Calculated. Syn. operation.	From osc. trans. conditions.
1	Turbo-Alt.	15800	3	1	1.622	1.572
2	" "	"	"	3	1.622	1.695
3	" "	15000	"	"	1.09	1.11
4	" "	20000	"	1	1.96	1.87
5	" "	15000	"	3	1.82	1.61
6	Syn. Cond.	4500	"	"	3.84	3.87
7	" "	"	"	1	3.28	3.95
8	Alt. Gen.	12500	"	3	1.22	1.25
9	" "	"	"	"	1.22	1.28
10	Syn. Cond.	15000	"	"	1.066	1.064

NOTE: Machines 1 to 5, inclusive, are of solid cylindrical rotor type; the remainder are of the laminated salient pole type; 6 7 & 10 are equipped with damping windings.

R. E. Doherty: If there is a single characterizing feature of all this discussion, it is disagreement. The authors naturally expected much disagreement on a subject of this nature, and of course were not disappointed on that score. They were disappointed, however, at the nature of the criticism. While some of it contains constructive suggestions, much of it is purely destructive. It says the method, and therefore the calculated result, is wrong; yet it does not show how to make it right, nor does it give adequate reasons for saying the method is wrong. For instance, Messrs. Newbury, Fechheimer and Laffoon, who, as designers, are in possession of data from which to form opinion, apparently all agree that the method of calculation is fundamentally wrong because they believe serious factors are omitted; that even if it were correct, the method employed to arrive at the "test" value is wrong, because, they believe armature reaction can not be calculated with sufficient accuracy: and therefore agreement of "test" and calculated results could not be of much significance. But not a word is said about the theoretical soundness of the method, given in Appendix B of the paper, for more accurate calculation of armature reaction, taking into account the influence of harmonics in the flux wave. This influence has always been the cause of the greatest uncertainty regarding armature reaction calculations. Neither do they show any way of including the factors which they believe we have omitted. They are therefore urged by the weight of that acknowledgment to suggest, in the way of an alternative, the old friend—the expedient of having different formulas and different empirical constants for different lines of machines. That is, they are apparently agreed that this thing which we are trying to do can not be done. Mr. Fortescue subscribes heartily to that viewpoint, giving reasons in mathematical terms which to those even moderately versed in mathematics, can not possibly have much significance, and which to others seem to be evidence of a more thorough knowledge of mathematics than of alternators, inasmuch as he fails to consider that "in a system like this" the integration constants for his different differential equations may turn out to be the same constants for all, and that he may as well have had one equation. This is not the first instance where a voice has been raised that "it can not be done", when at that very moment it was being done. It may not be possible to obtain a single reactance formula, which can be depended upon in practical application to synchronous machines of all ranges in speed, voltage and capacity now being manufactured, but as evidence that it might be possible, we submit the fact that one of the large manufacturing companies has actually used such a formula for about two years, and the predictions based on those calculations have proved to be dependable. And that formula is the one given in the paper.

That sums up in a general way the larger and more serious objections to the paper. There are, of course, several others which, although not of a nature to determine whether the results of the paper shall stand or fall, are yet very important. These, as well as the individual points included in the discussion referred to above, will now be taken up in order.

Prof. Karapetoff offers a number of criticisms. First, he contends that "field self induction" and "field reactance" are different concepts, and therefore the authors can not properly say "field self-induction or field reactance." If he had read the next line of the same sentence he would have seen that these terms are expressed as a ratio, *i. e.*, percentage, and therefore are numerically identical. This result is what the sentence which he criticizes, refers to.

He believes next that the Institute should demand of the author a list of symbols, instead of allowing him to define them in the context, and thereby increase the "readability." That may be worth while, but there are obvious advantages and disadvantages on both sides, which have been discussed so often we shall not enter it here.

Referring to Fig. 25 A, he asks why flux and equipotential lines are drawn through the field conductor, since he understands that the flux has no potential in space occupied by current. The paper says: "In Fig. 25 B the paths as indicated by the iron filings can not be traced to the pole body on account of the field winding. But it is clear from the direction at the edge of the field winding that they will terminate approximately as shown in Fig. 25 A." Further than that, we can not answer.

Prof. Karapetoff's other criticism is that the authors are guilty, with many other authors, of expressing inductance as a ratio of flux to ampere turns, instead of taking simply the permeance. He then calls attention to the fact that the permeance idea is followed consistently in his book on the "Magnetic Circuit." Our supposition is that because most of us were started off in college from the "interlinkages per ampere" viewpoint, we are inclined to fall back upon that viewpoint both in writing and in our effort to understand what is written. In order, therefore, not to decrease the "readability" of the paper, the authors were less interested in revolutionizing peoples' viewpoint to confirm to Prof. Karapetoff's book than in helping readers understand the problem in hand by using terms familiar to them.

Messrs. Newbury, Laffoon and Fechheimer have raised a fundamentally important question, what is "the quantity we are attempting to calculate." Also how are we to test for that quantity in the actual machine. What the authors consider as the "quantity," must be clear to any one who has studied the paper. They were not aware that there was so much uncertainty regarding what particular reactance should be considered in various circumstances of operation. Since such uncertainty and also the question of testing do exist, these should be cleared away to permit some approach to agreement.

As a starting point toward settling what armature reactance means, take Mr. Newbury's lucid explanation regarding armature reactance and armature reaction. We will take up transient reactance later. With that portion of his discussion (1st four paragraphs) covering the explanation, we are in entire agreement, as may be obvious from reading the paper. Figs. 27 A and 27 B show our conception of the fluxes he describes. There can be no question, therefore, about what leakage flux is involved when we say *armature reactance*. Fechheimer's discussion breaks the accord at this point, by injecting the old bogy of harmonics which produce distortion and upset all calculations; then he closes by admitting: "that the distortion is very much reduced at full load, zero power factor, is quite evident." Now in order to compose this particular difficulty, let's say we mean the armature reactance which exists under zero power factor condition—*i. e.*, armature and field m. m. f's. in direct opposition. Then the complication of distortion is eliminated, and we can start once more. If it happens, as actually does and as explained in the paper, that this same value may be applied in regulation calculation, etc. under watt load conditions also, that is another matter which has no direct bearing on this present problem of establishing a bench mark to work from. Permit us to say with much emphasis: that is the reactance which is called *armature reactance* in the paper, and which equation (**47**) in Part 1 is intended to calculate. It is the calculated value listed in Table I.

Before taking up the difficulties involved in testing this value consider further Mr. Fechheimer's discussion regarding the influence of the harmonics which remain, even at zero power factor. It is pointed out in the paper that the salient-pole construction causes variation in reluctance of the leakage path, producing principally a third harmonic, which, with other third harmonics, disappears in the terminal voltage of Y-connected machines for well known reasons. There are other higher harmonics naturally but how much influence do they exert in the reactance or impedance, *i. e.*, in determining, in conjunction with the armature reaction, the current which flows under sustained short circuit, where, as already mentioned, the field and armature m.m.f's. are in direct opposition. The armature reactance under this condition, remember, is the reactance we are concerned with. That current, as Fechheimer acknowledges, is substantially the fundamental, free from harmonics. A glance at any oscillogram of this current will confirm that. Whether these harmonics are damped out by eddies alone, as he suggests, or are partly suppressed by the higher reactance to the higher harmonics, matters little. The simple fact is they are not there. If we want to know what determines the current, which is the fundamental, it must be obvious therefore that the fundamental of the reactive voltage is the factor we are after. A fifth harmonic, or any other higher harmonic, may be in the reactive flux or voltage, as he says, but what if it is? If we were studying core loss, that fact might concern us, but not here.

Take up now the several methods of testing reactance which Mr. Fechheimer mentions. This is important, because much of the disagreement centers on the test method. On the same machine, Mr. Newbury obtains from test by Mr. Fechheimer's No. 2 method, which he adopts, a value of armature reactance different from the value the authors obtain by No. 1 method, which they adopt. Don't misunderstand. We are both after the same thing. The question is, which method of obtaining it by test is right. As we see it, the principle in each method is sound, and by his explanation referred to above, Mr. Newbury agrees. To draw the issue clearly, Messrs. Newbury and Fechheimer think that our respective "test" determinations of armature reactance on the same machine do not check, because they believe we are unable to calculate armature reaction accurately. Hence the difference between that wrong armature reaction and the test value of field m. m. f. (synchronous impedance $A\ T$) gives the wrong reactance. He agrees, however, that that difference, properly determined, gives *armature reactance*. Our only answer is that we have given in Appendix B a new method of accurately calculating armature reaction, and it seems that they should have felt obliged to show why that method is not right. To simply say it is inaccurate is not at all convincing. Not a word was said about the method in Appendix B; not a word as to *why* armature reaction can not be calculated. We are therefore not in position to know and discuss what is in their minds.

In applying No. 2 method, which Mr. Newbury recommends, one meets a number of difficulties. It involves, first a graphical solution in which a cut-and-try triangle is made to fit in at all points between two curves. One of those curves is calculated, and involves estimates of pole leakage and also estimates of how saturation is changed by having, under load, more flux in the pole body, but practically no more flux in the armature and a varying amount of increase in the teeth. Hence that calculated curve must rest considerably upon judgment. It is important to note that the same objection which was raised to No. 1 method regarding the large error resulting in the difference of two large quantities, from a small error in one of the large ones, applies equally to method No. 2; as does also the objection that one of the large quantities is calculated. It is our opinion, therefore, that method No. 1 involving a simple subtraction of ampere turns armature (in which the amperes are tested, the actual turns are known, and the only calculated factor is "effective" turns) from ampere turns field (in which the amperes are tested and the known turns are 100 per cent effective) is much more reliable than No. 2 method with its difficulties mentioned above; and that the discrepancy, mentioned by Messrs. Newbury and Laffoon, between their test determination and ours is due to their errors in applying No. 2 method. It may be asked: if the "test" value of armature reactance determined in error by No. 2 method is applied in excitation and regulation calculations, why will not these calculations also be in error. Simply because the steps by which the reactance was determined from load excitation curves are retraced in load excitation calculations. This pursuit and follow of one's self about a circle does not apply to the use of No. 1 method. Therefore, assuming that armature reaction calculations are even approximately correct, then it would seem that to obtain a value from test with which to compare the calculated value, No. 1 method is far superior. It is acknowledged that no method yet proposed, which is based on commercial tests of saturation curves and synchronous impedance curves, can be depended upon, for reasons already mentioned, to give in every case the armature reactance within, say, 10 or 15 per cent. This may mean only 2 or 3 per cent error in the tested curves. If that error is more serious, as sometimes happens, then the reactance error goes skyward. The error in reactance value incident to test error applies alike to both methods referred to. Hence, although accurate tests of course eliminate this point, it is yet true that where, as in our case, the experience available involves such tests, this fact must be weighed in the interpretation of the data. This has important bearing upon the data in Table I, which I shall refer to later.

Considering further Mr. Fechheimer's list of methods, No. 3 and No. 4 give nothing but the impedance of the stator under those test conditions, *i. e.*, with rotor out. It is absurd to take this as the *armature reactance*, since obviously the latter would be different in the same stator with rotors of different pole proportions. Methods 5 and 6, give, as did 3 and 4, the impedance of the machine under the conditions of the test, *i. e.*, with stationary rotor, where any given phase group has a fixed relation to the pole. If there happens to be a secondary winding, *i. e.*, a squirrel-cage, solid-rotor or short-circuiting collars on the poles, the transformer impedance is involved, that is none of these is the *armature reactance* which we have been discussing. This seems too obvious for comment. Method 7 may give some interesting results of the synchronous reactance in Mr. Fechheimer's investigation. However, it is probably as accurate and infinitely easier to read the synchronous reactance directly from the saturation and synchronous impedance curves.

Method 8 gives the *transient reactance* accurately.

Of the eight methods, therefore, either 1 or 2 (if 2 is correctly applied) should be used for obtaining *armature reactance*. Method 8 should of course, be taken for *transient reactance*.

Referring to Table I, Mr. Fechheimer urges caution in adopting the formula, because 40 per cent of the machines show a difference between calculated and test values of more than 20 per cent, and points out two others in which the difference is 145 per cent and 185 per cent. Please bear in mind these two facts: In that table of 138 machines, all of the bad ones, as well as all of the good ones are there, *i. e.*, all the cards are on the table. Secondly, the "test" values, with few exceptions, were determined from commercial tests of saturation and synchronous impedance, in which circumstance, something would be wrong, as already intimated, if all machines checked within 10 or 15 per cent. Many of those tests are all right; others may be off by 2 or 3 per cent giving 10 or 15 per cent in reactance; others, more. It would be nice to have 138 different, accurate tests to compare with, but they are not available. We are obliged to accept the commercial test records for the mass of test data. Where tests are known to be accurate, no radical disagreement in results occurs: But we submit that knowing the probable error that *may* result from test, and noting the consistency of the majority of the cases of widely different machine characteristics, in falling in line with calculations, the extreme cases mentioned, as well as a variation of 20 per cent in 40 per cent of the cases, are to be expected, and can not be taken, as Mr. Fechheimer suggests, as evidence that the formula is wrong. If the entire table is studied with these points in mind, I think it must be granted that the agreement is most encouraging. If the table contained only eight or ten machines of about the same characteristics, then it could not be offered as evidence that the formula is reliable. But that is not the case. There are 138 of them. We might add that during the seven months which have passed since the paper was written, additional data, including many accurate tests, have been obtained, all confirming the reliability of the formula.

Referring to transient reactance, Mr. Fechheimer submits an ingenius example to show that saturation prevents the field leakage from having much influence in limiting the initial short-circuit current, and says also that eddy currents in the spider should be considered as to their influence on transient reactance and attenuation factors. Now that example showing the tremendous increase in pole density at short circuit is most interesting. We agree that saturation would certainly be a "serious factor" in such a case. But Mr. Fechheimer is confused. It has apparently escaped him that the flux in the pole at short circuit is roughly what it was before short circuit. He adds the 10 times 12 per cent leakage, to the 100 per cent normal flux, giving a total of 220 per cent. He forgets that the field leakage can not possibly be more than a fraction of the original 100 per cent. As a matter of fact, the only difference after short circuit is that *part* of the 100 per cent which before short circuit crossed the air gap into the armature, now crosses the field leakage paths. The rest goes through the armature leakage paths. So you see he has accounted for 220 per cent which, if it were not for saturation, would cross the field leakage paths, and has not yet mentioned the armature paths, which, by the charitable assumption that the field leakage is 50 per cent as great as the armature leakage, and that the ratio of effective field interlinkages to the product of field turns times flux is about $^2/_3$, the armature paths would absorb $^2/_3$ times 440 per cent equals about 300 per cent more. That is, 5.2 times normal flux, which of course is absurd, since it means manifold increase in field interlinkages. To have remembered that these interlinkages can not change significantly at short circuit would surely have prevented such a weird example. A study of Part 2 of the paper will clear up this point.

A glance at Table II may save discussion about the influence of saturation. Short-circuit tests are shown at different voltages. It will be noted that this evidence is in accord with conclusion (**10e**) given at the end of Part 2.

Regarding the effect of eddies in a solid rotor rim upon transient reactance and upon the attenuation factors, this phase of the problem was considered, as mentioned briefly in the last paragraph of the abstract at the beginning of the paper. Specific data regarding this effect was not ready in time for the original copy of the paper, but has been included in the TRANSACTIONS.

In speaking of transient reactance Mr. Newbury states that in his experience "the armature reactance during synchronous operation (obtained from no-load and full-load saturation curves) is not materially different in value from the total reactance effective during sudden short circuit." He grants that they may not be the same reactances, and explains that their numerical equality may be due to the fact that whatever increase may occur in reactance due to the field leakage, is offset by "some other action." From what we have said in previous paragraphs, our explanation of that numerical coincidence is due less to "some other action" in the machine than to the fact that Mr. Newbury draws in the calculated modification of no-load saturation curve too near the no-load curve. It is a hopeful augury that he agrees in his last sentence that it "may be due to difference and errors in determining the true armature reactance and agreement can hardly be expected until this question of test method is settled."

It is very important to note at this point that from an exchange with Mr. Newbury of designs and test data for the purpose of removing all question regarding application of the formulas and interpretation of tests, calculations by equation (**47**) Part 1 and equations (**6**) and (**7**) Part 2 checked the tests on his machines, as shown in a following table, in connection with Mr. Laffoon's discussion. That is, the formulas apply alike to machines of different manufacture, when the calculated result is referred to a "test" value which the formula is intended to calculate, and not to some other "test" value.

Mr. Dawson suggests that the difference between the results given in the verbal discussion by Mr. Newbury, and those in the paper might be explained by different practises in end-coil support. I think we have made clear where the discrepancy lies. I remember a case where the same machine was tested first with steel-ring end-coil support, and then with brass support. The difference in reactance could not be detected. This was a moderate speed, about 1000-kv-a. machine, and the steel ring was about one inch (25.4 mm.) square.

Mr. Laffoon's comments center more upon the details of theory underlying the reactance formulas. But he prefaces those comments with some general objections. His reference to definition and "standardized reliable test", is repetition of Messrs. Newbury and Fechheimer's discussion of the same points, which has been fully answered.

He rather more fully amplifies their viewpoint regarding the extremely complicated nature of the problem. He believes that the problem is too intricate to treat analytically; that necessary flux diagrams, effect of eddy currents, change in permeability, indefiniteness of the flux paths and a few other things render the calculation hopeless. To take account of all these factors in an equation would be well nigh hopeless, we acknowledge. But if we understand the problem correctly, there is an important distinction to be drawn between a method involving a possible error of 10 per cent or so which for the practical purpose for which the reactance is used, is negligible, and a method required to include all factors to the extent of reducing the possible error to, say, 1 per cent. It is somewhat like the distinction between the method of a house carpenter who works to inches, and the factory mechanic who works to thousandths of an inch. In one the relative error may be many times larger than in the other; but, practically, they are of the same order. Our point is from our understanding of the problem in hand, and from our experience, we believe that practically all of those annoying and confusing factors which Mr. Laffoon refers to, are trifling and may be neglected in obtaining a practical estimate of reactance from the design. Where, for instance a factor can not be neglected, as harmonics in armature reaction, we have taken it into account. But to worry about most of the things he mentions is unwarranted, for reasons given in previous comments.

Regarding slot leakage and his Fig. 2, he reasons that there will be slight mutual action between phases due to saturation of teeth, then decides that there is some other effect that compensates for it. That is, agrees with the authors that there isn't any mutual effect of slot leakage.

Now as to the mutual induction of the end connections and tooth-tip leakage. Mr. Laffoon shows why in the case of the end connections the mutual effect could amount to "only a few per cent:" then shows that it would have to be less than that. Perhaps the shortest answer to his logic is that if exploring coils are put on the end connections following around the V-ends of a phase group, and then from an exterior source, the same current is put through the armature, first single phase and then three phase, the exploring coil voltage will be 50 to 60 per cent more in the latter case than in the former. This ratio for end leakage is the same whether the rotor is in or out. The following Table III gives data from such a test on a 125-kv-a. machine, four slots per pole per phase, and also on coils assembled in a wooden armature dummy.

He believes that the tooth-tip leakage is more complicated still, and also that the mutual induction between phases due to this leakage must be very small. Why? Because damping currents in the laminations, rivets, etc., etc., prevent the tooth-tip flux from entering the pole face. This restricts this leakage to the path along the air gap. This of course would not only prevent mutual induction with other phases, but also cut down radically the leakage itself. You see he is still allowing harmonics to cause undue confusion. Certainly there are no damping currents to the fundamental which we are after, and which being polyphase is stationary with respect to poles. Instead, the problem is rendered more simple that the harmonics are partially or totally damped out. We trust that the puzzling features he mentions will clear up if these points are kept in mind.

TABLE III
EXPLORING COIL TEST FOR END LEAKAGE ON 125 KV-A. ALTERNATOR

Test	* Rotor	Phase	† Exp. Coils in series	End Shield	Amps. in Phase group	‡ Exp. coil voltage	Column 7 corrected for 32 amps.	Ratio taken of tests	Ratio
1	Pos. 1	3	4	on	31.2	0.983	1.02	1 & 3	1.69
2	Pos. 2	3	4	on	33.6	1.09	1.04		
3	Pos. 1	1	4	on	31.4	0.595	0.605	2 & 4	1.52
4	Pos. 2	1	4	on	31.4	0.673	0.685		
5	Out	3	4	on	32.4	1.03	1.02	5 & 6	1.6
6	Out	1	4	on	31.5	0.627	0.637		
7	Out	3	4	off	31.3	1.04	1.07		1.67
8	Out	1	4	off	31.4	0.627	0.64		
Similar Test on Armature Dummy									
9		3	2		130	2.99		9 & 10	1.49
10		1	2		130	2.01			

*Position 1, phase group bearing exploring coils, symmetrically surrounding the pole. Position 2, phase group directly over pole face.

† Exploring coils were arranged as shown in Fig. 7, except there were four instead of two slots per pole per phase.

‡ Maximum of fundamental. Fig. 6 shows typical oscillogram of exploring coil voltage.

He adduces Mr. Newbury's test with rotor removed showing 16 per cent increase three-phase over single phase reactance to show that that increase can not be of the order of 50 per cent. At the time the problem was being studied, the authors of course made such tests, and agree that under that condition (rotor out) the ratio is about 1.16. But as we have emphasized before, we are not interested in what it is with the rotor out. What we are after is the reactance with the *rotor in and running in synchronism at zero power factor.* Under which circumstance (and relegating this damping action of the pole rivets etc., as mentioned above) the iron rotor permits mutual interlinkages by the fundamental with other phases. This idea is fully developed in the paper. In this respect, there is other possible evidence, available to those concerned, that has bearing on this point. Theoretical considerations indicated that this mutual induction factor, three-phase to single phase (terminal to neutral), obtains in the transient as well as the armature reactance. This led to equation (**13**) Part 2. While at the time the paper was printed, no tests were available, a test has since been made which confirms that point of view. It might be instructive for those interested to try this out. The results of that test which was made on a 1000-kv-a., 750 rev. per min., 50-cycle generator are as follows: Short circuit, three-phase, no load gave 1900, 1800, 1900 amperes (a-c. component) on three different tests. Short circuit, one phase, terminal to neutral, no load and same voltage, gave 2950 and 3060 amperes (a-c. component) on two different tests. Average ratio three phase to single phase is 1.6.

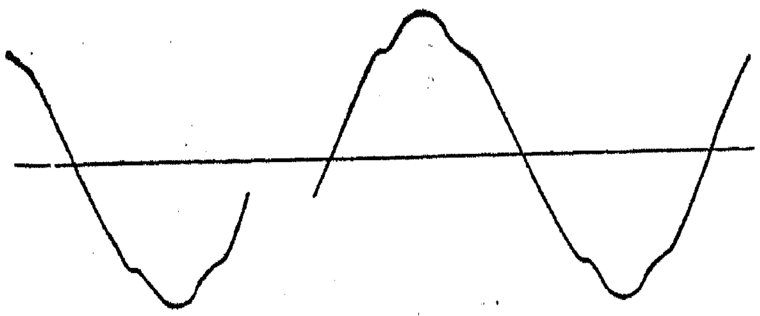

Fig. 6—Oscillogram of Exploring Coil Voltage of End Connection Leakage on 125 Kv-a. Alternator

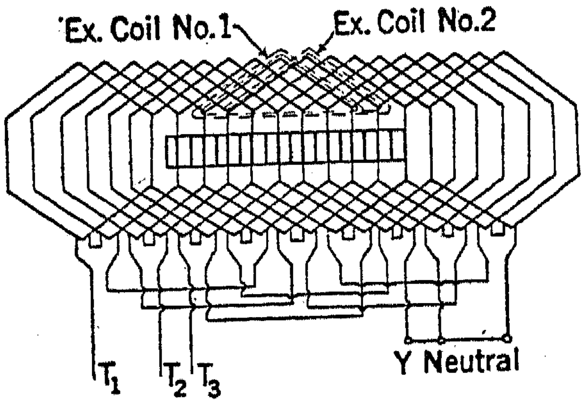

Fig. 7—Exploring Coil Test for End Leakage—Armature Coils Assembled in Dummy Indicated by Heavy Lines at Center of Figure

Mr. Laffoon believes we have made an error in the calculation of tooth-tip leakage by neglecting the important component of this leakage which goes along the air gap and does not enter the pole face. Now there may be a number of viewpoints from which the authors have not studied this problem, and there may be possibilities of significant factors which they have failed to search out and weigh, but may we not, in the face of this avalanche of criticism, properly give ourselves credit for having been pretty careful and painstaking to do these things. The question just mentioned about tooth-tip leakage was analyzed and the factor for including this "along-the-air-gap" component was put in the equation; but it was found to be practically negligible, and was omitted. We take it that Mr. Laffoon has not properly made such an analysis, or he would not have brought the question up. The space to discuss this criticism may be worth while, because it is a characteristic type. At first thought, the matter seemed to us very much the same as it still does to Mr. Laffoon. That is why we worked it out. It seems obvious that with slot width equal to air gap on a one slot per pole per phase, the leakage from the face of one tooth to the face of next tooth, will not all go across the gap to the pole face and return; part will go along the air gap. With two slots per phase group, the path across to the pole face has not changed, but the path of the component along the air gap is doubled. For three slots per group the first path between outside teeth is still the same, the path along the gap is trebled. The middle slot has its own leakage, but as explained by Gray, this contributes only one ninth to the total leakage of the group. And so on; the component along the air gap growing progressively less as the slots per phase group are increased. Then why does not that component have to be considered for one or two slots per pole per phase. Because no one, I think, ever saw an air gap equal to the slot width on such a machine. If one did, one saw a monstrosity. Assuming tooth and slot width equal, and three slots per pole, would mean an air gap equal to one-sixth of the pole pitch. A normal air gap is, say, one to 3 per cent of the pole pitch, possibly four per cent. Take four per cent, for instance. If the slot width is the same, it means eight per cent per tooth pitch, *i. e.* about 12 teeth per pole, or more. If two per cent gap had been taken, it would have meant 24 slots per pole. The more the teeth the less, by the square, do the middle ones count, and the middle slot is all that would be practically affected by this component we are accused of having omitted. Moreover, if there were such a machine where this factor was a significant portion of the total tooth-tip leakage, then the total tooth-tip leakage would be of practically negligible magnitude compared to end leakage and slot leakage; and the error would be of no practical significance in the total reactance. Those are reasons why we omitted that factor. We did not overlook it.

Regarding the application of Carter's Fringing Coefficient to tooth-tip leakage. Take any alternator and lay out the distribution of tooth-tip leakage over the pole. Determine the coefficient to account for the spread of flux. Then look up the value on Carter's curve. It is practically the same, as one might expect, except possibly in some monstrous example as Mr. Laffoon suggests. Even then, for reasons mentioned, the error, is harmless.

The title for the curves based on the ratio maximum to minimum air gap, which assume a constant pole arc radius for any given pole, has been modified to meet his objection regarding the difficulty of applying them. They can not properly be applied to a pole involving different arc radii.

Turn now to Mr. Laffoon's Table I. Before commenting on the comparison it gives, we would refer to the following table, giving the rest of the data which Mr. Laffoon should have felt obliged to include, but did not. We have already referred to an interchange of design and test data with Mr. Newbury for the purpose of eliminating the possibility of wrong application of formula and of different interpretations from tests. Mr. Laffoon's Table I is supposed to show the results of that interchange. In our Table IV only machine numbers 4, 6, 11, 12 and 13 appear, since we have not data on the others. But sufficient data is in Table IV for interesting comparison with Mr. Laffoon's Table I. These are the main points of interest:

1. Our calculation of *armature reactance* checks reasonably with the test value (No. 1 method), which the formula is intended to calculate, on Mr. Laffoon's machines, as well as on ours. Mr. Laffoon's Table I does not show the "test" value which our formula calculates, but something else.

2. His calculated and test values are, he avers, either *armature* or *transient* reactance, since they are the same. Note that those values agree with our calculated *transient* reactance. Fortunately, on one of the machines he submits, the 12,500-kv-a., results from short-circuit tests are available. In this case, we calculate 13.2 per cent armature, and 19 per cent transient. Test was 14.2 per cent armature, and 20.8 per cent transient reactance. His calculated and "test" values are 20.2 and 19.3 per cent respectively.

TABLE IV
PER CENT REACTANCE

Machine	Kv-a.	Phase	* Slot width air gap	Calc. D. & S.		Test †		Calc.	Test ‡
				Arm.	Transient	Arm.	Transient	L	L
D. & S. 4	1250	3	3.4	9.6	13.6	7.8	...	10.9	...
D. & S. 6	700	3	2.4	13.6	19.1	13.0	...	22.85	...
L 11	3750	3	1.6	10.2	14.5	12.9	...	13.63	13.25
D. & S.12	560	3	1.06	17.6	23.0	18.2	...	26.0	...
L 13	12500	3	0.94	13.2	19.0	14.2	20.8	20.2	19.3

* Minimum air gap is used in this ratio.

† *Armature reactance* from saturation and synchronous impedance curves. Method 1 in Mr. Fechheimer's discussion. *Transient reactance* from oscillograph records of short circuit.

‡Mr. Laffoon's application of test Method 2, as described in Mr. Fechheimer's discussion.

3. Our calculation of armature reactance on machine 11, is 10.2 per cent instead of 9.36 per cent given in Laffoon's Table I.

Therefore the alleged agreement of results at high ratio of slot width to air gap, and a gradual taper to radical disagreement at low ratio, due to our omission of the "along-the-air-gap" component of tooth-tip leakage, vaporizes with the rest of the facts and logic which have been submitted to show that the results of the paper are entirely wrong. Note in Table IV that, instead, the agreement happens to be better at low ratio (see *D* and *S* 12 and *L* 13) than at high rates. The results in these tables simply show again what has been brought out in all this discussion, that, in the first place, the soundness or the theory underlying the method given in the paper has not been disproved; and also that the difficulty Messrs. Newbury, Fechheimer and Laffoon have experienced in adjusting their viewpoints rests on the fact that they have been working on the unsound principle that the transient reactance and armature reactance are the same. And by the treacherous possibility in No. 2 test method of taking in excitation calculation any value one pleases for armature reactance, (so long as that value is applied in the same way it was derived from the zero power factor excitation curve) they have lead themselves to believe that the above principle is right. A different value of reactance obtained by No. 2 method simply means a change in judgment regarding where the calculated saturation curve should be placed. The coincidence of the value of armature and transient reactance probably had its origin in the view that they should be the same, and the full-load leakage correction of the saturation curve adjusted accordingly.

Mr. Diamant's comments center principally on three points: 1. The author's statement of disagreement with certain results of his own paper. 2. Priority claims. 3. Terminology.

Since Mr. Diamant states that "the authors really do not disagree with any of my conceptions, at least I agree with theirs . . . .", the first point would seem at once to be settled. We disagreed because we misunderstood his meaning. We misunderstood not by a lack of diligence in trying to understand, but because he used terms familiar to us, but to which he attached a meaning entirely different from that almost universally understood, and which is uniformly defined in leading text books. Take one glaring instance: by armature self-inductive reactance, he means synchronous reactance. Referring to Fig. 27B in the paper, his meaning of "self-inductive reactance" involves all the flux shown. What is generally meant by self-inductive reactance, involves only that portion of the total flux shown that does not link with the field winding. With the almost generally accepted definitions in one's mind Diamant's statement "the ratio of the true self-inductive reactance to the total leakage reactance reduced to the field is about three," is perplexing indeed. Being familiar with leading texts that define these terms and with the consequent general understanding, and yet not having stated his intention to use a different meaning, he could hardly have expected his words to be understood.

But after definition has been cleared up, one point remains. We obviously can not agree to the general statement in Diamant's paper that the initial short-circuit current is determined "by the armature impedance with good approximation." Although true in some cases, as tacitly granted in our paper, "this assumption would lead to serious error in many cases." We gather from Mr. Diamant's interesting comments confined to logic, that, after all, he agrees.

Just a word about our reference to his paper. In taking up this problem of reactance and short circuits, we naturally used as a starting point the most recent work on the subject, and that, of course, was Diamant's paper. He had covered the matter historically, which to us, was a considerable factor in the value of his paper, and had offered interesting and valuable test data, and also some extensions in mathematics, one of which was used in our paper, Fig. 21. As stated, there were in his work points with which we did not agree. Hence it was necessary and proper that that disagreement should be stated, in order to clearly proceed to further development; surely not in order that we might "triumphantly state" that some one was wrong. We regret that the latter interpretation was made by Mr. Diamant, because it has done much to spoil what otherwise might have been a masterful criticism.

We are accused of having attempted to offer, for the first time, what Boucherot proposed in 1912. The first paragraph of Part 2 states that Diamant's paper sums up the theories underlying short circuits as developed by Berg and by Boucherot. These are in complicated mathematical terms from which the average engineer finds difficulty in obtaining a physical conception of the phenomena. Moreover, to apply their equations it is necessary to know the reactance. In order to calculate the reactance which limits the initial short-circuit current, it is necessary to establish a physical conception of what happens to the magnetic fields under that condition. It is just as necessary to establish that conception in the reader's mind, if he is to understand clearly what is being calculated. And to the contempt of those who would say everything mathematically, we have described the phenomena "in terms as free as possible from mathematics." It is an interesting circumstance, therefore, that we are now obliged to acknowledge that we did not discover the principles underlying that exposition; nor did Boucherot, or Diamant. Faraday and Maxwell deserve some credit for them. They are stated in any good text on physics. However, there are one or two points which were brought out which we believe were not generally appreciated. To mention one, the increase in flux density at the bottom of the pole at short circuit. This is not only an interesting result, but one that has important bearing upon the variation in attenuation factors, as explained in the paper. This phenomenon is not all obvious from previous literature.

Before referring to terminology, we will answer one or two other points. We are asked to explain why a single-phase short circuit between terminals of a three-phase machine produces the same current as a three-phase short circuit, especially since a terminal to neutral, single-phase short circuit produces 50 per cent greater current than the three-phase. If it is assumed that the two phases involved may be considered as different phases, as in three-phase, and with the three-phase value of reactance per phase, then the short-circuit current would be

$$I = \frac{1.73\,E}{2\,X} = 0.86\,\frac{E}{X} = 0.86\,I_3$$

that is 86 per cent of three-phase value. That assumption, however, involves the three-phase mutual induction between phases, which is strictly true only at one instant, when the current is a maximum. Then the pole iron is in same position relative to the armature m. m. f. as at one particular moment under three-phase conditions. If that relative position were maintained, then the current would be as given above, *i. e.*, $0.86\,I_3$. However, as the pole moves on, the relation of the iron path to the leakage flux is changed to decrease slightly the fundamental of the leakage flux, and as tests show, this decrease is about 15 per cent.

Mr. Diamant says the following statements in the paper are wrong: "the alternating component depends upon the flux per pole" and "the direct component depends upon the flux linked with the armature circuit." He says those components depend rather upon *the difference between the mutual and sustained flux.* The above quotations obviously follow if one accepts the principle of constant interlinkages in a closed circuit without resistance, and grants what is the generally accepted view that for a practical study of the phenomena, the resistance is negligible for the first cycle or so. But even if one dissents from that view, we will take the magnitude of these components as determined by projection of wave of crests to zero time. But however obvious those statements may seem to those who look at these matters from a viewpoint of facts, it is acknowledged that where one, instead, reasons from somebody's equations, say Boucherot's, instead of from the background of facts Boucherot had in mind, it is possible for one to arrive at a wrong, sometimes dangerous, conclusion. Consider what happens. At short circuit the entire useful flux is absorbed in the armature and field leakage paths in proportion to their respective reactances. There is of course a very small per cent of it taken up in $R\,I$ drop, but this is negligible. As the flux dies down, the relative proportion of it in the armature leakage path at any time is progressively greater, that in the field path, less, until at the sustained value, it is all absorbed in the armature leakage (except of course the original pole leakage due to original exciting field current). Now the practically universal practise of all authors in expressing that fact mathematically (and it is a mathematical convenience not free from inconsistency) is to assume that the sustained value existed all the time, and that the transient condition involves a change from initial flux and current to sustained flux and current. Diamant's opinion probably rests on such a *formula*, otherwise it is difficult to understand the basis for his opinion that the a-c. component depends upon the difference between initial and sustained flux. But, as intimated above, such a formula does not explain certain facts. If the sustained value is assumed to have existed from the first instant, then it means that by the formula there is a greater percentage of flux taken in the armature paths at the initial moment than actually is taken. As an instance, assume a ratio of armature to field strength equals two. Also assume ten per cent armature, five per cent field leakage. That means 6.66 times normal current at short circuit. Actually at the first moment the division of the *total* flux, which is 110 per cent, is 66.6 per cent in armature, (33.3 per cent + 10 per cent) in field leakage path. At the sustained condition, the current is practically two times normal, that is, the flux division is 20 per cent in armature leakage, 10 per cent in field. By the well known formula referred to, assuming that the sustained values existed from the start, the initial division would be as follows: the sustained value accounts for armature 20 per cent, field 10 per cent. That leaves 80 per cent of the useful no-load flux to be absorbed in armature and field in the ratio of 10 to 5, *i. e.*, 53.3 per cent armature, 26.6 per cent field. That is a total of (20 per cent + 53.3 per cent) in the armature and (26.6 per cent + 10 per cent) in the field. That is, this equation says 73.3 per cent is in the armature leakage, 36.6 per cent in the field leakage; whereas, in fact, the division is 66.6 per cent in armature, 43.3 per cent in the field. Now obviously, by the well known equation referred to, this question would not come up if the total reactance is assumed to be in the armature. In other words, that equation involves the fundamental assumption that the total reactance is in the armature. If, therefore, one thinks in terms of that equation, one may go astray.

Another viewpoint, which seems more rational, yet perhaps less convenient, is that the sustained value of flux is zero at the initial instant, and by an exponential, reaches the sustained value; and at the same time the total initial flux decreases to zero by the same exponential. That is the flux at any time is

$$\phi = \Phi \epsilon^{-\alpha_f t} + \Phi_s \left(1 - \epsilon^{-\alpha_f t}\right)$$

where Φ = initial flux
Φ_s = sustained flux
α_f = field attenuation factor based on the total leakage of armature and field.

The flux which is caught in the armature circuit starts at that value and decreases to zero by

$$\phi_a = \Phi_a \epsilon^{-\alpha_a t}$$

The equation of short-circuit current crests would be, using notation in paper, and assuming wave completely offset,

$$i = \sqrt{2}\left\{ I_0\,\epsilon^{-\alpha_J t} + I_s\left(1 - \epsilon^{-\alpha_f t}\right) + I_0\,\epsilon^{-\alpha_a t} \right\}$$

These forms are consistent with the facts. They say that the division of flux in the above example is initially 66.6 per cent armature and 43.3 per cent, as facts require; and at sustained value, 20 per cent armature and 10 per cent field leakage. And at any time during the transient, the proper division is obtained from the equation.

While the other form may be used in practical calculation for practical results (and hence the familiar instead of the new forms were used in the paper) it yet becomes a serious difference if generalization is to be made from the formula to the extent of Diamant's statement that the initial a-c. component depends upon the difference between the initial and sustained flux. Think. If at the instant the short circuit occurs, the field terminals were also short-circuited, the initial armature and field current would necessarily be the same as if those terminals had not been short-circuited. In the former case, however, there would be no sustained value. The flux and currents would die to zero. So the difference between initial and sustained flux could not affect the initial current, either the a-c. or d-c. components; unless it is assumed always that the sustained value of flux is zero, and that is assuming the armature leakage to be zero.

Regarding terminology, we are critized for introducing the "new impressive, misleading name of transient reactance." It has escaped his notice that in previous A. I. E. E. papers and in other literature, this term has been proposed; and in the present paper at the beginning of Part 2, the origin of the term is recorded: Durgin and Whitehead in, 1912; F. D. Newbury in 1914. It may sound to Mr. Diamant as unscientific; but sometimes scientific terms are so long that commercial engineers will not use them, and it is necessary to have a shorter, if less accurate term. How many busy engineers could be persuaded to say, "I want 'the total leakage reactance reduced to armature terms.'" While we reaffirm our acceptance of the term, "transient reactance," as practical and descriptive, and also believe the terminology used in leading tests such as Steinmetz, Berg and Lawrence are scientifically sound, we believe that there should be some terminology which would mean the same to everybody, and avoid such confusion as is experienced in reading Diamant's 1915 paper. His discussion devotes considerable space to this matter. His terms are sound and consistent. Whether it is too late to ask the engineering world to change entirely to adopt Diamant's suggestions, or instead to adopt officially what is already generally used, is questionable.

We acknowledge the omission of Douglas' paper in the bibliography. It properly should have been included.

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PROTECTION FROM FLASHING FOR DIRECT-CURRENT APPARATUS

BY J. J. LINEBAUGH AND J. L. BURNHAM

Abstract of Paper

The equipment developed for the protection of direct-current apparatus as described in this paper is applicable to all direct-current apparatus and all methods of operation. Special means of protection for use only with particular apparatus or conditions of operation have not been mentioned. The principal steps in the experimental development of high-speed circuit breakers and flash barriers are briefly given.

The protection afforded by the high-speed breaker or barriers is sufficient for most apparatus and service, but *complete* protection for *any* direct-current apparatus and service requires both the high-speed breaker and flash barriers. Attention is directed to the importance of arranging the connections to the brush rigging so that the magnetic action on the arc will be a minimum, and properly directed, so the flash will do the least damage.

THE problem of protection from flashing has for many years confronted engineers who build and operate direct-current machines. Numerous schemes and suggestions have been put forward which it was hoped would overcome the tendency to flashover on extra heavy overloads or short circuits. Some time ago it was felt that the subject of prevention and protection from flashing had not received the study and investigation justified by the trouble experienced and it was decided to make a comprehensive study of the entire subject.

Some form of barrier has been the most common protection suggested, and different forms have been tried with a slight degree of success on some machines and absolute failure on others. It was the opinion of many engineers that barriers could not be designed to take care of a short circuit and that their value was doubtful. However, a special form of barrier, which gives the required protection, will be described later.

It was realized that the means for prevention of flashing at the commutator and brushes of direct-current machines must operate to remove the cause very quickly. The use of some form of high-speed device, which would open the circuit or insert resistance before the short-circuit current could reach a value which would cause flashing, seemed the most logical way to solve the problem, although it was appreciated that the action of the device must be much more rapid than any commercial circuit-opening device previously produced. An investigation was conducted along these lines and two distinct types of high-speed breakers developed, which will be described separately.

A flash at the commutator starts from excessive sparking. Sparking is produced by the breaking of current in the coils short-circuited by the brush as each segment of the commutator passes from under the brush. As the coil is inductive, the spark or arc tends to hold and, if the arc is of sufficient volume, the vapor produced thereby forms a low resistance path between segments and from brush to brush or to frame; through which a large current may pass. See Figs. 1 and 2.

Sparking may be prevented by providing a magnetic field of proper strength and distribution to influence the coils during reversal of their current as they pass through short circuit by the brushes. To provide the correct commutating field for all conditions of load has been the object of designers but success has been only partial. At high loads, saturation of magnetic circuits and distorting influence prevent attainment of the desired field, and for sudden changes in load the changes in field cannot be properly synchronized. It is more difficult to avoid sparking with rapidly varying loads than with gradually changing or steady load, but if a sudden load which would cause flashing is of short enough duration, the arcing at brushes may not produce enough conducting vapor to establish an arc supported by the main voltage. *The value of load that causes flashing when applied suddenly (short circuit) is a function of the time required to throw it off.* The quicker the circuit is opened the higher the value of current that will not cause arcing.

With the ordinary circuit breaker which begins to open in about 0.15 second, there is a certain maximum load which cannot be exceeded for each commutating machine without causing flashing. If feeders have sufficient resistance to limit the short-circuit current to this critical value, flashing will occur only on the rare occasion of a short circuit in a feeder itself. See Fig. 3. It has been the standard practise of nearly all manufacturers to recommend tapping the feeders, especially railway feeders, at a sufficient distance from the substation to insure enough resistance in the circuit to limit current in case of short circuit near the station.

Inductance may be added to the circuit to retard the rate of increase of current on short circuit to such an extent that the ordinary breaker will have time to trip before the current in the machine reaches a value that would cause flashing. The amount of inductance required to delay the rise in current sufficiently, however, introduces other disadvantages which make its use undesirable. When the current is interrupted, the increase in voltage from inductive "kick" is difficult for circuit breakers to handle and introduces the possibility of applying dangerous voltage stresses to the apparatus.

Reactors have been tried in a few instances with some success but it has always been a mooted question whether the resistance of the reactor did not give as much or more protection than the inductance of the coil, and if this is the case resistance only would be much cheaper to install. A coil to give the delay required is usually very large and expensive and occupies much valuable space, giving a total cost out of proportion to the cost of the machines protected or the protection obtained.

With special high-speed circuit-opening devices operating in about 0.005 second, the more sensitive machines, such as 60-cycle synchronous converters for railway voltages, may be short-circuited without flashing over, even though the maximum current is of higher value than would cause flashing with suddenly applied load and ordinary circuit-breaker protection.

The speed at which a circuit breaker must operate to prevent flashing depends on the amount of load thrown on the machine but, under worst conditions, our tests seem to confirm that it must be *quicker* than one-half cycle of the machine to be protected. The time of operation of the breaker would be measured between the time that the current reaches the flashing value to the time that the current is again reduced to the same value after the breaker opens. If the arc formed between two segments is not blown out as they pass from one set of brushes to the next and all following segments have similar arcs formed between them, the arc would completely bridge between positive and negative brushes in one-half cycle, which would complete the flashover. Complete flashover might also occur from gases being blown by windage, magnetically, or by expansion, to increase or decrease the half cycle time.

The time of operation of circuit breakers as given herein is measured from the beginning of short circuit to the instant the breaker begins to reduce the current rise.

Investigation covering these several schemes of protection was made, which it is believed will be of interest and will be described briefly with oscillograms, reproductions from photographs, etc., showing behavior under different loads and short-circuit conditions.

All short-circuit tests were made by connecting positive and negative terminals with a 500,000 circular mil cable; the only equipment in the circuit being the necessary current shunt for the oscillograph, a contactor to close the circuit, and a circuit breaker for overload protection, in addition to the protective device being investigated. Power for the 300-kilowatt, 25-cycle and 500-kilowatt, 60-cycle, 600-volt synchronous converters, used in fuse, barrier, reactor, and high-speed circuit breaker tests, was supplied from a 6000-kilowatt frequency changer set only a few feet from the test, so that there was very little drop in the voltage of the generator or from resistance, and the oil switch was set so that it did not trip out.

HIGH-SPEED CIRCUIT BREAKER

At the time this development was started it was felt that if a circuit breaker could be designed to operate *within* the time required for a commutator bar to pass from one brush to another; that is, within one half cycle, protection would be afforded against practically any short circuit. Designs were therefore begun on a circuit breaker which would open within 0.007 second, which would cover most commercial machines; *i.e.*, for 60 cycles and lower frequency.

High-speed breakers had been suggested and attempts made to produce such devices previous to this time but, as far as the writers know, had never been made to obtain as high speed as the discussion shows would be necessary.

Different types of construction were studied and samples of several preliminary models constructed without obtaining the speed desired. One of the most promising types of construction considered consisted of a knurled fly wheel operating continuously with a knurled cam, so designed and located that a current relay would insert a wedge between the wheel and the cam and trip a breaker attached to the cam by suitable toggle mechanism. This preliminary sample indicated that 0.035 second was the best speed that could be attained.

It was then decided to concentrate all energies on a circuit breaker using the well known principle of a latch, heavy spring

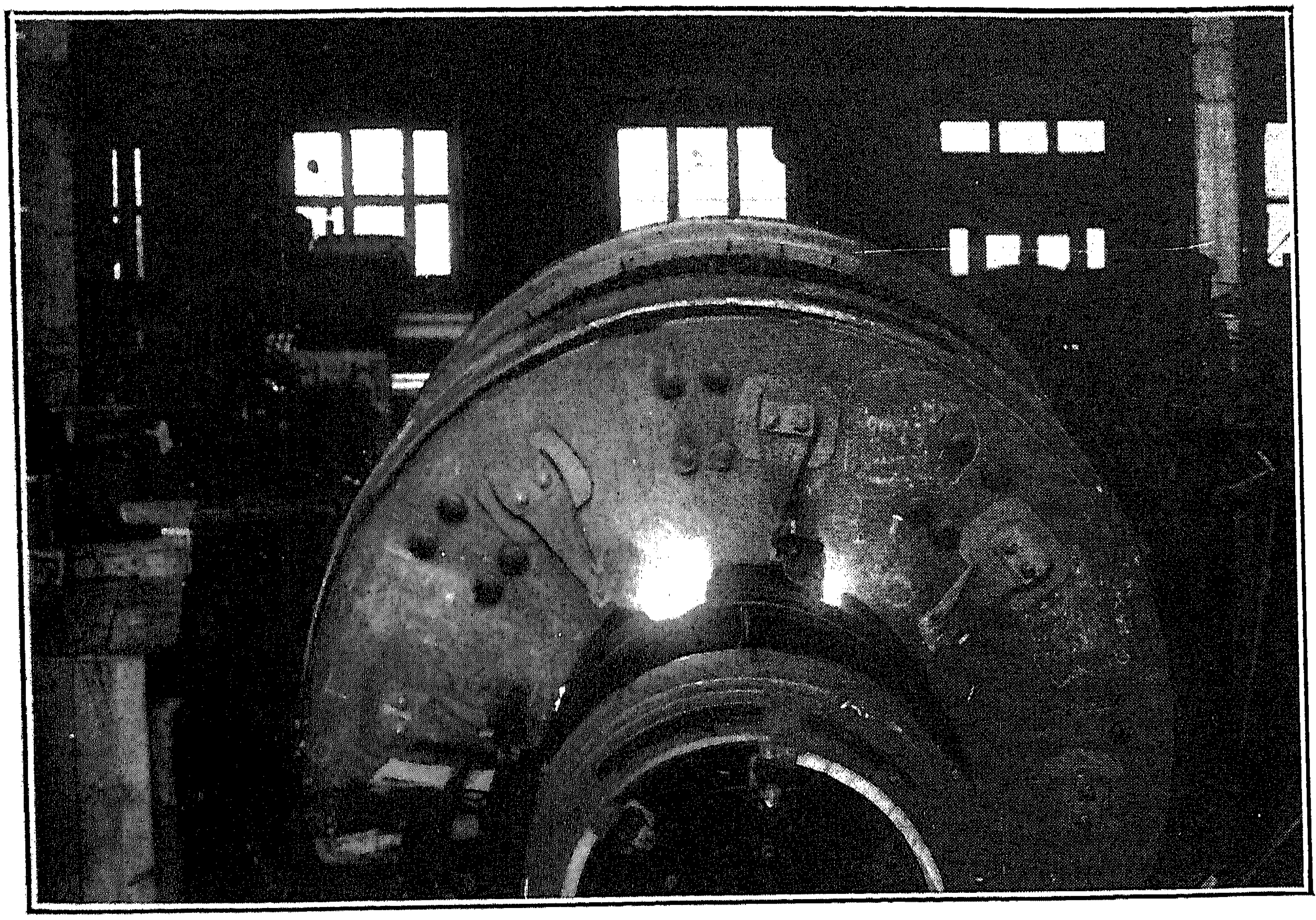


FIG. 1

Flashing at brushes on 1000-kw., 1500-volt generator forming part of 2000-kw., 3000-volt motor-generator set at five times load, showing different stages of arc formation.

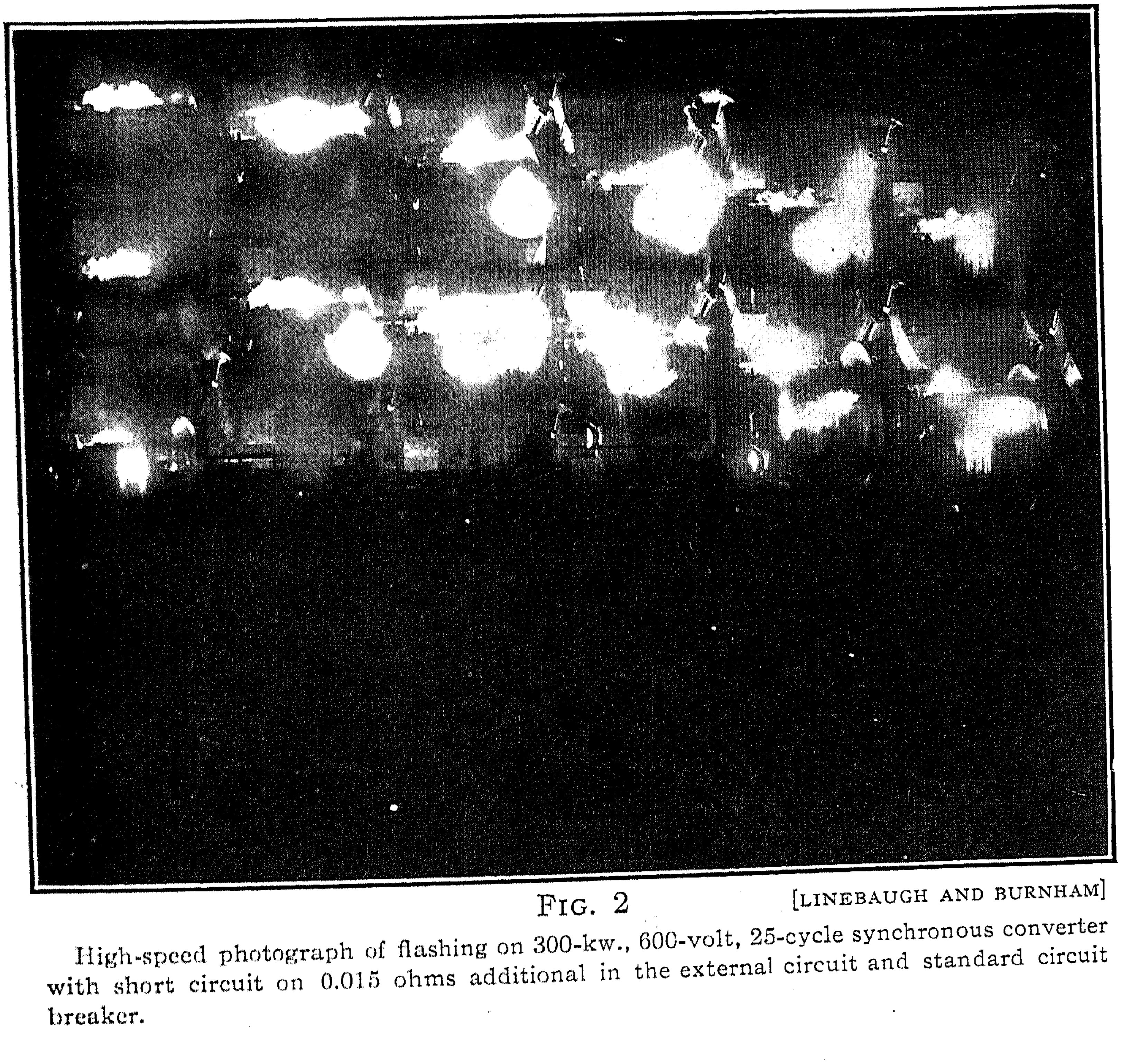


FIG. 2 [LINEBAUGH AND BURNHAM]

High-speed photograph of flashing on 300-kw., 600-volt, 25-cycle synchronous converter with short circuit on 0.015 ohms additional in the external circuit and standard circuit breaker.

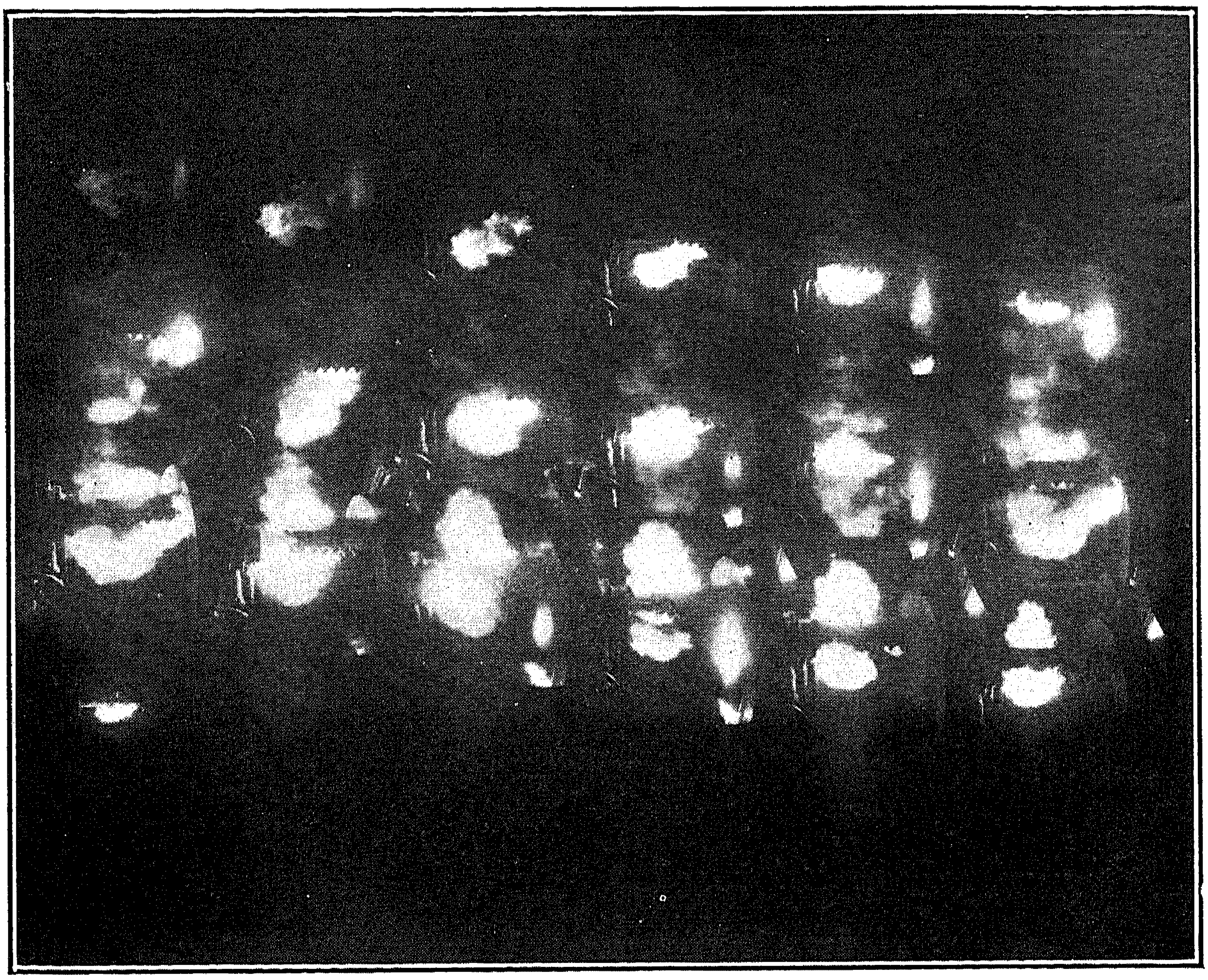


FIG. 3

High speed photograph of short circuit on 300-kw., 600-volt, 25-cycle, synchronous converter with standard circuit breaker.

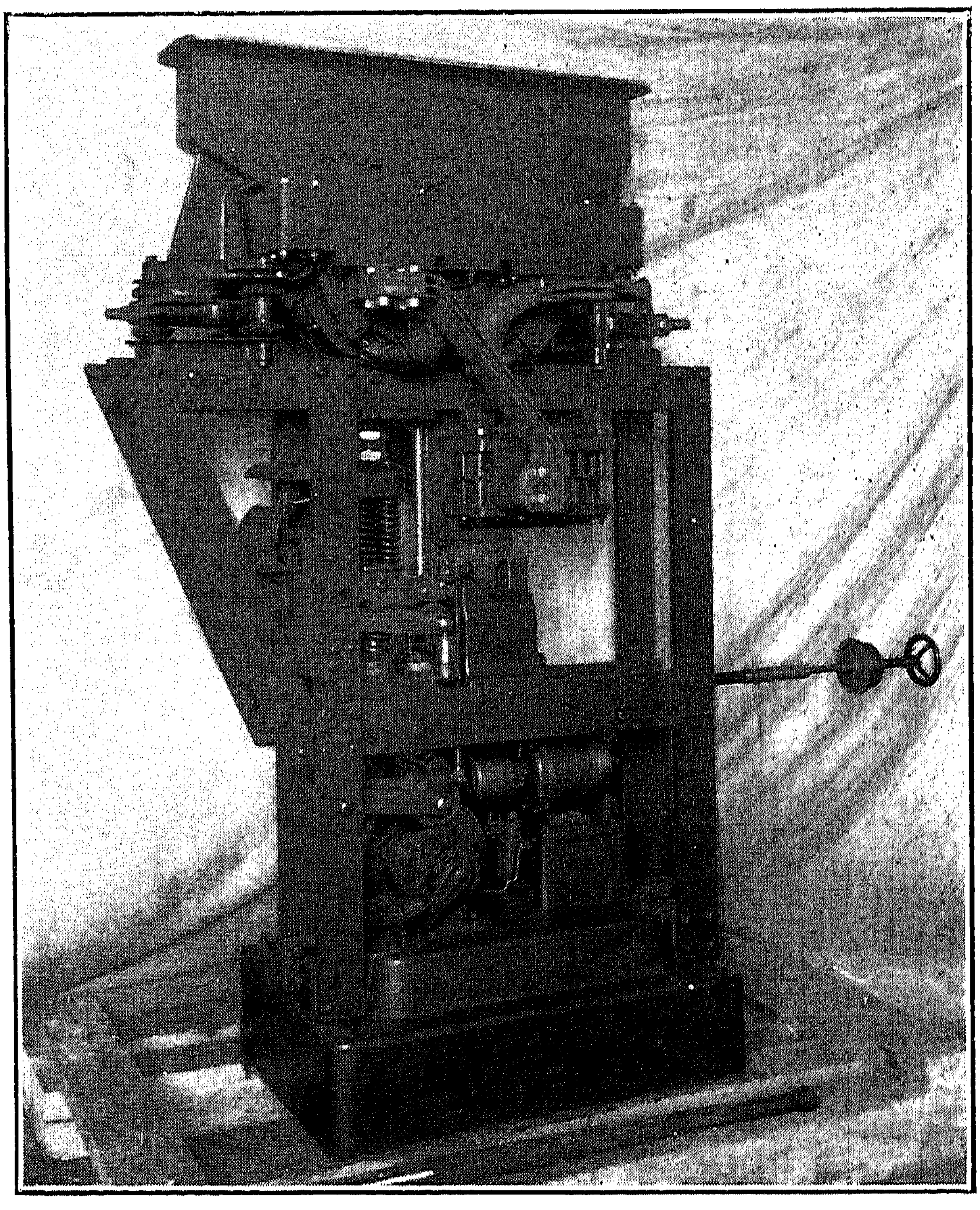


FIG. 4 [LINEBAUGH AND BURNHAM]

3000-ampere, 3600-volt, direct-current high-speed circuit breaker.

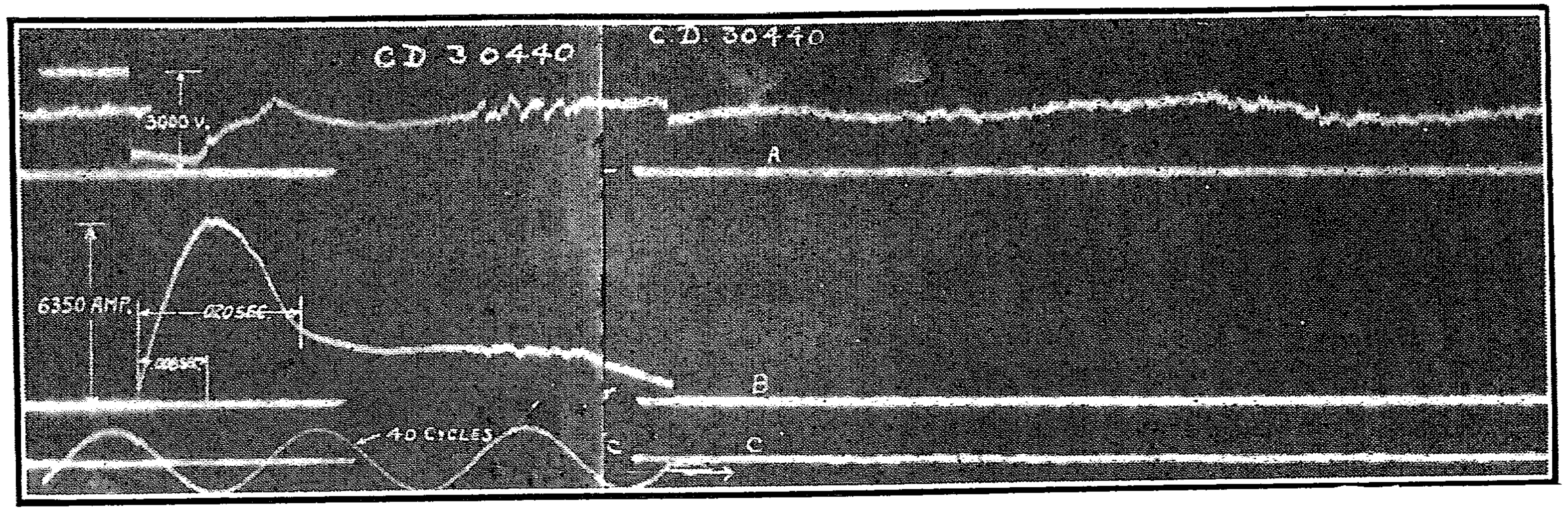


FIG. 5

Direct-current short circuit on 2000-kw., 3000-volt, motor-generator set with high-speed circuit breaker and standard 3000-volt switchboard type circuit breaker.

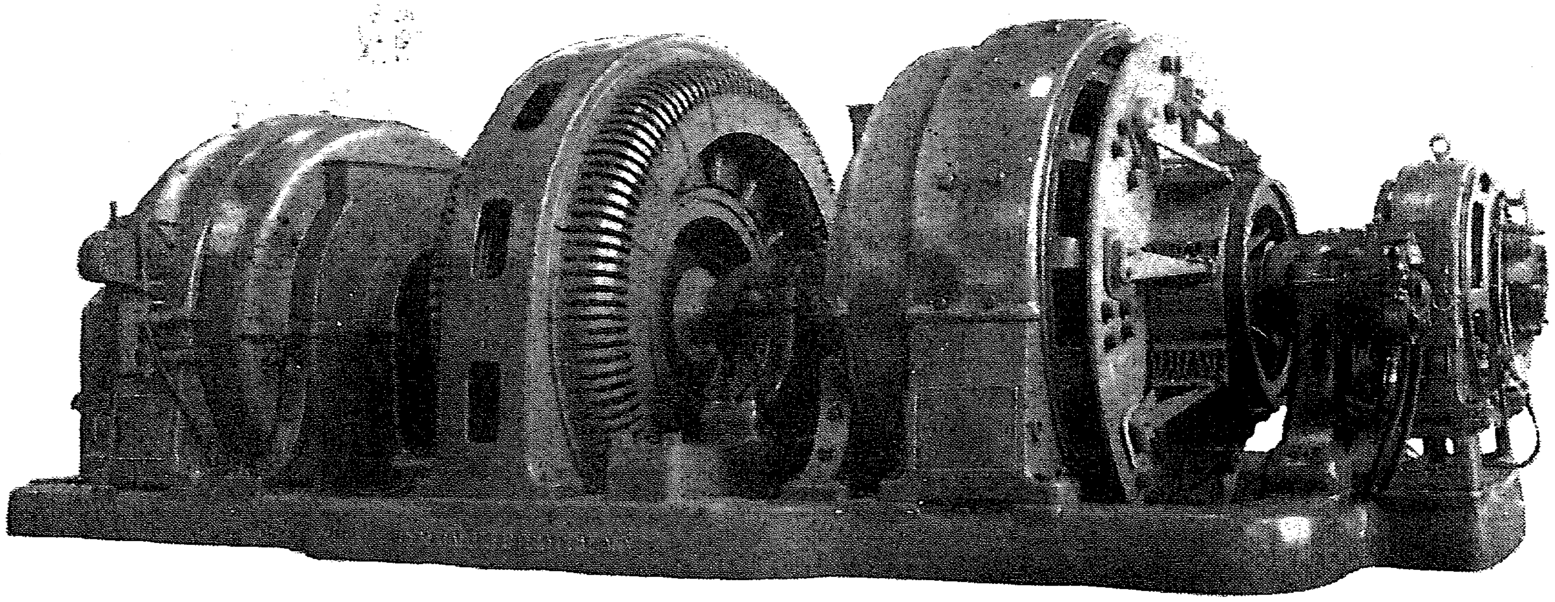
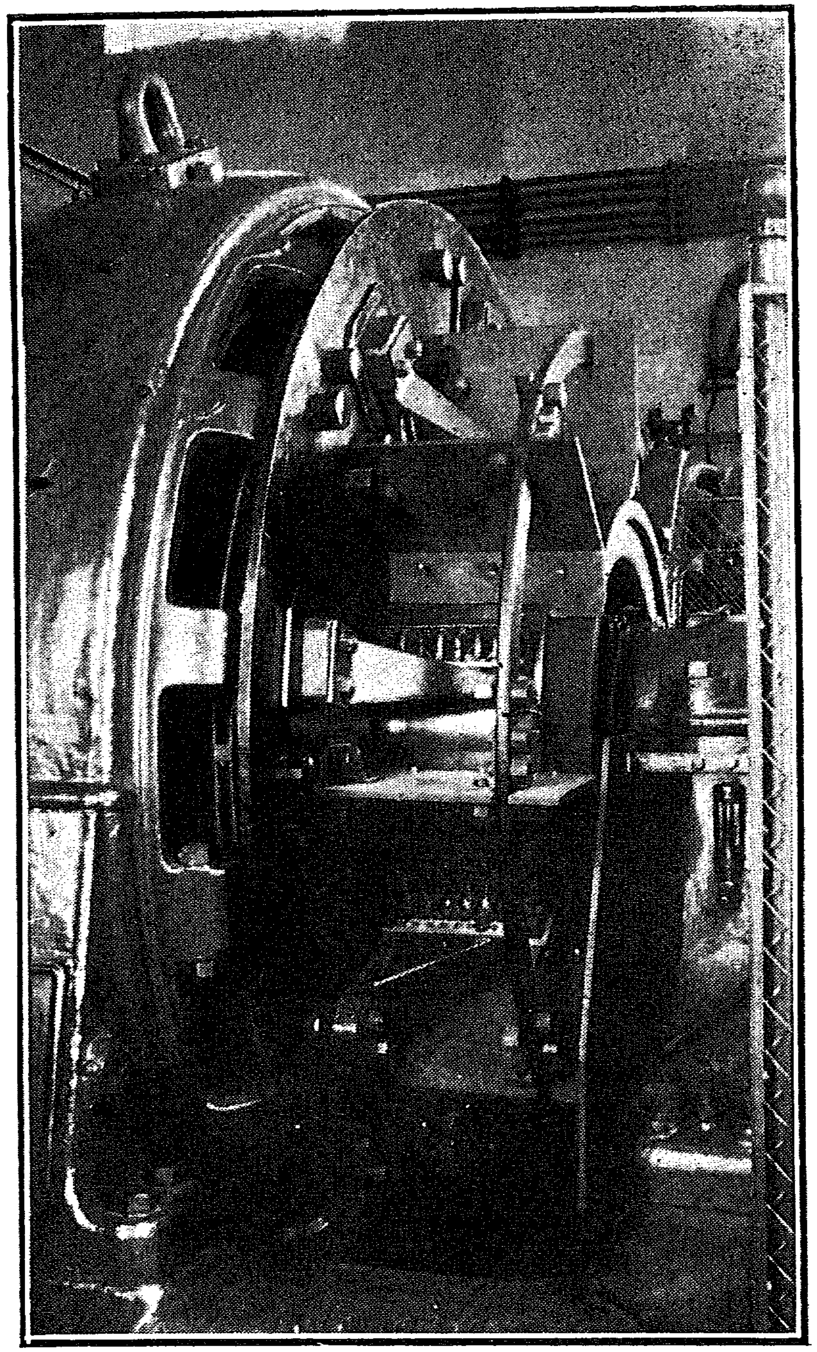


FIG. 6

2000-kw., 3000-volt, direct-current synchronous motor-generator set before assembly of flash-barriers.



[LINEBAUGH AND BURNHAM]

FIG. 7

Type of flash barrier installed on 2000-kw., 3000-volt, synchronous motor-generator set used in connection with high-speed circuit breaker.

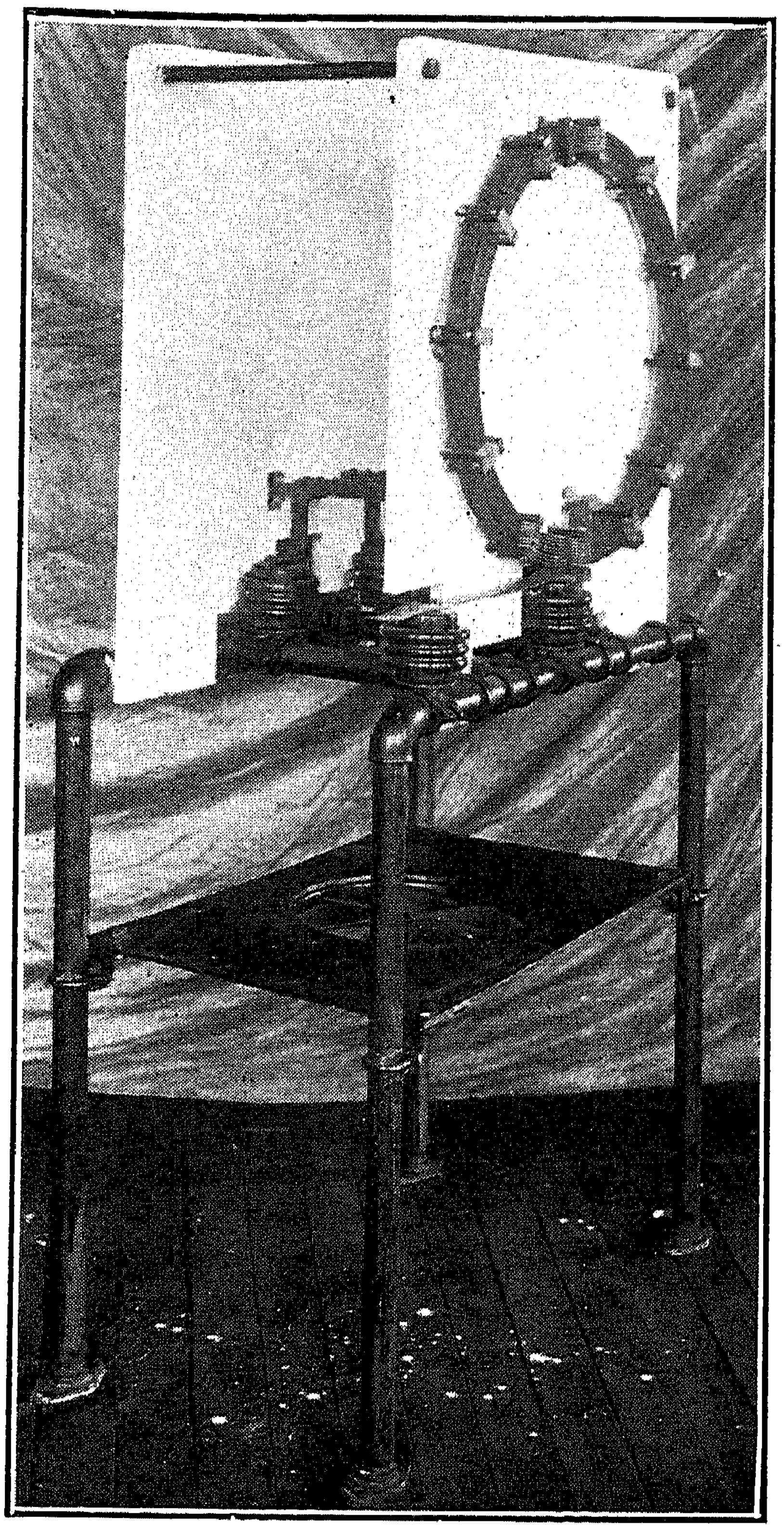


FIG. 9
High-speed air cooled fuse holder with magnetic blow-out used in test.

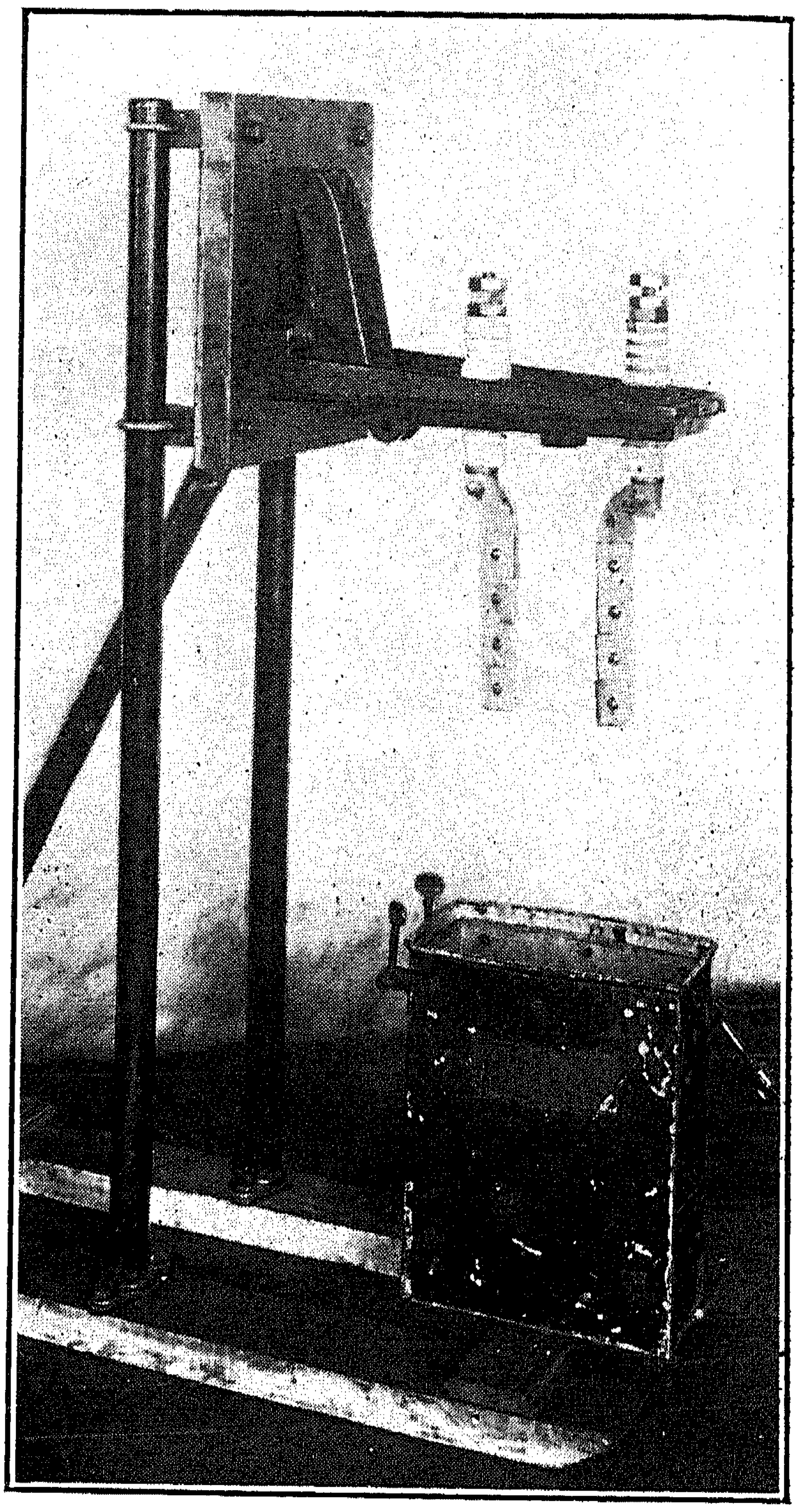


FIG. 10
High-speed oil-cooled fuse holder, used in test.

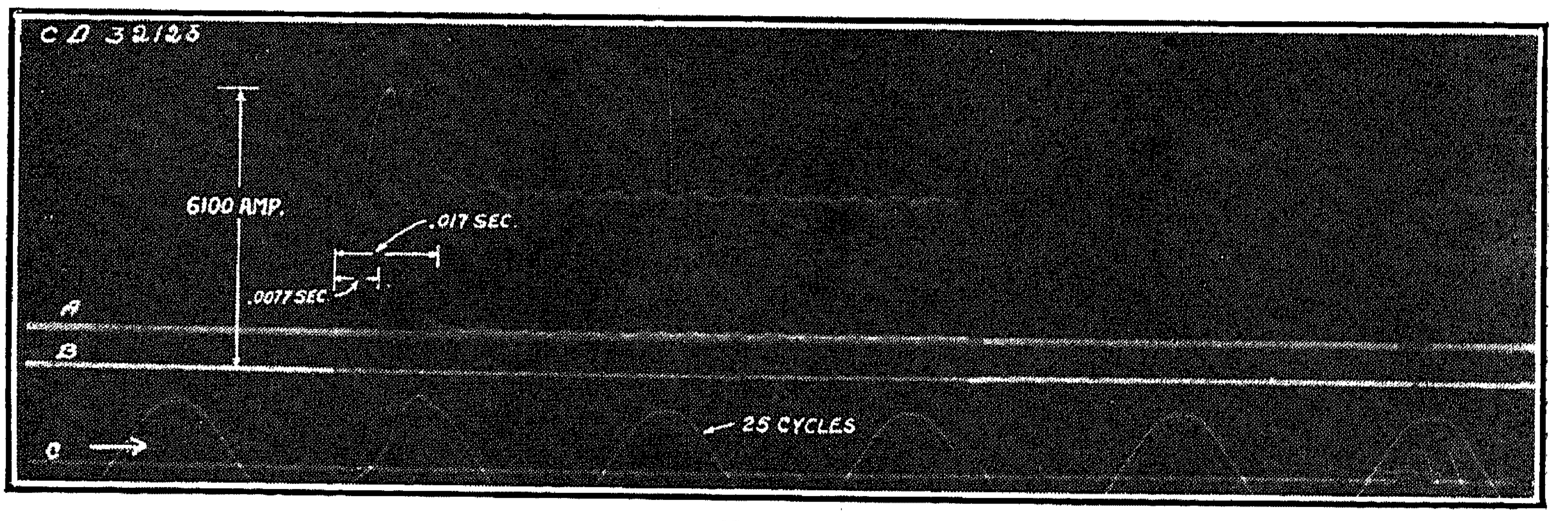


FIG. 11 [LINEBAUGH AND BURNHAM]

Short circuit on 300-kw., 600-volt, 25-cycle, synchronous converter protected by air-cooled high-speed fuse. Curve A, voltage across fuse; Curve *B*, line current; Curve *C*, collector-ring voltage.

and series tripping coil, and the high-speed breaker shown in Fig. 4 was finally built.

The problem was to obtain very quick tripping, rapid acceleration of contacts and a sufficient number of ampere turns in the magnetic blowout to insure rapid breaking of the arc. Previous ideas of design had to be abandoned when working for such high speed when a loss of 0.001 second meant a very serious increase in time of operation.

It was found that a series blowout coil had to be used, as sufficient time could not be allowed for the building up of a field after the contacts opened as is ordinarily done in circuit-breaker design, and the strength of this coil must be many times that usually used to rupture the circuit by giving the quick start and acceleration to the arc necessary for the speed desired. The breaker in question has a total of about 150,000 ampere turns at the maximum current obtained.

The moving parts must all be as light as possible, consistent with the great strength required, so that they can be started, accelerated and stopped in a very short space of time and distance. Even with this type of construction, it was found necessary to use somewhat high spring pressure; the spring being compressed to about 8000 pounds when the breaker was closed and ready for tripping.

A very special latch with very small tripping movement was designed somewhat similiar to the hair trigger on a rifle, in connection with a special high-speed tripping coil so that about 0.001 inch movement of the plunger would trip the breaker. It will assist in appreciating the speed attained when it is noted that the breaker must be arranged so that it will not trip under ordinary load condition and must be set above the tripping point of the regular substation breaker so that it will act while the current is increasing from say three and one-half times load to eight times load; current rising at the rate of about 1,000,000 amperes per second. Fig. 5 gives a very good idea of speed and limiting of current, from which it will be seen that the breaker starts to insert resistance in about 0.008 second and the load on the machine is reduced well below the flashing value in 0.020 second after the short circuit was applied.

A breaker was tested very exhaustively in connection with a 2000-kilowatt 3000-volt direct-current synchronous motor-generator set shown in Fig. 6, built for the Chicago, Milwaukee & St. Paul electrification, and found to give complete protection from damage or burning on short circuit when equipped with barriers shown in Fig. 7.

In connection with the test, it was found that even the speed of 0.008 second obtained would not completely protect machines from flashing on the most severe short circuit, and barriers shown were designed and installed. Tests referred to with high-speed breakers were taken with these barriers, which will be described later.

It is evident that it is preferable in case of short circuit to insert resistance by a high-speed breaker to quickly limit the current to some conservative value and then open the circuit. This type of protection has been adopted as standard. All tests, investigations, et cetera, were based on this theory, although some tests were taken by opening the circuit. It was found that there was a greater tendency for machine to flash if the circuit was opened completely at one time or if too high resistance was inserted, reducing the load to too low a value. For the sake of convenience and comparison, all tests were made throwing short circuit on the machine without load.

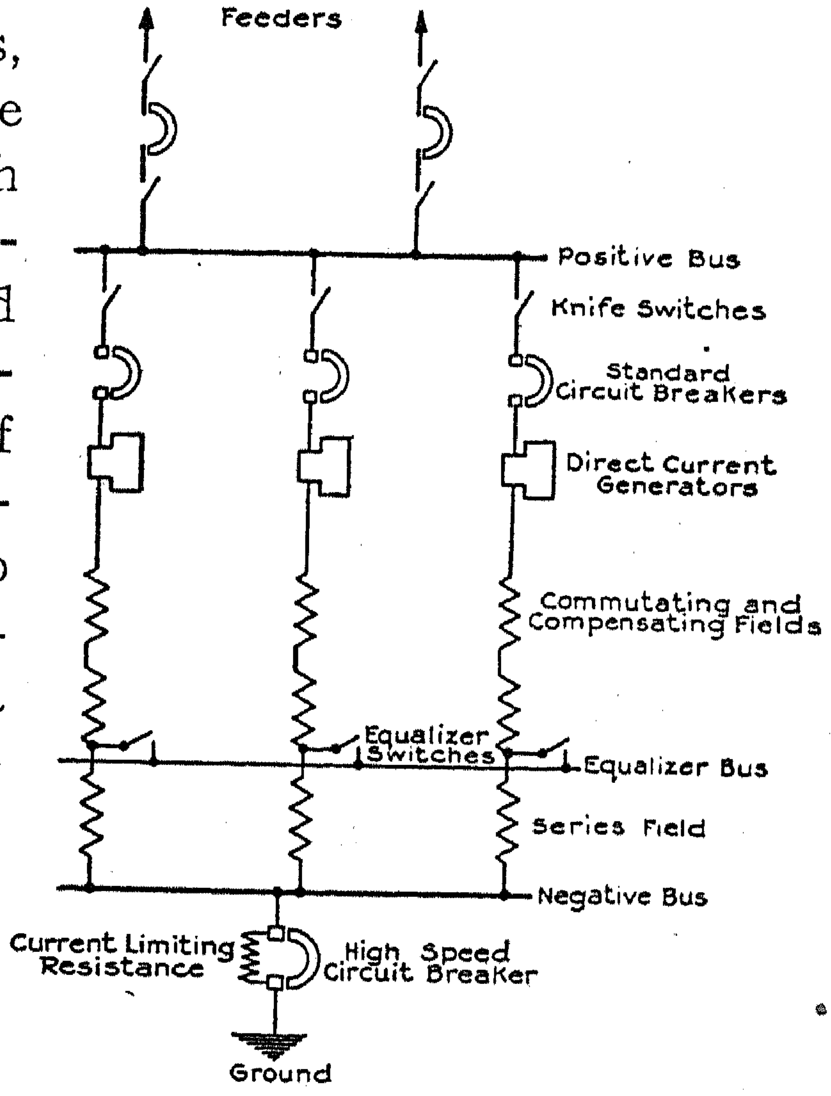

Feeders
Positive Bus
Knife Switches
Standard Circuit Breakers
Direct Current Generators
Commutating and Compensating Fields
Equalizer Switches
Equalizer Bus
Series Field
Negative Bus
Current Limiting Resistance
High Speed Circuit Breaker
Ground

FIG. 8

Diagram of direct-current connections for substation equipped with three motor-generator sets protected by one high-speed circuit breaker connected across limiting resistance

Some of these breakers have been in service since early in 1917 in the substations of the Chicago, Milwaukee & St. Paul Railroad and have amply justified the faith of the railroad company and the designers, as they protect the apparatus from all short circuits experienced, although all feeders are tapped directly to the overhead trolley system immediately at the substation.

One of these breakers is installed in each of the substations connected between the negative bus of the station and the ground or return circuit, as this location gives maximum protection, and one breaker can be used for each machine or one for the entire substation, as shown in Fig. 8.

HIGH-SPEED FUSE

It is evident that if a fuse could be developed that would melt at a very small increment of current above normal rating, it might be possible to obtain a speed which would limit the current on a short circuit along the same line as the high-speed circuit breaker just described.

A careful study of all available metals was made by Mr. P. E. Hosegood, who suggested using a silver fuse, and a number of silver fuses of different shapes were tried in the special fuse holders shown in Figs. 9 and 10. The oscillograph record, taken with air break fuse and magnetic blow-out, shown in Fig. 11, indicates that a very high speed is obtained, giving excellent protection and duplicating almost exactly the speed of the high-speed circuit breaker. It was found that a short circuit could be thrown on the 300-kilowatt, 25-cycle, 600-volt synchronous converter without flashing over and with very little sparking at the brushes. The oil-immersed fuse holder without magnetic blow-out gave practically the same result (Fig. 12), the operation being slightly better as far as speed was concerned but the mechanical difficulties of replacing the fuse, etc., being greater.

REACTORS

Oscillograph records of short circuit on the 300-kilowatt, 25-cycle, 600-volt synchronous converter show an average initial current rise of about 1,300,000 amperes per second. To protect by reactance, the amount required would depend on the rate of circuit-breaker action. With coils made of 1000 feet of 500,000-circular mil cable, wound on cable reels having an inductance of approximately 0.02 henry in circuit, this particular machine could be short-circuited without flashing when protected by a breaker opening in about 0.15 seconds.

An examination of records, Figs. 13 and 14, will show the severe duty on the circuit breaker and increase in voltage on the apparatus.

It was suggested that shunting the reactor by resistance might reduce duty on the circuit breaker. The coils were shunted by 14 and by 100 ohms and it was impossible to determine from either observation or oscillograph any effect due to the resistance.

The effect of an iron core in a reactor having an inductance of 0.00105 henry is shown in Fig. 15, from which it will be noted that the iron saturated at about 1000 amperes in about 0.007

seconds, after which the current rises abruptly, being limited only by the inductance of the coil as if there were no iron in its magnetic circuit. The delay of about 0.007 seconds due to the presence of iron in the coil, is far less than the time required for the usual breakers, now in use, to open. The weight of this reactor was 7 per cent of the weight of the synchronous converter and would have to be many times larger to give protection with an ordinary breaker.

SECOND FORM OF HIGH-SPEED CIRCUIT BREAKER

Mr. J. F. Tritle has more recently suggested a design for a high-speed circuit breaker which is simple and substantial in construction. This device was built as shown in Fig. 16 and test indicated that the speed was even faster than the large breaker previously described, as will be seen by comparing oscillograms, Fig. 5, on the large breaker, and Fig. 17 taken with the later breaker. This device is essentially a contactor having a laminated structure with electric holding coil and series bucking coil so that it opens when the load current reaches a value sufficient to offset the ampere turns of the holding coil. Tests on the 300-kilowatt, 25-cycle synchronous converter with this device showed that a short circuit could be thrown on the machine without any tendency of the machine to flash over, and the only sparking obtained extended not over one-half inch from the brushes. Similar tests, Fig. 18, on the 60-cycle, 500-kilowatt synchronous converter showed more sparking and, although it protected the machine at times on short circuit, there were other times when the machine flashed over. When the machine was equipped with barriers, dead short circuit could be thrown on with impunity, there being no tendency to flash over and scarcely sufficient sparking to be noticeable.

This later type of high-speed breaker is a part of the more recent equipment being furnished the Chicago, Milwaukee & St. Paul Railway.

BARRIERS

The barriers shown in Fig. 7 in connection with the description of the high-speed circuit breaker were developed to delay time of flashover, so that the breaker would give complete protection. Such satisfactory and promising results were obtained without the breaker that it was decided to continue investigation to ascertain if it would be possible to devise barriers

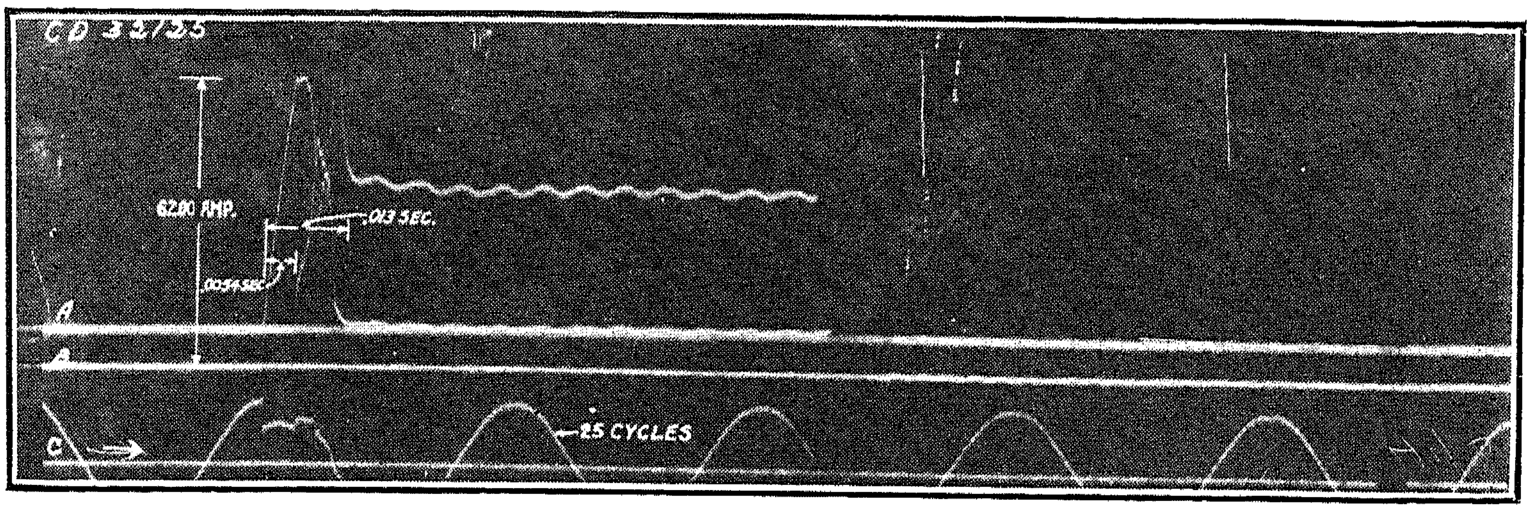


FIG. 12

Short circuit on 300-kw., 600-volt, 25-cycle, synchronous converter protected by oil-cooled high-speed fuse. Curve *A*, voltage across fuse; Curve *B*, line current; Curve *C*, collector-ring voltage.

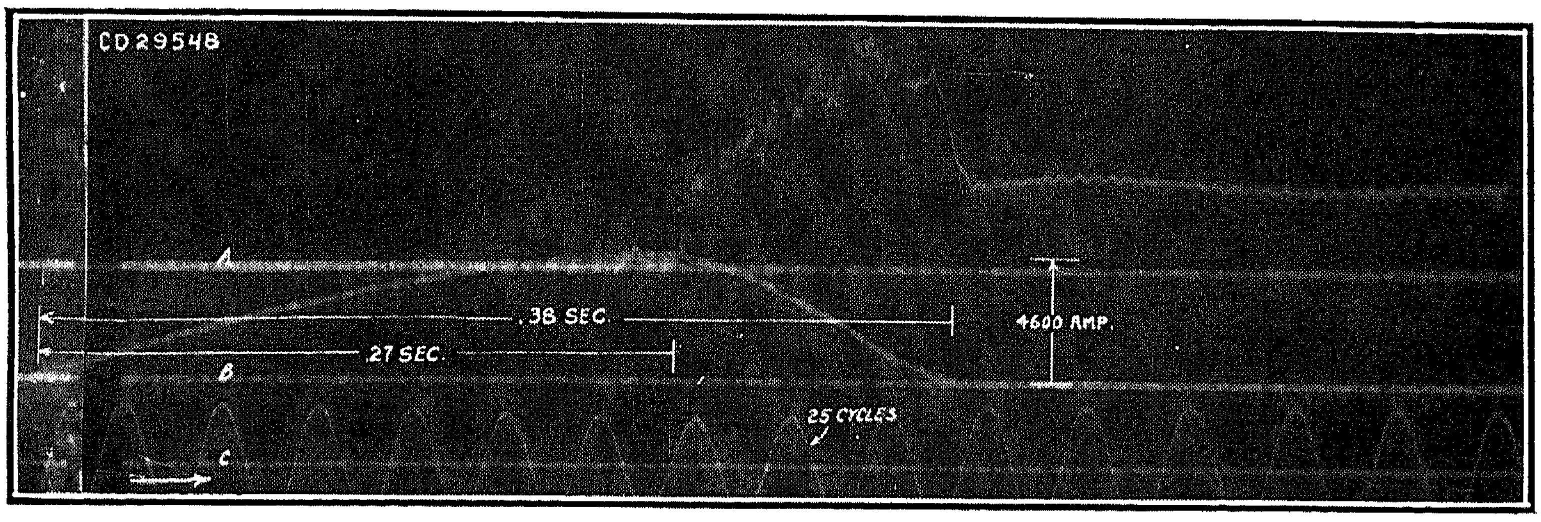


FIG. 13

Short circuit on 300-kw., 600-volt, 25-cycle, synchronous converter protected by air-core reactor in direct-current circuit and standard circuit breaker. Curve *A*, voltage across circuit breaker; Curve *B*, line current; Curve *C*, collector-ring voltage.

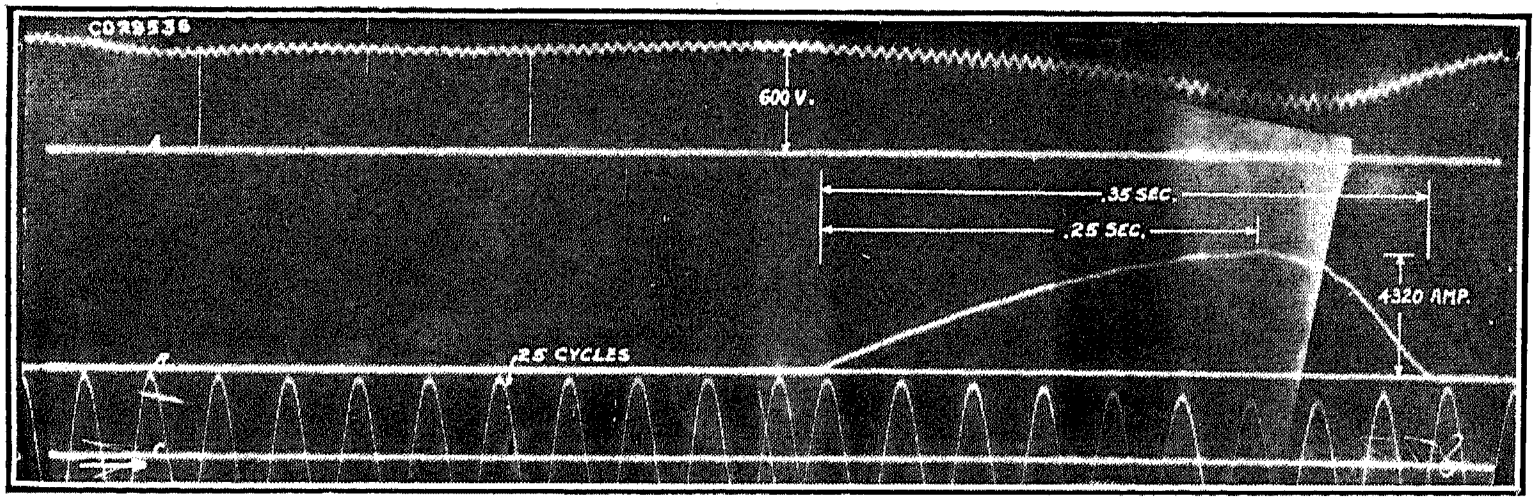


FIG. 14

Short circuit on 300-kw., 600-volt, 25-cycle, synchronous converter protected by air-core reactor in direct-current circuit and standard circuit breaker; Curve-*A*, voltage across armature; Curve *B*, line current; Curve *C*, collector-ring voltage.

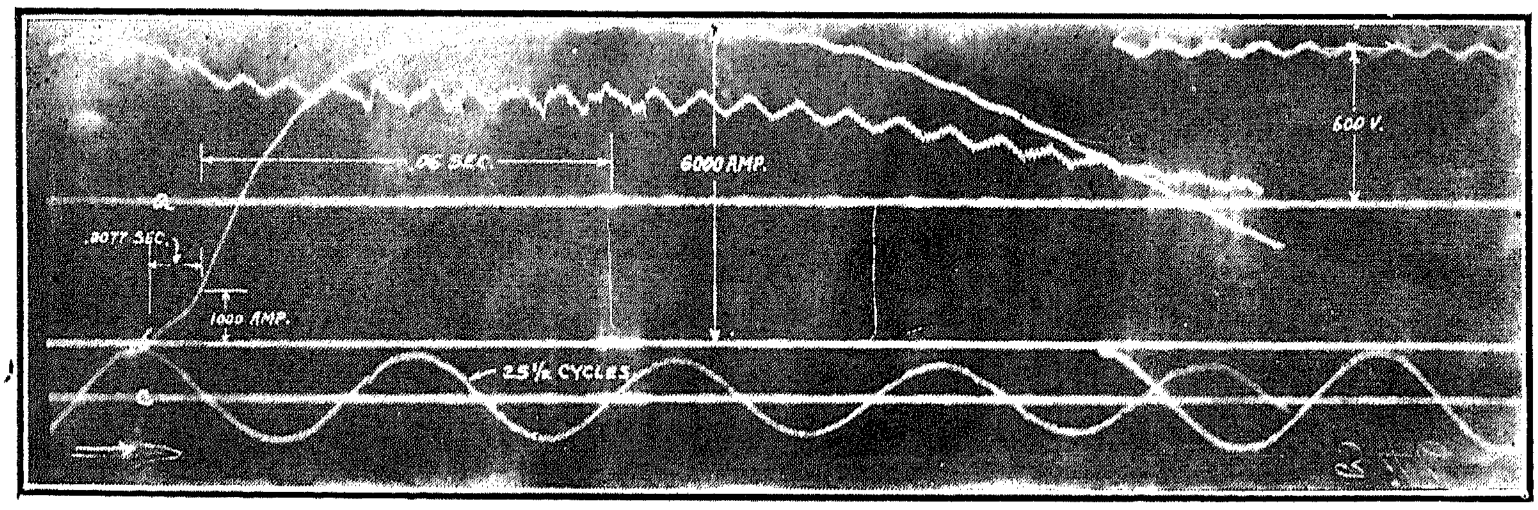


FIG. 15 [LINEBAUGH AND BURNHAM]

Short circuit on 300-kw., 600-volt, 25-cycle, synchronous converter, protected by iron-core reactor in direct-current circuit and standard circuit breaker. Curve *A*, voltage across the armature; *C*urve *B*, line current; *C*urve *C*, collector-ring voltage.

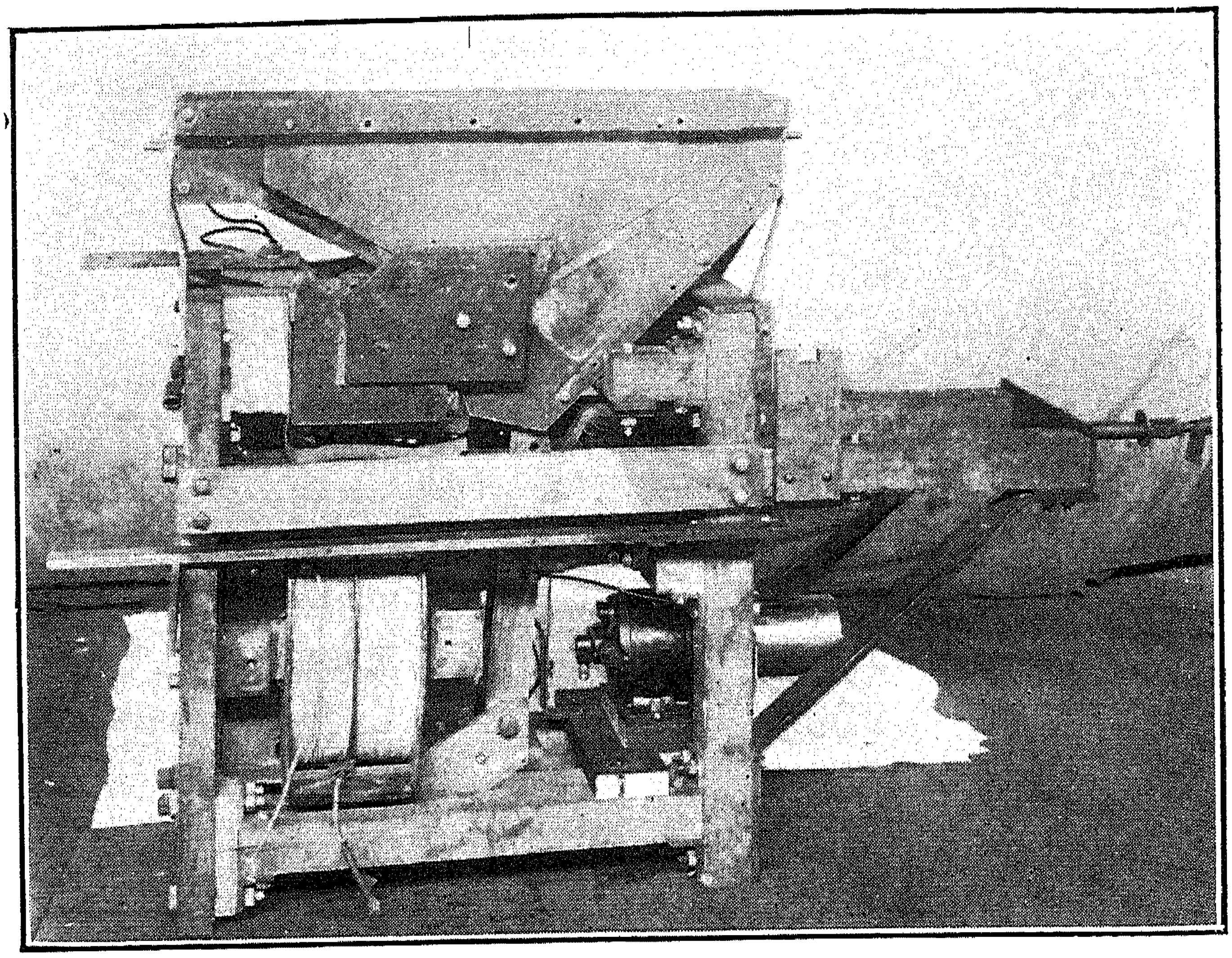


FIG. 16

Second form of high-speed circuit breaker, capacity 1500 amperes, 600 volts.

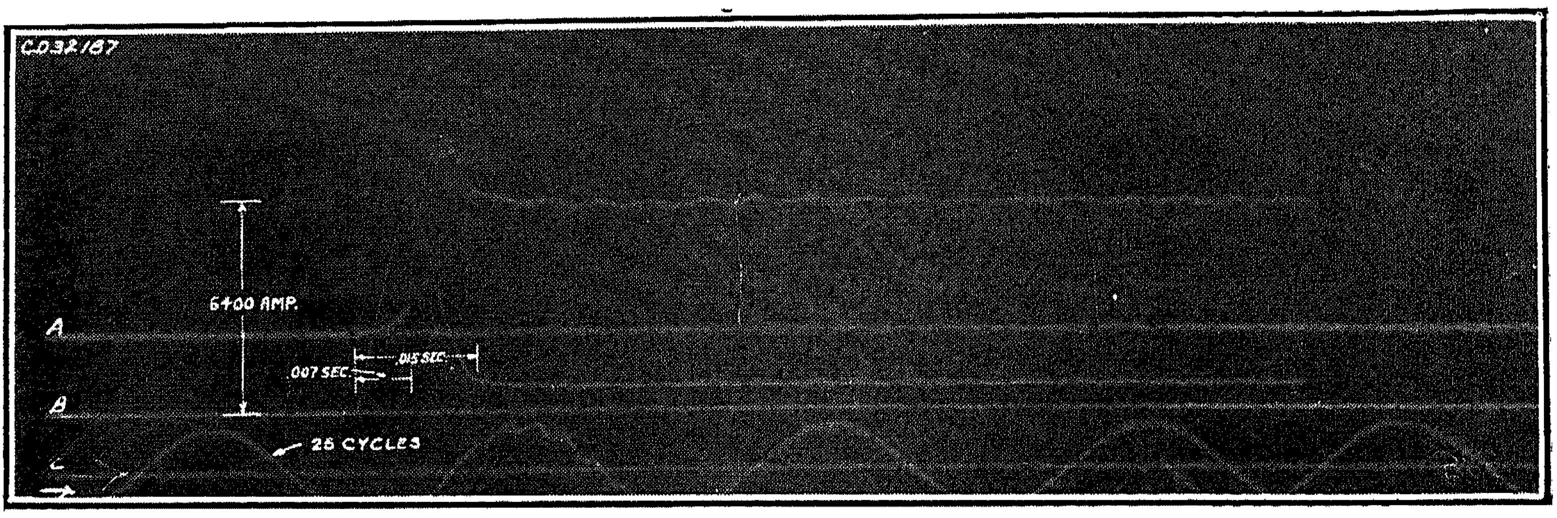


FIG. 17

Short circuit on 300-kw., 600-volt, 25-cycle synchronous converter protected by second form of high-speed circuit breaker. Curve *A*, voltage across circuit breaker; Curve *B*, line current; Curve *C*, collector-ring voltage.

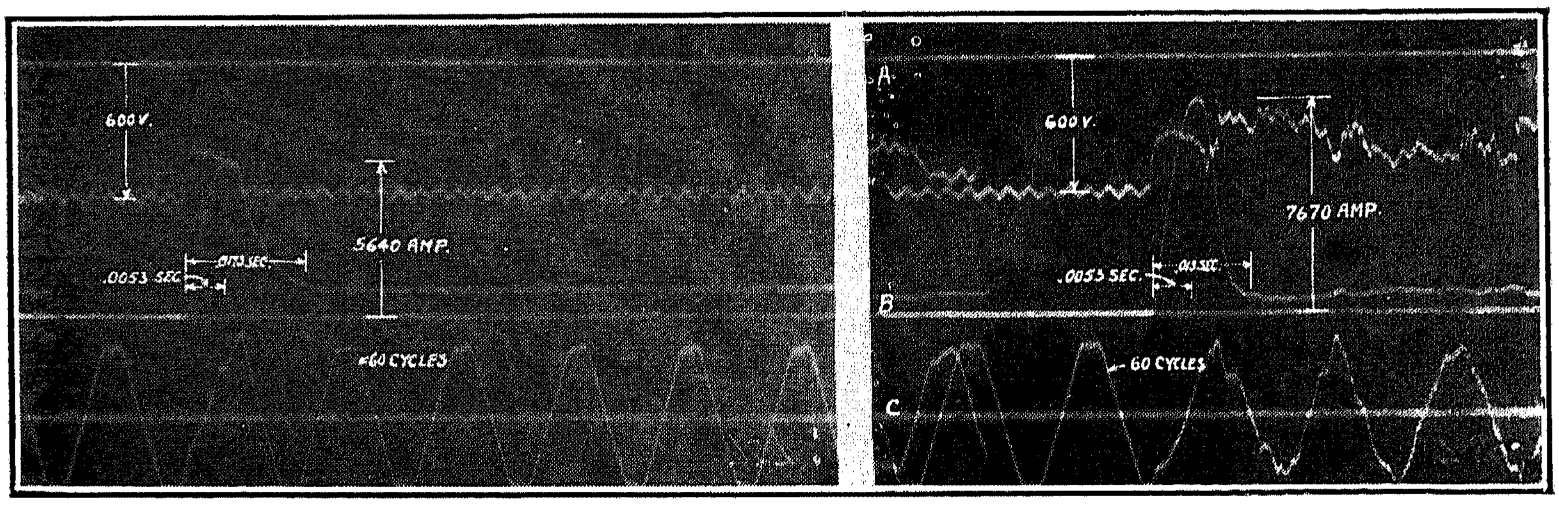


FIG. 18 [LINEBAUGH AND BURNHAM]

Short circuit on 500-kw., 600-volt, 60-cycle, synchronous converter protected by second form of high-speed circuit breaker.

Left hand curve	Right hand curve
Load of 0.03 ohms	Short circuit.

Curve *A*, armature volts
" *B*, line current
" *C*, collector-ring voltage.

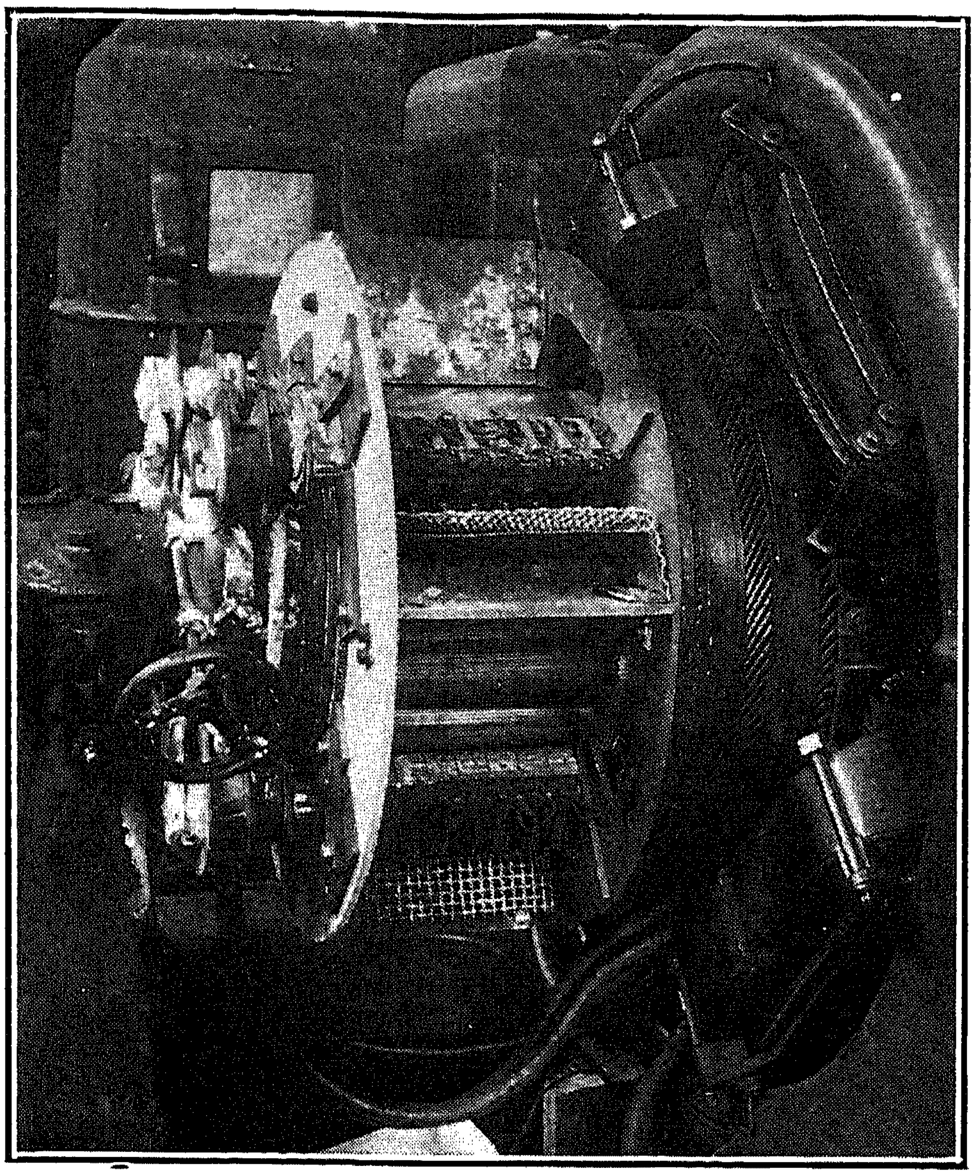


FIG. 19
Final development of flash barriers on 300-kw., 25-cycle, 600-volt synchronous converter.

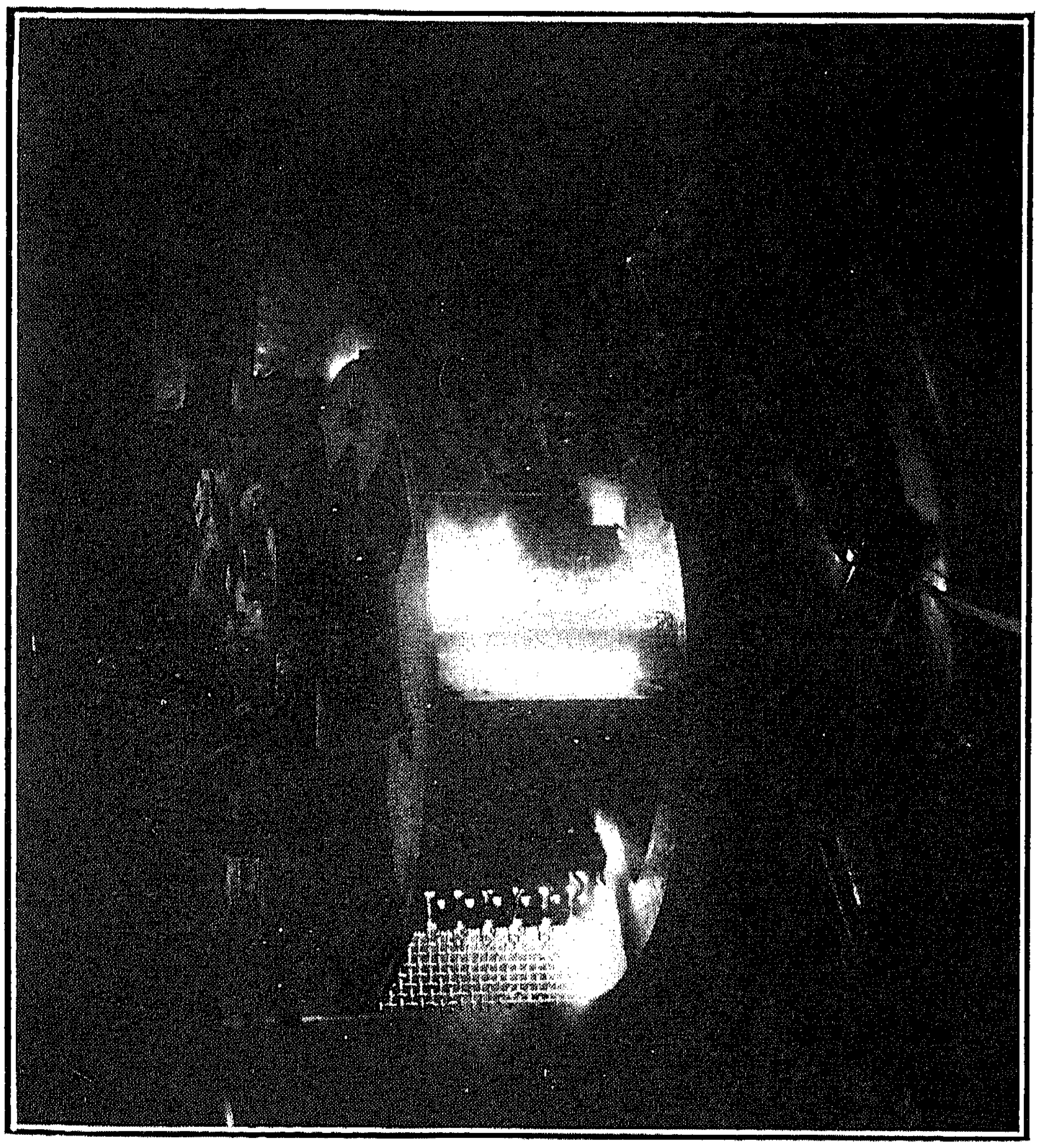


FIG. 20 [LINEBAUGH AND BURNHAM]
Short circuit on 300-kw., 25-cycle, 600-volt, synchronous converter protected by flash barriers and standard circuit breaker.

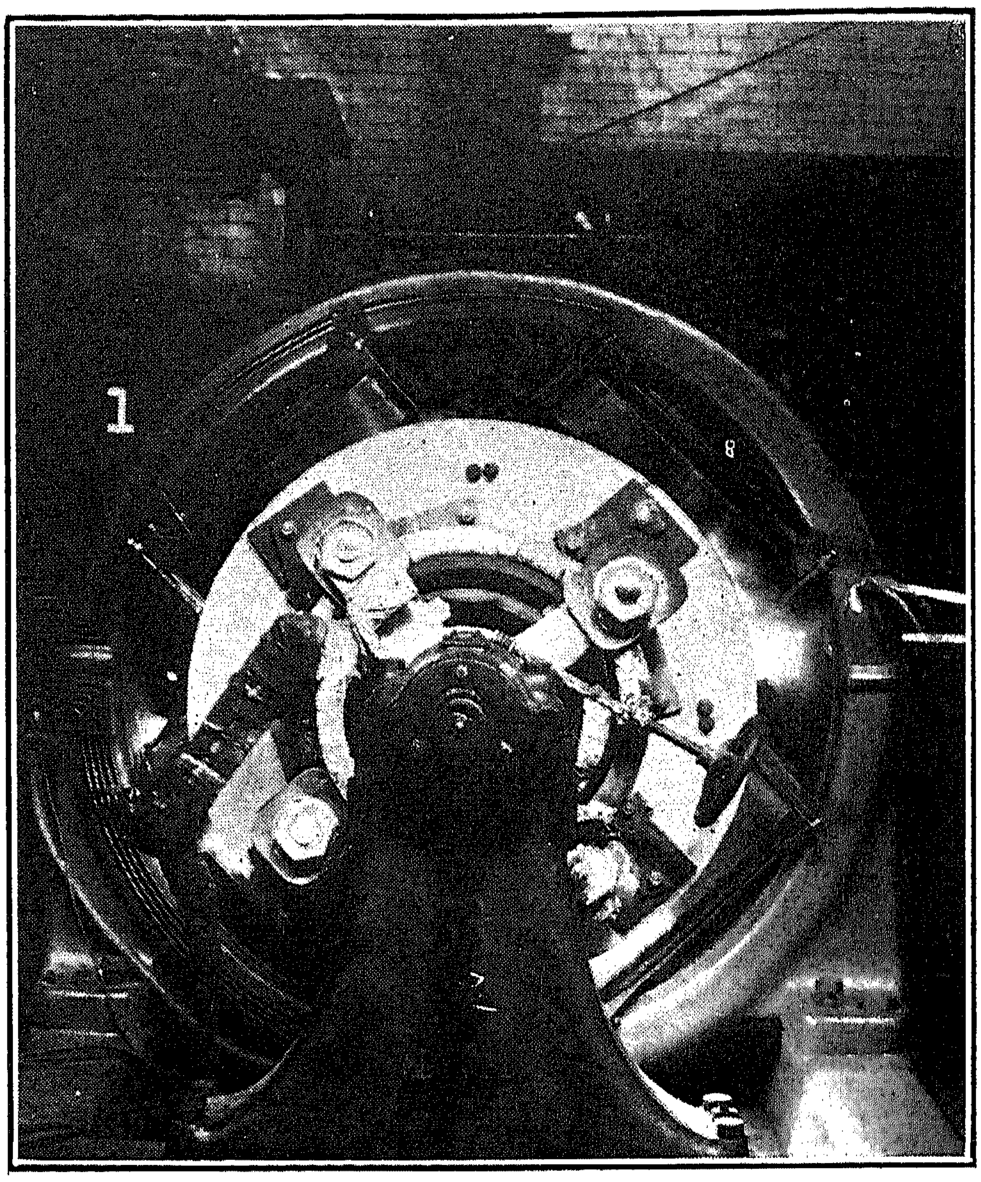


FIG. 21

Short circuit on 300-kw., 25-cycle, 600-volt synchronous converter protected by flash barriers and standard circuit breaker.

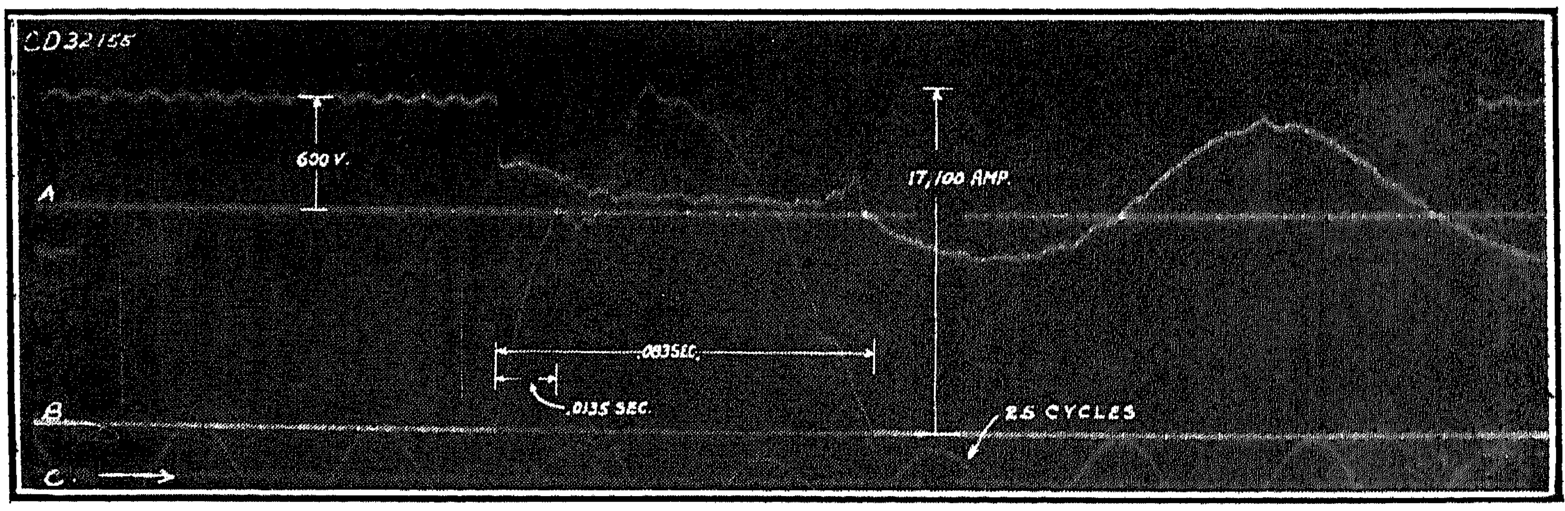


FIG. 22 [LINEBAUGH AND BURNHAM]

Short circuit on 300-kw., 600-volt, 25-cycle, synchronous converter equipped with flash barriers and standard circuit breaker. Curve *A*, armature volts; Curve *B*, line current; Curve *C*, collector-ring voltage.

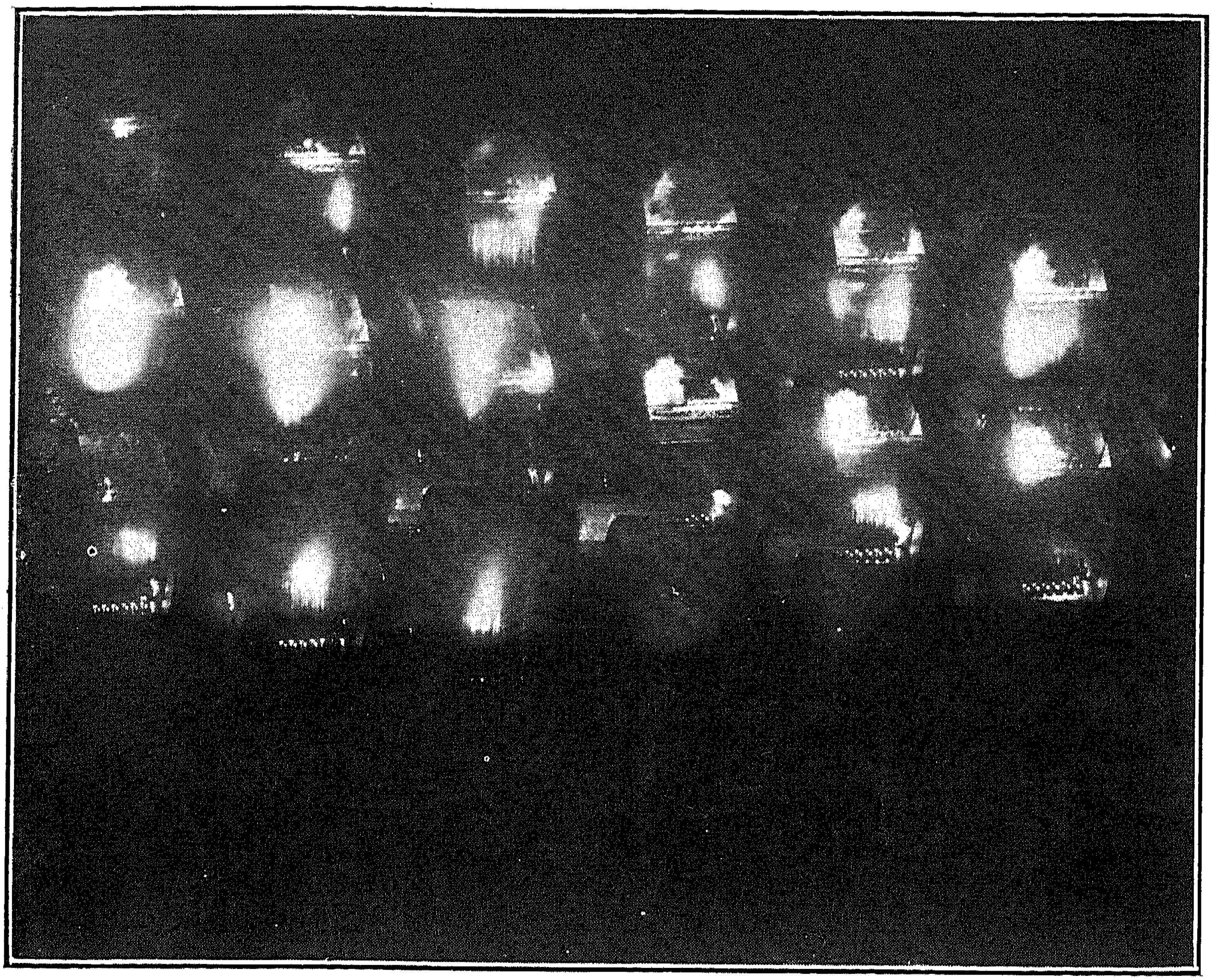


FIG. 23
High-speed photograph of short circuit on 300-kw., 600-volt 25-cycle synchronous converter protected by flash barriers and standard breaker.

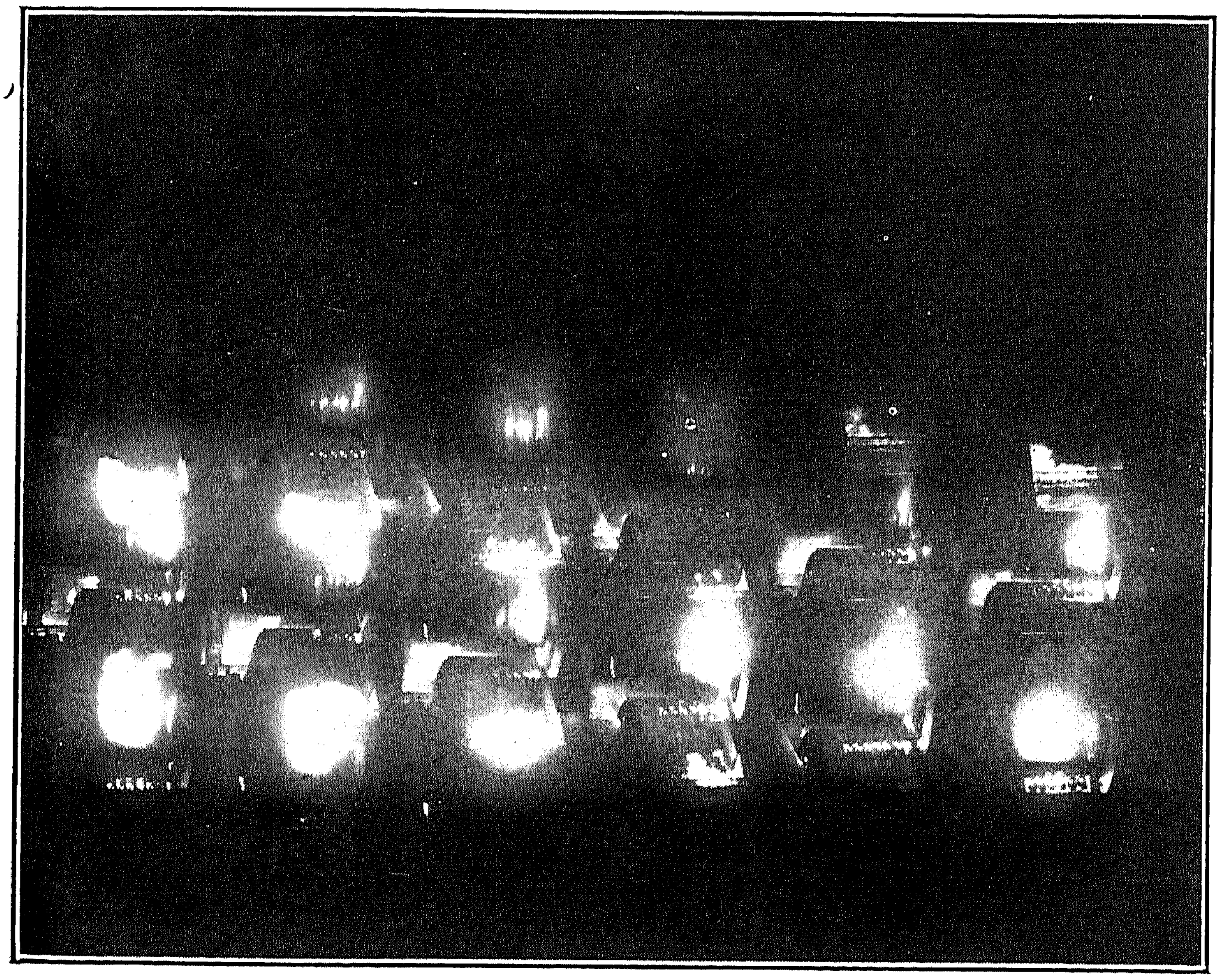


FIG. 24 [LINEBAUGH AND BURNHAM]
High-speed photograph of short circuit on 300-kw., 600-volt, 25-cycle, synchronous converter protected by flash barriers and standard circuit breakers.

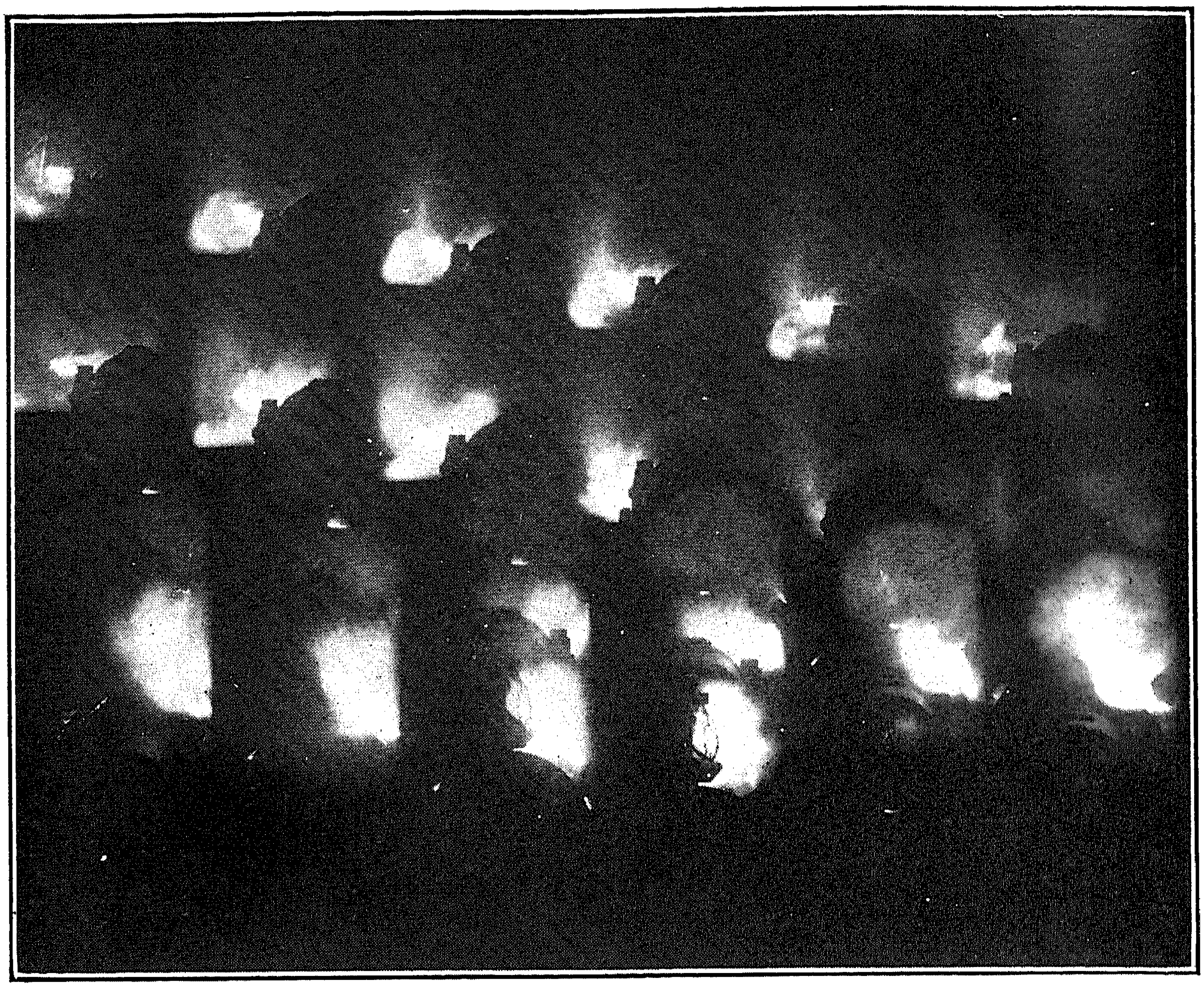


FIG. 25
High-speed photograph of short circuit on 500-kw., 600-volt, 60-cycle synchronous converter without protection.

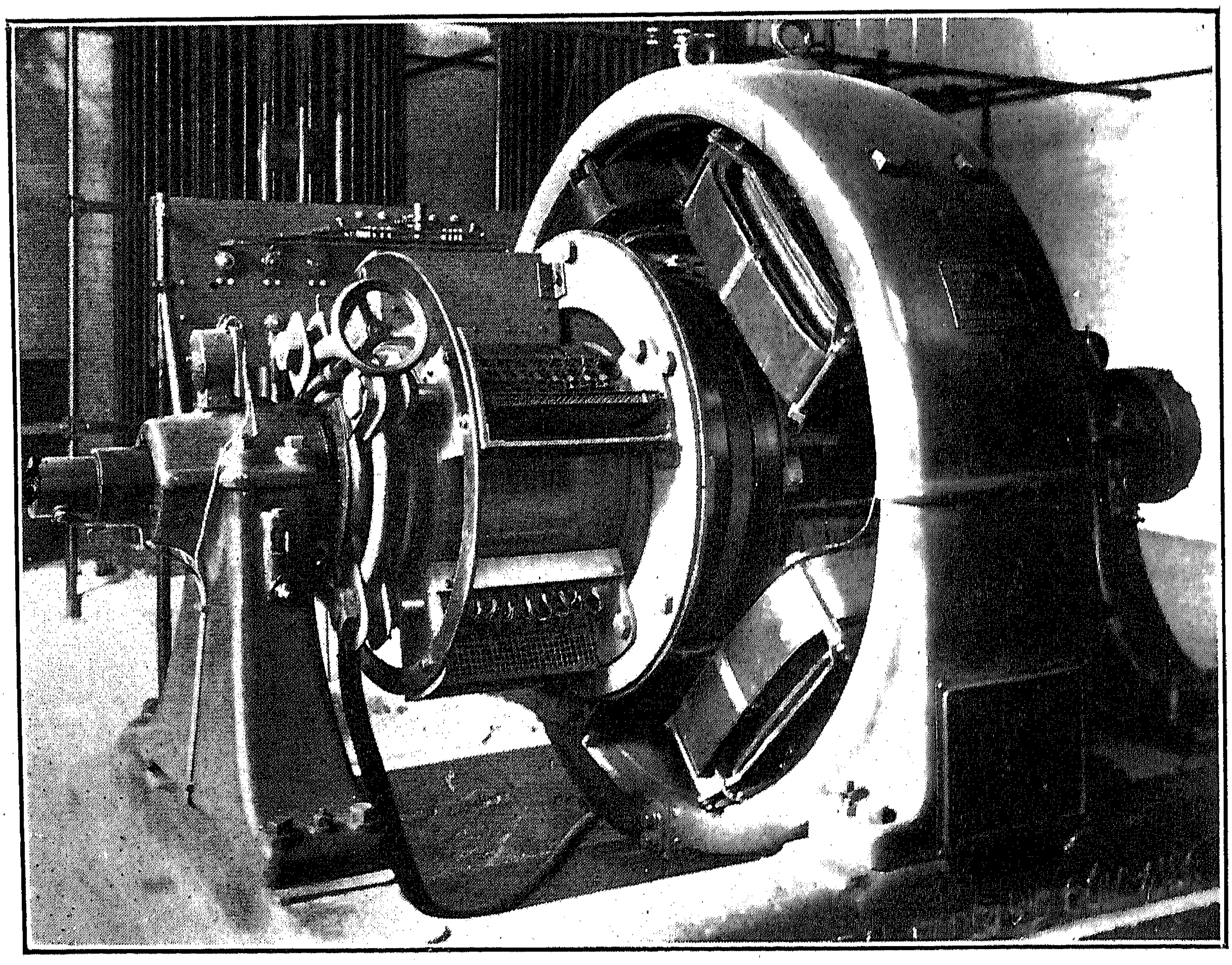


FIG. 26 [LINEBAUGH AND BURNHAM]
500-kw., 25-cycle, 600-volt, synchronous converter, installed in automatic substation, equipped with commercial form of flash barrier.

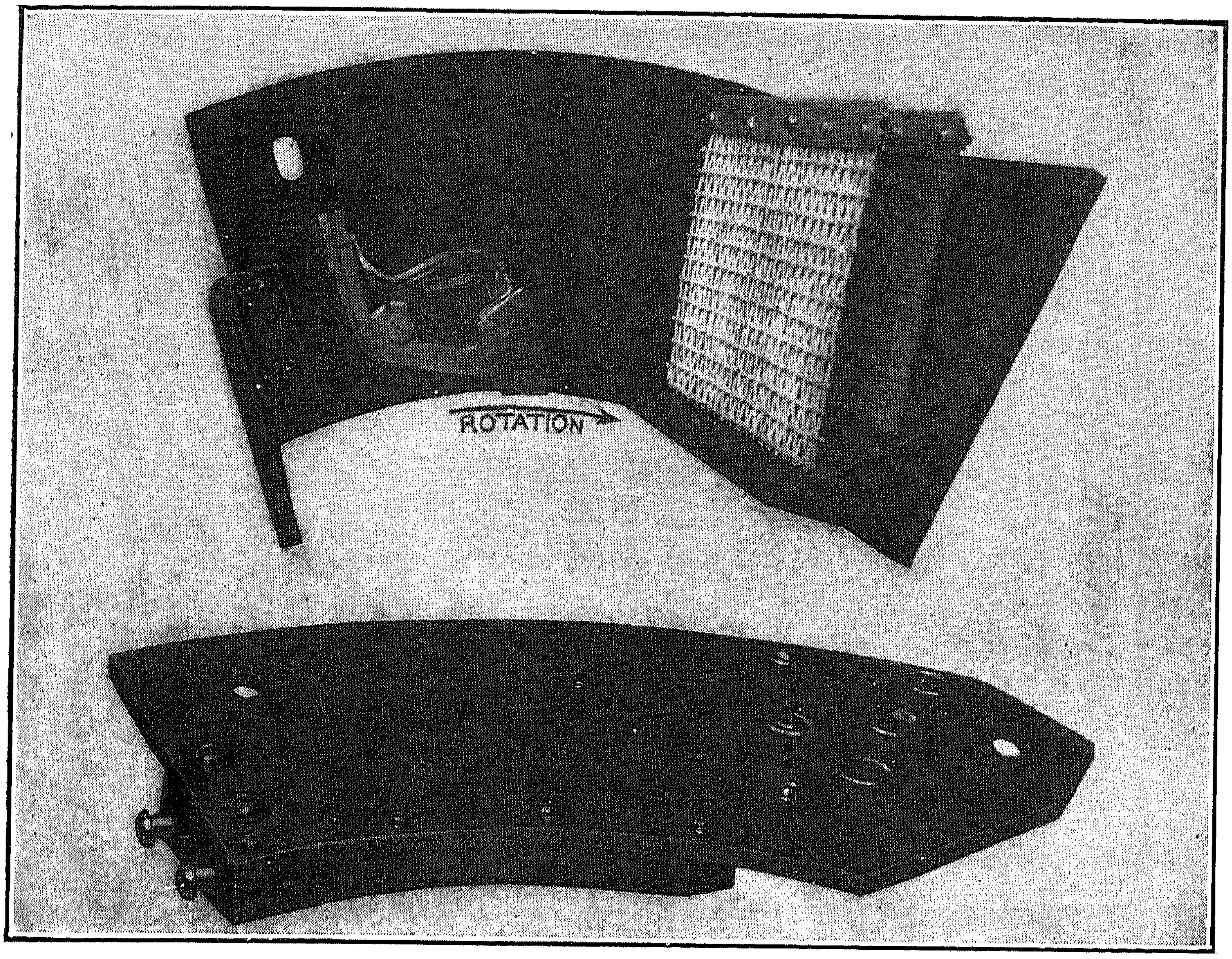


FIG. 27
Flash barrier with front removed to show location and construction of arc scoop and wire-mesh arc coolers.

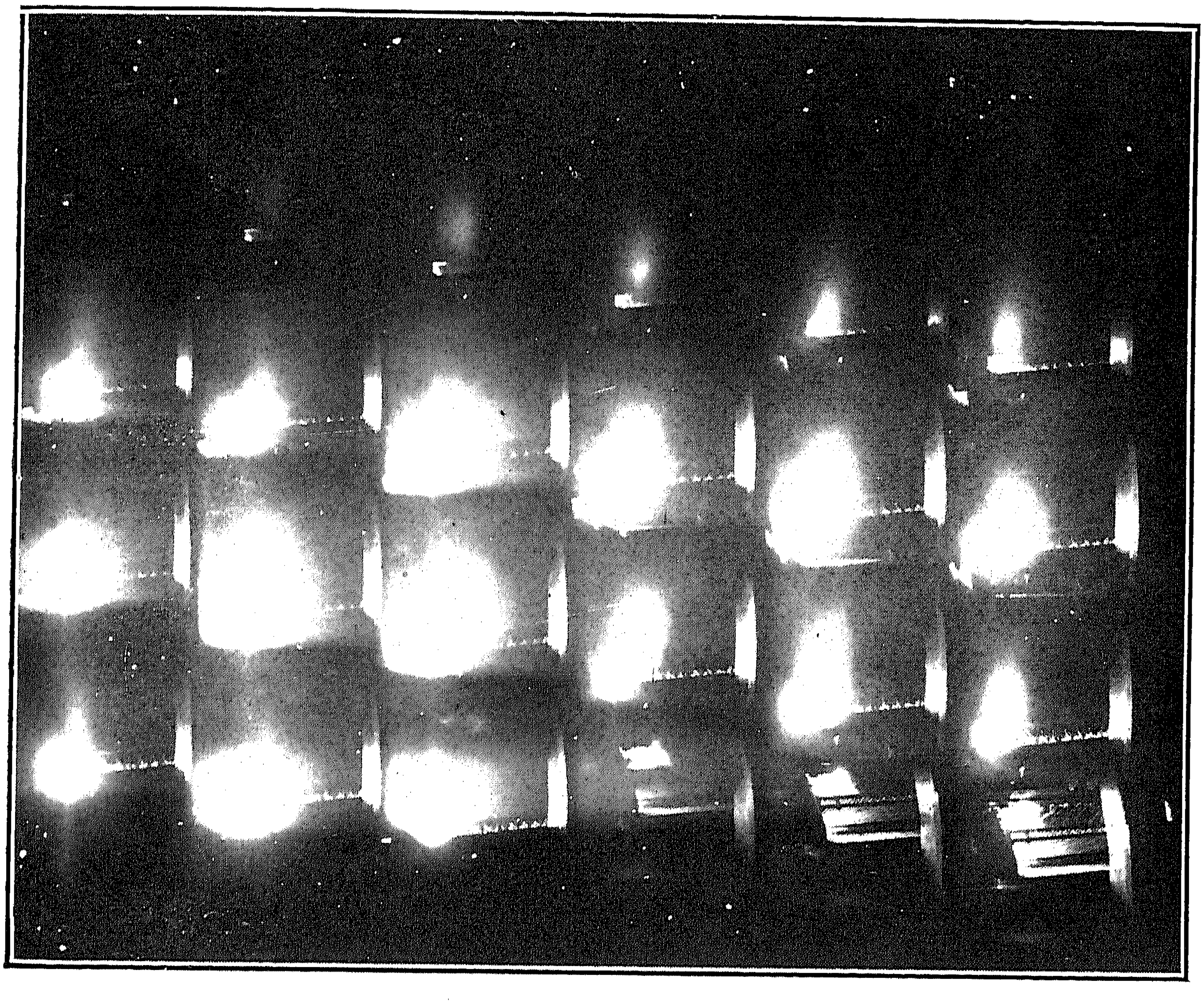


FIG. 28 [LINEBAUGH AND BURNHAM]
High-speed photograph of short circuit on 500-kw., 600-volt, 60-cycle, synchronous converter with flash barriers and standard circuit breaker with preliminary arrangement of brush rigging. Arc at outer end of commutator.

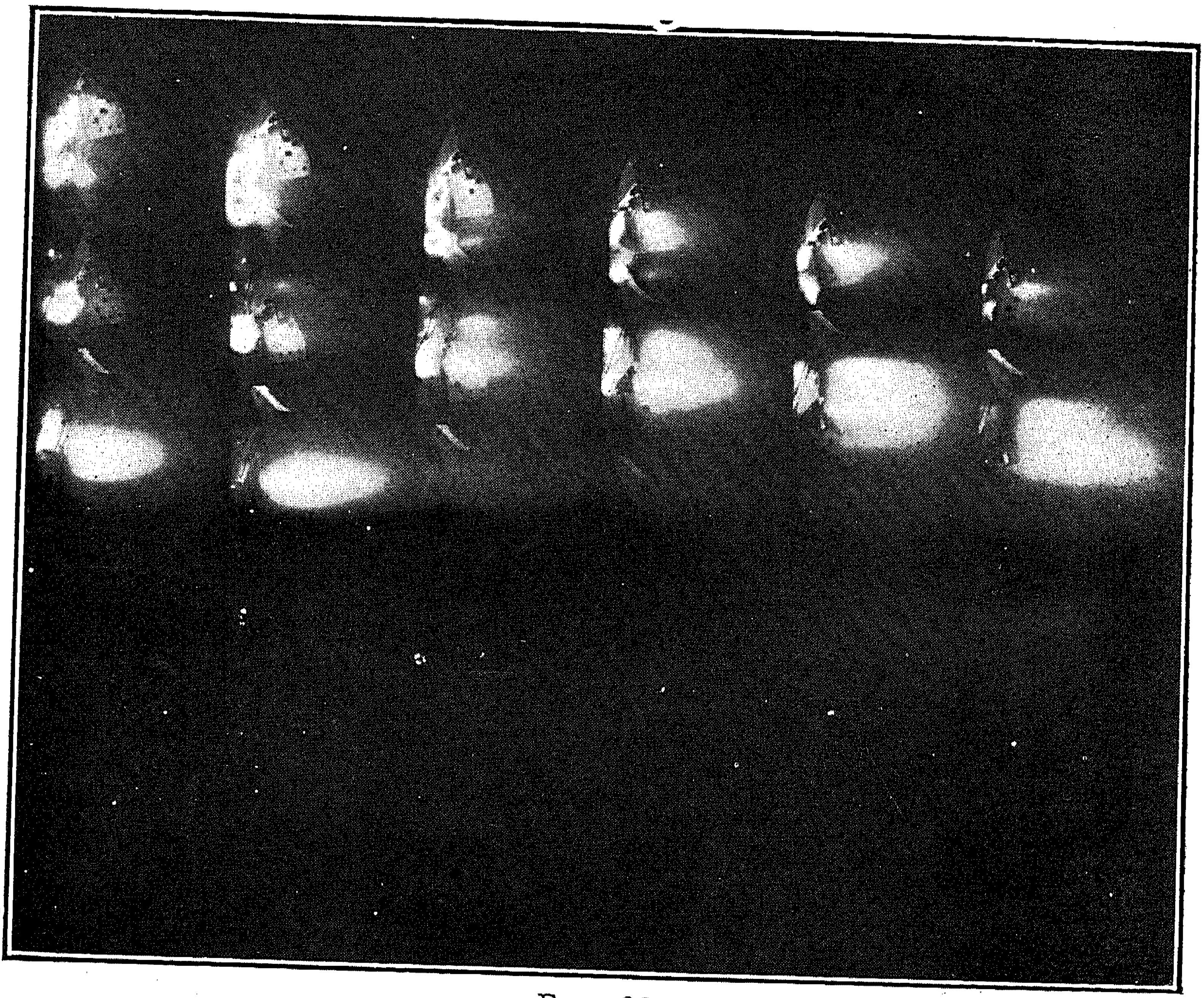


FIG. 29
High-speed photograph of short circuit on 500-kw., 600-volt, 60-cycle synchronous converter with flash barriers and standard circuit breaker with preliminary arrangement of brush rigging. Arc at outer end of brush rigging.

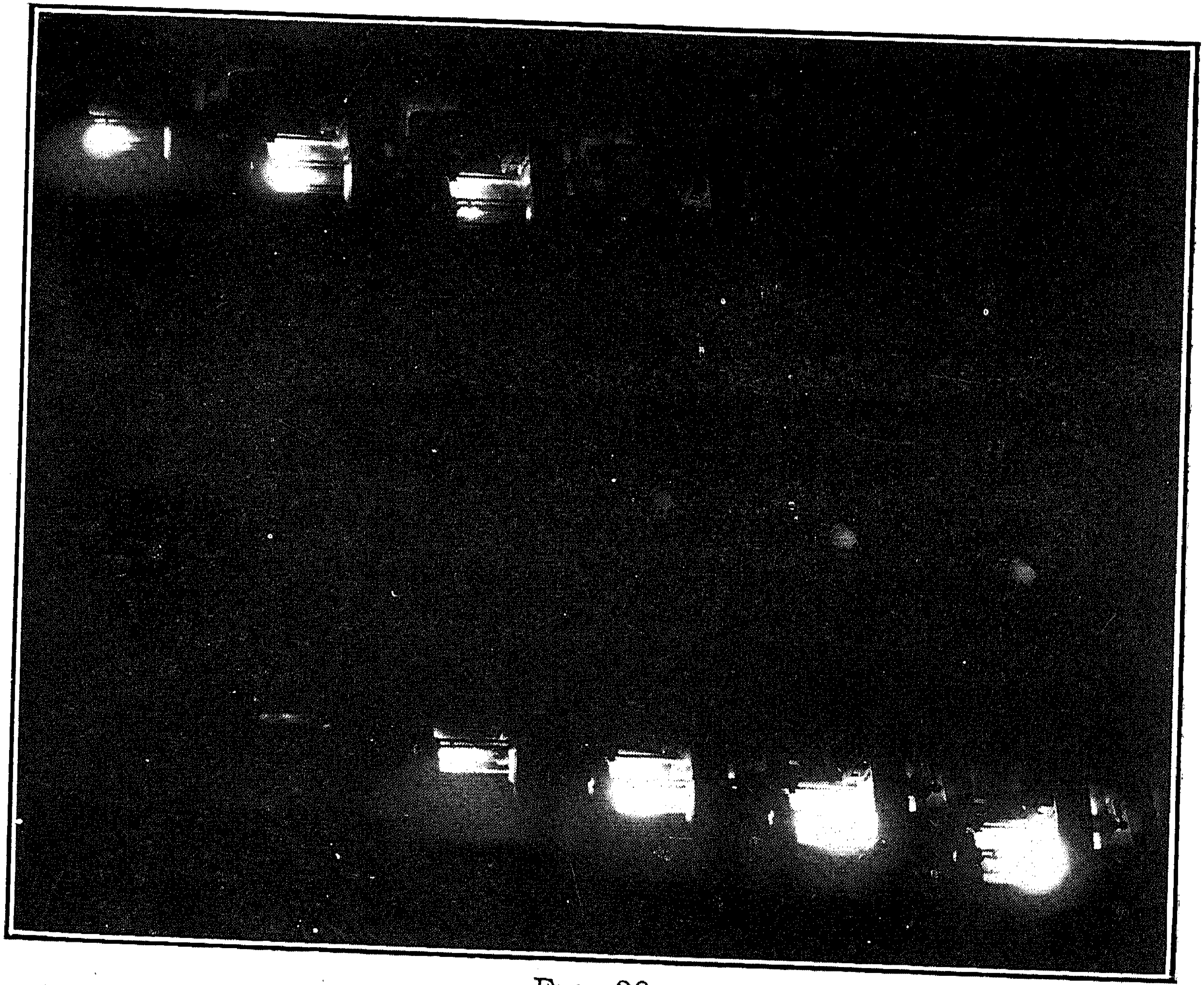


FIG. 30 [LINEBAUGH AND BURNHAM]
High-speed photograph of short circuit on 500-kw., 600-volt, 60-cycle synchronous converter protected by flash barriers and standard circuit breaker after arrangement of brush rigging has been changed. Uniform distribution of flashing.

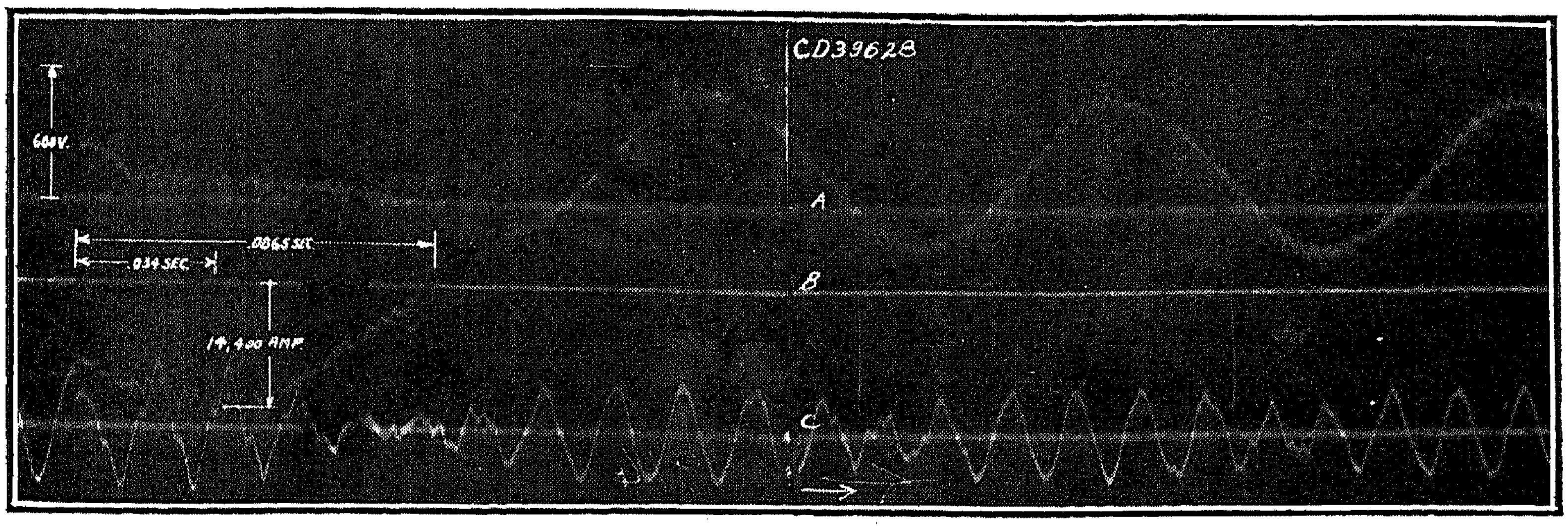


FIG. 31

Short circuit on 500-kw., 600-volt, 60-cycle, synchronous converter protected by flash barriers and standard circuit breaker after arrangement of brush rigging had been changed.

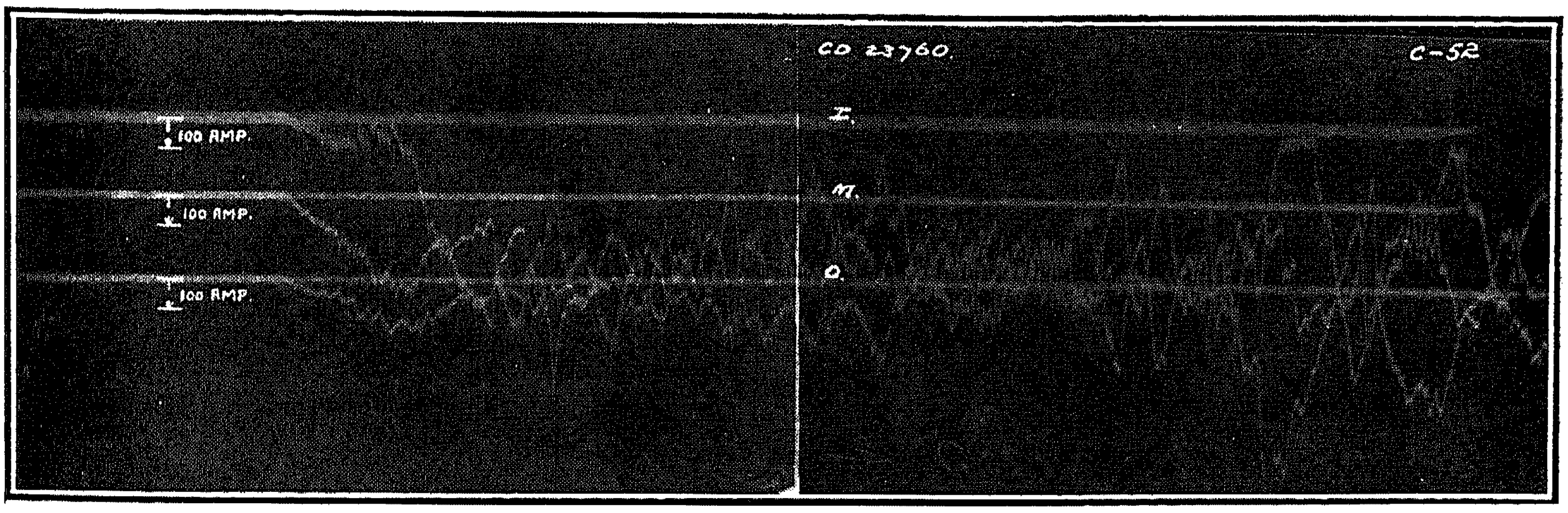


FIG. 32

Short circuit on 50-kw., 600-volt generator. Curve *O*, current in outside brush; Curve *M* current in middle brush; Curve *I*, current in inside brush.

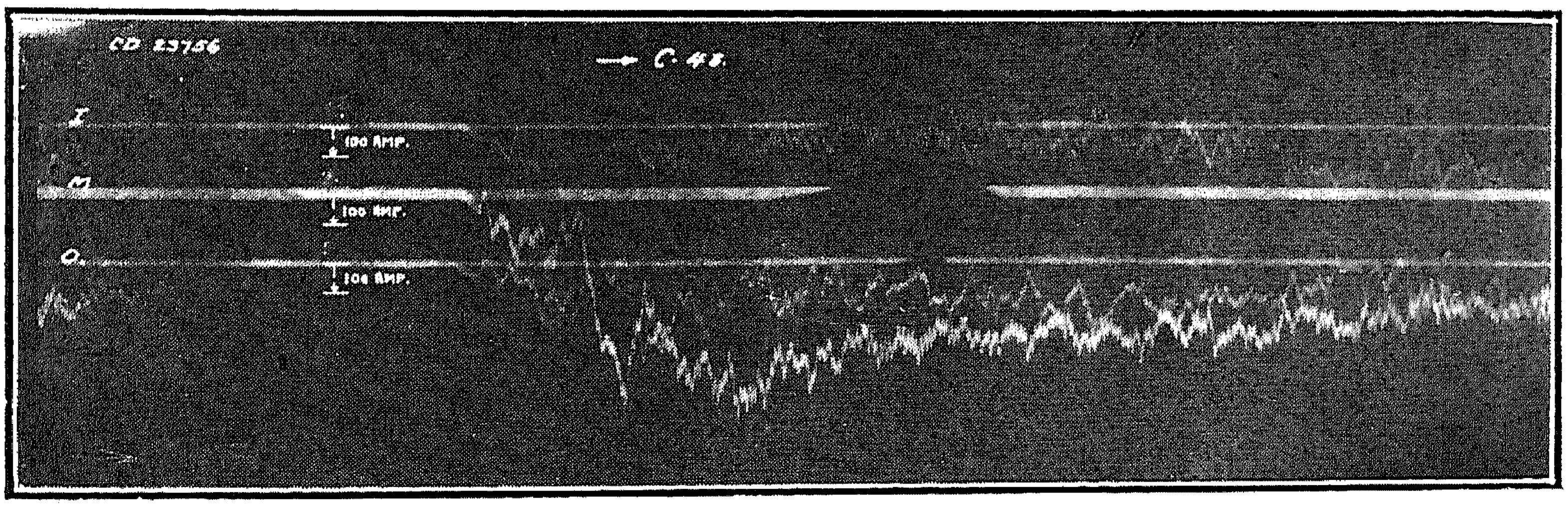


FIG. 33 [LINEBAUGH AND BURNHAM]

Short circuit on 50-kw., 600-volt generator. Curve *O*, current in outside brush; Curve *M*, current in middle brush; Curve *I*, current in inside brush.

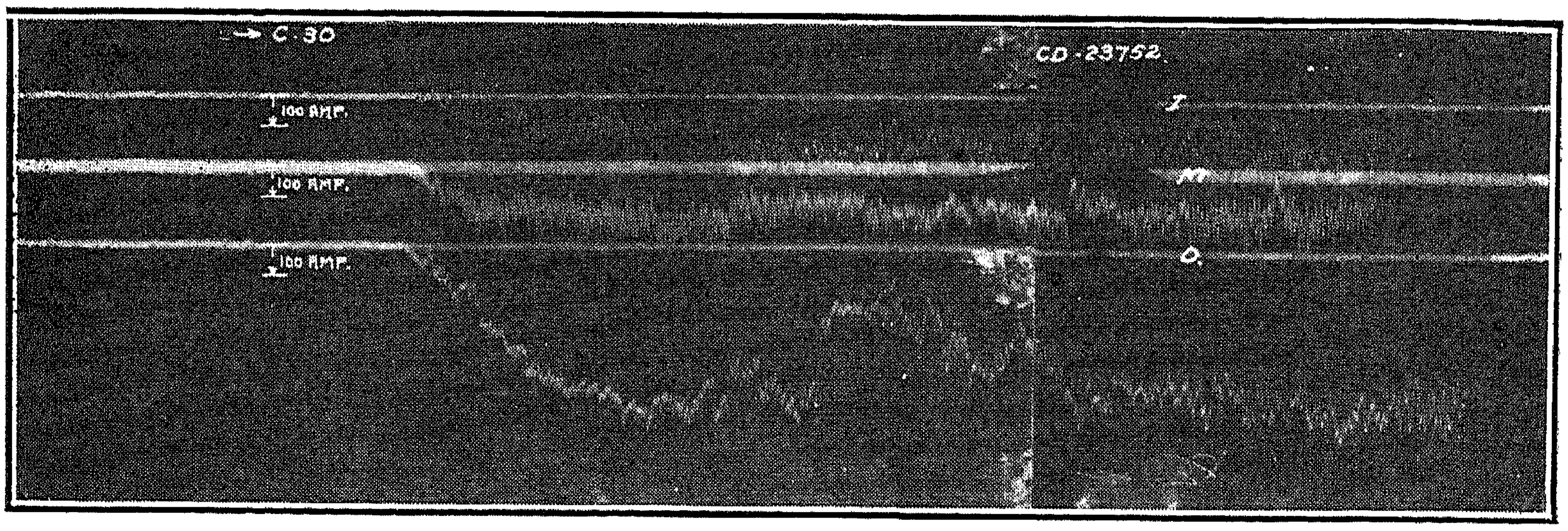


FIG. 34

Short circuit on 50-kw., 600-volt generator. Curve *O*, current in outside brush; Curve *M*, current in middle brush; Curve *I*, current in inside brush.

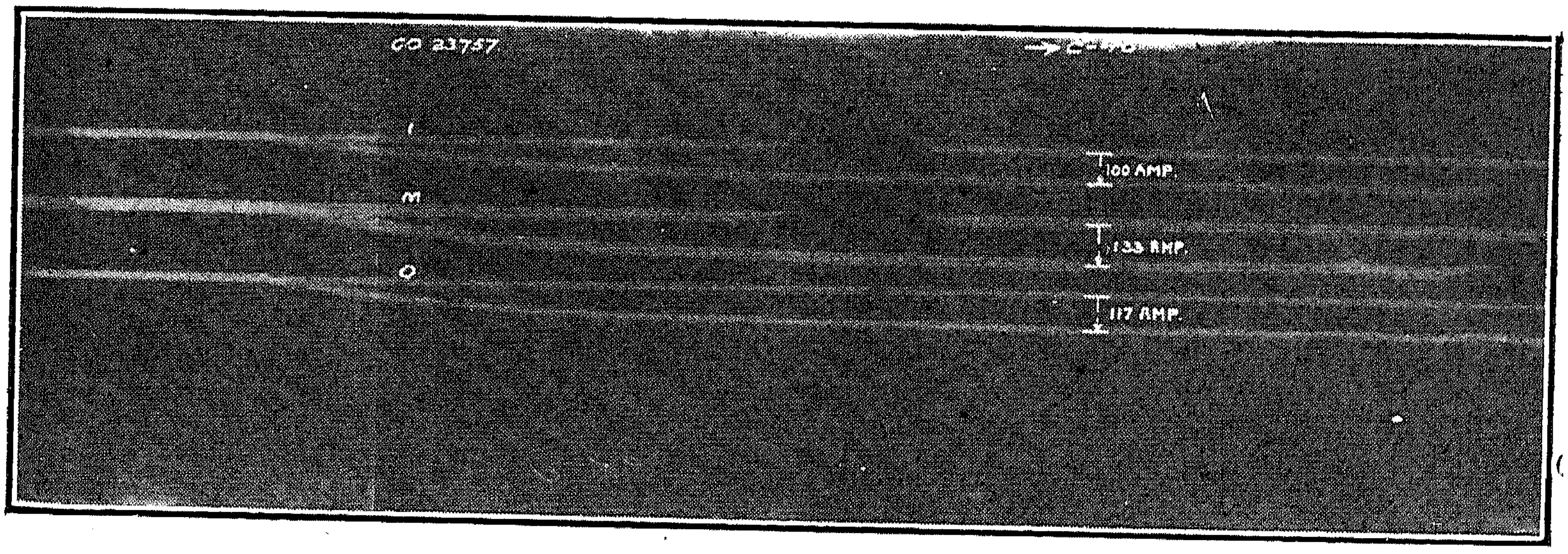


FIG. 35 [LINEBAUGH AND BURNHAM]

Current passed through 50-kw. 600 volt generator from an external source.

that would take care of all short circuits experienced in actual service.

Under certain conditions it might be desirable to supplement rather than replace, appliances already installed or to protect from disturbances other than direct-current load which cause flashing. For instance, a synchronous converter could not be protected by a high-speed direct-current circuit breaker if flashing is caused by a.c. phase displacement. For this reason additional protection, such as barriers, to dissipate the arc when started was also needed.

Many different forms of barriers were tried on the 300-kilowatt, 25-cycle, 600-volt synchronous converter, previously mentioned. With increasing success as improvements were made to meet failures, the barriers shown in Fig. 19 were evolved. These barriers gave complete protection from flashover or damage on short circuit. Fig. 20 shows machine on short circuit giving a good idea of flashing and protection afforded, while Fig. 21 shows clearly the small amount of flash which extends beyond the barrier.

About 65 short circuits were thrown on the 300-kilowatt, 600-volt, 25-cycle machine without burning of brushes, brush connections or rigging, or damages of any kind to commutator or machine. Oscillogram, Fig. 22, shows a record of current reaching 34 times full load and gives a good idea of the protection afforded. Many of these short circuits were applied at very short intervals, even as close as one minute apart, without failure to hold and extinguish the arc when the breaker opened the circuit.

Figs. 23 and 24 are very interesting high-speed pictures of the same short circuit analyzed by means of a special high-speed camera devised by Lieut. Chester Lichtenberg, and the successful high-speed pictures we are able to show in this paper are mainly due to his efforts. This camera made it possible to obtain as high as 24 complete pictures of one short circuit, while the best results it was possible to obtain with a moving picture camera were two under-exposed and therefore indistinct pictures.

A little explanation is necessary to read these photographs as, due to the construction of the camera, the lower right hand picture is the first picture of the short circuit; the next picture being the one immediately to the left, and so on to the end of the plate; the first picture at the right of the the next row being the next picture in the same order and until the end of the plate and the number of rows of pictures. These pictures show very clearly the growth of the arc, disposition on commutator and dissipation of the arc as the regular breaker opens. These permanent records eliminated the personal factors of memory and observation and showed the way for changes to give improvements in barriers. Fig. 25 illustrates very clearly what happens if the machine is short-circuited without protection.

The general arrangement of a successful barrier, Fig. 26, is shown herewith.

A close fitting box of fire-proof insulating material surrounds each set of brushes and is located so as to give a small clearance between the box and the commutator.

On the side of the box towards which the commutator rotates after leaving the brush is fastened a V-shaped "scoop", Fig. 27, of fire-proof insulating material, preferably having good heat conductivity, pointing toward the brush and having small running clearance from the commutator.

Radially above the scoop, about one inch apart, are two metal screens, one coarse and one fine mesh, through which the arc is successively forced and cooled.

It was found that a moderate amount of material is required to give the necessary thermal capacity to prevent an arc from passing beyond a screen of this kind. The scoop running very close to the commutator with narrow edge and small clearance picks up the arc from the commutator and deflects it into the arc coolers which, from their construction, allow free passage of all gases generated by the arc. The cooling and condensing of the arc reduces the gas pressure so that shields at the end of the commutator, to prevent the arc being thrown from the end of the commutator and communicated to pillow block and frame, are permissible. It will be noted from the illustrations that the commutators extend beyond the end of the barrier as it was found that the arc must be prevented from being communicated to the end of the bars.

Investigation was then transferred to a 500-kilowatt, 60-cycle, 600-volt synchronous converter and barriers of similar type, but without continuous end shields, were tried.

Tests showed that these barriers did not give protection on short circuit although they prevented machine from flashing over on very high overload. The high-speed camera record indicated that the arc was being thrown to the outer end of the commutator for some reason, causing such high gas pressure at the outer end of the commutator that the arc was blown under the barrier and the machine flashed over. Figs. 28 and 29.

The differences in performance were ascribed to differences of magnetic fields acting on the arc.

To demonstrate the effect of the magnetic field, various arrangements of connections of brush rigging were made, each to produce a different field where the arcing occurs. The results indicate that it is possible to arrange the brush rigging and connection to make a barrier, as described above, effective on practically all commutating machines and to prevent complete flashover. Figs. 30 and 31 show the effects of change in connection on arc distribution, giving the uniform distribution most favorable to good barrier performance.

Other tests were made to record the simultaneous short-circuit current in the outer, middle and inner brushes by the oscillograph. The records in Figs. 32, 33 and 34 show typical variations of current distribution produced by different connections to brushes. The distribution of current is principally dependent on the magnetic field surrounding the brushes where the arc is formed. To show that differences of impedance have very little influence, record Fig. 35 was taken with current supplied from an exterior source with no flashing. It will be seen that current is practically the same in all brushes. With some connections the deflection of the arc can be plainly seen to follow the well-known relation, as given in Fig. 36, but with the more complicated connections the difficulty of determining resultant field from many sources makes it difficult to determine the direction of deflection of the arc except by experiment.

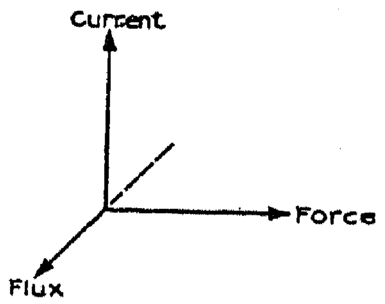


FIG. 36

Direct-current machines for use in * automatic substations are being equipped with these barriers and short-circuit tests at the substations have been taken, indicating that they will take care of any short circuit experienced in actual service. These barriers are in operation and short-circuit tests were taken on a 500-kilowatt, 600-volt, 25-cycle synchronous converter of the Des Moines Electric Railway, Des Moines, Iowa, a 500 kilowatt, 600-volt, 60-cycle synchronous converter of the Coumbus Electric Railway & Light Company, Columbus, Ohio, and a 500-kilowatt, 30-cycle, 1200-volt synchronous converter at Montieth Junction, Michigan, and other installations are now in service.

The investigations and tests indicate that if any commutating machine is equipped with barriers and the last high-speed circuit breaker described, complete protection will be given against external short circuits of all kinds so that interruption to service will not be of any greater duration than necessary for closing the circuit breaker and main switch as in ordinary overload operation.

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*See paper by Taylor and Allen, A.I.E.E. TRANSACTIONS Vol. 34, 1915, page 1801.

DISCUSSION ON "PROTECTION FROM FLASHING FOR D-C. APPARATUS" (LINEBAUGH AND BURNHAM), ATLANTIC CITY, N. J., JUNE 28, 1918.

Charles L. Fortescue: I would like to congratulate the authors on the fine results they have been able to obtain with a mechanically-operated high-speed breaker. I can assure you, gentlemen, that to attain a speed of 0.005 of a second on a mechanically operated breaker is no small achievement. The very nature of the problem and the materials available set a limitation to the possible speed obtainable.

I am very much interested in the subject, because I have been associated with various men of our company in a similar line of development. In 1911, in association with the late Mr. William Cooper, the problem of preventing flashing of railway motors came to my attention. On one of the elevated lines in New York City, they were having some trouble with flashing of the motors, due to short circuits and it was proposed to prevent it by operating the breakers quickly. Mr. Cooper conceived the idea that we could obtain a quick operating relay, which would operate the valve in the air-operated breakers, depending upon the magnetic energy released due to the short circuit. With this idea, we built a relay-transformer. The transformer had a small air gap in it, and we depended on the discharge of the magnetic energy stored in this gap to operate the relay. With this little device we were able to obtain such satisfactory results that we managed to overcome our troubles on that particular road. We obtained, as I recall, speeds of from 0.025 to 0.030 of a second, and that encouraged us to go a step further, and we decided to design a direct-operated circuit breaker of the carbon type. The second design was a carbon breaker with a magnetic blowout, and it had a rather powerful spring, somewhere around 300 or 400 lb. This breaker proved quite satisfactory. We were able to get speeds down to 0.015 of a second with the latch operated by the transformer device.

Later on I conceived the idea of using a condenser instead of a transformer for storing the energy required to operate the trip, in other words, we placed the condensers in series with the trip coil across the direct-current terminals. When a short circuit occurs, there is an initial fall of potential, and this discharges the condenser through the trip coil and operates the trip magnet. The frequency of the discharge current will depend on the capacity of the condenser and the self-inductance of the magnet. This arrangement was very satisfactory, in fact, with a circuit-breaker of very ordinary design, nothing very special about it, except it had a rather stronger spring than usual, we were able to obtain a speed of 0.011 of a second.

I would like to take exception to a few points in the authors' discussion of the question of reactors used for retarding the growth of current. The problem, as I see it, is not that the choke coil is expensive, it is a balance between the cost of the choke coil and the high-speed breaker. As it increases in speed the breaker becomes more and more expensive, and as I said before, it is a big problem to produce a mechanical device which will always operate satisfactorily at high speed. There is a chance of breakage and crystallization of parts, the springs are tremendously powerful, in some cases several tons, and the working parts are subject to heavy jarring action, so that the cost and maintenance of a device like that is necessarily very high. By using a choke coil, it is possible to use a lower speed and less costly device. It has been estimated that the economic speed was somewhere around 0.009 of a second.

A description of the early tests referred to in this discussion was published some time around 1913 in "*Electric Traction*" by Mr. Funk. Later on, 1914, Mr. Yardley presented a paper on the "Use of Reactors with Synchronous Converters," in which he showed a set of films that were taken with converters operated with one of these breakers. We also have films showing them successfully operated with 1500-volt direct-current machines.

I might mention that in these mechanically-operated breakers the capacity required to operate the tripping device was quite small. In the first breaker we made we were able to operate very satisfactorily at 500 volts, with a condenser of 1-microfarad capacity and at 1500 volts with a condenser of one-third of a microfarad capacity. A little later we used a stronger operating trip coil, which requires 2 or 3 microfarads at 500 volts and still later we have used a tripping device of very much the same pattern as that which the authors have described.

Another device that we have used to obtain quick operation is a relatively small choke coil, shunted by the trip coil. In this device the impulse current in the trip coil, when short circuit occurs operates the breaker enabling us to get very quick action.

We have had some of the condenser tripped apparatus in actual operation. We have had one installed at the plant of the Youghiogney Coal Company, operating in mine service, and the operation had been very satisfactory. In fact, most of our work has been done with the idea of developing something that could be used for such service and to protect generators and motors from short circuits on feeder circuits. We have tried to obtain something that could be sold at a moderate price, to be used for the protection of motors, etc., in industrial work.

We have also had another circuit breaker installed at the New River Coal Company. I saw a day's record of that system— I think it was last fall—which they say is quite an ordinary record: There were thirty shorts in one day, and in every case the breaker operated so that the motor or generator was protected. In this case we have a small choke coil in series with the machine in addition to the high-speed breakers.

This is a rough summary of the work that we have done in connection with high-speed breakers. We have not reached as high speed as Messrs. Linebaugh and Burnham have obtained with their very special mechanical construction but the results have been very satisfactory with ordinary forms of breakers.

F. C. Hanker: The use of higher direct-current voltages on railway systems has increased the difficulties in connection with the supply of power, particularly with commutation during the short-circuit conditions. In the early days, somewhat similar conditions obtained on 600-volt railways and forced improvements in the commutating characteristics of direct-current generators and converters, so that the modern machines, with commutating poles, and, for more severe service, with pole-face windings, are very rugged and have given satisfactory service.

The advent of the higher voltages has made the problem of the designer more difficult and the resultant increase in flashing troubles has required concerted attention on improvements. The authors have described in detail the manner in which they have attacked the problem and outlined the various forms of breakers and barriers they have developed after extended experimental testing. This method of solving the problem is the logical one, as pointed out by the authors, when we take into consideration that the remedy must be applied to existing installations. In other words, it may be classed as an external attack.

In the form of breaker described in the paper as being installed in the stations of the present electrified zone of the C. M. & St. P., the tripping mechanism is complicated, and extremely heavy spring pressures for rapid opening are used. Such a circuit breaker will be subject to relatively rapid deterioration and, unless kept at the maximum point of efficiency, will slow up in its speed, then if it fails to open the circuit before the flashing starts, it will be too late to stop it.

Another method of solving the problem of flashing that sounded rather revolutionary when it was first proposed was suggested by Mr. N. W. Storer, who is well known for his work in the railway field. It differed radically from the previous methods using circuit opening or limiting schemes and attacked the cause of the flashing. It effectively suppresses it while all other schemes only minimize the possibility of flashing or reduce the resulting effect.

The flash suppressor consists of a circuit closer arranged to short-circuit the armature winding of the direct-current generator through collector rings. It is tripped in a manner similar to circuit breaker practise with the addition of the anticipatory feature developed in previous high-speed circuit breaker investigations. It is also feasible to supplement this method of tripping by taking advantage of the increase in field current under the short-circuit conditions for operating an auxiliary tripping coil. The moving parts are small and as it is only necessary to move them a relatively short distance the device can easily be closed in from 0.006 to 0.008 sec. without using abnormal spring pressures. Higher speed can be obtained, if necessary, without undue complication, but tests show conclusively that the flashing is suppressed effectively after intervals several times longer, so that it was not considered necessary to secure higher speed of operation.

The flash suppressor really performs a double function; first, short-circuiting the collector rings and thus simultaneously reducing the voltage between commutator bars practically to zero, or, in other words, all of the voltage generated in the winding is absorbed in that winding so that no difference of potential between commutator bars will remain; and second, in reducing the field flux. Short circuits such as are applied by a flashing suppressor, result in a heavy wattless current flowing through the armature winding. The magnetizing force of the armature ampere turns opposes that of the field winding and cuts down the voltage at the maximum safe rate without opening either the field or armature circuit. In this way, the stored energy not only of the line but of the machine itself is harmlessly short-circuited through the machine and dissipated in the armature and field windings.

During the studies made of this method of suppressing flashing numerous tests were made on different types of machines and it may be of interest to review some of the more representative oscillograms.

The first oscillogram Fig. 1 shows a direct-current short circuit limited by resistance that is typical of a standard direct-current generator. The armature current rises to about 13,000 amperes or about eight times full-load current. The actual current obtained was about 60 per cent of the theoretical current that would be obtained by dividing the terminal voltage by the circuit resistance. The initial rate is about 2500 amperes per 0.001 sec. and the current rose to practically its final value in the first 0.010 sec. The shunt field current undergoes an increase in value, due to the transformer action between the armature and field winding.

Fig. 2 shows the effect of the application of the flash suppressor 0.006 sec. after a short circuit was applied to the direct-current side. In this case, the short circuits were made without resistance or reactance of any kind in the circuit. The effect on the commutator was confined to a small explosive puff of very short duration which did no burning on either the brush holders or commutator surface. Many duplicate oscillograph records of combined short circuits were made which established that the suppression of the flashing was independent of the point on the alternating-current wave at which the short circuit occurred and that all armature positions give essentially the same effect. The longer delays resulted in a somewhat heavier

PLATE LV.
A. I. E. E.
VOL. XXXVII, 1918

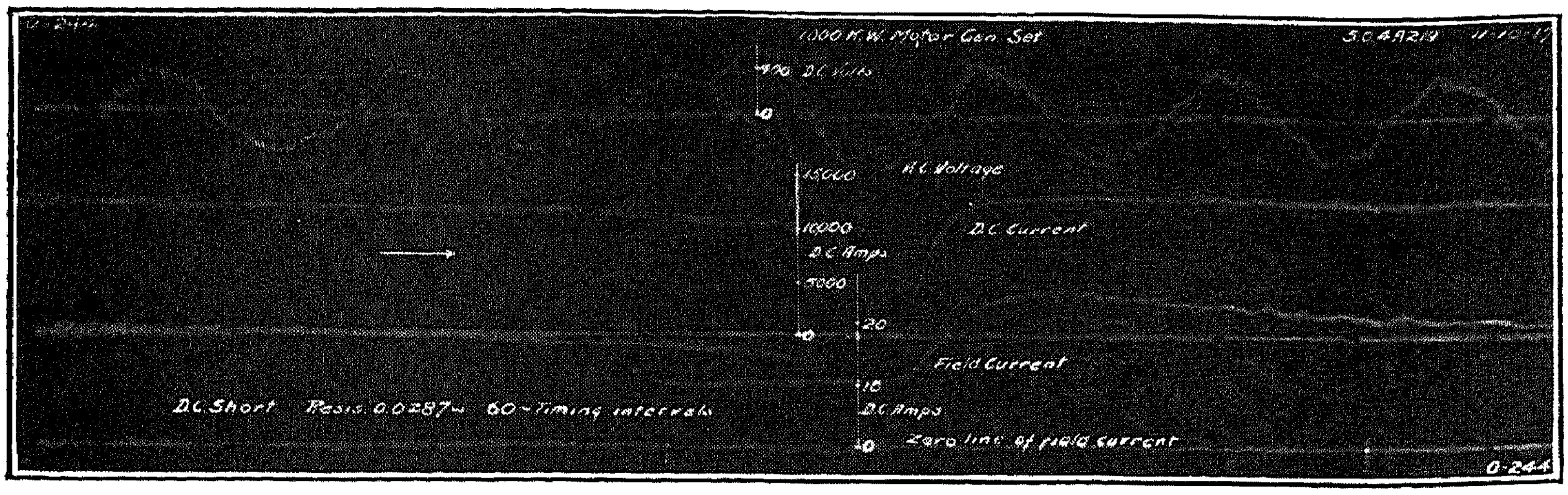

FIG. 1

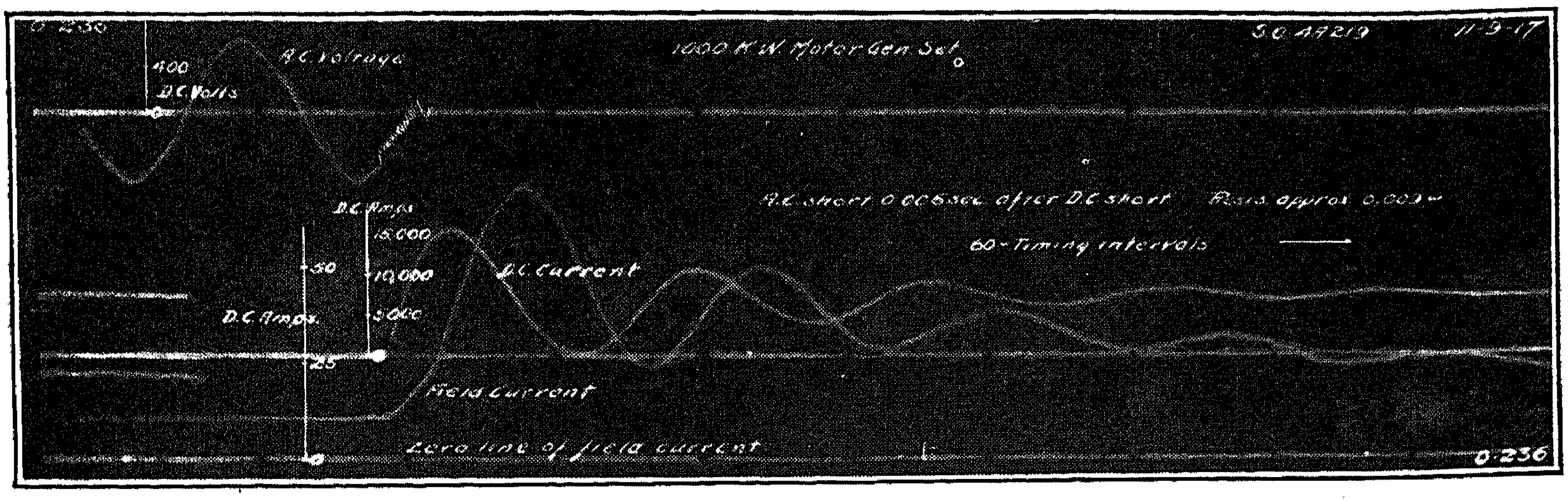

FIG. 2

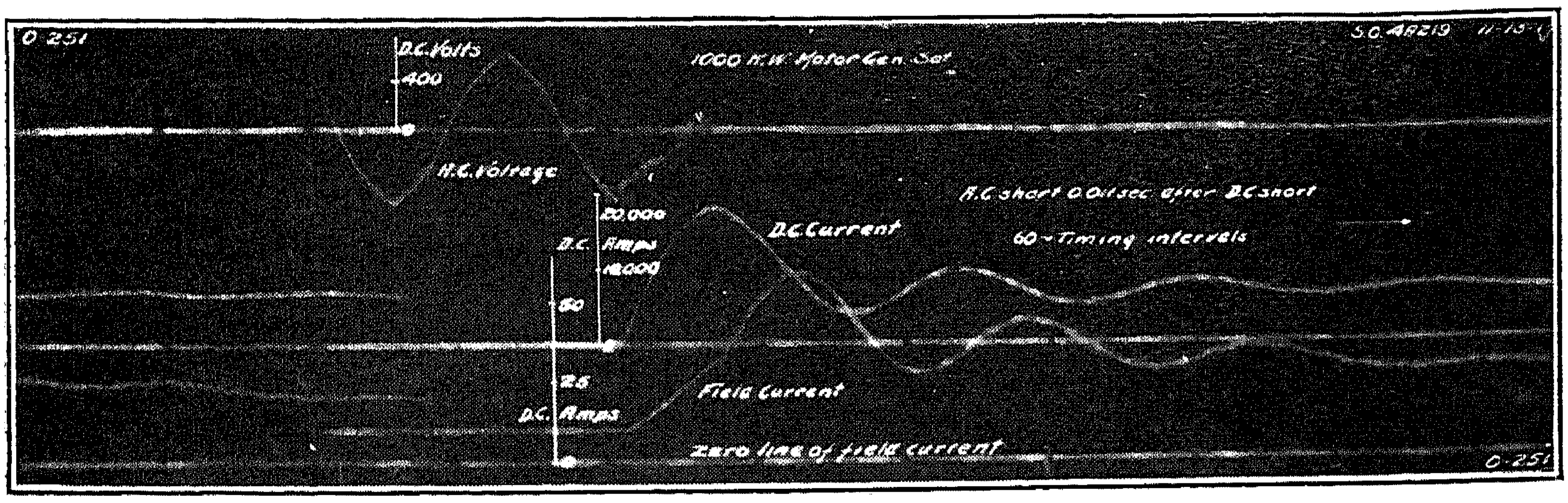

FIG. 3

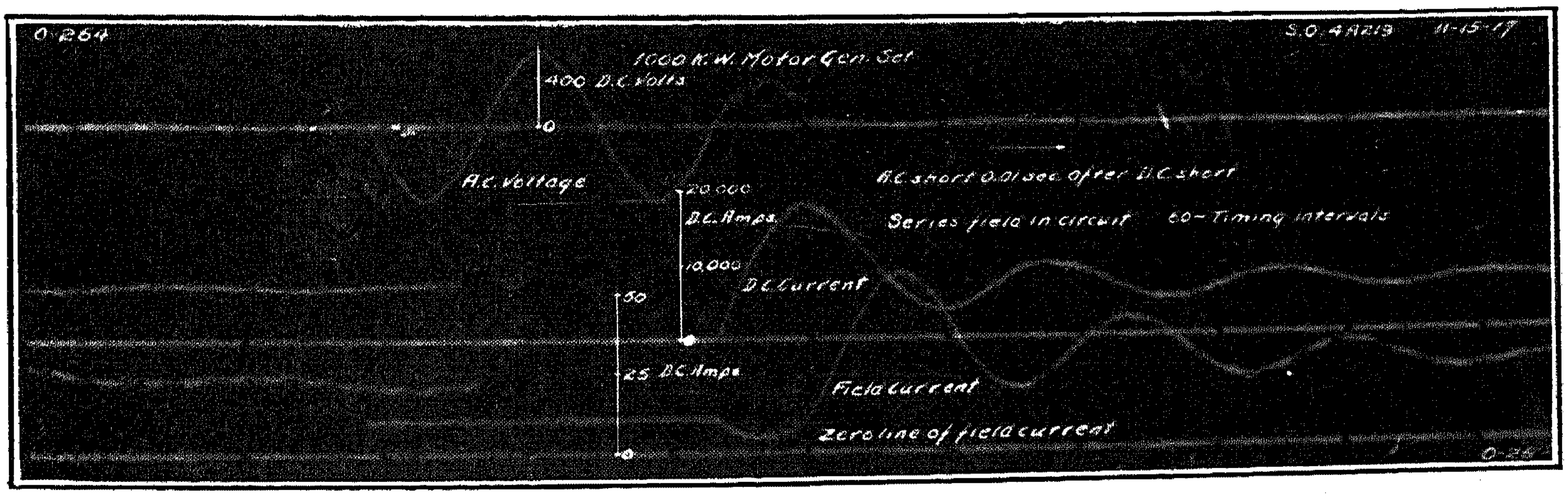

FIG. 4

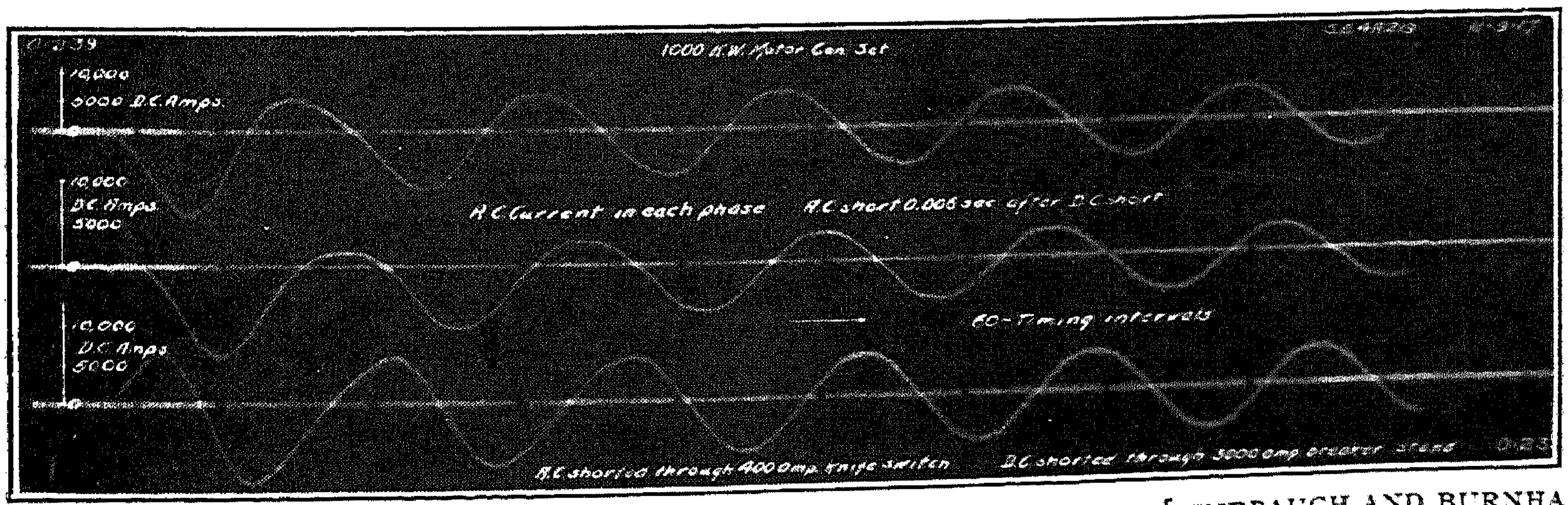

FIG. 5

[LINEBAUGH AND BURNHAM]

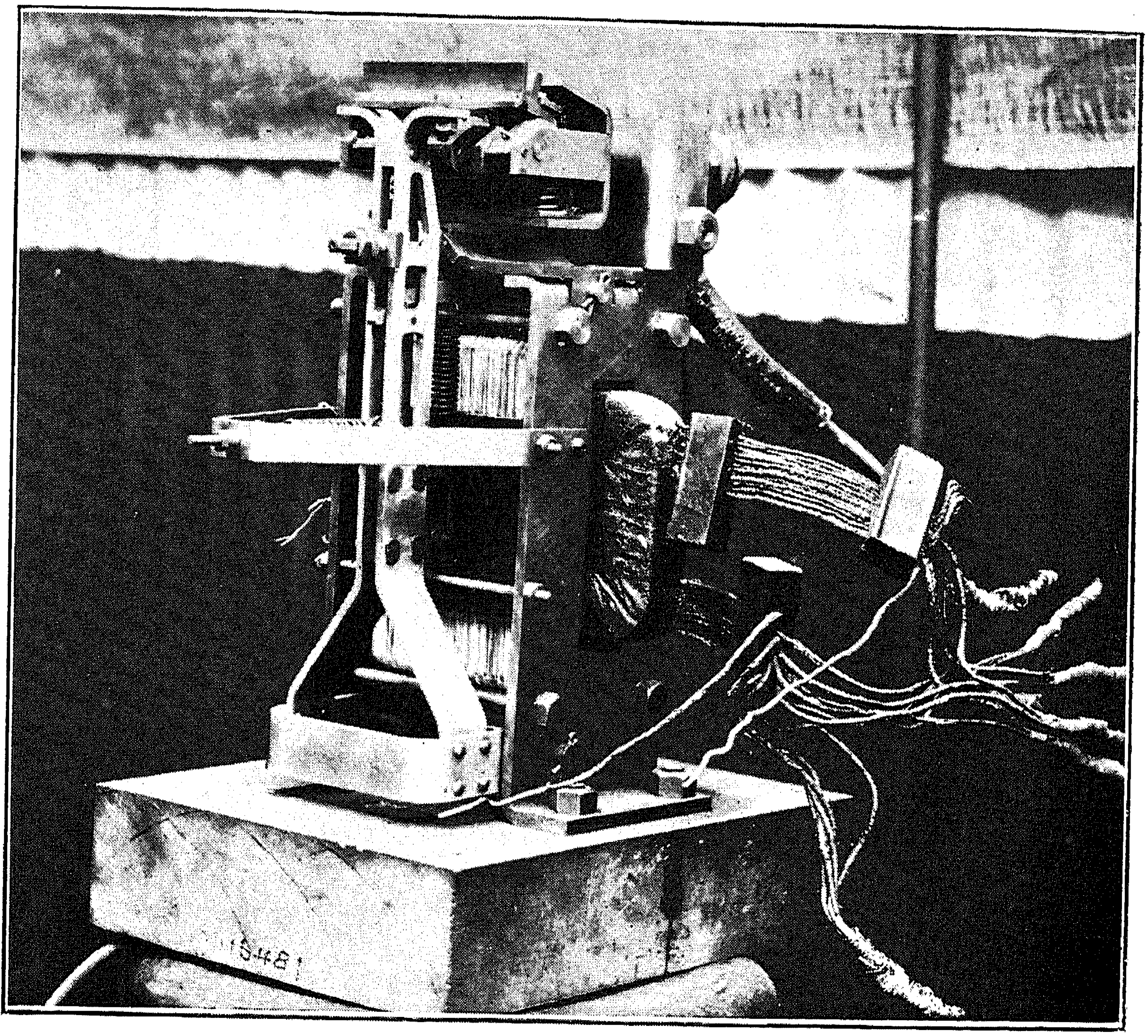

FIG. 6

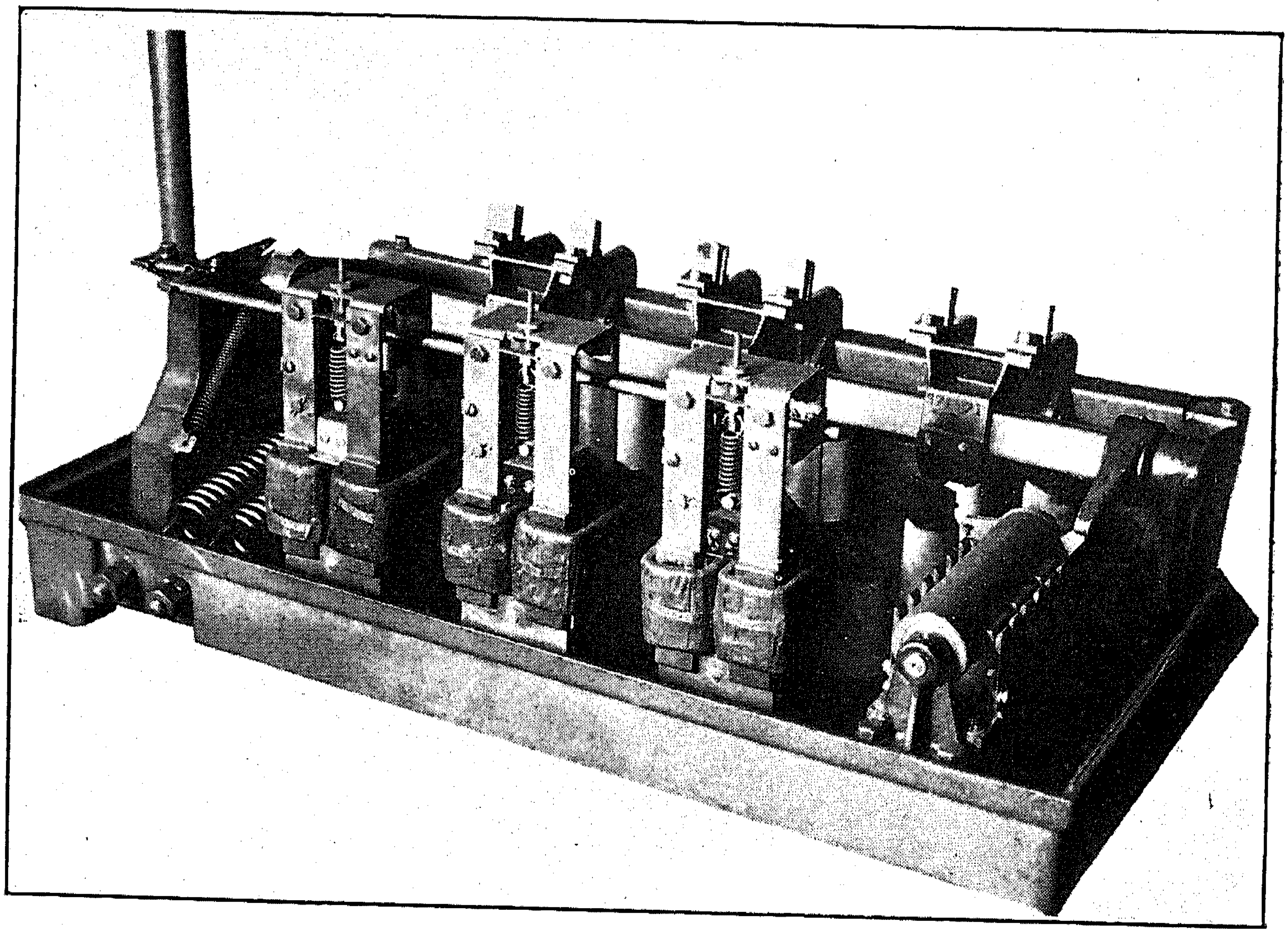

FIG. 7 [LINEBAUGH AND BURNHAM]

flash at the brushes, but none of them of sufficient duration to result in burning.

Fig. 3 is essentially the same as the previous, differing only in the time the operation of the flash suppressor is delayed.

Fig. 4 gives the results of a test with a compound wound generator.

Fig. 5 shows the alternating current in the three phases when short-circuited at full voltage. An analysis of the current of all phases indicates that the resultant current rises uniformly in magnitude until it reaches its maximum value in one-half cycle after the instant of the application of the alternating-current short circuit.

The operation of the suppressor immediately stops any further increase in the direct-current short circuit and in the next half cycle reduces the current to practically zero. The large decrease in the direct current in the entire cycle following the application of the alternating-current short circuit is well understood when it is recognized that an armature winding can deliver only a definite current on short circuit. This current is limited by the electrical constants of the armature winding and is not affected by the number of points at which a short circuit is applied to the winding. This means that the alternating-current short circuit is subtracted directly from the direct-current output and the decrease of direct current corresponds to the building up of the alternating short-circuit current. The alternating component disappears when the direct current finally settles down to a constant value of about 4½ times full load in about 0.075 sec.

In the various tests made by short-circuiting the direct-current side without external resistance, the short-circuit current reaches about 30 per cent of its theoretical short-circuit value. In other words, the normal design of high-speed compensated generator would deliver from twelve to fifteen times or even as high as twenty times full-load current on direct short circuit, that is, the percentage in basing the total on the circuit resistance, including the brush contact.

Fig. 6 is the single-unit type of suppressor consisting essentially of a high-speed circuit-closing contactor. With this design, a closing time of 0.0045 sec. was obtained. Fig. 7 illustrates a design for simultaneously short-circuiting several units, and on test, the closing time was between 0.006 sec. and 0.008 sec. These two photographs illustrate the simplicity of the circuit closing device.

It must be recognized that there are disadvantages in any of these special applications and they are only justified where the complication is necessary, due to the severity of the service. The flashing suppressor will be installed in the substations on the C. M. & St. P. Ry. that are being supplied by the Westinghouse Company while we understand that the new design of high-speed breaker will be installed in the substations that are to be furnished by the General Electric Company. In this way, both methods will be placed in actual operation on similar service, and a true comparison of the merits of both can be obtained. Engineers, generally, will be interested in watching the operation of these two types and, obviously such a practical test is of far more value than any theoretical discussion.

H. B. Dwight: I would ask the authors if they will say a word or two about the features of design of the brush rigging and brush-holder cross-connections required in order to prevent the troublesome crowding of the flash-over arc toward one end of the commutator, produced under some conditions by the magnetic effect of the current in the brush rigging. They mentioned, in connection with Figs. 30 and 31, that some simple change cured that trouble.

Felix Wunsch: I wish to ask the authors if it would not be practicable to insert blank dividing bars into high-voltage machines to prevent flashing.

F. D. Newbury: The authors make the statement that "reactors" are of questionable benefit. There is no uncertainty as to the effect of reactance. It is simply a question of the reactance voltage generated under the conditions obtaining in a particular case. At the first instant of short circuit, the rate of current increase is high, and a considerable opposing voltage is generated in the reactance. As time progresses, the rate of current increase falls off, and the reactance voltage correspondingly decreases. Finally, when the current reaches its steady value, there is no reactance voltage opposing it. Thus all that reactance does is to reduce the rate of current increase; that is, to reduce the slope of the current curve. In Figs. 13, 14, 15, the current curve has practically reached its steady value before the circuit-breakers started to open. Obviously, in these cases the reactance coil used had no effect at the time of circuit opening except that due to its resistance, a very appreciable effect, however, in Figs. 13 and 14.

In comparison with these results, I wish to show two oscillograms taken from a paper presented before the Institute by Yardley in 1914.* These oscillograms were taken in June, 1912, just six years ago, on a 600-kw., 1500-volt, 60-cycle synchronous converter unit, consisting of two 750-volt converters in series. These machines, by the way, represented the first application of 60-cycle synchronous converters to 1500-volt railway service. They did not have commutating poles which accounts for the relatively low current values at which they flashed over.

The first oscillogram shows results with reactance and a standard circuit-breaker. The current reached its final value of about 2500 amperes before the breaker opened, and the converter flashed when the circuit-breaker tripped. The second oscillogram shows the same conditions, except that a much quicker acting circuit-breaker was used. The circuit breaker opened in time to limit the current to 1000 amperes and the converter did not "buck."

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*"Use of Reactance with Synchronous Converters" by J. L. McK. Yardley, A. I. E. E. TRANS., 1914, Vol. XXXIII, Part II, p. 1521, Figs. 15 and 16.

From these cases it is clear that the function of reactance is to delay the rate of current increase. This is of value only when the circuit breaker opens before the current reaches a steady value at which time the reactance becomes ineffective.

The statement is also made that a generator or converter will often safely carry loads that will cause flashing when the circuit breaker is tripped. This statement has been confirmed in several installations that I am familiar with, and its truth is also shown by the oscillograms in Yardley's paper. The converter did not flash until the circuit breaker tripped. The point of practical importance to the operating engineer is that circuit breakers should be set so that feeder breakers trip before machine breakers, and the latter should trip only when absolutely necessary.

In Fig. 5, the line voltage curve drops to a low value at the instant of short-circuit, and when the current begins to decrease, the voltage begins to rise. So far, the action is reasonable, but beyond this point the voltage does not increase above half normal, even though the current falls to zero. Unless something abnormal happens, the voltage returns to normal when the breaker opens the circuit, as shown in Fig. 14. How is this voltage curve to be explained?

In Fig. 15, there is also a falling voltage curve co-incident with the falling current curve. This also calls for explanation.

The two parts of Fig. 18 are interesting in this same connection. In the left-hand curve, with 0.03 ohm in circuit, the current reached 5640 amperes and only a small temporary dip in armature voltage results. In the right-hand curve, on dead short circuit, the current rises to 7670 amperes, and the voltage does not return to a steady value until a considerable time after the current drops to its minimum value. What caused this voltage disturbance?

Fig. 22 illustrates a short circuit on a 300-kw., 600-volt converter, equipped with flash barriers and standard circuit breaker. The direct-current voltage (after the short circuit) becomes alternating, caused apparently by the converter dropping out of step. Was this the case? The value of current given in this oscillogram also calls for explanation.

In Figs. 13 and 14, the load circuit of the converter is closed through a reactor having a resistance of approximately 0.025 ohm (1000 ft. of 500,000-c.m. cable). The current practically reaches its steady value before the circuit breakers opens, so that only the resistance of the coil is effective. The oscillograms show that the armature voltage (using collector-ring voltage) drops to about 80 per cent of its initial value at the time of maximum current. Using these data, the resistance of the complete circuit is approximately 0.105 ohm and the resistance of the converter circuit is 0.08 ohm.

In Fig, 22, the converter is short-circuited. The collector-ring voltage falls to approximately 60 per cent of its initial value at the time of maximum current. Using this voltage and the current of 17,100 amperes, the effective resistance would appear to be only 0.02 ohms, or one-fourth the value derived from Figs. 13 and 14.

In Fig. 31, showing the behavior of a 500-kw., 60-cycle converter under the same condition as the 300-kw., 25-cycle converter, Fig. 22, the current is less than in the smaller converter, although it is reasonable to expect that the resistance in the larger machine would be materially lower. Apparently this 500-kw. converter flashed over—the alternating-current voltage as well as the direct-current voltage falling to zero—and the converter apparently dropped out of step.

The various methods of protecting commutating machines against flashing are valuable under conditions justifying their use. Their advantages are not secured without counter-balancing disadvantages. High-speed breakers are expensive, and barriers considerably decrease the accessibility of the commutator and brushes.

In general, higher direct-current voltages—1500-volts and higher—require protection from flashing. I question whether the usual 600-volt installation requires either high-speed breakers or barriers, or that these measures should be employed in preference to the usual protection afforded by a proper disposition of feeder taps. There is no justification, in the usual case, for locating taps immediately at the substation. The increase in losses caused by omitting taps at the substations is negligible.

In this connection, it should be remembered that feeders are usually connected to the trolley wire every thousand feet. Omission of one or two taps nearest the substation only affects appreciably the losses for that part of the energy used in that section, which obviously is a small part of the total energy. The protection provided by a proper disposition of tap is secured at a lower cost, considering cost broadly, than by the methods discussed in the paper. These more elaborate methods, while exceedingly valuable in extreme cases, should be used with discrimination. I would like to have the authors' opinion on this point: Do they advocate the general use of the barriers they have described on 600-volt converters, and are they now indifferent to the location of taps?

W. F. Dawson: The photographs appear to show that all of the flashing is the result of excessive sparking of the brushes, and that it all starts at the edge of the brush, and no flash starts in between the brushes, as might possibly be expected, due to the distortion of the main flux wave shape, with consequent very high potential between adjacent commutator bars under the edge of the leading pole.

J. F. Tritle: Fig. 8 illustrates the general scheme of operation of the second form of high-speed breaker. This breaker is radically different from the first form in that all mechanical latches and devices of that kind are entirely eliminated and the breaker, instead of being tripped mechanically, is tripped electrically. F^1 and F^2 represent a laminated field structure something like that of an ordinary alternating-current magnet. The poles of F^1 and F^2 are bridged by a very light armature A pivoted at P which is held in contact with the field by a shunt coil S^1 energized from any convenient direct-current source such as the exciter circuit or the main bus. A series bucking bar S^2 which electrically trips the breaker is located between the poles of the field magnet in a plane perpendicular to the plane of the laminations and in very close proximity to the armature, so that a given current flowing in it produces a maximum change in the armature flux with a minimum change in the flux interlinking the shunt winding S^1. The tension spring attached to the armature provides a means of adjusting the breaker, and also gives the high-speed opening of the contacts. The main contact tips C^1 and C^2 are of the solid copper type used so successfully on railway contactors. The blowout coil S is of the series type and designed to give a very intense magnetic field at the contacts.

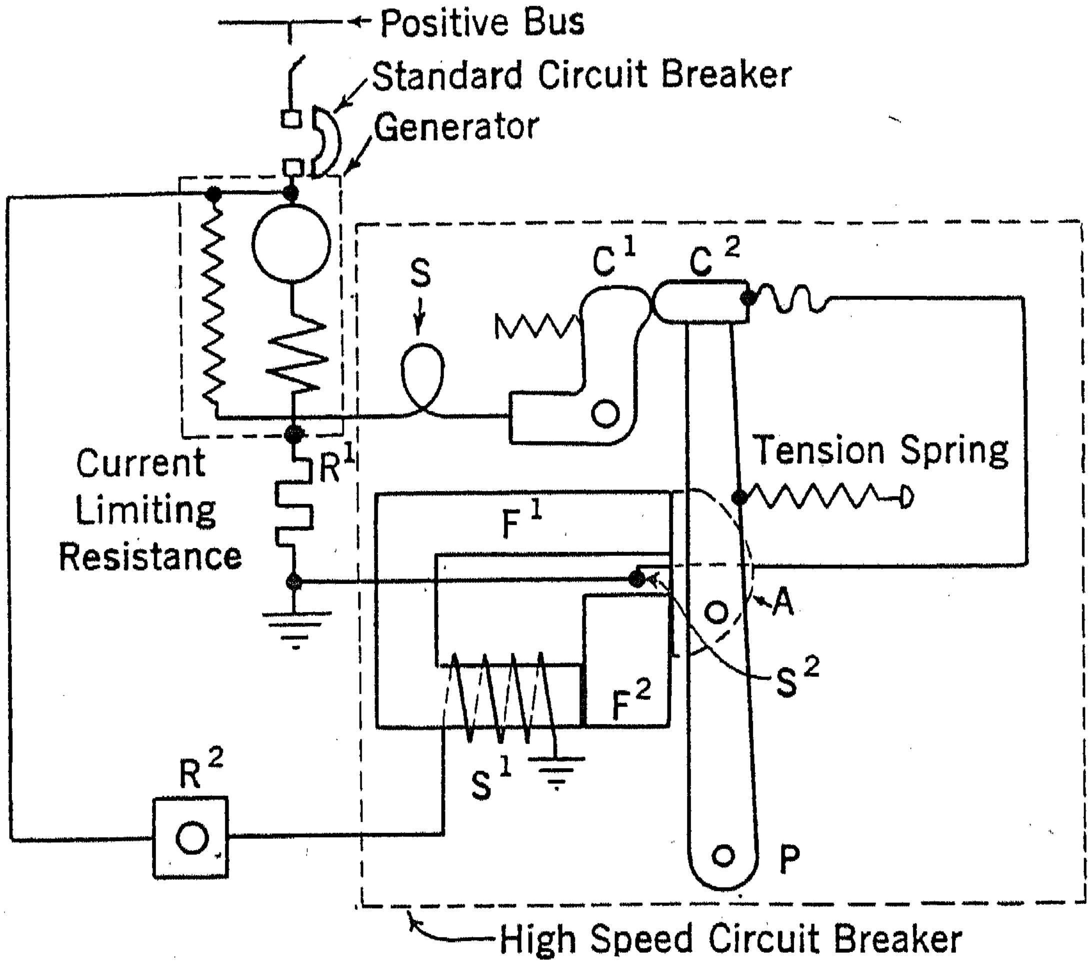

Fig. 8

Assume that it requires 2000 ampere turns in the holding coil to hold the armature in contact with the field with 0 current in the bucking bar and it is desired to set the breaker to trip on 2000 amperes. The current in the holding coil is simply adjusted to give approximately 4000 ampere turns by means of the calibrating rheostat R^2, and it is obvious that the armature will release as soon as the flux from the series bucking bar neutralizes or offsets the additional 2000 ampere turns of the shunt coil. If a short circuit occurs on the line or across the terminals of the machine the current may rise to several times the normal tripping point in which case the flux in the armature will be reduced to a very small value or even reversed. This condition gives the maximum speed of operation, because the armature starts to move at the instant the flux is reduced to normal drop-out value and by the time the flux reaches zero the armature is moving at a fairly high rate of speed. Several hundred short-circuit tests on three sizes of machines proved that owing to the steep pull curve and high-speed characteristics of the armature there is no danger of it pulling back in, due to the maximum currents in the series bucking bar, as might appear possible at first sight.

The speed of the breaker is even faster than the first form of breaker. On dead short circuits on 600-volt machines the contacts C^1 and C^2 break contact and begin to cut in the resistance R^1 in about 0.003 sec. from the start of the short circuit. The contacts are apart far enough in 0.0045 sec. to measurably reduce the rate of current rise, and the maximum peak occurs and the current starts down in a little less than 0.006 sec. The oscillograms of Fig. 18 show the current starting down in 0.0053 seconds.

H. E. Trent: With regard to the high-speed breaker shown in Fig. 4, it would be of interest to know the proportion of time taken to operate the different parts of the breaker on a short circuit, viz:

(1) From time of short circuit to releasing of latch.
(2) From time of short circuit to opening of main contacts.

A circuit breaker designed for such exacting service, one imagines, would require fairly frequent renewals with regard to arcing contacts, even taking into consideration the powerful magnetic blow-out provided.

The operation of the silver fuse with magnetic blowout coils is very satisfactory in point of speed, and it would be of interest to know, if information is available, as to how the shape of the fuse influenced the speed of opening the circuit and whether the authors made similar tests with higher d-c. voltage.

J. L. Burnham: Before replying, there is one question I would like to ask Mr. Fortescue, and that is how they measured the time of the three breakers mentioned, *i. e.*, are intervals measured the same as defined in our paper? In the case of the breaker described by Yardley several years ago I remember he said in his paper that the breaker flashed over (probably due to inductance in circuit for protection) and I do not know that the breaker was ever put in commercial operation. In reference to the impulse method of quick tripping, I think you would have the breaker doing things you did not want it to do, because it would depend on the rate of increase of current rather than actual value of current as to what it would do.

Mr. Hanker mentions possible deterioriation of the first type of circuit breaker we described and stated had been in successful operation over one year. Would add that the maintenance has been very low, and the adjustments have been of very minor importance. In addition severe factory endurance tests were made and none of the performance confirms his prediction. However, the second type of breaker described, which is a later development, is the one that we will now recommend on account of its greater simplicity and ruggedness.

In reference to the short-circuiting scheme of protection for killing the voltage, as described by Mr. Hanker, we still have to learn how this works out in actual service operation. In describing the trial switch he states that it is made up of light parts and has short movement. It may be found that the parts will have to be strengthened or enlarged for greater movement to meet service requirements and the resulting added weight would decrease the speed.

This method of suppressing flash-over has certain objections. The short-circuiting of the machine after every disturbance makes it essentially not universally applicable. That is, it cannot be used on converters as it would short-circuit the a-c. system; it cannot be used on individual feeders; it cannot be used to protect individual machines in parallel,—it must protect the station as a whole, which seems to be its particular application; it is difficult or impossible to apply it to old machines; every operation (additional short circuit) gives a maximum shock to the apparatus; killing the voltage by this method may cause a comparatively long interruption to service; the addition of collector rings and brushes to a direct-current machine involves added risk, especially on the higher voltage machines, and increase in cost of maintenance. There will also be certain losses with these collector brushes, which will decrease the efficiency. A machine equipped with rings is inherently longer. Instead of limiting the rise of current in the armature it adds thereto; regardless of magnitude of short circuit or distance from substation, maximum short circuit is thrown on adjacent substations when suppressor operates; an impulse from a distant short circuit or moderate but sudden increase of load may trip this type of breaker which subjects the apparatus to maximum strains that are not at all commensurate with the original load. It may even be possible for a short circuit in one station to cause adjacent stations to be short-circuited by their flash suppressor and so on until all substations are out of service. For comparison of the two methods of protection would refer you to the last paragraph of our paper.

Answering Mr. Dwight, as to the arrangement of brush holder connections, it involves the question as to whether the brushes are ahead or behind the stud that carries the current from the brushes, and also whether the connections are taken from one end of the commutator or the other, and I think if he will refer to the part of the paper dealing with the magnetic conditions and forces which affect the arc that it will explain in a general way what he desires to know. There are many combinations of conditions affecting the direction of flashing.

In reference to the placing of alternate "dead" segments between the regular commutator segments, suggested by Mr. Wunsch, it is, of course, equivalent to doubling the width of mica between segments. There may also be some cooling action of the dead segments which would tend to decrease any sparking that occurs. This might increase slightly the load that a machine could carry without flashing over. If this arrangement is effective it must be at the expense of increased diameter of commutator with corresponding increase in speed. The same increase in size of commutator, if possible, would also reduce the tendency to flash if the "dead bars" were not used.

In reference to Mr. Newbury's remarks on the various curves; it will be noted that the curves referred to are made under different conditions, that is, in the case of the generators, we had the voltage killing device operated by a main slow breaker which inserted resistance in series with the field after the slow breaker was tripped. The slow breaker, of course, operated after the fast breaker which explains the decrease in d-c. voltage in Fig. 5. Then, some of the tests were taken with barriers, and others without barriers and some with slow-speed breakers and others with high-speed breakers and also with many combinations of protection.

Fig. 13 was given only to show the rise in voltage across the circuit breaker when reactance is in circuit. The breaker was tripped by hand.

Fig. 15 is a record in which the breaker did not trip. The reactance reduced the synchronizing power of the converter enough to cause it to gradually drop out of synchronism and thus reduce the d-c. voltage. The total time was very short as indicated by the 25½-cycle timing curve.

In Fig. 18 the disturbance in voltage was caused by some tendency to flash, but complete flash-over did not occur as will be seen by the record of current which was reduced by insertion of resistance connected across the high-speed circuit breaker. The complete record shows the voltage back to a steady value in 11 cycles or 11/60 of a second.

The short circuit recorded in Fig. 22 shows that the converter pulsated and reversed polarity to small values. When it became steady the polarity was correct.

Regarding the comparison of short-circuit values of current on the 500-kw. commutating-pole machine and the 300-kw. non-commutating pole machine, I believe there are too many variables in the transient phenomena to arrive at true values of internal resistances especially when the current is sufficient to throw the machines temporarily out of step as shown in Figs. 22 and 31. Considerable variations in values are obtained on a given machine even under apparently the same conditions.

The apparent discrepancy in resistance values of armature resistance as calculated by Mr. Newbury from data in Figs. 13, 14 and 22, is due to assuming resistance as of one protective coil only. It will be noted that the text refers to coils and there were actually three coils used in the test. This will bring the values of armature resistance into reasonably close agreement considering the rough method of determination.

I endorse Mr. Newbury's suggestion that the operating companies should not tap their feeders into the trolley less than 1000 feet and preferably further from the station. That is the simplest form of protection and in the majority of cases would make the equipment much more reliable. I would much prefer this method to using the other means which have been described, if it proves suitable in any individual case.

Mr. Dawson asked the question whether the arc always starts at the edge of the brushes. The high-speed photographs seem to indicate that this is generally so. In two instances out of many trials a converter flashed back of the brushes at a point where no protection had been provided. This shows, I think, that the conditions back of the brush are favorable for supporting a flash but the direction of motion of the commutator causes the arc to form and be carried around with it from the front of the brush.

Presented at the 34th Annual Convention of the American Institute of Electrical Engineers Atlantic City, N. J., June 28, 1918.

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THE AUTOMATIC HYDROELECTRIC PLANT

BY J. M. DRABELLE AND L. B. BONNETT

Abstract of Paper

The automatic hydroelectric generating station of the Iowa Railway and Light Company at Cedar Rapids, Iowa, is a radical step in advance in the elimination of operator's wages in a station of appreciable size, without sacrificing complete control.

This station consists of three 400-kw., 500-kv-a., 60-rev. per min., 2300-volt, vertical generating units, tied in to a system, of which the main generating station contains about 20,000 kv-a. in steam turbo-generators. One striking feature is the entire omission of the usual governors, the waterwheel gates being motor driven and controlled by contact-making ammeters. Each unit has its individual control panel, consisting of the necessary contactors and relays to connect it to the bus at the proper time. A motor-driven drum controller gives the proper time element between the different steps in the operation of placing the generator on the line. Any generator can be started either by a float switch when the pond level reaches the proper height or by a remote control button in the steam station. The starting of the first generator throws on the line the motor of one of the two exciter sets, and the generator cannot be connected to the bus until the excitation voltage has reached the normal value. The waterwheel gates are then partly opened and the generator comes up to approximately normal speed. It is then connected to the bus without field through an iron-core reactance. Then a weak field is applied. Next it is raised to full normal value, and then the reactance is short-circuited. The contact-making ammeter opens the gates to full gate opening and the generator then carries full load in about 40 seconds after either the control button is closed or the float switch is closed.

On October 2nd there was placed in operation at Cedar Rapids, Iowa, the automatic hydroelectric plant of the Iowa Railway and Light Company. This plant represents a further development of the automatic substation first described in a paper by Messrs. Allen and Taylor in the A.I.E.E. TRANSACTIONS, Volume XXXIV, Part 2, page 1801.

The station operates with no attendant and the different generating units start or stop, depending on the flow of the river or can be started and kept running by remote control from the main steam station, as described in detail later.

The building is 112 ft. (34.13 m.) long, 41 ft. (12.49 m.) wide and has a depth to rock bottom at the bottom of the draft tubes of 37 ft. (11.27 m.). The substructure is of concrete; the superstructure is 30 ft. (9.14 m.) high above the water table of the building and is constructed of Bedford Limestone. The interior of the building is finished in yellow brick and the floor in red tile. The exterior is shown in Figs. 1 and 2. There are installed three 500-kv-a. 80 per cent power factor, 60-cycle two-phase, 2300-volt, 60-rev. per min. generators, of the vertical type. They are driven by waterwheels rated 540 b. h. p., 60 rev per min. under a 10 ft. (3 m.) head, the operating head varying from 8 ft. (2.43 m.) to 11 ft. (3.35 m.). The runners are of the Francis type, having a diameter of 171 in. (4.33 m.). The rotating element is supported by a thrust bearing at the top, supplied with oil by a pump driven from the generator shaft.

The dam consists of nine spillway sections, each 60 ft. (18.28 m.) long with automatic flashboards, the total over-all length being 600 ft. (182.88 m.). The dam is shown in Fig. 3.

There are two General Electric Induction motor-driven exciter sets rated 100-kw. 125-volts each capable of carrying the entire excitation of the total development, namely, that of four generators. The motors are rated 150-h.p., 2300-volt, two-phase, 60-cycle, 1200-rev. per min. and are designed to be started by the application of full voltage at standstill.

The plant is designed to operate in parallel with the 19,000-kw. steam turbo-generating station of the Iowa Railway and Light Company, the steam plant being located at a distance of 3300 ft. (1005.84 m.) from the hydroelectric plant. The stations are tied together by one underground line consisting of two 600,000-cir. mil, two-conductor, concentric, 7500-volt, varnished cambric, lead-covered cables.

The different generating units may be started and stopped either automatically, depending on the rise or fall of the water level in the river; or by remote control from the steam station when the capacity is needed. To accomplish the latter, a bench board section, shown in Fig. 5, was installed in the steam plant. On this, for each generator, there are provided three single-throw and one double-throw control buttons, the purpose of which will be explained later. All control wires between the stations are bunched in a multi-conductor lead-covered cable consisting of fifty No. 12 and four No. 4 B. & S., rubber-covered wires.

The same benchboard contains for each generator an in-

PLATE LVII.
A. I. E. E.
VOL XXXVII. 1918

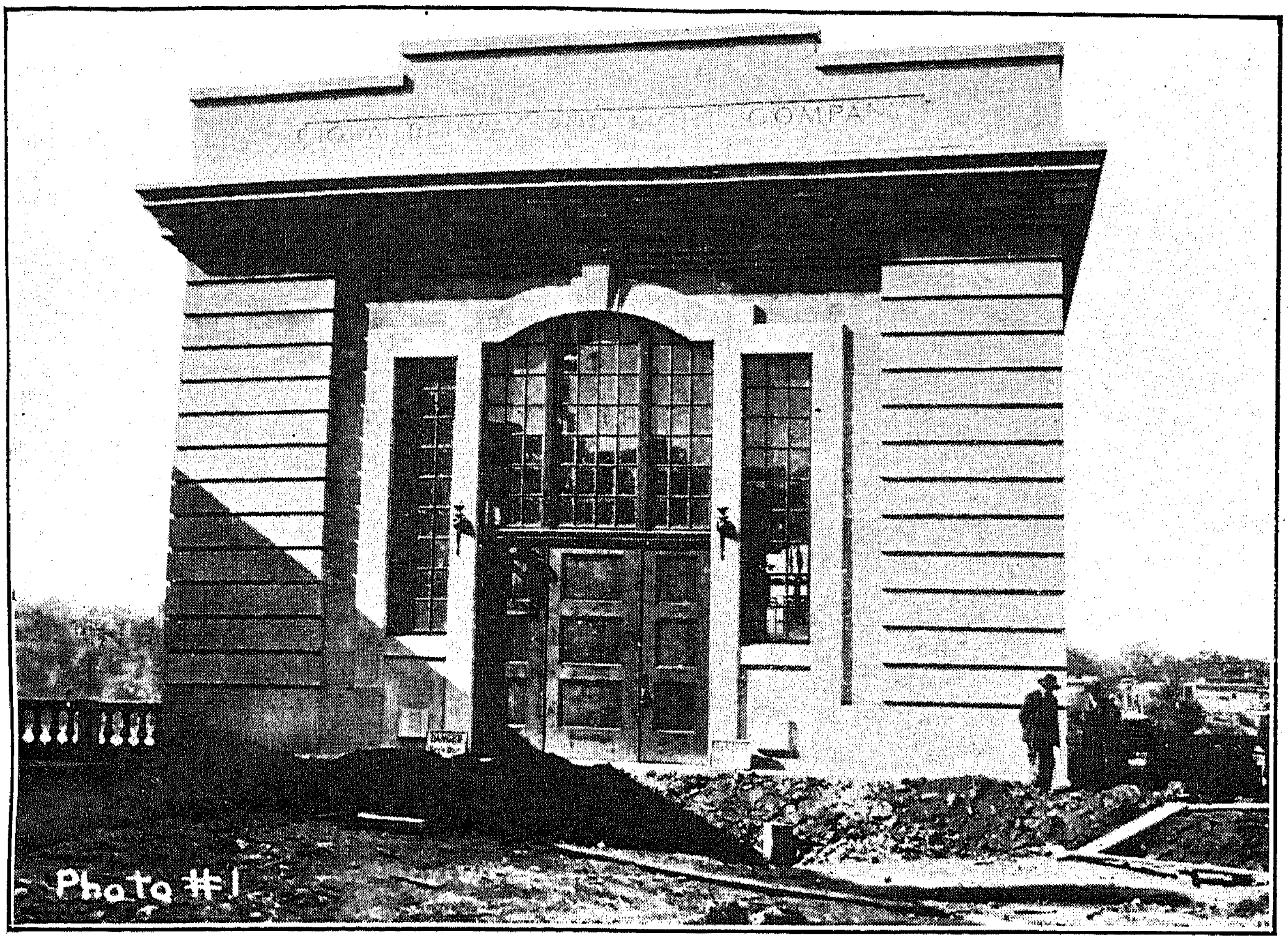
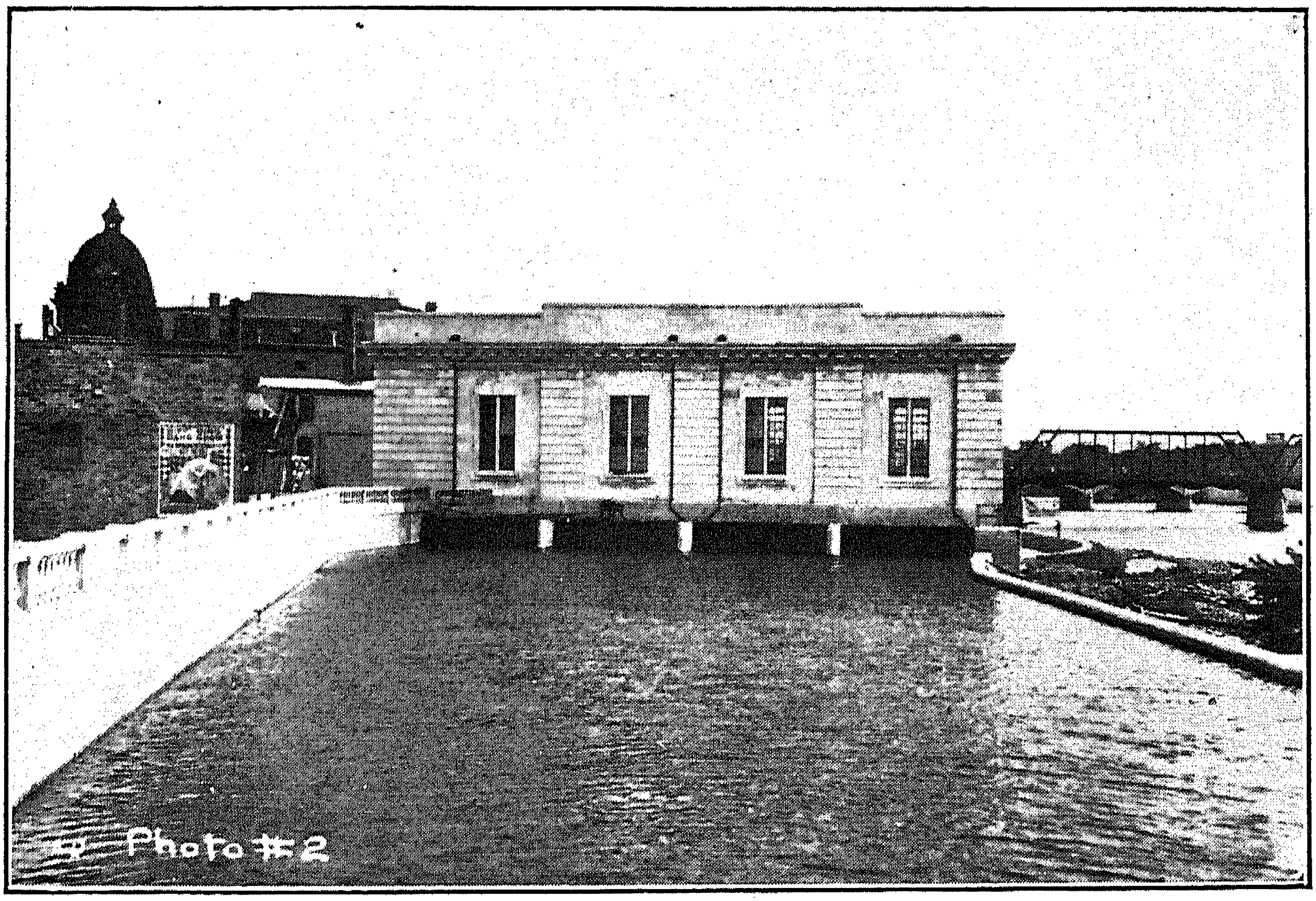


FIG. 1—EXTERIOR OF STATION—FRONT VIEW



[DRABELLE AND BONNETT]

FIG 2—EXTERIOR OF STATION—REAR VIEW

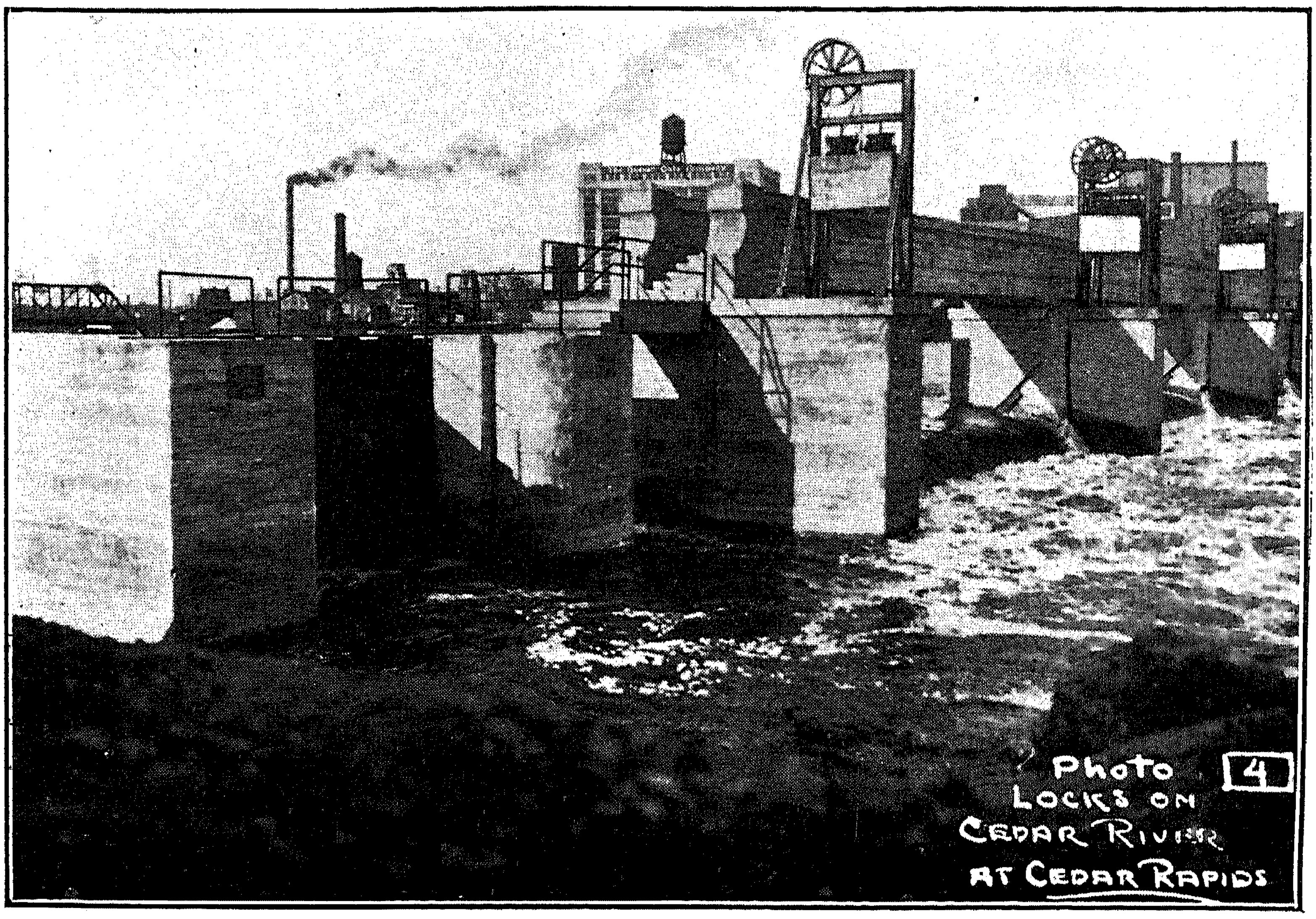
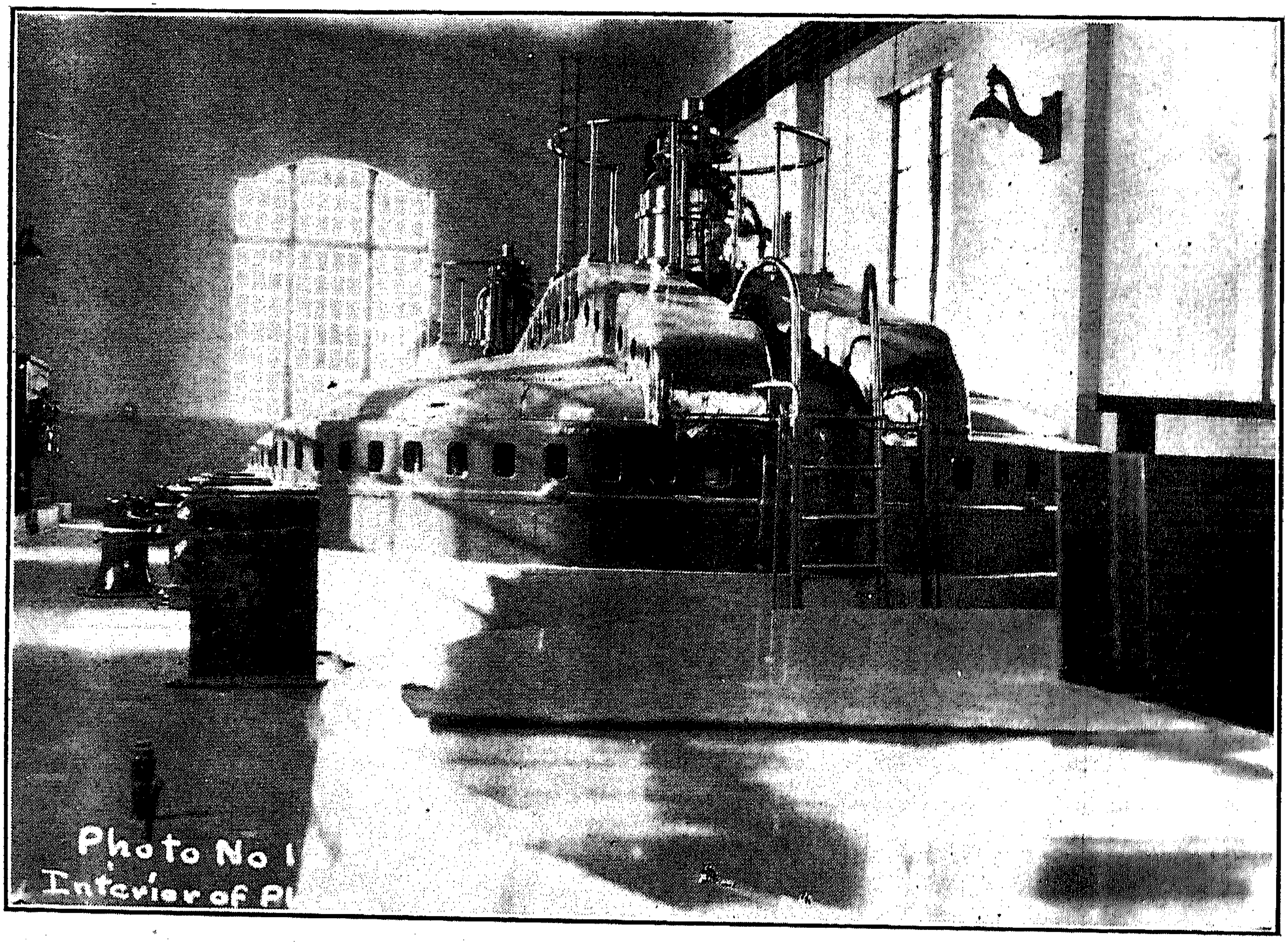


FIG. 3—LOCKS ON CEDAR RIVER AT CEDAR RAPIDS



[DRABELLE AND BONNETT]

FIG. 4—INTERIOR OF STATION

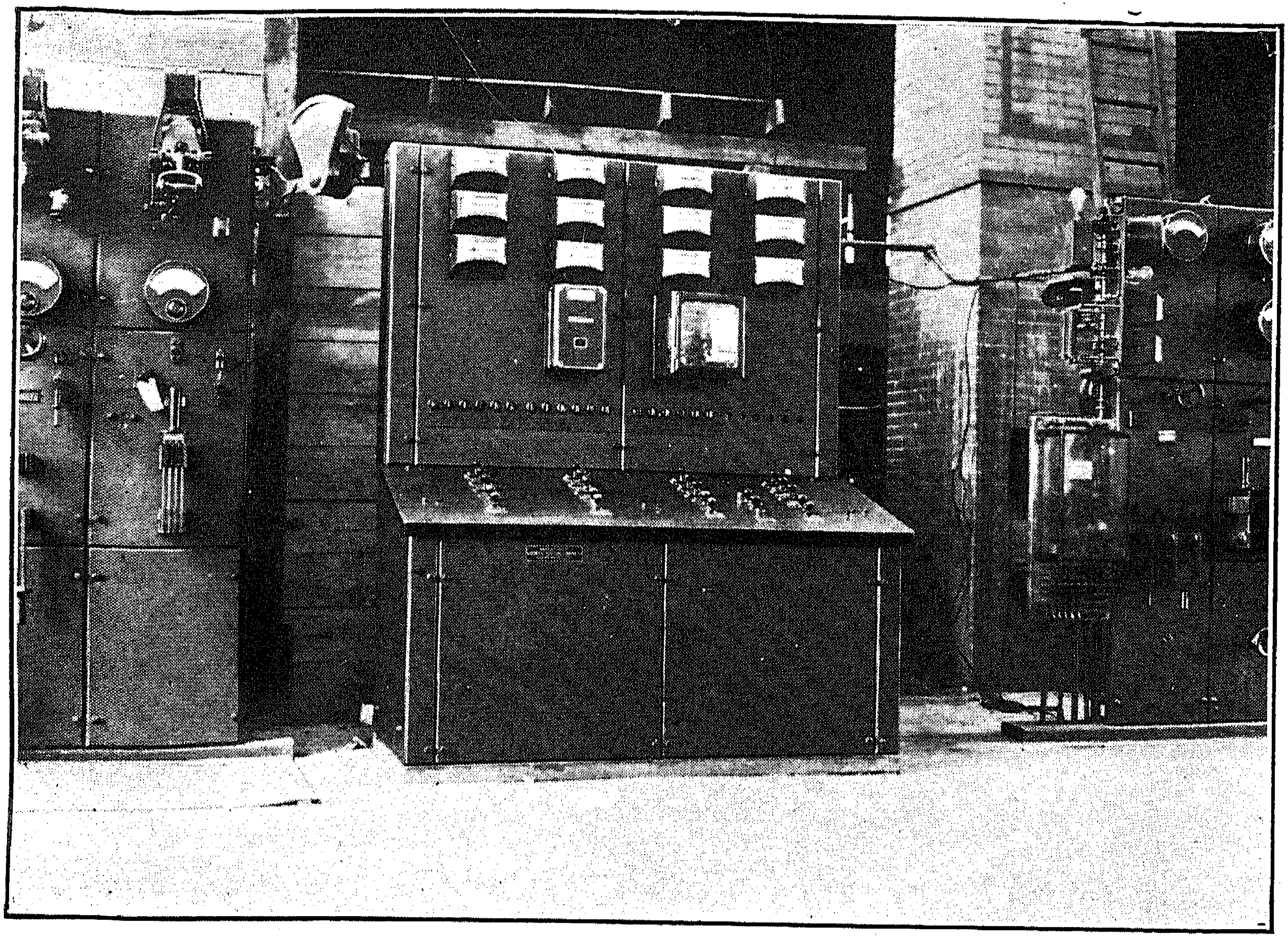
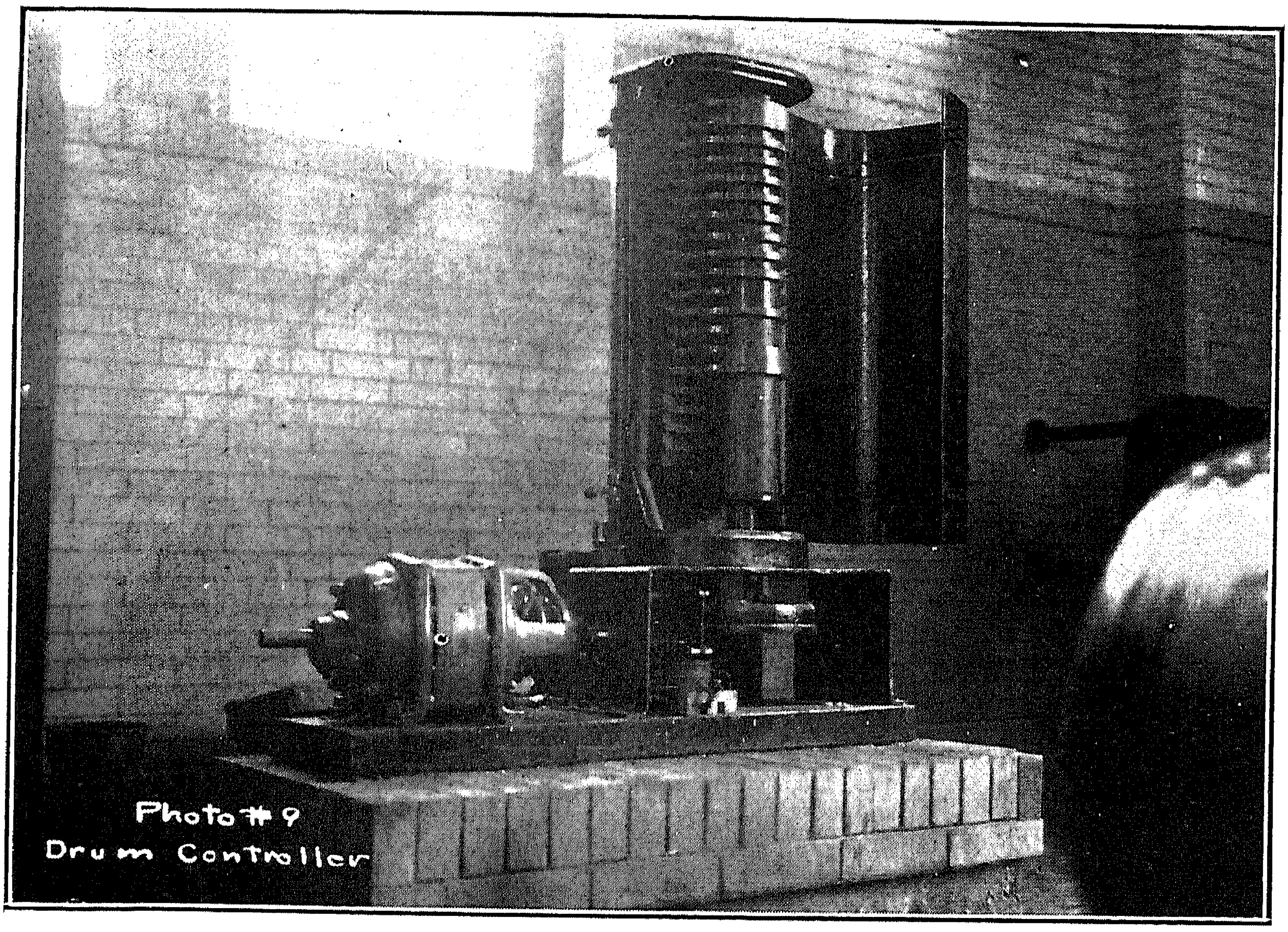


FIG. 5—CONTROL BOARD AT STEAM STATION



[DRABELLE AND BONNETT]

FIG. 6—DRUM CONTROLLER

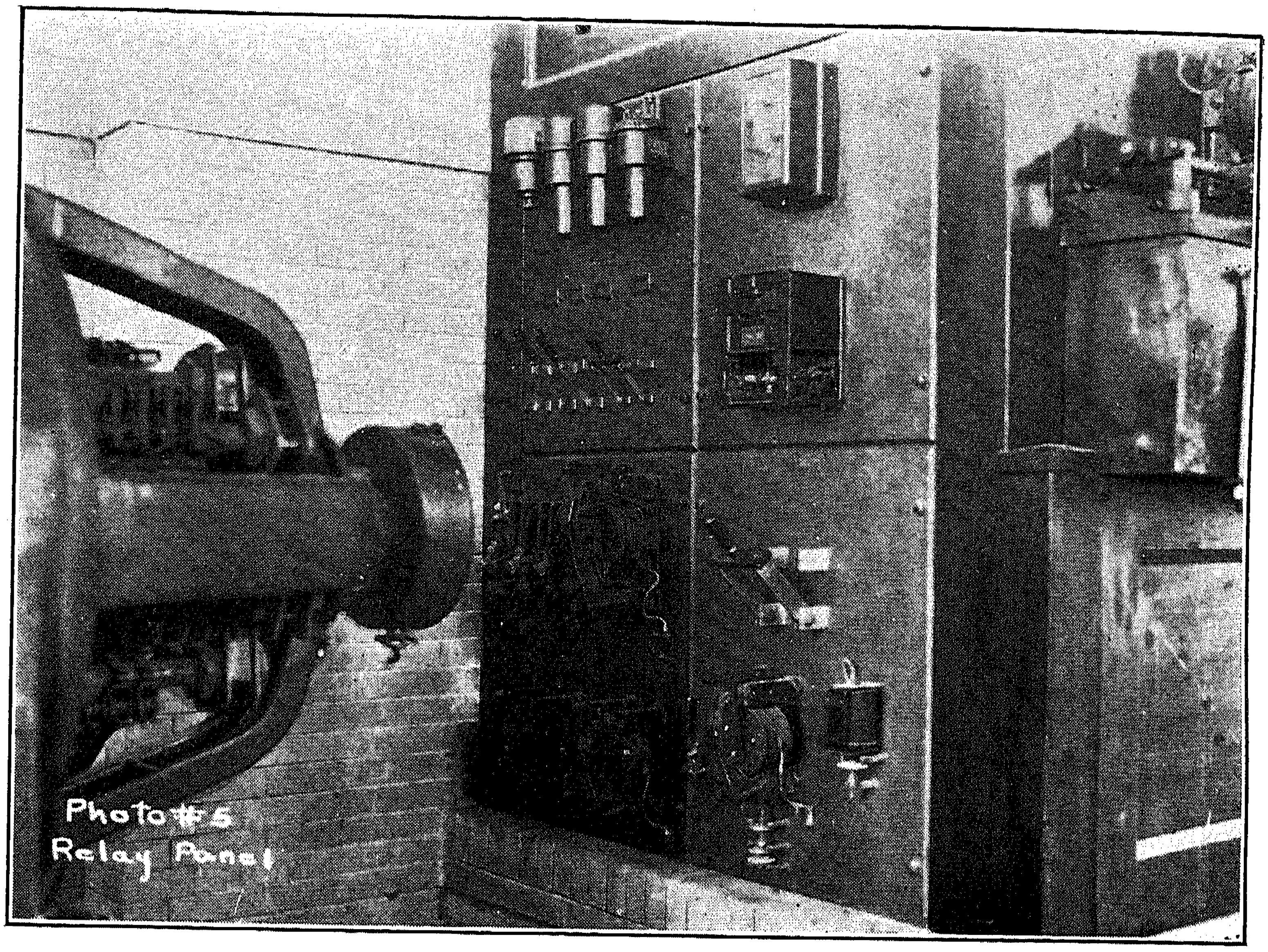
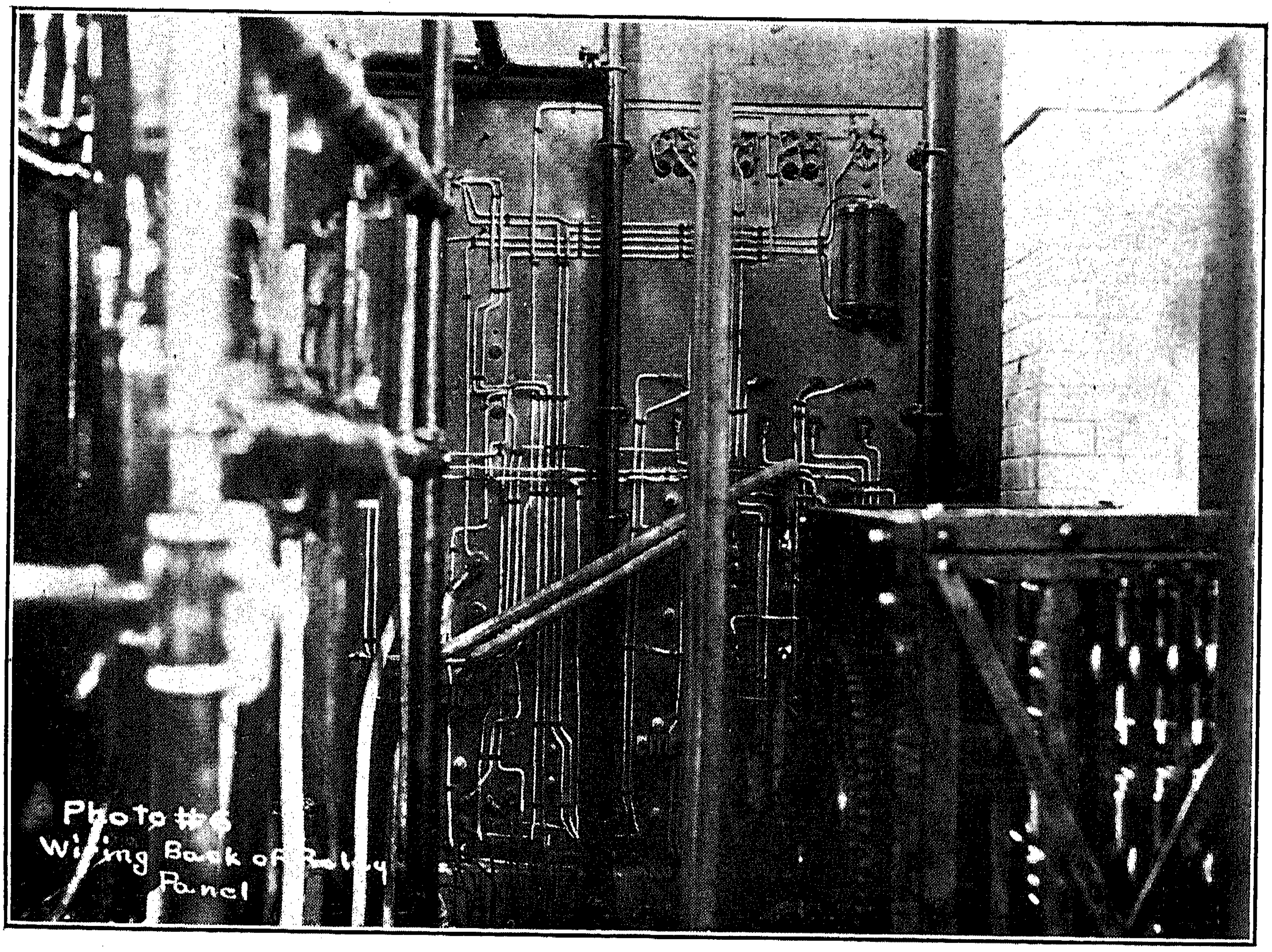


FIG. 7—RELAY PANEL



[DRABELLE AND BONNETT]

FIG. 8—WIRING BACK OF RELAY PANELS

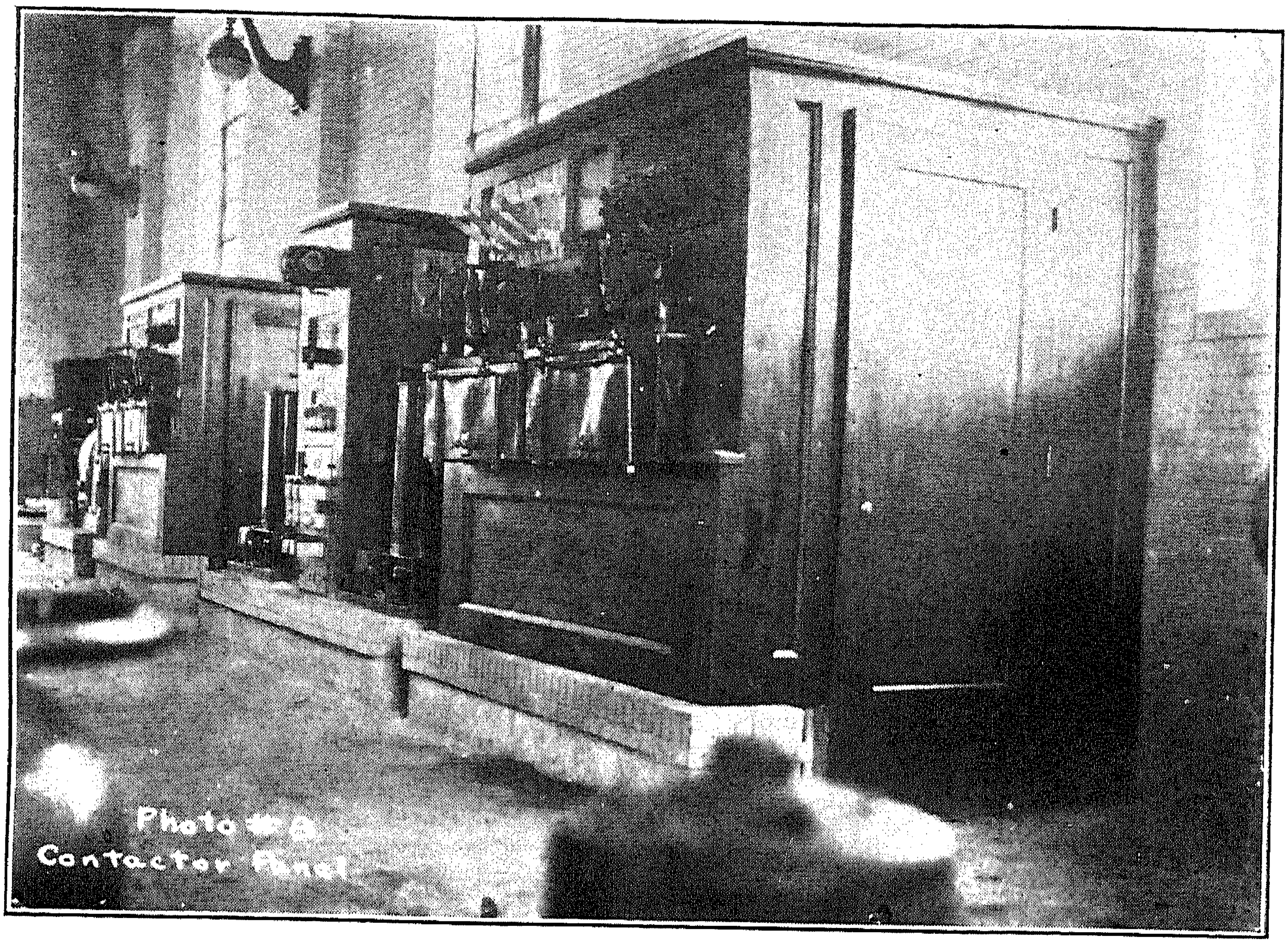
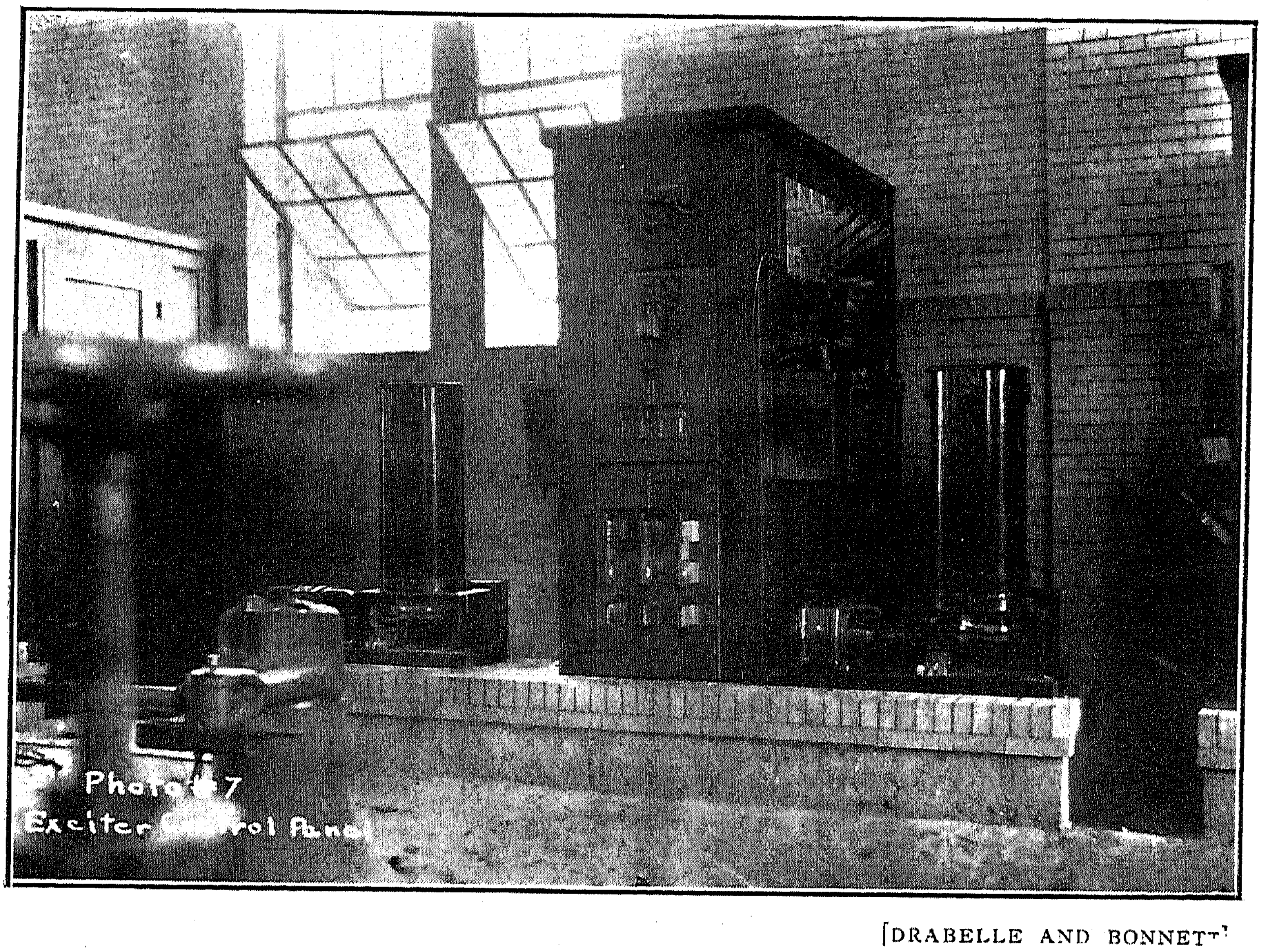


FIG. 9—CONTACTOR PANEL



[DRABELLE AND BONNETT]

FIG. 10—EXCITER CONTROL PANEL

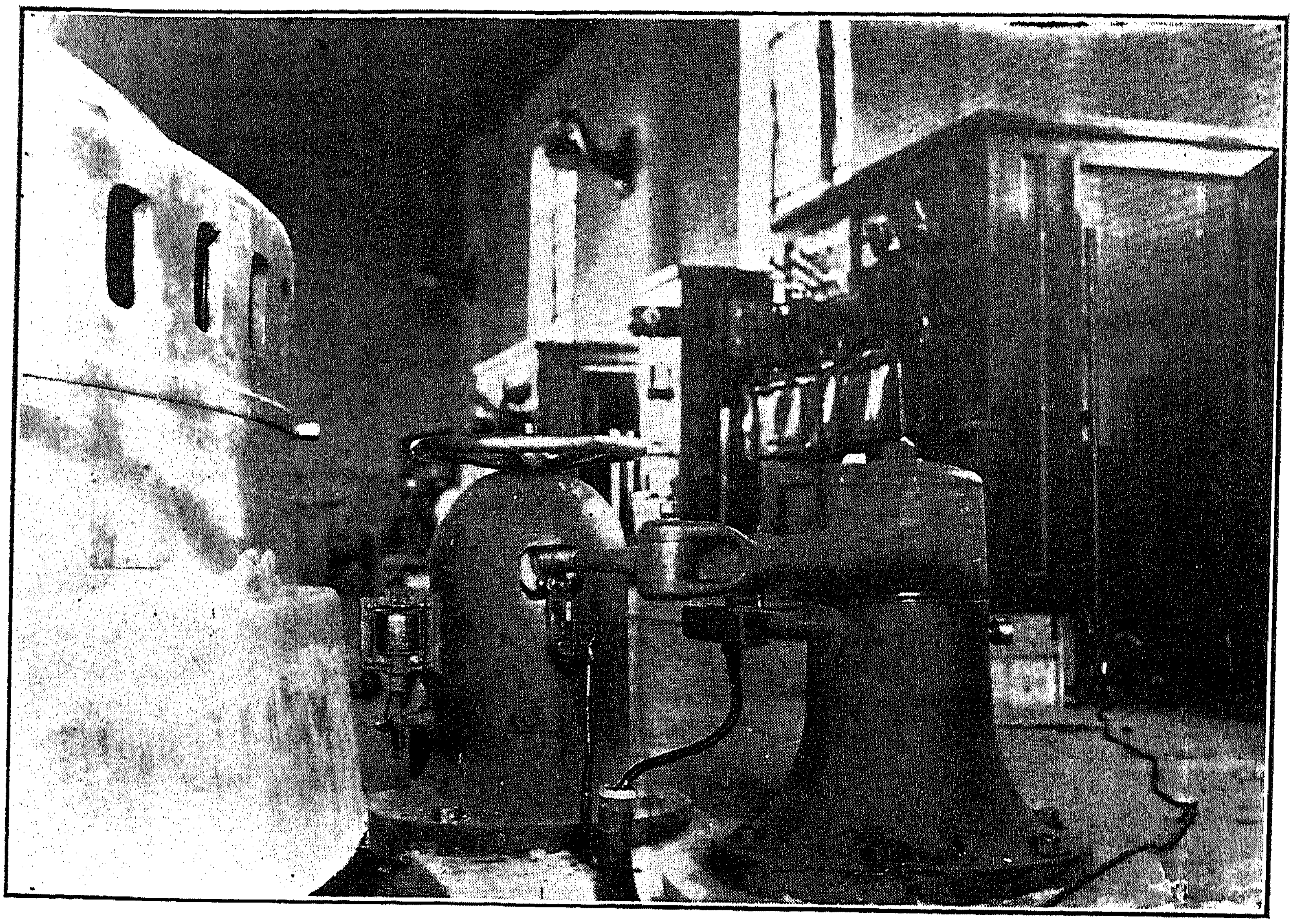
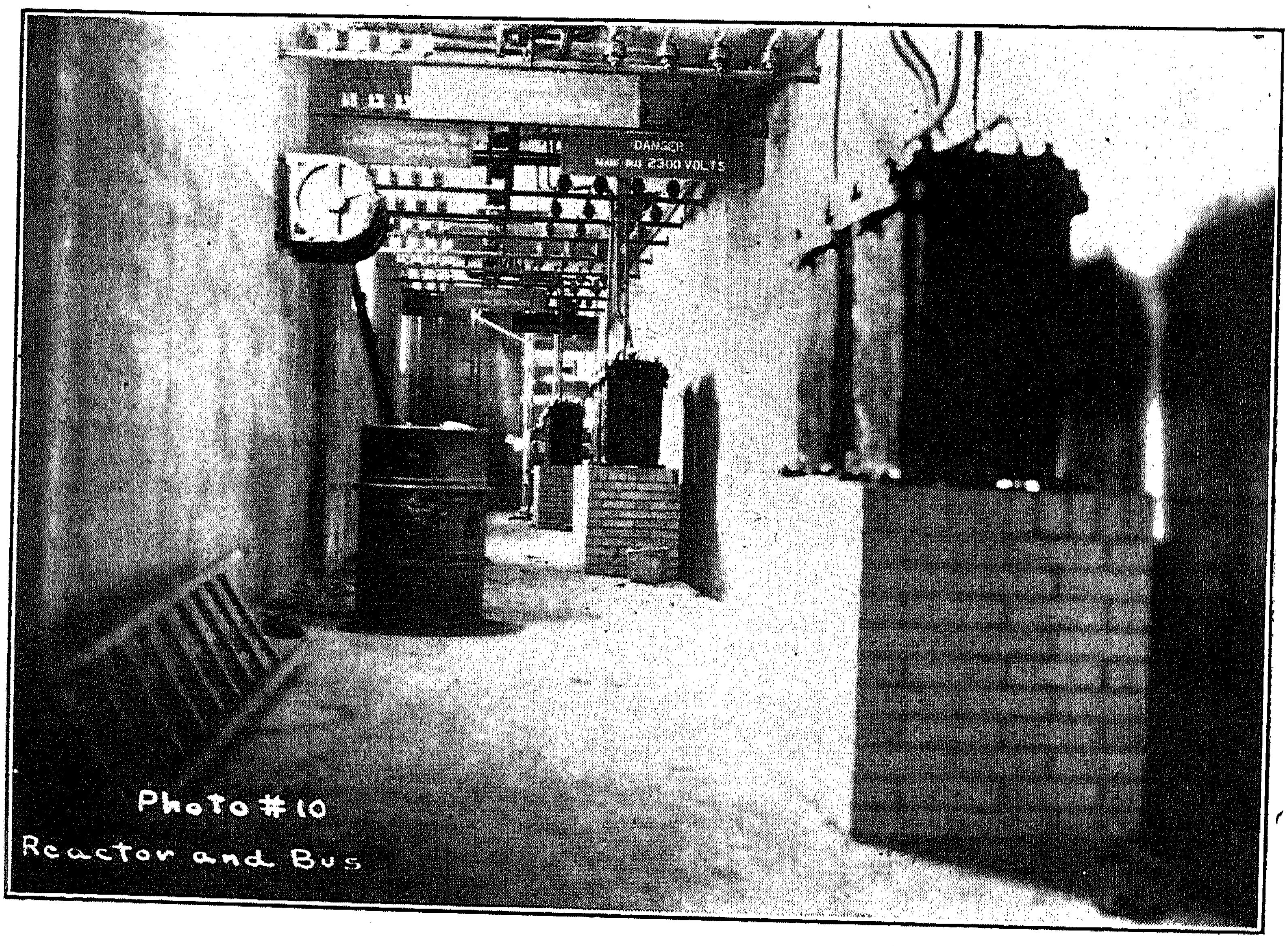


FIG. 11—GATE CONTROL



[DRABELLE AND BONNETT]

FIG. 12—REACTOR AND BUS

dicating ammeter (150-ampere scale) and wattmeter (600-kw. scale), there being no instruments in the hydroelectric plant. The leads for these meters from the current transformers in the hydroelectric plant are bunched in an eleven conductor, No. 4, lead-covered cable. These leads are wired through calibrating terminals at the hydroelectric plant so that portable meters can be wired in if desired at any time. The lower row of instruments on the benchboard is a totalizing equipment, reading the total incoming power from the hydroelectric plant. This consists of two ammeters, a power factor indicator, an indicating wattmeter, a curve-drawing wattmeter and a watthour meter.

The waterwheel gates are not supplied with the usual hydraulic governor; but instead the opening and closing of the gates is obtained by a 5 h. p. induction motor through suitable reduction gearing. The gear ratio is such that it requires 25 seconds for the gates to completely open, starting from rest in the closed position. The motor is equipped with a solenoid brake which releases when voltage is applied to the motor. This prevents overshooting by quickly stopping the motor after the power supply is interrupted. Each gate mechanism is equipped with a contact-making device operating indicating lamps on the benchboard at the Sixth Street station, showing the exact amount of gate opening.

The correct sequence of operations in the control of each generator is obtained by properly placed segments on the drum of a controller which is driven by a single-phase one h. p. repulsion-type motor. This controller definitely determines the proper time spacing between the different steps of starting the unit and connecting its generator to the bus. One of these is shown in Fig. 6.

The contactor equipment and protective relays are mounted in cabinets as shown in Figs. 7, 8 and 9. Fig. 8 shows the wiring back of the panels.

For controlling the exciter sets, two oil-immersed contactors are used, also a double-throw, triple-pole switch, which connects either one or the other of the exciters to the exciter bus. The middle blade of this switch selects the proper contactor for starting the exciter set which is connected to the excitation bus. This equipment is shown in Fig. 10. At the top of the cabinet will be noticed a small induction motor. This carries a centrifugal device on the end of the shaft which in case the frequency of the system rises to 64 cycles, will open, and shut down the plant, thereby preventing runaway.

The units have individual float switches set for slightly different levels. If the water rises to a certain level it closes the float switch of the first unit, and that machine is started and put into service. If the flow of the river is such that the water continues to rise the second float switch closes, starting the second machine. The third switch is set still higher. The machines are shut down in the reverse order as the water level falls.

In general the sequence of operation of each generator is as follows: The closing of the float switch or the proper remote-control button in the steam station, starts the exciter set (provided it has not already been started with another generator), and opens the waterwheel gate. When the generator comes up nearly to synchronous speed it is connected to the bus without field but in series with a 20 per cent reactance. A weak field is then applied pulling the machine into synchronism. The field is then strengthened to full field, the reactance short-circuited and the water wheel gates opened to carry full load on the unit.

The generators operate with a fixed excitation adjusted for normal full-load value. If the water level is so low that with full gate opening normal load cannot be obtained, the generator, due to its high excitation, takes care of part of the wattless kilovolt-amperes of the system. The speed and voltage are, of course, determined by the steam turbine generators.

Details of Operation

Detailed operation of this automatic equipment is as follows, referring to the wiring diagram:

Connected to the 2300-volt bus there are two small transformers stepping down to 220 volts, two-phase, for an operating bus. One phase is taken from this bus through the double pole switch No. 29, for controlling the contactors of each generator equipment. In the line between these transformers and the 220-volt bus there is an emergency throwover switch which in case of failure of voltage on either phase is thrown over by a spring to a separate source of power brought in from the other station.

The closing of float switch No. 1, as shown on the right-hand side of the drawing, or control button No. 3 in the steam station, closes contactor No. 4 provided that control button No. 2 is also closed. This energizes segment No. 14 of the controller. Segment No. 17 is in contact with its finger and a circuit is completed through the coil of contactor No. 6, which starts the motor driving the drum controller. A circuit is also made at No. 16 through the contacts of relay No. 41, which is not energized, through the coil of the contactor No. 5, through the contacts of relay *E*, the inverse-time relays No. 5 and the thermostat relay No. 10. The thermostat relay is connected across one phase of the control circuit and is of course, picked up if all the thermostats are closed, indicating that there is no overheating. Relay *E* is connected across the other phase and is picked up unless the frequency of the bus exceeds 64 cycles. Contactor No. 5 now closes energizing segment No. 1 of the controller. A circuit is now made through segment No. 2 picking up relay No. 42. Either one or the other of the two exciters sets can be started, depending on whether the switch *K* is closed up or down. The picking up of relay No. 42, completes the circuit through switches No. 28, which are closed, through the coil of contactor *G*, throwing full 2300 volts on the stator of the induction motor.

Contactor No. 43, with its coil across the armature of the exciter, short circuits with its contacts the field rheostat until the exciter voltage is built up to 75 or 80 volts. The set comes up to speed and the voltage up to normal very quickly and the d-c. voltage relay No. 11 picks up. The holding circuit of contactor No. 5 is now closed through this low-voltage relay No. 11, the lower interlock of contactor No. 5 and through segment No. 15-A, so that No. 5 stays closed after No. 16 has left contact unless the exciter voltage drops.

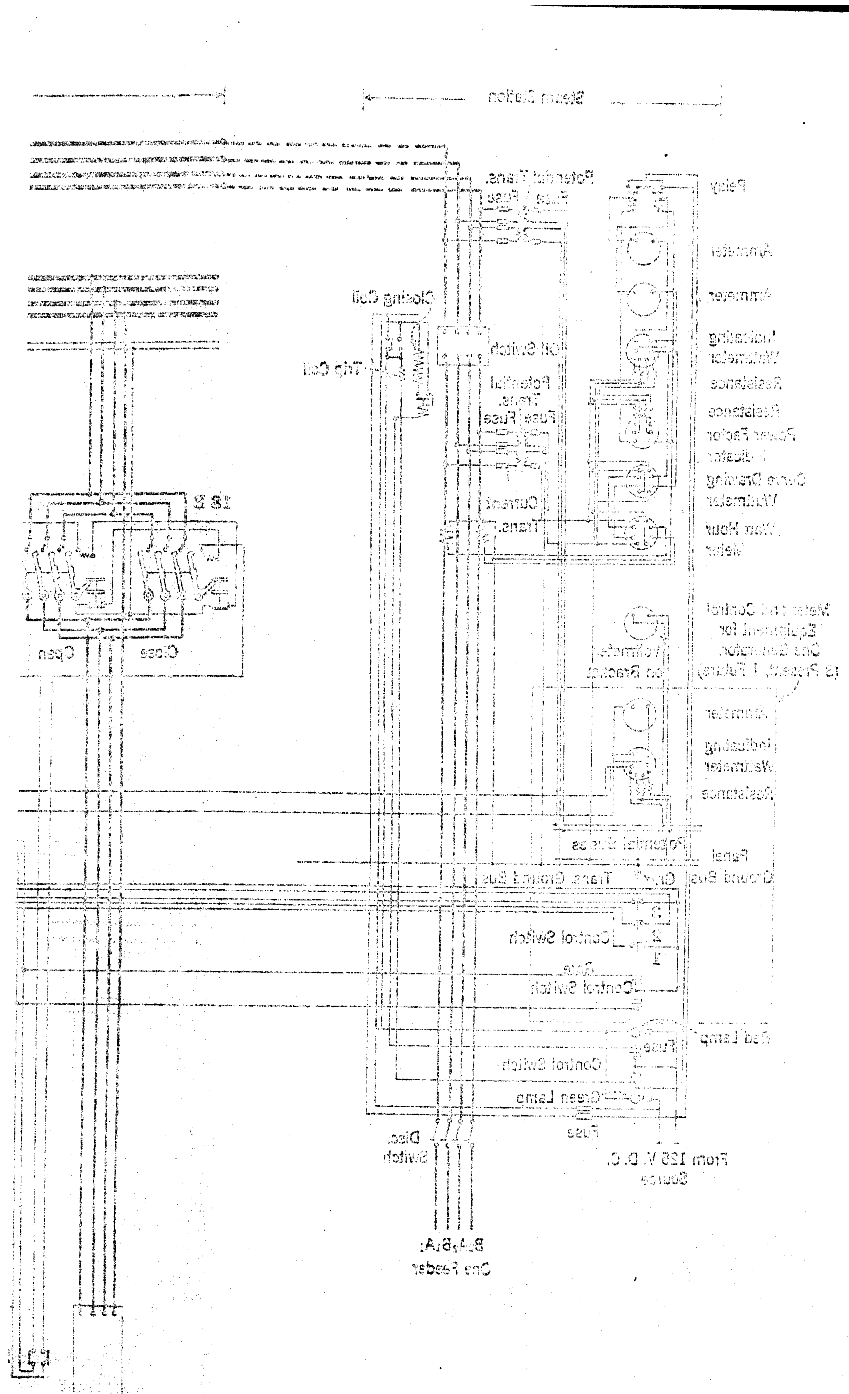
In the meantime, the controller has been rotating and a circuit is made through finger No. 3 to the middle of point of relay No. 20. As the generator is not connected to the line there is no current flowing in the contact-making ammeter No. 21 and its contacts are closed in the up position. If No.1 button in the steam station is closed the left hand side of relay No. 20 is picked up. The energizing of finger No. 3, therefore, completes the circuit through the coil of contactor No.18-A, which starts the gate motor in the proper direction for opening the gate of the waterwheel. Finger No. 3 is in contact a definite length of time which is sufficient to give a gate opening on the waterwheel of such amount that the no-load constant speed would be about 65 or 70 rev. per min.

The controller motor has been receiving its energy through contact No. 6 held closed by finger No. 17. This finger next comes to the break in its segment and the controller stops in this position, waiting for the generator to come up to speed. On each generator there is a centrifugal device No. 13, set to close at about 55 rev. per min. that is, five rev. per min., below normal. When this switch closes, a connection is made from finger No. 17-A to No. 19 and through the controller to No. 18 to contactor No. 6, starting the controller again.

The gate opening is such that the generator is slowly rising in speed, so that by the time the segment No. 4 on the controller comes into contact, the speed is approximately normal. This segment closes contactor No. 23 connecting the generator to the bus without field and with a reactance in series. Finger No. 5 next closes the field contactor No. 24 putting a weak field on the generator and pulling it into synchronism. Finger No. 6 then closes contactor No. 40, short-circuiting a part of the field rheostat, thus strengthening the field to full normal value. Finger No. 8 closes contactor No. 39, short-circuiting the reactance. In the meantime finger No. 3 has come into contact again and the contact-making ammeter No. 21 opens the gate until the current of the generator is up to normal value.

The generator is now properly connected to the bus and carrying full load. Finger No. 17 comes to the second break in its segment and controller stops in the full running position.

The equipment can be shut down in several different ways. If the pond level falls, due to the fact that the demand for water is greater than the flow of the river; float switch No. 1 will open. If it is desired that the steam station shut down and start from the water level, button No. 4 will be open and the opening of both this button and the float switch will trip contactor No. 4. If it is desired to shut down any generator and prevent its starting again, button No. 2 in the steam station should be opened, thus also tripping contactor No. 4. This de-energizes segment No. 14 of the controller which in turn drops out contactor No. 5. This de-energizes segment No. 1 and drops out all other contactors connecting the machine to the bus. When contactor No. 5 drops out, the upper interlock makes a circuit through segments No. 19 and 18, picking up contactor No. 6 and running the controller to the off position. The middle interlock of contactor No. 5 is also closed, energizing contactor No. 18-B and closing the waterwheel gate. As



Steam Station
Potential Trans.
Fuse
Relay
Ammeter
Ammeter
Indicating Wattmeter
Resistance
Resistance
Power Factor Indicator
Curve Drawing Wattmeter
Watt Hour Meter
Closing Coil
Trip Coil
Oil Switch
Potential Trans.
Fuse
Current Trans.
Meter and Control Equipment for One Generator (3 Present, 1 Future)
Voltmeter on Bracket
Ammeter
Indicating Wattmeter
Resistance
Potential Buses
Panel Ground Bus
Trans. Ground Bus
Control Switch
Gate Control Switch
Red Lamp
Fuse
Control Switch
Green Lamp
Fuse
Disc. Switch
From 125 V. D.C. Source
B₁ A₁ B₁ A₁ One Feeder
Open
Close
Gate Motor (3 Present, 1 Future)

FIG.

the controller rotates to the off position fingers No. 20 and 21 are bridged. These contactors are in parallel with the middle interlock on contactor No. 5 to give additional contact surface.

Connected to the operating bus is a one-h. p. two-phase motor driving a speed-limit device which opens its contacts when the speed exceeds 64 cycles. If for any reason, the hydroelectric plant becomes separated from the steam station, the generators will, of course, speed up increasing the frequency of the bus. This in turn will trip the speed-limit device *D*, dropping out the four relays *E*, which in turn drop contactors No. 5 on all generators. This drops out all other contactors, and as the bus voltage then drops to zero the automatic throw-over switch No. 46 connects a separate source of power to the control bus thus furnishing energy for closing the gates of the water wheels to shut down. The wiring is so arranged that the relays *E* and No. 10 are connected outside this throw-over switch so that the generators cannot be started until the main cable between the station is re-energized.

If for any reason the generators have gone above speed and the main tie cable has been re-energized, relay No. 41 which is picked up whenever the centrifugal switch on the generator is closed, prevents the picking up of contactor No. 5, until the speed of the generator has dropped to a value somewhat below normal. This prevents the generator from being thrown on the line again when running at a considerable speed above normal.

The foregoing description covers only one generator; but also applies equally to the other generators. Each controller has a relay No. 42 so that the exciter can be started from any controller; that is, any generator can be started first and its controller starts the exciter.

The steam station is equipped with control buttons so that, if desired, the gate opening, and therefore the load on the hydroelectric generators, can be controlled from that place. The ammeter control is cut out by opening button No. 1.

TESTS

A considerable number of tests were taken to determine what would happen in the automatic equipment in case of incorrect operation of the control buttons, or failure of some part to operate correctly.

First, one of the No. 1 control buttons was opened and the load on the generator controlled by hand. It was found impossible to overload the generator, even with the gate opening to the full amount allowed by its limit switch. Button No. 1 was then closed, putting the ammeter again in control, and at the same time the operator tried to control the load from the gate control buttons. It was found that he could increase or decrease the load as desired; but when the hand control button was released the ammeter returned the load to the proper amount determined by its setting.

Buttons No. 2 and 3 were opened and immediately reclosed. The opening of these buttons tripped out all contactors and started the gate motor closing. The reclosing of the buttons caused the controller to rotate but since the generator was running at normal speed or a little above, relay No. 41 was held up and contactor No. 5 could, therefore, not pick up. This prevented any other contactors from closing. The controller made about one and one-half revolutions before the generator speed dropped so that relay No. 41 could reset. The next time around the controller picked up contactor No. 5, and went through the usual sequence of starting. This proved that if the operator accidentally opened one of these buttons and immediately closed it again, the generator would not be thrown immediately back on the bus, but would go through its proper sequence of operations.

In the steam station there is a voltmeter connected on the hydroelectric side of the oil switch. We next tripped this oil switch and found that the voltage dropped to zero in about two seconds, indicating that in this time the contactors in the hydroelectric plant had all tripped out, thus clearing the generators from the bus. Upon tripping of this oil switch the generators lost their load and tended to run away. Within the two seconds mentioned the speed-limit device trips relay *E* at 64 cycles and all contactors drop out. The automatic throw-over control switch connects the separate source of power to the control bus and the dropping out of contactors No. 5 closes the water wheel gates. We did not read the maximum speed reached, but it was very apparent from watching the machines that the speed rose to not more than 20 to 25 per cent above normal. The action of the gate is quite rapid and the inertia of the machines is such that the gates closed before the wheels had time to accelerate seriously.

With the generator running under load we shut down the exciter set. The armature current of the generator immediately increased and the contact making ammeter started to close the gate. The contactors connecting the generator to the bus dropped out, due to the tripping of the direct-current, low-voltage relay No. 11, which dropped because of the failure of the exciter voltage. The generator, after loss of field, broke from synchronism and tended to run above speed, but the contact-making ammeter had already closed the gate far enough to prevent serious overspeed, and the drop out of the contactors completed the closing.

Since the hydroelectric plant has been put into regular operation, the exciter in the main steam generator station failed one day. The load on the system, of course, was much greater than the hydroelectric plant could carry by itself, so that this plant should shut itself down and clear from the steam plant. As soon as the exciter voltage in the steam station began to drop the current of the hydroelectric generators, of course, rapidly rose and the contact-making ammeters started the closing of the waterwheel gates. Before the voltage dropped too low, the ammeters had almost completed the closing of the gates. All contactors dropped out because of low voltage, disconnecting the generators from the bus. The hydroelectric plant was ready to start again upon return of voltage on the tie cable between the two plants.

One of the accompanying illustrations shows an oscillogram taken of the starting conditions. This oscillogram is marked, showing the different contactors operated. The first part of the film shows the generator connected to the line without field. Next a weak field is applied, pulling the machine into step. The field is then strengthened to normal. There now seems to be a tendency for the current to pulsate, presumably caused by the heavy 20 per cent reactance in series with the machine. The short-circuiting of this reactance quickly steadies the current into a stable condition. The wave shown at the end of the film is something less than normal load, as the contact-making ammeter, the day this film was taken, was set for holding less than full load. The time from the throwing on the line to the short-circuiting of the reactance is about eleven seconds.

Although the generators are low speed and have considerable inertia, only 39 seconds are required from beginning of the opening of the gate until the generator is connected to the bus, with a reactance short-circuited. Within 45 seconds, the generator is carrying full load. In spite of the speed with which these generators are connected to the bus, there is no serious mechanical jar perceptible.

The plant has now been in operation from October 2nd until the present, and so far, no troubles, other than the small ones which always go with any new development, have been experienced, and these have all been of a very minor nature. The changes that have been made, consist essentially of substituting heavier and more reliable relays at one or two points on the control system, and the expectations and hopes of those who have worked on this development have been more than realized. The plant has successfully withstood short circuits, exciter failures in the steam plant, low water and high water, and all of the tests that those in charge of the work could conceive of, wrong operations being brought about artificially. It has also been operated by the regular operating force at the steam plant, and there has been no occasion to keep men, nor have they been kept in the hydroelectric plant to watch its operation.

The automatic development, if it means anything at all, means that it will now be possible to develop a large number of small low-head plants and tie them in on a high-tension system, leaving their operation entirely to the float switch and voltage relays. If there is voltage on all three phases of the high-tension line and water for the turbines, they will start up and go on the line without wrecking themselves or disturbing the operation of the rest of the system. In these days of scarcity of coal and scarcity of labor, together with its high price, the utilization of our water power is of national importance, for, every pound of coal that can be saved by the water of our rivers is just that much more that can be used by our country for the successful prosecution of the War.

With the elimination of the excessive labor cost of operation of a small plant, many water power sites are capable of development as a paying investment.

As this paper goes to press, the station has been in operation about seven months. Some notes as to experience with it might be of interest. During this time, the total flow of the river has been used at all times, and a total of approximately 3,000,000 kw-hr. has been fed into the system. There have, of course, been a few failures to start, as no automatic apparatus can be

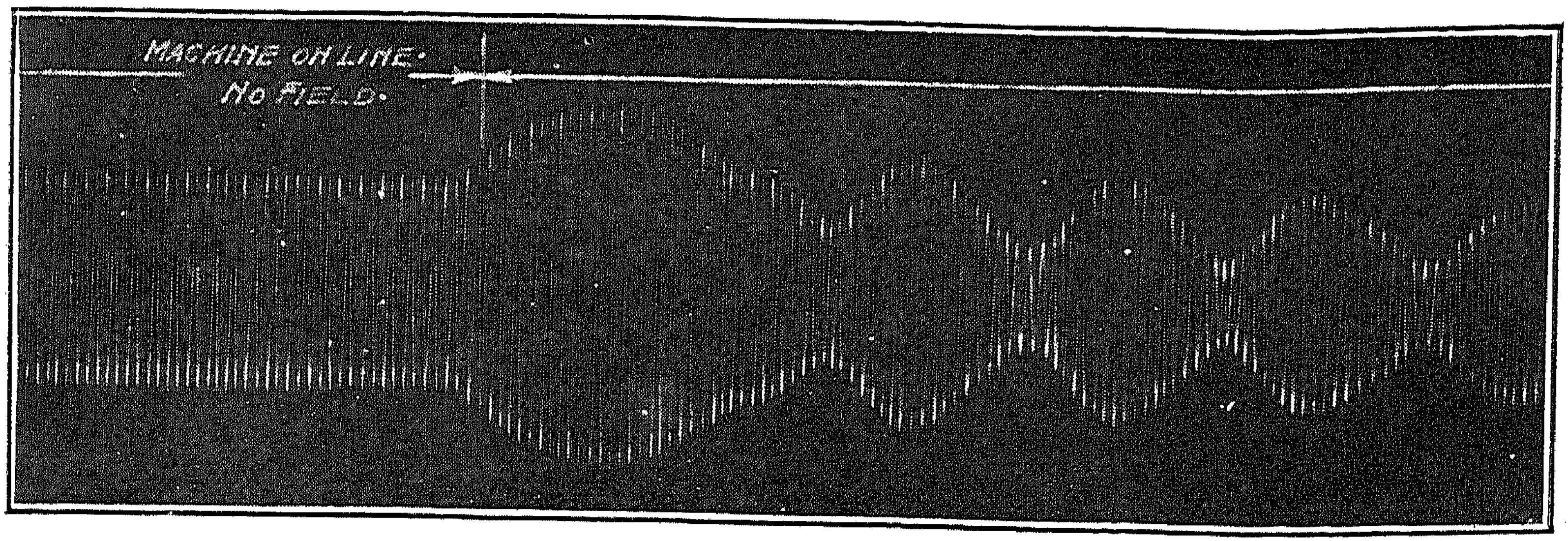


FIG. 14—GENERATOR ON LINE WITHOUT FIELD

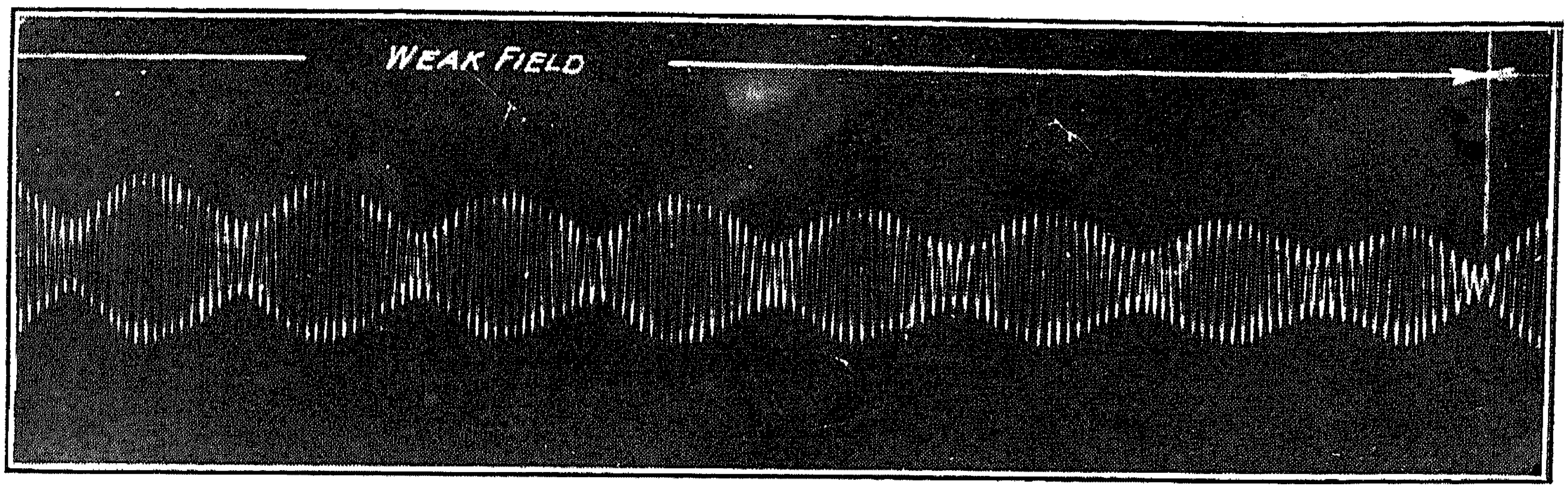


FIG. 15—GENERATOR WITH WEAK FIELD APPLIED

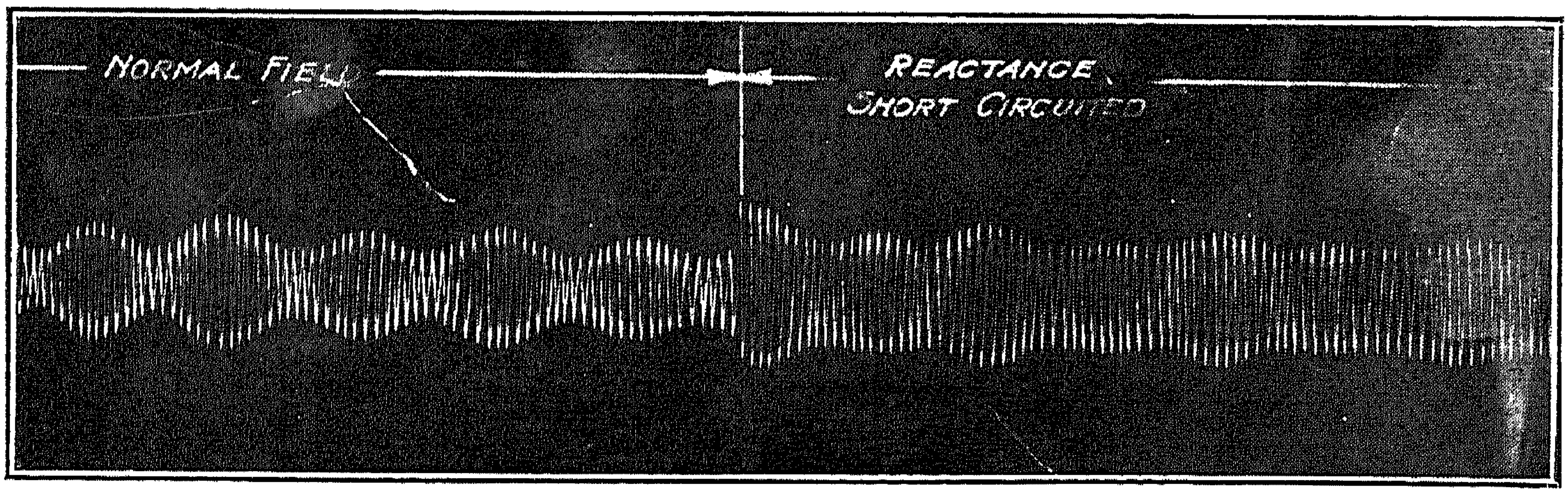


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FIG. 16—GENERATOR WITH NORMAL FIELD

absolutely infallible. The source of such failures has been easily found and corrected. In this connection the equipment is designed to shut down in case of abnormal conditions, so that all the machines will be protected against injury. It is also of interest to note that owing to the pressure of other work and the shortage of help, this generating station ran continually for ten weeks, with no attention to automatic equipment of any kind. Of course, a more frequent inspection and cleaning of the contacts should be made in order to keep the equipment in the best of condition. There have been several cases of exciter trouble at the main steam station, which resulted in a complete shut down. As soon as bus voltage was restored, the automatic equipment put the hydroelectric plant back on the line without attention of any kind, and with no damage or injury to any of the machines. The expense for operators is a large per cent of the total cost of the operation of small hydroelectric plants. Many plants have considerable trouble in keeping their operating forces, particularly in these days of shortage of labor. With the present high price of coal and the shortage of labor, many small water powers might be developed with a profit as auxiliary plants on a fairly large steam generating system.

DISCUSSION ON "THE AUTOMATIC HYDROELECTRIC PLANT." (DRABELLE AND BONNETT), ATLANTIC CITY, N. J., JUNE 28, 1918.

H. R. Summerhayes: How often is it necessary to send an inspector around? What is the regular routine?

J. J. Linebaugh: The success of this station indicates a means has been found of tying into a large network the small waterpower developments which are available throughout the country, being somewhat along the lines of the paper given by Dr. Steinmetz last evening. The small cost of the automatic equipment makes it possible to use a standard synchronous alternator, for the exciter and auxiliaries, at an automatic substation, without going to the induction generator.

Mr. Bump: I notice the gates were all closed by individual motors on the water wheels. I would like to know what would happen if you had a short-circuit on the line between the waterpower and the power station. How would you ever close those gates?

Oliver C. Traver: I would like to register belief in the possibilities of the automatic station in the future, this faith being based on the results as obtained and as recorded in this paper, and also as observed in other cases. One reason for this faith, however, I will give, and that is, that even with a hand-operated station it is necessary for the operator to rely on some form of indicating device or some signal device by means of which he could be given instructions, and every one of these indicating or signalling devices can be duplicated as an automatic device, to do what the operator would be supposed to do if he had seen the indication or if he had received the instructions, and do it in a more orderly and correct manner than the operator could himself.

J. M. Drabelle: In regard to the question raised about the frequency of the inspection. We have one record we are not very proud of, it came about due to the accumulation of a lot of trouble elsewhere on the system, but this plant ran approximately for ten weeks without an inspection of any of the electrical parts. The trash racks are cleaned once every twenty-four hours by one of the day laborers over at the steam station, and that is all the attention that the plant gets. We try, however, to look at the drum controller and contacts about once a week, or once in two weeks, but the plant did successfully stand ten weeks' operation without an inspection of any kind.

L. B. Bonnett: As to Mr. Bump's question, if you get a dead short-circuit on the line, between the two stations, which I think is meant, there would be no way of reenergizing that line, and you would have to send a man to the station to turn the gates down by hand. There is a hand wheel on the mechanism for doing that. It is a reasonably long job, but it can be done. The generators and water wheels in this particular plant are designed to operate at runaway speed for one hour. That is reasonably simple, on a low-head proposition. These wheels run 60 rev. per min. normally.

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SKIN EFFECT IN TUBULAR AND FLAT CONDUCTORS

BY H. B. DWIGHT

ABSTRACT OF PAPER

A method is presented for calculating the skin effect resistance ratio of a tube, which is a form of conductor to be recommended for high-frequency work. A formula is also developed by means of which the asymptote to the curve of the ratio R'/R may be drawn, and thus the magnitude of the skin effect at extremely high frequencies may be obtained.

The values of the ratio R'/R for tubes of various thicknesses, are plotted in a set of curves (Fig. 3) which may be used for the solution of practical problems.

A similar method is described for the calculation of skin effect in a thin strap. Although the calculations cannot be carried out for as high frequencies as the calculation for tube, it indicates a method of coordinating the test results for straps, which have been published. A set of empirical curves for straps is given in Fig. 7, from which approximate values of R'/R for any case may be read.

IN any conductor carrying alternating current, the magnetic field around the axis of the conductor produces variations in the current density. In an isolated conductor, the current tends to flow more densely in the outermost parts, farthest from the axis of the conductor. The effect is more pronounced as the frequency becomes high, or as the section of the conductor becomes large. The result with a round wire or tube when the frequency is high, is that the current is concentrated in the outer skin of the conductor—hence the name "skin effect". In an isolated flat strap, the current crowds mainly toward the edges of the strap, and term "edge effect" has been suggested[1], although the general term "skin effect" is often used for all cases of the phenomenon.

The formula for the magnitude of the skin effect in a round, non-magnetic wire may be obtained by assuming an infinite series for the current density at any point of the section[2]. The formula may also be obtained by forming a differential equation connecting the current and the dimensions of the section[3]. An alternative method has been described[4] by which the effect of the magnetic field is traced step by step. Successive increments of current and voltage drop are calculated, to keep the voltage uniform over the section, as of course it really is. The increments become smaller and smaller, forming convergent series. This method gives the usual result for round wire, and is useful for the special cases of tubes and straps, which are calculated in this article.

---

[1] Skin Effect Resistance Measurements, by A. E. Kennelly and H. A. Affel, Proc. Inst. of Radio Engineers, May, 1916.

[2] Clerk Maxwell, *Electricity and Magnetism*, Vol. II, para. 689.

[3] A. Russell, *Philosophical Mag.*, Vol. 17, 1909, p. 524, and A. E. Kennelly, F. A. Laws and P. H. Pierce, TRANS. A. I. E. E., 1915, p. 1953.

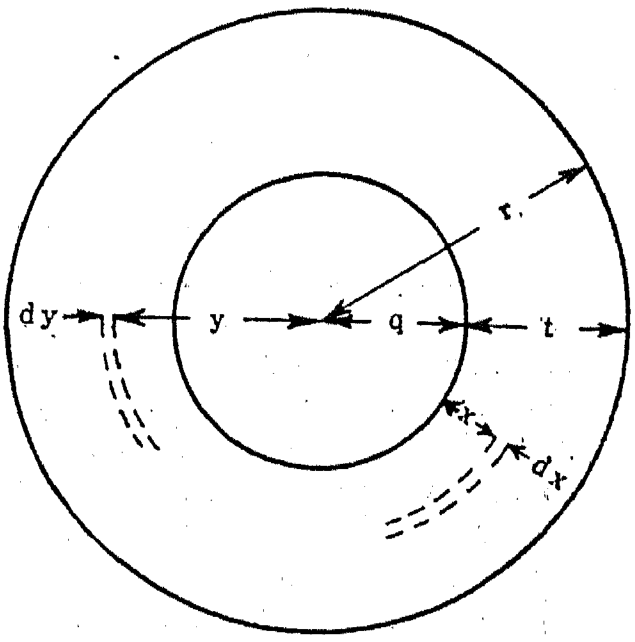

FIG. 1—SECTION OF TUBULAR CONDUCTOR

Assume that a uniform current of density a_0, in absolute electromagnetic units, that is, in absamperes per square centimeter, flows at all parts of the section of the tube indicated in Fig. 1. The current inside the circle dx is

$$I_{(x)} = \pi\, a_0\, (2\, q\, x + x^2) \quad \text{absamperes} \qquad \textbf{(1)}$$

The flux density at dx is

$$\frac{2\, I_{(x)}}{q + x}$$

$$= 2\,\pi\, a_0\, q \left[2\, \frac{x}{q} - \frac{x^2}{q^2} \left(1 - \frac{x}{q} + \frac{x^2}{q^2} - \frac{x^3}{q^3} + \ldots \right) \right]$$

$$= 2\,\pi\, a_0\, q \left(2 \frac{x}{q} - \frac{x^2}{q^2} + \frac{x^3}{q^3} - \ldots \right) \text{ lines per sq. cm.}$$

The flux outside of $d\,x$ and inside the metal of the tube, per centimeter of tube, is

$$\varphi_{(x)} = 2\,\pi\, a_0\, q \int_x^t \left(\frac{2\,x}{q} - \frac{x^2}{q^2} + \frac{x^3}{q^3} - \ldots \right) d\,x \text{ lines}$$

The reactive drop at $d\,x$ due to a_0 is

$$j\,\omega\,\varphi_{(x)} = j\,\omega\, 2\,\pi\, a_0\, t^2$$
$$\left(1 - \frac{1}{3}\,\frac{t}{q} + \frac{1}{4}\,\frac{t^2}{q^2} - \ldots - \frac{x^2}{t^2} + \frac{1}{3}\,\frac{x^3}{t^2\, q} - \frac{1}{4}\,\frac{x^4}{t^2\, q^2} + \ldots \right)$$

---

[4] Transmission Line Formulas, by H. B. Dwight, 1913, Chap. X.

$$= \frac{j\, m^2\, t^2\, \rho\, a_0}{2}\left(1 - \frac{1}{3}\,\frac{t}{q} + \frac{1}{4}\,\frac{t^2}{q^2} \dots - \frac{x^2}{t^2} + \frac{1}{3}\,\frac{x^3}{t^2\, q} - \frac{1}{4}\,\frac{x^4}{t^2\, q^2} + \dots\right) \text{abvolts} \quad (\mathbf{2})$$

putting $$m^2 = \frac{4\,\pi\,\omega}{\rho} \quad (\mathbf{3})$$

where ω is equal to $2\,\pi$ times the frequency in cycles per second, and where ρ is the specific resistance of the metal in absolute units. The specific resistance in ohms is $\rho \times 10^{-9}$

Let a current of density a_1 flow, such that $a_1\,\rho$ will be equal and opposite to the above terms in x. Then

$$a_1 = \frac{1}{2}\, j\, m^2\, t^2\, a_0 \left(\frac{x^2}{t^2} - \frac{1}{3}\,\frac{x^3}{t^2\, q} + \frac{1}{4}\,\frac{x^4}{t^2\, q^2} - \dots\right) \text{absamperes per sq. cm.} \quad (\mathbf{4})$$

By the above process, the total current due to a_1 is found to be

$$\frac{j\, m^2\, t^2\, a_0\, 2\,\pi\, q\, t}{\underline{/3}}\left(1 + \frac{1}{2}\,\frac{t}{q} - \frac{1}{4 \times 5}\,\frac{t^2}{q^2} + \dots\right) \text{absamperes,} \quad (\mathbf{5})$$

and the reactive drop at $d\,x$, due to flux in the metal caused by a_1, is

$$\frac{(j\, m^2\, t^2)^2\, a_0\, \rho}{\underline{/4}}\left(1 - \frac{2}{5}\,\frac{t}{q} + \frac{3}{10}\,\frac{t^2}{q^2} - \dots - \frac{x^4}{t^4} + \frac{2}{5}\,\frac{x^5}{t^4\, q} - \frac{3}{10}\,\frac{x^6}{t^4\, q^2} + \dots\right) \text{abvolts} \quad (6)$$

By assuming a current of density a_2, such that $a_2\,\rho$ will neutralize the terms of x in (**6**), it is found that the total current due to a_2 is

$$\frac{(j\, m^2\, t^2)^2\, a_0\, 2\,\pi\, q\, t}{\underline{/5}}\left(1 + \frac{1}{2}\,\frac{t}{q} - \frac{1}{14}\,\frac{t^2}{q^2} + \dots\right) \text{absamperes} \quad (\mathbf{7})$$

and the reactive drop at $d\,x$, due to flux in the metal caused by a_2 is

$$\frac{(j\, m^2\, t^2)^3\, a_0\, \rho}{\underline{/6}}\left(1 - \frac{3}{7}\,\frac{t}{q} + \frac{9}{28}\,\frac{t^2}{q^2} - \dots - \frac{x^6}{t^6} + \frac{3}{7}\,\frac{x^7}{t^6\, q} - \frac{9}{28}\,\frac{x^8}{t^6\, q^2} + \dots\right) \text{abvolts} \quad (\mathbf{8})$$

By continuing this process, the following results are obtained:

$$I\,Z' = a_0\,\rho\left[1 + b_1\frac{j\,m^2\,t^2}{\underline{|2}} + b_2\,\frac{(j\,m^2\,t^2)^2}{\underline{|4}} + b_3\frac{(j\,m^2\,t^2)^3}{\underline{|6}} + \ldots\right] \text{abvolts} \qquad (9)$$

where I is the total current in the tube, where Z' is the effective impedance due to resistance and inductance caused by flux inside the metal of the tube, and where

$$b_1 = 1 - \frac{1}{3}\,\frac{t}{q} + \frac{1}{4}\,\frac{t^2}{q^2} - \ldots$$

$$= \frac{1}{2} + \frac{q}{t} - \frac{q^2}{t^2}\,logh\left(1 + \frac{t}{q}\right) \qquad (10)$$

$$b_2 = 1 - \frac{2}{5}\,\frac{t}{q} + \frac{3}{10}\,\frac{t^2}{q^2} - \frac{17}{70}\,\frac{t^3}{q^3} + \ldots \qquad (11)$$

$$b_3 = 1 - \frac{3}{7}\,\frac{t}{q} + \frac{9}{28}\,\frac{t^2}{q^2} - \frac{11}{42}\,\frac{t^3}{q^3} + \ldots \qquad (12)$$

$$b_4 = 1 - \frac{4}{9}\,\frac{t}{q} + \frac{1}{3}\,\frac{t^2}{q^2} - \ldots \qquad (13)$$

$$b_5 = 1 - \frac{5}{11}\,\frac{t}{q} + \frac{15}{44}\,\frac{t^2}{q^2} - \ldots \qquad (14)$$

$$b_n = 1 - \frac{n}{2\,n+1}\,\frac{t}{q} + \ldots \qquad (15)$$

If R is the resistance of the tube per unit length, in absolute units,

$$I\,R = \frac{I\,\rho}{\pi\,(2\,q\,t + t^2)}$$

$$= \rho\,a_0\left[1 + c_1\,\frac{j\,m^2\,t^2}{\underline{|3}} + c_2\,\frac{(j\,m^2\,t^2)^2}{\underline{|5}} + c_3\,\frac{(j\,m^2\,t^2)^3}{\underline{|7}} + \ldots\right] \qquad (16)$$

where $c_1 = 1 - \frac{1}{20}\,\frac{t^2}{q^2} + \frac{1}{20}\,\frac{t^3}{q^3} - \frac{11}{280}\,\frac{t^4}{q^4} + \ldots$

$$(17)$$

$$c_2 = 1 - \frac{1}{14}\ \frac{t^2}{q^2} + \frac{1}{14}\ \frac{t^3}{q^3} - \ldots \qquad \textbf{(18)}$$

$$c_3 = 1 - \frac{1}{12}\ \frac{t^2}{q^2} + \ldots \qquad \textbf{(19)}$$

and

$$c_4 = 1 - \frac{1}{11}\ \frac{t^2}{q^2} + \ldots \qquad \textbf{(20)}$$

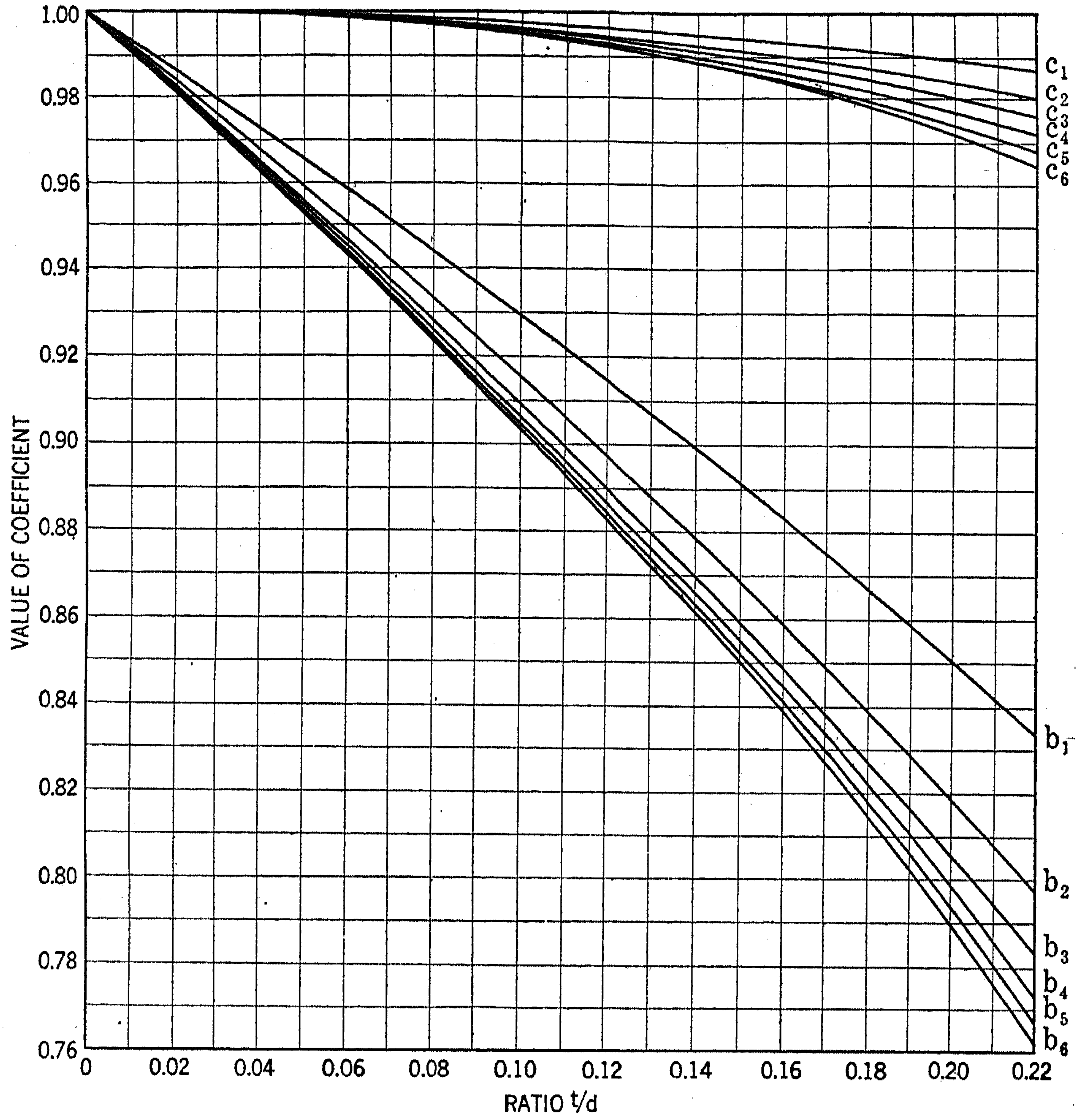

FIG. 2—COEFFICIENTS FOR CALCULATING SKIN EFFECT IN TUBES

The values of the coefficients b_1, b_2, c_1, etc., are shown in Fig.2. These have been used in calculating the curves of $\frac{R'}{R}$ at the lower frequencies shown by the full lines in Fig. 3. The complex quantity $\frac{Z'}{R}$ is found from (**9**) and (**16**). The denominator is rationalized, and then the real part of $\frac{Z'}{R}$ is equal to $\frac{R'}{R}$.

The formula for the skin effect in a tube of infinite radius is of the same form as that for a return circuit of two adjacent straps,[5] but different constants are involved.

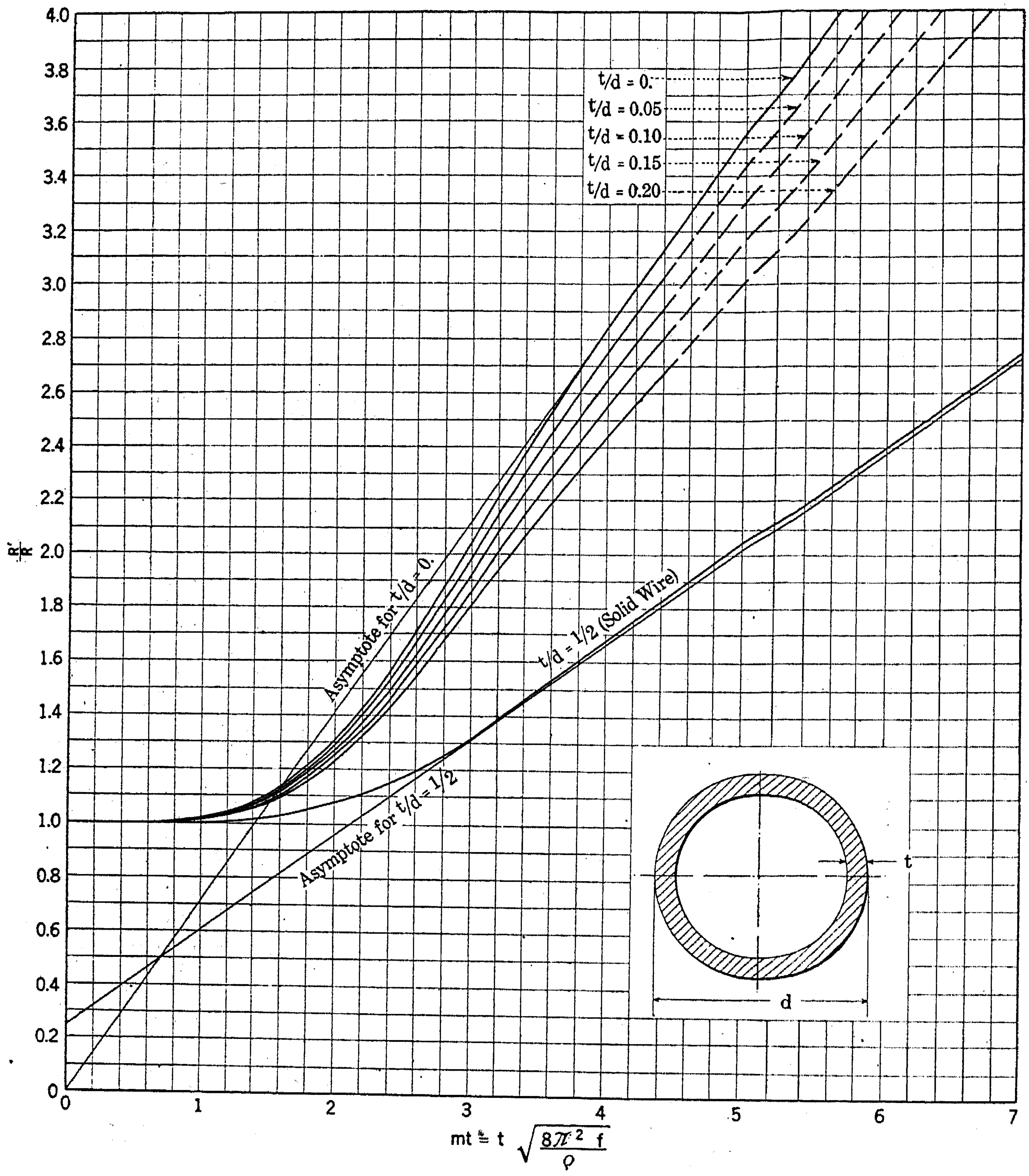

FIG. 3—SKIN EFFECT IN TUBES OF VARIOUS THICKNESSES

The above calculation can be carried to fairly high frequencies and it can be shown by it that the curve of $\frac{R'}{R}$ of a tube becomes approximately a straight line for high frequencies when $\frac{R'}{R}$ is plotted against the square root of the frequency. The angle of slope of the asymptote, that is, the line which the curve approaches as a tangent, when $\frac{R'}{R}$ is large, may be calculated as follows, in a manner similar to that used by Dr. A. Russell in deriving a method for determining the skin effect of a concentric main, that is, a tube containing an insulated return wire.[6]

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[5] Skin Effect of a Return Circuit of Two Adjacent Strap Conductors, by H. B. Dwight, *The Electric Journal,* April, 1916.

The skin effect in an isolated non-magnetic tube, in which the current returns at a considerable distance from the tube, is quite different from the skin effect in a tube containing a return wire, and so a new calculation is required, using different constants.

Let i be the current density at radius y, Fig. 1. The drop per cm. of the tube at radius y is

$$e = \rho\, i + \frac{d\,\varphi}{d\,\tau} \tag{21}$$

where ρ is the specific resistance of the metal in the tube, where τ is time, and where

$$\varphi = \int_y^r \frac{2\,I_{(y)}}{y}\,d\,y = -\int_r^y \frac{2\,I_{(y)}}{y}\,d\,y \tag{22}$$

$$I_{(y)} = \int_q^y 2\,\pi\,i\,y\,d\,y \tag{23}$$

Therefore, $$\frac{d\,\varphi}{dy} = -\frac{2\,I_{(y)}}{y} = -\frac{2}{y}\int_q^y 2\,\pi\,i\,y\,dy \tag{24}$$

Differentiate (24) with respect to τ

$$\frac{d}{d\,\tau}\,\frac{d\,\varphi}{dy} = \frac{d}{dy}\,\frac{d\,\varphi}{d\,\tau} = -\frac{4\,\pi}{y}\int_q^y y\,\frac{di}{d\,\tau}\,dy$$

Now e is constant over the section. Therefore

$$\frac{d\,e}{dy} = 0 = \rho\,\frac{di}{dy} + \frac{d}{dy}\,\frac{d\,\varphi}{d\,\tau} \qquad \text{from (21)}$$

$$= \rho\,\frac{d\,i}{dy} - \frac{4\,\pi}{y}\int_q^y y\,\frac{di}{d\,\tau}\,dy \tag{25}$$

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[6] *Philosophical Mag.*, Vol. 17, 1909, p. 524.

When $y = q$, $\quad 0 = \rho \dfrac{di}{dy} - 0$

Therefore, when $y = q$, $\quad \dfrac{di}{dy} = 0 \qquad (26)$

From (25), $\quad y \dfrac{di}{dy} = \dfrac{4\pi}{\rho} \displaystyle\int_q^y y \dfrac{di}{d\tau} dy \qquad (27)$

Differentiate (27) with respect to y. Then

$$y \frac{d^2 i}{dy^2} + \frac{di}{dy} = \frac{4\pi}{\rho} y \frac{di}{d\tau}$$

$$\text{Put} \quad m^2 = \frac{4\pi\omega}{\rho} = \frac{8\pi^2 f}{\rho} \qquad (28)$$

where f = frequency in cycles per second.

Then $\quad \dfrac{d^2 i}{dy^2} + \dfrac{1}{y} \dfrac{di}{dy} = \dfrac{m^2}{\omega} \dfrac{di}{d\tau}$

and therefore $\dfrac{d^2 i}{dy^2} + \dfrac{1}{y} \dfrac{di}{dy} - j m^2 i = 0 \qquad (29)$

when the current has a sine-wave shape.

The solution of this differential equation may be written

$$i = (A + jB) J_0 (my\sqrt{-j}) + (C + jD) K_0 (my\sqrt{-j}) \qquad (30)$$

where $J_0 (my\sqrt{-j})$ is a Bessel function of the first kind and of order zero, where $K_0 (my\sqrt{-j})$ is a Bessel function of the second kind and of order zero, and where A, B, C, and D are constants to be determined. Thus

$$i = (A + jB)(\mathit{ber}\, my + j\, \mathit{bei}\, my) \\ + (C + jD)(\mathit{ker}\, my + j\, \mathit{kei}\, my) \qquad (31)$$

that is, $i = A\, \mathit{ber}\, my - B\, \mathit{bei}\, my + C\, \mathit{ker}\, my - D\, \mathit{kei}\, my$
$+ j(A\, \mathit{bei}\, my + B\, \mathit{ber}\, my + C\, \mathit{kei}\, my + D\, \mathit{ker}\, my)$[7] $\quad (32)$

From (26), $\quad \dfrac{di}{dy} = 0$ when $y = q$.

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[7] See equations (29), (44) and (77) of the paper by Kennelly, Laws and Pierce, and the paper by A. Russell, referred to in the second paragraph of this paper, reference 3.

Therefore,

$$A \text{ ber}' mq - B \text{ bei}' mq + C \text{ ker}' mq - D \text{ kei}' mq = 0 \qquad \textbf{(33)}$$

and $$A \text{ bei}' mq + B \text{ ber}' mq + C \text{ kei}' mq + D \text{ ker}' mq = 0 \qquad \textbf{(34)}$$

Let the total current in the tube be the quantity to which all phase relations are to be referred, and let it be equal to $I + j\,0$. Then, since

$$\int y \text{ ber } my\, dy = \frac{y}{m} \text{ bei}' my, \quad \int y \text{ bei } my\, dy$$

$$= -\frac{y}{m} \text{ ber}' my, \quad \int y \text{ ker } my\, dy = \frac{y}{m} \text{ kei}' my$$

and $$\int y \text{ kei } my\, dy = -\frac{y}{m} \text{ ker}' my,$$

we have

$$I = \int_q^r 2\pi y\, i\, dy$$

$$= \frac{2\pi}{m}\Big[A\, y \text{ bei}' my + B\, y \text{ ber}' my + C\, y \text{ kei}' my + D\, y \text{ ker}' my\Big]_q^r$$

$$+ j\frac{2\pi}{m}\Big[-A\, y \text{ ber}' my + B\, y \text{ bei}' my - C\, y \text{ ker}' my + D\, y \text{ kei}' my\Big]_q^r$$

The quantities in the brackets are equal to zero when $y = q$, by (**33**) and (**34**). Therefore,

$$I + j\,0 = \frac{2\pi r}{m}\Big[A \text{ bei}' mr + B \text{ ber}' mr + C \text{ kei}' mr + D \text{ ker}' mr\Big]$$

$$+ j\frac{2\pi r}{m}\Big[-A \text{ ber}' mr + B \text{ bei}' mr - C \text{ ker}' mr + D \text{ kei}' mr\Big]$$

Therefore,
$$-A\,\mathrm{ber}'\,mr + B\,\mathrm{bei}'\,mr - C\,\mathrm{ker}'\,mr + D\,\mathrm{kei}'\,mr = 0 \qquad (35)$$

and
$$A\,\mathrm{bei}'\,mr + B\,\mathrm{ber}'\,mr + C\,\mathrm{kei}'\,mr + D\,\mathrm{ker}'\,mr = \frac{I\,m}{2\,\pi\,r} \qquad (36)$$

Let Z' be the effective impedance per centimeter of the tube at a certain frequency due to its effective resistance R' and its inductance caused by flux inside the metal. Since the flux does not cause any drop where $y = r$, we have, from (**21**),

$$e = I\,Z' = \rho\, i_{(r)}$$

Therefore, by (**32**),

$$I\,Z' = \rho\left[A\,\mathrm{ber}\,mr - B\,\mathrm{bei}\,mr + C\,\mathrm{ker}\,mr - D\,\mathrm{kei}\,mr\right] + j\,\rho\left[A\,\mathrm{bei}\,mr + B\,\mathrm{ber}\,mr + C\,\mathrm{kei}\,mr + D\,\mathrm{ker}\,mr\right] \qquad (37)$$

The drop in phase with the current I is

$$I\,R' = \rho\left[A\,\mathrm{ber}\,mr - B\,\mathrm{bei}\,mr + C\,\mathrm{ker}\,mr - D\,\mathrm{kei}\,mr\right] \qquad (38)$$

Now, by (**36**),
$$I\,R = \frac{I\,\rho}{\pi\,(r^2 - q^2)} = \frac{2\,\pi\,r\,\rho}{\pi\,(r^2 - q^2)\,m}\left[A\,\mathrm{bei}'\,mr + B\,\mathrm{ber}'\,mr + C\,\mathrm{kei}'\,mr + D\,\mathrm{ker}'\,mr\right] \qquad (39)$$

Therefore,
$$\frac{R'}{R} = \frac{m\,(r^2 - q^2)}{2\,r}\;\frac{(A\,\mathrm{ber}\,mr - B\,\mathrm{bei}\,mr + C\,\mathrm{ker}\,mr - D\,\mathrm{kei}\,mr)}{(A\,\mathrm{bei}'\,mr + B\,\mathrm{ber}'\,mr + C\,\mathrm{kei}'\,mr + D\,\mathrm{ker}'\,mr)} \qquad (40)$$

The four equations (**33**), (**34**), (**35**) and (**36**) are sufficient to determine the four constants A, B, C, and D.

A similar equation for the skin effect inductance ratio could easily be written down, but would not be of much practical interest in the case of a tube since the flux inside the metal is extremely small compared with the flux outside the tube.

Equation (**40**) could be used to calculate the skin effect resistance ratio of a tube at any frequency if very complete tables of the eight functions, *ber*, etc., were available.

The result is the same as in Dr. Russell's equation (**89**) for a concentric main, except that the inner and outer radii of the tube are interchanged, due to the different position of zero flux. Thus by interchanging the inner and outer radii in Dr. Russell's low-frequency formula (**101**), which is derived from (**89**), a low-frequency formula for an isolated tube would be obtained. In this article, equation (**40**) will be used only for obtaining the slope of the asymptote to the curve of $\frac{R'}{R}$, thus giving approximate values of $\frac{R'}{R}$ for high frequencies by a very simple formula.

For large values of *mr*, approximate formulas for the eight functions *ber*, etc., are given in Dr. Russell's paper referred to above, of which the following is an example:

$$ber\, mr = \frac{\epsilon^{\frac{mr}{\sqrt{2}}}}{\sqrt{2\,\pi\, mr}} \cos\left(\frac{mr}{\sqrt{2}} - \frac{\pi}{8}\right) \tag{41}$$

The values given by these approximate formulas are more accurate as *mr* is larger.

If these formulas are substituted in the well known equation for a solid wire,

$$\frac{R'}{R} = \frac{mr}{2}\,\frac{(ber\, mr\; bei'\, mr - bei\, mr\; ber'\, mr)}{(ber'^2\, mr + bei'^2\, mr)} \tag{42}$$

we obtain

$$\frac{R'}{R} = \frac{mr}{2\sqrt{2}} \tag{43}$$

which gives the slope of the asymptote of the curve of $\frac{R'}{R}$.

In order to obtain the equation of the asymptote (see Fig. 3), namely,

$$\frac{R'}{R} = \frac{mr}{2\sqrt{2}} + \frac{1}{4} \tag{44}$$

it is necessary to use values of the form

$$ber\, mr = \frac{\epsilon^{\frac{mr}{\sqrt{2}} + \frac{1}{8\, mr\, \sqrt{2}}}}{\sqrt{2\, \pi\, mr}} \cos\left(\frac{mr}{\sqrt{2}} - \frac{\pi}{8} - \frac{1}{8\, mr\, \sqrt{2}}\right) \qquad (45)$$

given in Dr. Russell's paper. If further terms of the series in $\frac{1}{mr}$ are added, an asymptotic formula for $\frac{R'}{R}$ is obtained, which is very accurate except at low frequencies.

It is thus evident that the substitution of the formulas similar to (**41**) in equation (**40**) will give only the slope of the asymptote of the curve of $\frac{R'}{R}$. The result of the substitution, after a certain amount of trigonometrical work, is

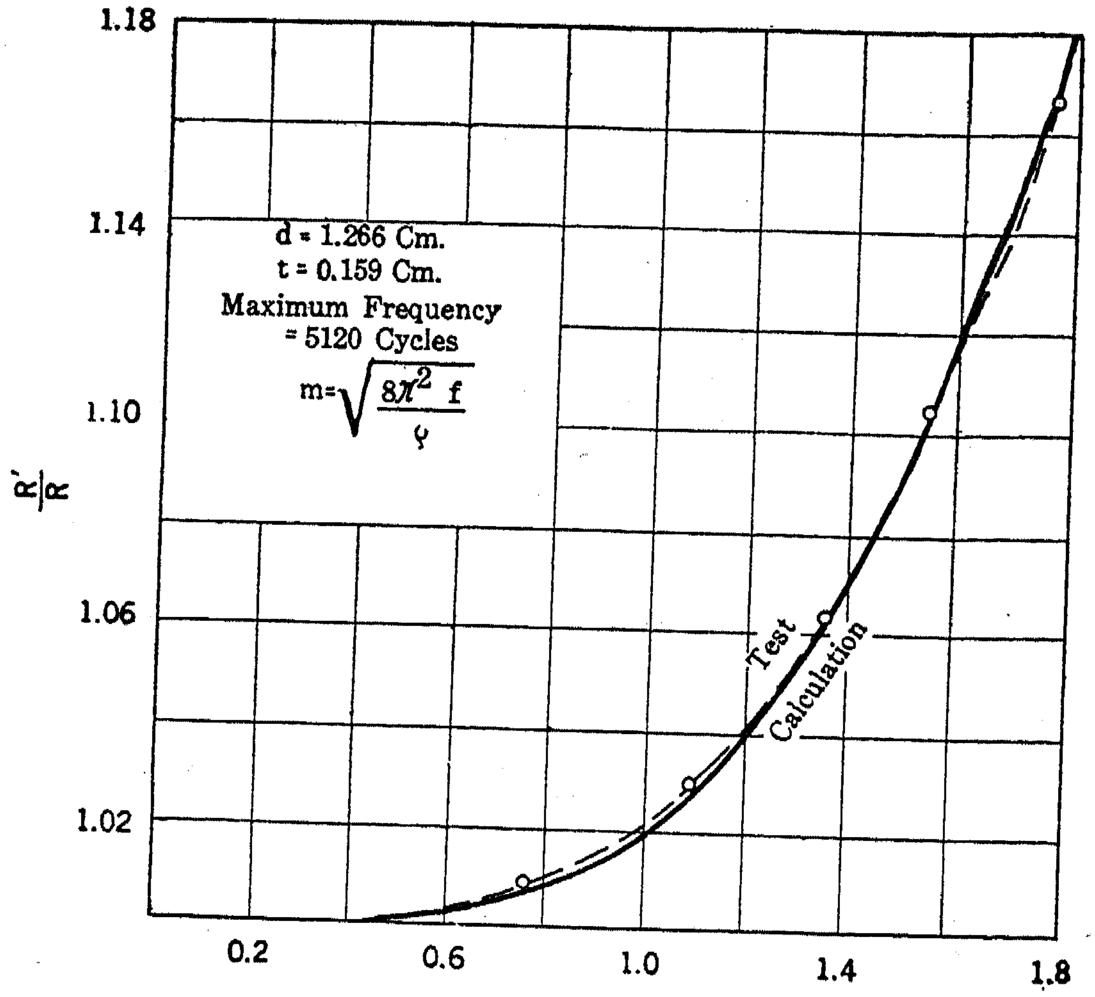

FIG. 4—COMPARISON OF CALCULATION WITH TEST

$$\frac{R'}{R} = \frac{mt\,(q + r)}{2\, r\, \sqrt{2}} \; \frac{\{\sinh\,(mt\, \sqrt{2}) + \sin\,(mt\, \sqrt{2})\}}{\{\cosh\,(mt\, \sqrt{2}) - \cos\,(mt\, \sqrt{2})\}} \qquad (46)$$

This reduces to

$$\frac{R'}{R} = \frac{mt\,(q + r)}{2\, r\, \sqrt{2}} \qquad (47)$$

which gives the required slope of the asymptote. If the inner radius $q = 0$, we obtain equation (**43**) for a solid wire. If q is practically equal to r, (**47**) becomes

$$\frac{R'}{R} = \frac{mt}{\sqrt{2}} \tag{48}$$

This happens to be the actual equation of the asymptote, which in this case passes through the origin. (See equations (**100**) and (**110**) of the paper by Kennelly, Laws and Pierce, reference 3.)

The useful fact is therefore deduced that the asymptote of the curve for $\frac{R'}{R}$ of a thin tube passes close to the origin. It is therefore possible to plot $\frac{R'}{R}$ closely for tubes at high frequencies, as is done in the dotted lines in Fig. 3. The values found from the high-frequency, Bessel function calculation are in agreement with those found by the low-frequency calculation.

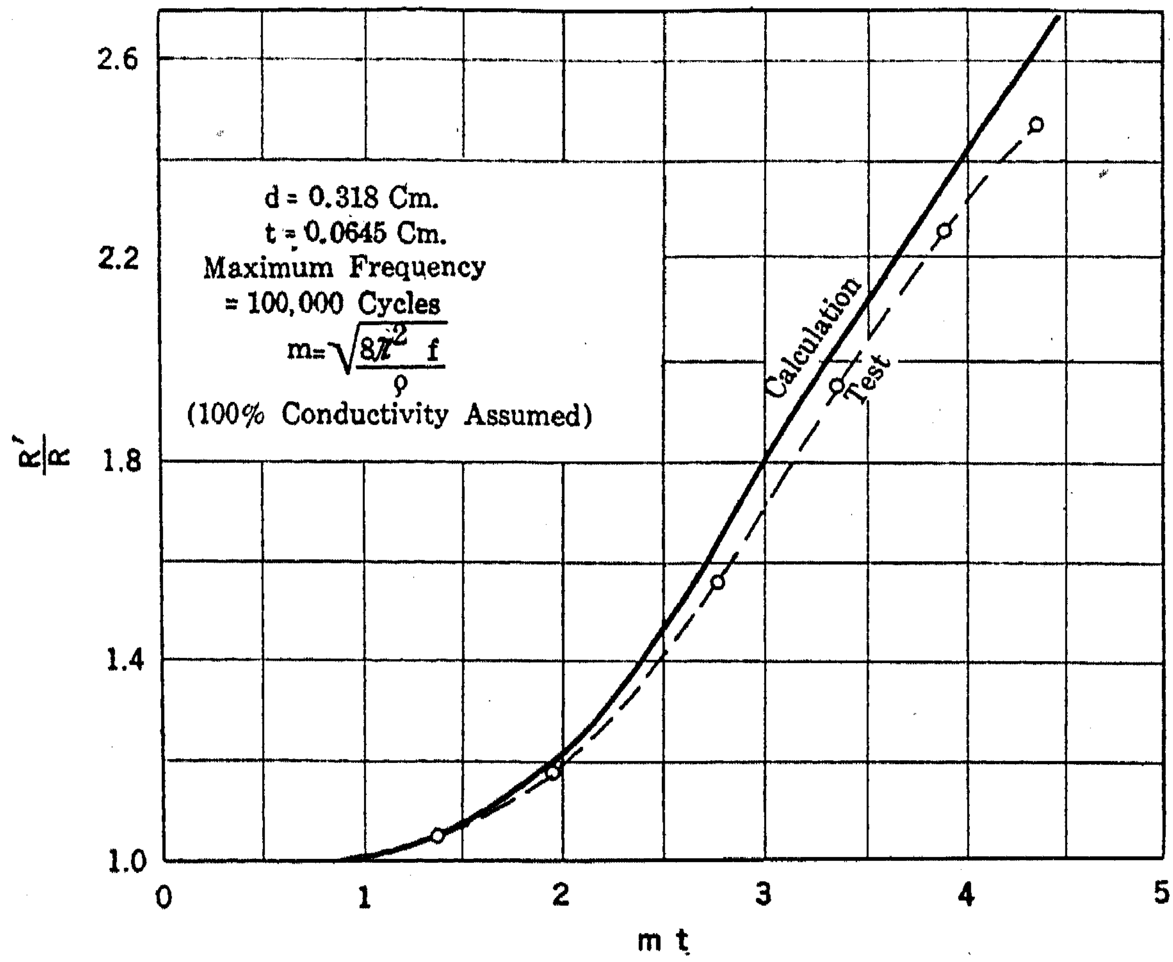

FIG. 5—COMPARISON OF CALCULATION WITH TEST

As examples, the results for two tubes tested by Dr. Kennelly, described in the papers mentioned above, are shown in Figs. 4 and 5. From the data given, the d-c. conductivity of the hard-drawn copper tube in Fig. 4 was about 50 per cent, and the curves have been drawn accordingly. The test curve of the soft-drawn copper tube of Fig. 5 was calculated on the basis of 100 per cent conductivity, in the absence of data regarding the conductivity. If the conductivity were lower, the test curve would be drawn farther to the left.

Skin Effect in Strap. The process of calculating corrective currents to keep the voltage drop uniform over the section of the conductor may be applied to a thin isolated strap as well as to a tube. In this calculation, the strap is assumed to be so thin that the only appreciable action is the crowding of the current to the edges. The test curves shown in Fig. 7 indicate that this assumption is allowable for very thin straps.

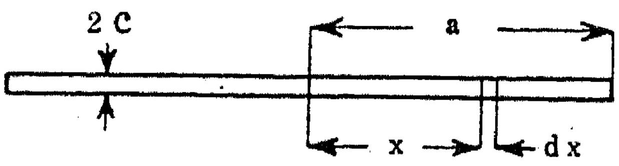

FIG. 6—SECTION OF STRAP CONDUCTOR

Since the magnetic flux around the strap does not lie in circles, the voltage drop at any part of the section is calculated from the effect of the various elements of current in the section. Thus, omitting terms involving the distance to the return conductor, which can be shown not to affect the skin effect resistance ratio when the distance is large, it is found that the reactive drop at dx, Fig. 6, due to a current of uniform density b_0, is

$$j\,\omega\,4\,ac\,b_0\left[2-\left(1+\frac{x}{a}\right)\log h\left(1+\frac{x}{a}\right)-\left(1-\frac{x}{a}\right)\log h\left(1-\frac{x}{a}\right)\right]$$

$$=\frac{1}{2}\,j\,p^2\,\rho\,b_0\left[1-\frac{1}{1.2}\,\frac{x^2}{a^2}-\frac{1}{3.4}\,\frac{x^4}{a^4}-\frac{1}{5.6}\,\frac{x^6}{a^6}-\cdots\right] \qquad (49)$$

where $$p^2=\frac{4\,\omega}{\rho}\times 4\,ac=\frac{4\,\omega}{R}$$

and where R is the resistance of the conductor per centimeter in absolute units. The resistance in ohms is $R \times 10^{-9}$.

Also, the reactive drop at dx due to current $b_1\,\dfrac{x^2}{a^2}$

$$=\frac{1}{6}\,j\,p^2\,\rho\,b_1\left[\frac{1}{3}+\frac{x^2}{a^2}\right.$$

$$-\frac{1}{2}\left(1+\frac{x^3}{a^3}\right)\log h\left(1+\frac{x}{a}\right)$$

$$- \frac{1}{2} \left(1 - \frac{x^3}{a^3}\right) \log h \left(1 - \frac{x}{a}\right)\Bigg]$$

$$= \frac{1}{2} j p^2 \rho b_1 \left[\frac{1}{3.3} + \frac{1}{1.2} \frac{x^2}{a^2} - \frac{1}{1.4} \frac{x^4}{a^4} - \frac{1}{3.6} \frac{x^6}{a^6} - \frac{1}{5.8} \frac{x^8}{a^8} - \ldots\right] \quad (50)$$

The reactive drop due to current $b_n \frac{x^{2n}}{a^{2n}}$

$$= \frac{1}{2} j p^2 \rho b_n \left[\frac{1}{(2n+1)^2} + \frac{1}{(2n-1)2} \frac{x^2}{a^2} + \frac{1}{(2n-3)4} \frac{x^4}{a^4} + \ldots \right.$$

$$\left. \ldots + \frac{1}{1(2n)} \frac{x^{2n}}{a^{2n}} - \frac{1}{1(2n+2)} \frac{x^{2n+2}}{a^{2n+2}} - \frac{1}{3(2n+4)} \frac{x^{2n+4}}{a^{2n+4}} - \ldots\right] \quad (51)$$

The total current in the strap due to b_0 is $4 ac b_0$. The total current due to $b_1 \frac{x^2}{a^2}$ is $\frac{1}{3} b_1 4 ac$ and the total current due to

$$b_n \frac{x^{2n}}{a^{2n}} \text{ is } \frac{1}{2n+1} b_n 4 ac \quad (52)$$

Assume that a uniform current i_0, flows in the strap. The reactive drop at $d x$ is given by (**49**). Next, assume a current whose resistance drop is equal and opposite to the terms in x of (**49**). The total current in the strap can be found by applying (**52**). By repeating the above process, and carrying the various series out to five terms or more in each case, the following results were obtained. Drop in the strap due to alternating current

$$= I Z' = i_0 r [1 + j \tfrac{1}{2} p^2 - 0.0149994 p^4 + j 0.003122 p^6 + 0.000532 p^8 - j 0.0000779 p^{10} - 0.0000096 p^{12} + j 0.00000094 p^{14} + 0.00000005 p^{16}] \quad (53)$$

Drop in the strap due to a direct current of the same amperage

$$= I\,R = i_0\,r\,[1 + j\,0.0965736\,p^2 + 0.0152178\,p^4 - j\,0.00218\,p^6 - 0.000259\,p^8 + j\,0.0000237\,p^{10} + 0.00000100\,p^{12} + j\,0.00000022\,p^{14} + 0.00000008\,p^{16}] \tag{54}$$

Some of the coefficients of the above series can be expressed exactly by algebraic functions. Thus, in (**53**) the term in p^4 is

$$-\left(\frac{1}{4}\log h\,2 - \frac{5}{16} + \frac{\pi^2}{64}\right)p^4 = -\,0.0149994\,p^4$$

In (**54**) the term in p^2 is

$$j\left(\frac{1}{2}\log h\,2 - \frac{1}{4}\right)p^2 = j\,0.0965736\,p^2$$

and the term in p^4 is

$$\left[\frac{1}{8}\log h\,2 - \frac{1}{4}\left(\log h\,2\right)^2 + \frac{5\,\pi^2}{192} - \frac{5}{24}\right]p^4 = 0.0152178\,p^4$$

The fraction $\frac{Z'}{R}$ can be calculated from (**53**) and (**54**) for various values of p. Then, after rationalizing the denominator, the real part is equal to $\frac{R'}{R}$, the skin effect resistance ratio.

The values of $\frac{R'}{R}$ for various values of p are plotted in Fig. 7. It is seen from this figure that the curve of $\frac{R'}{R}$ of a strap, when plotted on the square root of the frequency, has the same general shape as the curves for wire, tube, and a return circuit of two adjacent straps, and it seems to approach a straight line as an asymptote in the same way that they do.

The calculated curve has been carried only as far as $p = 2$. The other lines in Fig. 7 are test curves published in the papers by Kennelly and Affel, and by Kennelly, Laws and Pierce, referred to in the first part of this article. The test curves, especially at frequencies up to 70,000 cycles, show approximately straight lines, when plotted on the square root of the frequency. The curve, $\frac{a}{c} = 240$, Fig. 7, has the lowest position of any of the published curves when they are plotted according to Fig. 7. A conductivity of 100 per cent was assumed for this copper strap. If the conductivity were lower, the curve would be slightly higher. The section of this strap was 0.016 by 3.81 cm.

The curve for a round wire of the same sectional area as the strap lies below the curve for strap up to p equal to about 2.1, but at higher frequencies than that, the wire has a greater ratio than the strap. (See Fig. 8). It is to be expected that the curves for thick straps would lie intermediate between the curve for thin strap and that for wire. Such curves are represented by test curves $\frac{a}{c} = 16$ (0.158 × 2.52 cm.) and $\frac{a}{c} = 8$ (0.1575 × 1.26 cm.)

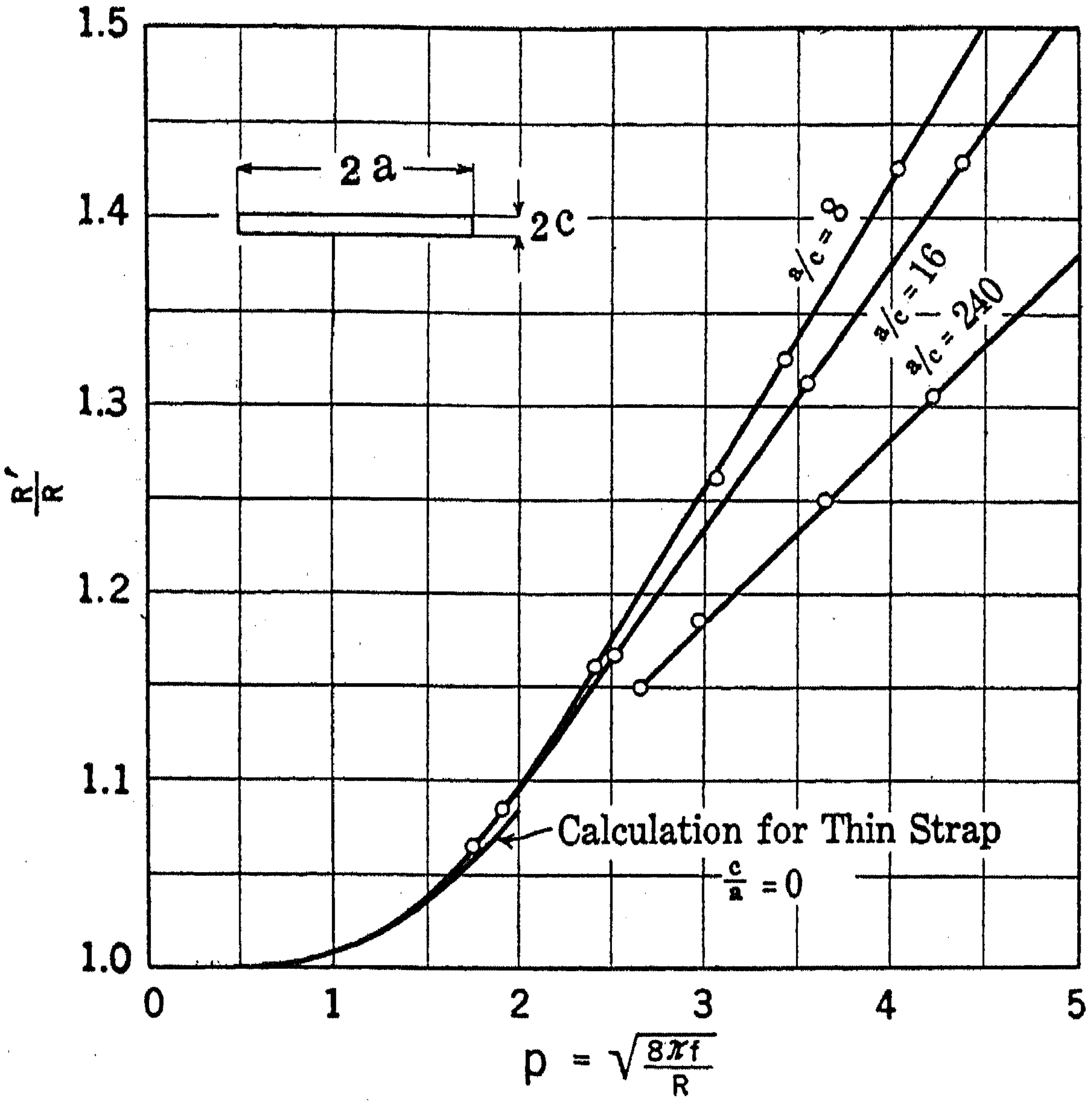

FIG. 7—SKIN EFFECT IN STRAP CONDUCTORS

The curves of Fig. 7 may be considered as an extension in further detail of the article by the writer in the *Electrical World* of March 11, 1916, page 593, giving an empirical curve for skin effect in straps. The calculated curve now given tends to confirm the theory of the former article.

It would appear from Fig. 7 that rolling a strap thinner and wider, but keeping the same sectional area and conductivity always reduces the skin effect resistance ratio for values of p greater than 2:1. However, it seems probable that the improvement is inappreciable after $\frac{a}{c}$ is more than about 50, and that the strap then approximates to an infinitely thin strap.

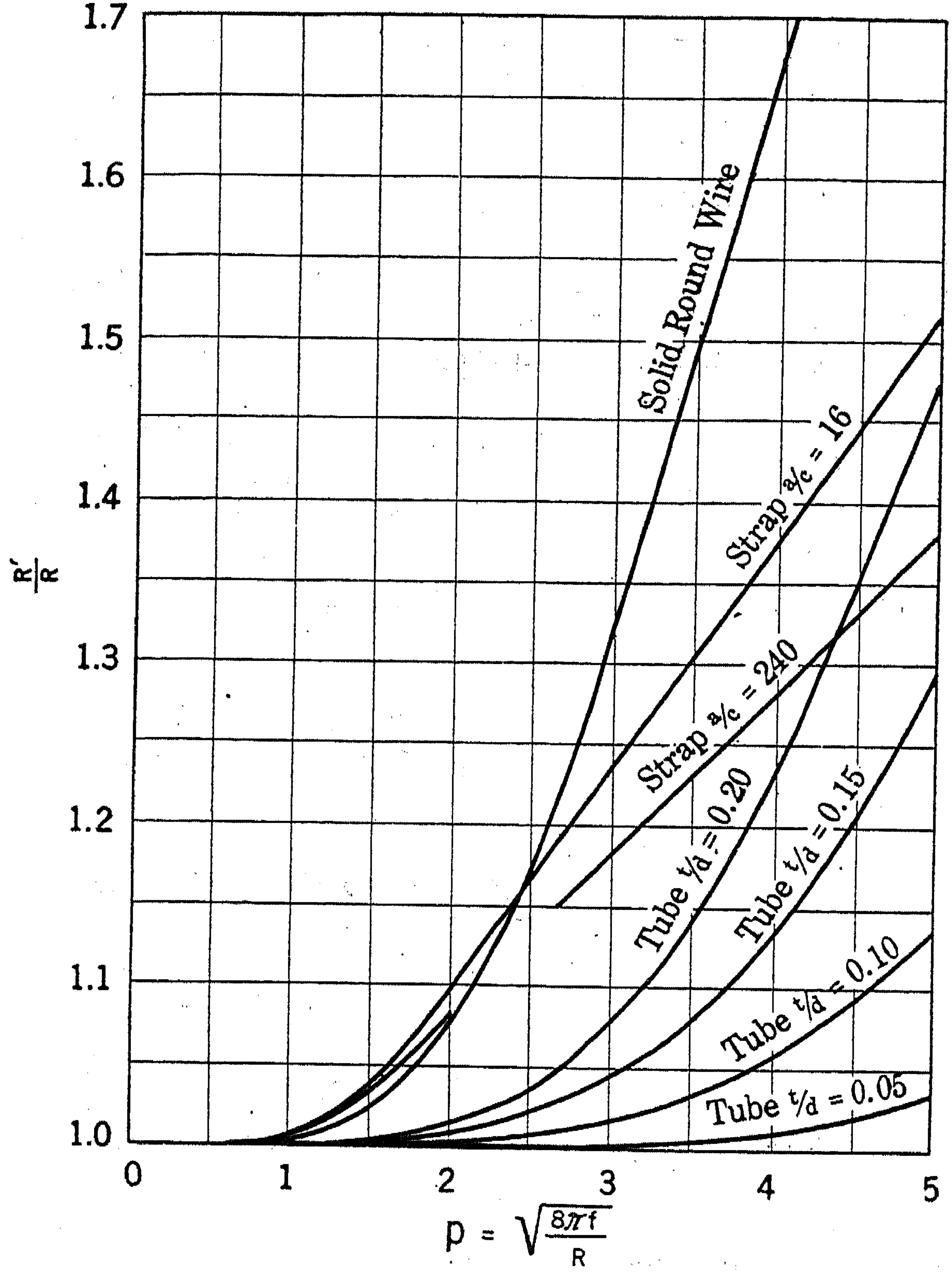

Fig. 8—Skin Effect of Various Conductors of the Same Sectional Area

Thus a strap a foot wide would have the same skin effect resistance ratio as a thin strap one inch wide, if $\frac{a}{c}$ is in both cases greater than 50, and if both straps have the same resistance per 1000 feet, due to the wide strap being thinner or being made of material of lower conductivity.

By dividing series (**53**) by (**54**) a series is obtained which is convenient for calculations when p is comparatively small. This is

$$\frac{R'}{R} = 1 + 0.0087432\, p^4 - 0.000384\, p^8 + 0.0000189\, p^{12} - \ldots \qquad (55)$$

This series is not so convergent as (**53**) and (**54**). It should not be used unless p is small enough to make the last term quite small.

It should be remembered that the discussion on skin effect in strap in this article refers only to a strap which is not close to the return conductor, for in that case the proximity effect changes the value of $\frac{R'}{R}$, generally increasing it when the straps are in an edgewise position, and decreasing it when they are in parallel planes.

A copper strap $\frac{1}{8}$ by 5 inches, carrying current at 60 cycles, may be taken as an example of the calculated formula of skin effect in a thin strap. By changing the dimensions to centimeters, and taking the specific resistance of copper in absolute units to be 1724 at 20 deg. cent., we obtain

$$p^2 = 8\,\pi \times 60 \times \frac{1}{8} \times 5 \times \frac{6.45}{1724} = 3.53$$

Therefore $\quad p = 1.88$ and $\frac{R'}{R} = 1.071$, from Fig. 7.

For a strap $\frac{1}{16} \times 10$ inches,

$$p^2 = 8\,\pi \times 60 \times \frac{1}{16} \times 10 \times \frac{6.45}{1724} = 3.53$$

Therefore the two straps which have the same resistance per foot, have the same value of $\frac{R'}{R}$. Both straps can be considered to be practically the same as an infinitely thin strap, since the thicker one has a width 40 times the thickness.

In order to show graphically what types of conductors are most advantageous for high-frequency work, or for heavy currents in electric furnace circuits, the curves of Fig. 8 have been drawn. These show the skin effect to be expected from using a given weight of copper in the form of wire, strap, or tubes of various proportionate thicknesses. In the case of a tube

$$\frac{8\,\pi\,f\,A}{\rho} = m^2\,t^2\left(\frac{r^2 - q^2}{t^2}\right) \tag{56}$$

where A is the sectional area, and this equation may be used to obtain the curves of Fig. 8 from those of Fig. 3. From the equation of the asymptote to the curve for a tube, it may be found that a tube in which $\frac{t}{d}$ is less than about 0.025 will always have less skin effect than a flat strap of the same sectional area and conductivity. A tube whose proportionate thickness is greater than the above amount may have a greater skin effect resistance ratio than a strap of the same sectional area at very high frequencies.

The curves of Fig. 8 are in agreement with the conclusion expressed in the paper by Kennelly and Affel, that a thin tube is the most economical form of conductor for high-frequency currents, when the return conductor is a considerable distance away.

As a general principle, it may be stated that a conductor, or a combination of conductors, of a certain proportionate shape and a certain value of $\frac{f}{R}$, will have a definite value of $\frac{R'}{R}$. This is true of tubes and round wires, as is shown by Fig. 3, where the skin effect of a tube of a certain ratio of thickness to diameter, but of any size, is given by a single curve. That the principle is true also of any shaped conductor, or any combination of conductors, may be indicated as follows:

Assume that an irregularly shaped conductor carries current at such a frequency, f, that $\frac{R'}{R} = (1 + a)$. The current will be crowded to certain parts of the section of the conductor, and the exact extent of this crowding may be indicated by plotting on the section, the lines for 95 per cent, 100 per cent and 105 per cent of average current density. In the case considered, the increase in resistance by the factor $(1 + a)$ is caused by a certain location of the current density lines.

Suppose that the conductor heats up so that its resistance increases 25 per cent, and suppose that the frequency also increases 25 per cent. The unbalance of current is equivalent to a circulating current which goes with the main current at parts of the section, and returns at other parts of the section. This circulating current is driven by a voltage proportional to the frequency, and its voltage drop is due to resistance and is accordingly proportional to the resistance. Since both the frequency and the resistance have increased 25 per cent, the circulating current will be unchanged. Therefore, the position of the current density lines and the value of $\frac{R'}{R}$ will be unchanged.

Now, instead of the heat in the conductor raising its resistance, let a conductor of the same shape but 80 per cent as large be substituted, and let the frequency still be 25 per cent higher than at first. The resistance of the conductor is 25 per cent higher than at first. Thus the path for circulating current has 25 per cent higher resistance, and the frequency is 25 per cent higher, so that the circulating current is unchanged. Therefore, the relative position of the current density lines will be the same as before, and the value of $\frac{R'}{R}$ will be the same quantity, $(1 + a)$.

Thus in any conductor or combination of conductors or in any return circuit, of a certain proportionate shape, a given value of $\frac{f}{R}$ corresponds to a certain value of $\frac{R'}{R}$. A tested curve of $\frac{R'}{R}$ plotted on $\frac{f}{R}$ will therefore apply to conductors of different size from those tested, when the shape and relative position are the same.

The practical application of this principle is seen by considering that skin effect and proximity effect values are very often desired to be known for very heavy conductors at commercial frequencies. In such cases the currents are too large and the voltage drops are too small, to be conveniently or precisely measured. Precise results could be obtained by constructing miniature conductors to scale, and measuring the skin effect with conveniently small currents at high frequency.

In this way, the current distribution in three-phase circuits of ventilated busbars or of several cables in parallel, could be studied with exactness.

By considering a ventilated busbar to be approximately equivalent to a solid conductor of the same outside dimensions and the same total resistance, a simple extension of Fig. 7, containing curves for various proportionate thicknesses, would show the approximate skin effect of any isolated, rectangular, nonmagnetic conductor, whether solid or ventilated, hot or cold, large or small, and whether copper or aluminum. Such a set of curves would be useful in designing heavy busbars and conductors, not placed close together.

DISCUSSION ON "SKIN EFFECT IN TUBULAR AND FLAT CONDUCTORS" (DWIGHT). (PRESENTED BY PUBLICATION ONLY.)

Joseph Slepian: Mr. Dwight has certainly done a very creditable piece of work in solving so completely and in such usable form the skin effect problem in the two important cases of tubular and strap conductors. The paper is, however, really of very much broader scope than its title would indicate, because the method used is one which appears to be applicable to any shape of conductor, and the tube and strap solutions given here are only clearly explained examples of the working of the method. It is probable that with a shape of any great complication the method would become rather laborious, but it would remain simple and straightforward in theory at least and be capable of being carried to a numerical conclusion. The solution by means of expansions in series of orthogonal functions similar to Bessel's functions while in a sense more elegant is of little use for numerical computation and so from the engineer's point of view is not a solution at all.

The similitude principle given on page 1398 is obviously of great importance. It permits the determination of skin effect in large conductors by experiments on small models. A mathematical proof of this principle may be of interest as verifying the informal though convincing proof given.

Suppose a set of parallel conductors given, so that the specific resistance, $\rho\ (x\ y)$, in space is a function of x, y, coordinates in a plane perpendicular to the conductors.

Let $E\ (x, y, t)$ be the impressed e. m. f. per unit length of conductor; let $I\ (x, y, t)$ be the current *density* at any point.

Then we have the relations:

$$E\ (x\ y\ t) = \rho\ (x\ y\ t) \cdot I\ (x\ y\ t) + \frac{\partial A}{\partial t} \qquad (1)$$

$$\text{Where } A\ (x\ y\ t) = -\,2 \int\!\!\int I\ (\xi\ \eta\ t) \log r\ d\,\xi\ d\,\eta \qquad (2)$$

$$r = \sqrt{(x-\xi)^2 + (y-\eta)^2}$$

Of course we must assume $\int\!\!\int I\,dx\,dy = 0$ if A and E are to remain finite.

If $E\ (x\ y\ t)$ is a given harmonic function of the time then these equations serve to determine uniquely the corresponding steady state of alternating current, $I\ (x\ y\ t)$.

Now consider a new configuration of conductors obtained by expanding linear dimensions of the old configuration in the ratio k to 1. Then for the new configuration the conductivity, $\rho'\ (x\ y)$ is related to the conductivity in the old configuration $\rho\ (x\ y)$ by the equation:

$$\rho'\ (x\ y) = \rho\ (x/k;\ y/k) \qquad (3)$$

Now suppose that in the new configuration we have the current density $I'(x, y, t)$ the same as that at corresponding points in the old, except that frequency is reduced in the ratio 1 to l, thus

$$I(x, y, t) = I(x/k, y/k, t/l) \tag{4}$$

What must be the corresponding impressed e. m. f. in the new configuration?

We have:

$$E'(x\,y\,t) = \rho'(x, y)\, I'(x\,y\,t) + \frac{\partial A'}{\partial t} \tag{5}$$

$$A' = -2\int\!\!\int I'(\xi, \eta, t) \log r \, d\xi \, d\eta \tag{6}$$

Substituting (4) in (6)

$$A'(x\,y\,t) = -2\int\!\!\int I(\xi/k, \eta/k, t/l) \log r \, d\xi \, d\eta$$

If now we change variables in the integral

$$\xi = k\,\bar{\xi}$$
$$\eta = k\,\bar{\eta}$$

$$A'(x\,y\,t) = -2\int\!\!\int I(\bar{\xi}, \bar{\eta}, t/l) \log\sqrt{(x - k\bar{\xi})^2 + (y - k\bar{\eta})^2}\; k\,d\bar{\xi}\,k\,d\bar{\eta}$$

$$= -2\int\!\!\int k^2 I(\bar{\xi}, \bar{\eta}\ t/l) \log k\sqrt{(x/k - \bar{\xi})^2 + (y/k - \bar{\eta})_2}\; d\bar{\xi}\,d\bar{\eta}$$

$$= -2k^2\int\!\!\int I(\bar{\xi}, \bar{\eta}, t/l) \log\sqrt{(x/k - \bar{\xi})^2 + (y/k - \bar{\eta})^2}\; d\bar{\xi}\,d\bar{\eta}$$

$$- 2k^2 \log k \int\!\!\int I(\bar{\xi}, \bar{\eta}, t/l)\, d\bar{\xi}\, d\bar{\eta}$$

Therefore, $A'(x\,y\,t) = k^2 A(x/k, y/k, t/l) - 0$ (7)

Hence, $\dfrac{\partial A}{\partial t}(x\,y\,t) = \dfrac{k^2}{l}\;\dfrac{\partial A}{\partial t}(x/k,\ y/k,\ t/l)$ (8)

Substituting (8), (4) and (3) in (5)

$$E'(x, y, t) = \rho(x/k, y/k) \cdot I(x/k, y/k, t/l) + k^2/l\,\frac{\partial A}{\partial t}(x/k, y/k, t/l) \tag{9}$$

Comparing equation (9) with (1), we see that if $l = k^2$. so that $k^2/l = 1$,

$$E'(x\,y\,t) = E(x/k, y/k, t/l) \tag{10}$$

Hence in the new configuration, a similar system of impressed e. m. f.'s will produce a similar system of current distribution if the frequency is varied inversely as the square of the linear dimensions. Since, however, the resistance per unit length conductor varies inversely as the square of the linear dimensions, we may say that the frequency must vary directly with the resistance.

H. B. Dwight: Since writing the paper, the author has derived an asymptotic formula for the skin effect in a tube, by using expressions for the *ber* and *ker* functions given by H. G. Savidge.* This formula, which is an extension of equation (**47**), is

$$\frac{R'}{R} = \frac{mt\,(q + r)}{2\,r\,\sqrt{2}} \left(1 + \frac{1}{mr\,\sqrt{2}} + \frac{3}{8m^2\,r^2} + \frac{0}{m^3\,r^3}\right)$$

For values of *mt* greater than 4, this formula gives curves which are the same as those of Fig. 3. The equation of the asymptote of each curve is

$$\frac{R'}{R} = \frac{mt\,(q + r)}{2\,r\,\sqrt{2}} \left(1 + \frac{1}{mr\,\sqrt{2}}\right)$$

which may be used for obtaining practical values of the skin effect ratio for *mt* greater than 4. It may be observed that the term in $\frac{1}{m^2\,r^2}$ is positive, and therefore the curves lie a little above their asymptotes when *mt* is greater than about 4.

To check the above formula, it may be applied to the case of a solid wire by putting $q = 0$ and $t = r$, so that it becomes

$$\frac{R'}{R} = \frac{mr}{2\,\sqrt{2}} + \frac{1}{4} + \frac{3}{16\,mr\,\sqrt{2}} + \frac{0}{m^2\,r^2}$$

This is in agreement with the value of $R'/R = x/2 \times W/Y$ given by equation (**31**) of the paper by Dr. Russell in the *Philosophical Magazine*, Vol. 17, 1909, page 524.

I am much obliged to Mr. Slepian for his contribution. His mathematical proof seems to me to add greatly to the force of the statement made in my paper that skin effect tests made at different frequencies can be corrected mathematically for frequency and compared with each other with confidence that they ought to agree, and in particular, that tests made on miniature conductors at a high frequency such as 5000 cycles can be corrected for frequency so as to be of great use in the design of 60-cycle bus bars and other conductors.

**Philosophical Magazine*, Vol. 19, 1910, page 49.

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A DIRECT-CURRENT GENERATOR FOR CONSTANT POTENTIAL AT VARIABLE SPEED

BY S. R. BERGMAN

ABSTRACT OF PAPER

The standing of this problem in the past is briefly reviewed. As far as is known to the author the type of machine described presents a new solution, the advantages of which are shown to be:

1. The machine is self-excited, *i. e.*, regulates independent of any other source of potential.
2. The machine regulates independent of speed and load and may be compounded.
3. The regulation is inherent, *i.e.*, no external regulating device is used.
4. The regulation is instantaneous.
5. The regulation is approximately independent of the heating.

The theory of the machine is described and diagrams of connections given. Performance curves obtained from tests are shown.

A method is described whereby instantaneous regulation of the voltage is obtained, which method also secures approximately a constant voltage independent of the heating.

Finally there follows a discussion of the efficiency of the new machine as compared with a standard machine of the same speed and output.

A PROBLEM in direct-current engineering, especially met with in traction, is that of producing constant voltage at variable speed. Train lighting from axle-driven generators is one of its oldest applications. The transformation from a variable to a constant voltage can be accomplished by revolving machinery provided that the principle of producing constant potential, independent of the speed, is known. Of late years this problem has received new impetus with the introduction of electric starting and lighting of automobiles and a great number of solutions have been offered, most of them depending upon the presence of a storage battery which greatly simplifies the conditions, since the steadying influence of the battery results in a source of nearly constant potential from which a constant excitation may be drawn. The regulation in the great majority of such charging sets depends upon the load current and is spoiled if the load is disconnected; *i. e.*, if the battery is not in the circuit the generator voltage varies greatly with the speed. Other systems exist employing automatic shunt regulators either operated magnetically or depending on combinations containing resistors with negative temperature coefficients.

In railway and automobile applications it is desirable to avoid all automatic apparatus if possible and it occurred to the writer some years ago that it would be possible to produce a generator which would regulate on constant potential independent of the speed and load, due to inherent properties of the windings, that is, without the use of any kind of regulator. The conditions to be fulfilled appear as follows:

1. The machine should be self-excited, *i. e.*, should regulate independent of any other source of potential.

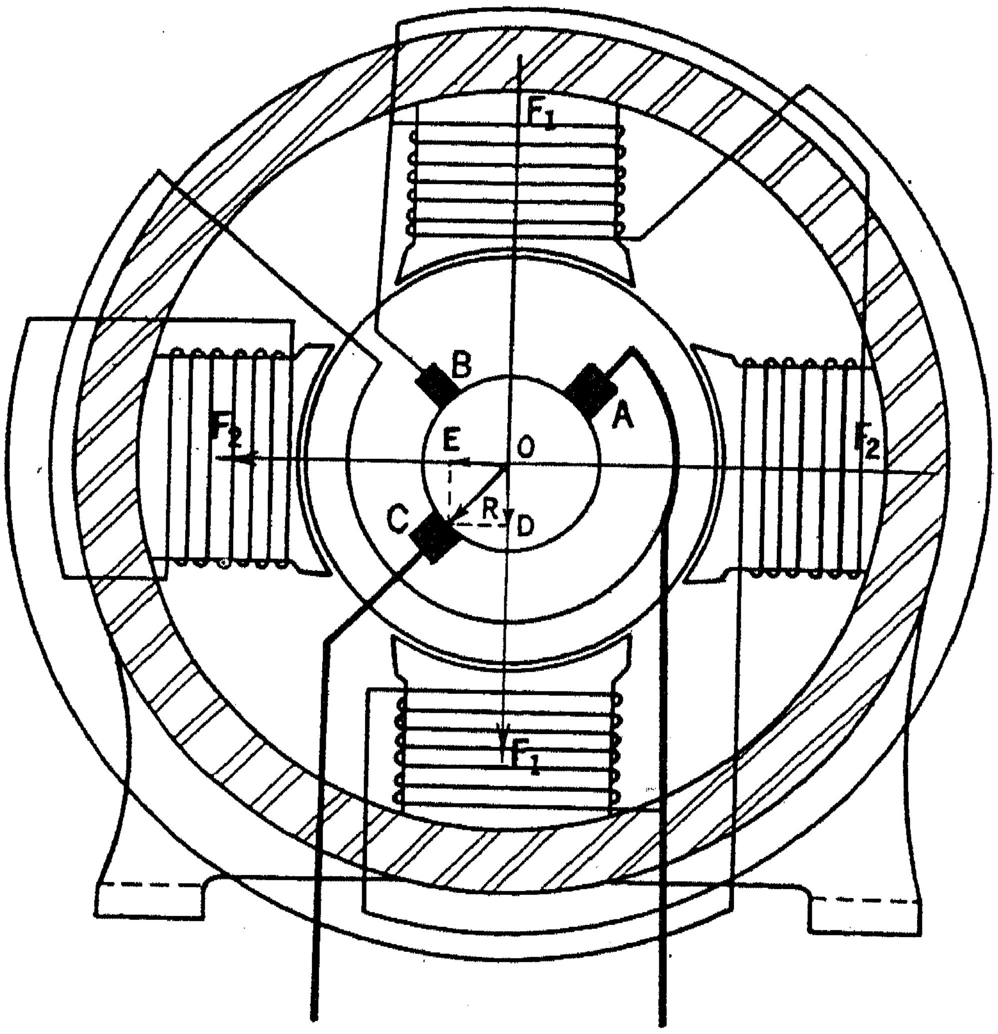

FIG. 1—CONSTANT POTENTIAL GENERATOR

2. The machine should regulate independent of the load and should possess the properties of compounding.
3. The regulation should be inherent.
4. The regulation should be instantaneous, *i. e.*, no over- or under-shooting of voltage should occur at sudden speed variations.
5. The regulation should be independent of the heating, *i. e.*, same potential generated when machine is cold as when hot.

A number of machines fulfilling the above conditions have been built and are now in successful service. In the following pages the principle of operation will be described and illustrated by characteristic curves obtained from tests.

Fig. 1 illustrates, diagramatically, the principle of the machine. The armature is series wound and the field contains twice as many poles as the number of poles for which the armature is wound. In the case illustrated, the armature is wound for two poles and the field contains four poles, symmetrically located. The load is taken from the brushes A and C and in addition to these load brushes there exists a third brush B, placed 90 electrical degrees from the load brushes. The field may be considered to consist of two independent magnetic circuits, one of these circuits $F1$ is saturated and the corresponding flux ϕ_1, will be called the main flux of the machine. The second magnetic circuit $F2$ is not saturated and the corresponding flux ϕ_2 will be called the cross flux. The flux ϕ_1 generates, between the brushes A and B, an e. m. f. which will be called the main voltage of the machine. The flux ϕ_1 does not generate any e.m.f. between the brushes B and C. Similarly the flux ϕ_2 generates, between the brushes B and C, an e.m.f., which will be called the cross voltage.

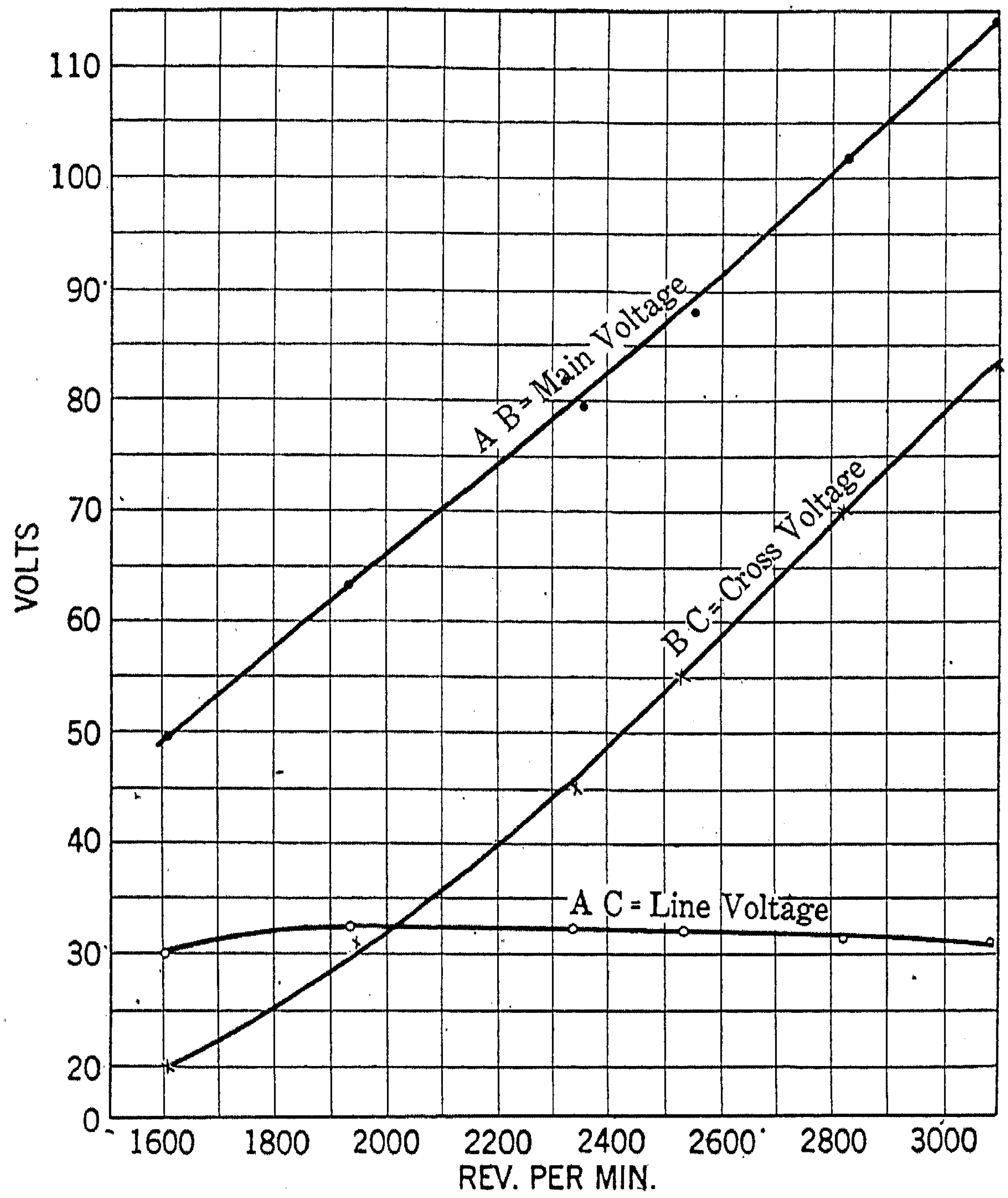

FIG. 2—CONSTANT-POTENTIAL GENERATOR
1.5 KW.—32 VOLTS—1600/3200 REV. PER MIN.

This flux does not generate any e.m.f. between the brushes A and B. The excitation of the machine is taken from the brushes A and B for which reason the brush B will be called the exciting brush. The excitation consists, as shown, of two multiple branches, one branch exciting the main poles and the second branch exciting the cross poles. The two fluxes ϕ_1 and ϕ_2 are entirely independent of each other, which may be verified by separately exciting the machine, and tests show that if the main excitation is varied only the main voltage is affected, the cross voltage remaining constant; and if the cross excitation is varied only the cross voltage is affected, the main voltage remaining constant.

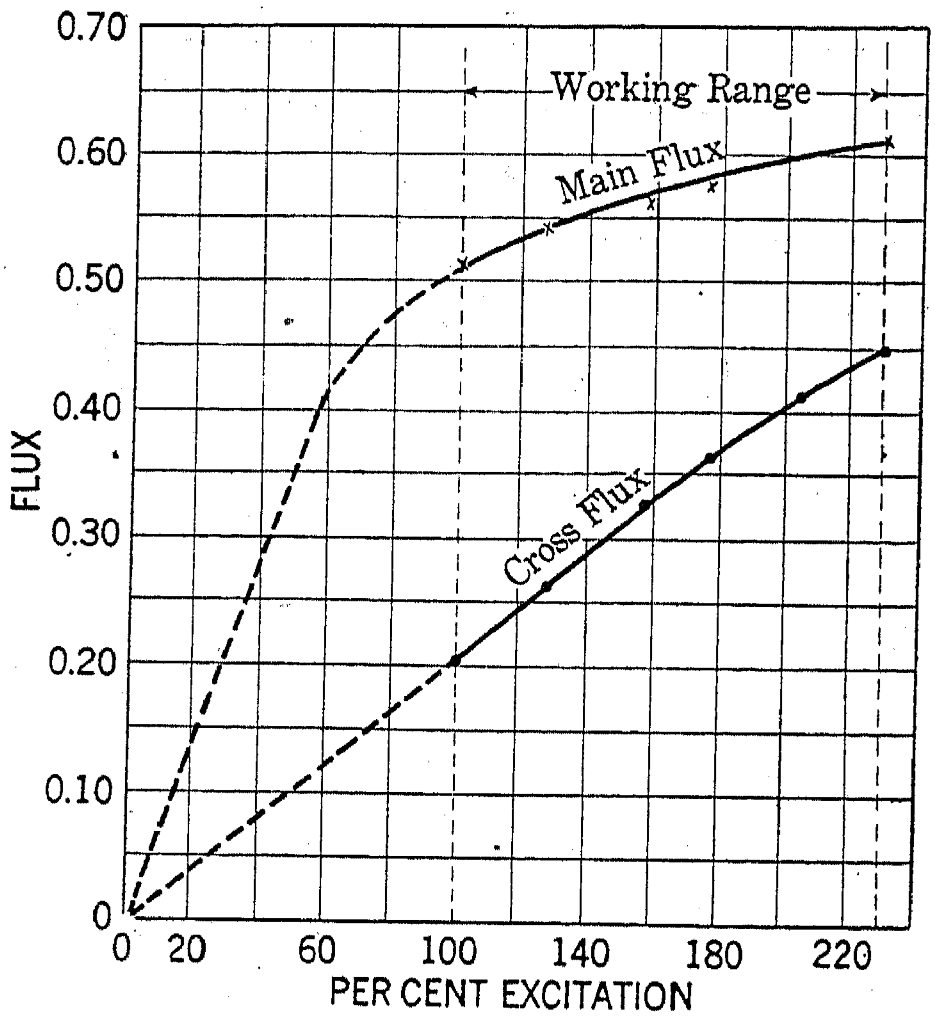

FIG. 3

The directions of the main and the cross fluxes are such that the difference between these fluxes is interlinked, with the line brushes A and C, as shown in the figure, and thus the line voltage AC is the difference between the main voltage AB and the cross voltage BC. Hence, the voltage $AC = AB - BC$. Since the main circuit is saturated the flux ϕ_1 remains constant and the main voltage AB is proportional to the speed. Therefore, the excitation of both the main and the cross fields is proportional to the speed. The cross circuit not being saturated the cross flux ϕ_2 will increase in proportion to the speed and, hence, the cross voltage BC must increase with the square of the speed. As will be shown by saturation curves the cross circuit should not be entirely unsaturated since then the cross voltage would increase too fast and the machine voltage would decrease with increasing speed. The cross flux should approach saturation and it is possible to choose the saturation of the cross magnetic circuit, so that the line voltage *A C* remains constant. In Fig. 2 is shown the variation of the different voltages with the speed. Since the excitation is taken from the brushes *A* and *B* the variation of the exciting ampere-turns follows the main voltage *A B* and may, therefore, in proper scale, be read from the curve *A B*. From these curves may be determined the amount of flux in each magnetic circuit for any given excitation and in

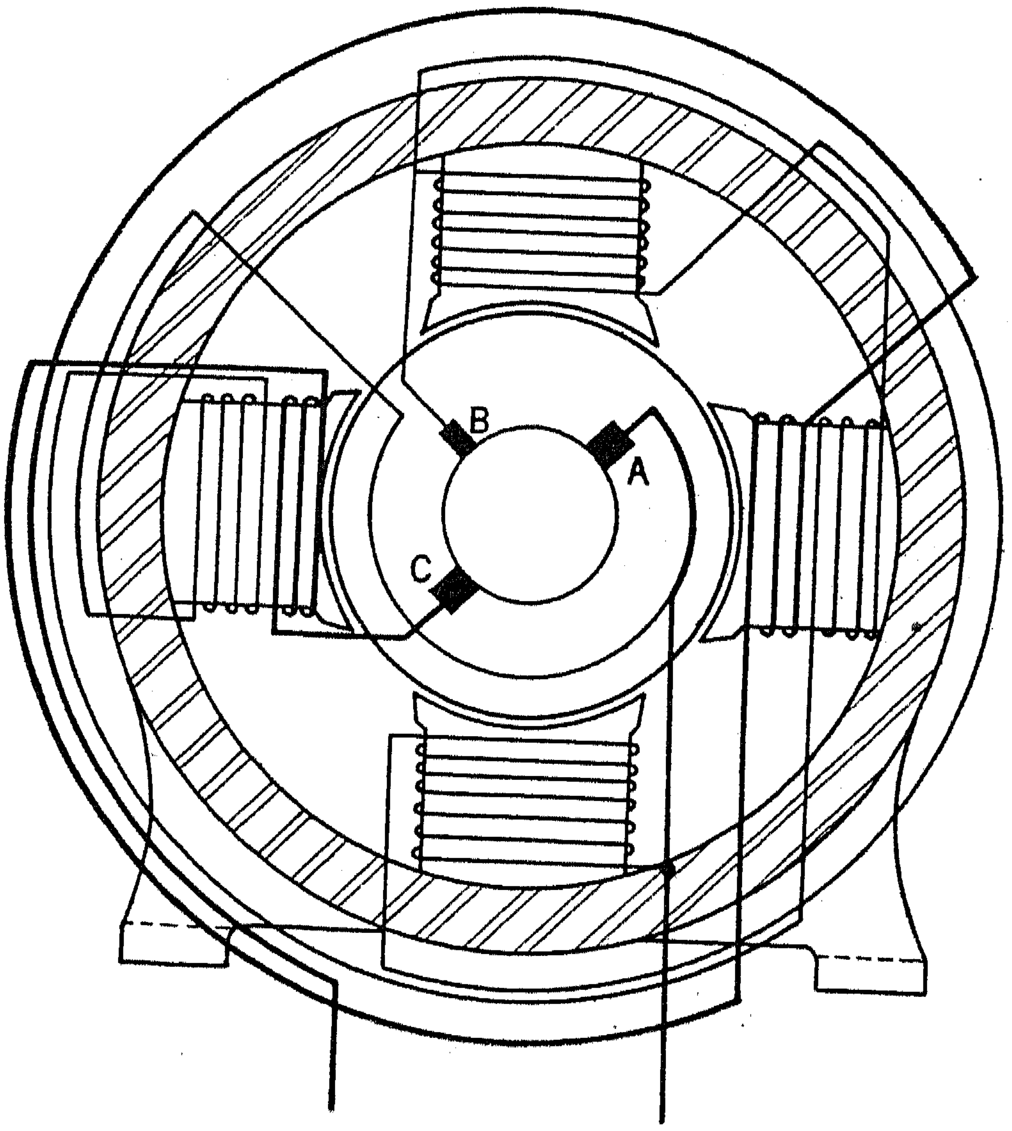

FIG. 4

Fig. 3 are plotted the two fluxes against the excitation. These saturation curves show that over the working range the main magnetic circuit is nearly saturated and the cross magnetic circuit at first is unsaturated but from a certain point, where the curve bends, this circuit starts to approach saturation.

Since the line current is taken from the brushes *A* and *C*, Fig. 1, there exists an armature reaction *O R* in the direction of *A C*. This armature reaction may be resolved in two components, one *O D* in the direction of the main flux and the other *O E* in the direction of the cross flux. The main magnetic circuit being saturated, the additional excitation, due to the armature reaction, cannot add anything to the main flux. The component *O E* in the direction of the cross flux will, however, interfere with this flux and would disturb the regulation of the machine and in order to overcome this influence a series winding is added to the cross poles. This winding should have an equal and opposite strength to the armature reaction working in this direction, and will be called the compensating winding. The location of this winding is shown in Fig. 4. It will be easily understood that by changing the strength of the compensating winding it is possible to obtain either rising or falling machine voltage with increasing load. By over-compensation the voltage will rise with the load and with under-compensation it will fall. Obviously the strength of the compensating winding can be made to compensate for the *I R* drop resulting in a flat compounded generator.

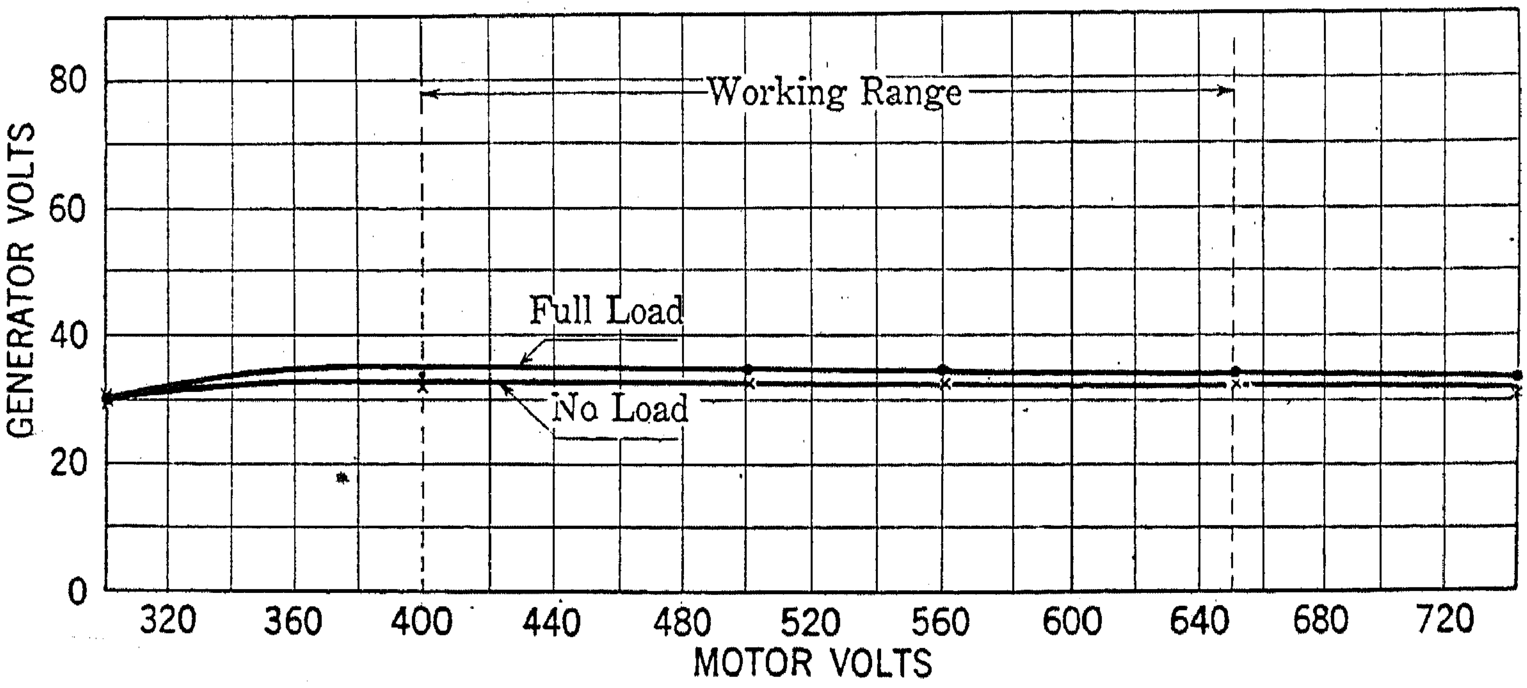

FIG. 5

If a generator of the type under consideration is coupled to a motor such a motor-generator is capable of transforming from a variable voltage to a constant voltage. Fig. 5 shows the performance from test of a motor-generator, the motor voltage varying from 400 to 650 and the generator voltage being maintained constant at 32 volts. As will be seen from Fig. 5 the generator voltage rises slightly with the load showing that the machine is over-compounded.

If the motor of a motor-generator of this type is thrown directly on the line a voltmeter connected to the generator shows that the voltage, due to sudden acceleration, exceeds the rated voltage. The amount of this over-shooting of the voltage depends upon the relative values of the time constants of the two multiple exciting circuits. For example; assuming that the rate of increase of the exciting current is nearly the same in both circuits then the flux in the unsaturated circuit will appear at a later time than in the saturated circuit. Consequently the negative voltage will not develop fast enough, resulting in the phenomenon which may be called over-shooting of the voltage. It is possible, however, to speed up the rate at which the cross magnetizing current increases by insertion of resistance in this circuit. The complete diagram of connections is shown in Fig. 6 in which the permanent resistance is designated as *R*. Tests have demonstrated that if this resistance is properly chosen it is possible to entirely eliminate over-shooting of the voltage. In this figure has also been added commutating poles to assure perfect commutation.

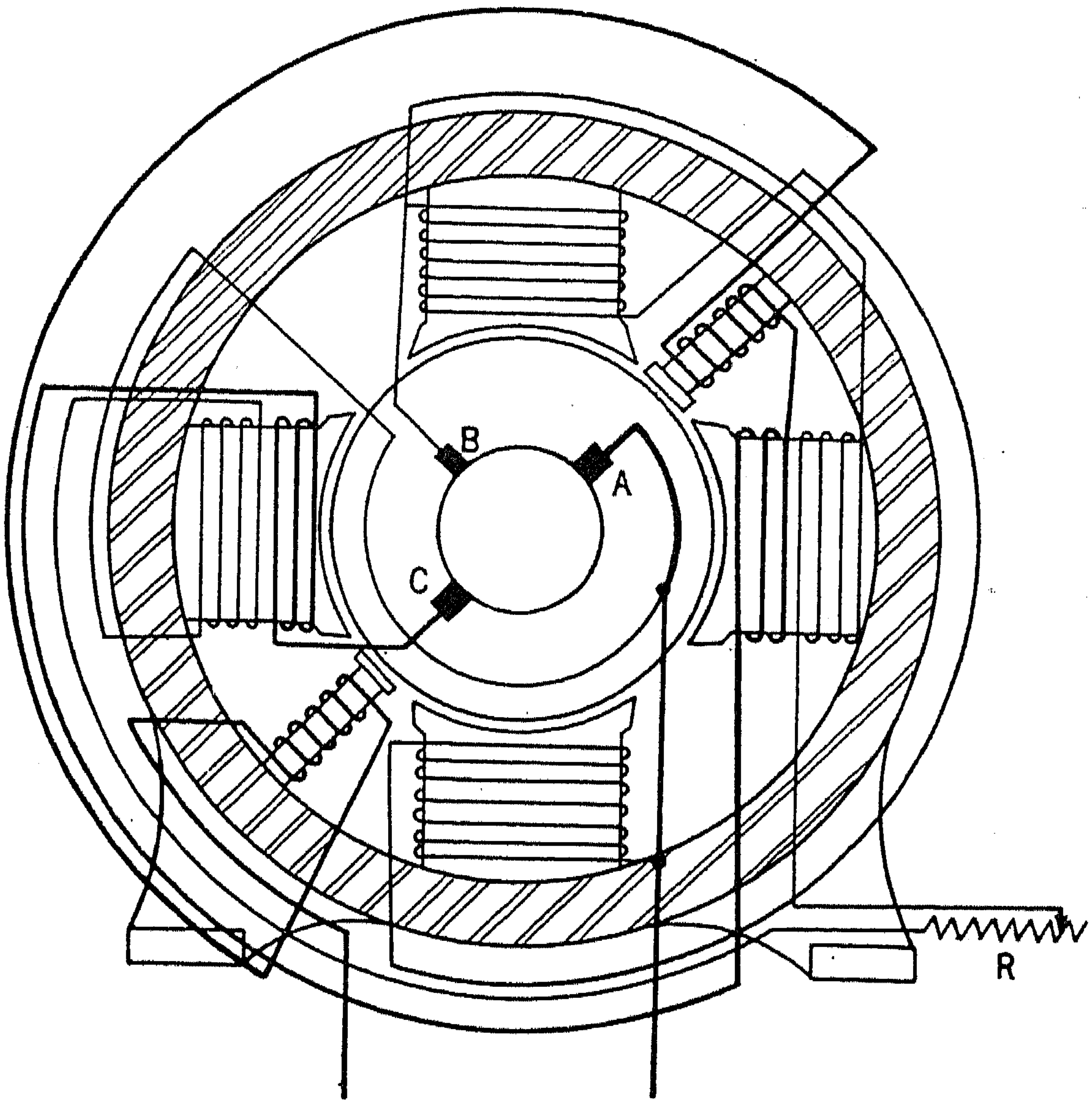

FIG. 6

The resistance *R* also serves another purpose, namely, to keep the regulation of the machine approximately the same when the generator is cold as when hot. Assuming for the moment that the generator was built in accordance with Fig. 4, without any resistance in the cross circuit, it is obvious that when the generator is cold the current in both exciting circuits will be larger than when hot. The main flux will not be influenced by this increased excitation, since saturation exists. On the other hand, the increased exciting current of the cross circuit will cause an increase in the cross flux resulting in an increase of the cross-voltage. Since this voltage is negative it follows that when the machine is cold the line voltage will be too low. By inserting a resistance of zero temperature coefficient the variation of the cross excitation is limited and tests show that if the resistance R is approximately $\frac{1}{3}$ or more of the resistance of the cross exciting circuit, the change of excitation, due to heating, will be limited to such an extent that the variation of voltage is less than 5 per cent, corresponding to a temperature range of 100 deg. cent.

As has already been explained this generator is in reality a combination of two independent generators, one bucking the other, which leads to the conclusion that the machine must become somewhat larger in size and weight than a standard machine of same speed and output. On the other hand, there are several other conditions which tend to make this machine more economical than the standard. When the speed increases all flux densities and hence the stability increases. In a machine of standard make in which the shunt excitation is regulated in order to maintain constant voltage, the flux densities and hence the stability decreases with increasing speed. Therefore, a standard machine requires much larger gaps than this new machine and furthermore, the armature reaction in the standard type must be chosen with due consideration to the speed range. In order to give an approximate idea of the size and weight of this new type of generator, as compared with a standard type of machine, it may be stated that by a sacrifice in the efficiency of approximately 10 per cent, the new machine will have the same weight as a standard type of same rating.

As has already been pointed out this new type of constant potential generator shows some distinct advantages over a standard generator with automatic voltage control. Where reliability of service is concerned this new type is superior since there does not exist automatic devices of any kind for maintaining the regulation. The new machine has also the advantage of being instantaneous in action even at violent and large speed variations. Therefore, it is thought that this new machine will create a field of its own and satisfy a long felt need in certain applications of the electrical industry.

DISCUSSION ON "A DIRECT-CURRENT GENERATOR FOR CONSTANT POTENTIAL AT VARIABLE SPEED" (BERGMAN,) (PRESENTED BY PUBLICATION ONLY).

J. G. De Remer: In the paper by S. R. Bergman, attention should be called to an apparent misstatement contained on the lower part of page 1408 thereof.

The reasoning therein given relative to the variation of the main voltage *AB* and the cross voltage *BC* is based upon assumptions which do not apply. For if the main voltage varies with the first power of the speed and the cross voltage with the second power of the speed their difference *AC* can not be constant.

The figures two and three bear this out as observed by extending the curves of figure two to 3200 revolutions per minute (double 1600). The curves so extended then indicate that doubling the speed from 1600 to 3200 increases the main voltages from 50 to approximately 120 which is 140 per cent increase instead of 100 per cent. The cross voltage is increased from 20 to about 90 by increasing the speed from 1600 to 3200 which is 350 per cent instead of 300 per cent as would be required by the square law.

The foregoing is further evidenced by the flux curves shown in Fig. 3 where the main flux is not constant as assumed in the author's statement.

Figs. 2 and 3 indicate that neither the main flux nor the cross flux were maintained at the saturation point. On the contrary, the conditions of the two fluxes were such as to give a resulting flux which decreases with the first power of the speed. Such a flux variation would give constant potential across *AC* and is the condition which doubtless existed when the data of Fig. 2 were obtained.

S. R. Bergman: In answer to a communication by Mr. J. G. DeRemer, I wish to give the following reply.

The best plan to follow in order to thoroughly understand an electro-magnetic problem is to draw up a picture of the flux distribution. (Fig. 1). If Mr. DeRemer would attempt to do so I predict he will find that the theory he suggests leads nowhere.

The theory which I have advanced in my paper is based on the physical existence of two independent fluxes, the main flux and the cross flux, the distribution of which is shown in Fig. 1. It is now obvious without further consideration, that the corresponding voltage may be expressed as follows:

$$\left.\begin{aligned} AB &= k \times n \times \Phi_1 \times 10^{-8} \\ BC &= k \times n \times \Phi_2 \times 10^{-8} \end{aligned}\right\}$$

If saturation exists in the main magnetic circuit the main flux is constant and the voltage *AB* is proportional to the speed as stated in the paper.

Complete saturation never exists in any dynamo electric machine but a slight increase of the flux always takes place when the excitation is increased, as may plainly be seen from Fig. 3 in my paper. The main flux is, therefore, in reality not constant but is a function of the excitation and it would not be difficult, although laborious, to solve this problem analytically by aid of the Fröhlich formula—(compare Steinmetz' "Transient Electric Phenomena and Oscillations" page 33.) Such a calculation would so complicate the problem that it would becloud the

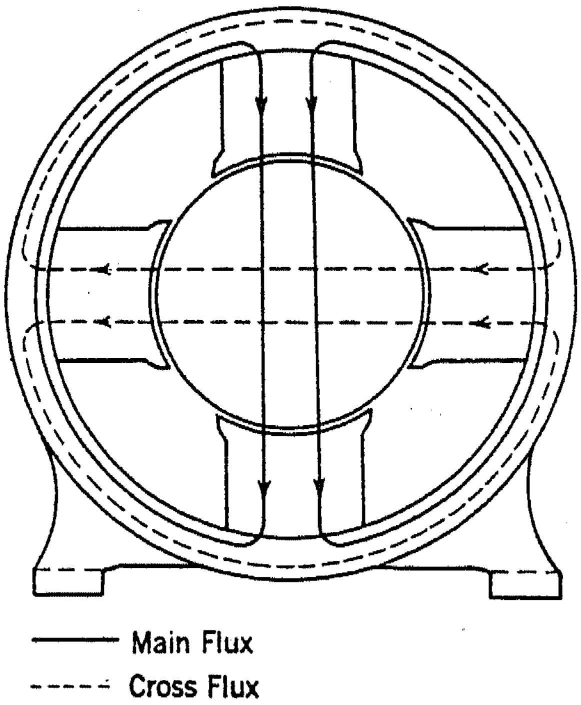

FIG. 1—FLUX DISTRIBUTION. CONSTANT POTENTIAL GENERATOR

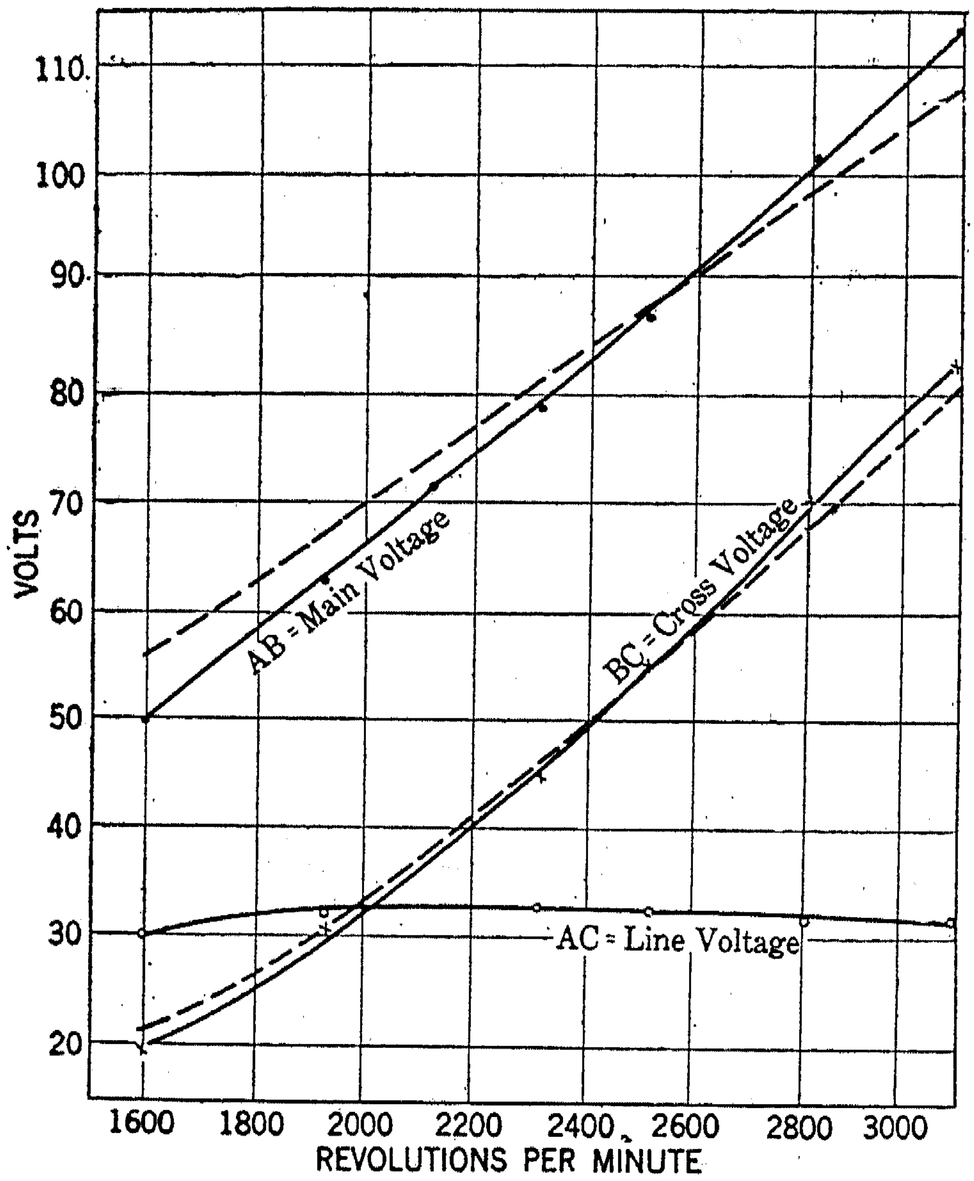

FIG. 2—CONSTANT POTENTIAL GENERATOR 1.5 KW.—32 VOLTS—1600/3200 REV. PER MIN.

physical phenomenon instead of clearing it up. Hence, I prefer in my discussion to state that the saturation keeps the flux constant. Tests justify this assumption with fair approximation as may be seen from Fig. 2 which is a reproduction of Fig. 2 in my paper with the addition of a dotted straight line representing proportionality.

In regard to the cross voltage *B C*, this voltage increases with the square of the speed if we assume that the main voltage varies directly with the speed, and also, that no saturation exists in the cross circuit. Also, in this case can the problem be solved accurately by aid of the Fröhlich formula. However, the square law holds true with good accuracy as shown in Fig. 2 in which a parabolic curve is shown in dotted lines, representing the square law.

Mr. DeRemer states: "If the main voltage varies with the first power of the speed and the cross voltage with the second power of the speed their difference *A C* cannot be constant." In order to avoid this very misunderstanding I stated in my paper "As will be shown by saturation curves, the cross circuit should not be entirely unsaturated since then the cross voltage would increase too fast and the machine voltage would decrease with increasing speed. The cross flux should approach saturation and it is possible to choose the saturation of the cross magnetic circuit so that the line voltage *A C* remains constant."

In order to further emphasize the independence of the two fluxes, I wish to point out that the cross flux branches out from points of equal magnetic potential located on the path of the main magnetic circuit and vice versa, the main flux branches out from points of equal potential on the path of the cross flux.

Very similar conditions exist in a commutating-pole machine, having the same number of commutating poles as main poles. An excellent discussion of the independent existence of the main flux and the commutating flux may be found in Arnold's classical work, "Die Gleichstrom Maschine."

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CRITICAL REVIEW OF THE BIBLIOGRAPHY ON UNBALANCED MAGNETIC PULL IN DYNAMO-ELECTRIC MACHINES

BY ALEXANDER GRAY AND J. G. PERTSCH, JR.

ABSTRACT OF PAPER

The purpose of this paper is to serve as an introduction to the succeeding comprehensive article by Mr. E. Rosenberg, on the subject of Magnetic Pull in Electric Machines. The Maxwell equation, upon which all of the formulas for unbalanced magnetic pull are based, is first established. A critical review is then given of the articles pertaining to this subject, which have appeared in the technical press to date. It is pointed out wherein the various expressions which have been proposed differ from one another and to what extent they are consistent.

ALL of the formulas are based on Maxwell's equation:

$$\text{Pull, in dynes} = \frac{B^2 A}{8 \pi}$$

where B = flux density, in lines per sq. cm.

A = cross section, in sq. cm. of the flux path in the gap between the pulling surfaces.

This formula may be established in a simple manner for the case of a saturated magnetic circuit, as follows:

The magnetic circuit shown in Fig. 1 is symmetrical about the axis $O\,O'$, and therefore the action of each pole may be considered separately. With n turns per pole and e volts across each coil, the excitation $n\,i$ produces a magnetic flux of ϕ lines.

In Fig. 2, $O\,a$ shows the magnetization curve for each half of the complete magnetic circuit of Fig. 1 when the air gap clearance between the armature and each pole-face is g cm. If now, the armature is allowed to move by a virtual displacement $\Delta\, g$ under the action of the magnetic pull P, so as to reduce the gap, then the curve for the new magnetic circuit becomes $O\,b$ (Fig. 2).

In order to establish the initial flux ϕ at an excitation $n\,i$, the total energy supplied to the coil after closing the switch $s\,s$, is

$$\int_0^t eidt = \int_0^t (n\frac{d\,\phi}{d\,t} \;.\; 10^{-8})\, idt + \int_0^t iridt$$

In this expression, the stored energy

$$\int_{0}^{\phi} ni\, d\phi \quad 10^{-8} = \text{Area } Oac \text{ (Fig. 2), in watt-sec.}$$

If, now, the armature is allowed to move under the action of the pull P, so as to reduce the gap by an amount Δg, the excitation $n\, i$ being maintained constant, the flux changes from ϕ to $\phi + \Delta \phi$. The increase in flux $\Delta \phi$ induces a counter e. m. f. in the winding and since, by assumption, the current is held constant, the applied voltage must temporarily increase. The amount of energy so brought into the electromagnet is

$$\int_{\phi}^{\phi+\Delta\phi} \left(n \frac{d\phi}{dt} \cdot 10^{-8}\right) i dt = ni \times \Delta \phi \,.\, 10^{-8} = \text{Area } cabd$$

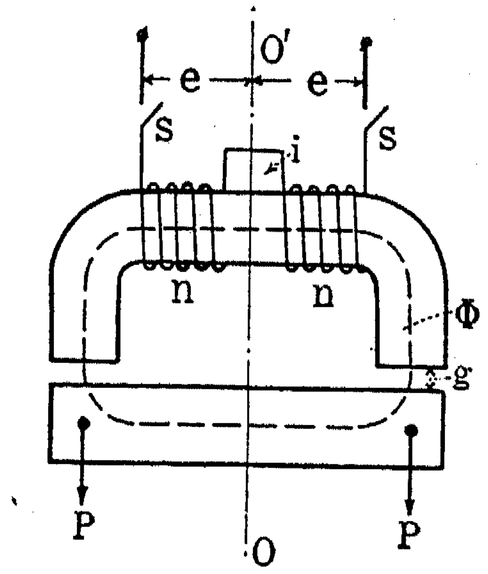

FIG. 1

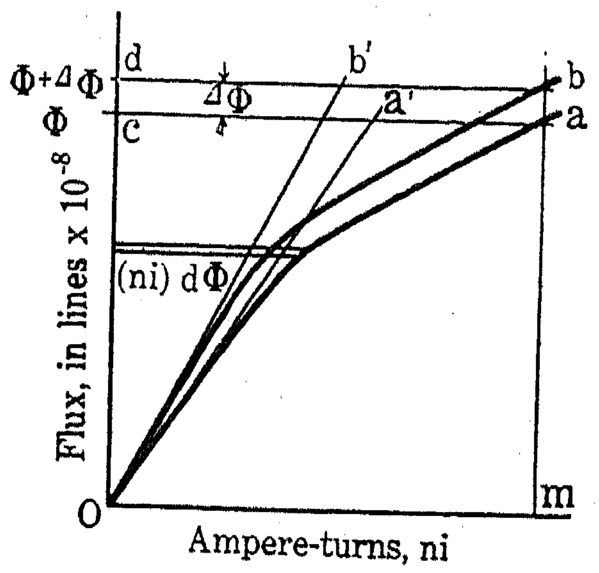

FIG. 2

The energy which is thus supplied by the source goes partly to increase the stored energy and the rest lifts the armature.

The energy stored in the magnetic circuit under the new conditions, with flux $\phi + \Delta \phi$,

$$= \text{Area } Obd.$$

Hence, mechanical work done = initial stored energy + energy supplied from the line − final stored energy

$$\begin{aligned} \text{or } P\,.\,\Delta g &= \text{Area } Oac + \text{Area } cabd - \text{Area } Obd \\ &= \text{Area } Oab \\ &= \text{Area } Oa'b' \end{aligned}$$

where Oa' and Ob' in Fig. 2 represent the magnetization curves for the air gaps g and $g - \Delta g$ respectively.

But, for very small values of $\Delta\,\phi$,

$$\text{Area } Oa'b' = \frac{1}{2}\cdot \text{Ampere-turns for gap } \Delta\, g \times \phi \cdot 10^{-8} \ldots \text{watt-sec.}$$

$$= \frac{1}{2}\left(\frac{10}{4\,\pi}\cdot\frac{\phi}{A}\cdot\Delta\, g)\right) \times \phi \,.\, 10^{-8}$$

$$= \frac{\phi^2}{8\pi A} \times \Delta\, g \times 10^{-7} \ldots\ldots\ldots \text{watt-sec.}$$

$$= \frac{\phi^2}{8\pi A} \times \Delta g = P \times \Delta\, g \ldots\ldots\ldots\ldots \text{ergs.}$$

or, Pull in dynes, $P = \dfrac{\phi^2}{8\pi A} = \dfrac{B^2 A}{8\pi}$

This formula is thus true whether the magnetic circuit is saturated or not. It will also be seen from the above what is meant by A in the formula, and how problems involving slotted armatures are to be solved.

The subject of magnetic pull in dynamo-electric machines has not received much attention in recent years; in fact, it appeared as if the whole field had been covered by 1911.

Fisher-Hinnen[1], in his "Dynamo Design" published in 1899 derives the following formula:

$$\text{Pull, in dynes} = \left(\frac{B^2 A}{25 \times 10^6}\cdot\frac{x}{g\,\alpha}\right) C$$

where B = density of flux in the air gap, in lines per sq. cm.
A = section of the pole face, in sq. cm.
x = rotor displacement
g = air-gap clearance
α = ratio of the reluctance of the total magnetic circuit to that of the normal air gap alone.

C is a calculated coefficient, given in the following table:

Value of C	Number of poles	Remarks
2	4	Displacement along axis of field magnets.
2.8	4	Displacement along neutral line.
6	6	Displacement along axis of magnet.
9	8	Displacement along neutral line.
15	12	Displacement along neutral line.
20	16	Displacement along neutral line.

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[1] See Bibliography.

B. A. Behrend[2] in 1900 gives a mathematical discussion of the case of a machine with a very large number of poles and derives the formula

$$\text{Pull} = \frac{B^2 S'}{8\pi} \cdot \frac{2x}{g} \cdot$$

where $S' = \pi \times R \times l = \frac{1}{2} \times$ total air-gap area, and shows that this is exactly one-half the value obtained by assuming that half of the poles have a gap $g - x$ and the other half a gap $g + x$.*

If we substitute for $S' = \frac{1}{2} \times A \times$ poles, Behrend's expression becomes

$$\text{Pull} = \frac{B^2 A}{8\pi} \cdot \frac{x}{g} \cdot \text{poles.}$$

Edgar Knowlton,[3] about the same time, derives an expression for a machine with a definite number of poles and obtains the formula

$$\text{Pull in lb.} = \left(\frac{B^2 A}{77{,}134{,}000} \cdot \frac{x}{g\alpha}\right) C$$

where B = normal gap density in lines per sq. in.
A = area of pole-face in sq. inches
x = displacement
g = normal air gap
α = ratio of the total reluctance of the magnetic circuit to that of the normal gap alone.
C = a coefficient, obtained by calculation
= 2 for 4 poles
= 4.7 for 6 poles
= 7 for 8 poles
= 9.6 for 10 poles

Above 10 poles, C = number of poles, and the formula then checks with that of Behrend.

Knowlton's expression is of exactly the same form as that of Fisher-Hinnen, but the constants given by the two authors for 6 and 8 poles differ considerably. Knowlton, moreover, states that it makes but little difference whether the plane of deflection is taken through a pole or between two poles.

Hans Linsenmann[4], in 1902, uses the saturation curve directly

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* Note that in the following paper Rosenberg formulates a similar statement.

(as is done in Fig. 3 of the following article by Rosenberg) for determining, at a given excitation, and with sine distribution of flux under the pole, the variation of the average magnetic pull per pole with the length of airgap. He expresses this variation by an approximate equation and introduces it into the general equations connecting the bending moments of the deflecting forces in large alternators.

J. Rey[5], in 1904, states that Behrend's and Fisher-Hinnen's formulas are not consistent and sets out to derive a new formula especially applicable to induction motors with eccentric rotor. Rey's formula is

$$\text{Pull in dynes} = \frac{1}{8\pi^2}\,(B_{eff})^2\,S \times C$$

where $B_{eff} = B_{av} \times 1.1$ for a sine distribution of flux
S = rotor surface = $2\pi Rl$
$C = f(\epsilon)$
ϵ = eccentricity, expressed as a fraction, in terms of the single air gap = x/g]

and for

$\epsilon = x/g =$	0.05	0.1	0.15	0.2	0.25	0.3	0.4	0.5
$C =$	0.157	0.319	0.488	0.688	0.866	1.084	1.63	2.415

For an eccentricity less than 20 per cent, this formula reduces to

$$\frac{(B_{eff})^2}{8\pi^2} \times S \times \frac{0.488}{0.15} \times \frac{x}{g}$$

$$= \frac{(B_{eff})^2}{8\pi} \times S \times \frac{x}{g},\ \text{approximate within one per cent,}$$

which is identical with Behrend's formula, because with a sine distribution of flux, as in an induction motor, $(B_{eff})^2$ is the average value of $(B)^2$, where B is the density at any point along the gap.

J. K. Sumec[7] derives a similar formula to that of Behrend and Rey, but by certain transformations reduces it to the form

$$\text{Pull in dynes} = \frac{B^2}{8\pi} \cdot S\,\frac{x}{g}\,\frac{1}{[1-(x/g)^2]^{3/2}}$$

This is the same as Behrend's formula except for the last term which for an eccentricity as large as 25 per cent, is equal to 1.1 or a difference of only 10 per cent, while for eccentricities below

10 per cent, the correction term differs from unity by only about one per cent.

Sumec's formula, as might be expected, gives the same values as that derived by Rey. It is to be noted that in most of the formulas derived mathematically, the effect of the reluctance of the iron part of the magnetic circuit is neglected; the assumption made that the flux density is inversely as the airgap is therefore only true in unsaturated machines

In 1905, B. Soschinski[8] gives an account of some tests made to check the formulas of Rey and Sumec. Very good agreement was obtained for small airgaps and iron feebly saturated, taking the area and flux density at the top of the teeth but with increasing saturation, the calculated results (iron reluctance neglected) were higher than experimental ones. With larger values of air gap, however, the test values were throughout higher (up to 100 per cent). The latter result was accounted for by the fact that as the armature moves to one side the lines become concentrated at the tooth tip on the side of the reduced gap and spread out on the side of the increased gap.

In the same year (1905), Niethammer[9], by a transformation of Sumec's expression, obtains the following formula which permits taking into account the reluctance of the iron path by determining the flux density from the saturation curve, as is done by Rosenberg. This expression is regarded by him as giving the most reliable results.

$$\text{Resultant pull, in } kg. = \frac{1}{2}\left\{\left(\frac{B_{max}}{5000}\right)^2 - \left(\frac{B_{min}}{5000}\right)^2\right\} p\,A$$

where p = number of pairs of poles

A = mean effective cross sectional area of gap per pole in sq. cm.

In 1907, Picou[10], starting with the relation for stored energy, derives the expression for magnetic attraction, and gives a modification of the formula due to Sumec for unbalanced pull.

C. R. Moore[12] in 1911, gives a graphical method for studying the unbalanced magnetic pull, by using Maxwell's formula and the given saturation curve of the machine, and summing up the pull for various adjacent halves of adjoining pole faces. He takes the airgap density for these half arcs as corresponding to the average airgap lengths across these faces and to the given excitation. Upon plotting the unbalanced magnetic pull for various field excitations he establishes the important fact noted by Rosenberg that, for low saturations the pull increases with the excitation; at a critical density, however, corresponding to about the knee of the magnetization curve, the unbalanced pull reaches a maximum for all eccentricities, and for larger excitations then decreases again. Therefore a machine, which normally operates at high saturation, may be subjected to greater stresses while it is building up than under normal field excitation.

Miles Walker[14] in his book on "Specification and Design of Dynamo-Electric Machinery" (1915), points out that the amount of the unbalanced pull for a given displacement will depend on the extent to which the iron parts are saturated, and that the effect of increased saturation is to reduce the pull for a given airgap clearance and magnetic induction. He first assumes that all the ampere-turns are expended in the airgap and by the usual method derives an expression for the unbalanced pull exactly similar to that of Behrend (see above). He then shows how the saturated magnetic circuit may be replaced by an equivalent airgap obtained by means of a graphical construction on the saturation curve of the machine, and then uses this equivalent airgap in the formula for the unbalanced pull.

R. E. Hellmund[11], Miles Walker[14] and others have pointed out the effect of series and parallel windings on the unbalanced pull and have also discussed how the unbalanced magnetic pull in the induction motor may be reduced to a minimum, by using in the stator winding two paths in parallel which lie on opposite halves of the frame and thus make it impossible for the flux on these two halves to be very unequal.

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MAGNETIC PULL IN ELECTRIC MACHINES

BY E. ROSENBERG

List of Symbols

A	Pole face area in square centimeters
B	Magnetic induction (flux density in the airgap.)
B_1	Reduced induction with increased airgap
B_2	Increased induction with reduced airgap
B_m	Critical induction, causing maximum unbalanced pull
$\frac{d\,B}{d\,H}$	Differential quotient of induction and magnetomotive force in lines per sq. cm. and Gilbert's ($4\pi/10$ ampere-turns) respectively
D	Rotor diameter
E	Voltage
$\frac{d\,E}{d\,i}$	Differential quotient of voltage and exciting current
f	"Gravity deflection"; deflection of the horizontally placed shaft under the static influence of the rotor weight
F_x	Unbalanced magnetic pull of the machine, due to a displacement x
F_f	Unbalanced magnetic pull of the machine, due to a displacement f
$(F_x)_{neutr.}$	In two-pole machines: Unbalanced pull due to displacement x in the direction of the neutral diameter.
$(F_x)_{axis}$	In two-pole machines: Unbalanced pull due to displacement x in the direction of the field axis
g	"Correct" air gap, taken as an average around the machine
g_1	"Virtual air gap", which, for low induction, would require the same m.m.f. as the magnetic half circuit of the machine
g_2	In two-pole machines: Phantom air gap of the interpole (air gap of an imaginary interpole with same section as the main pole, which would require the same m.m.f. for the passing of a certain flux as the real interpole)

G	Rotor weight
H	Magnetomotive force
i	Exciting current
L	Effective length of rotor (air ducts subtracted)
n	Revolutions per minute
$n_{crit.}$	Critical speed in rev. per. min.
$2p$	Number of poles
q	Ratio between the combined deflection of all the machine parts, caused by unbalanced pull, to the displacement, causing the unbalanced pull
q_r	Ratio between rotor deflection caused by unbalanced pull to the displacement causing the unbalanced pull
q_s	Ratio between combined deflection of the other machine parts to the displacement causing the unbalanced pull
$T_{circ.}$	Ampere-turns per pole required to produce a certain induction (within the straight line characteristic) in magnetic circuit of the machine with "correct" air gap
T_x	Ampere turns required to produce the same induction in the air gap x
$\tan \alpha$	Gradient of the magnetization characteristic
x	Displacement of rotor and stator centers
X	Final static displacement
$X_{mom.}$	Final momentary displacement
α	Local angle between magnetization curve and horizontal
θ	Angle between point of periphery and symmetry diameter

NOTE: Lengths are given in centimeters, forces and weights in kilograms, inductions in lines per square centimeter, unless otherwise stated.

Designers who figure in inches and pounds and who use "Kapp Lines per square inch" (one Kapp Line being equal to 6000 c.g.s. lines), substitute for $\left(\frac{B}{5000}\right)^2$ the value $\frac{1}{2} \cdot$ (B Kapplines/sq. inch)^2, to obtain the pull in pounds per square inch in formulas (**1**) to (**10a**).

The critical speed, formula (**11**), changes to

$$n_{crit.} = \sqrt{1 - q} \cdot \frac{188}{\sqrt{f_{inch}}}$$

if f_{inch} denotes the gravity deflection in inches.

INTRODUCTION

THE aim of the present paper is to investigate the occurrence and the effect of unbalanced magnetic pull in electric machines and to derive simple practical formulas for the use of the designer. There is an interesting chapter on the subject in Miles Walker's excellent book "Specification and Design of Dynamo Electric Machinery" in which for a given induction the influence of saturation on the unbalanced pull is considered. The present author has endeavored to carry the matter to its logical conclusion and to find, whether in a given machine there is a "critical induction" which gives a higher unbalanced pull than any other, smaller or larger, induction; further, to find a simple rule for determining this critical induction, and to investigate the permissible deflection of the machine parts in connection with the unbalanced pull, and the influence of the latter on the critical speed. Multipolar and bipolar machines on the one hand, and those with salient poles and cylindrical fields with distributed winding on the other hand, show very striking differences although the maximum of the forces occurring can be covered by the same formula.

A. MAGNETIC PULL OF A POLE AND OF A SERIES OF POLES

The magnetic attraction on a square centimeter of a pole face is

$$\frac{B^2}{8\,\pi}\ \text{dynes}$$

or, with an accuracy of nearly 1 per cent,

$$\left(\frac{B}{5000}\right)^2\ \text{kilograms,}$$

if B, the flux density in the airgaps, immediately adjoining the pole face, is expressed in c.g.s lines per sq. cm. To get the pull per pole, provided the pole extends over a comparatively small part of the periphery, we have to multiply by the area A of the pole face, measured in square centimeters. In general, in electric machines the magnetic pulls of the different poles are equal and are arranged symmetrically around the center. With a stator of the same stiffness in all diameters the magnetic pull would only cause a certain well defined and moderate strain all around the machine. With a split machine, however, especially if the joints are not very well stiffened, the deflection of the frame in the vertical direction will be different from that in the horizontal direction and a distortion may result which actually reduces sensibly the airgaps in the diameter perpendicular to the plane of the joint. Each half of the stator can be considered as a beam, the support and fixing of the upper half being different from that of the lower half which contains the feet.

It can happen that, through error or accident, only part of the poles are excited. The worst case will be when all the poles in one half of the machine are over-excited, the poles in the other half being without excitation. The mechanical parts of the machine should be strong enough to withstand this condition without being overstrained, although it will not be possible in large machines with small airgaps to prevent entire pulling over of the rotor against the stator core. Considering the resulting force on the one half of the rotor or stator, it is clear that every pole, the center line of which describes an angle θ with the symmetry line (Fig. 1), will contribute a component in the direction of the symmetry line

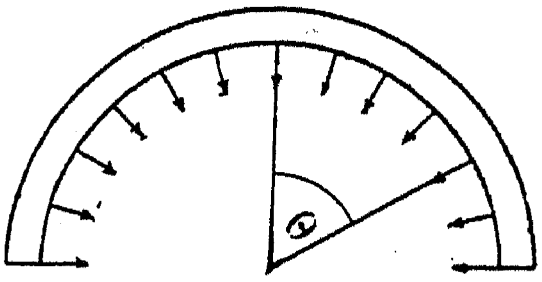

FIG. 1

$$\left(\frac{B}{5000}\right)^2 \cdot A \cdot \cos\theta$$

The average value of the cosine function over a quadrant is $2/\pi$. If therefore the machine has $2p$ poles, only p neighbouring poles being excited, the resulting force, both on the stator and rotor, will be

$$\frac{2}{\pi} \cdot \left(\frac{B}{5000}\right)^2 \cdot A \cdot p \qquad (1)$$

If the air gap density B is not constant over the whole pole face, but graded, the mean value of B^2 should be taken.

The unbalanced pull, which appears when only half of the machine is excited, grows with the second power of the air gap density or of the flux. With an air gap density of 5000 it is one kilogram per sq. cm.; with a density of 10,000 it is four kilograms per sq. cm., and with a density of 12,250, six kilograms per sq. cm. The highest possible, not the normal, saturation has to be considered to provide for this case, for it is likely that the accident or error which causes the non-excitation of one half of the poles, will cause the over-excitation of the other half as for instance, when all the exciting coils are connected in series, and wires leading to opposite coils become accidentally short-circuited. It must be ascertained whether both the shaft and frame etc. will withstand the force found by formula (**1**) without overstraining, or whether the rotor core will pull over hard against the stator core with some lower force which is not sufficient to cause overstrain.

B. UNBALANCED PULL DUE TO ECCENTRICITY

I. *Multipolar Salient-Pole Machines.* An entirely different and very general case is that of a machine, the coils of which have an equal excitation, but cause a different flux under the various poles due to differences in the air gap. Many causes contribute to such a condition. Frequently the outer surface of the rotor and the inner surface of the stator are not perfectly cylindrical. Even if they are perfectly cylindrical and concentric while the machine is cold, a noticeable deviation may occur due to the difference in temperature between stator and bedplate when the machine is heated. With a machine of five meters diameter, for instance, a difference of 20 deg. cent. in the average temperatures of frame and bedplate would correspond to approximately one mm. difference in length. If both feet of the hotter frame are rigidly bolted down to points of the cooler bedplate, a distortion of the frame will result; if only one foot is rigidly fixed, a shifting of the stator center will occur.

Another factor is the clearance between shaft and bearings. Even with perfectly new bearings this clearance can be measured by tenths of a millimeter. While the machine is at rest, the oil is squeezed out from underneath the shaft, and all the clearance will be between the top part of the shaft and the upper bearing shell. In this position the machine is erected and centered. When the machine is running, the bearing is flooded with oil and the clearance divides equally around the shaft. Very frequently of course, imperfect erection, a bending of the shaft or slight subsidence of the foundations with consequent distortion of the bedplate may be the cause of a displacement of the rotor and stator centers.

We shall now investigate the case of a rotor and stator each with a cylindrical surface, but with the centers displaced by an amount x (the eccentricity). Without displacement, the radical airgap would have a constant value g. The diameter drawn through the two centers we shall call "symmetry diameter". Assume, first of all, a multipolar machine with salient poles, all the exciting coils giving the same number of ampere-turns. In the symmetry diameter the air gap will in one place be reduced to $g - x$, in the opposite place increased to $g + x$. At points, the radius through which describes an angle θ with the symmetry diameter, the increase or reduction of the airgap will be, with close approximation, $x \cdot \cos \theta$. At right angles to the symmetry diameter the increase or reduction will be practically zero.

The flux emitted by one field pole returns through the contiguous halves of the neighboring poles. In a bipolar machine the flux of each pole is the same in spite of air gap differences; in a multipolar machine the poles of one half of the machine will carry a greater flux than those of the other half, unless the winding arrangements prevent this (equalizing connections). These apart, the flux of each pole will be different from that of the others, dependent upon the local air gap. Poles in the diameter at right angles to the symmetry diameter will carry the normal flux, for their neighbor poles in the one half (with reduced air gap) will tend to increase their flux just as much as those in the other half (with increased air gap) will tend to reduce it. The result is that in a multipolar machine each pole may be considered as if it would create its own flux and not be dependent upon adjacent poles.

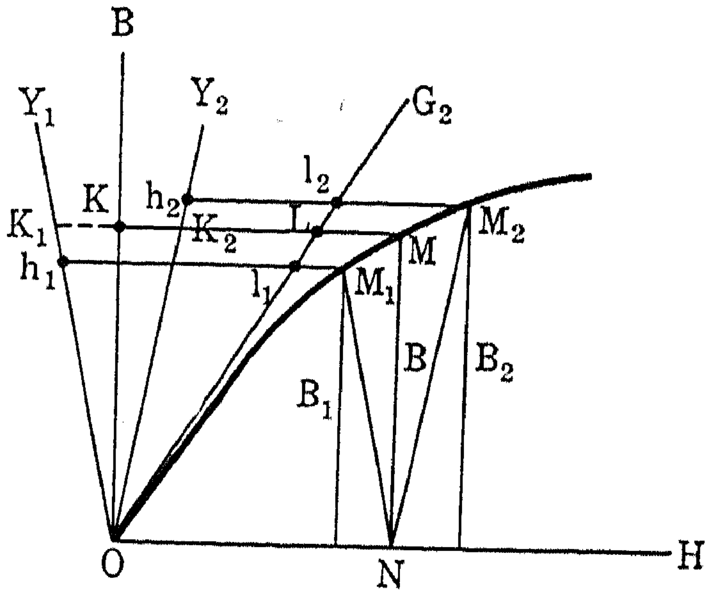

FIG. 2

We assume the machine is excited to give, with the mean air gap, a flux density B lines per sq. cm. of pole face. Where the air gap is reduced to $g - x$, the density will have a greater value B_2, and where the air gap is increased to $g + x$, a smaller value B_1. The first gives a greater magnetic pull in one direction than the second in the opposite direction, and the difference of the two pulls or the "local unbalanced pull" in kilograms per sq. cm. of the pole surface is given by the formula

$$\left(\frac{B_2}{5000}\right)^2 - \left(\frac{B_1}{5000}\right)^2 = \left(\frac{B_2 + B_1}{5000}\right) \cdot \left(\frac{B_2 - B_1}{5000}\right) \qquad (2)$$

It is easy to determine the flux densities B_2 and B_1, if the magnetic characteristic of the machine is given. In Fig. 2 the abscissas H of the curve represent the exciting m.m.f. per pole, while the ordinates B, represent the flux density per sq. cm. of the pole face, with the normal air gap. $O\,N = K\,M$ represents a certain m.m.f., required to produce the induction B. $O\,G$ is the "air gap line", the portion $K\,L$ representing the m.m.f. required for the magnetization of the air gap g, the part $L\,M$ the m.m.f. for the magnetization of the iron. The real existing air gap, however, is not g but is $g + x$ in one case and $g - x$ in the other. We draw through O two lines $O Y_1$ and $O Y_2$ such that for any induction B ($= N\,M$), $K_1\,L$ represents the m.m.f. required for the air gap $g + x$ and $K_2\,L$ the m.m.f. for the air gap $g - x$. We then draw through N two lines respectively parallel to these new axes $O\,Y_1$ and $O Y_2$. The points M_1 and M_2 where lines so drawn intersect the magnetization curve then give the actual air gap densities B_1 and B_2 under the poles with the increased and with the reduced air gap, for a given excitation $O\,N$, because

$$h_1\,M_1 = h_2\,M_2 = O\,N.$$

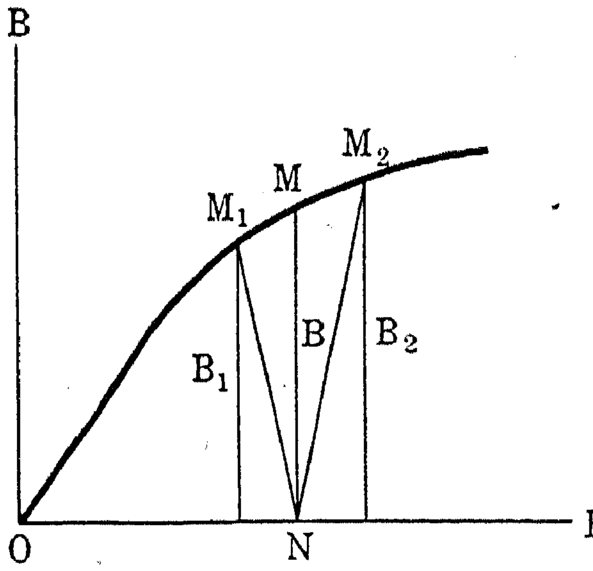

FIG. 3

Fig. 3 is a repetition of Fig. 2 without unnecessary lines.

In general, the abscissas of the lines $O\,Y_1$ and $O\,Y_2$ (Fig. 2) will bear to the abscissa of $O\,G$ for the same flux density very nearly the ratio of the displacement x to the air gap length g. Only if the displacement is very great compared with the average air gap and if the machine has open slots, will the proportionality be markedly disturbed due to the well known crowding of the lines in the air gap near the teeth. Thus, if teeth and slots have equal width and the normal radial air gap is equal to half the width of the slot, then in the extreme case of a machine pulling hard over, the abscissa of the line $O\,Y_1$ will be equal to about 0.85 of the abscissa of the line $O\,Y_2$ which latter one, of course, then coincides with $O\,Y$.

For small displacements, $O\,Y_1$ and $O\,Y_2$ can always be drawn symmetrical with reference to the ordinate, and their m.m.f. for a given induction B is approximately $\pm\, x\,.\,B$, where the displacement x is measured in centimeters and the m.m.f. in gilberts

$$\left(\frac{4\,\pi}{10}\text{ ampere-turns}\right)$$

In formula (2) there appears the sum and the difference of the two inductions B_2 and B_1. In Fig. 4 the part M_1MM_2 of Fig. 2 is drawn on a greater scale and also the chord M_1M_2 which cuts the ordinate of point M in m. The chord describes with the horizontal an angle α. The difference $B_2 - B_1$ is in the figure represented by

$$P_1 P_2 = P_1 m + m P_2 = (M_1 P_1 + P_2 M_2) \cdot \tan \alpha$$

$M_1 P_1$ and $P_2 M_2$ are the abscissas of the x characteristics for inductions B_1 and B_2 respectively, are therefore equal to $x\,B_1$ and $x\,B_2$. We therefore have

$$B_2 - B_1 = (B_2 + B_1) \cdot x \cdot \tan \alpha$$

and the formula (2) can be written in the form

$$\left(\frac{B_2 + B_1}{5000}\right)^2 \cdot x \cdot \tan \alpha$$

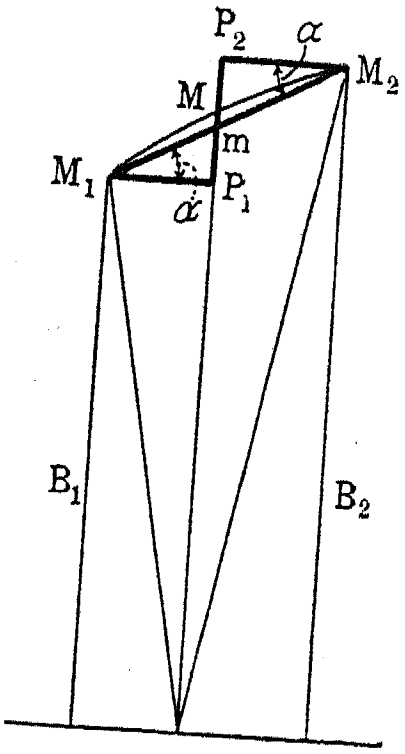

FIG. 4

If x is not very great, the sum $B_2 + B_1$ can be accurately replaced by $2\,B$, and the angle which the chord forms with the horizontal can be replaced by the angle formed by the tangent in point M with the horizontal. Tan α is then the *gradient* of the curve in point M and can be expressed as a differential quotient $\frac{dB}{dH}$ or the ratio of an infinitesimal increase in induction to the increase in m.m.f. causing it.

The unbalanced pull in kilograms per sq. cm. is, therefore,

$$4\left(\frac{B}{5000}\right)^2 \cdot x \cdot \tan \alpha = 4\left(\frac{B}{5000}\right)^2 \cdot x \cdot \frac{dB}{dH} \qquad (3)$$

The expression $x \cdot d\,B$ in formula (3) represents also the m.m.f. necessary to produce an increase $d\,B$ in induction in an air gap x, while $d\,H$ represents the m.m.f. per pole necessary to produce in the magnetic half circuit of the machine with the correct air gap the same increase $d\,B$ in induction. The ratio $\frac{x \cdot dB}{dH}$ is therefore a ratio of magnetomotive forces and can also be replaced by the ratio of the corresponding ampere turns; ampere turns required to drive through the air gap x an infinitesimal increase $d\,B$ in flux density, divided by the ampere turns per pole required to drive through the magnetic circuit of the machine with the correct air gap the same increase in flux density.

Looking closer into the approximations which we introduced in formula (**3**), we find: If x is large and if the magnetization characteristic were a straight line (Fig. 5), the sum $B_2 + B_1$ would actually be greater than $2B$. In Fig. 5 the value $\frac{B_2+B_1}{2}$ is shown dotted in. If, however, the characteristic is strongly curved (Fig. 6), $B_2 + B_1$ will be slightly smaller than $2B$. For hard pulling over, when x reaches the greatest possible value, the saturation will in any case be high enough that the curvature of the characteristic is marked. We are therefore certain that, on this score, formula (**3**) does not give too low values for the extreme case.

A very important question is, now: How does the expression of formula (**3**) change with growing excitation? The first part of almost every magnetization curve is a distinct straight line going through the origin of the ordinates. For every straight line characteristic, whether or not going through the origin, the gradient $\frac{dB}{dH}$ is constant, the unbalanced pull therefore will grow with the square of the induction B. As soon, however, as the saturation of the iron parts is noticeable, the quotient $\frac{dB}{dH}$ will constantly diminish, and the question is how to find the induction B for which the product $B^2 \cdot \frac{dB}{dH}$ is a maximum. In Fig. 7 a graphical procedure is shown. For a point M of the saturation curve I the ordinate $B = NM$ is drawn and a line MP perpendicular to the direction of the curve in point M. The angle NMP is identical with the angle α formed between the curve in point M and the horizontal.

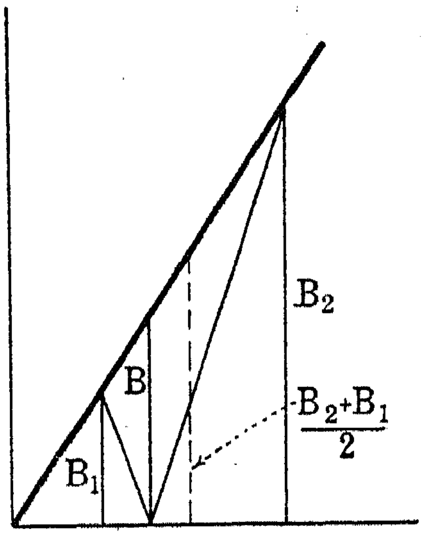

FIG. 5

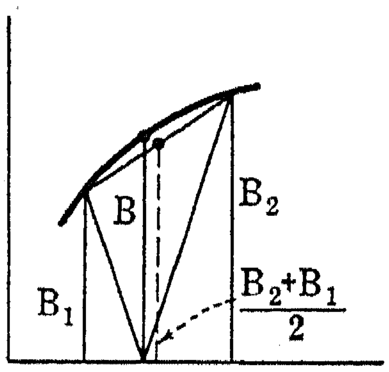

FIG. 6

$$\tan \alpha = \frac{dB}{dH}$$

$$NP = NM \tan \alpha = B \tan \alpha$$

and the area of the triangle NMP is

$$\frac{1}{2} B \cdot B \cdot \tan \alpha = \frac{B^2}{2} \cdot \frac{dB}{dH}$$

If such triangles are drawn for different points of the curve, the triangle area will be proportional to the function $B^2 \cdot \frac{dB}{dH}$ for the various points.

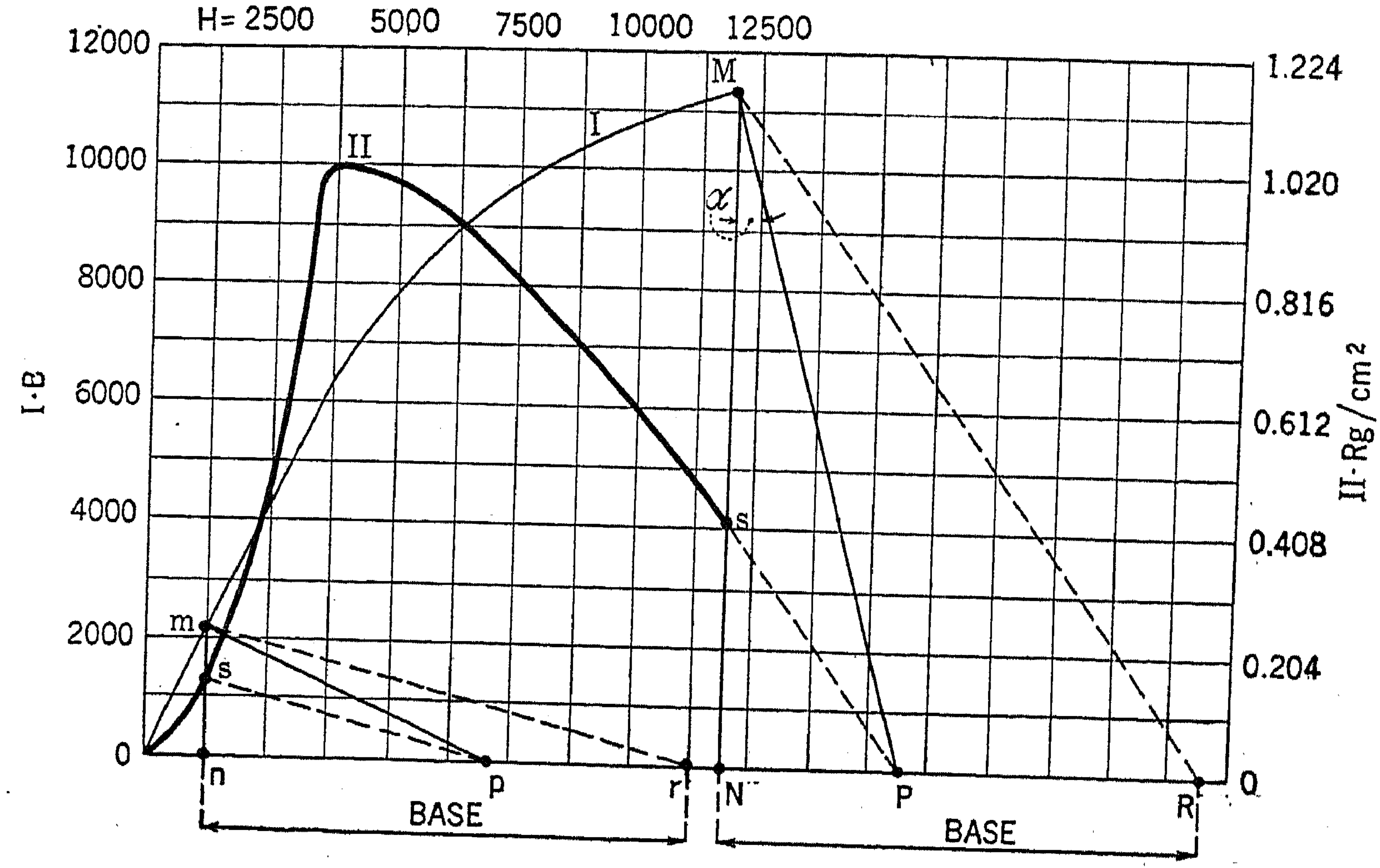

FIG. 7

In order to obtain a curve representing the magnetic pull for every excitation, it is now only necessary to replace all the triangles NPM by others which have a suitable arbitrary base NR. Connecting R with M and drawing a line PS parallel to RM, it follows from the elements of geometry that the triangles NPM and NRS are equal in area. (The line RS of the latter triangle is not drawn in Fig. 7). NS is therefore a measure of the function $B^2 \cdot \frac{dB}{dH}$. The scale is determined by the base NR. If the construction is repeated point for point (for a second point m the construction is shown), we obtain the bold line curve II in Fig. 7, the first part of which, corresponding to the straight portion of the magnetization characteristic, consists of a parabola with vertical axis. Nearly immediately as the magnetization characteristic begins to bend, the curve of the pull reaches its maximum and from then falls steadily.

This teaches us a thing of the utmost importance. In salient-pole machines which are not *entirely* unsaturated, it is not sufficient to calculate the unbalanced magnetic pull for a high excitation. The "critical" excitation for which the unbalanced magnetic pull is a maximum, is reached at the very beginning of the "knee" in the magnetization curve; as generators and motors must be able to stand abnormal changes in voltage, the unbalanced magnetic pull must be calculated for this critical excitation.

In some machines, exciters for instance, and also turbo-generators with cylindrical fields, very high saturation occurs in a small part of the magnetic circuit (saturation plates or teeth), before appreciable saturation occurs in the other parts of the magnetic circuit, and in this case, after a very short initial straight part, a nearly continuous bending of the saturation curve or even two distinct bends can be observed. At even very small induction the curve bends, (the saturation plates becoming saturated), then there is a straight line characteristic and then there is a second bend, (when the saturation of the iron circuit as a whole becomes marked). In this latter case the straight line characteristic does not go through the origin of the coordinates. The first bend may cause a *local* maximum of the unbalanced pull, that is to say, a point higher than the neighbor points, but the important maximum occurs at the second bend of the characteristic. The "short cut" explained later on for the determination of the "critical induction" does not refer to these cases. The character of the unbalanced pull as shown in the bold curve II of Fig. 7 is obtained for most electric machines.

It is not necessary really to construct the whole curve of unbalanced pull, point by point. A few points at the end of the straight line and at the beginning of the curved characteristic are sufficient. What we are interested in is only the *maximum* value of the unbalanced pull and this is, with such characteristics, obtained immediately after the straight line part of the saturation curve is ended. The only task is, then, to find an official "terminus" of the straight line part.

A rule which the author suggests and which seems to give sufficiently accurate results, is to draw a tangent to the saturation curve with a gradient equal to 5/6 of the gradient of the straigh part of the curve. In Fig. 8 the simple construction is show for the characteristic of Fig. 7. Through the origin a line O Z is drawn which has an ordinate of 5000 for the same abscissa fo which the straight line characteristic has an ordinate of 6000. T U is the tangent parallel to O Z; the point of contact with the curve is T. V, the point on the extended straight line, which has the same ordinate B_n as T, is what we will call the end point or terminus of the straight line characteristic.

We now see the two factors which enable us to reduce in a machine with fixed principal dimensions the unbalanced magnetic pull caused by a definite displacement. The first is to increase the air gap, because $\frac{dB}{dH}$ for the straight part of the magnetization curve is very nearly in inverse proportion to the air gap. The second means, however, is to reduce the iron section, for example that of poles and teeth, because that causes the saturation characteristic to bend earlier, or in other words it cuts short the straight part of the magnetization curve.

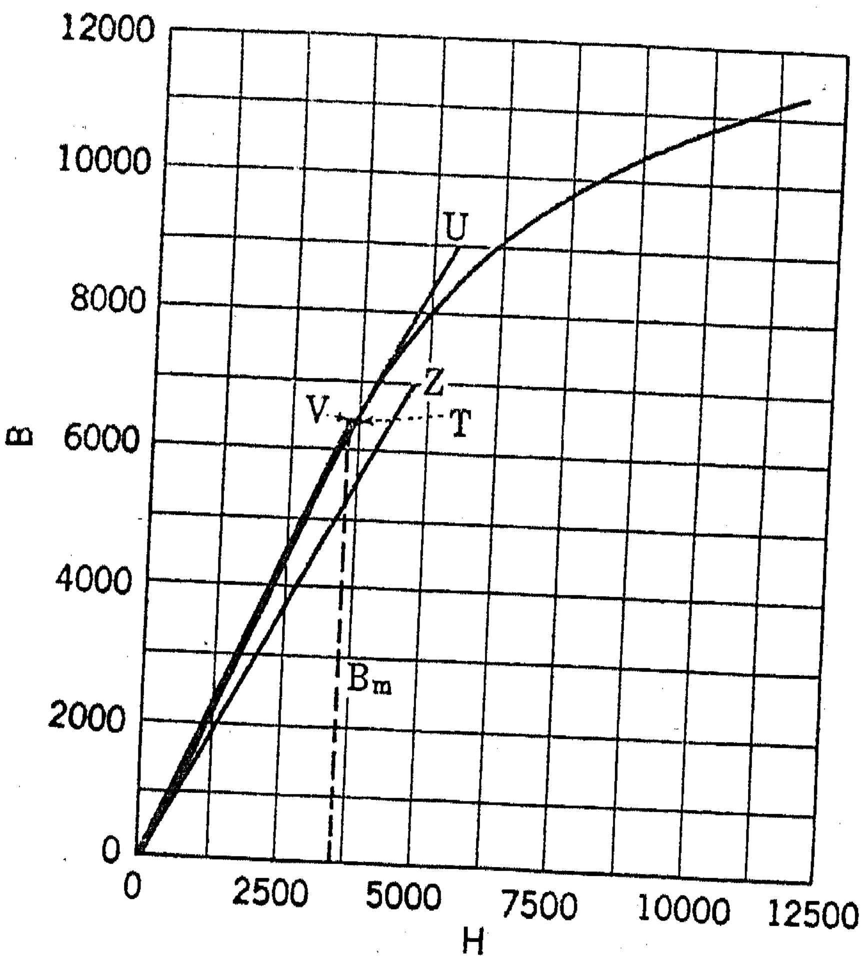

FIG. 8

Up to now we have calculated the radial unbalanced pull per sq. cm. for an eccentricity x. The eccentricity changes in a quadrant from x to zero according to the expression $x \cos \theta$.

The radial magnetic pull per sq. cm. in the different parts of a multipolar machine will therefore be very nearly proportional to $x \cos \theta$ and the component of this pull, working along the "symmetry diameter" will be proportional to $x \cos \theta \cdot \cos \theta = x \cos^2 \theta$.

In machines with a number of poles divisible by 4, we may always consider together two points, separated by a full quadrant one with an angle θ, the other with an angle (90 deg. + θ). The *sum* of the components of these pulls per sq. cm. is

$$4 \cdot \left(\frac{B}{5000}\right)^2 \cdot \frac{dB}{dH} \cdot [x \cos^2 \theta + x \cos^2 (90 \text{ deg.} + \theta)]$$

$$= 4 \cdot \left(\frac{B}{5000}\right)^2 \cdot x \cdot \frac{dB}{dH},$$

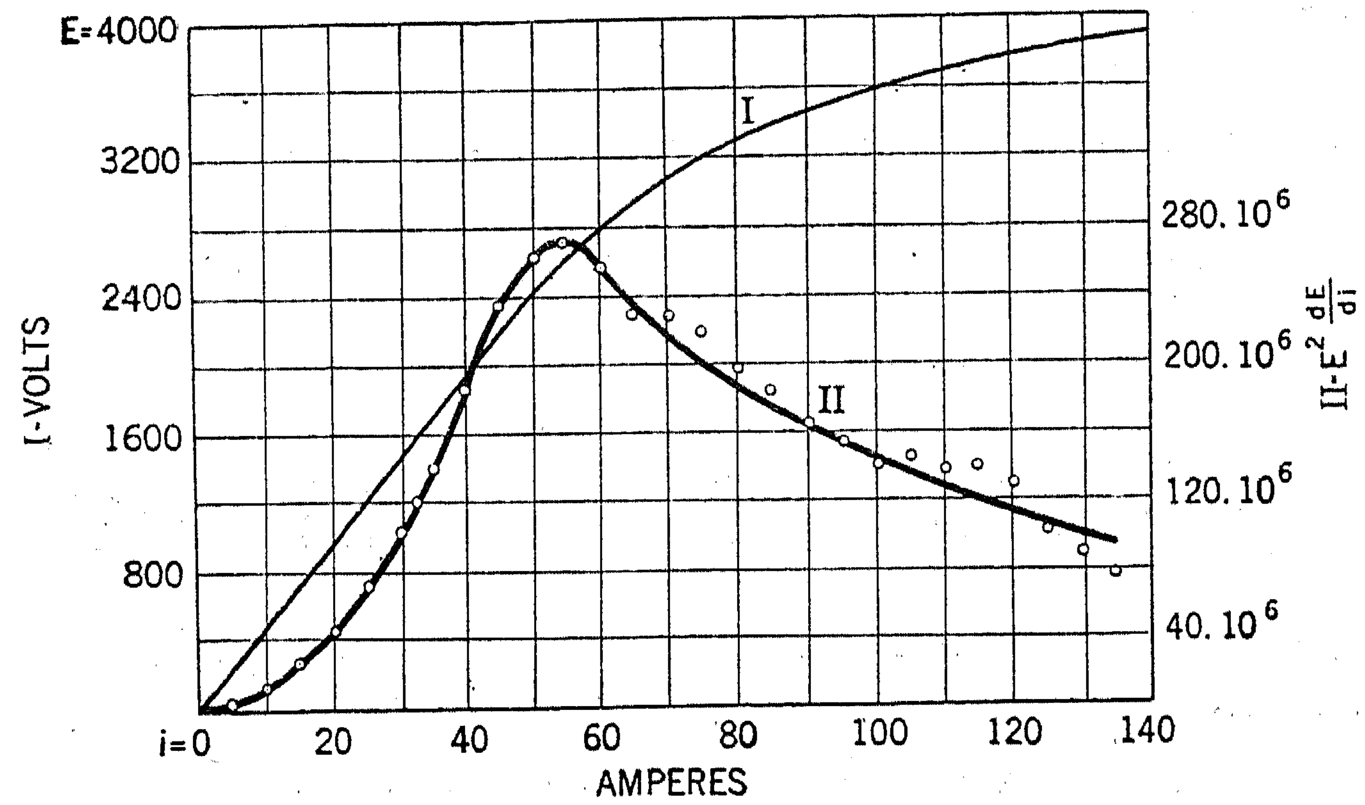

FIG. 9

as cos (90 deg. + θ) = − sin θ and therefore $\cos^2 \theta + \cos^2 (90 \text{ deg.} + \theta) = 1$.

In such machines we can therefore always find two pole pairs which, taken together, develop the same unbalanced pull as a single pole of identical dimensions and induction, the axis of which is parallel and the pole face of which is perpendicular to the symmetry diameter.

In a six-pole machine three pole pairs together develop $1\frac{1}{2}$ times the unbalanced pull of an imaginary pole pair with an axis parallel and a pole face perpendicular to the symmetry diameter, as is proved by the following consideration:

In a six-pole machine we can consider at the same time three places with angles θ, 60 deg. + θ, 120 deg. + θ, and we have $\cos^2\theta + \cos^2(60\text{ deg.} + \theta) + \cos^2(120\text{ deg.} + \theta)$

$$= \frac{1+\cos 2\theta}{2} + \frac{1+\cos(120\text{ deg.} + 2\theta)}{2} + \frac{1+\cos(240\text{ deg.} + 2\theta)}{2} = \frac{3}{2}$$

The same investigation for a machine with 10, 14. . . . .poles would confirm that *the total unbalanced pull can be calculated in all multipolar machines, as if the air gap were reduced by an amount x in one quarter of all the poles, and increased by x in another quarter of all the poles, and left alone in the remaining two quarters.* As poles of electric machines are symmetrical, there is no difficulty in calculating the pull belonging to one half pole.

The total unbalanced magnetic pull F_x of the whole machine will therefore be obtained, if we multiply the pull per sq. cm. from formula (3) with $\frac{p \cdot A}{2}$, or one quarter of the total pole face area of the machine.

$$F_x = \frac{pA}{2} \cdot 4 \cdot \left(\frac{B}{5000}\right)^2 \cdot x \cdot \frac{dB}{dH} = 2pA \cdot \left(\frac{B}{5000}\right)^2 \cdot x \cdot \frac{dB}{dH} \quad (4)$$

Let us call B_m an induction, situated on the extension of the straight line characteristic, which gives the same magnetic pull as is actually obtained by the "critical" induction. In reality B_m will be slightly smaller than the critical induction, but we shall speak, with a small inaccuracy, of B_m as critical induction.

For the straight line part of the characteristic, going through the origin, we can replace $\frac{x \cdot dB}{dH}$ by $\frac{x \cdot B}{H}$ or $\frac{T_x}{T_{circ.}}$ if T_x stands for the ampere turns required to produce in an air gap x any flux density within the straight line characteristic and $T_{circ.}$ stands for the ampere turns per pole, required to produce in the magnetic circuit of the machine with the correct air gap the same flux density.

The formula takes therefore the following simple form:

$$F_x = 2pA \cdot \left(\frac{B_m}{5000}\right)^2 \cdot \frac{T_x}{T_{circ.}} \quad (5)$$

or

$$F_x = 2pA \cdot \left(\frac{B_m}{5000}\right)^2 \cdot \frac{x}{g_1} \qquad (6)$$

if g_1 is the "virtual air gap", that is to say, an air gap which would take, for an induction within the straight line characteristic, as many ampere turns as the real air gap and the iron path of the magnetic half circuit together.

For machines with small air gap, as induction motors, the "virtual air gap" is appreciably higher than the real air gap, say 30 per cent higher or more; while in generators, especially turbogenerators, the virtual air gap is nearly equal to the actual one.

Instead of the total pole face area $2pA$, we may substitute the cylindrical field surface $c \cdot \pi \cdot DL$, if c represents the "pole factor" or ratio of the pole arc to the pole pitch, D the diameter and L the effective length of the field (air ducts, if any, excluded). The formula is then written

$$F_x = c \cdot \pi\, DL \cdot \left(\frac{B_m}{5000}\right)^2 \cdot \frac{x}{g_1} \qquad (6a)$$

The factor c is mostly in the neighborhood of 2/3.

The magnetization curve I shown in Fig. 7 is a reproduction of Fig. 347 from Miles Walker's book. The machine in question is an alternator with 40 poles, each with a pole face area of 650 sq. cm. The radial air gap is 0.51 cm.

$$2p = 40 \qquad A = 650 \qquad g = 0.51 \text{ cm.}$$

The curve II of the unbalanced pull shows that the maximum is 1.02 kilograms per sq. cm. for a displacement of 0.1 cm. The scale for the "pull curve" II is marked on the right of Fig. 7 and it can be verified by figuring the pull for any particular induction on the straight part of the magnetization characteristic. Taking the simpler form of the determination of the maximum (Fig.8) we obtain the critical induction $B_m = 6300$. The straight part of the line goes through a point with an ordinate $B = 4300$ and an abscissa $H = 2500$. The ratio

$$\frac{x \cdot dB}{dH} = \frac{T_x}{T_{circ.}} = \frac{4300 \cdot x}{2500} = 1.72 \cdot x$$

The maximum radial unbalanced pull per sq. cm. is therefore according to formula (3)

$$4 \cdot \left(\frac{B_m}{5000}\right)^2 \cdot \frac{dB}{dH} \cdot x = 4 \cdot \left(\frac{6300}{5000}\right)^2 \cdot 1.72 \cdot x = 10.8 \cdot x$$

or for a displacement of $x = 0.1$ cm., 1.08 kilograms. This method of calculation gives here a result 6 per cent high, which is entirely satisfactory. It is, of course, not necessary to calculate at first the force for one sq. cm. Formula (**5**) gives us direct the unbalanced pull for the whole machine

$$F_x = 2pA \cdot \left(\frac{B_m}{5000}\right)^2 \cdot \frac{T_x}{T_{circ.}} = 40.650 \cdot \left(\frac{6300}{5000}\right)^2 \cdot 1.72 \cdot 0.1$$

$$= 7100 \text{ kg.}$$

If we had used formula (**6**) instead of formula (**5**), we should have obtained at first from the straight line characteristic

$g_1 = \frac{2500}{4300} = 0.58$ cm. (*i. e.* 0.07 cm., or 14 per cent greater than the actual air gap)

$$F_x = 2pA \cdot \left(\frac{B_m}{5000}\right)^2 \cdot \frac{x}{g_1} = 40.650 \cdot \left(\frac{6300}{5000}\right)^2 \cdot \frac{0.1}{0.58}$$

$$= 7100 \text{ kg.}$$

It may be mentioned that Walker figures for the machine in question, which works with a normal air gap density of 9160, an unbalanced pull of 5400 kilograms. If we make the calculation in our way for normal saturation, we should also get approximately the same result. But, as our investigations have shown the normal induction does not give the greatest pull, which must be considered by the designer.

The flux density in the most saturated iron parts (sheet steel) corresponding to an air gap density of 6300 is not more than about 11,000 to 13,000 lines per sq. cm.

A salient-pole machine with given dimensions of the iron parts and with a given air gap experiences for a certain displacement the greatest unbalanced pull with an excitation, which is, as a rule, well below the normal working excitation. There is a passage in Walker's book, which, although it was most likely correctly understood by its author, is apt to be misleading. Walker says on page 56 that in large alternators with a great number of poles and a small air gap, the flux density in the gap must be limited to a moderate value, such as 50 kilolines per square inch (7750 lines per sq. cm.) to keep the unbalanced pull down. Most designers believe indeed that in a given machine with fixed iron dimensions the unbalanced magnetic pull is reduced by lowering the saturation. This is contrary to the result of our investigation, (unless the induction be reduced below the end point of the straight line characteristic.) As long as the iron dimensions are not changed, the flux density may be increased to any desired value, and the effect will in fact be a reduction of the magnetic pull at the normal voltage, although the *possible* maximum unbalanced pull which has to be considered for the mechanical design remains the same. The unbalanced magnetic pull can only be reduced by a reduction of the flux, if a reduction of the iron sections goes hand in hand with the flux reduction.

II. *Multipolar Machines with Distributed Exciting Winding.* Up to the present we have considered machines with salient poles in which the induction over the whole pole face would be constant but for the displacement of the rotor and stator centres. In the case of induction motors, however, and turbo-alternators, the induction changes gradually from zero to a maximum. At first we shall deal with multipolar machines of this description. The rule already arrived at, that the unbalanced pull is the same as if the air gap had been reduced by the full amount x in one quarter of the poles and increased by x in the opposite quarter holds good, from its derivation, also for machines with distributed winding.

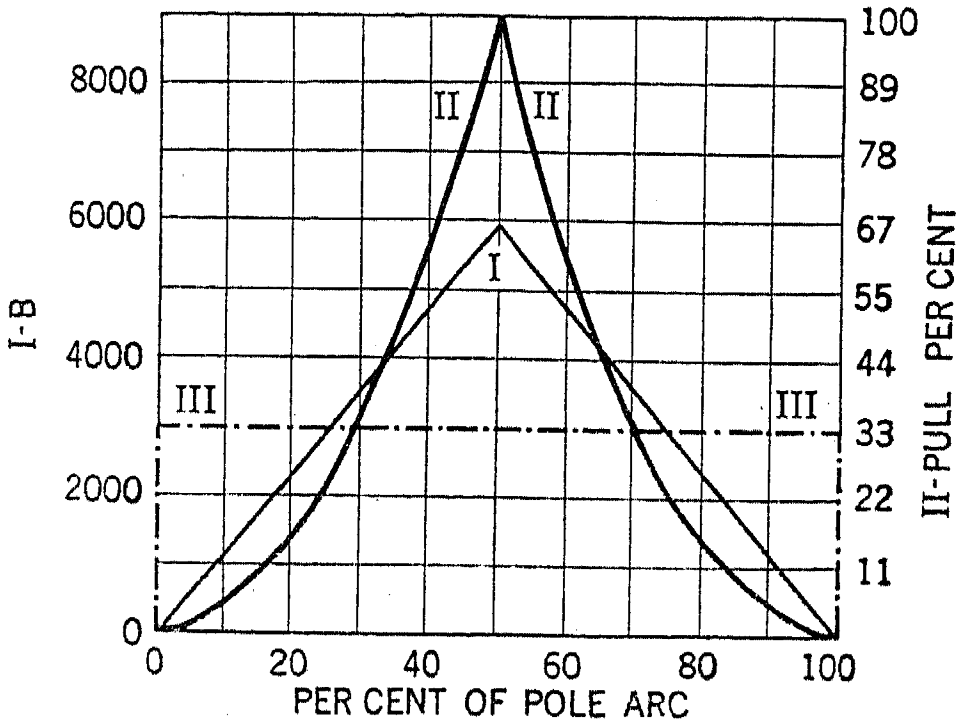

FIG. 10

Let us assume that the curve I in Fig. 7 represents the curve of the air gap saturation of such a machine, while the ordinates of the bold line curve II represent the magnetic pull in kilograms per sq. cm. for a displacement of 0.1 cm. The induction over the pole face may change according to a straight line law, (I in Fig. 10), if the machine is entirely unsaturated and if the exciting winding (in case of turbo-generators for instance) is equally distributed around the whole pole face. These two conditions are hardly ever fulfilled, but the case forms a starting point. For every induction of curve I in Fig. 10 the magnetic pull per sq. cm. can be taken from Fig. 7 and so we obtain as curve representing the local pull over the pole face the bold line parabolas II in Fig. 10. The average value of the pull taken over the whole pole face is in this case exactly 1/3 of the value corresponding to the maximum induction. The area of the dotted rectangle III in Fig. 10 is equal to the area enclosed by the base and the double parabola. This, however, does not represent by any means, the *maximum* unbalanced pull possible in this machine. In Fig. 11 the field form I and the "pull curve" II are drawn for the case that the induction in the center of the pole is increased to 10,000. The average value of the pull, line III, is here 66 per cent of the maximum value. A full review of the possible changes in the average value of the pull is given in curve III of Fig. 12. I and II in Fig. 12 are repetitions of the curves I and II in Fig. 7. The ordinate of curve III for any given abscissa gives the average value of all the ordinates of the pull curve II starting from the abscissa O up to the abscissa in question. As long as the pull curve II follows the law of a parabola, the ordinates of curve III are exactly 1/3 of those of curve II. The further points are obtained by point to point calculation. With this curve it is quite

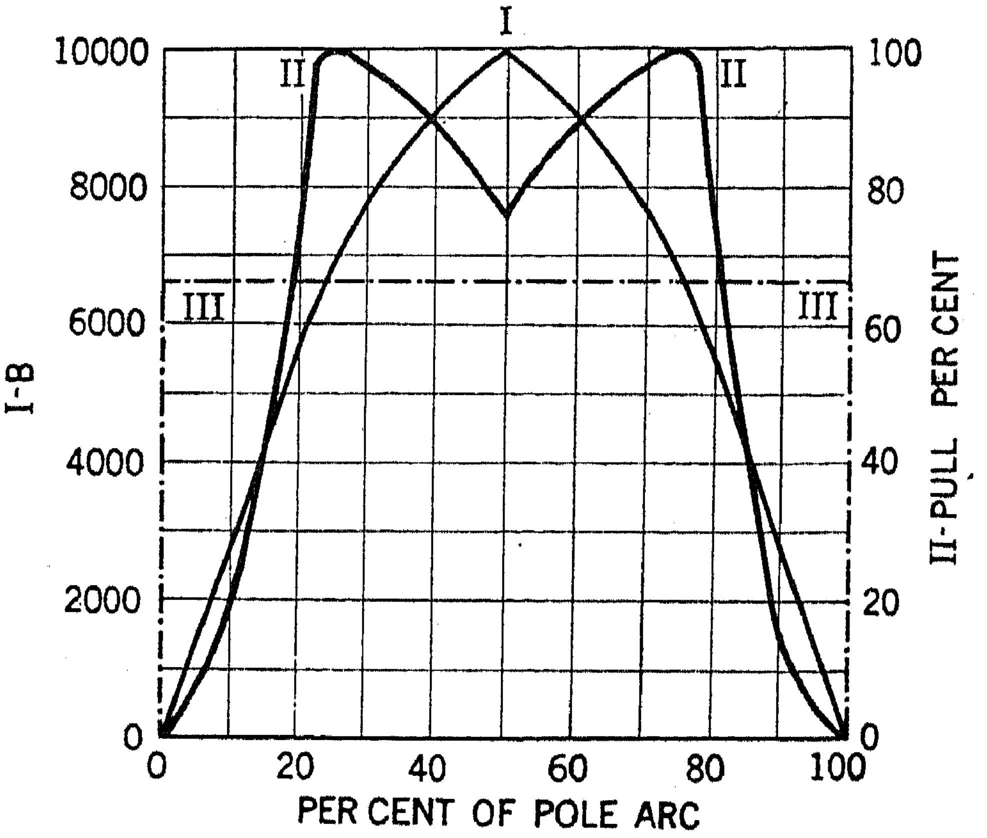

FIG. 11

easy to give the average value of the pull over one half-pole (and therefore over the whole pole), if the total m.m.f. per pole or the highest induction is known. The curve III has a maximum of about 66.5 per cent of the maximum of curve II for a highest induction of 10,500 lines per sq. cm. The curve III is very much flatter in its upper part than the curve II and its ordinates exceed 60 per cent for abscissas varying from $H = 6250$ to $H = 12,000$. In a machine with distributed winding there is not by any means the marked falling off of the unbalanced magnetic pull, after a certain comparatively low induction is reached. On the contrary, the magnetic pull remains very nearly constant over a wide range of possible inductions.

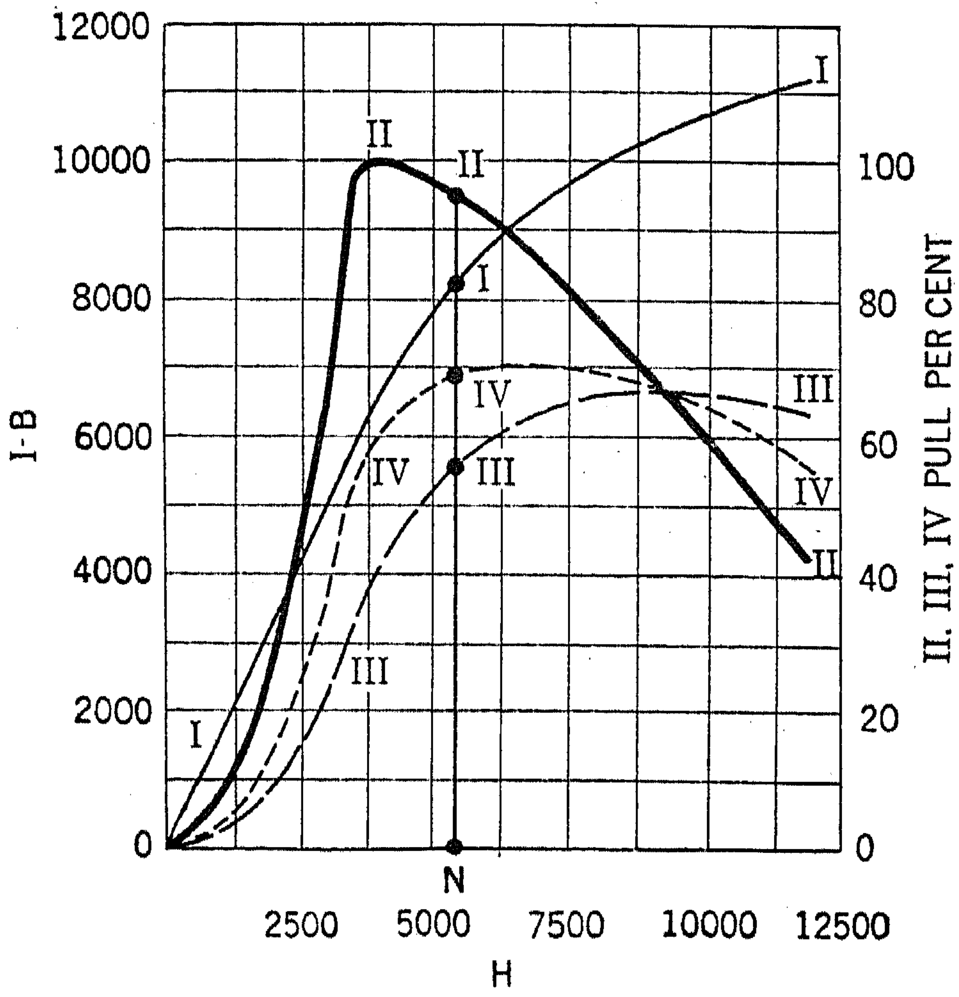

FIG. 12

As a rule, the winding in turbo-generators and induction motors is distributed so as to give a more sinusoidal shape of the flux. For instance, the middle third of a pole face is left without exciting winding, and the winding is only distributed in the first and third part of the pole. If rotor and stator are slotted evenly all around the periphery, as is the case in induction motors, a-c. commutator motors and sometimes also in turbo-generators, the field form can be represented by curve I in Fig. 13, while curve II shows the local pull and the dotted straight line IV the average value of the pull in this case, the area of the dotted rectangle IV being equal to the area enclosed by curve II and the base. It is quite easy to determine the average pull value over the pole for *any* saturation by combining the curves II and III in Fig. 12. It is shown as an example for the abscissa *ON*, which corresponds to an induction *N* I (curve I). Two-thirds of the pole has an induction gradually rising from zero to the value mentioned before. The average value of the pull in this part is given by the ordinate *N* III. One-third of the pole face has a constant induction *N* I, its pull per sq. cm. being represented by the ordinate *N* II. To get the average over the whole pole, we take 2/3 of *N* III and add 1/3 of *N* II, thus obtaining the ordinate *N* IV. The distance III · IV is one-third of the distance III · II. If we know the maximum value of the induction or the total m.m.f. per pole, the ordinate of curve IV gives at once the average value of the pull for this field form. A value of 2/3 of the peak ordinate

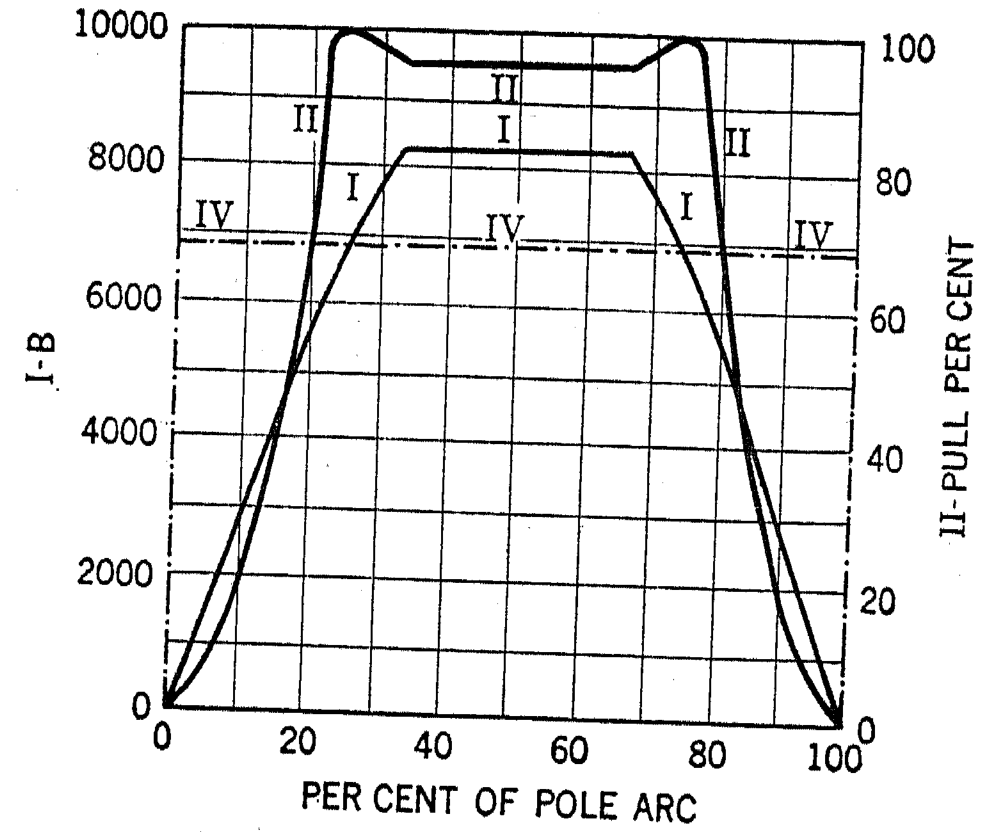

FIG. 13

of curve II is in the example of Fig. 12 already obtained for an induction of approximately 7700, the highest value of IV is 70.5 per cent of the peak value of II and corresponds to an induction of 9300; the line of 2/3 is reached again for an induction of 10,500.

Other magnetization curves were investigated by the author and a slightly higher value was found for the maximum pull of a pole with such distribution of winding.

We can very well say, in a machine with a field slotted all round and a distributed exciting winding, the unbalanced magnetic pull for a given displacement remains very nearly constant with saturations as they are practically applied, and it is approximately 2/3 of the value which would be obtained, if the whole active cylindrical surface of the rotor were equally excited, so as to give the "critical induction". In semi-closed-slot rotors the active rotor surface is very nearly equal π . DL, L representing the effective length, after deduction of air ducts. For such machines we can therefore use the formula obtained for salient-pole machines

$$F_x = c \cdot \pi DL \cdot \left(\frac{B_m}{5000}\right)^2 \cdot \frac{x}{g_1} \qquad \textbf{(6a)}$$

the value of c or "pole factor" being roughly 2/3 for closed-slot rotors. In a salient-pole machine we have called "pole factor" the ratio of the pole arc to the pole pitch and this value as a rule is also about 2/3. *The maximum unbalanced pull in a machine with a field slotted all round and with distributed winding is therefore approximately equal to the maximum pull in a machine with salient poles having a pole factor of* 2/3.

With open-slot rotors (turbo fields) the surface $\pi D L$ should be reduced by the area of the slot openings. It must be born in mind, that, with the reduction of the surface, goes hand in hand a nearly proportional increase of the value of B_m, the critical air gap saturation, because the saturation curve starts to bend when the *iron* saturation, and not when the air gap induction, reaches a certain value. As B_m appears in the second power, an open-slot rotor will give a slightly higher value for the unbalanced pull than the closed-slot rotor.

To get an accurate result in turbo-generator fields which are not slotted in the middle part of the pole, two magnetization characteristics should be considered: The one for the solid part (say 1/3 of the pole width) with the highest induction, and the other for the slotted part, with induction grading down to zero. The critical induction of the slotted part will be appreciably lower than that of the solid part. The unbalanced pull contributed by the slotted parts, will only be about half of that contributed by the solid part. The latter corresponds to a salient pole with a pole factor 1/3. The factor for the whole machine, if B_m *for the solid part only* is put into the formula, will then be about 0.5.

On the other hand, it is possible to adhere to our accustomed pole factor 2/3, if one constructs a *mean* magnetization curve about the average between that of the slotted and that of the plain part and considers the B_m and g_1 of this mean curve.

The case is similar to that of a salient pole with graded airgap.

III. *Salient Pole Machines with Graded Air gap.* Often in salient-pole machines the air gap is not constant but a minimum in the middle of the pole and increased gradually towards the tips. The induction therefore cannot reach the critical value in all parts of the pole face at the same time. The maximum pull will not be reached when the middle part of the pole has the critical induction, because it would mean that all the other parts have a lower induction and therefore contribute an appreciably smaller magnetic pull than the maximum. It will be just as in machines with distributed winding, that the maximum resulting pull is obtained with a higher saturation in the pole center and that the upper part of the curve, representing the magnetic pull dependent on the excitation, is flatter than for a machine with constant induction over the pole face. If in one part of the pole the induction reaches the critical value, this part gives the maximum pull. The parts from there to the corner which have lower values of induction, give a much smaller pull per sq. cm.; the parts towards the center of the pole with higher inductions also give a pull below the maximum, but not falling so rapidly. If the excitation is increased, the point of the critical induction will shift towards the corner, increasing the pull of the corner parts and reducing that of the center parts, so that the total change in pull is not so very marked. If the excitation is lowered, the critical induction will shift more towards the center, increasing the pull there and reducing the pull of the corner parts. In general, it will be possible to change the excitation within comparatively wide limits without very great change of the resulting unbalanced magnetic pull. The calculation of the greatest value of the unbalanced pull can be made, with sufficient accuracy for practical purposes, as if the air gap (without displacement of the rotor center) had a constant mean value.

IVa. *Bipolar Salient-Pole Machines.* In multipolar machines the flux leaving the field at a place where the air gap is reduced by the eccentricity, also returns into the field in a place where the air gap is reduced. The sum total of all the fluxes taken over one-half of the machine will therefore be greater than the sum total of all the fluxes taken over the other half of the machine.

The same possibility exists in a two-pole machine, if the eccentric displacement is at right angles to the field axis. In Fig. 14 this case is shown. A rotor is assumed with salient poles built in the manner of the Siemens *H* armature, with one exciting coil, the pole face covering a greater arc than is usual in actual machines, but presenting a distinct neutral zone. The field axis $A\ A_1$ has a distance x from the stator center. It is clear that in this machine, lines of force leaving the left corner of the upper pole, marked 1, will after passing through the armature, re-enter the field at the left corner of the lower pole, marked 4, while lines leaving the field at the right hand corner 2 of the upper pole will re-enter at the right hand corner 3 of the lower pole. In the half machine on the left side of the axis $A\ A_1$ the air gaps are throughout smaller than on the right side, and as the two sides represent two parallel magnetic paths with different reluctance, the total flux on the left side will be greater than the total flux on the other side. This case is similar to that of a multipolar machine except for the fact on the one hand that through the neutral zone a considerable part of the magnetic pull is cut out, while on the other hand the saturation of the stator and rotor *cores* is hardly influenced by the unequal distribution of the flux in the two machine parts. In order to find out the air gap densities B_2 and B_1 corresponding to a certain displacement x, a magnetic characteristic of the air gap, teeth and pole tips should be drawn, excluding the ampere turns required for the cores, and therefore the critical induction B_m will be higher and the local unbalanced pull resulting from the eccentricity will be greater than it would be if the magnetic characteristic of the whole machine had to be considered.

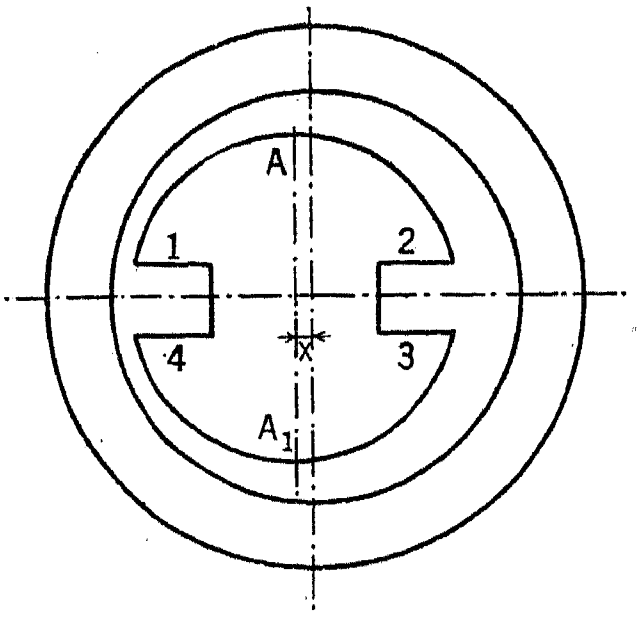

FIG. 14

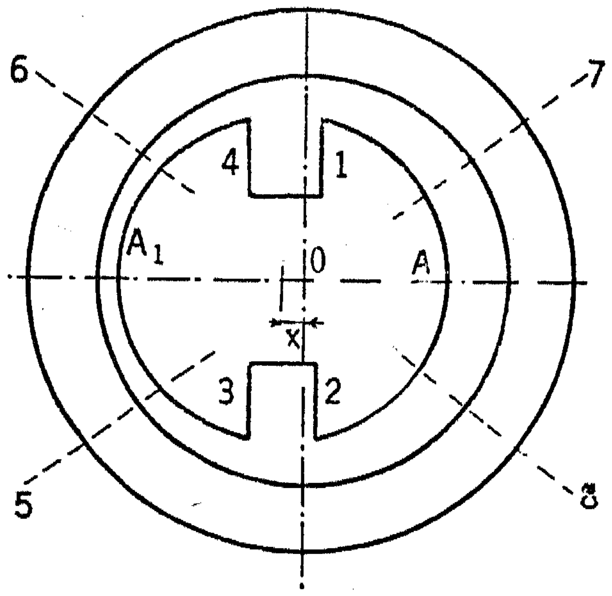

FIG. 15

A different state of affairs exists if the rotor center is displaced with regard to the stator center by an amount x in the direction of the field axis. (Fig. 15). Although the air gap in the left half is smaller than in the right half, the total flux must be equal in both halves, if no flux can return through the neutral zone.

The lines of force pass in series through the smaller and the larger air gaps. The only effect of the eccentricity will be a different distribution of the flux over the pole faces. Let us assume for simplicity's sake that the rotor in a central position the air gap induction over the whole pole face would be equal. Then it is clear, that with the rotor shifted as in Fig. 15 the flux density will be a minimum in the center A of the right pole, where the air gap is a maximum, while it will increase gradually to the corners 1 and 2 of the pole. In the left pole, however, the flux density will be a maximum in the pole center A_1 and will grade down towards the corners 3 and 4. Therefore, considering a square centimeter in the middle A of the right pole and one in A_1 of the left pole, there will indeed act on the rotor an unbalanced magnetic pull directed to the left.

The air gap changes, going in a quadrant from the horizontal to the vertical position, in the left half from the value $g - x$ to nearly the value g; in the right half from the value $g + x$ to nearly g. The average values of the air gaps in the left and right halves are about $g - 0.7\,x$ and $g + 0.7\,x$ respectively.

Within the sector $5 \cdot O \cdot 6$ shown in Fig. 15 the flux density will be greater, and in the corner parts 3 and 4 smaller than the average. In the opposite sector $7 \cdot O \cdot 8$ the flux density will be smaller, and in the corner parts 1 and 2 greater than the average. The sectors mentioned will contribute an unbalanced pull directed to the left, the corner parts a smaller component directed to the right. It is clear that the total unbalanced pull in this case is smaller than in the case of Fig. 14. A two-pole rotating field will therefore, if its center is displaced with regard to the stator center, experience an unbalanced pull which changes twice during a revolution from a maximum to a minimum value.

In practise, a salient-pole of a two-pole machine, covers about 120 deg. In the case corresponding to Fig. 14 (displacement parallel to the neutral diameter) the limits for O from 1 to A are 30 deg. and 90 deg. or $\pi/6$ and $\pi/2$. An element of the surface covering an infinitely small angle $d\,\theta$ has an area $\frac{1}{2}\,DL \cdot d\,\theta$. The component of unbalanced pull in the horizontal direction contributed by this element is

$$\frac{1}{2} \cdot DL \cdot d\theta \cdot 4 \cdot \left(\frac{B_m}{5000}\right)^2 \cdot x\,\frac{dB}{dH} \cdot \cos^2 \theta$$

$$= \frac{1}{2} \cdot DL \cdot d\theta \cdot 4 \cdot \left(\frac{B_m}{5000}\right)^2 \cdot \frac{x}{g_1} \cdot \cos^2 \theta$$

The integration is to be made from + 30 deg. to 90 deg and from − 30 deg. to − 90 deg., or the integration made from 30 deg. to 90 deg. must be doubled. This gives

$$4\,DL \cdot \left(\frac{B_m}{5000}\right)^2 \cdot \frac{x}{g_1} \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \cos^2\theta \cdot d\,\theta$$

The value of the integral is $\frac{\theta}{2} + \frac{\sin 2\,\theta}{4}$ taken between above limits, or $\frac{\pi}{6} - \frac{\sqrt{3}}{8} = 0.307$. Therefore

$$(F_x)_{neutr.} = 1{,}228 \cdot DL \cdot \left(\frac{B_m}{5000}\right)^2 \cdot \frac{x}{g_1}$$

$$= 0.39 \cdot \pi\,DL \cdot \left(\frac{B_m}{5000}\right)^2 \cdot \frac{x}{g_1} \qquad \textbf{(7a)}$$

The pole factor is here 0.39 instead of the accustomed value of 2/3 We see that the unbalanced pull in case of a two-pole salient pole turbo-rotor with 120 deg. pole angle is only 59 per cent of that of a multipolar rotor of the same dimensions and total pole area, provided that the critical induction and the virtual air gap are the same in both cases.

IVb. *Bipolar Salient-Pole Machines with Interpoles.* Bipolar salient-pole machines are used mainly for continuous current, while for two-pole turbo-alternators the cylindrical field and distributed winding is the rule.

If there are interpoles in a two-pole constant-current machine, the total flux of one main pole can be different from that of the other main pole, and the case of Fig. 15 requires a revision for such machines. If, for example, the air gap of the north pole is smaller than that of the south pole, there will be a greater flux in the north pole and the difference in flux will return through the iron of the interpoles, creating in the latter ones south magnetism, whereas normally they would be neutral as long as the armature gives no current. The two interpoles together carry the difference between the actual fluxes of the main poles. One interpole carries the "excess flux" viz. the difference between the greater flux and the average flux. In a four-polar machine which has poles in and at right angles to the symmetry diameter, no m.m.f. is required to drive the excess flux back, for in that case there is, for instance, just as much increase of flux in the upper pole as reduction of normal flux in the lower pole. In the present case, however, a certain amount of m.m.f. is required for the return of the excess flux through the interpole. This will depend mainly upon the section of the interpole and the air gap under the interpole. In a multipolar machine one per cent vertical displacement causes one per cent excess m.m.f., therefore one per cent excess flux in the one pole and likewise one per cent deficit flux in the opposite pole. Here one per cent excess m.m.f. has to drive the excess flux not only through the upper air gap, main pole and half yoke, but also through the interpole. If the magnetic reluctance of the interpole were equal to that of the main pole, the percentage of the excess flux would obviously be one-half of the percentage of excess m.m.f. By introducing

$$\frac{x}{2g_1} \text{ instead of } \frac{x}{g_1} \text{ for } x \cdot \frac{dB}{dH} \text{ in formula (3).}$$

we would obtain the local radial pull for a displacement in the direction of the field axis. In general we may introduce the idea of a "phantom interpole air gap" (Symbol g_2), or the air gap length of an imaginary interpole with the same section and pole tip as the main pole, which would require the same m.m.f. for the passing of a certain flux as the real interpole. While g_1 is only slightly greater than the real main pole air gap g, the phantom gap g_2 will be a multiple of the real interpole air gap, if the section of the interpole is only a fraction of that of the main pole.

Introducing $g_1 + g_2$ instead of g_1, we obtain the element of the unbalanced pull component parallel to the field axis for an induction B_m:

$$\frac{1}{2} DL \cdot d\theta \cdot 4 \cdot \left(\frac{B_m}{5000}\right)^2 \frac{x}{g_1 + g_2} \cdot \cos^2 \theta$$

We assume a pole factor of $^2/_3$ or a pole arc of 120 deg. The total unbalanced pull therefore is given by integrating twice between 0 and $\pi/3$

$$(F_x)_{axis} = 4\, DL \cdot \left(\frac{B_m}{5000}\right)^2 \cdot \frac{x}{g_1 + g_2} \int_0^{\frac{\pi}{3}} \cos^2 \theta \,.\, d\,\theta$$

The solution of the integral is $\frac{\theta}{2} + \frac{\sin 2\,\theta}{4}$, taken between above limits, that is $\frac{\pi}{6} + \frac{\sqrt{3}}{8} = 0.740$

$$(F_x)_{axis} = 2.96 \ DL \cdot \left(\frac{B_m}{5000}\right)^2 \cdot \frac{x}{g_1 + g_2}$$

$$= 0.94 \ \pi \, DL \cdot \left(\frac{B_m}{5000}\right)^2 \cdot \frac{x}{g_1 + g_2} \qquad \textbf{(7b)}$$

If g_2 is larger than $2g_1$ (generally it *is* larger), the pull calculated according to this formula is smaller than the pull due to a displacement at right angles to the field axis, formula (**7a**), which is for interpole machines the same as for other two-pole machines, as long as the armature gives no current.

V. *Bipolar Machines with Cylindrical Field and Distributed Winding.* The case of bipolar cylindrical fields lends itself to easy mathematical, treatment, if we assume an entirely unsaturated sine-shaped field and a rotor and stator slotted evenly around the periphery. $x \cdot \frac{dB}{dH}$ in formula (**3**) is then constant and can be replaced, for a displacement in the direction of the neutral diameter, by x/g_1. The induction at any point, (starting to count the angle θ from the neutral diameter bb_1 in Fig. 16) is $B \sin \theta$. For a displacement x in the direction of the neutral diameter the unbalanced pull is

$$(F_x)_{neutr.} = 2 \int_0^{\frac{\pi}{2}} \frac{1}{2} \cdot DL \cdot d\theta \cdot 4 \cdot \left(\frac{B \sin\theta}{5000}\right)^2 \cdot \frac{x}{g_1} \cdot \cos^2\theta$$

$$= 4 \, DL \cdot \left(\frac{B}{5000}\right)^2 \cdot \frac{x}{g_1} \cdot \int_0^{\frac{\pi}{2}} \sin^2\theta \cos^2\theta \cdot d\,\theta$$

$$= \frac{1}{4} \cdot \pi \, DL \cdot \left(\frac{B}{5000}\right)^2 \cdot \frac{x}{g_1}$$

For a displacement in the direction of the field axis, the conditions are in principle the same as in two-pole machines with interpoles; the phantom air gap of the interpole can be regarded here as equal to the virtual air gap of the main pole. We write

$2\,g_1$ instead of $g_1 + g_2$ and consider further that, if we start to count the angle from the field axis, the local induction is $B \cos\theta$. The unbalanced pull is then

$$(F_x)_{axis} = 2 \int_0^{\frac{\pi}{2}} \frac{1}{2} \cdot DL \cdot d\,\theta \cdot 4 \left(\frac{B \cos\theta}{5000}\right)^2 \cdot \frac{x}{2\,g_1} \cdot \cos^2\theta$$

$$= 2\,DL \cdot \left(\frac{B}{5000}\right)^2 \cdot \frac{x}{g_1} \cdot \int_0^{\frac{\pi}{2}} \cos^4\theta \cdot d\,\theta$$

$$= \frac{3}{8} \cdot \pi\,DL \cdot \left(\frac{B}{5000}\right)^2 \cdot \frac{x}{g_1}$$

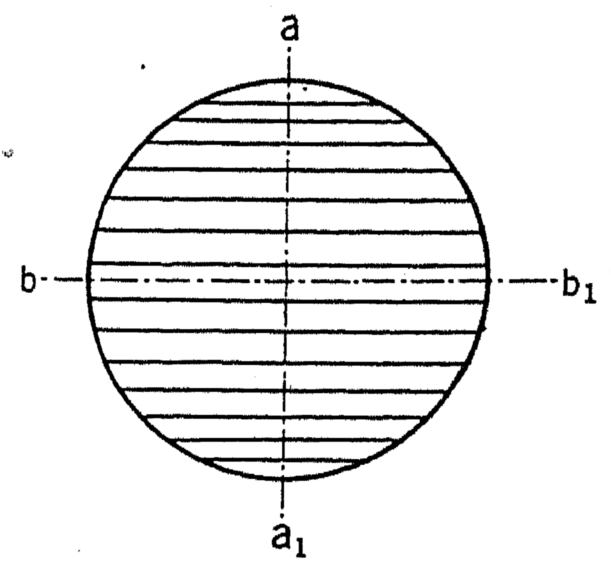

FIG. 16

In this case, a displacement in the direction of the field axis would actually cause a greater unbalanced pull than a displacement in the neutral diameter. The upper limit for B in these formulas would be B_m and we obtain pole factors of 0.25 and 0.375 respectively. The relation of these pulls changes, however, if we increase the saturation. As long as the induction is on the straight line characteristic, $B^2 \cdot \frac{dB}{dH}$ is highest for the point a in the field axis (Fig. 16); this point contributes, for a displacement in the direction of the axis, the full value of its radial pull as vertical component; for a displacement in the direction of the neutral diameter, the horizontal component of the radial pull is zero. If, however, we increase the saturation above B_m, than the point of the maximum $B^2 \cdot \frac{dB}{dH}$ will shift the farther away from the field axis the higher the saturation goes, and the horizontal component of this maximum pull will grow, while the vertical declines.

As here the saturation curve comes into play, we shall abandon the analytical treatment and the assumption of a sine-shaped field and return to a field form like that represented in Fig. 13.

It is clear to us from the beginning that to obtain the highest possible unbalanced pull for a given displacement in the neutral diameter, we shall have to choose the excitation high, so as to obtain the value B_m already for a small angle θ, the cosine of which is near unity.

A four-pole machine with winding distributed evenly over the whole surface and with a zero value of field in the horizontal has a field maximum under an angle of 45 deg. to the horizontal. The maximum pole factor for such a machine was 66.5 per cent for the saturation curve of Fig. 12. By far the greatest part of the magnetic pull, caused by a displacement in horizontal direction, is contributed by the rotor zone lying between 0 deg. and 45 deg., and if we are free to choose the induction of the two-pole rotor for the zone from 0 to 45 deg. exactly equal to that of the four-pole rotor, the unbalanced pull of the two-pole machine will not greatly differ from that of the four-pole machine. In the zone from 45 to 90 deg., it is true, the induction of the four-pole machine will fall again, while that of the two-pole machine will rise from 45 to 60 deg. and remain constant from 60 to 90 deg. But the contribution for the total pull from this zone does not count for very much. The maximum value of the unbalanced pull will be reached in the two-pole machine for an appreciably higher peak saturation than in the four-pole machine.

This is born out in Fig. 17. Curve I represents the induction for one-half pole (from 0 deg. to 90 deg.). The peak induction (from 60 to 90 deg.) is very high viz. 11,100 lines per sq. cm. or 75 per cent above the critical induction. The latter one is reached already at an angle of 20 deg. Curve II represents the local radial pull per sq. cm. for a constant displacement. Both curves I and II are transferred from Fig. 12, only the horizontal scale being changed. To obtain the component of the radial pull in the neutral diameter, we multiply every ordinate of curve II with $\cos^2 \theta$ and obtain thus curve V. For instance, the ordinate of V for an abscissa 10 deg. is equal to that of curve II multiplied with $\cos^2 10$ deg. $= 0.97$. For small angles, the curve V therefore hugs closely the curve II. The greater the angle, however, the smaller the ordinates of V become compared with those of II.

The average value VI of the ordinates of V taken from 0 deg. to 90 deg. is, in this case, 31 per cent of the peak value of the pull curve II. This also would not change appreciably for an *equally* distributed winding, which would reach still higher saturation from 60 to 90 deg.

The same calculation carried through for a peak induction of 9700 gives 28 per cent as the average value.

To compare this with multipolar machines we must consider that there the average of 66 to 70 per cent which we took from the curves III and IV of Fig. 12 was to be multiplied with one quarter of the total pole-face area, while here we have to multiply with one half of the total pole-face area. To bring the result into line with the formula for multipolar machines, the value 0.31 is to be doubled to give the "pole factor". The pole factor is therefore 62 per cent which is very near the value of $^2/_3$ obtained in multipolar machines with distributed winding.

For displacements in the direction of the pole axis we have to multiply the ordinates of curve II by $\sin^2 \theta$ instead of $\cos^2 \theta$, and to half the values thus obtained. The parts of the ordinates between the curves V and II in Fig. 17, if halved, represent this value. The average value of the ordinates between V and II equals here just about the average value of curve V ordinates. The result is, therefore, that for the case represented by Fig. 17 the unbalanced magnetic pull for displacement in the direction of the field axis is about half of the pull for displacement in the neutral diameter.

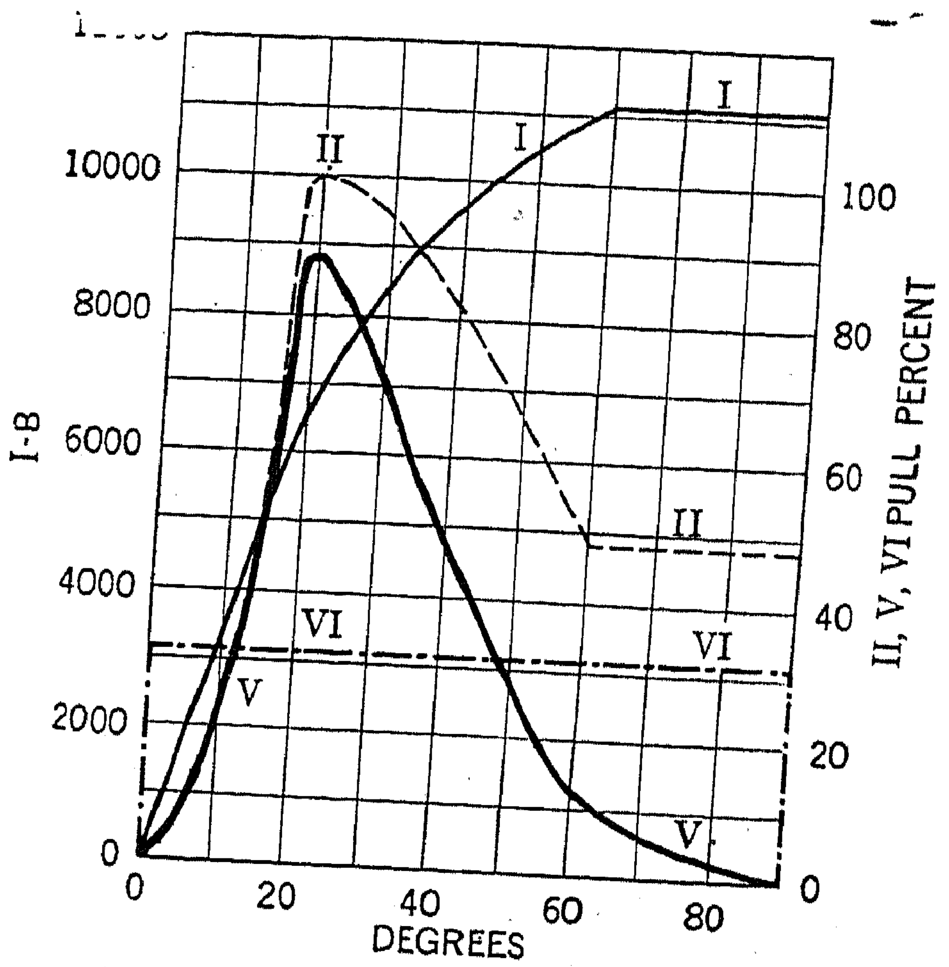

FIG. 17

If the middle part of each pole (60 deg. to 120 deg.) remains unslotted, this will have on the unbalanced pull for a displacement in the direction of the neutral diameter, a small effect which may be deemed to be covered already by the small increase (from 62 per cent to $^2/_3$) which we have made in the value of the pole factor.

The ultimate result is that for multipolar salient-pole and cylindrical fields and also for two-pole cylindrical fields the maximum magnetic pull can be figured from the same formula (**6a**) *and, with approximate accuracy, with the same pole factor, viz.,* $^2/_3$.

C. INCREASE OF ECCENTRIC DISPLACEMENT DUE TO UNBALANCED PULL

A displacement of the rotor from its true central position causes a one-sided magnetic pull. A one-sided magnetic pull, on the other hand, causes a further displacement. There is a force working on the rotor and stator pulling them towards each other in the direction where the air gap is a minimum. This will cause not only an elastic deflection of the rotor shaft but also of the stator frame, of the bedplate and of the pedestals. If the machine is not excited, all its parts experience a certain deflection due to the weights. The elastic deflections of all parts will change if a one-sided magnetic pull occurs, whatever its direction may be. A magnetic pull on the rotor directed upwards will reduce the apparent weight of the rotor and increase the apparent weight of the stator. The transfer of this weight from the rotor bearings to the feet of the stator frame will also change the elastic deflection of the bedplate. The absolute value of the difference in deflection will be the same, whether the unbalanced pull on the rotor is directed up or downwards. But if the pull is directed sideways, the deflection of the bedplate and of the bearing pedestals will be entirely different. All parts of the machine contribute their share to the combined deflection. In many cases the contribution of pedestals and bedplate may be negligible. Rotor shaft and frame, however, must be calculated in *every* case, so as to be strong enough not to increase unduly by consequent deflection an existing initial displacement. All the deflections work in the same sense, so as to reduce the air gap in the spot where it was originally reduced and to increase it where it was originally increased.

The force F_x of the one-sided magnetic pull, is proportional to the displacement x. The elastic deflections of all the machine parts caused under influence of the force F_x are proportional to this force, therefore also proportional to the initial displacement x. The sum of these deflections constitute the "first increment" of the original displacement. This increment in displacement will cause an increment in the one-sided magnetic pull which bears the same ratio to the original pull as the increment in displacement bears to the original displacement. The increment in magnetic pull will cause a second increment in displacement, this a second increment in pull, the latter a third increment in displacement etc. etc. As long as the proportionality between displacement and pull exists, every following increment will have the same ratio to its immediate predecessor as this to its own predecessor. In other words, the initial displacement and its increments are represented by a geometric series which we may write in this form:

$$x + q\,x + q^2\,x + q^3\,x + \ldots.$$

The sum of the series can only have a finite value, if $q\,x$, the first increment of the displacement, is smaller than the initial displacement x, or if q is smaller than unity. The sum of the geometric series of the final static displacement is represented by the formula

$$X = \frac{x}{1 - q} \tag{8a}$$

By substituting into the previous formulas X instead of x, the final static force of the magnetic pull is obtained.

Up to now we have considered static forces and displacements. If, however, a machine is *suddenly* excited, as it is generally the case with an induction motor, the shaft, frame and other parts of the machine will not only deflect so as to increase the initial displacement x to the final value X, but all the parts of the machine being elastic will overshoot the position of the new static equilibrium like a spring or a chord. The combined distance travelled by the parts of the machine from the position of the old into the position of the new equilibrium is $X - x$, and they will overshoot this position by approximately the same amount. The *momentary* displacement therefore will be

$$X_{mom.} = x + 2\,(X - x) = 2\,X - x = x \cdot \frac{1 + q}{1 - q} \tag{8b}$$

The original mechanical displacement must therefore be multiplied with the factor $\frac{1 + q}{1 - q}$ to obtain the displacement occurring at the moment of switching in.

We can now formulate the requirement, that a machine, say an induction motor, should be free from "pulling over," even if the initial displacement of the rotor center is equal to one-half of the mean air gap. In other words, the displacement at the moment of switching in, should be less than twice the initial displacement.

$\frac{1 + q}{1 - q}$ must be smaller than 2. This gives the result:

q smaller than $\frac{1}{3}$.

The smaller q, the safer is the machine against pulling over. If, for instance, $q = 0.2$, the displacement at the moment of switching in, will be only $\frac{1 + 0.2}{1 - 0.2} = 1.5$ of the original mechanical displacement. In other words, the machine would only pull hard over, if the original displacement were two-thirds of the original air gap.

This calculation considers only elastic deflections. It is, however, quite feasible that there is, at the moment of switching in, at first a movement of the machine parts through a space without any appreciable resistance being encountered. Suppose, for instance, that there is a clearance in the bearings and that nevertheless the air gap between stator and rotor core is smaller at the top than at the bottom. At the moment of excitation the rotor will travel freely throughout the clearance of the bearing. Only after the clearance is taken up, an opposing force will be caused through the elastic deflection of the machine parts. The rotor travelling through the clearance has obtained a kinetic force which carry it further than according to formula (**8b**).

A "momentary" appearance of the field does not only occur in induction motors but also in generators, if a circuit breaker opens after a short circuit has taken place. While the short circuit lasts, the actual value of the magnetic field is very small; at the moment of the opening of the circuit breaker, the field suddenly rises and it passes certainly also through the value of the "critical" induction.

A measure for the stiffness of the shaft is the deflection, produced by the action of gravity. If the machine has a vertical shaft, we must, for the purpose of this comparison, figure out the deflection which *would* occur, if the shaft were placed horizontally. We shall call this deflection "gravity deflection." Suppose the frame and the other parts of the machine were infinitely stiff, so that we need not consider any deflection of these parts caused by the magnetic pull. We then could allow such an unbalanced pull as to produce in the rotor shaft a first increment in displacement equal to ⅓ of the original displacement. Let us call f the gravity deflection caused under the static influence of the rotor weight G. If now we take f as original arbitrary displacement, a first increment $f/3$, produced by the magnetic pull, would be permissible. A deflection of $f/3$, however, is produced by a force equal to $G/3$, provided the second force attacks in the same part of the shaft as the rotor weight. The latter condition would be exactly fulfilled, if we exclude from G the weight of the ends of the shaft protruding over the rotor body. But even if this is not excluded, the inaccuracy is very small. We can therefore say: The gravity deflection of the shaft must be within such limits, that the unbalanced magnetic pull produced by a displacement equal to that deflection is less than one-third of the rotor weight.

The deflections of the stator frame and the other parts may be negligible in small machines and also in turbo-generators and high-speed induction motors. In machines with large diameters, however, as low-speed induction motors and engine type generators, they cannot be neglected. There we may divide the quotient q into two parts

$$q = q_r + q_s,$$

q_r being the ratio of the rotor deflection caused by unbalanced pull to the displacement causing the unbalanced pull, and q_s being the ratio of the combined deflection of the other machine parts (principally the stator frame) to the displacement. Then the force of unbalanced pull caused by a deflection f is

$$F_f = q_r \cdot G \tag{9}$$

If we allow

$$q_r = q_s = \frac{q}{2} = \frac{1}{6}$$

we may say that *the shaft must be strong enough so that a displacement equal to the gravity deflection should not produce an unbalanced pull more than* 1/6 *of the rotor weight, provided that the combined deflection of the other parts of the machine under a certain unbalanced pull does not exceed the deflection of the rotor under the same pull.* Should the deflection of the other machine parts under the influence of a certain pull be doubly as great as that of the rotor shaft, the highest permissible value for q_r would be 1/9.

For the calculation of the deflection of the stator frame the force, of course, must not be taken as attacking concentrated in one point, but must be distributed, the parallel components attacking under an angle θ from the symmetry diameter being proportionate to $\cos^2 \theta$.

From formula (**9**) the permissible gravity deflection can be directly deduced. The machine, the characteristic of which is given in Fig. 7, has a rotor weight of approximately 9000 kg. We want to allow an unbalanced pull of not more than say $\frac{1}{6} \cdot 9000 = 1500$ kg. for a displacement equal to the gravity deflection. If now the unbalanced pull for 0.1 cm. displacement is 7100 kg., the permissible gravity deflection is $\frac{1500}{7100} \cdot 0.1 = 0.021$ cm. and the shaft must be made strong enough so that this deflection is not exceeded.

We can establish a direct formula to give us the permissible gravity deflection from the point of view of unbalanced magnetic pull without at first having recourse to the "arbitrary displacement" x.

We use formula (**6a**) for our purpose which is applicable to all machines, and substitute the gravity deflection f for the arbitrary displacement x.

$$F_f = c \cdot \pi D L \cdot \left(\frac{B_m}{5000}\right)^2 \cdot \frac{f}{g_1}$$

On the other hand is

$$F_f = q_r \cdot G$$

It follows

or
$$f = \frac{q_r \,.\, G}{c \cdot \pi D L \cdot \left(\frac{B_m}{5000}\right)^2} \cdot g_1 \qquad (\mathbf{10})$$

$$q_r = \frac{f}{g_1} \cdot \frac{c \cdot \pi D L \cdot \left(\frac{B_m}{5000}\right)^2}{G} \qquad (\mathbf{10a})$$

In the previous example $c \,.\, \pi D L$ is 40.650 and we obtain

$$f = \frac{1/6 \cdot 9000}{40.650 \cdot \left(\frac{6300}{5000}\right)^2} \cdot 0.58 \text{ cm.} = 0.021 \text{ cm.}$$

Suppose the same frame had to be used also for an induction motor with a radial air gap of 0.2 cm. In the generator the virtual air gap was 0.07 cm. larger than the real airgap. Let us assume the same for the motor, then g_1 is 0.27 cm. The critical value of B_m is likely to be reduced in an induction motor, as there, in general, a lower flux is applied and therefore the teeth section correspondingly reduced. This, as a rule, is already necessitated by the greater number of slots which are usual in induction motors. Let us assume that B_m is 5000 for the motor. The rotor core has a diameter of 385.6 cm. and a gross length of 33 cm. from which 5 ventilation ducts of together 6.5 cm. are deducted, giving a net length of 26.5 cm. The total pole surface $\pi\ D\ L$ is therefore

$$\pi \cdot 385.6 \cdot 26.5 = 32{,}100 \text{ sq. cm.}$$

The permissible gravity deflection for $B_m = 5000$ and $q_r = 1/6$, assuming the rotor weight to be again 9000 kg. is

$$f = \frac{1/6\ .\ 9000}{\frac{2}{3} \cdot 32{,}100 \cdot \left(\frac{5000}{5000}\right)^2} \cdot 0.27 \text{ cm.} = 0.019 \text{ cm.}$$

In the assumed case the permissible gravity deflection is nearly the same as the one for the generator, but everything depends on the sections of yoke, rotor and stator teeth, for they determine the value at which the saturation makes itself felt. If the critical induction of the motor were 6000 instead of 5000, the permissible deflection would be reduced to 0.013 cm., and this would also require a stiffer stator frame to comply with the condition that the deflection of the other machine parts, caused by the unbalanced pull, should not exceed, the rotor deflection caused by the same pull. If the stator cannot be made stiff enough, q_r would have to be reduced, and the permissible deflection would be less than 0.013 cm. Should it be impossible to provide such stiffness in the shaft, then the machine should always be provided with a *winding with equalizing connections.*

We see that in all machines (apart from windings with equalizing qualities) the consideration of the unbalanced magnetic pull fixes a limit to the permissible deflection of the shaft under the influence of the rotor weight, just as in high-speed machines the consideration of the critical speed fixes such a limit.

D. INEQUALITIES IN EXCITING COILS

If, *in a multipolar machine*, one pole is excited with a greater or smaller number of ampere turns than the others, the induction in the pole in question and in the adjoining parts of the two neighbor poles will be higher or lower than in the rest of the machine, and an unbalanced pull is set up. The induction is, in a working machine, produced by the action of the primary exciting coil and the reaction of the armature coil combined. Whether the ampere turns of one of the primary coils are different from those of the others and all the armature coils give the same number of ampere turns, or whether the primary coils work identically amongst themselves and one of the armature coils different from the others, the effect will be an unbalanced pull which is stationary, if the disturbing part is stationary, and rotating, if the disturbing part is rotating. Specially, if a coil of the rotating part is accidentally short-circuited, heavy vibrations will be set up. A disturbance in the balance of the machine which only occurs when the machine is excited, accompanied by the necessity of an increase in exciting current in order to obtain a given voltage, is an indication of a partial or complete short circuit of a rotor coil.

In *two-pole turbo-generators* with cylindrical field and distributed winding, the flux density is highest in the middle part of the pole and is gradually reduced to zero, when approaching a line at right angles to the pole axis. As long as this distribution is the same in two opposite poles and the rotor is in a central position, no unbalanced pull occurs. If, however, one of the coils of one pole is short-circuited, the flux distribution in one pole will be different from that of the other pole, and an unbalanced pull will be set up, rotating with the rotor and producing vibrations of the stator.

E. EQUALIZING CONNECTIONS

In Fig. 18 the winding of an a-c. machine is indicated, consisting of 8 coils, the four coils of the upper half being connected in series, as are also the four coils of the lower half. The two paths are connected in parallel. The voltage and periodicity impressed on the two parts are alike, and as both parts consist of the same number of identically situated coils, the total flux passing the upper half will be nearly identical with the total flux in the lower half. Any dissymetry would be practically wiped out by a slight increase of magnetizing current in the one and corresponding reduction of the magnetizing current in the other.

An arrangement like this will therefore prove effective against the consequences of a vertical displacement of the rotor from its true central position. It is, however, entirely ineffective with regard to horizontal displacement. If the rotor is displaced to the left from the true central position, the coil 1_a through which the same magnetizing current is flowing as through the coil 4_a, will cause a stronger magnetic field than the latter, and so will the coil 4_b create a stronger field than the coil 1_b. The result therefore will be an unbalanced magnetic pull tending to create an increment of the displacement which is directed to the left of the rotor.

In a two-phase machine it is possible to arrange the windings of one phase with a horizontal symmetry line, the windings of the other phase with a vertical symmetry line, and so to provide to a certain extent for two components of the magnetic pull.

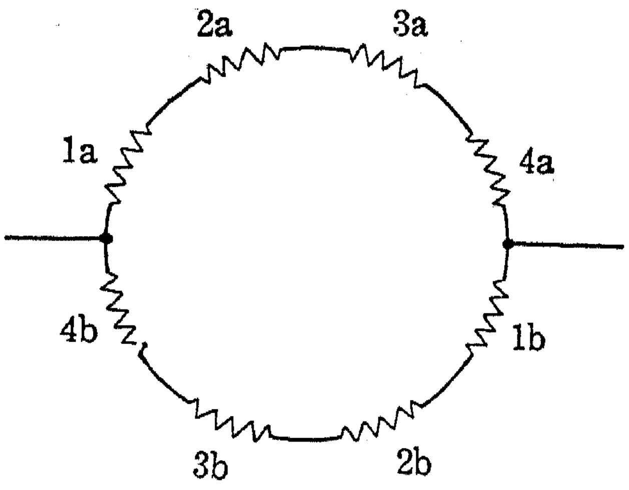

FIG. 18

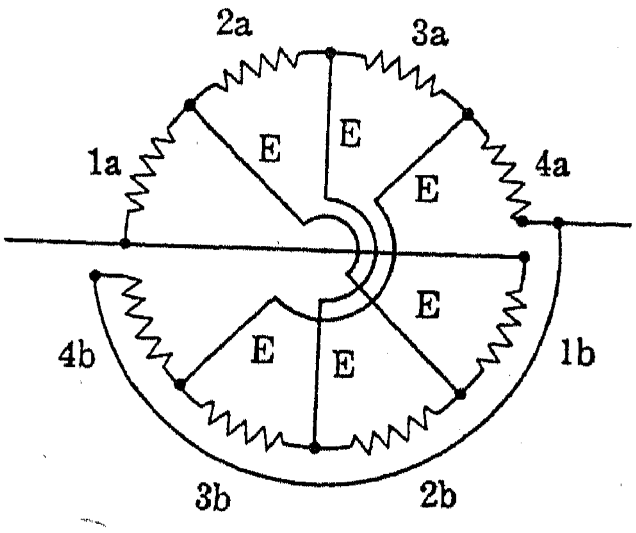

FIG. 19

In a three-phase machine the symmetry lines of the windings can be arranged mechanically under a 60-deg. angle.

A far more effective arrangement, however, is obtained, if equipotential connections *E E E* are added to the windings of every phase, strictly equalizing diametrically opposite coils, as shown in Fig. 19. The coil 1_a is equalized against the diametrically opposite coil 1_b. On both coils the identical voltage is impressed, the flux and the magnetic pull on both parts will therefore be very nearly identical. This is true of all the coils around the whole circumference and therefore the unbalanced pull will amount to a negligible figure.

The necessary requirement for magnetic equalization is of course, that the coils which are equalized against each other, contain the same number of identically situated turns. If, however, a turn should accidentally become short-circuited or open-circuited, the magnetic balance would of course immediately be upset. Equalizing arrangements in the windings therefore do not free the designer from the duty to design the mechanical parts strong enough so as to deal, in case of accidents, with the force calculated in formula (**1**), but for ordinary running conditions the unbalanced pull need not be considered.

In multipolar *continuous current machines* with multiple winding but without equipotential cross connections and with as many brush arms as poles, an incomplete magnetic equalization between pole pairs is produced by currents flowing between the brushes of the same polarity. The part of the armature winding moving in a stronger field generates more current, the reaction of which weakens the field. As, however, large continuous currents are required to produce any marked weakening effect by armature reaction, the commutation becomes, as is well known, very seriously impaired, if there are any sensible magnetic inequalities. The tendency to equalization of pole strength refers here to all north poles and equally to all south poles round the machine.

If equipotential cross connections are applied, equally situated armature coils are subjected to the same *alternating* voltage, just as the coils of Fig. 19.

The current flowing through the equalizing or equipotential connections as "wattless" current, is as a rule only small, as comparatively few ampere turns in the armature are sufficient to correct the difference in field strength caused by small differences in the air gap. If, however, by accident an exciting coil is short-circuited, a very large current will flow into the part of the armature winding influenced by the pole in question, and therefore it is not wise to stint in the section of the equalizing connections. Cases are known of four-pole constant-current turbo-generators in which the equalizing connections of the armature have been burnt out due to accidental occasional short circuit of a series field coil.

Equalizing connections between coils excited with continuous current are, of course, useless for our purpose, as the current distribution in such coils is not influenced by the field strength, but solely by the ohmic resistance of the parallel paths. In a machine at rest, excited with direct current, no equalization of flux or pull takes place.

F. EFFECT OF UNBALANCED PULL ON ROTATING MACHINE

The effect of the unbalanced magnetic pull due to eccentricity may be of a twofold character. If the outer rotor surface and the inner stator surface are truly cylindrical but not concentric, while the rotor taken by itself is running true, the forces exercised by the unbalanced magnetic pull have always the same direction in space independent of the rotation and are in the case of multipolar machines constant, and only in the case of two-pole rotating fields variable.

If, however, the rotor itself is untrue, no matter whether the field or the armature is rotating, then the place of the smallest air gap is rotating round with the rotor and therefore the force of the unbalanced pull rotating round. As far as the rotor is concerned, the effect will be the same, as if its weight were not balanced mechanically, only with the difference that the unbalance is dependent upon the excitation of the machine. The rotating force acts, however, also on the stator and will set up in low-speed machines visible "breathing"; in high-speed machines sensible vibrations of the frame, the frequency of vibrations being equal to the number of revolutions. We shall not go, in this paper, into the question what "phase difference" exists between the passing of the eccentric part of the rotor and the radial oscillations of a point of the stator, but shall assume in the following section that the stationary parts of the machine are infinitely stiff, so that the above question is eliminated from the following investigation.

G. INFLUENCE OF UNBALANCED PULL ON CRITICAL SPEED

If the center of gravity of a rotor does not coincide with the axis of rotation, but has a distance x from the same, a centrifugal force will be developed which is proportionate to the displacement x and to the square of the number of revolutions per minute. At a certain speed the centrifugal force will be so great that it would cause (in the absence of friction etc.) a deflection of the shaft equal to the original displacement. The displacement being doubled, the centrifugal force would be doubled causing a double increment in deflection etc. At this speed therefore, the so-called "critical" speed, an original displacement, if not infinitesimal, would cause an infinite final displacement, if there were no resistance to these movements. Although in reality the resistance will keep the displacement down to finite values, the great increase of vibrations at the critical speed is highly objectionable and there is always careful calculation in the design of high-speed machines to make sure that the running speed is well above or well below the critical speed.

In an electric machine, the unbalanced pull due to the rotor displacement will work in the same direction as the centrifugal force and is itself proportionate to the displacement, but independent of the speed. If we call x the displacement in centimeters, the displacement measured in meters will be $\frac{x}{100}$. Let G be the rotor weight in kilograms, then the mass of the rotor in terrestrial units is $\frac{G}{9.8}$, the acceleration of gravity being 9.8 m. per second per second. Let n be the number of revolutions per minute. Then the centrifugal force is

$$4 \cdot \frac{G}{9.8} \cdot \frac{x}{100} \cdot \pi^2 \cdot \left(\frac{n}{60}\right)^2 = x \cdot G \cdot \left(\frac{n}{300}\right)^2$$

The unbalanced magnetic pull for a displacement equal to the gravity deflection f is $q_r \cdot G$ and for a displacement x is

$$\frac{x}{f} \cdot q_r \cdot G$$

Assuming that the stationary machine parts are infinitely stiff (so that $q_r = q$), the same formulas hold good for the rotating machine.

The sum of centrifugal force and unbalanced pull is

$$x \cdot G \cdot \left(\frac{n}{300}\right)^2 + \frac{x}{f} \cdot q \cdot G$$

The elastic gravity deflection, caused by the weight G is f, the elastic deflection caused by one kilogram is f/G. To obtain the elastic deflection caused by the above sum of forces, we have to multiply by f/G.

$$\frac{f}{G} \cdot \left[x\,G\left(\frac{n}{300}\right)^2 + \frac{x}{f} \cdot q\,G\right] = x \cdot \left[f \cdot \left(\frac{n}{300}\right)^2 + q\right]$$

For the critical speed this deflection will be equal to the original displacement x, the expression in brackets therefore will be unity for the critical value of n

$$f \cdot \left(\frac{n_{crit.}}{300}\right)^2 + q = 1$$

$$n_{crit.} = \sqrt{1 - q} \cdot \frac{300}{\sqrt{f}} \qquad \textbf{(11)}$$

For the unexcited machine, q is zero and the critical speed is $\frac{300}{\sqrt{f}}$, (f given in cm.), which formula, of course, is well known. The critical speed is reduced, if the machine becomes excited and will be the lowest for that excitation which gives the highest value of the unbalanced magnetic pull. If q is equal to 1/6, the critical speed will be

$$\sqrt{\frac{5}{6}} \cdot \frac{300}{\sqrt{f}} = 0.912 \cdot \frac{300}{\sqrt{f}}$$

If q is equal 1/3, the critical speed will be

$$\sqrt{\frac{2}{3}} \cdot \frac{300}{\sqrt{f}} = 0.817 \cdot \frac{300}{\sqrt{f}}$$

An electric machine without equalizing connections has not *one* definite critical speed, but *a range* of critical speeds according to its excitation. In two-pole turbo-generators it must also be considered that the unbalanced pull in the diameter of the field axis is different from that at right angles to the field axis, and therefore such a machine will even for a definite excitation have different critical speeds in respect to these two diameters, similar to a rotating shaft with rectangular section.

To illustrate the numerical influence of the unbalanced pull on the critical speed, we may take a rotor of 60-cm. diameter and 100-cm. rough length of the iron core. After deduction of air-ducts and slot openings, 80 per cent of the cylindrical surface, that is $0.8 \cdot \pi \cdot 60 \cdot 100 = 15{,}100$ sq. cm. may be considered as effective. We will assume that the rotor belongs to a two-pole machine with distributed field winding, running at 3000 rev. per min., that the actual radial air gap is 1.5 cm., the virtual air gap 1.6 cm., and will consider as alternatives first, a mechanical critical speed of 3600, then of 1800. The weight of the rotor may be 3000 kg. We will further assume that the critical induction for which the unbalanced local pull is a maximum, is 6000 lines per sq. cm.

As the mechanical critical speed is given, we can figure directly the gravity deflection of the rotor shaft

$$f = \left(\frac{300}{n_{crit.}}\right)^2 = \left(\frac{300}{3600}\right)^2 = 0.0070 \text{ cm. and } \left(\frac{300}{1800}\right)^2$$

$$= 0.0278 \text{ cm. respectively}$$

We can figure $q = q_r$ directly from formula (**10a**). The pole factor c we will take as $^2/_3$.

$$q = q_r = \frac{f}{g_1} \cdot \frac{c\,\pi\,D\,L \cdot \left(\frac{B_m}{5000}\right)^2}{G} = \frac{f}{1.6} \cdot \frac{\frac{2}{3} \cdot 15{,}100 \cdot 1.44}{3000}$$
$$= 3.02 \cdot f$$

that is, 0.021 and 0.083 respectively.

$\sqrt{1-q}$ is $\sqrt{0.979} = 0.989$ and $\sqrt{0.917} = 0.956$ respectively. The influence of the magnetic pull on the critical speed is here small, it lowers the cricical speed only 1.1 per cent in one case, 4.4 per cent in the second case.

If the rotor belonged to a four-pole machine with half the virtual air gap, and a mechanical critical speed of 1800, q would be 0.166,

$$\sqrt{1-q} = \sqrt{0.834} = 0.913,$$

which would reduce the critical speed to 0.913 . 1800 = 1640 rev. per min. a value which is already uncomfortably near the running speed of 1500.

In the case of an induction motor with wound rotor and with the same core dimensions, the same rotor weight, a virtual airgap of 0.3 cm. (mechanical 0.2 cm.), and a critical induction of 5000 lines per sq. cm. and a critical speed of 3600 and alternatively 1800, q would be

$$q = \frac{f}{0.3} \cdot \frac{\frac{2}{3} \cdot 15{,}100 \cdot 1}{3000} = 11.2\,.\,f$$ that is 0.078 and 0.31 respectively. In the latter case, the critical speed would be reduced by the magnetic pull to $\sqrt{0.69} \cdot 1800 = 1490$ rev. per min. This could not be accepted for a machine running at 1500 rev. per min. and it would be necessary to stiffen the shaft.

This example has been simplified by assuming the same rotor weight, whether the rotor is destined for 1500 or 3000 rev. per min., for a two- or four-pole machine, generator or motor. In reality, for an exact calculation, differences in weight would have to be considered, also certain minor differences in effective surface, critical induction, pole factor etc., etc.

CONCLUSIONS

The main conclusions arrived at in this paper are as follows:

1. The mechanical parts of a multipolar machine, whether its winding is provided with equalizing connections or not, should

be designed strong enough to withstand, as an emergency condition, without overstraining, the magnetic pull produced, if only one-half of the machine is magnetized up to full saturation of the iron, while the other half is unexcited. In machines with small air gap the rotor core may pull hard over towards the stator core, under these conditions.

2. Under normal working conditions, a reduction x of one air gap and an increase x of the diametrically opposite air gap causes in multipolar machines locally an unbalanced magnetic pull per square centimeter, given by the formula

$$4 \cdot \left(\frac{B}{5000}\right)^2 \cdot \frac{x \cdot d\,B}{d\,H} \text{ kilograms} \qquad (3)$$

This pull has a maximum value for a comparatively *low* state of saturation. For higher saturation the unbalanced pull falls off. If the magnetic characteristic of the machine is represented by an initial straight part of appreciable length, followed by a distinct "knee", the "critical induction" is in the very beginning of the knee. For an approximate determination of the maximum pull take the induction B_m of that point of the magnetization curve in which the gradient is 5/6 of that of the straight line part and consider B_m as the end point of the straight line characteristic. The maximum unbalanced pull in kilograms per square centimeter is

$$4 \cdot \left(\frac{B_m}{5000}\right)^2 \cdot \frac{x}{g_1}$$

g_1 being the "virtual" air gap of the machine.

(Virtual air gap, an air gap which would take for an induction within the straight line characteristic as many ampere turns as the real air gap and iron path of half the magnetic circuit together.)

3. In multipolar machines, the total unbalanced pull, for a given displacement x of rotor and stator centers, is independent of the rotor position. Its maximum value in kilograms is

$$F_x = c \cdot \pi\, D\, L \cdot \left(\frac{B_m}{5000}\right)^2 \cdot \frac{x}{g_1} \qquad \textbf{(6a)}$$

c, the "pole factor", being approximately $^2/_3$, both for machines with salient poles and for those with cylindrical fields and distributed windings. In machines with salient poles and constant air gap the maximum unbalanced pull is reached at a low saturation and falls off distinctly at higher saturations.

In machines with distributed field winding and also in salient-pole machines with graded air gap, the maximum pull is reached for higher saturations and changes less with changes in saturation.

In machines with cylindrical fields which are not slotted evenly around the periphery, and machines with salient poles and graded air gap, the saturation curve from which B_m and g_1 are taken, is a mean of the component saturation curves.

4. In bipolar machines, the unbalanced pull has different values for displacement in the direction of the field axis and at right angles thereto. The highest value for a cylindrical field with distributed winding is reached for high peak saturation, and can be calculated from formula (**6a**) with a pole factor of approximately $^2/_3$. Only the saturation curve of the *slotted* part should be taken to determine B_m and g_1, in bipolar machines which contain a non-slotted middle part of each pole. For salient-pole machines with 120 deg. pole arc, the pole factor is about 0.4. The saturation curve, from which B_m and g_1 are taken, should exclude in bipolar machines the m.m.f. required for the bodies of stator and rotor cores.

5. Assuming that under the influence of unbalanced magnetic pull the air gap experiences equal changes through the deflection of the rotor shaft and through the combined deflection of all other machine parts, the magnetic pull caused by a displacement equal to the "gravity deflection" of the shaft must not exceed 1/6 of the rotor weight, to ensure that the displacement, at the moment of sudden excitation, does not exceed twice the value of the original mechanical displacement.

If the other parts of the machine were infinitely stiff, the magnetic pull caused by a displacement equal to the gravity deflection could be allowed to approach ⅓ of the rotor weight.

("Gravity deflection" is the static deflection of the *horizontally situated* shaft under the influence of the rotor weight.)

6. The critical speed is lowered by the unbalanced magnetic pull and varies with changes in excitation. For the greatest permissible value of unbalanced pull, mentioned in the preceding rule, the reduction of critical speed amounts to 18 per cent. In machines, the winding of which has full equalizing qualities, no reduction of the critical speed takes place.

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ECONOMIC PROPORTION OF HYDROELECTRIC AND STEAM POWER

BY FRANK G. BAUM

ABSTRACT OF PAPER

This paper describes a new method of determining for any power system, what proportion of generation should be hydroelectric, and what steam, from the standpoint of economics.

A method is outlined for obtaining a curve showing "Total cost per kilowatt-year for hydroelectric and steam power", for any percentage combination of generation.

With system load curve, fixed charges on steam and hydroelectric plants, cost of fuel and other steam energy charges all known quantities, this curve can readily be calculated, and one can see at a glance the limiting economical percentage of steam power for the given conditions.

IT IS, of course, well known that steam power is usually less expensive for low-load factors than hydroelectric power, and the latter becomes economical only when the load factor is favorable. To determine the economical division between the two there are usually given curves of cost varying with load factor. Such curves show that at certain load factors the cost of steam power exceeds the cost of water power, but the actual yearly cost of power for any assumed proportion between water power and steam power must be calculated for each case. This becomes laborious.

The results can, however, be shown in a much more illuminating way if presented as shown in Fig. 1. In this figure, abscissas from left to right (from O to O^1) show percentage of total load carried by water power and from right to left (from O^1 to O) the abscissas show the percentage of total load carried by steam power. The sum of the steam power and water power must of course equal 100 per cent for every condition, hence the sum of the two abscissas is 100 per cent at any point.

If now we take the yearly cost per kilowatt of hydroelectric power as $O^1 h$, taken in the figure at $22 per kilowatt-year, and draw the straight line $O h$, this line will represent by any ordinate the yearly charge per kilowatt against the water power for any proportion between steam and water power. For it is clear that if we have one-half water power then the yearly charge per kilowatt against the entire 100 per cent load is $11.

(In comparing the cost of power we must of course include total cost of delivery to center of load.)

Similarly, if we take the yearly fixed cost per kilowatt of steam power as *O s*, taken in the figure as $11 per kw-year, and draw the line O^1 *s*, this will represent by any ordinate the yearly charge per kilowatt against the steam power for any proportion between steam and water power.

The straight line *s h* then represents the total fixed charge against the steam and hydroelectric power for any proportion of steam and water power. (*O h* and O^1 *s* being straight lines and *s h* being derived by adding the ordinates, gives another sraight line.)

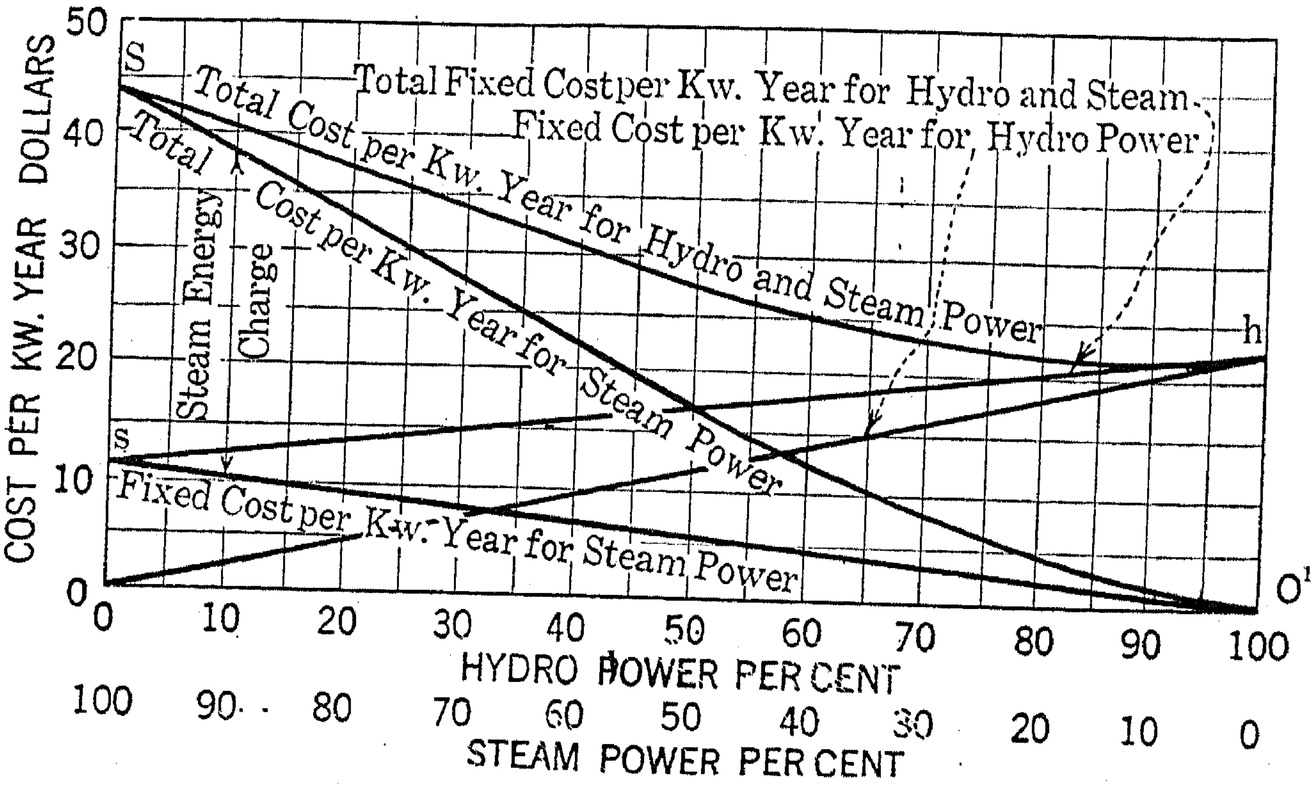

FIG. 1—PROPORTION OF WATER TO STEAMPOWER

Water power cost per kw-yr..............	$22.00
Steam, fixed cost per kw-yr..............	11.00
Steam energy charge per kw-yr, and 100% load factor......................	44.00

For example, let total load equal 100,000 kilowatt, divided 70 per cent hydroelectric and 30 per cent steam; then the yearly charge against the water power and steam will be

Water power, fixed charge..	70,000 kw. × $22	$1,540,000
Steam power, fixed charge..	30,000 kw. × $11	330,000
Total power, fixed charge....................		$1,870,000

or $18.70 per kw-year, as shown by the ordinate of the line *s h* at 70 per cent water power, 30 per cent steam power. All water power fixed charge would cost $2,200,000 and all steam power fixed charge $1,100,000 per year. All steam power costs $4,400,000.

For any other assumption of cost per year of water power or steam power, it is only necessary to determine the yearly fixed charge against steam power (*O s*) and water power (*O h*) and draw the line *s h*, and we have immediately the total fixed charge for any proportion of water power and steam power. This very much simplifies the problem and visualizes the results.

To determine the total charge per kw-year against the combined steam and water power, it is necessary to add the kw-hr. charge against the steam power. (It is of course assumed that all charges against the water power are fixed charges.)

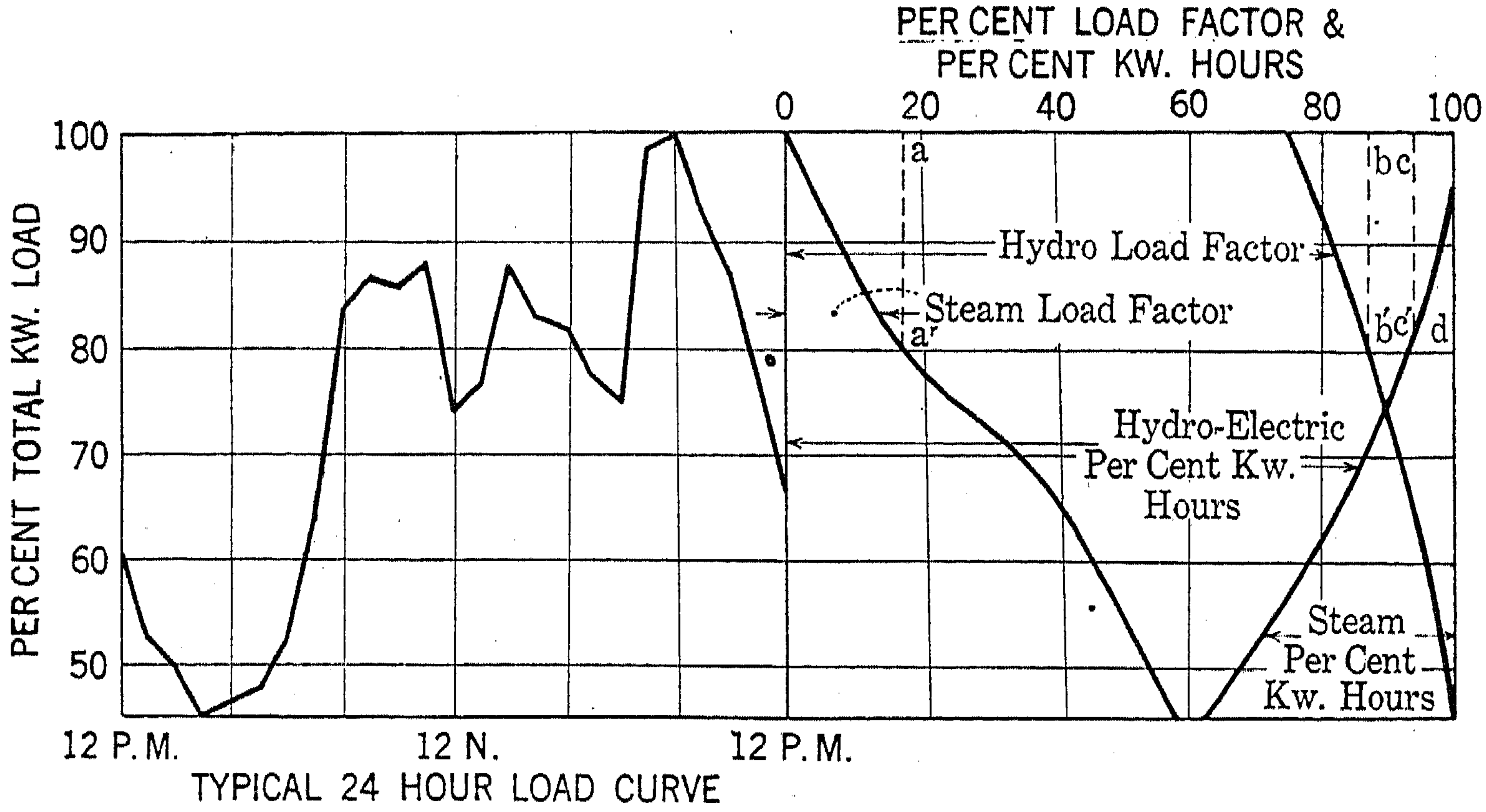

FIG. 2—LOAD FACTORS AND KW-HR. FOR STEAM AND WATER

—EXAMPLE—

For 20% peakload carried by steam and 80% by water the curves show

Steam load factor = 17% = *o a*
Water load factor = 87% = *o b*
Water kw-hr. = 94% = *o c*
Steam kw-hr. = 6% = *c d*

NOTE—Steam carries everything above 80% line in example, and the curves will show per cent load factor and per cent kw-hours for any other per cent load taken.

To determine the yearly charge against steam power for any load factor we must start with the load curve of the power system. The load curve assumed is that shown on the left of Fig. 2. The ordinates, it will be noted, are plotted in percentage of the load, the peak load being 100 per cent.

Now we must determine the kw-hours carried by steam for any percentage of the total load carried by steam, it being assumed of course that the steam power takes the load off the top of the curve. For this purpose we determine the curve of kilowatt-hours to be carried by steam power when 10 per cent, 20 per cent, etc. of the top of the load curve is carried by steam. To do this we take the area of the load curve above 90 per cent for example, and determine what percentage this is of the entire area of the load curve. Similarly for areas above 80 per cent, 70 per cent, etc. The results are shown in the curves to the right of the load curve.

For example, if all load over 80 per cent is carried on steam, we get the steam load factor = 17 per cent, and steam kilowatt-hours 6 per cent of the total as shown by *o a* and c^1 *d*. Also we get a hydroelectric load factor 87 per cent and kilowatt-hours 94 per cent of the total as shown by *o b* and *o c*.

From the curves in Fig. 2 and the cost per kilowatt-hour of fuel and other strictly steam energy charges, we determine for any percentage of load carried by steam power the yearly energy charge per kilowatt-year. Assuming $33 per kilowatt-year for energy charge where all the energy is supplied by steam for the particular load curve under consideration (which corresponds practically to $44 per kilowatt-year, or 0.5 cent per kilowatt-hour for 100 per cent load factor, as the load factor of total load is 75 per cent) and adding the energy charge to the fixed charge for steam power, we obtain the total cost of steam power as shown by the curve "cost steam power" O^1 *S* in Fig. 1.

To obtain now the total cost of all power for any proportion of steam power to water power, we add the ordinates of *o h* the "fixed cost of hydroelectric power" to the ordinates o^1 *S* the "Total cost of steam power" and obtain the curve *h S* the "total cost per kilowatt-year hydroelectric and steam power." This curve starts at $22 per kilowatt-year if all power is water power. By adding some steam power to take off the peaks, we see there is a slight decrease in the yearly charge until the steam power carries about 15 per cent of load. At 20 per cent of load the cost again comes to about $22 per year, and then a gradual increase in power cost results.

At 30 per cent load carried by steam for this particular load curve there is little difference in the yearly charge per kilowatt and we would not for this case be warranted in installing less than 30 per cent steam; for naturally, unless very material savings result, the decision will always be to install steam power because of the smaller capital cost.

At 50 per cent of total load taken by steam installation and 50 per cent by water power, we have the yearly charge as follows:

50 per cent × \$22 = \$11.00. . yearly fixed charge water power
50 per cent × \$11 = 5.50. . yearly fixed charge steam power
Total. . . . . \$16.50. . yearly fixed charge total power

At 50 per cent load factor practically 33⅓ per cent of the kilowatt-hours are carried by steam and this adds the energy charge of \$33 × 33⅓ per cent or \$11. Therefore the total yearly charge is \$16.50 + \$11 = \$27.50, as shown by the curve *h S* by the ordinate at 50 per cent. The added charge over all water power costs here is \$5.50 per kw. year, but on a system with 100,000 kw., the yearly excess charge is \$550,000, a very substantial sum.

The curve of total yearly power costs per kilowatt shows graphically what we want to know, and after we have the curve of energy cost of steam power at various load factors, we can very quickly make up total costs per kilowatt-year for any assumption of fixed cost of hydroelectric and steam power. It is believed this method will assist engineers in their work. The actual proportion of steam to hydroelectric power will of course be somewhat influenced by service insurance conditions.

This paper is merely an outline of the general principles and gives the general method to follow. There are, of course, many details and different conditions in different sections of the country. It is hoped that the method presented will appeal to engineers and managers.

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APPLICATION OF HARMONIC ANALYSIS TO THE THEORY OF SYNCHRONOUS MACHINES

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ABSTRACT OF PAPER

It is shown that the flux distribution in the air gap of a synchronous machine consists of a series of component distributions that are simple harmonic wave trains, either stationary or moving at constant velocities. Methods are suggested for determining the effects of slots and the saturation of the magnetic circuit on the magnitudes of these component distributions. Expressions for the voltage generated and the power developed thereby are given. The theory is applied to the operation of a three-phase synchronous machine under different conditions of load, both qualitatively and quantitatively. A table of comparative calculated field currents is appended.

A COMPLETE theory of the performance of alternating-current machinery must recognize the presence of harmonics in the current and voltage and in the distribution of the air gap flux. The principle of the following analysis is well known but the author believes it has never been so fully developed. It seems best to show how the method may be applied in a variety of cases rather than to carry any one case into all of its manifold refinements. Much of the analysis is qualitative rather than quantitative. It is useful to know, however, what harmonics may be present even when their magnitudes cannot be calculated. Where it is possible to determine certain coefficients with sufficient accuracy the method should prove of considerable practical importance. The fundamental principle upon which the analysis is based is that of the superposition of magnetic fields. For example, if a constant magnetizing force produces at any point a certain magnetic flux density when it alone is acting, and another constant magnetizing force produces at the same point another flux density when it alone is acting, then when both magnetizing forces act together the resultant flux density at the point in question is the vector sum of these two flux densities, provided the permeability at every point in the region concerned is the same in each of the three cases. Thus the component flux densities should be calculated for a condition of the magnetic region which is assumed to be identical with that existing when the components act simultaneously. When the magnetic forces are variable the theorem does not hold if eddy currents are produced. If the resultant magnetizing force is constant, however, the eddy currents that may be produced by the component magnetizing forces should be neglected. With the laminated magnetic masses that are usually employed when the resultant flux density is variable, the effect of the eddy currents can often be neglected without undue error. Such is the case at 25 or 60 cycles with laminations no thicker than are ordinarily used in transformers. At high frequencies or with laminations of two or three times this thickness, however, the effect of the eddy currents may be comparatively great. This would occur with the practically solid rotors of large turbo-alternators.

The procedure followed in this analysis is to decompose the actual flux distribution in the air gap into a series of simple sinusoidal distributions and then to determine how each of these is effective in the generation of voltage and in the development of power. At any instant the flux distribution, or flux density, in the air gap of an alternating current machine may be represented by a curve such as shown in Fig. 1. The abscissas, x, of this curve are measured in electrical radians from some point fixed of reference, O, in the airgap. In the case of a synchronous machine it is convenient to choose this point as half way between the centers of adjacent field poles. It is also simpler to assume that this point of reference is stationary and that the armature is rotating. This will be done throughout the analysis. The ordinates represent the normal flux density in the air gap at the surface of the armature—or, at the surface of the field poles if this is desired. A distribution of this sort may be of constant magnitude and wave shape, or both may vary periodically, and in either case it may be fixed or moving at a constant or a variable angular velocity. In any case the most complex distribution can be decomposed into a series of simple sinusoidal dis-

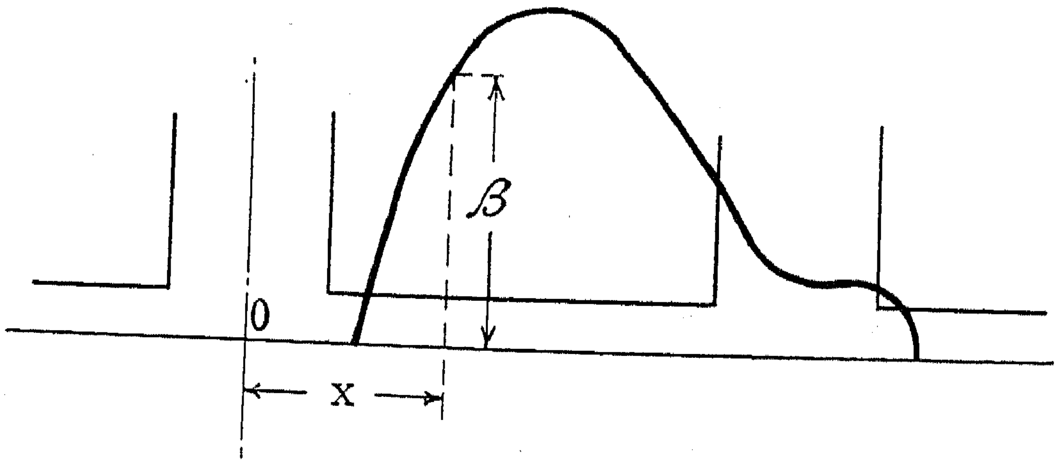

FIG. 1

tributions of different wave lengths and amplitudes all of which move at a constant but not the same velocity. In fact, some move in one direction through the air gap and some in the opposite direction.

As an illustration of this process of decomposition, consider the case of a salient-pole alternator in which but one armature coil per pole is carrying current. At any instant the flux distribution per ampere (see Fig. 1 for example) can always be represented by a Fourier series consisting of both sine and cosine terms. As the coil moves through the air gap the shape of this distribution will vary periodically due to the non-uniformity in the reluctance of the magnetic circuit. If, therefore, the coefficient of each term of the distribution series is plotted against time its value may be represented by another Fourier series. That is, the air gap flux distribution per ampere may be represented by a series of products of sine and cosine terms. Each of these products may be broken up into a sum or difference of sine or cosine terms of constant amplitude, wave length and angular velocity. If instead of there being one ampere in the coil the current in it should vary periodically, each of these sine or cosine terms must be multiplied by still another Fourier series, representing the current. The resulting products of sine and cosine terms would again be broken up into a series of sine and cosine terms which represent simple harmonic wave trains that are either stationary or moving at constant angular velocities.

GENERAL EXPRESSIONS FOR VOLTAGE AND POWER

Let the equation for one of these component wave trains be

$$\beta = B_q \cos (q\,x - j\,\omega\,t + \alpha_{qj}) \qquad \textbf{(1)}$$

This represents a sinusoidal distribution of maximum value B_q and of wave length $2\,\pi/q$, such as might be produced by $q \cdot P$* poles moving through the airgap at an electrical angular velocity of $j\,\omega/q$ in the same direction as does the armature. See Fig. 2. If the sign before $j\,\omega\,t$ is positive the direction of rotation is opposite to that of the armature. If the coefficient j is zero the distribution is stationary with respect to the field poles. The angle α_{qj} determines the phase of the distribution.

When $\omega\,t$ equals zero or any multiple of π the value of the flux density at the reference point, O, due to this distribution alone is $\pm B_q \cos \alpha_{qj}$.

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*P represents the actual number of field poles. q is throughout the harmonic order of the flux distribution and is usually 1, 3, 5, 7, etc. j is usually 0, 2, 4, 6, etc.

The frequency of the voltage generated in the field winding by this distribution is proportional to the equivalent number of poles, $q \cdot P$, and the relative velocity, $j\,\omega/q$. The frequency is $j f$, where f is the fundamental frequency. If the field circuit is closed a current of this frequency will be produced in it and a torque will be developed tending to move the field in the direction in which the distribution is moving. This alternating current in the field winding will also produce a harmonic variation in the field flux which will in turn cause a higher and a lower harmonic in the armature electromotive force.

The rotating flux distribution will also directly produce an electromotive force in the armature winding, the frequency of which is proportional to the equivalent number of poles, $q \cdot P$, and the relative angular velocity, $(\omega - j\,\omega/q)$. The frequency is $(q - j) f$. If there exists a current of this frequency in the armature winding, power will be developed if the phase relation of the current and the generated voltage is favorable. The equation for the voltage produced in the armature winding by this flux distribution of the q^{th} order is

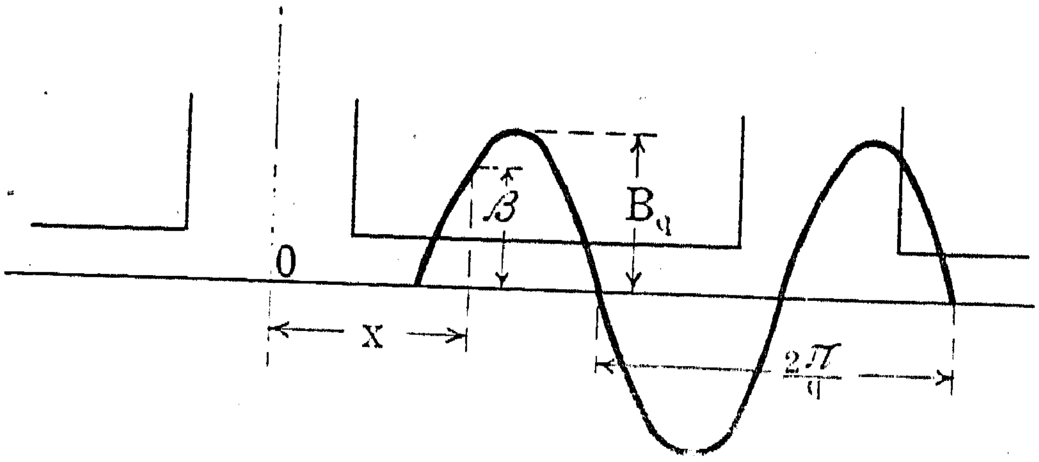

FIG. 2

$$e_{(q-j)} = \sqrt{2}\, E_{q-j} \left[\cos (q - j)\, \omega t + \alpha_{qj}\right]^* \qquad (2)$$

The average power developed in a single-phase winding by

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*The effective value of this voltage is

$$E_{q-j} = \frac{K_q}{K_1} \; \frac{B_q}{B_1} \; \frac{q-j}{q} \; E_1$$

where E_1 is the effective voltage due to the fundamental flux distribution, whose maximum value is B_1, and K_q and K_1 are the voltage reduction factors for the qth harmonic and the fundamental flux.

this electromotive force and a harmonic current of the s order whose equation is $i_s = \sqrt{2}\, I_s \sin(s\, \omega\, t + \theta_s)$† is

$$P = \pm E_{q-j} \cdot I_s \sin(\alpha_{qj} \pm \theta_s)\ddagger \qquad (3)$$

The average power is zero, however, unless $(q - j \pm s)$ is zero, that is to say, unless the voltage and current have the same frequency. When the average power is positive electrical power is developed, but when it is negative mechanical power is developed.

In a balanced three-phase alternator, power is developed only when $q \pm s = j =$ 0, 6, 12 etc. if there are no even harmonics present or 0, 3, 6 etc. if there even harmonics in the current. The power is

$$P = \pm 3\, E_{q-j} \cdot I_s \sin(\alpha_{qj} \pm \theta_s) \qquad (4)$$

Electrical power is developed when $(\theta_s - \alpha_{qj})$ is an angle of lead, and mechanical power is developed when it is an angle of lag. For this reason it may happen that though power is developed in each of the phases of a three-phase alternator, the net power developed is zero. In this case at least one of the phases would develop electrical power while at least one of the other two would develop mechanical power. That is, the windings of a three-phase alternator may act as a self-contained motor-generator set.

Reduction Factors for Flux. Most frequently the flux in the air gap is due to the action of several magnetizing coils per pole. The resultant flux distribution is the sum of the distributions due to the individual coils acting separately. If these component distributions are decomposed into their elementary sinusoidal parts, as can always be done, those of the same wave length, which are stationary or which travel in the same direction at the same velocity can be combined according to the principles of vector addition. This process of addition is simplified if the components are of the same amplitude and are equally displaced from one another in space-phase position. In this case the method of addition is the same as that for a number of equal voltages of the same frequency differing from each other in time phase by the same amount. With n such distributions of wave length $2\,\pi/q$ differing from each other in position by an angle γ on the fundamental scale, in which the pole pitch equals

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†When the conductors of a phase belt are at the zero point midway between the poles, the s harmonic component of the current in that belt has a value $\sqrt{2}\, I_s \sin\ \theta_s$

‡In this equation the signs are either both plus or both minus.

π radians, the sum is what it would be if they were coincident multiplied by

$$K_b = \frac{\sin \frac{q\,n\,\gamma}{2}}{n \sin \frac{q\,\gamma}{2}}$$

This is the belt reduction factor.

If it is further desired to add two such distributions which are equal in magnitude but differ in phase position by an angle λ measured on the fundamental scale, their sum is what it would be if they were coincident multiplied by $K_p = \cos \frac{q\,\lambda}{2}$. The angle λ is some multiple of γ. This is the pitch reduction factor.

The total reduction factor for the qth harmonic flux distribution is

$$K_q = \frac{\sin \frac{q\,n\,\gamma}{2}}{n \sin \frac{q\,\gamma}{2}} \cdot \cos \frac{q\,\lambda}{2}$$

This holds true only when the flux per ampere is independent of the position of the coil, that is, when the air gap is uniform.

Reduction Factors for Voltage. Having calculated the harmonic components of the resultant flux distribution in the airgap, the electromotive force generated in either the stator or rotor winding that links this flux can be found. The belt reduction factor for voltage depends upon the space displacement of the coils and upon the order of the harmonic in the flux distribution. It does not depend upon the velocity of the distribution. The frequency of the voltage produced in the winding, however, depends upon the order of the harmonic in the distribution and upon its velocity with respect to the winding. Thus neither the belt nor the pitch reduction factor for voltage depends upon the frequency of the voltage. If there are n equal coils per pole displaced from one another by an angle γ and having a pitch deficiency of λ, the total reduction factor for voltage for the qth harmonic in the flux distribution is

$$K_q = \frac{\sin \frac{q\,n\,\gamma}{2}}{n \sin \frac{q\,\gamma}{2}} \cos \frac{q\,\lambda}{2}$$

Both γ and λ are measured on fundamental scale.

Flux vs. Voltage Reduction Factor. The angles γ and λ in the voltage reduction factors are always equal to the angles between the coils in which the voltage is produced. It is only with a uniform air gap, however, that the angles γ and λ in the flux reduction factor are equal to the angles between the coils which are producing the flux. If a winding is designed to eliminate a given harmonic from the flux distribution that it produces, a harmonic flux distribution of the same order will generate no voltage in the winding no matter what its relative velocity may be. There may be produced, however, an electromotive force of this order of harmonic frequency by another component in the distribution which is moving at the proper speed with respect to the winding.

Distributing a winding is thus no sure means of reducing or of eliminating a given harmonic in the voltage. For example, a fifth harmonic in the flux distribution moving at fundamental velocity will produce a fifth harmonic in the voltage, as will also a third harmonic in the flux distribution moving at five-thirds of the fundamental velocity. A winding designed to eliminate the harmonic voltage due to the first of these distributions will fail to eliminate that due to the second. All that can be accomplished by distributing the winding is in the reduction or elimination of the harmonics caused by given harmonics in the flux distribution. Another method of reducing the harmonics in the voltage would be to reduce the troublesome harmonics in the flux distribution. If the harmonic flux is in motion with respect to the field, this can be accomplished by means of a short-circuited winding set in the field poles. A winding of this sort is most effective when it is of low resistance and is brought into close proximity to the field winding it protects. The total variable flux that links this amortisseur or damper is just sufficient to account for the resistance drop in it.

In some cases, particularly when there is not an integral number of slots per pole, the reduction factors for flux and voltage must be calculated for each individual case, as no general formula is applicable.

Control of Flux Distribution. The distribution of flux in the air gap of an alternator may be controlled to a certain extent in either of two ways,—by shaping the pole surface if salient poles are used, or by properly distributing the magnetizing winding if the air gap is uniform. The first method will not be discussed.

The first and simplest case assumes that the air gap is uniform, as it approximately would be in a turbo-alternator. The direct effect of the slots in both the field and armature in producing harmonics is neglected. This effect can be largely eliminated by closing the slots with magnetic wedges and by choosing the proper relative numbers of stator and rotor slots. As is customary the reluctance of the field and armature cores is neglected. An approximate method that corrects for this omission is given later.

A single field coil with a pitch equal to the pole pitch will then produce a rectangular flux distribution in the air gap which may be represented by the Fourier series

$$\beta = \frac{4}{\pi} B_0 \sum \frac{1}{q} \sin q\,x \qquad \textbf{(5)}$$

q is the order of the harmonic in the distribution. β is the flux density in the gap at an angular distance x radians from one side of the coil, and B_0 is the average flux density in the gap. See Fig. 3. The relative magnitudes of the harmonics are: 1.00: 0.33 : 0.20 : 0.14., etc.

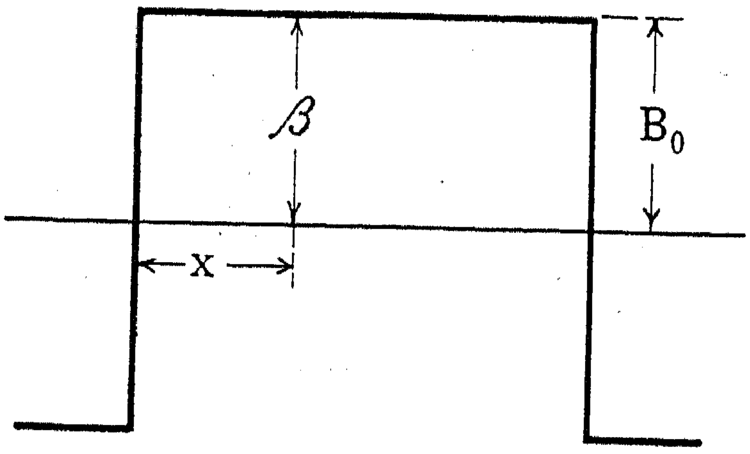

FIG. 3

If the field winding is distributed in more than one slot per pole the relative magnitudes of the harmonics are reduced. The amount of this reduction depends upon the method of distributing the field winding. With n coils per pair of poles, each of unity pitch* and having the same number of turns, and equally displaced by an angle γ, the flux density at an angular distance x from the middle of a belt is

$$\beta = \frac{4\,B_0}{\pi} \sum \frac{\sin q\,\frac{n}{2}\,\gamma}{n \sin q\,\frac{\gamma}{2}}\;\frac{1}{q} \sin q\,x \qquad \textbf{(6)}$$

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*As a matter of fact the field coils are arranged concentrically but the magnetic effect is the same as if the end connections were made to give unity pitch. See Fig. 4.

If the winding were concentrated the average flux density would be B_0. The relative magnitudes of the harmonics depend upon the number of magnetizing coils, n, and the angle, γ, between them. If, for example, there are six coils per pair of poles displaced by 20 degrees, the relative magnitudes of the harmonics are 0.83 : 0.0 : 0.038 : 0.022. The third harmonic in the distribution vanishes and the phase of the fifth harmonic is reversed.

The qth harmonic or any odd multiple of it will vanish if $\frac{q\,n\,\gamma}{2} = \pi$ or any multiple of π. Thus, by properly choosing the breadth of the winding, any harmonic in the flux distribution may be theoretically eliminated, although practical winding conditions will limit this possibility. Since the harmonics are much less prominent than with a concentrated winding the distribution is more nearly sinusoidal. The fundamental flux is now 83 per cent of what it would be if the winding were concentrated. This might be called the efficiency of the winding.

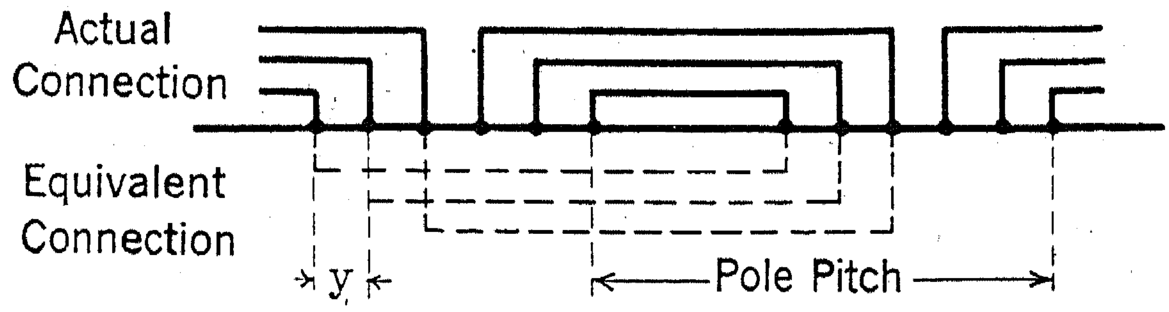

FIG. 4

If the field winding is equivalent to two similar distributed windings displaced by an angle λ as shown in Fig. 5, the relative magnitudes of the harmonics are still further reduced. By properly choosing the pitch deficiency angle, λ, any harmonic in the distribution may be theoretically eliminated. The angle, λ, is always some multiple of the angle, γ, between the slots. The distribution is represented by

$$\beta = \frac{4}{\pi} B_0 \sum \frac{\sin \frac{q\,n\,\gamma}{2}}{n \sin \frac{q\,\gamma}{2}} \cdot \cos \frac{q\,\lambda}{2} \cdot \frac{1}{q} \sin q\,x \qquad (7)$$

The average flux density for a concentrated winding having the same number of turns is B_0. Fig. 5 shows two windings, each like that shown in Fig. 4, which are displaced by two slots or 40 degrees. The relative magnitudes of the harmonics in this case are:

0.78 : 0.0 : 0.0066 : 0.017

The effect of the pitch factor is to reverse both the fifth and seventh harmonics so that the fifth now appears in phase with, and the seventh in opposition to the fundamental. The harmonics are very small. The efficiency of this winding is 78 per cent.

By properly choosing the breadth of one belt or layer, $n\,\gamma$, and the pitch deficiency, λ, any two harmonics may be eliminated theoretically. There is a practical limitation to this, however, fixed by the minimum allowable angle between the slots. If the angle between the slots is $\pi/15$, and the breadth of one layer of the winding is 10 slots and the pitch deficiency is three slots, the third harmonic will be eliminated by the distribution, and the fifth harmonic by the pitch, of the winding. In this case the seventh harmonic is 1.4 per cent of the fundamental. The efficiency of this winding is 0.79.

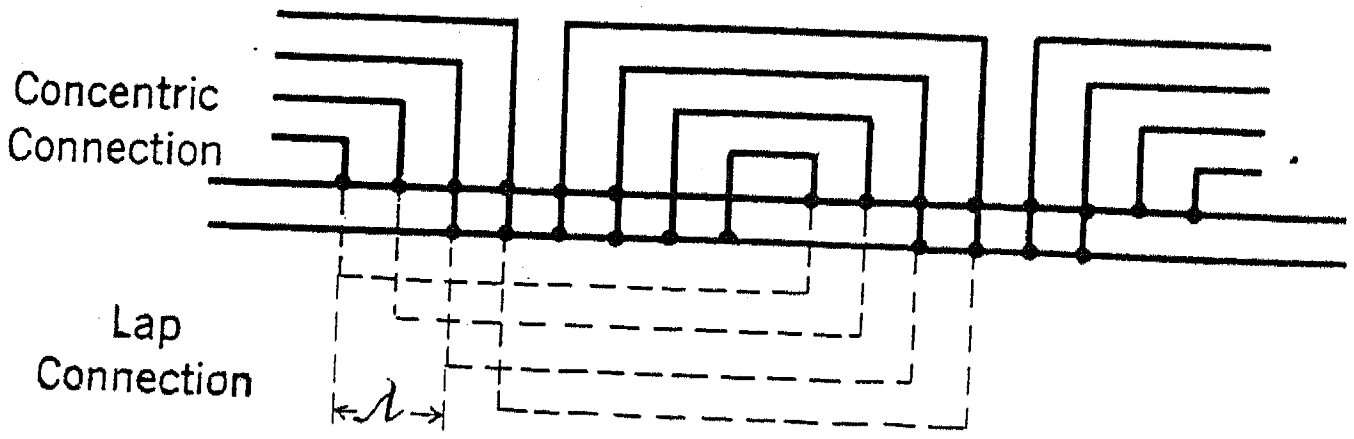

FIG. 5

Correction for Saturation. Thus far the reluctance of the field and armature cores has been neglected. The following method of correction is an approximate one designed to compensate for this omission. Both the field and armature are assumed to be uniformly slotted. The flux distribution in the air gap due to a single magnetizing coil instead of being rectangular, is less at the center than at the conductors, as shown in Fig. 6* The simplest equation that can be given such a curve between $x = 0$ and $x = \pi$ is

$$\beta = B_0 \left(1 + \frac{b_1}{B_0} \cos 2\,x\right)$$

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*Because some parts of the magnetic circuit are more highly saturated than others the component distributions due to the individual coils will not be alike. This effect, however, is disregarded, although it may be of prime importance in determining the magnitudes of the harmonics. Other assumptions as to the shapes of the component distributions might be made which would increase the precision of the results.

B_0 is the average flux density in the gap and $(B_0 + b_1)$ and $(B_0 - b_1)$ are the maximum and minimum flux densities respectively. The ratio of b_1 to B_0 is approximately equal to the ratio of the ampere turns required for the field and armature cores to those required for the air gap and the teeth.

The Fourier series that represents this distribution is

$$\beta = \frac{4\,B_0}{\pi} \sum \left(\frac{1}{q} + \frac{b_1}{B_0}\,\frac{q}{q^2 - 4} \right) \sin q\,x \tag{8}$$

If the ratio $\frac{b_1}{B_0}$ is 10 per cent, the relative magnitudes of the harmonics produced by a single magnetizing coil are

$$1.0 : 0.406 : 0.232 : 0.163$$

instead of

$$1.0 : 0.333 : 0.200 : 0.143$$

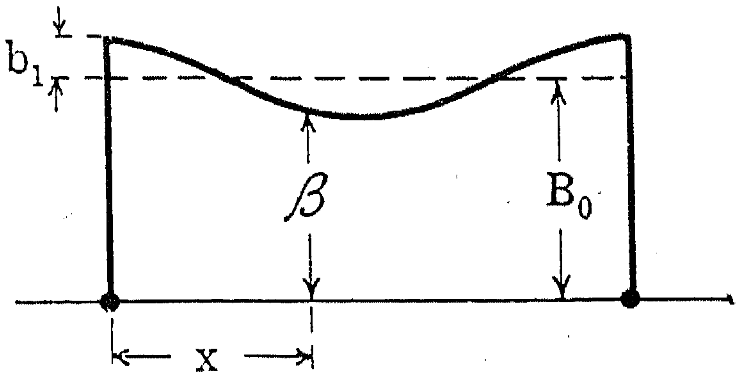

FIG. 6

as they are on the assumption that the reluctance of the field and armature cores is negligible.

The corrected series for a distributed fractional pitch winding is

$$\beta = \frac{4\,B_0}{\pi} \sum \frac{\sin \frac{q\,n\,\gamma}{2}}{n \sin \frac{q\,\gamma}{2}}$$

$$\cdot \cos \frac{q\,\lambda}{2} \left(\frac{1}{q} + \frac{b_1}{B}\,\frac{q}{q^2 - 4} \right) \sin q\,x \tag{9}$$

At higher flux densities the ratio $\frac{b_1}{B_0}$ is greater and the harmonics become more prominent. The general effect of this is to make the flux distribution in the air gap flatter than it is at lower saturations.

Correction for Unslotted Portion of Field Surface. Usually the field winding is distributed over only a portion of the polar pitch as shown in Fig. 7. In this case the flux density between the points a and b is greater than it would be if the field were slotted in this region. The full ampere turns of one pole are acting to produce the flux density here and the increase in the density between a and b over what it would be provided the entire field were slotted may be represented roughly by the curve shown in Fig. 8. The length of the unslotted portion is η. The Fourier series that represents this curve is:

$$\beta = \frac{4\,B_0}{\pi} \sum \frac{b_2}{B_0} \sin \frac{q\,\pi}{2} \cdot \sin \frac{q\,\eta}{2} \cdot \frac{1}{q} \sin q\,x \, ^* \qquad \textbf{(10)}$$

B_0 is the average flux density with a uniformly slotted field produced by a concentrated field winding having the given number of turns per pole. This series represents the flux density which should be added to that calculated on the assumption that the entire field is slotted.†

Large turbo-alternators are now built with a field winding arranged as shown in Fig. 9. The method of analysis, including

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$$\frac{b_2}{B_0} = \frac{(N\,I)_1 - (N\,I)_2}{(N\,I)_2}$$

where: $(NI)_1$ = ampere turns required to produce B_0 with a uniformly slotted field.

$(NI)_2$ = ampere turns required to produce B_0 with an unslotted field.

†The magnitude of the correction is shown by the following example in which $\frac{b_2}{B_0}$ is taken equal to 0.20 and η equal to $\frac{\pi}{3}$.

$$\beta = \frac{4\,B_0}{\pi}\,(0.10 \sin x - 0.067 \sin 3\,x + 0.020 \sin 5\,x + 0.014 \sin 7\,x + \ldots)$$

The corrected distribution for the winding shown in Fig. 4 if $\frac{b_1}{B_0} = 0.10$ and $\frac{b_2}{B_0} = 0.20$ is:

$$\beta = \frac{4\,B_0}{\pi}\,(0.91 \sin x - 0.067 \sin 3\,x - 0.023 \sin 5\,x + 0.039 \sin 7\,x + \ldots)$$

The third harmonic which had been cut out by the breadth factor is now 7.4 per cent. of the fundamental.

The corrected efficiency of the winding is 85 per cent.

The uncorrected efficiency as previously calculated was 83 per cent.

the corrections for the effects of saturation and of the unslotted portions of the field, is similar to that already outlined.

The magnitude of any harmonic in the flux distribution depends not only upon the pitch and breadth reduction factors but also upon the degree of saturation and the breadth of the unslotted portion of the field. Harmonics that have been entirely eliminated by the first two factors will still appear on account of the last. It is only with a uniform air gap that harmonics can be surely eliminated by the distribution of the field winding. Field slots that are closed by magnetic wedges tend to produce this condition. In general the effect of increased saturation is to flatten the flux distribution curve.

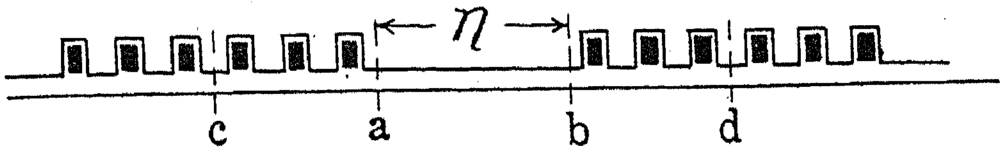

FIG. 7

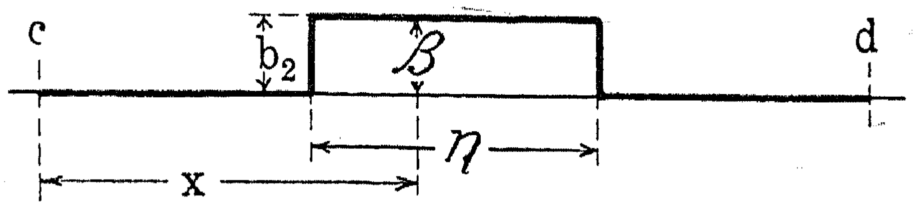

FIG. 8

Flux Distribution with Special Winding. It has been shown that if the equivalent field winding consists of equally spaced coils having the *same* number of turns, *any two* harmonics or their odd multiples can be eliminated from the flux distribution by properly choosing the breadth of the winding and the pitch of the coils. Thus if a winding is chosen that eliminates the third and fifth harmonics, it will also eliminate the ninth, fifteenth, twenty-first, twenty-fifth, etc. On the other hand, if the field winding consists of coils having *unequal* numbers of turns more can be accomplished in the elimination of the harmonics. Only the case in which the field is uniformly slotted will be considered. the second correction does not then have to be applied. The illustration here given is for a field winding consisting of four

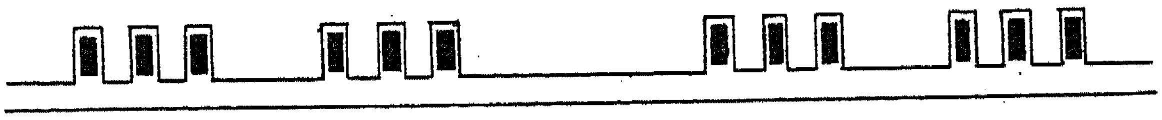

FIG. 9

coils per pole contained in slots displaced by 20 degrees as shown in Fig. 16. See Appendix I. The central slot is not occupied. If the relative numbers of turns in the coils 1 to four are respectively 1.0, 0.88, 0.65 and 0.35, all of the harmonics in the flux distribution will be eliminated up to and including the fifteenth. The first harmonics that do occur, viz. the seventeenth and nineteenth, have the same relative magnitudes as they do with a concentrated winding.*

Analysis of Air gap Flux. The flux distribution in the air gap of a synchronous machine can be modified to a considerable extent by shaping the pole surfaces, or, as has been shown with non-salient poles, by properly distributing the field winding. In either case the value of the flux density in the air gap may be represented by a Fourier series which may be written:

$$\beta = i_f\,(\,\Sigma\, A_q \sin q\, x + \Sigma\, C_q \cos q\, x) \tag{11}$$

x is the electrical angle measured from a point midway between the poles, i_f is the value of the exciting current, and the coefficients, A_q and C_q are the maximum values of the various harmonic distributions per ampere of field current at the particular saturation considered.

With a smooth-core armature the magnetic circuit is constructed so that the flux distribution is at all times symmetrical about the center of the field poles. This disregards the effect of the cross magnetizing action of the armature currents which increases the saturation of one-half of the magnetic circuit above that of the other half. With a symmetrical flux distribution the C coefficients are all zero. When the armature is slotted, as it almost invariably is, the magnetic circuit is symmetrical only when a slot or a tooth is exactly at the center of a field pole. The C coefficients are no longer zero but vary periodically with the time.

There are three cases to consider. The first is for a smooth armature and a constant exciting current. The flux distribution is then independent of the position of the armature and i_f and the coefficients A_q are all constant. The second case is for a smooth core armature and a periodically varying field current. i_f is then a periodic function of the time but the coefficients A_q are still assumed to be constant. The third case is for a slotted armature core but with a constant field current. Here i_f is constant but the coefficients A_q and C_q are periodic functions of the time.

The armature winding in which the electromotive force is generated is distributed in n slots per pole per phase, displaced by an angle γ and with a pitch deficiency, λ.

I. *Constant Flux.* In the first case, which is the simplest, the electromotive force generated in any armature conductor has the same wave form as the flux distribution. This is not true of the other cases.

If the fundamental flux per pole is ϕ_1 the q^{th} harmonic flux per pole is

$$\phi_q = \frac{1}{q} \frac{A_q}{A_1} \phi_1$$

The voltage produced in the armature winding due to this harmonic flux is

$$E_q = 4.44\, K_q\, N\, q\, f\, \phi_q\, 10^{-8}$$

N is the number of turns in one phase of the armature winding and f is the fundamental frequency. K_q is the total reduction factor for the q^{th} harmonic in the flux distribution.

By properly choosing the distribution and the pitch of the winding it is theoretically possible to eliminate any two harmonics from the electromotive force. A three-phase winding arranged in 15 slots per pole, having a phase belt of 10 slots and a pitch deficiency of three slots will entirely eliminate the third and fifth harmonics. In a well distributed three-phase winding, a pitch of five-sixths reduces the fifth and seventh harmonics to about four or five per cent of their full value. Special windings can be designed to eliminate several harmonics. For example, a single-phase winding such as previously described in connection with the flux distribution will eliminate all of the harmonics up to and including the fifteenth.

Power will be developed in the armature only when there is a harmonic in the current of the q^{th} order and then only if the time-phase displacement is not $\pi/2$.

Since the flux is of constant magnitude it produces no voltage in the field winding and therefore no reaction with it.

II *Variable Field Current.* In the second case it is simplest to assume, though incorrectly, that the distribution of the flux in the air gap has a fixed wave form. If the variations in the field current are not large the approximate effect of a periodic field current may be represented by

$$i_f = I_0 + k \Sigma I_s \sin (s\, \omega_1\, t - \theta_s)$$

The frequency of the fundamental variation in this current is $f_1 = \frac{\omega_1}{2\pi}$. It may or may not be equal to the frequency of the fundamental in the armature electromotive force. The constant term I_0 produces the harmonics that were calculated in the first case. Any harmonic in the field current produces a flux distribution in the air gap which is equivalent to two distributions of the same magnitude and wave length, which rotate in opposite directions with respect to the field poles. The frequencies of the voltages produced in the armature by these rotating fluxes are $(q f \pm s f_1)$. Thus in a 60-cycle alternator, a 25-cycle current in the field winding produces voltages having frequencies of 35 cycles and of 85 cycles, due to the variation in the fundamental component of the flux distribution.

If the flux distribution contains a third harmonic, the 25-cycle field current produces voltages in the armature winding with frequencies of 155 cycles and of 205 cycles. This method of resolving a pulsating field into oppositely rotating fields was used by Ferraris in analyzing the operation of the single-phase induction motor.

III *Effect of Slots*. In the third and last case to be considered the field excitation is constant but the *permeance* of the magnetic circuit varies periodically on account of the slots in the surfaces of the field and armature. The wave form of flux distribution in the air gap varies cyclically while the armature is advancing the distance of the armature tooth pitch.* The periodic variation in the shape of this flux distribution produces a corresponding variation in the coefficients of its harmonic components. Each of these coefficients may then be represented by a Fourier series.

If n represents the number of slots per pole the series which represent any of the coefficients such as A_q and C_q are

$$A_q = \Sigma\ {}_p a_q \cos 2\, n\, p\, \omega\, t$$

and

$$C_q = \Sigma\ {}_p c_q \sin 2\, n\, p\, \omega\, t$$

The first is a cosine and the second is a sine series for the same reasons as outlined in the analysis of the armature reaction.

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*Time is measured from the moment that an armature slot is directly opposite the zero point, that is, midway between the field poles.

(See Appendix II). The general expression for the flux distribution in the air gap reduces to

$$\beta = \frac{I_0}{2}\ \{ \Sigma\,({}_p a_q + {}_p c_q) \sin (q\,x + 2\,n\,p\,\omega\,t) + \Sigma\,({}_p a_q - {}_p c_q) \sin (q\,x - 2\,n\,p\,\omega\,t)\ \}^{\dagger} \qquad (12)$$

${}_p a_q$ is the coefficient of the p^{th} harmonic term in the Fourier series which represents the variation in the coefficient of the q_{th} harmonic sine term, A_q, in the flux distribution, and ${}_p c_q$ is the coefficient of the p^{th} harmonic term in the Fourier series which represents the variation in the coefficient of the q^{th} harmonic cosine term, C_q, in the flux distribution.

The flux distribution in the air gap is thus resolved into two components one of which (for $p = 0$) is stationary with respect to the field poles and the other of which (for $p = 1, 2, 3$ etc.) is rotating. This second component can be further resolved into two wave trains which rotate in opposite directions through the air gap. The effect of the stationary component has already been analyzed. The harmonics existing in it may be due to the shape of the pole surface or, in the case of non-salient poles, to the distribution of the field winding or to the field slots. The rotating fluxes are due wholly to the presence of the armature slots although their magnitude is influenced by the shape of the field poles or the arrangement of the field winding. The fundamental slot harmonics, *i. e.*, $p = 1$, are probably the most important.

The periodic variations in the A coefficients indicate corresponding variations in the flux per pole, while the periodic variations in the C coefficients indicate corresponding distortions of the flux distributions. The latter are equivalent to an oscillation of the flux across the pole face.

The coefficients ${}_p a_q$ and ${}_p c_q$ can be determined by analyzing the flux distribution for successive positions of the armature with respect to the field poles. If the values of the coefficients of any particular harmonic are then plotted against time, this curve can be analyzed for the coefficients of its harmonic components, *i. e.*, ${}_0 a_q$, ${}_1 a_q$, ${}_2 a_q$, etc.

The order of the harmonic in the generated voltage depends upon the order of the harmonic in the flux distribution which is affected by the slot pulsations, and the number of slots per pole. It is $(q \pm 2\,n\,p)$. The most important slot harmonics are due to the variation in the fundamental flux and are of the orders $(2\,n \pm 1)$.

---

† I_0 is the field current.

Distributing an armature winding has no effect in eliminating the harmonics caused by the pulsations due to the slots except in so far as it modifies the reduction factor for any particular harmonic in the flux distribution.

Harmonics due to the armature slots may be very prominent in the flux distribution and yet generate little or no voltage in the armature winding. This is because these harmonics move synchronously with the armature and will thus be effective in producing voltage only when their magnitudes vary so that they produce a net variation of flux through the armature coils.

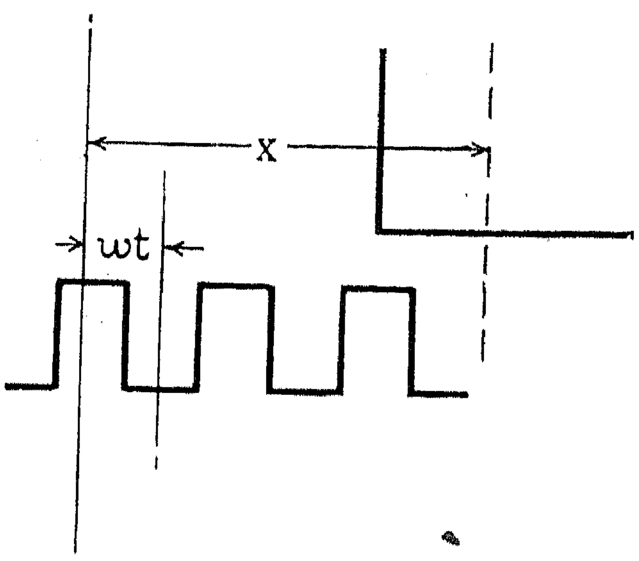

FIG. 10

In Fig. 10 assume that the flux distribution in the air gap is represented by the expression

$$\beta = B_0 \sin x \, [1 - a \cos 2\, n\, (x - \omega\, t)\ ]$$

where n is the number of slots per pole. The value of a determines the magnitude of the slot harmonics. This flux distribution may be thought of as the result of a sinusoidal distribution of magnetomotive force acting on a magnetic circuit whose permeance varies with the time as shown by the bracketed term. The flux linking a coil of unity pitch is

$$\phi = 2\, B_0 \left(1 + \frac{a}{4\, n^2 - 1}\right) \cos \omega\, t\, ^*$$

Thus, in this case the flux through the coil varies sinusoidally and there are no higher harmonics in the voltage, although the armature slots are the cause of large harmonics—depending upon the value of a—in the air-gap flux distribution. These harmonics are of the orders $(2\, n + 1)$ and $(2\, n - 1)$, but their amplitudes vary with a frequency of $2\, n$ times the fundamental frequency. The net effect of this is to produce a fundamental voltage only. Further, the flux from one pole is

$$\phi_0 = 2\, B_0 \left(1 + \frac{a}{4\, n^2 - 1} \cos 2\, n\, \omega\, t\right)^{\dagger}$$

---

*This assumes that the center line of the slot marks the division between the flux which enters the tooth at the right and the one at the left. This is of course not strictly true.

† $\phi_0 = \int_0^{\pi} \beta\, d\, x$ (assumed)

The flux is greatest when a slot is midway between the poles, and least when a tooth is midway between them. The slots may thus cause a periodic variation in the total flux per pole and yet there may be no corresponding harmonics in the generated voltage.

Armature Reaction. The flux produced by the armature current will be divided into two distinct parts; that which exists in the air gap and that which does not. The magnitude and distribution of the first varies with the excitation and power factor, but the second portion is very nearly independent of each of these factors, and for the sake of simplicity it is assumed to vary only with the armature current.

The effect of the armature current of an alternator in modifying the air gap flux generally influences the operation of the machine more than anything else. The method here used to analyze this effect is to first obtain an expression which shows how the flux density in the air gap, due to a single armature coil of unity pitch, varies when the coil is placed in *any* position relative to the field poles. The resultant effect due to a single-phase winding is the sum of the separate effects due to the individual coils of the winding. An extension of this method of combination will give the resultant reaction for a three-phase alternator.

In Fig. 1, β is the normal component of the flux density in the air gap due to a single coil in which there is a current of i amperes. The equation of this distribution may be written in the form of a Fourier series.

$$\beta = \frac{N_c}{N_f} i \ \{ \Sigma A_q \sin q \ (x - \omega t) + \Sigma C_q \cos q \ (x - \omega t) \} \tag{13}$$

N_c and N_f are respectively the turns per armature coil and the turns per field pole. A_q and C_q are respectively the instantaneous maximum values of the q^{th} harmonic sine and cosine flux distributions for an armature current equivalent to one ampere in the field circuit.

As the coil moves through the air gap, *i. e.*, as ωt varies, the coefficients A_q and C_q of the series will vary periodically at double frequency. If the magnetic circuit is assumed to be symmetrical about the center line of the poles the series for A_q will consist of cosine terms only and the series for C_q will consist of sine terms only. The proof of this is given in Appendix II.

The series for A_q and C_q may be written

$$A_q = {}_0a_q + \Sigma\, {}_pa_q \cos 2\, p\, \omega\, t$$
$$C_q = \Sigma\, {}_pc_q \sin 2\, p\, \omega\, t$$

where p may equal 1, 2, 3, 4, etc.

If these values are substituted in equation (**13**) and the products of sine and cosine terms are expanded, the result gives

$$\beta = \frac{N_c}{N_f}\, i \left\{ \sum {}_0a_q \sin (q\, x - q\, \omega\, t) + \right.$$
$$\frac{1}{2} \sum \sum ({}_pa_q + {}_pc_q) \sin (q\, x - (q - 2\, p)\, \omega\, t) +$$
$$\left. \frac{1}{2} \sum \sum ({}_pa_q - {}_pc_q) \sin\, (q\, x - (q + 2\, p)\, \omega\, t) \right\} \quad \textbf{(14)}$$

This represents the flux distribution in the air gap due to a single armature coil carrying a current of i amperes. The angular displacement of the coil from the center of the pole is $\omega\, t$. ${}_0a_q$ is the average value of the amplitude of the q^{th} harmonic in the flux distribution. ${}_1a_q$ and ${}_1c_q$ are respectively the fundamental amplitudes of the variation in the maximum values of the q^{th} harmonic sine and cosine distributions.

With a distributed winding the values of $\omega\, t$ are different for the individual coils and the total reduction factors are

$$K_{q \pm 2\, p} = \frac{\sin \dfrac{(q \pm 2\, p)\, n\, \gamma}{2}}{n \sin \dfrac{(q \pm 2\, p)\, \gamma}{2}} \cdot \cos \frac{(q \pm 2\, p)\, \lambda}{2}$$

If the expression for the current is $i = \Sigma\, \sqrt{2}\, I_s \sin (s\, \omega\, t + \theta_s)$ the general expression for the flux distribution, due to the armature current alone, in the air gap of a single-phase alternator reduces to

$$\beta = \frac{1}{\sqrt{2}} \frac{N_P}{N_F} \left\{ \sum \sum K_q \cdot {}_0a_q \right.$$
$$\cdot\, I_s\, [\, \cos (q\, x - (q + s)\, \omega\, t - \theta_s)$$
$$- \cos\, (q\, x - (q - s)\, \omega\, t + \theta_s)\,]$$

$$+\sum\sum\sum K_{q-2p}\cdot\frac{({}_pa_q + {}_pc_q)}{2}$$

$$\cdot I_s\,[\cos(q\,x - (q - 2\,p + s)\,\omega\,t - \theta_s) - \cos(q\,x - (q - 2\,p - s)\,\omega\,t + \theta_s)\,]$$

$$+\sum\sum\sum K_{q+2p}\cdot\frac{({}_pa_q - {}_pc_q)}{2}$$

$$\cdot I_s\,[\cos(q\,x - (q + 2\,p + s)\,\omega\,t - \theta_s) - \cos(q\,x - (q + 2\,p - s)\,\omega\,t + \theta_s)\,]\Big\}^{*} \qquad (15)$$

Three-Phase Armature Reaction. In a three-phase alternator the resultant flux distribution due to the armature current alone is the combination of three such distributions. With a balanced load the currents are equal in magnitude and differ in phase by 120 degrees. The three armature windings also differ in their relative positions by 120 degrees. The sum of the three single-phase distributions will then be zero except when the coefficients of $\omega\,t$ in equation (**15**) are zero, three, or some multiple of three. In the coefficient of $\omega\,t$, q is always odd; $2\,p$ is always even and ordinarily the current contains no even harmonics and s is therefore odd. $(q \pm 2\,p \pm s)$ is thus either zero or even. If, however, the current contains even harmonics, this coefficient may be odd.

The general expression for the armature reaction flux in a three-phase alternator is given in equation (**15**). If the current contains no even harmonics only those terms can appear in which the coefficient of $\omega\,t$ is zero, six, or some multiple of six. If the armature current contains none but even harmonics only those terms can appear in which the coefficient of $\omega\,t$ is zero, three or some odd multiple of three.

In a turbo-alternator with a uniform air gap, the armature flux is independent of the position of the armature coil with respect to the field poles. In this case all of the coefficients except ${}_0a_q$ vanish, *i. e.*, p has no value but zero, and the complete expression for the armature reaction is much simplified, consisting as it does of only the first two terms in equation (**15**).

---

*N_P and N_F are the armature and field turns per pole.

For the method of calculating the coefficients ${}_pa_q$ and ${}_pc_q$ and their values see Appendix III.

The only assumption made in deriving this expression for the armature reaction flux is in regard to the symmetry of the magnetic circuit. But for this it is perfectly general.

Armature Flux, Voltage and Power. In an alternator with a uniform air gap the armature reaction is stationary with respect to the field poles only when the harmonic orders of the component flux distribution and the component armature current are the same. With a salient-pole alternator, however, the fundamental current may produce a stationary third harmonic in the armature flux, or a third harmonic component in the armature current may produce a stationary fundamental component in the armature flux distribution. With a salient-pole alternator there is likely to be a considerable third harmonic component in the armature reaction which is stationary with respect to the field poles and is due solely to the fundamental part of the armature current.* If the armature is delta-connected, a third harmonic current will exist in the windings which may of itself produce a stationary fundamental component in the flux distribution.† The latter would affect both the fundamental voltage and the power developed.

With a uniform air gap, in which case $p = o$, no average power is developed. That is, no power is developed in an alternator having a uniform air gap if the field excitation is zero. With a salient-pole alternator, however, in which case p is an integer, power may be developed due to the armature reaction flux alone, or, in other words, when there is no field excitation. This power is proportional to the sine of twice the lead angle of the fundamental armature current. The power is greatest with a phase angle of 45 degrees. Thus an unexcited synchronous machine may develop considerable power as a generator if the current leads, and considerable power as a motor if the current lags. These are well known facts. Furthermore a current of one harmonic order may produce a flux that reacts with a current of a different harmonic order and produces power.

When a synchronous motor is being brought up to speed by hysteresis and eddy current action, the fundamental current is relatively a fifth harmonic at one-fifth of synchronous speed, a third harmonic at one-third of synchronous speed and a second harmonic at one-half of synchronous speed. If one of these harmonics produces a torque greater than that due to hysteresis and eddy currents, the motor will tend to lock in at that speed and not come up to synchronism. Whether it does or does not will depend upon whether the momentum of the rotating part is sufficient to carry it into a phase position that is unfavorable for the development of motor torque due to the reaction of the air gap flux and the current. The hysteresis and eddy current torque is greater at one-fifth speed and the synchronous torque developed by the current is smaller than at either one-third or one-half speed. The tendency of the motor to lock in at one-half speed is greater than at either one-third or one-fifth speed. It depends upon the design and principally upon the ratio of pole arc to pole pitch.

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*See fourth term, equation (15) $q = 3$, $p = 1$, $s = 1$.

†See sixth term equation (15) $q = 1$, $p = 1$, $s = 3$.

Harmonics in Transient Short-Circuit Condition. The transient short circuit in an alternator gives a good illustration of the effect of even harmonics in the armature current. We will consider only the case of a three-phase turbo-alternator which has its field and armature windings so distributed that the harmonic components in the flux distributions they produce are negligible compared with the fundamental components.

The general equation for this current is

$$i = \Sigma A\, e^{-\alpha t} + \Sigma B\, e^{-\beta t} \sin s\, \omega\, t + \Sigma C\, e^{-\gamma t} \cos s\, \omega\, t \qquad \textbf{(16)}$$

As far as the writer is aware the constants in this equation have never been evaluated.

Due to the first term in this expression there is a decaying flux distribution which rotates at synchronous speed with respect to the field poles. There will thus be produced a current of fundamental frequency in the field winding. The periodic flux produced by this current will generate a voltage of double frequency in the armature winding, causing a second harmonic current, which will react on the field circuit and produce a third harmonic current in it. This third harmonic variation of the field flux will produce a second and fourth harmonic in the armature winding. The effect of a second harmonic has already been accounted for. The fourth harmonic in the armature current will produce only a third harmonic in the field winding, the effect of which has already been considered. The fifth harmonic which would have been produced in a single-phase alternator is not present, since the component fluxes which would produce it sum up to zero in the case of a three-phase alternator.* There will thus be a fundamental and a third har-

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*First term, equation (**15**) $q = 1$, $s = 4$.

monic in the field current, and a fundamental, a second and a fourth harmonic in the armature current, but none higher. The magnitude of these harmonic components will be influenced to a considerable extent by the eddy currents produced in the practically solid rotor.

Calculation of Field Current. One of the most interesting applications of this theory, is to the problem of calculating the field current of an alternator for a given load condition. A three-phase, Y-connected generator, with neutral free, is chosen so that there can be no troublesome third harmonics in the armature current. There is generally little error in neglecting the effect on the terminal voltage of the fifth or higher harmonics in the armature flux if the magnetic circuit is designed so that there is no prominent fifth harmonic in the flux distribution due to the field. If it be assumed that the armature currents are sinusoidal and balanced, that the harmonics in the armature flux above the third are negligible and that the harmonic variations in these components above the second are also negligible,* the expression for the stationary armature reaction flux distribution reduces to

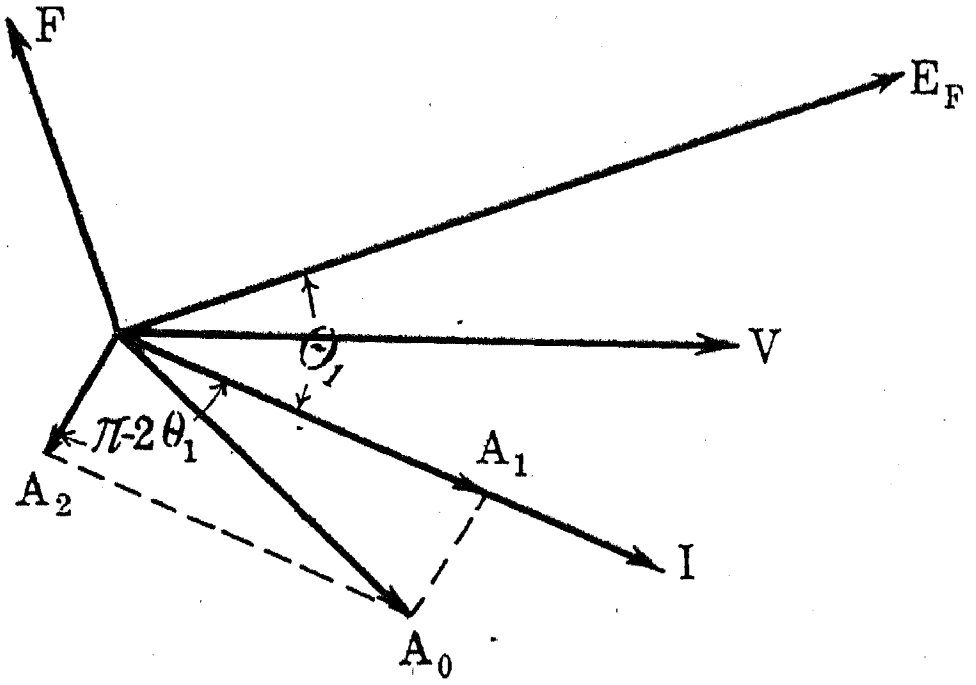

FIG. 11

$$\beta = \frac{1}{\sqrt{2}} \frac{N_P}{N_F} K_1 I_1 \left\{ -{}_0a_1 \cos(x + \theta_1) + \frac{{}_1a_1 + {}_1c_1}{2} \cos(x - \theta_1) - \frac{{}_1a_3 + {}_1c_3}{2} \cos(3x + \theta_1) + \frac{{}_2a_3 + {}_2c_3}{2} \cos(3x - \theta_1) \right\} \qquad (17)$$

---

*In equation (**15**) $s = 1$, $q = 1$ and 3 and $p = 1$ and 2 only.

The first of these terms is that ordinarily given for the armature reaction and it is the only one present when the air gap is uniform. The other terms are due to the non-uniformity of the magnetic circuit for different positions of the armature coils on account of the spaces between the salient poles. The third and fourth terms are due to the presence of variable third harmonics in the flux distributions produced by the armature coils. They have no direct effect on the fundamental armature voltage but they do alter the saturation of the main magnetic circuit and may be important on this account. If they increase the flux from each pole it requires more field current to give a specified armature electromotive force than is shown by the no-load saturation curve.†

The vector diagram showing the phase relations of the fundamental armature reaction, the armature current and the air gap flux is given in Fig. 11. F and E_F are the impressed field and the voltage it would produce on open circuit. V is the terminal voltage. The armature current, I, lags behind the excitation voltage E_F by an angle θ_1. A_1 and A_2 are the first and second terms of the resultant armature reaction A_0. From equation (**17**) it is seen that

$$A_1 = \frac{1}{\sqrt{2}} \cdot \frac{N_P}{N_F} \cdot K_1 \cdot I_1\ ({}_0a_1)^*$$

and $$A_2 = \frac{1}{\sqrt{2}} \cdot \frac{N_P}{N_F}\ K_1 \cdot I_1 \cdot \frac{{}_1a_1 + {}_1c_1}{2}^{\,*}$$

A_1 is in phase with the current while A_2 lags behind the current by $(\pi - 2\,\theta_1)$ if $({}_1a_1 + {}_1c_1)$ is positive, as it usually is. Thus the resultant armature reaction lies more nearly in line with the impressed field than if the air gap were uniform. This condition occurs for two reasons; first, because an armature coil produces more flux when opposite a pole than when half way between poles $({}_1a_1 > 0)$, and second because the axis of the armature flux tends to conform with the axis of the field poles $({}_1c_1 > 0)$. At unity power factor, A_1 and A_2 are in opposition, while at zero power factor they are in conjunction. Thus the magnitude of the armature reaction flux is much greater at zero power factor than at unity power factor.

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†See *Effect of Third Harmonic on Saturation.*

*Values of the coefficients ${}_0a_1$, ${}_1a_1$ and ${}_1c_1$ etc. are given in appendix III.

Fig. 12 shows the vector diagram which may be used in calculating the regulation of an alternator. The power factor (external) at which the alternator is operating is cos θ.* The sum of the terminal voltage, V, the resistance drop, $I\,r_e$†, and the leakage reactance drop, $I\,x_a$†, gives the voltage E_a, which is produced by the resultant fundamental flux in the air gap. This flux is due to the effect of the field and armature currents acting in conjunction on the magnetic circuit. If the permeability of all parts of the magnetic circuit is now assumed to be constant, the air gap flux which produces E_a may be replaced by the component fluxes which would be produced in the air gap by the independent action of the field and armature currents. With such an assumption the fluxes and the equivalent field currents producing them are proportional. I_R is the field current corresponding to E_a, obtained from the open-circuit saturation curve. A_1 and A_2 are the equivalent field currents corresponding to the two parts of the armature reaction. I_F is the actual field current. It is readily proved by geometry that the vector drawn from the terminus of I_R to the intersection of A_1 and I_F is equal to $(A_1 - A_2)$ and that the distance from this intersection to the terminus of I_F is $2\,A_2 \sin \theta_1$. Thus the phase position of I_F is determined by the armature reaction at unity power factor.

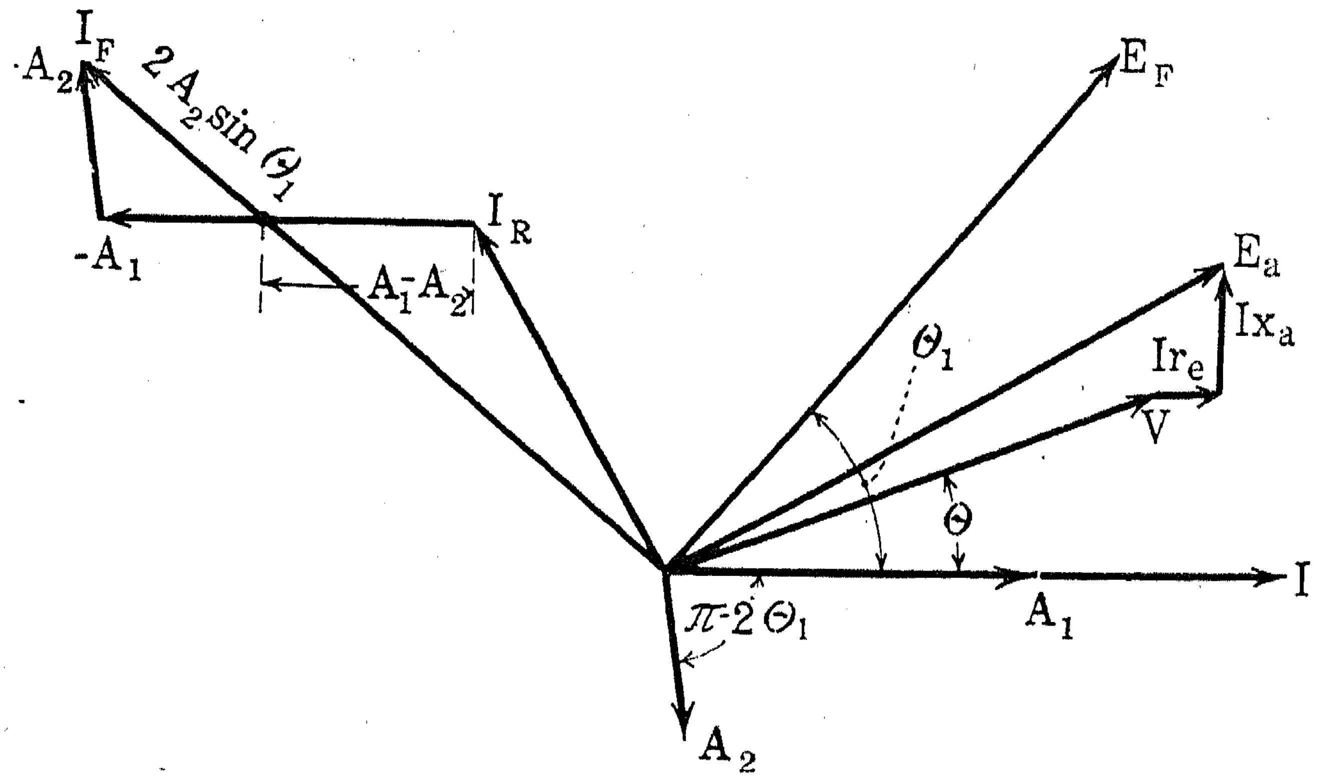

FIG. 12

For the purpose of calculation it is more convenient to rotate I_R and I_F through 90 degrees clockwise so that the vector diagram appears as in Fig. 13.

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*An angle of lead is positive and an angle of lag is negative.

†r_e and x_a are the equivalent single-phase values.

The armature leakage reactance may be determined from the open- and short-circuit characteristics. At short circuit the internal power-factor angle, θ_1, is essentially 90 degrees. Calculate the field current equivalent to the total armature reaction at zero power factor, viz. $A_1 + A_2$. Observe the voltage E_{AO} that would be produced on open circuit by this field current. Referring to Fig. 14 the leakage reactance is shown to be

$$x_a = \frac{E_F - E_{AO}}{I_a}$$

There are two minor corrections which will increase the precission of the results. They are the effect of the third harmonic and the increased pole leakage.

Effect of Third Harmonic on Saturation. The flux in the air gap and in the main magnetic circuit is produced by the magnetizing action of the field current and the armature current. Part of this flux is sinusoidally distributed and produces a measurable voltage across the terminals of the armature, while a small part of it is a third harmonic distribution and produces no voltage across the terminals. The saturation of the magnetic circuit, however, is determined by the entire flux. The armature voltage, E_a, (See Fig. 13) is due to the combined action of the field current and the sinusoidal portion of the armature reaction. The third harmonic portion of the armature reaction increases the flux per pole, when the current is a lagging one, and therefore more net field current is required to produce E_a under load conditions than at no-load. The higher harmonics in the armature flux may also increase the saturation, but their effect is

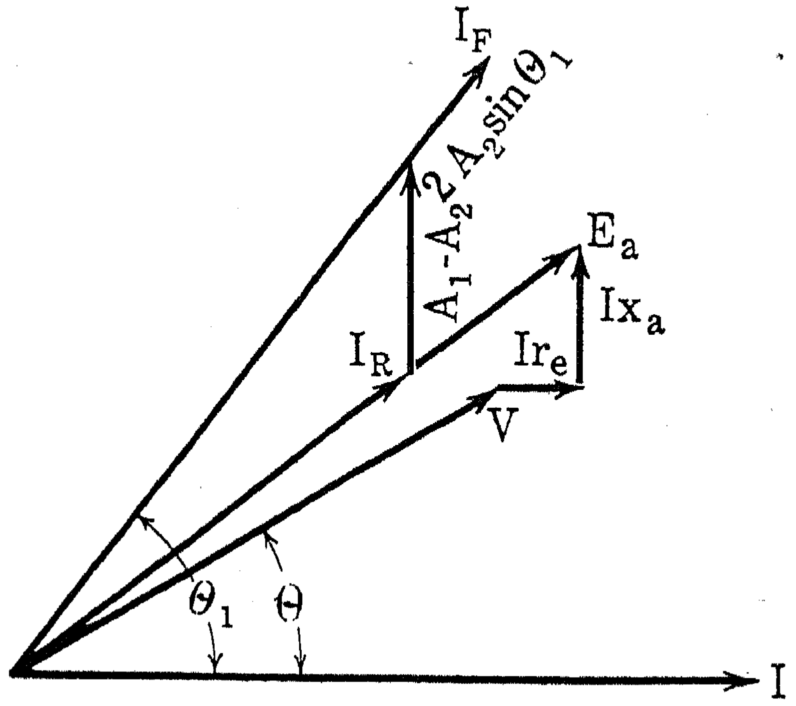

FIG. 13

usually small. The magnetizing action of the third harmonic component, in equivalent field current, is somewhat less than·

$$A_{03} = -\frac{1}{3}\,\frac{1}{\sqrt{2}}K_3\,\frac{N_P}{N_F}\,I_1\left(\frac{{}_1a_3 + {}_1c_3}{2} + \frac{{}_2a_3 + {}_2c_3}{2}\right)\sin\theta_1{}^*$$

In Fig. 15 E_a is the armature voltage which would be produced by I_R on open circuit. Under load, because of the increased saturation due to the third harmonic, it requires I'_R amperes to produce this same voltage. The entire magnetizing action for this condition is equivalent to a field current of $(I'_R + A_{03})$ amperes. On open circuit this would produce a voltage E. Perhaps the simplest method of determining the field current,

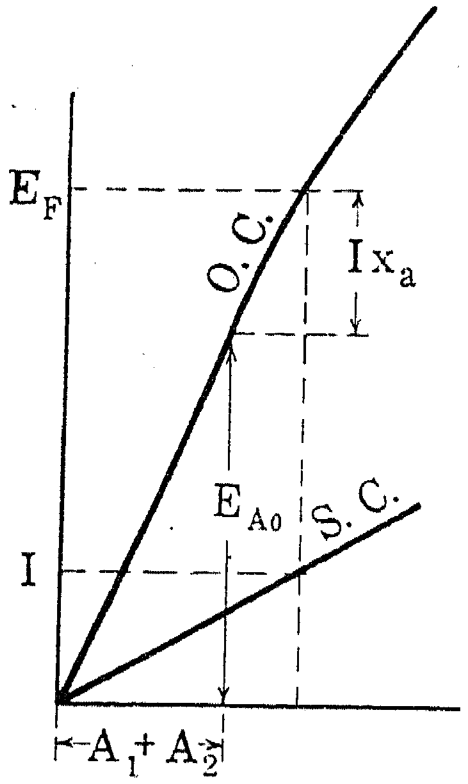

FIG. 14

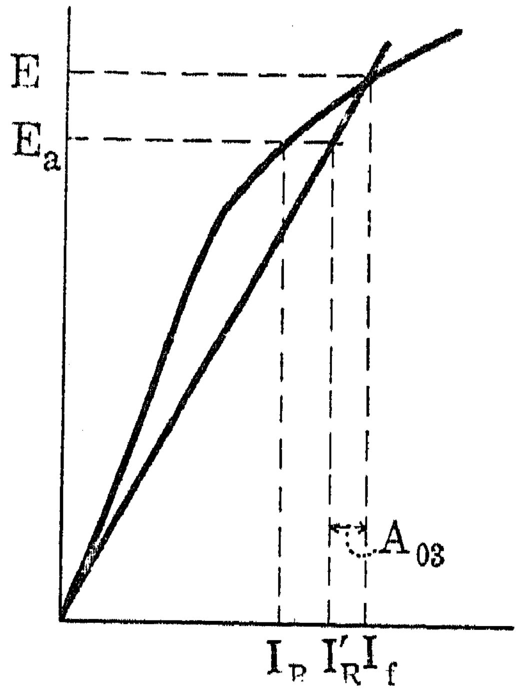

FIG. 15

I'_R, necessary to give the required armature voltage is to assume two values of E somewhat above the actual armature voltage, E_a, to calculate the corresponding values of E_a and then to interpolate between them. The values of I'_R and E_a corresponding to an open-circuit voltage E are

$$I'_R = I_f - A_{03} \text{ and } E_a = \frac{I'_R}{I_f}E$$

It is suggested that values of I_f be chosen equal to $I_R + A_{03}$ and $I_R + 3\,A_{03}$. This correction is quite small unless the saturation is high and the power factor is low. Except in a few cases to which attention is called no account of this effect is taken in the calculations that follow.

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*For a lagging current $\sin\theta_1$ is negative.

Increased Field Leakage. On open circuit the field current produces not only the flux in the airgap but also the leakage flux between the poles. Under load conditions the field current must usually be increased if the flux is to be maintained constant, on account of the demagnetizing effect of the armature reaction. Since the pole leakage is nearly proportional to the field current, it will usually be greater under load conditions than at no-load for the same armature voltage. The increased leakage requires additional field current, which is proportional to the difference between the calculated value of the field current, I_F, and that required to produce the armature voltage, namely I'_R. An approximate value of this added field current is

$$\left(\frac{v-1}{v} u\, y\right) \cdot (I_F - I'_R)$$

in which v is the leakage coefficient of the field, u, the saturation factor for the field poles and yoke, and y, the ratio of ampere turns required for the field poles and yoke to those required for the rest of the magnetic circuit. These coefficients should all be calculated for a condition of the magnetic circuit corresponding to the field current I'_R.

NUMERICAL CALCULATION OF FIELD CURRENT

1080-h p., 3-phase synchronous motor.

Volts	6600
Poles	6
Slots	108
Series conductors per slot	14
Coil pitch	0.83
Turns per field spool	462
Pole arc: pole pitch	0.642
Field Leakage coefficient	1.083
Ratio $\frac{\text{amp. turns, poles and yokes}}{\text{amp. turns airgap, teeth and arm. core}}$	0.026
Saturation factor—field poles and yokes (E_a = 6025)	5.6
${}_0a_1$	0.803
$\frac{{}_1a_1 + {}_1c_1}{2}$	0.269

Calculation of field current when 98.5 amperes is delivered at a power factor of 0.42, leading

$$0.707\, K_1 \frac{N_P}{N_F} I_a = 0.707 \times 0.923 \times \frac{18 \times 7}{462} \times 93.1$$

$$\begin{aligned} &= 16.57 \qquad \text{for } I_a = 93.1 \\ &= 17.52 \qquad \text{for } I_a = 98.1 \end{aligned}$$

Armature reaction at zero power factor

$$\begin{aligned} A_0 &= 1.072 \times 16.57 \\ &= 17.76 \text{ equivalent field amperes.} \end{aligned}$$

Field current on short circuit for armature current of 93.1 amperes is 19.2 amperes. Armature voltage on open circuit for field current of 50 amperes is 2250 volts.

$$I\, x_a = \frac{19.2 - 17.76}{50} \times 2250$$

$$\begin{aligned} &= 648 \text{ volts for armature current of } 93.1 \text{ amperes} \\ &= 686 \quad \text{“} \quad \text{“} \quad \text{“} \quad \text{“} \quad \text{“} \quad 98.5 \quad \text{“} \end{aligned}$$

$$\begin{aligned} V &= 6600 \cos \theta - j\, 6600 \sin \theta \qquad \cos \theta = 0.42 \\ &= 2772 - j\, 5990 \end{aligned}$$

$$\begin{aligned} I\, z_a &= \underline{92 + j\, 686} \qquad I\, r = 92. \text{ volts (between terminals)} \\ E_a &= 2864 - j\, 5304 \\ &= 6025 \text{ volts} \end{aligned}$$

Corresponding field current from saturation curve.

$$\begin{aligned} I_R &= 14.2 \text{ amperes} \\ &= \frac{2864}{6025} \times 14.2 - j \frac{5304}{6025} \times 14.2 \\ &= 6.75 - j\, 12.49 \end{aligned}$$

Armature reaction at unity power factor for armature current of 98.5 amperes.

$$\begin{aligned} A_1 - A_2 &= 0.535 \times 17.52 \\ &= 9.38 \\ I_R + j\,(A_1 - A_2) &= 6.75 - j\, 3.11 \\ &= 7.43 \\ 2\, A_2 \sin \theta_1 &= 0.537 \times 17.52 \times \frac{3.11}{7.43} \\ &= 3.94 \\ I_F &= 7.43 - 3.94 \qquad \theta_1 \text{ is an angle of lead} \\ \frac{v-1}{v}\, u\, y &= \frac{8.3}{108.3} \times 0.026 \times 5.6 \\ &= 0.011 \end{aligned}$$

$$I'_F - I_F = 0.011\,(3.49 - 14.2)$$
$$I'_F = 3.37$$

Measured value of field current = 2.0

It is difficult to predetermine accurately the field current of a synchronous generator for a given load condition, particularly when it is delivering a leading current. If the so-called magnetomotive force method or the method proposed by the A. I. E. E. is taken as the standard of simplicity, then it can be said that there is no simple method of calculating the field current that will give reliable results for every load condition. The effect of the armature current is much too complicated to be calculated so easily. The general method, in which the leakage reactance and armature reaction are determined by the Potier triangle, is perhaps as good a compromise as any between accuracy and simplicity.

TABLE OF COMPARATIVE FIELD CURRENTS.

Output	Voltage	P.F.[1]	FIELD CURRENT				
			Gen. method	M. M. F.	A.I.E.E.	Proposed	Observed
500 kv-a.	240	−0.0	....	121.5	....	134	135.5
1,500 kv-a.	2,300	−0.0	....	120.5	....	135.7	130.
2,000 kv-a.	2,300	−0.0	....	199.	....	214.	227.
2,000[2] kv-a.	2,100	−0.0	....	52.5	....	60.5	64.5
2,000[2] kv-a.	2,100	+0.0	−12.4	−5.6	....	−8.7	−10.0
2,500 kv-a.	2,300	−0.0	....	257.	....	310 327[4]	330.
2,500 kv-a.	2,300	+0.0	−12.	10.[5] −42.[6]	67	−6	−4.8
5,000 kv-a.	2,300	−0.0	....	235	....	258	251.
5,000 kv-a.	6,600	−0.0	....	221	....	246[4,7]	250.
5,000 kv-a.	6,600	1.0	173	171	165	170	169.
83.5 amp.[3]	11,150	1.0	40.0	39.5	36	41.1[4,7]	42.8
68.0 amp.[3]	11,550	+0.784	53.	48.1	56	58.2	59.
12,000 kv-a.	6,600	−0.0	....	372	....	406	410.
93.1 amp.	6,600	−0.0	....	35.2	....	36.8	36.9
110 amp.	6,600	+0.72	16.6	16.5	16.7	13.	12.0
98.5 amp.	6,600	+0.42	9.2	9.0	16.1	3.37	2.0

1. + sign indicates a leading current; − sign, a lagging current.
2. Two-phase.
3. As a motor.
4. Corrected for effect of third harmonic.
5. Armature m.m.f. from short-circuit data.
6. Armature m.m.f. from zero power factor data at rated voltage.
7. Not corrected for increased field leakage.

The author wishes to aknowledge his indebtedness to the General Electric Company and Westinghouse Electric and Manufacturing Company for the data from which these results were calculated.

APPENDIX I

Refer to Fig. 16. N_1, N_2, N_3 and N_4 indicate the numbers of turns in the coils. Let B_0 be the average flux density per ampere turn when one coil alone is acting. The flux density per ampere at a distance x, measured from the center of a belt of conductors, reduces to

$$\beta = \frac{8\,B_0}{\pi} \sum \left(N_1 \cos \frac{q\,\gamma}{2} + N_2 \cos \frac{3\,q\,\gamma}{2} + N_3 \cos \frac{5\,q\,\gamma}{2} + N_4 \cos \frac{7\,q\,\gamma}{2}\right) \frac{1}{q} \sin q\,x$$

If any harmonic is to be eliminated its coefficient in this series must be zero. Since there are three independent variables in each coefficient, viz. $\frac{N_2}{N_1}$, $\frac{N_3}{N_1}$, $\frac{N_4}{N_1}$, at least three harmonics and their odd multiples may be eliminated. Choose these variables so that the third, fifth and seventh harmonics will vanish. Since there are 9 slots per pole the angle $\gamma/2$ equals 10 degrees. The equations are

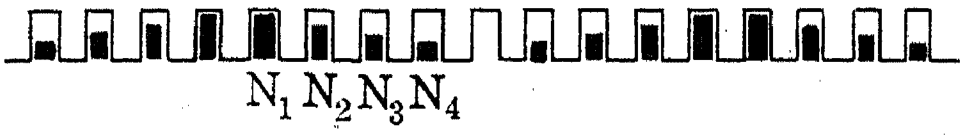

FIG. 16

$$N_1 \cos 30 + N_2 \cos 90 + N_3 \cos 150 + N_4 \cos 210 = 0$$
$$N_1 \cos 50 + N_2 \cos 150 + N_3 \cos 250 + N_4 \cos 350 = 0$$
$$N_1 \cos 70 + N_2 \cos 210 + N_3 \cos 350 + N_4 \cos 490 = 0$$

The ninth harmonic vanishes for any number of turns in the coils. The equation for the eleventh harmonic coefficient is identical with that for the seventh except that the signs are changed. It therefore vanishes. The same is true for the thirteenth and fifteenth harmonics. Thus the first harmonic that appears is the seventeenth. The other higher harmonics that may be present are the nineteenth, twenty-third, twenty-ninth, etc. The first harmonics that appear, viz. the seventeenth and the ninteenth are of the same order as would be produced by the rotor slots. They could be reduced to small proportions by closing the slots with magnetic wedges. The solution of the above equations for the relative numbers of turns in the field coils gives

$$\frac{N_2}{N_1} = 0.88$$

$$\frac{N_3}{N_1} = 0.65$$

$$\frac{N_4}{N_1} = 0.35$$

The efficiency of the winding is 80 per cent.

The reduction factor for the seventeenth harmonic is the same as that for the fundamental but the harmonic is reversed in phase. The same is true of the nineteenth harmonic. Each of these harmonics thus occurs in the same relative magnitude as it would in the distribution due to a concentrated winding. Making the correction for saturation, neither alters the efficiency of the winding nor the reduction factors for the seventeenth and nineteenth harmonics, but does alter their relative magnitudes. If $\frac{b_1}{B_0}$ is 0.10, they are both about thirteen per cent greater than at low saturation.

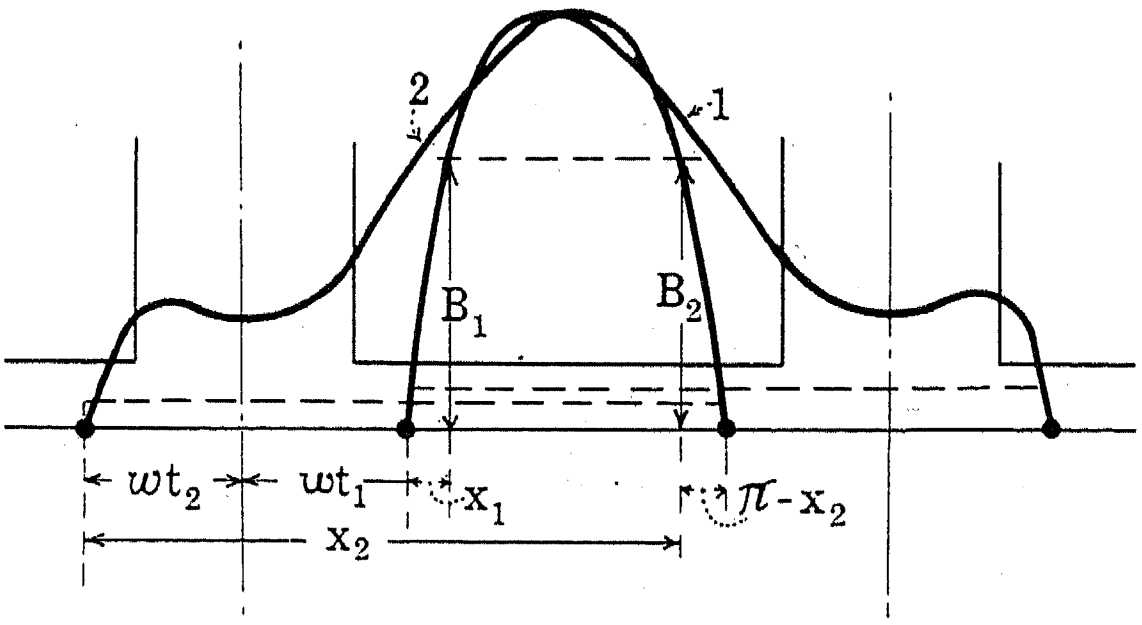

FIG. 17

APPENDIX II

In Fig. 17, $B_1 = B_2$ if $x_1 = \pi - x_2$ and $\omega\, t_1 = -\ \omega\, t_2$ and the magnetic circuit is symmetrical about its center line.

$B_1 = \Sigma\, A_q' \sin q\, x_1 + \Sigma\, C_q' \cos q\, x_1$

$B_2 = \Sigma\, A_q'' \sin q\, x_2 + \Sigma\, C_q'' \cos q\, x_2$

But since $x_1 = \pi - x_2$, and q is always odd

$B_2 = \Sigma\, A_q'' \sin q\, x_1 - \Sigma\, C_q'' \cos q\, x_1$

From this it follows that $A_q' = A_q''$ and $C_q' = -\ C_q''$.

But A_q and C_q are periodic functions of $2\, \omega\, t$ and since $\omega\, t_1 = -\ \omega\, t_2$, A_q must be a cosine series and C_q a sine series.

If one tip of the field pole is more highly saturated than the other, as it usually would be, the magnetic circuit will not be quite symmetrical about its center line. This distortion is probably never very great since most of the reluctance is in the air gap. Its effect would be to introduce sine terms into the series for A_q and cosine terms into the series for C_q. The final result would be to produce small quadrature electromotive forces in the armature winding which would otherwise be absent and which are neglected throughout this analysis.

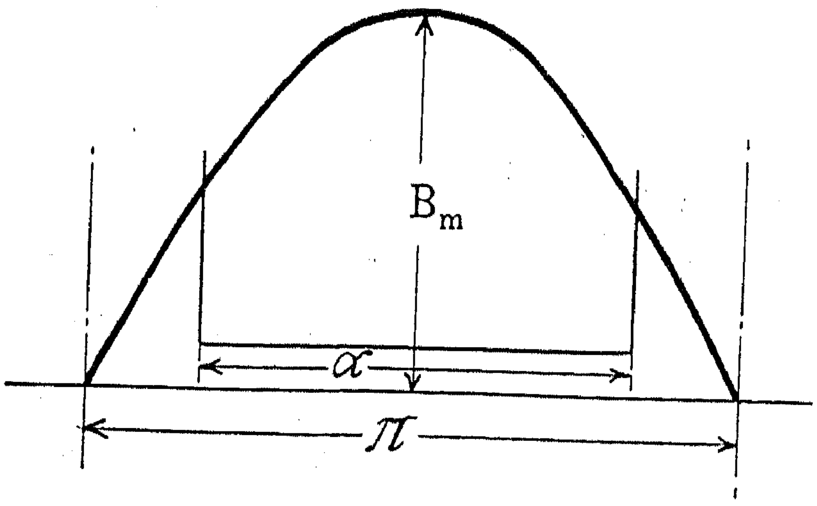

FIG. 18—FLUX DISTRIBUTION DUE TO FIELD CURRENT

APPENDIX III

Calculation of Coefficients. The following assumptions in regard to the distribution of flux in the air gap are considered ideal. They are chosen to represent the actual conditions as nearly as possible and still allow relatively simple calculations to be made. With these assumptions, however, it requires about fifteen pages of close calculation to obtain the desired coefficients. In practise, it may happen that the departure of the actual distributions from these ideal forms may be so great as to warrant new calculations that will more nearly fit the

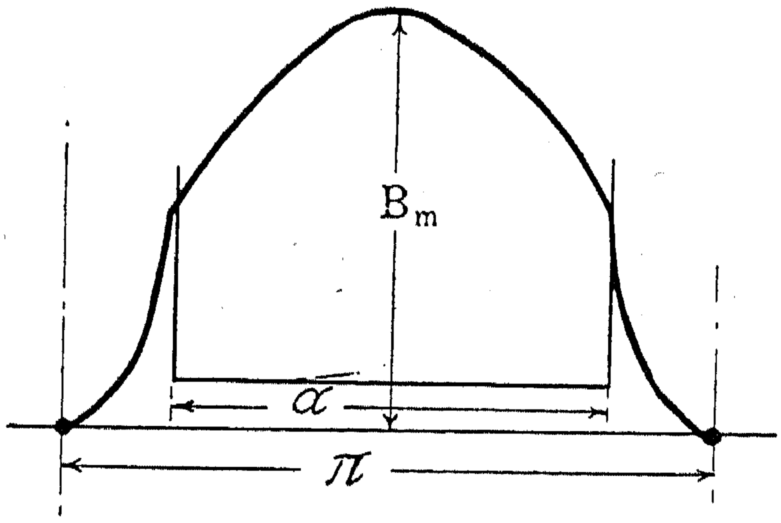

FIG. 19—FLUX DISTRIBUTION DUE TO ARMATURE CURRENT. ZERO DEGREES.

special case. The flux distributions can also be determined experimentally and the coefficients deduced therefrom.

With one ampere-turn acting per field pole the flux distribution in the air gap is assumed to be sinusoidal, with a maximum value of B_m. See Fig. 18. For the same saturation of the main magnetic circuit with one armature turn carrying one ampere, the flux distributions for different positions of the turn are shown in Figs. 19, 20 and 21. At zero degrees, with the armature coil directly opposite a field pole, the distribution is an inverted sine curve between the poles and coincides with the distribution due to a field ampere-turn under the poles. The maximum value is B_m. At 45 and at 90 degrees, the distribution is also an inverted sine curve between the poles and coincides with the distribution due to a single field ampere-turn under the poles. Taking the maximum value in each of these four cases as B_m assumes that the reluctance of the armature core, pole and yoke is negligible compared with that of the air gap and teeth. This is nearly so at low saturation. For operating conditions the

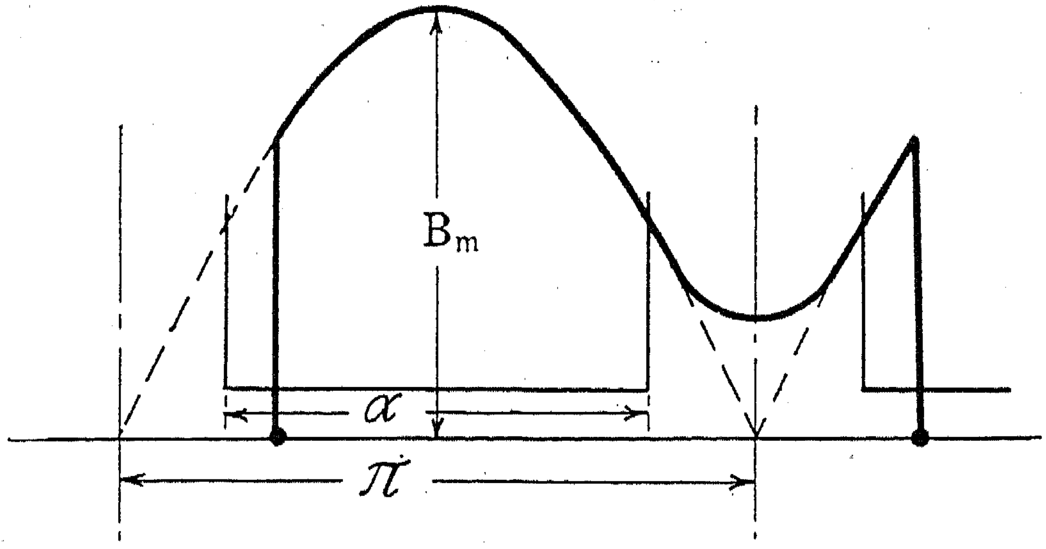

FIG. 20—FLUX DISTRIBUTION DUE TO ARMATURE CURRENT. 45 DEGREES

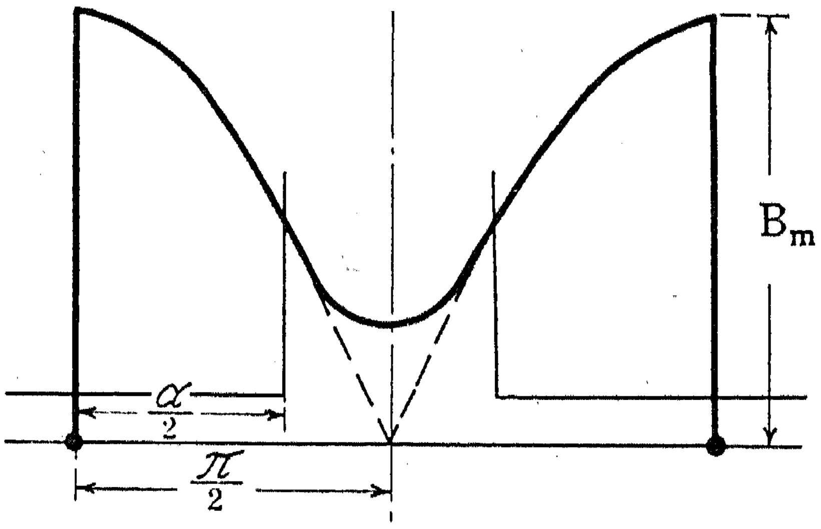

FIG. 21—FLUX DISTRIBUTION DUE TO ARMATURE CURRENT. 90 DEGREES.

saturation is higher and the flux densities due to an armature coil will be relatively greater for those positions that give flux paths that do not include the field poles, yokes and core. In order to make the calculations as simple as possible it is assumed that this applies only to the distribution at 90 degrees. It should also apply to the distribution at 45 degrees. The method of estimating this increase in the flux density, however, assumes that the condition of the magnetic circuit for a coil position of 90 degrees is the same as at low saturation. This assumption neglects the increase in reluctance of the armature core and pole faces and thus makes a compensating error. The method of estimating the increase in the flux density for a coil position of 90 degrees is illustrated in Fig. 22. If the saturation of the main magnetic circuit is determined by the voltage E_a on the open-circuit saturation curve, the value of the maximum flux density should be taken as $\frac{E'}{E_m} B_m$ for the distribution at 90 degrees.

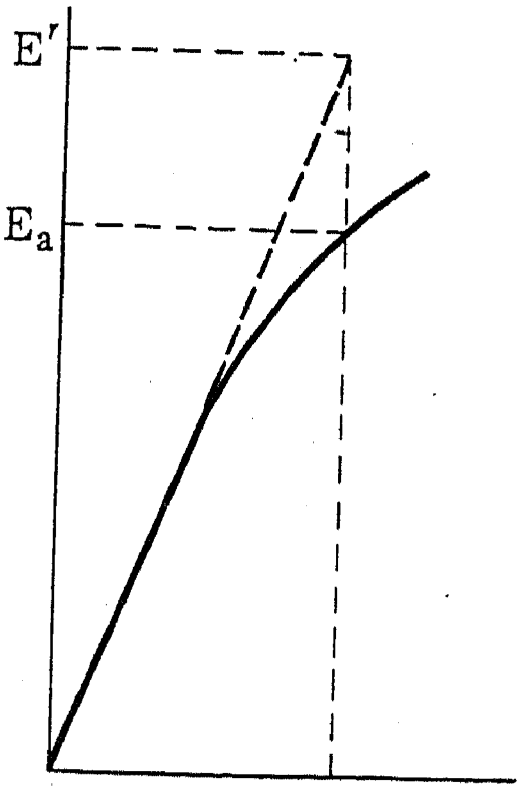

FIG. 22

In the two latter positions of the armature coil the amplitude of the inverted sine curve is taken as $1 - \frac{2}{\sqrt{4 + \frac{b^2}{g^2}}}$ of the density at the pole tip*, where b is the width of the interpolar space and g is the length of the air gap at the pole tip. An average value of this amplitude is about 0.85 and it is taken as such in the calculations. The Fourier analysis of these assumed flux distributions gives formulas for the coefficients of the sine and cosine terms of the fundamental and third harmonic distributions. If the coefficients of the third and higher harmonics in the *variations* of these coefficients are assumed to be negligible, the approximate values of ${}_0a_1$, ${}_1a_1$, ${}_1c_1$, ${}_1a_3$, ${}_2a_3$, ${}_1c_3$ and ${}_2c_3$ can be calculated. At the end of this appendix these coefficients are tabulated for a ratio of pole arc to pole pitch of 0.70 for different saturations. For ratios of pole arc to pole pitch between 0.65 and 0.75 the armature flux is nearly constant.

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*This formula is due to Mr. F. W. Carter, who, however, uses it in connection with the problem of determining the flux distribution opposite an armature slot placed in an otherwise uniform air gap.

Different assumptions in regard to the shape of the flux distributions for different positions of the armature coils would give different values for these coefficients. Considerable research may be necessary before reliable representative values can be chosen. Oscillographic measurements made at low saturation on a 7.5-kw. generator give a ratio of third harmonic to fundamental component at unity power factor of 52 per cent, and at zero power factor of 18 per cent. The ratio between the fundamental fluxes for unity power factor and zero power factor is 45 per cent. Referring to the table of coefficients it will be seen that the calculated values of these ratios are respectively 64 per cent, 22 per cent and 50 per cent. The coefficients given in this paper are only roughly indicative of the general results that may be expected, and the following conclusions are liable to some error. Within its usual values the ratio of pole arc to pole pitch has apparently little effect on the armature reaction. At unity power factor the reaction is about 3 per cent greater for a ratio of 0.75 than for one of 0.65. For a ratio of the length of the air gap at the pole tip to the length of theinterpolar space of 0.05 the reaction at unity power factor is about 2.5 per cent less than for a ratio of 0.10. Saturation has no effect on the armature reaction at zero power factor but at unity power factor the armature reaction increases with the saturation. At higher saturations the third harmonic in the armature reaction is more prominent than at lower saturations at unity power factor, but at zero power factor the third harmonic component is practically constant.

TABLE OF COEFFICIENTS.*

Saturation..........	1.0	1.11	1.2	1.3
${}_0a_1$..................	0.80	0.82	0.83	0.85
$\frac{{}_1a_1 + {}_1c_1}{2}$	0.27	0.26	0.24	0.23
$\frac{{}_1a_3 + {}_1c_3}{2}$	−0.29	−0.30	−0.32	−0,33
$\frac{{}_2a_3 + {}_2c_3}{2}$	0.05	0.07	0.09	0.10

*To obtain the best results the coefficients should be calculated for the proper ratio of pole arc to pole pitch and for air gap to interpolar space.

DISCUSSION ON "APPLICATION OF HARMONIC ANALYSIS TO THE THEORY OF SYNCHRONOUS MACHINES" (LYON). (PRESENTED BY PUBLICATION.)

N. S. Diamant: I would like to call attention to a few points in connection with this paper. As the author states very clearly, the principle of the analysis given in the paper is well known and all that the author claims and expects to do is to develop it more fully and put it in a form more usable for the average engineer. It is difficult to see whether the author has succeeded in this for the following reasons.

In a paper of this nature it is well, if not necessary, to connect up one's work with that of previous investigators by giving references at the proper places. This, however, the author has not done. Therefore, it may be of interest to state that a very complete resumé of generation of voltage in dynamo electric machines was given by S. P. Smith and R. H. Boulding, *Journal I. E. E.*, 1915, Vol. 53. In connection with the general method of analysis referred to by the author in the first three paragraphs of the paper, the following references might prove useful to the reader who wants to go further into the subject: *Journal de Physique* 1897, 3 series, T. VI. p. 341 and 483. A. Blondel, *L'Eclairage Electrique*, 1895, T. IV, pp. 241, 308 et 358. T. V. p. 268. See also P. Janet, *Lecons d'Electrotechnique generale*, T. III, chp. V. Alex Russell, *Altern. Currents*, Vol. II, Chp. XIV.

The above remarks are not made in order to detract from the value of the paper, but because during the last few years, in connection with the study of similar subjects, I felt the need of a paper or book summarizing in a general and clear way the best results of work that had been done along this line of the application of harmonic analysis to the theory of synchronous machines. Thus, in a paper* presented at the Atlantic City Convention on "Sustained Short-Circuit Phenomena and Flux Distribution of Salient-Pole Alternators," I was compelled to include some introductory material bearing on the flux distribution and flux pulsation and oscillation. See Section 3 and Supplementary Notes at the end of the paper referred to. It is rather disappointing to find that the author has not quite filled this need of a general critical summary of the subject.

I think it would be well in a paper of this nature to make perfectly clear when one refers to time harmonics called multiple fields. For example, referring to page 1184 and following the author's illustration it can be shown that an ordinary armature winding with N conductors per slot and one slot per pole per phase, supplied with a sinusoidal, in time, current, will be exactly equivalent to: (1) flux, sinusoidally distributed, in space, having a pitch of τ cm. or inches. (2) a flux sinusoidally distributed having a pitch of $\tau/3$ and 1/3 as many turns as the first, etc. In general, in case of a polyphase alternator with a sine-shaped armature current with regard to time, and any ordinary type of winding, non-sinusoidally distributed in space, the resultant m.m.f. can be considered as consisting of series of m.m.f. curves, all of fundamental frequency, in time, and having a pole pitch τ/k and traveling at a speed of k times the speed of the fundamental wave. Thus, it is extremely important and makes it very easy reading if the author in every case would make it perfectly clear whether he refers to time or space harmonics.

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*TRANS., A. I. E. E., 1918, Vol. XXXVII, p.

Mathematically speaking, it is also very well to make clear whether the author refers to traveling waves or stationary waves.

In connection with the effect of slots given on page 1492 it would have been well if the author had explained more in detail the origin and production of the tooth harmonics, particularly as there seems to be considerable difference of opinion in regard to this as referred to in my paper on 1493. The author states that "the periodic variations in the shape of this flux distribution produce a corresponding variation in the coefficients of its harmonic components. Each of these coefficients may then be represented by a Fourier series." On the next page after some mathematical transformations he is contented to state further "that the periodic variations in the A coefficients indicate corresponding variations in the flux per pole, while the periodic variations in the C coefficients indicate corresponding distortions of the flux distribution. The latter are equivalent to an oscillation of the flux across the pole face."

These statements I am afraid are at their best too abstract and mathematical, and personally I would greatly appreciate having a little more detailed interpretation and explanation of the underlined phenomena. I would be glad for example to have the author explain how he starts with periodic variations in the flux and after some mathematical transformations he gets a variation plus a to-and-fro oscillation of the flux across the pole face. The coefficients A and C may indicate this but it would help very much if he will explain to us the physical mechanism involved.

With reference to the harmonics in sudden short-circuit conditions I would appreciate having an explanation of the derivation of equation (**16**) and just what is meant by the remark that "so far as he is aware the constants in this equation have never been evaluated." This statement may be true but for the sake of the reader who wants usable data on the subject as to theory I may refer him to Boucherot and Berg, and as to experimental data and methods of calculation to papers published during the last few years in the TRANSACTIONS A. I. E. E., *Electrical World*, *General Electrical Review*, and *Electric Journal*.

W. V. Lyon: The author agrees with Mr. Diamant that a comprehensive bibliography would be useful to the student of this subject.

The fundamental equation (**1**) on page 1479 represents a sinusoidal distribution of flux in the air gap that may be stationary or traveling with respect to the field poles, depending upon whether j is or is not zero. For any fixed value of time the equation shows that the flux density varies sinusoidally as we progress around the gap. The wave length is $1/q$ of the wave length of the fundamental distribution, *i. e.* $1/q$ of twice the polar pitch. The equation also shows that the density at any fixed point in the gap varies sinusoidally with the time. Should we progress around the gap at such a rate that $q\,x$ is always proportional to the time and equal to $j\,\omega\,t$ the density at this traveling point is constant. In this way the velocity of the distribution is determined. The velocity, x/t, thus equals $\frac{j\,\omega}{q}$ as stated on page 1479. When j is zero the distribution is stationary with respect to the field poles. Otherwise it is moving in the same direction as the armature if j is a positive quantity and in the opposite direction if j is a negative quantity. As stated, the poles are assumed stationary. Relative motion is of course the only thing that counts. The advantage of this form of expression for the flux density is that it at one and the same time indicates the shape of the distribution and its relative velocity with respect to the field poles.

Unfortunately the question of the harmonics in the e. m. f. due to the action of the slots cannot be handled with mathematical exactitude due to the fact that as the slots come under the pole tip the division of the flux between that which enters the tooth at the right and that which enters the tooth at the left is not at the center line of the slot. Thus it is not possible to calculate the instantaneous flux linkages with the armature coil lying in this and its corresponding slot by integrating the flux density function between limits, since the limits are not known. They might be determined experimentally by exploring the field within a slot when it occupies different positions with respect to the pole. In the absence of such data the author integrated from the center of the slot, but noted the assumption, page 1494. Referring to equation (**11**) on page 1490 it is seen that the flux per pole due to the field current alone is approximately equal to the integral of this expression from 0 to π. This is not of course strictly true since the density may not be zero at these two limits. The integral of the cosine terms is zero between these limits, *i. e.* the cosine terms contribute nothing to the polar flux. They do however alter its distribution, and, if variable, account for oscillations of the flux across the polar surface. On the other hand if the A coefficients vary the entire polar flux pulsates in magnitude. These oscillations and pulsations are due to the action of the slots as is well known. They are not always the cause of harmonics in the armature e. m. f. And although the question has been the subject of considerable investigation, the author has seen diametrically opposed conclusions in regard to the cause and suppression of these harmonics. Some combinations of pulsations and oscillations may produce no net effect in the winding as shown on page 1494. A simple method of reducing these variations in flux density is to make the ratio of slot opening to air gap as small as possible.

The author believes that the statement in regard to the transient short-circuit current is correct. While much work has been done in this important subject no solution has been given that is based on as rigorous theory as the question merits. The general form of equation (**16**) is of course determined by a consideration of the result produced by a disturbance in two inductively coupled circuits like the field and armature windings of an alternator. If further proof is needed it is supported by oscillographic records such as Mr. Diamant has taken.

Presented at a Section Meeting of the American Institute of Electrical Engineers, Philadelphia, Pa., September 16, 1918.

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ELECTRIC WELDING—A NEW INDUSTRY

BY H. A. HORNOR

ABSTRACT OF PAPER

This paper covers a brief review of the uses of electric spot and arc welding in this country prior to the formation of the Electric Welding Committee of the Emergency Fleet Corporation. It compares the status of the art at the present time and emphasizes the developments that have been made in apparatus in the last six months. It treats of the activities of the Welding Committee in applying electric welding processes to the shipbuilding industry and points the way to the general applicability of this method to other industries. It shows that the results obtained by investigation and physical tests prove that the applications of this process to heavy work are satisfactory.

INTRODUCTION

ABOUT a year ago the Chairman of the Standards Committee of the Institute was requested to investigate and standardize spot welders and the apparatus connected with them. It occurred to the members of this committee that electric welding could perform an important function in increasing the progress of steel ship construction. The work which was started by the Standards Committee was then transferred to the General Engineering Committee of the Council of National Defense. Last winter the Council of National Defense abolished all advisory committees but at this time the Emergency Fleet Corporation of the U. S. Shipping Board had become so much interested in the subject that they decided to adopt the Committee. The Committee is composed of representatives covering broadly the whole field of welding activities in this country and, although electric welding has been the subject of all the investigations up to the present time, it is now proposed to include gas welding with representatives from all the gas welding associations and companies connected with this industry.

The two main processes of electric welding, namely, arc welding and spot welding, were found by this committee applied in the first case to repairs and in the second case to certain factory quantity production jobs. The work done was in the case of spot welding only on light material, and in neither case very extensive. The processes to be successful in their application to the construction of merchant vessels would have to show reliability in the joining of steel plates from a half-inch to one inch in thickness. To this and kindred problems the committee immediately turned its attention.

The work had all been done in the field where it had been applied by practical men. It was first necessary to formulate the proper nomenclature and symbols. This was thoroughly investigated and a very comprehensive set of symbols has been approved by the committee and is in daily use by those now actively engaged in this new application. The approved nomenclature introduces the subject to the designing and calculating engineer and gives him the instrument by means of which he is able to place his thoughts rapidly and conveniently on drawings.

The manufacturers of apparatus joined the practical man in the study of the problems of electric welding. Apparatus and so-called processes introduced various types of machines suitable for the conversion of electrical supply to the proper values of current and voltage needed at the arc or at the spot. The manufacturer in his eagerness to meet the problem naturally encountered many difficulties. These difficulties increased until a point was reached as referred to above where he demanded some standards upon which his apparatus could clearly be rated. Therefore, the manufacturer was only too pleased to co-operate with the Welding Committee and is today conscientiously aiding in straightening out the difficulties in which he was involved prior to last year.

Arc welding in this country has largely been done in the railroad repair shops. It was discovered that the process was much cheaper and could be performed more rapidly than by any of the gas welding methods. It also could be applied without preheating and in many cases without the expense of disassembling complicated pieces of machinery. Spot welding besides being used in many different industries was sought for by the railroad man and there has been built a gondola car which has seen some seven or eight years of service. It is interesting to note here the difference in practise between Great Britain and the United States. The former knowing little or nothing about spot welding had the practise and application of arc welding very well under way; the latter exactly the reverse.

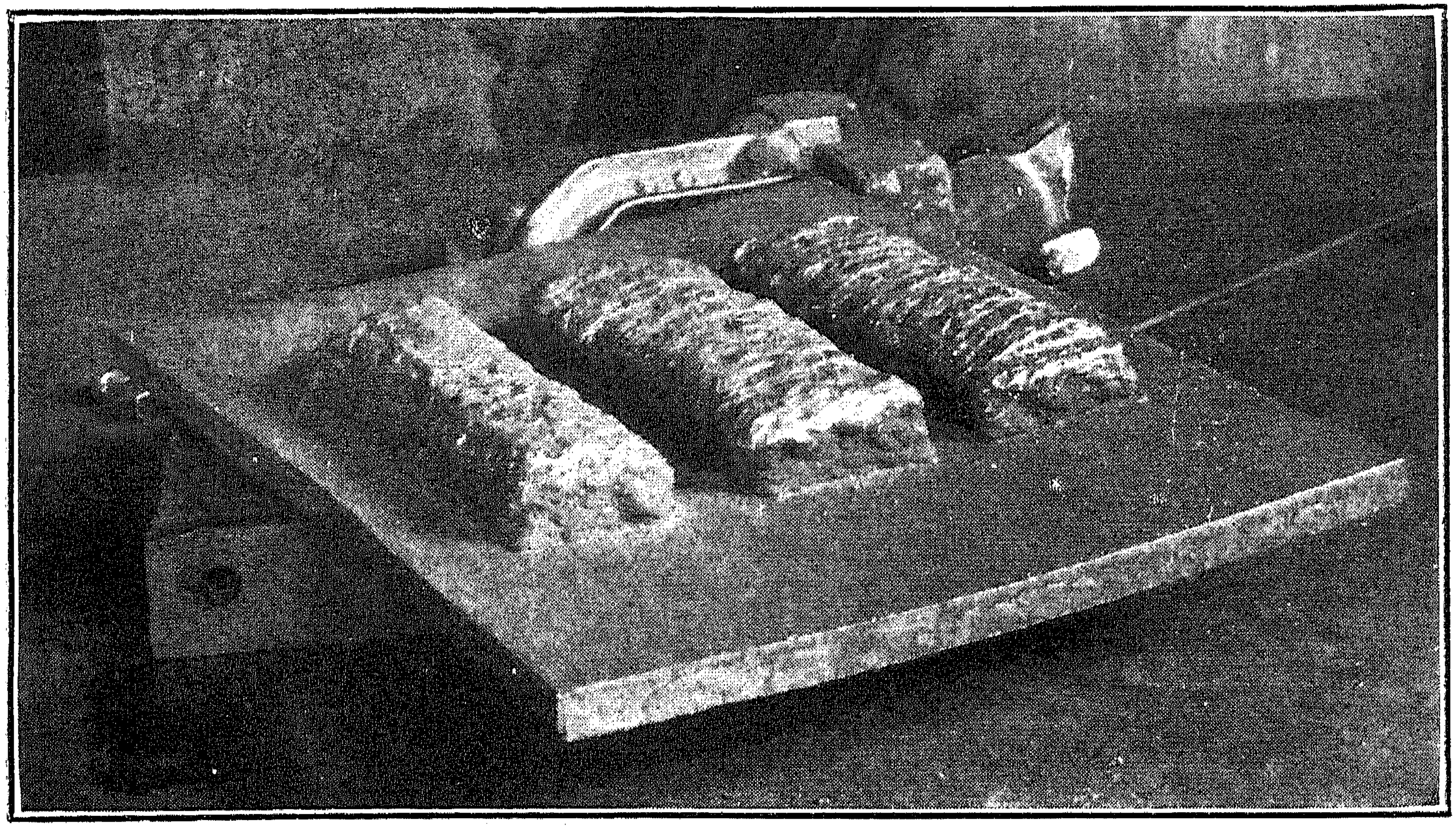
Apparently the attempts to train operators were rather crude and it was early observed that the reliability of the electric weld depended substantially upon the skill of the welder. The manufacturers of apparatus and the superintendents in railway shops had struggled with the problem of training operators but intensive study had not been given the subject so that there existed in this respect a great deal of groping in the dark.

PRESENT STATUS OF ELECTRIC WELDING

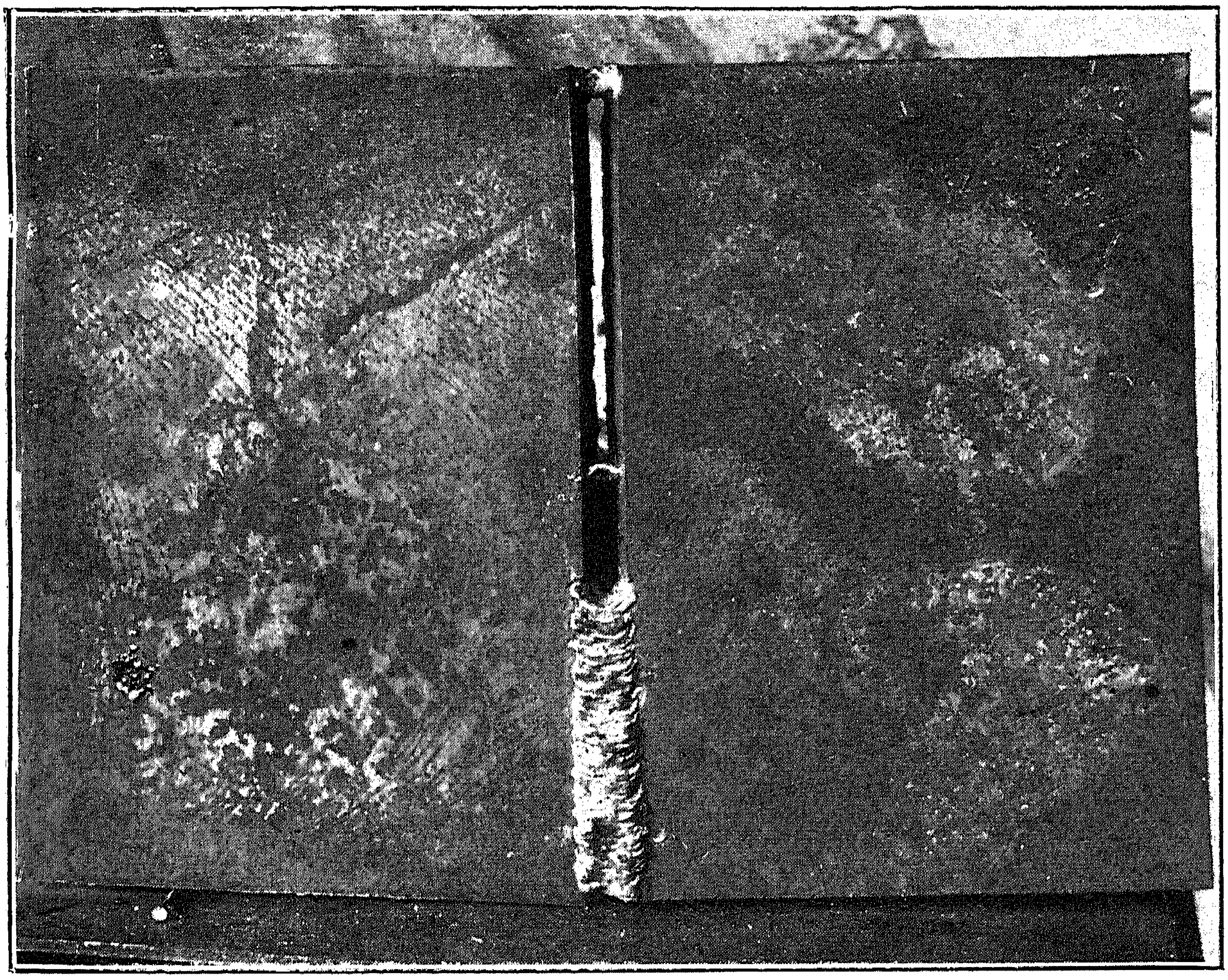
Investigations were immediately undertaken to answer the question whether spot welding could be successfully accomplished using one-inch thick steel plates. An experimental apparatus of large size was erected and put into operation, the results showing that no difficulty was encountered with half-inch and three-quarter inch plates. The same remark applies to one-inch steel plates. In fact, this experimental machine was successful in welding three thicknesses of one-inch plate, a condition which far exceeds the requirements of merchant ship construction. This operation has its historical significance in that this was the first time that any spot welding of this magnitude had been performed. The successful outcome of these experiments has led to the design and construction of large spot welders to be used in the fabrication of ship sections. The practical application of a large five-foot gap spot welder will be made at a demonstration of a forty-foot section of a standard 9600-ton ship to be built at the plant of the Federal Shipbuilding Company, Kearney, New Jersey. This is the largest portable spot welder ever built. It will prove two points in ship construction by the electric method, namely, the clamping of the ship's structural parts for assembly thereby reducing the time in working the material as well as for the erection of the ship material; and secondly, by the speed of spot welding it will prove the decrease in time for joining the material together. The consensus of opinion is that the large stationery spot welder of five- or six-foot gap will undoubtedly play an important part in increasing the speed of fabricating sections of standard steel vessels. Further investigations are being made and designs are being worked out for special spot welders for use in the construction of bulkheads. The designs proposed are chiefly for shop processes, but it can be asserted that such apparatus will be of undoubted value in the saving of time and man power.

Arc welding had been tried in a great variety of work but there was no conclusive evidence that it could be developed to the stage of joining ship plates with the certainty of full strength. The first stage of this investigational work is now almost completed. Sample welds of half inch ship structural steel were taken by a special sub-committee to fourteen or fifteen different places where electric welding was being performed. This sub-committee saw the welding done, noted the conditions of current, voltage, electrode, operator, etc., and then prepared the welded samples for tests. The samples were forwarded to the Bureau of Standards in Washington so that the tests should be conducted by parties absolutely disinterested and without knowledge of how the samples were obtained. The results of these tests showed a remarkable similarity especially when it is realized that they were made by several firms with different electrode materials and under varying conditions of the electrical circuit. Practically all of the welds pulled at over 50,000 pounds per square inch and several over 60,000 pounds the average being about 58,000. On the bending test one of the samples was bent to an angle of 78 degrees before a crack started and final failure reached 80 degrees. In another case the sample was bent to 65 degrees before the crack started and final failure did not occur until 86 degrees. The point of importance here is that all the welds showed a reliability and satisfactoriness which makes conclusive the opinion that electric arc welding is applicable for the joining of steel where the structure is submitted to live loads, bending strains, static pressure, or the like. The Sub-committee on Research is pursuing this subject and practical samples are being prepared for similar tests using three-quarter and one-inch stock material. The results of these tests will be available as soon as the reports are presented and approved by the Welding Committee. The Research Committee is also preparing various types of joints in heavy plating. These will be submitted to all the regulation tests and in addition to shock and fatigue tests and tests to destruction.

To give a further indication of the large size practical tests which are being carried on at the present time it may be stated that three 12-foot cube electrically welded tanks are now being constructed. These tanks are built in such a way that from twelve to fifteen different designs of joints are used in their construction. After these tanks are built they will be subjected to a static strain and the deflection of the seams will be directly measured. Afterwards they will be tested by external shock and crushed to destruction. Portions of the joints will be cut, sent to the Bureau of Standards, and again tested for the sake



SAMPLE SHOWING INDIFFERENT AND GOOD BUILT UP WELDS



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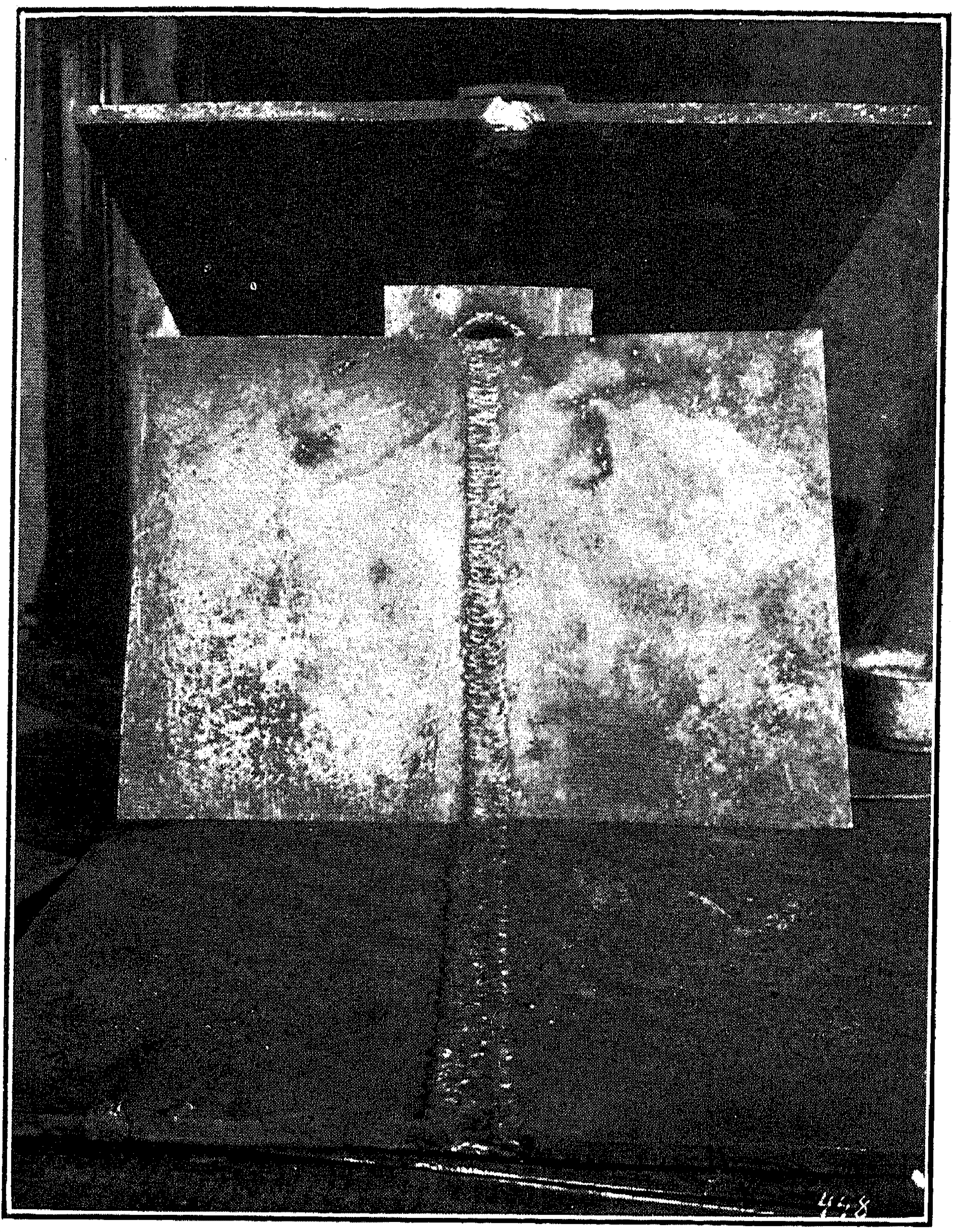
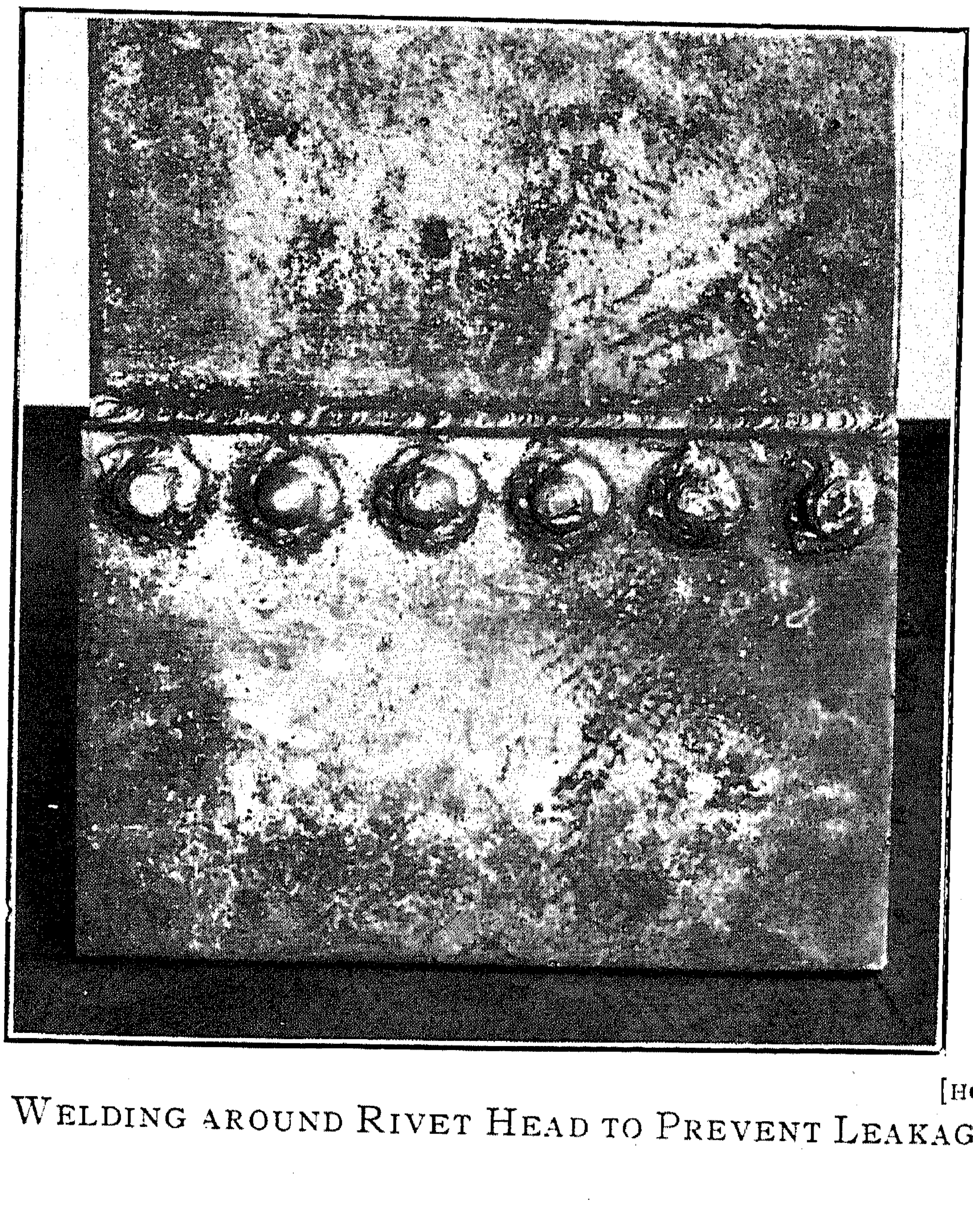
PARTIALLY COMPLETED ARC WELDED SEAM—NOTICE TACK WELDS TO HOLD PLATES TOGETHER

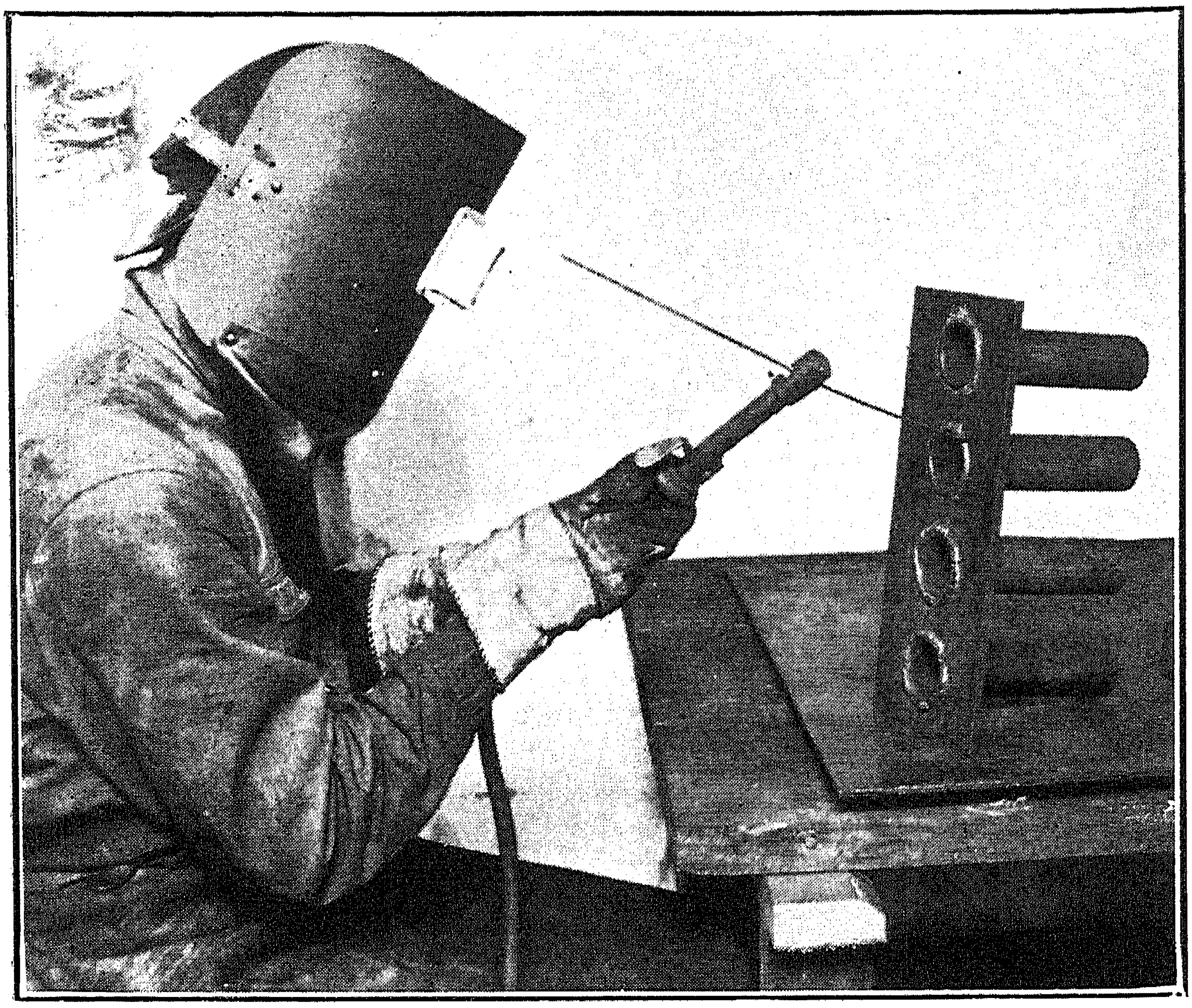


ARC WELDING IN THREE DIFFERENT POSITIONS—FLAT—VERTICAL AND OVERHEAD

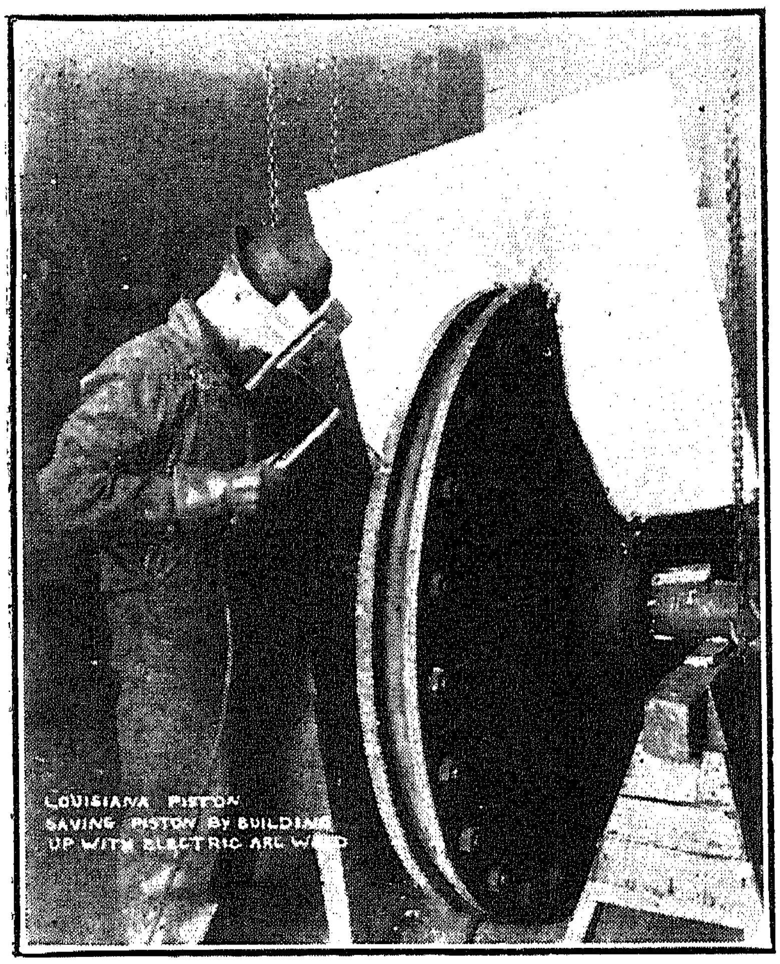


[HORNOR]

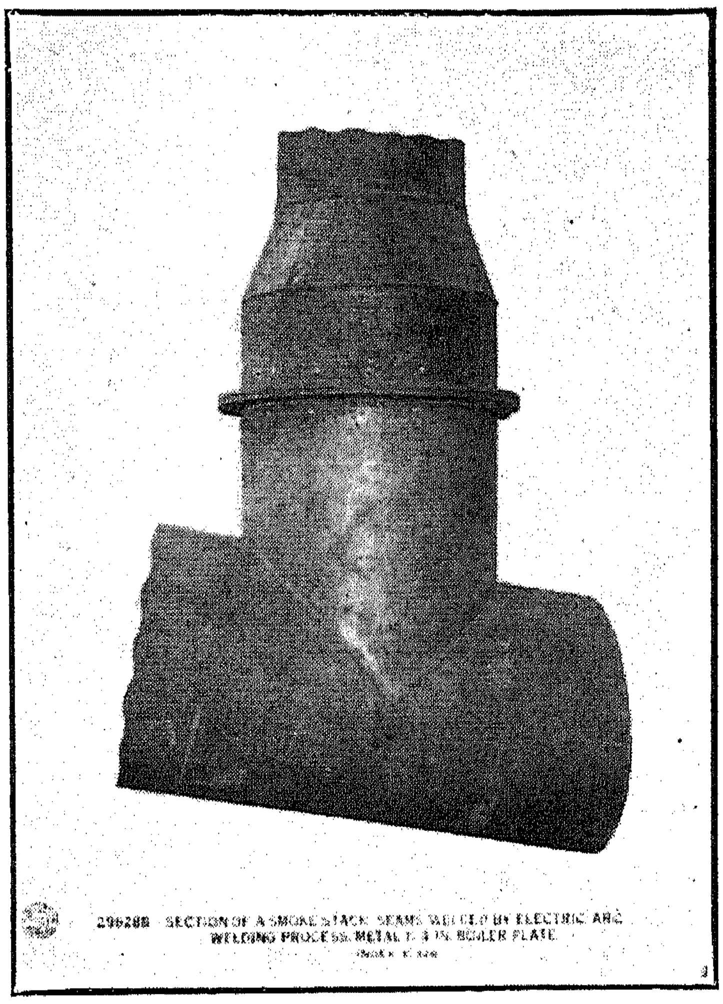
ARC WELDING AROUND RIVET HEAD TO PREVENT LEAKAGE



ARC WELDING BOILER FLUES

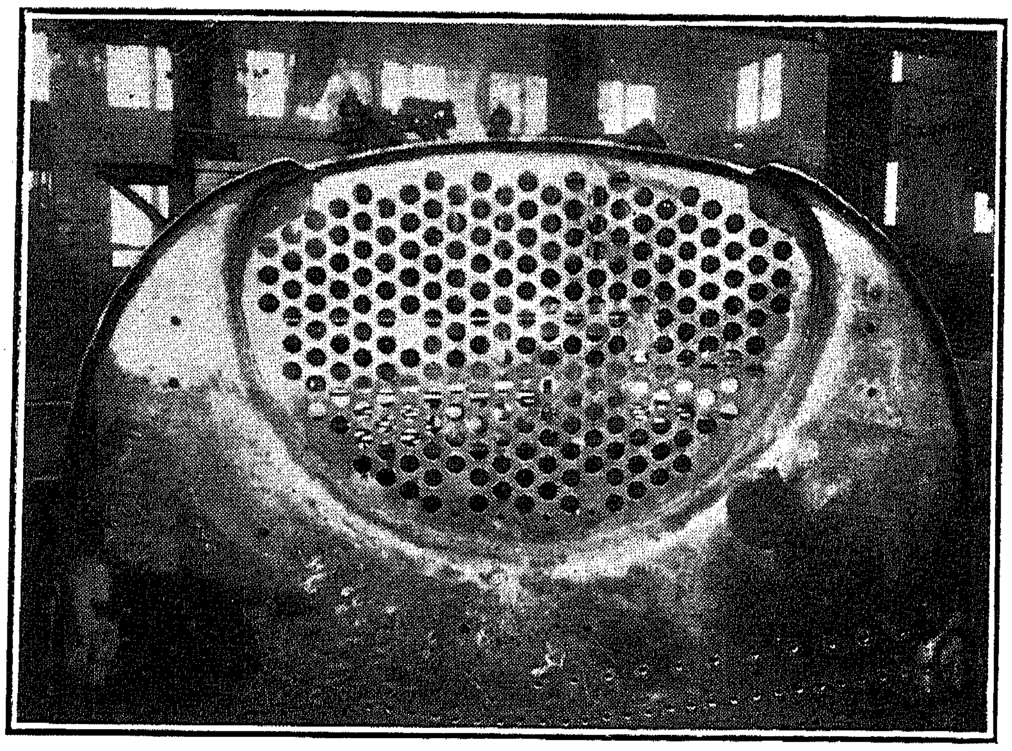


SAVING PISTON BY BUILDING UP WITH ELECTRIC ARC WELD

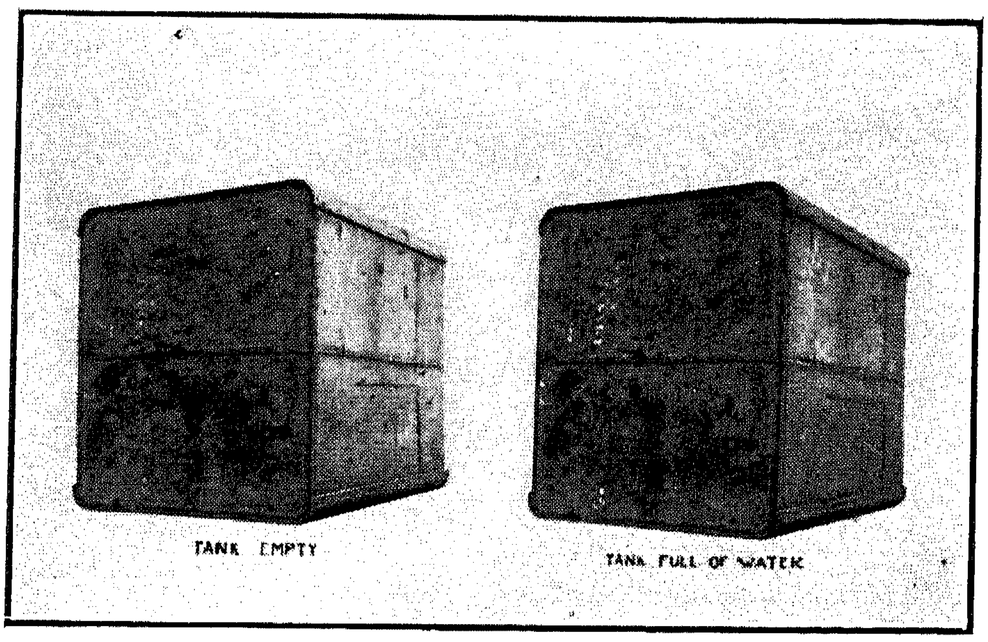

296288 SECTION OF A SMOKE STACK SEAMS WELDED BY ELECTRIC ARC
WELDING PROCESS. METAL 1/4 IN. BOILER PLATE

[HORNOR]

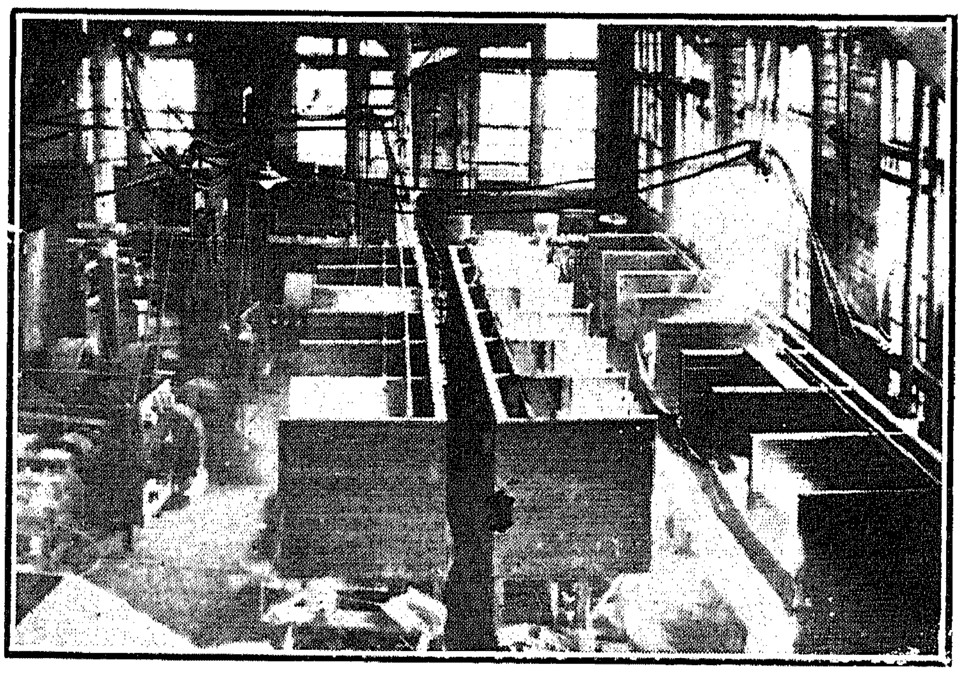
SECTION OF A SMOKE STACK—SEAMS WELDED BY ELECTRIC ARC WELDING PROCESS—METAL 1/4 IN. BOILER PLATE



BACK TUBE SHEET WELDED TO HEAD



TWELVE FOOT CUBE TANK ALL SEAMS ELECTRICALLY WELDED FOR EXPERIMENTAL PURPOSES—BUILT AT PITTSFIELD, MASS.



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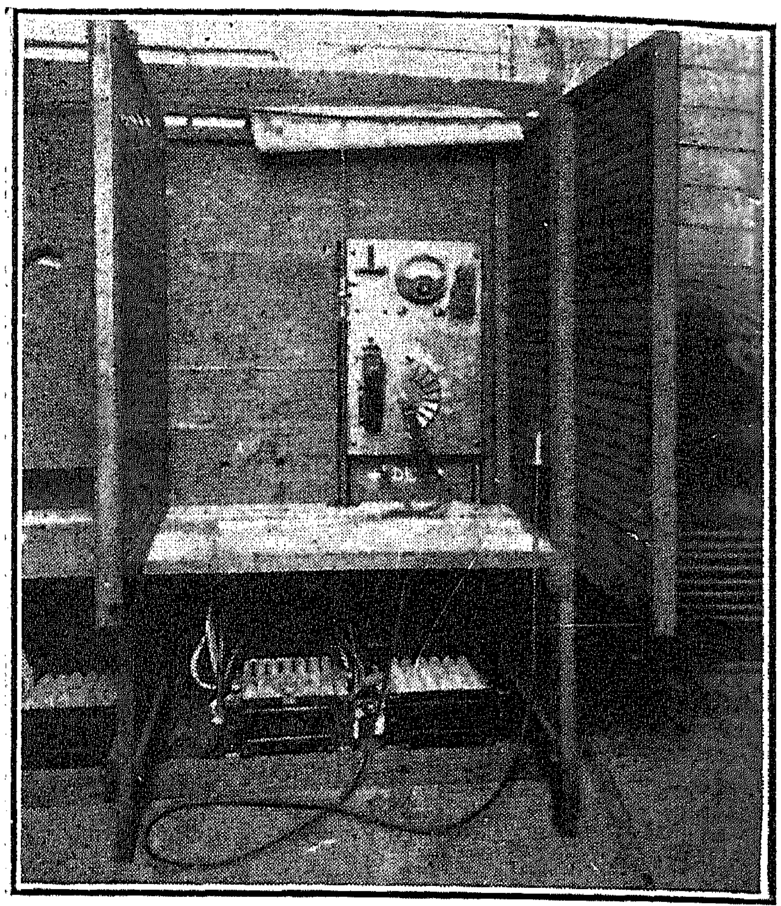
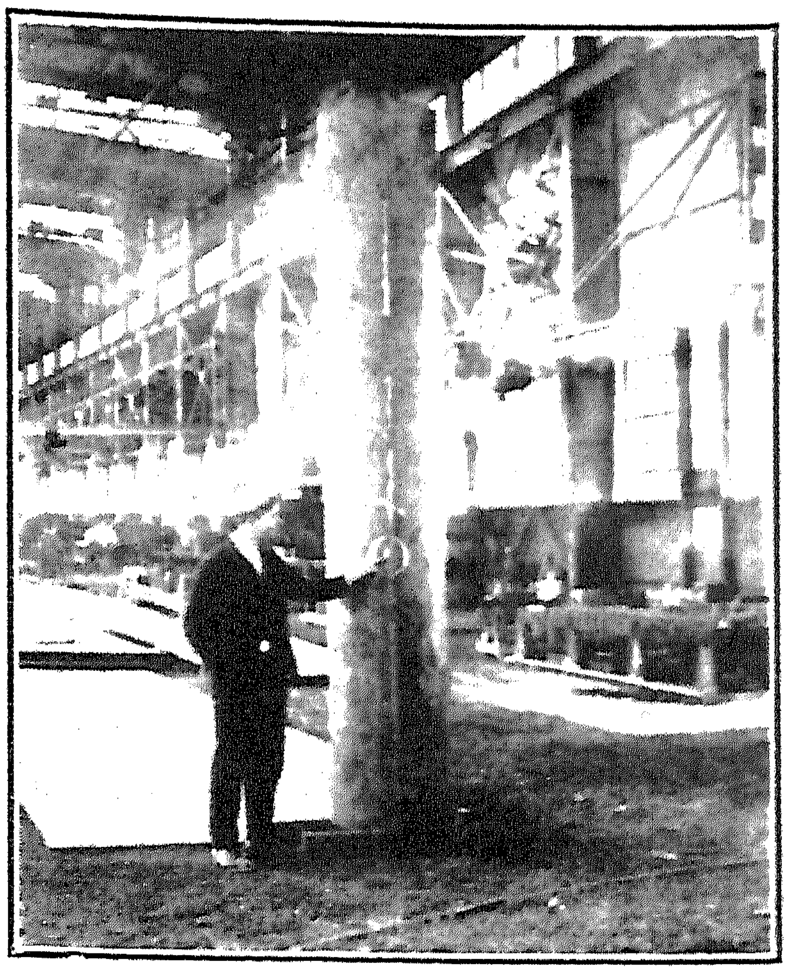
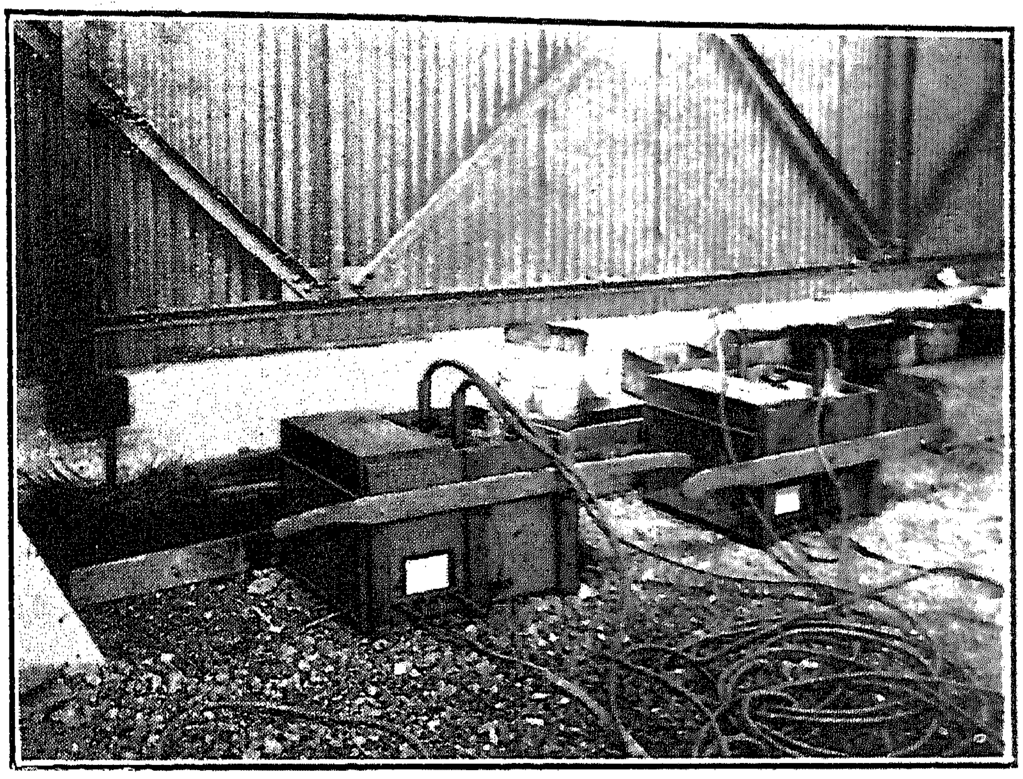
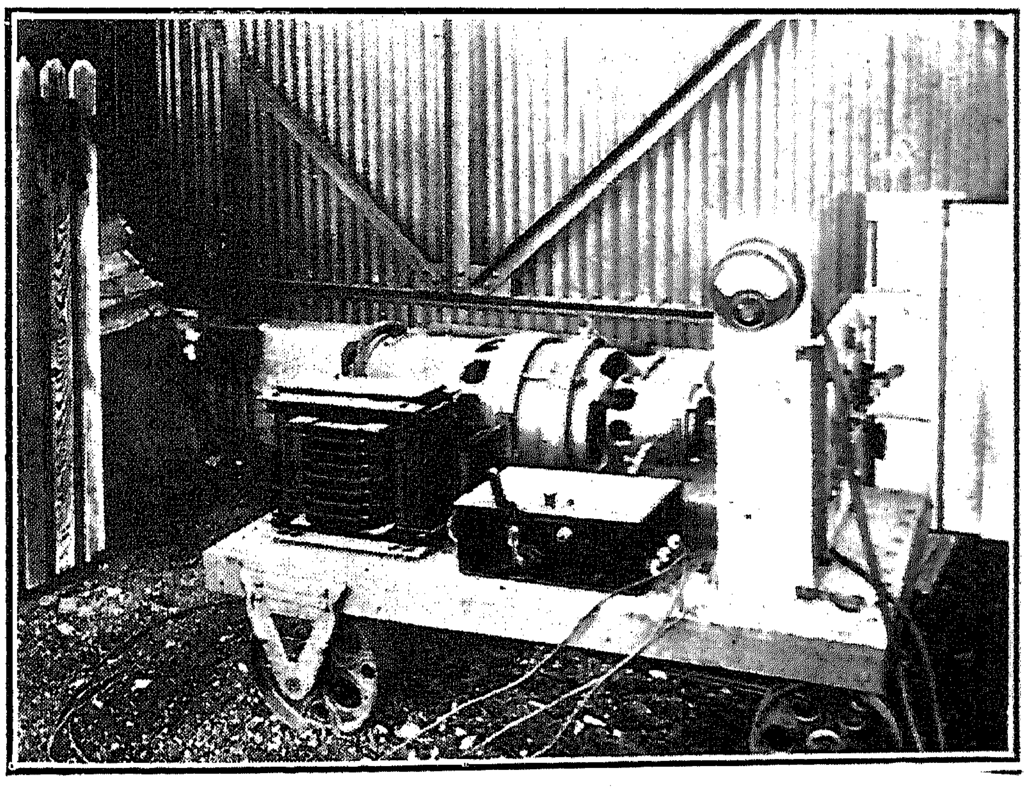
SCHOOL FOR ELECTRIC ARC WELDERS ESTABLISHED BY THE EMERGENCY FLEET CORPORATION AT THE PLANT OF THE GENERAL ELECTRIC COMPANY


FRONT VIEW WELDING BOOTH—G. E. WELDING SCHOOL


THIRTEEN FOOT MAST ELECTRICALLY WELDED


ONE-MAN A. C. TRANSFORMER OUTFIT FOR ARC WELDING


ONE-MAN PORTABLE D-C. ARC WELDING OUTFIT

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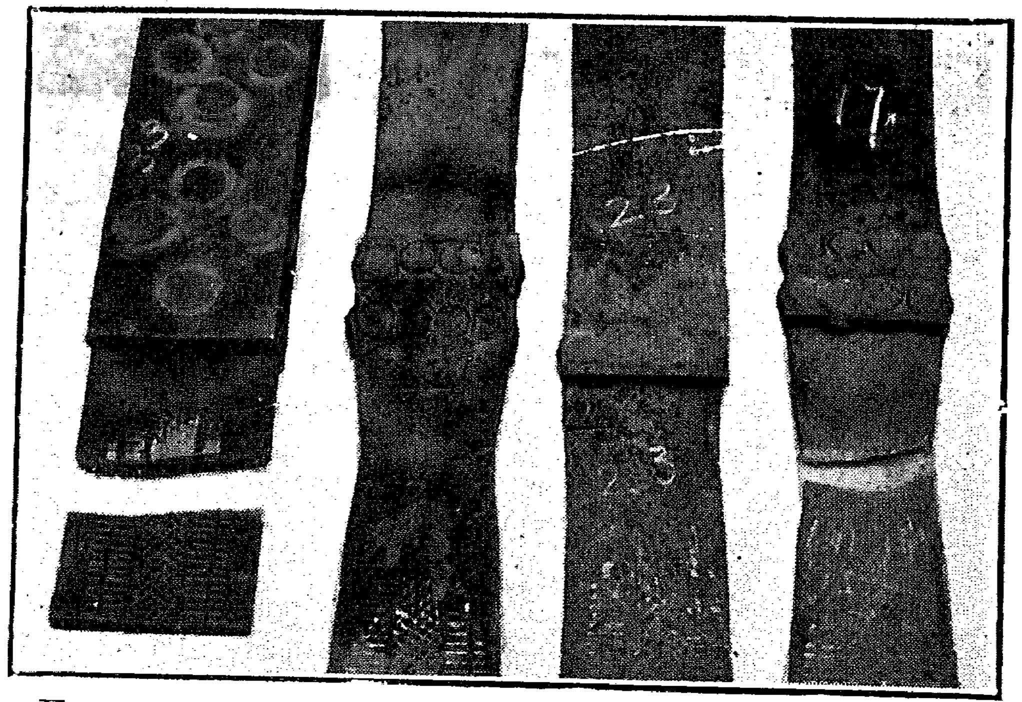
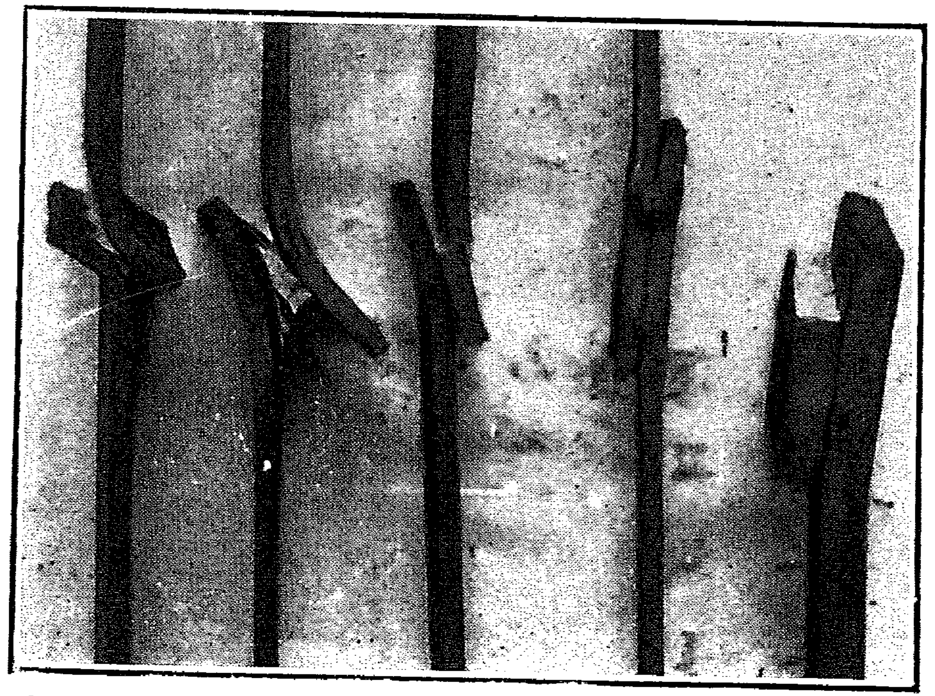
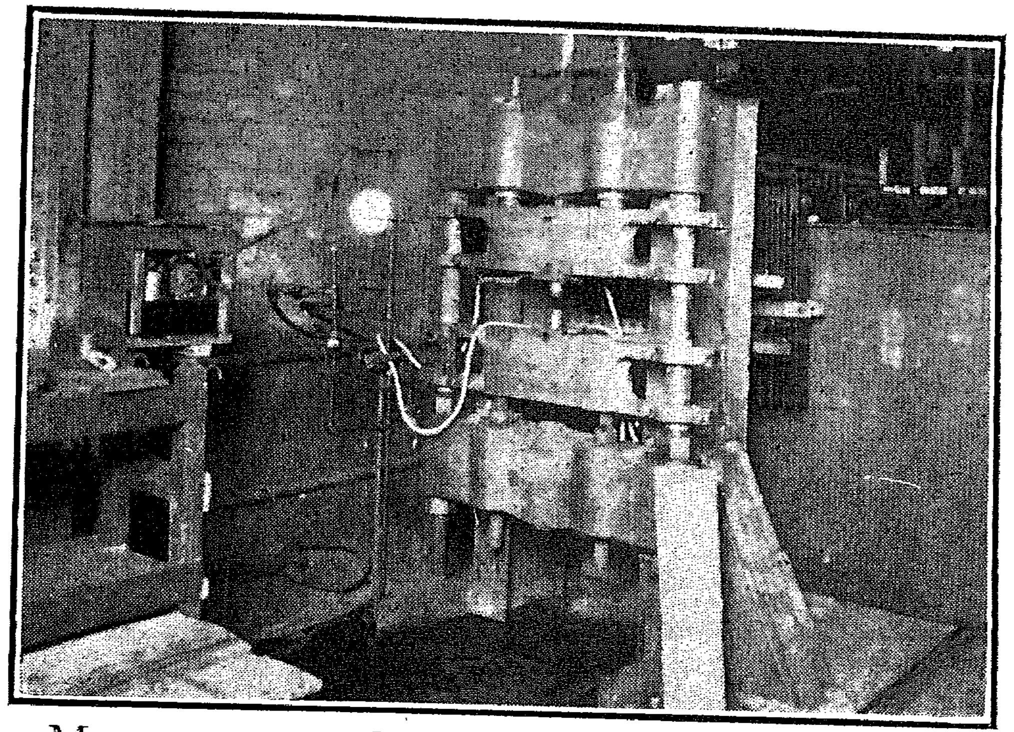


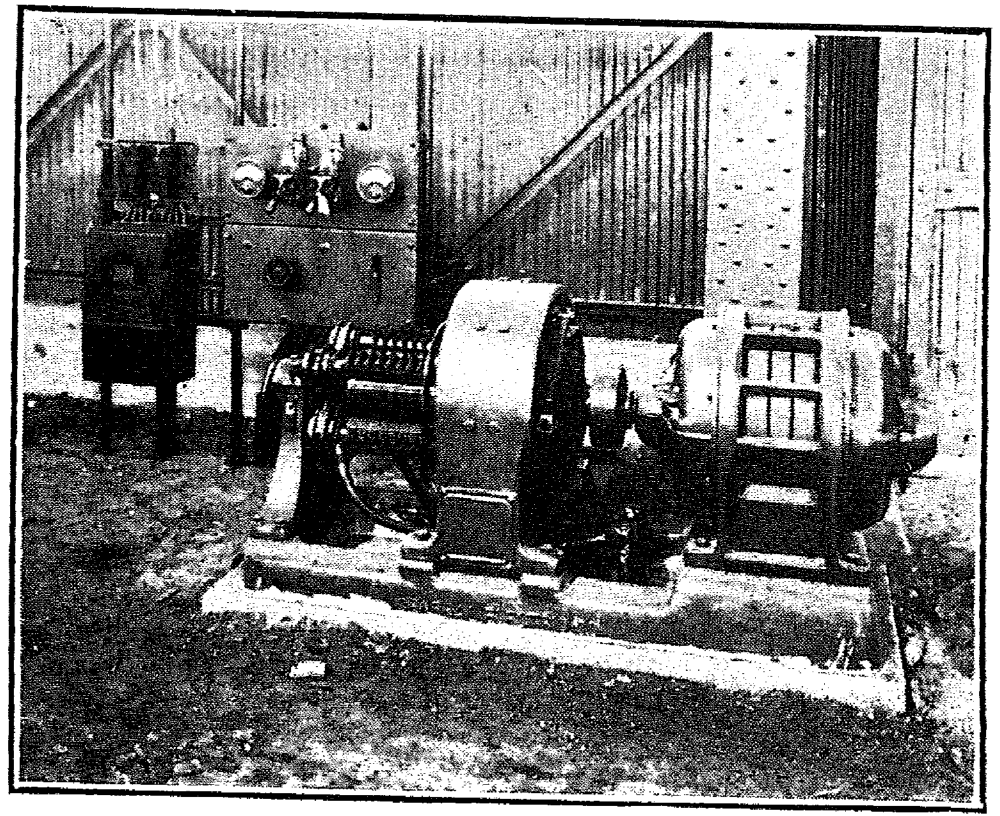
Elctrically Welded Samples—Failures in Tension Occurring Away From Weld—Steel Plates 3/4 in. by 7 in. and 1/2 in. by 6 in.



Electrically Welded Steel Plates 1/2 Inch and 1 Inch Thick after Being Tested in Tension—Showing Effect of Compound Stresses Due to Effect in Line of Tension Caused by the Lapping of the Poles



Machine for Experimental Welding for Heavy Steel by Electricity—Current Capacity 100,000 Amperes— Pressure Capacity 36 Tons



Direct-Current Motor Generator Set for Supplying 8 Arc Welders of accumulating precise data. In this connection there is being built at the Norfolk Navy Yard a battle-towing target. The keel of the target 110 ft. long will be entirely electrically welded and the results of this practical demonstration will be carefully recorded after it has been put in regular service.

It is to be expected that the manufacturers of apparatus being keenly observant of the increased interest in electric welding as well as in the future, which is probably now unquestioned, would be active in their desire not only to improve their present facilities and their design of apparatus but also to proceed themselves to follow the trend of the investigations made by the Welding Committee. The consequence of this has been a large increase in output of apparatus and it may be unhesitatingly stated that there are no difficulties in the way of obtaining all the electrical welding apparatus that is needed. One interesting point is that certain manufacturers who were decidedly of the opinion that direct current was the only proper current to use for arc welding have within a very recent period changed their point of view and are willing to admit that alternating current may have certain advantages in the development of this art.

The electric arc requires a reduced voltage and this is difficult to attain with direct current without relatively expensive machines or a useless expenditure of energy. The practise in this country in manufacturing establishments of any size has been toward an increase in the supply voltage so that very few large manufacturing plants use less than 220 volts direct current. With this voltage the only economical method of transformation is in the use of a motor-generator set. The efficiency in this case is in the neighborhood of 50 to 60 per cent. It is possible to use a supply voltage of 110 volts with a variable resistance which cuts down the voltage to the arc volts. This gives a very poor efficiency. In the case of alternating current the supply voltage can be reduced by a transformer which will supply as in the case of direct current a sufficient voltage for striking the arc and a satisfactory reduction when the arc has been struck. On the other hand, if a low voltage alternating current is provided a simple reactance may be introduced which has some of the same wasteful characteristics of the resistance used with the direct current. The average apparatus will permit of electric arc welding consuming about six to eight kilowatts per welder but if low voltage is provided there are certain outfits which will reduce the consumption as low as three and one-half kilowatts per welder, or even less.

Without entering into an elaborate analysis of the relative costs of electric welding, it may be broadly stated that there is hardly any question that the electric process is cheaper than any other. The same may be said as regards speed and also reduction of man power. In a recent discussion of this subject President Adams stated that at one of the Eastern shipyards the total number of parts on the welding program of the standard riveted ships now building at that yard amounted to 225,000. The labor cost for riveting these pieces is about 245,000 dollars and for welding about 99,000 dollars making a saving of 146,000 dollars. But this is only a drop in the bucket when compared to what might be profitably done in this line. He stated further that in certain particular instances the saving is as great as 90 per cent.

One of the interesting questions discussed with some fervor by the members of the Welding Committee is the advantages of the bare and covered electrode. Regarding this discussion no definite facts can be stated. In England the practise has been to use the covered electrode which protects the welding arc from contact with the air thus guarding against too great a formation of oxide. The practise in the United States up to the present time has been largely bare wire. Recently, American investigators have discovered the important fact that there are advantages in the covered electrode and many experiments are now being made, some with results. It is important to observe that in the above mentioned tests of welds, the best one of these samples was made with a coated (not an asbestos covered) electrode using alternating current. The point in this case seems to rest upon the question of the ductility of the weld and it would seem that the bare electrode does not make as ductile a weld or at least one as easily bent as the coated or covered electrode. The question of the ductility of the weld is one of much importance in the application to ship construction and will doubtless be of importance in other allied industries. It is, therefore, a question of serious importance and constitutes an important part of the work of the Subcommittee on Research.

No matter what the type of electrode is nor its composition, no matter what kind of shank material is to be welded, no matter what kind of apparatus is employed, the reliability of the weld rests mainly upon the man who makes it. This man if he has been properly trained and is skilled in the art knows instantly whether he is making a weld or not. He becomes after much practise able to judge fairly well upon looking on a finished weld whether it is a good weld or not. The work of training electric welding operators early became a part of the functions of the Education and Training Section of the Emergency Fleet Corporation. The men connected with this work are members of the Welding Committee. Schools for the training of operators as well as for the conversion of operators into instructors, are established in many parts of the country. The objects held in view by the training department are first to give the man intensive practise work so that he becomes a good craftsman. The methods are simple to start with, as the exercise of the right arm muscles must become flexible enough to permit the operator to give the required movement to the electrode. By a graduated series of exercises this is accomplished in about eight weeks. The man is allowed to do production jobs in the shop which gives him confidence through responsibility. It becomes desirable at this time to give the man some outside work on ships and where this is practicable, it is done. The man is then turned over to an instructor who gives him an intensive course in pedagogics lasting from five to six weeks. At first sight it would not seem necessary to so instruct a man but it is not generally understood that teaching after all is itself a trade. The experience with the men in this respect is most interesting. In nearly every case the man has resented this course at the start but at the end has turned completely around and in many cases has desired an even more extensive training. What is really accomplished is to give the man the necessary confidence to impart the knowledge that he has gained to another green man. The men under training are taken from the various industries especially the shipbuilding industry and after they have finished their instructor training course are returned to their employer to carry on the instruction, in their own plant. The men who go through this training as provided by the Emergency Fleet Corporation are certificated when they have shown themselves to be entirely proficient. It is not possible nor expedient for the Emergency Fleet Corporation to require the certification of all electric welders. It is the consensus of opinion that all industries doing serious work with the electric arc should use men who are certified as to their ability in the art of electric welding. The main reason for this opinion is that the operator must be a conscientious workman or the weld will not be of perfect quality.

This brings forward another problem upon which a great deal of experimental work has been and probably will continue to be done, namely, a practical and scientific method of testing a welded joint after it has been made. There have been a number of suggestions made for the solution of this problem. They are briefly, as follows:

(a) Mechanical. By hammering the weld or by chipping at frequent intervals.

(b) Electric. By means of resistance or voltage drop.

(c) Magnetic. By means of the permeometer or the change of conditions of the magnetic circuit.

(d) X-ray. By means of an exposure on an X-ray plate.

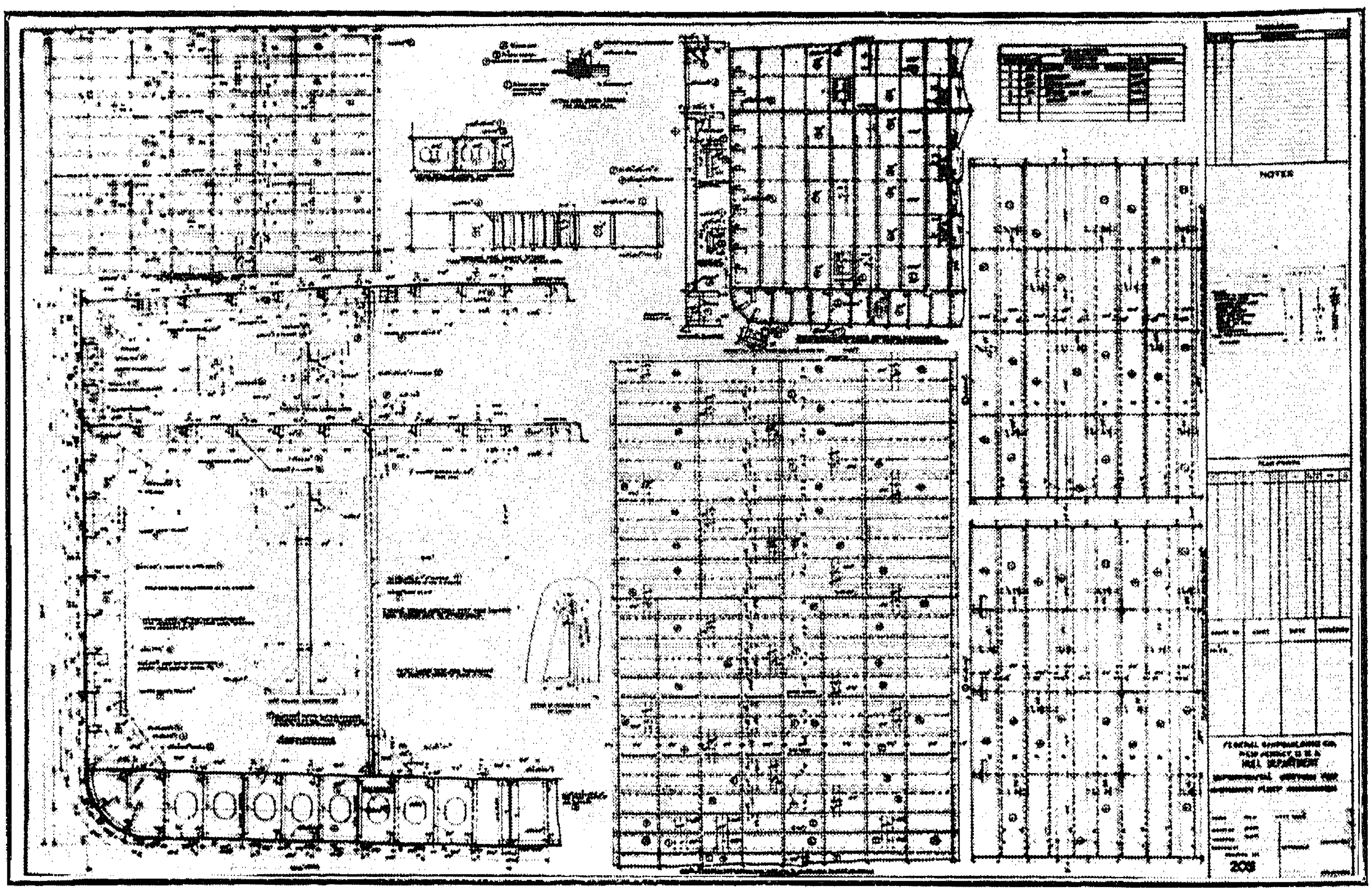
At the present time none of these suggested methods have been productive of conclusive results and recourse must be had to the purely mechanical methods of striking heavy blows on, or adjacent to, the weld or by using a chipping hammer and making intermittent examinations. It would seem by far the best procedure to make the inspector proficient in the art so that he may closely observe the welders while at work. This may be accomplished by a two or three weeks attendance of inspectors at any one of the electric welding training centers.

METHODS OF ELECTRIC WELDING

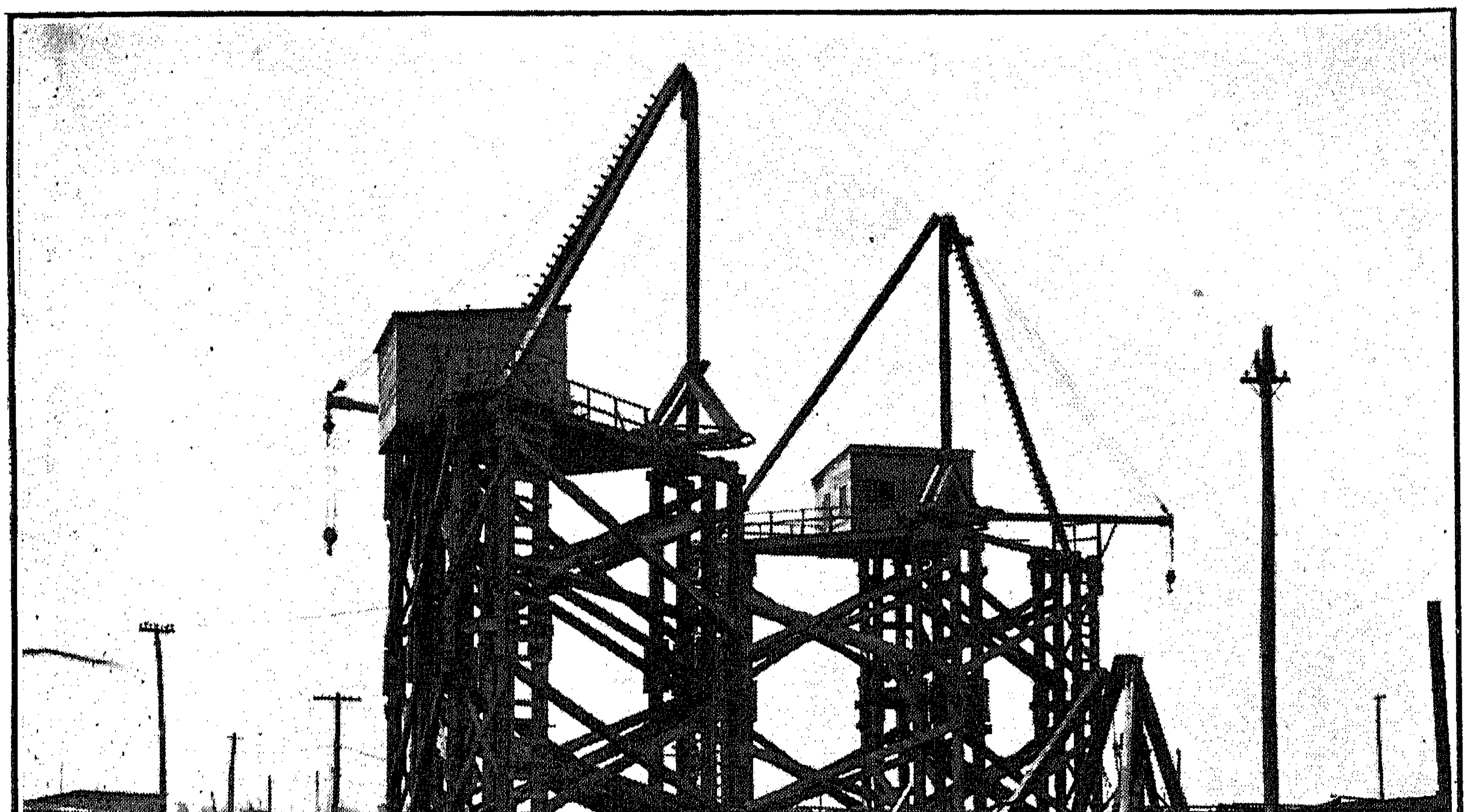
There are many methods and processes of electric welding but the two main ones that interest the committee at the present time and alone have been mentioned so far are the spot welding and arc welding. It may be a surprise to some of the old time welders to consider electric welding as a new industry. In substantiation of this statement it may be well to describe briefly what is meant by electric welding as it is practised today.

Spot welding is not much different in the methods of procedure or in design of apparatus as when it was first introduced. Copper electrodes, water-cooled in the heaviest machines, are placed on opposite sides of the material to be welded together. The joint is a lap joint. Machines are now so designed that two spot welds may be made at one time. The routine of the operation is as follows:

The electrodes are brought into contact with the materials to be joined, current is supplied sufficient to give the required heat, pressure

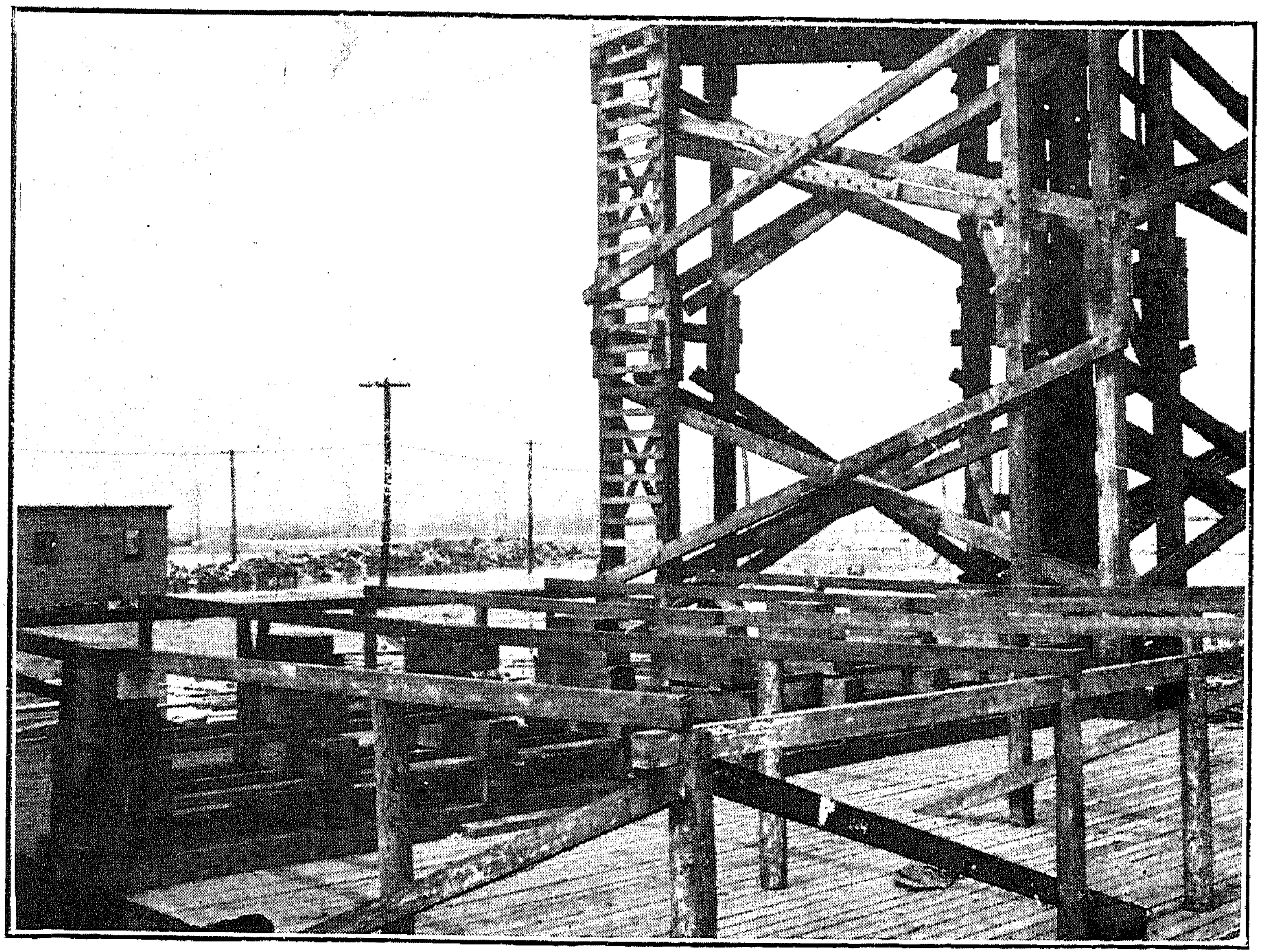


STRUCTURAL DRAWINGS OF EXPERIMENTAL SECTION OF STANDARD SHIP BEING BUILT AT KEARNEY, NEW JERSEY, UNDER THE DIRECTION OF MR. A. J. MASON

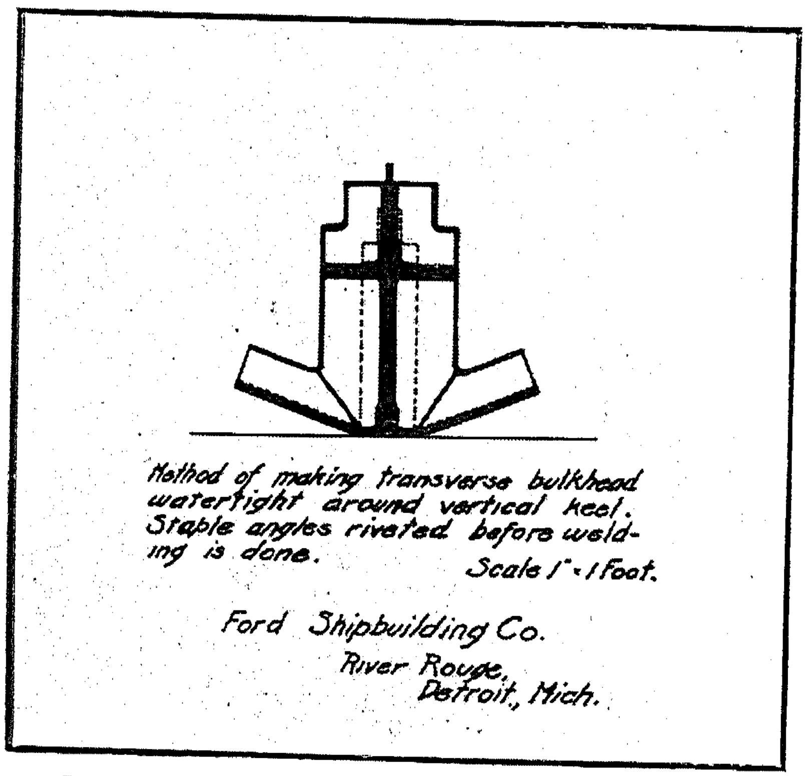


[HORNOR]

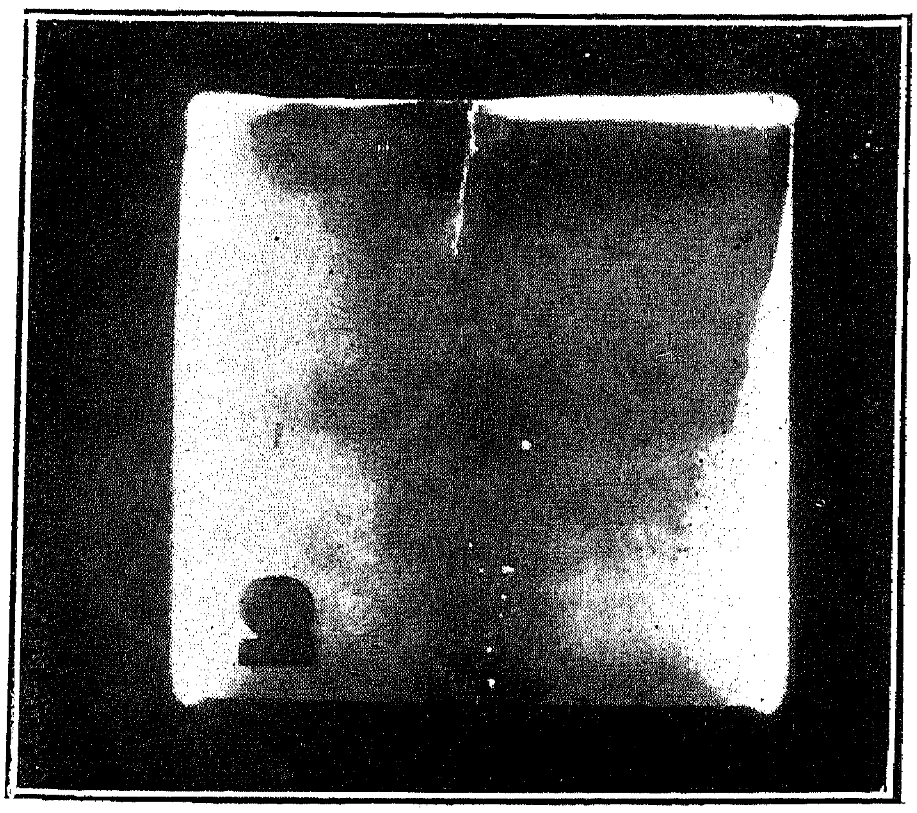
ARRANGEMENTS AT KEARNEY, NEW JERSEY, FOR THE BUILDING OF EXPERIMENTAL SECTION OF STANDARD SHIP



ARRANGEMENTS AT KEARNEY, N. J., FOR THE BUILDING OF EXPERIMENTAL SECTION OF STANDARD SHIP—NEAR VIEW SHOWING KEEL BLOCKS


Method of making transverse bulkhead watertight around vertical keel. Staple angles riveted before welding is done.
Scale 1" = 1 Foot.
Ford Shipbuilding Co.
River Rouge,
Detroit, Mich.

METHOD OF MAKING TRANSVERSE BULKHEAD WATERTIGHT AROUND VERTICAL KEEL—STAPLE ANGLES RIVETED BEFORE WELDING IS DONE



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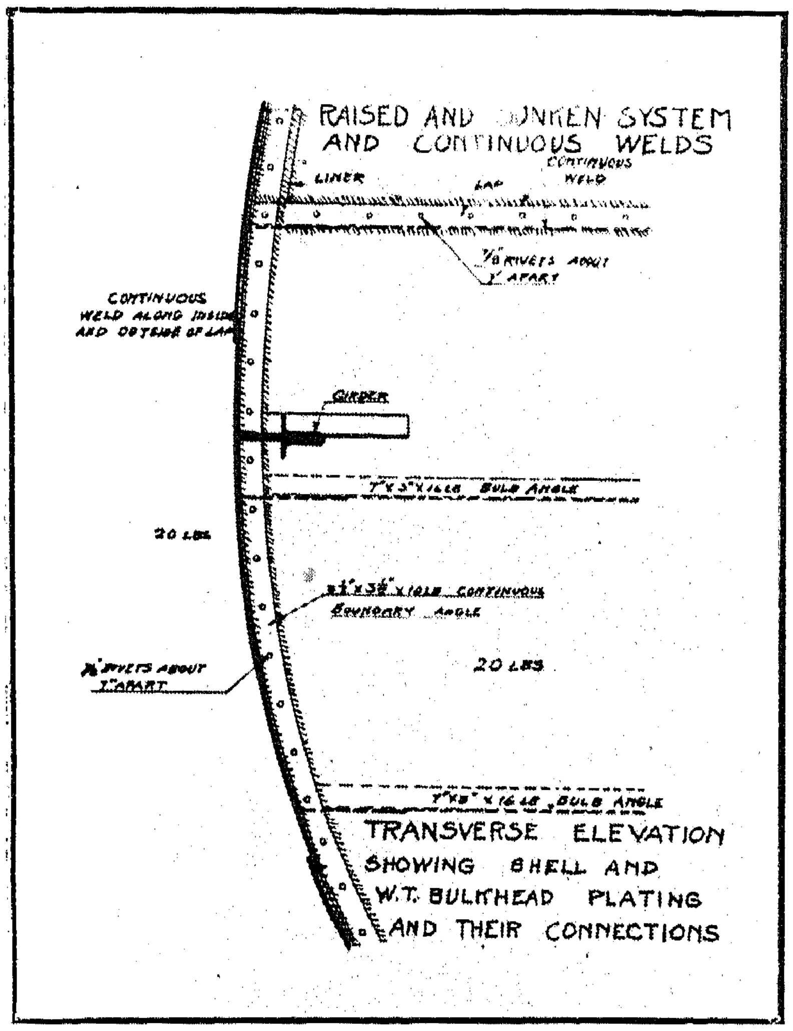
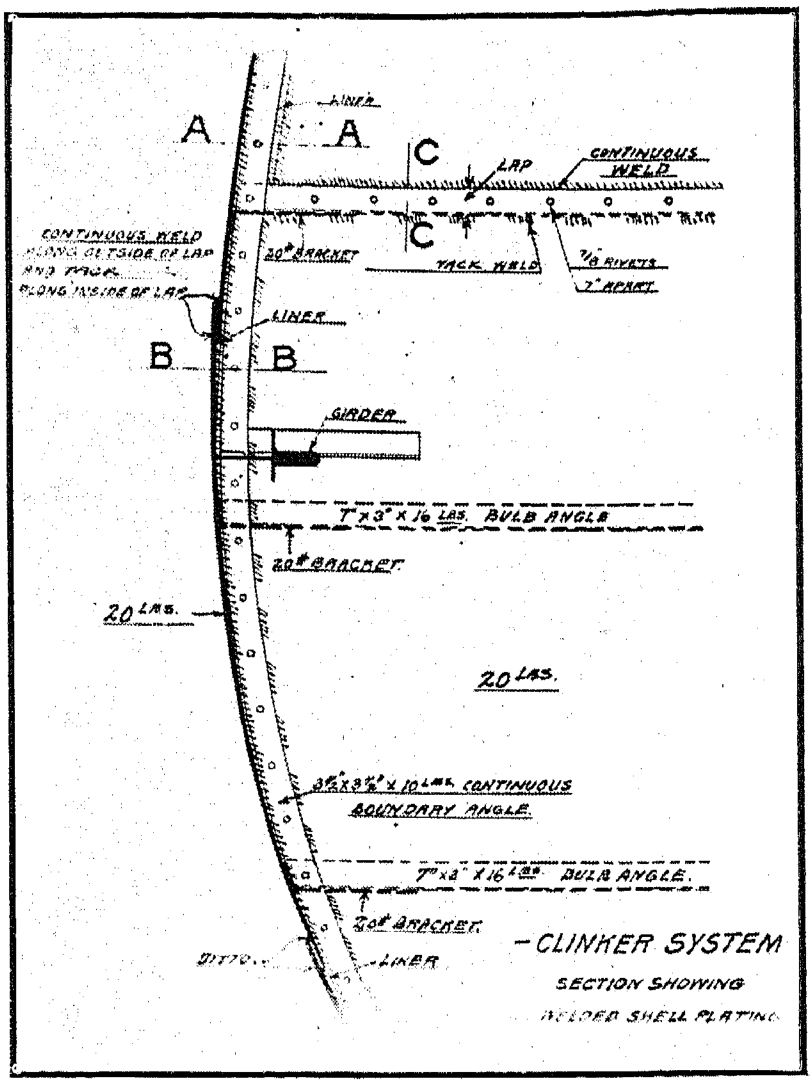
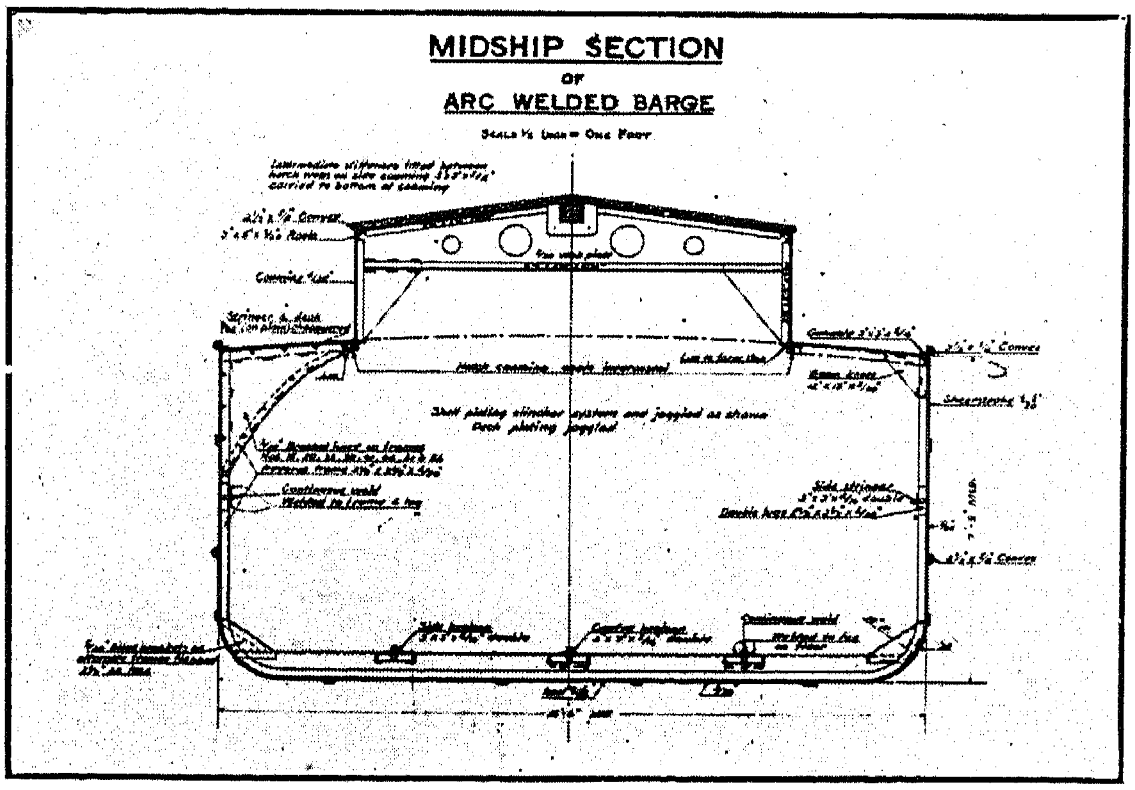
X-RAY PHOTOGRAPH OF WELD


RAISED AND SUNKEN SYSTEM AND CONTINUOUS WELDS
LINER
LAP
CONTINUOUS WELD
7/8" RIVETS ABOUT 7" APART
CONTINUOUS WELD ALONG INSIDE AND OUTSIDE OF LAP
GIRDER
7"×3"×16 LB. BULB ANGLE
20 LBS.
3½"×3½"×10 LB. CONTINUOUS BOUNDARY ANGLE
¾" RIVETS ABOUT 7" APART
20 LBS.
7"×3"×16 LB. BULB ANGLE
TRANSVERSE ELEVATION SHOWING SHELL AND W.T. BULKHEAD PLATING AND THEIR CONNECTIONS

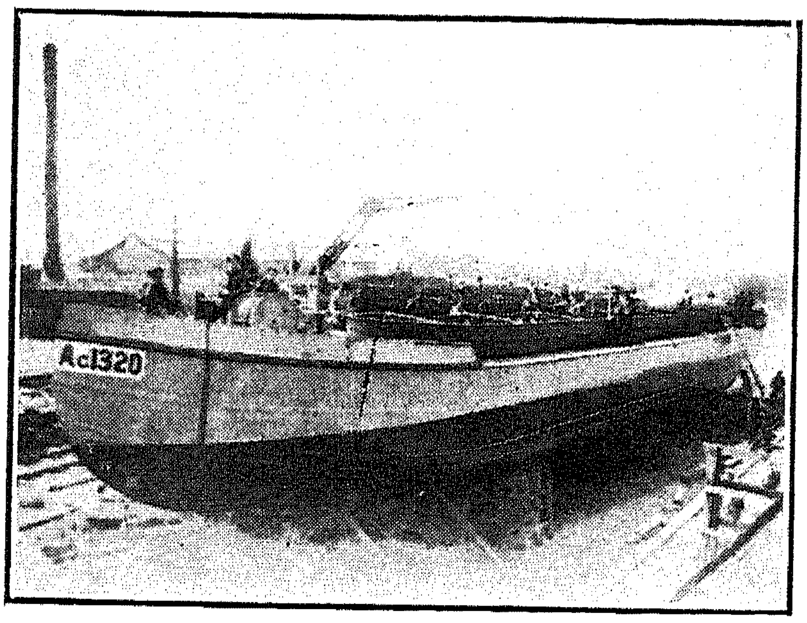
Raised and Sunken System and Continuous Welds


LINER
A
A
C
C
LAP
CONTINUOUS WELD
CONTINUOUS WELD ALONG OUTSIDE OF LAP AND TACK ALONG INSIDE OF LAP
20# BRACKET
TACK WELD
7/8 RIVETS 7" APART
LINER
B
B
GIRDER
7"×3"×16 LBS. BULB ANGLE
20# BRACKET
20 LBS.
20 LBS.
3½"×3½"×10 LBS. CONTINUOUS BOUNDARY ANGLE
7"×3"×16 LBS. BULB ANGLE.
20# BRACKET
DITTO
LINER
CLINKER SYSTEM
SECTION SHOWING WELDED SHELL PLATING

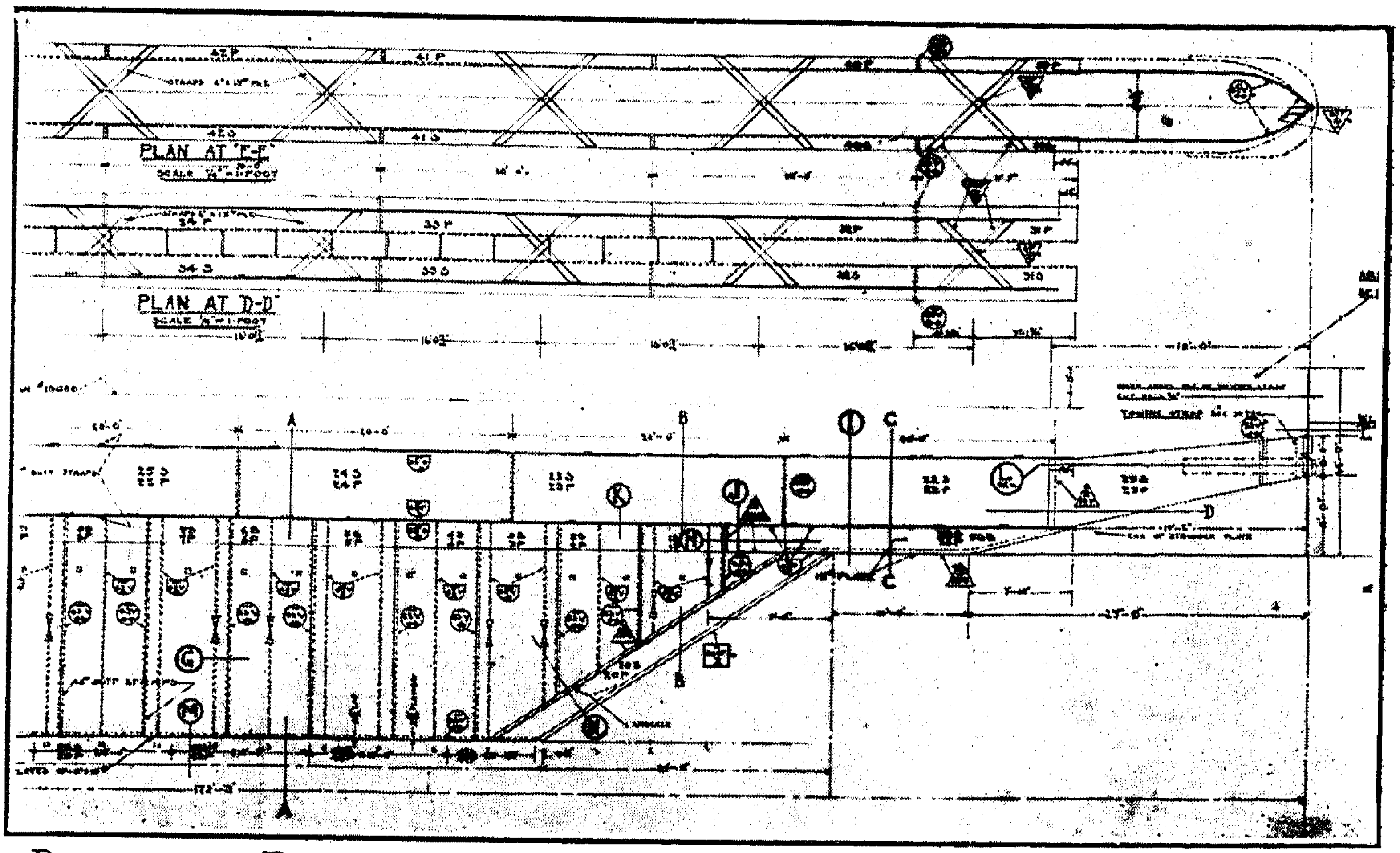
Clinker System—Section Showing Welded Shell Plating



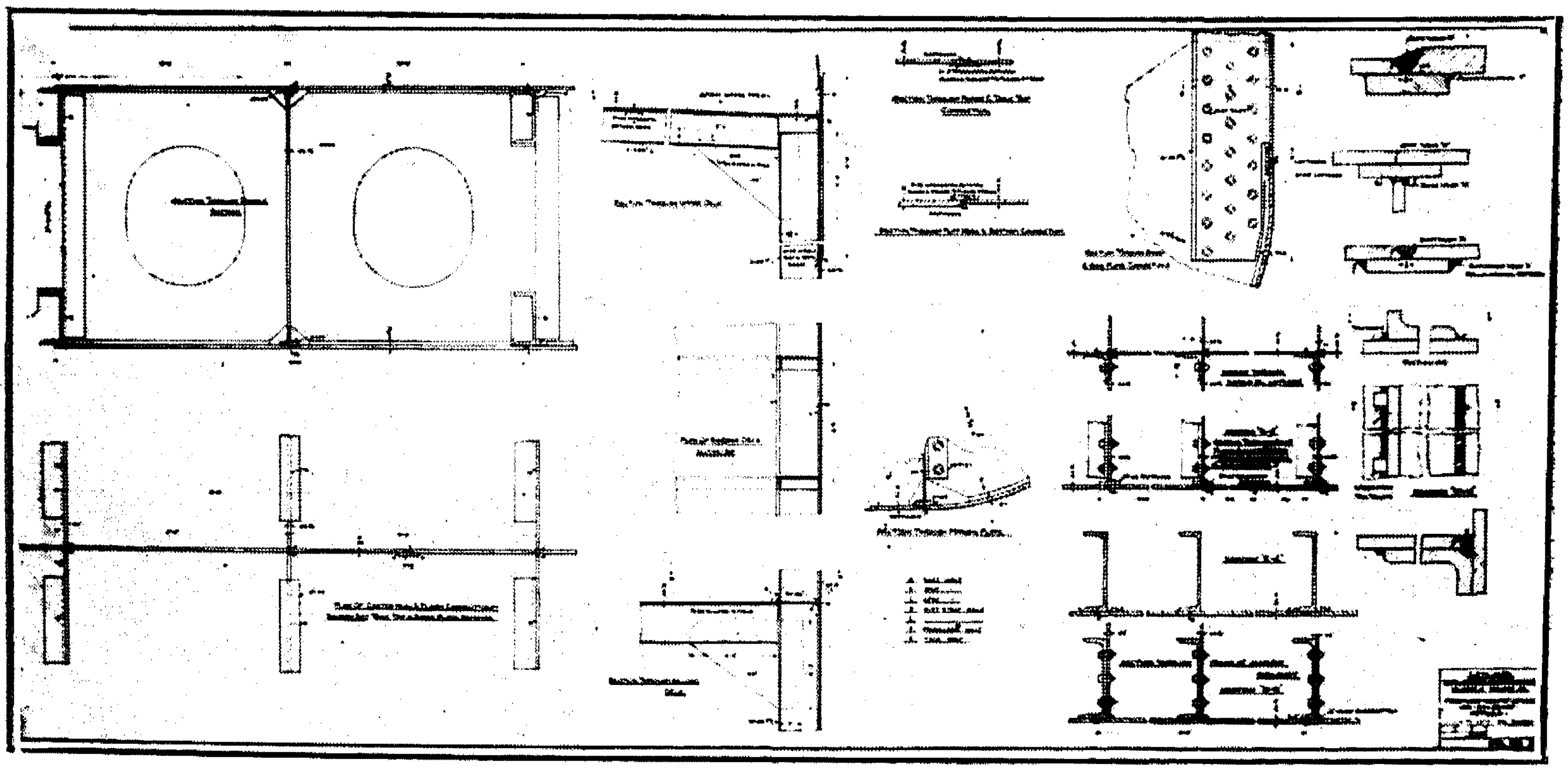
Midship Section of Arc Welded Barge [HORNOR]



Totally Electrically Welded Barge Built in England



Portion of Drawing for Keel of Battle Towing Target Showing the Use of Approved Symbols and the Simplicity of the Drawing



[HORNOR]

Detailed Design for Proposed Electrically Welded Ship is then applied, the current is removed, and the pressure is removed, the weld is then complete.

The operator has a perfect indication of making a good spot weld by the use of a button placed under the electrode observing which he knows exactly the proper timing of the operation. There is therefore no question as to a good, bad, indifferent, spot weld. Automatic spot welders have been designed and built but it is the general opinion that they add complication to a process which in itself is very simple.

The process of arc welding is as follows:

> One side of the electric circuit is connected to the material to be welded, the shank material is usually prepared by beveling the edge of the pieces to be welded together. The other side of the electric circuit is connected to the electrode. The operator is provided with a holder which carries the electrode. By touching the electrode to the shank material the arc is drawn. The skilled operator now moves the electrode from side to side of the groove giving a semi-circular motion while at the same time moving the electrode along the groove.

It is important that the arc "bite" into the shank metal creating a perfect fusion along the edges and the movement of the electrode is necessary for the removal of any mechanical impurities that may be deposited. In the coated electrode it is further necessary that the slag which forms for the protection of the pure metal be worked up to the surface and it is extremely important in the event of a second or third layer that the slag or impurities be carefully scraped away before the virgin metal is again laid on.

The operator in arc welding is protected with either a hand screen or a covering for his face with special glass through which to observe his work. The electric arc emits dangerous invisible rays in both the upper and lower spectrum scale and it is quite evident that both the infra-red and ultra-violet are dangerous in their effect, the former is pathological the latter actinic. The operator further uses gloves for his hands and for the very difficult work of overhead welding it is necessary for him to use a helmet which partly covers his breast.

Developments

The tendency of developments in spot welding has already been slightly touched upon. In their nature as applicable to shipbuilding the advancement will naturally have to proceed toward means for accomplishing spot welding in very cramped locations. This makes an exceedingly difficult problem as the power requirements are such as to preclude any very small device. In riveting one-half of the apparatus is on one side of the work and the other half on the opposite side and it is difficult to conceive of any method of spot welding that will admit of such an arrangement. In shipbuilding it is quite probable that designs may be made that will permit of a large or at least increased amount of spot welding in the actual construction of the vessel. Certainly, present designs of riveted ships will not allow of this to any great extent. As already stated, spot welding can now take its place in the fabricating shops and it is to be expected that within a few months spot welding will begin to supplant riveting in this field. The only drawback to this will be the sufficient production of spot welding apparatus.

The tendency of development in arc welding is toward the automatic machine to obviate the responsibility that has to be placed upon the skilled operator. Intensive work has been done within the last few months in the line of automatic arc welding machines and at the present time sample tests of welds made by such apparatus have been sent to the Bureau of Standards. These machines will occupy a very important position in repetition work. They will not immediately supersede the skilled operator in repair work, or in special jobs but it may be expected that the development of such machines will bring about apparatus which can be man-handled and will eventually take the place of most of the hand work as it is now known.

Of the scientific advancement in the art of electric welding there is so much to be treated that only a general outline can be considered at this time. The research work has only just begun. Practise has preceded the scientific investigation. The field, therefore, is full of most interesting problems. Those who have been following the development of the past six months are deeply interested to know the fundamental reasons. The investigational questions may be grouped into three main divisions:

1. Metallurgical.
2. Physical.
3. Electrical.

The metallurgist has yet to tell us what the conditions of the metals are after the electrode material has fused with the parent metal, and to determine what the proper conditions must be to produce a good weld. This problem has in it a great many variables. The physicist must explain the atomic or electronic conditions which permit of the combinations at the high temperatures involved and must explain the phenomenon of overhead welding. The electrical investigator must determine all the various phenomena connected with the preferences between and the advantages of the use of different forms of electrical energy and the varying characteristics of the electric circuit in producing different types of welds.

CONCLUSION

From the preceding remarks it must be conceded that the Welding Committee of the Emergency Fleet Corporation has already crystalized the problems connected with this art. The working functions of this Committee have been laid down upon the broadest possible lines. Liberal opportunity has been given every one to state in detail his opinion and to express the reasons for his preference on every point connected with this subject. The Committee goes even further than this. It furnishes those interested with every new idea that is brought to bear upon the subject after sifting from the suggestions any question of doubt or misstatement of fact. All suggestions of improvement or problems of special application are gladly taken in hand, thoroughly investigated, and reports made. It will welcome any comments that those connected with the industries may desire to lay before it. The personnel is at the present time such that it can devote not one but many minds to the solution of any specific problem that is laid before it.

The Committee early discovered that the literature of electric welding was very much clouded by misstatement of fact or half-baked theory and much of the time of the Committee has been taken up in disproving such statements. In order to spread the results of this work to all quarters a handbook is now being prepared which will contain only definite facts and results of investigations as are approved by the whole Committee. This handbook will be made available to all those who desire to acquaint themselves with the proper means of accomplishing good and reliable electric welding.

Presented at the 343d Meeting of the American Institute of Electrical Engineers, New York, November 8, 1918.

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ELECTRIC MOTORS IN THE CEMENT INDUSTRY

BY R. B. WILLIAMSON

Abstract of Paper

This paper has been compiled by the Committee on Industrial and Domestic Power to give general information regarding the different classes of machinery used in the cement industry and the sizes and types of motor best adapted for the work. The paper gives, first, a brief description of the process. This is followed by an outline of the various kinds of machinery used together with data as to power requirements. The types of motor best suited to each application together with starting characteristics, overload capacity, torque and other features are indicated.

PRACTICALLY all modern mills manufacturing Portland cement are driven by electric motors, and since the process is mainly one of crushing and grinding, a large amount of power is required.

Before considering the various motor applications, it may be advisable to give a brief outline of the process of manufacturing Portland cement, which constitutes about 99 per cent of the cement of all kinds manufactured in the United States.

General Description of Process

Portland cement is generally formed by an artificial combination of calcareous and argillaceous materials. These materials are mixed either before grinding or during the process of grinding. After being ground so that from 80 to 90 per cent of the material will pass through a screen having 200 meshes per lineal inch, it is burned to incipient fusion in a rotary kiln, after which the resulting clinker is again ground so that at least 78 per cent will pass the 200-mesh screen.

The Portland cement clinker, after it leaves the rotary kiln, requires no further treatment than grinding, excepting the addition of about 2 per cent of gypsum to retard the setting. In some parts of the United States natural cement rocks are found which contain nearly the proper proportions of materials to produce Portland cement; but, even in these localities it is generally necessary to add either limestone or shale in order to get the proper mixture. Certain deposits of cement rock so nearly approach the proper proportions of materials, that at one time limestone must be added and at another time shale.

In the United States, limestone furnishes the largest supply of calcareous material for Portland cement. Other sources of supply for this class of material, in addition to cement rock mentioned above, are marl, chalk and alkali waste. Shale and clay are most commonly used for obtaining the argillaceous material for Portland cement. Cement rock mentioned above, as well as blast furnace slag, are also used as a source of argillaceous material. Blast furnace slag, like cement rock, contains a considerable proportion of lime and has the further advantage that this lime is found as calcium oxide instead of the carbonate; therefore, the heat necessary to dissociate the carbon dioxide gas from this part of the lime is not required.

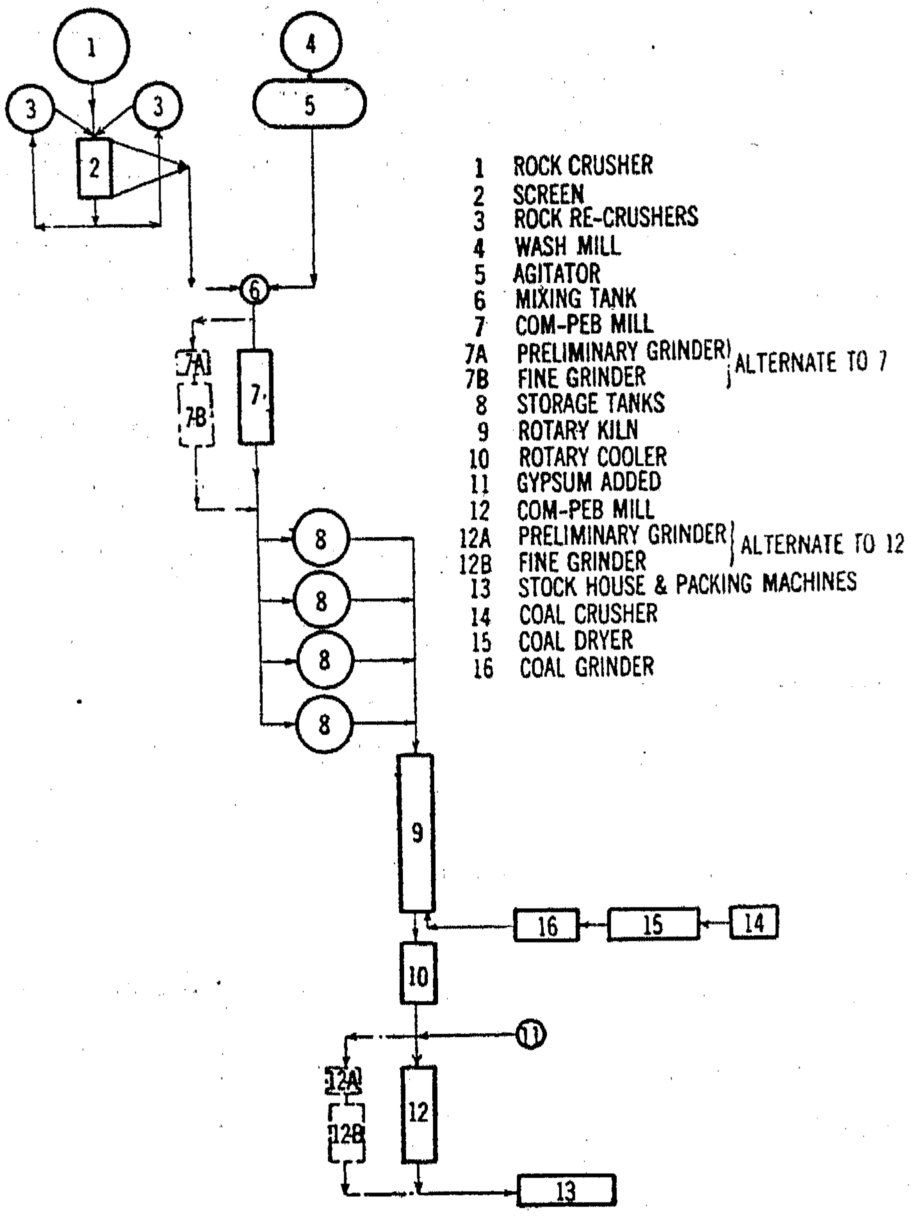

FIG. 1

The preliminary crushing of limestone, cement rocks and shale is ordinarily done in the crushers of the gyratory, jaw or roll type. Materials may be ground either in the wet or dry state. Fig. 1 shows a typical flow sheet for a wet mill and Fig. 2 for a dry grinding mill. In a dry grinding plant, the two classes of materials are preferably dried separately and then weighed, after which they are ground. In a wet grinding plant, the materials are generally measured or weighed in their natural state as excavated, and then ground. Where shale or cement rock is used as the argillaceous material, it must be crushed in rock crushers. In the past, it has been the practise in certain wet grinding cement plants to dry the clay in order better to store and handle it, and also to eliminate gravel and other foreign material. The better practise, however, is to dump the clay in its natural state into a wash mill where sufficient water is mixed with it to allow the heavy pieces of gravel to settle out, and also to form a slurry which can easily be stored in tanks. The flow sheet, Fig. 1, for a wet grinding cement plant shows a wash mill installed for handling the argillaceous material. Wash mills are used only for clays, marls and similar materials, which, by mixing with water, can be kept in suspension. If shale were used as argillaceous material, a crusher of some type would have to be provided instead of a wash mill.

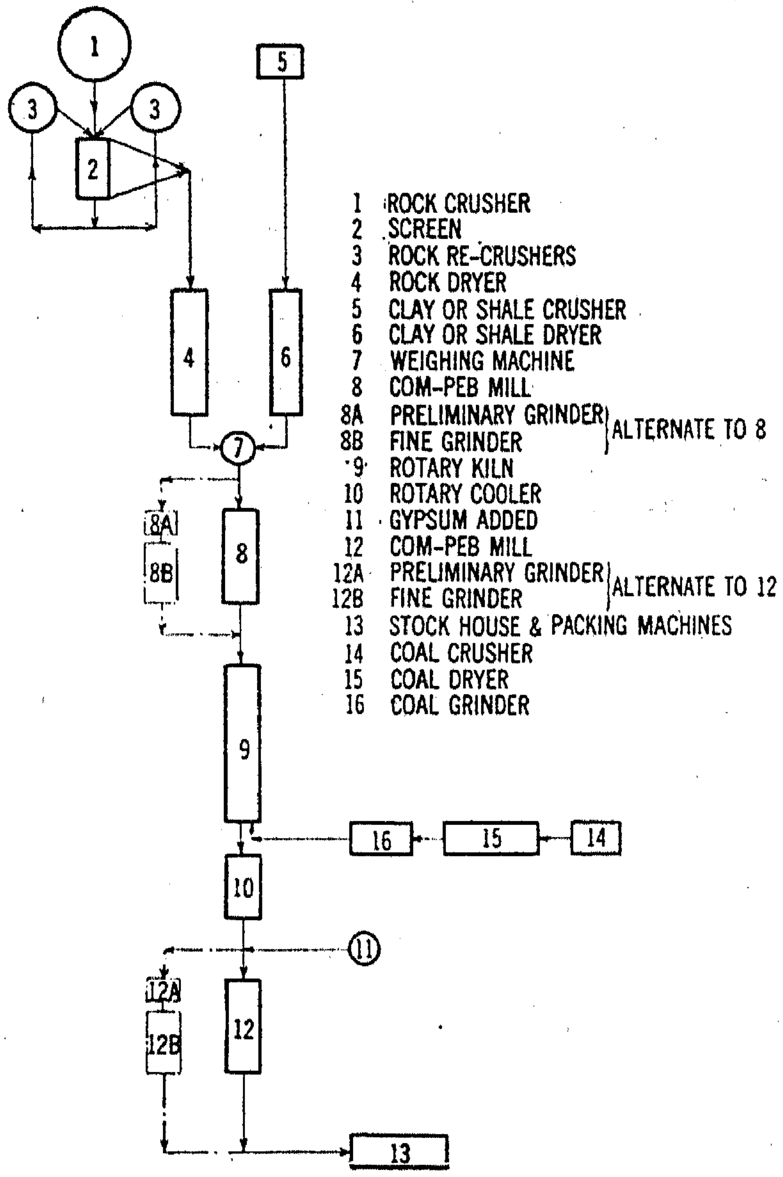

FIG. 2

The same general type of grinding machinery can be used for either wet or dry grinding. In the past, it has been customary to use a preliminary and a finishing grinder; but, the latest development in cement practise is to use a combination mill which combines the preliminary and finishing grinder in one machine, eliminating considerable expense in cost of installation and reducing materially the cost of operation. The combination mill has its greatest advantage in eliminating one set of storage bins and a great deal of elevating and conveying machinery. Combination mills consisting of a preliminary compartment charged with steel grinding balls and a finishing compartment charged with flint pebbles were tried many years ago, but mills of this type have never been successful until the introduction of the iron grinding body in the finishing compartment. The introduction of iron grinding bodies in the finishing compartment has also increased the capacity of mills of a given size, resulting in greatly simplifying the design of a cement plant, also reducing the cost of installation and operation.

While most mills have in the past been arranged for dry grinding, there is a strong trend towards wet grinding in new installations. The principal advantages of the wet process are, cleanliness, less tendency for different materials to segregate, and better opportunity to correct the mixture to obtain the exact proportions desired.

In the early days of the manufacture of Portland cement, the ground raw material was made into briquettes and then charged into a vertical kiln and burned. This process of burning was more economical in fuel consumption than rotary kilns, except of the very latest type; but had the disadvantage that a great deal of the material was wasted on account of being improperly burned and the labor cost was very high. In the United States, fuel was comparatively cheap and the cost of labor high, and, as might be expected, the rotary kiln is an American invention. The improvements made in the rotary kiln have resulted in such saving in the cost of burning cement that it is now generally used throughout the world.

The clinker leaves the rotary kiln at a high temperature, and before grinding it must first be cooled. Most modern cement plants use a rotary cooler after the kiln. The cooler is preferably placed below the kiln in order to conserve the sensible heat of the clinker, the air for cooling the clinker passing up into the kiln to support combustion. In addition, however, to cooling the clinker, it is desirable to age it before grinding, as aging not only improves the quality of the cement, but also increases the "grindability" of the clinker. The aging of cement can either take place before grinding or after grinding; but, the cement can be more cheaply stored in the condition of unground clinker than after it is ground.

The grinding of the clinker into finished cement requires more power than any other step in the process, as the clinker ordinarily is much harder to grind than the raw materials. It is common practise in American cement plants to grind the raw material and the clinker to about the same fineness. There is at the present time, a tendency to demand more finely ground cement, specifications for Portland cement in the United States having been changed in 1916 from 75 per cent through 200 mesh to 78 per cent through 200 mesh. There is a greater tendency, however, to grind the raw material extremely fine. There are three advantages in the extremely fine grinding of raw material; fuel is saved, a better cement is produced, and the clinker from finely ground raw material is more easily ground than clinker from the same material not so finely ground.

Natural gas and fuel oil are used in a few cement plants, but the most commonly used fuel for burning Portland cement is finely pulverized bituminous coal. Bituminous coal is crushed in a preliminary crusher, or the screenings from the coarse coal are used, after which it is passed through a rotary dryer and then ground so that about 85 per cent passes through a 200 mesh sieve. Finely pulverized coal is blown into the rotary kiln by an air blast; the pressure used varies greatly but is generally from two to six ounces.

GENERAL CONDITIONS AFFECTING MOTORS

The same reasons that hold for the preference of a-c. to d-c. motors in general industrial work make the a-c. motor more desirable for cement mill operation. There are two main reasons for this. First, the relatively higher alternating voltage that can be used, *i. e.* 440 or 550 volts or even 2200 volts for some of the motors in large mills; and second, the greater mechanical simplicity of the a-c. motor, particularly that of the squirrel cage type. The great majority of cement plants use alternating current, though there are a number of mills equipped with d-c. motors which have given excellent service. There is nothing inherently injurious about the action of cement dust on the commutators of d-c. machines, and they are successfully employed in some plants, but, all in all, the a-c. motor is preferred and used. The following features should be borne in mind in selecting motors for a cement mill.

Service. Continuous. Shut down for repairs only.

Starting Conditions. Severe on many applications and making phase-wound-rotor motors preferable or even essential.

Rating. May be affected by heavy accumulations of dust on the motor windings, thus interfering with ventilation. Machines should be frequently blown out and cleaned off.

Bearings. Should also be made dust proof by the use of felt and steel washers, as cement has a highly abrasive action and greatly increases bearing wear when present in the bearings.

Outboard Bearings. It is good practise to supply outboard bearings for belted motors of 75 h. p. and larger. Pulleys larger than usual in both diameter and length are often specified since the cement dust causes more than normal belt slippage and makes necessary tighter belts and increased strain on bearings.

Drive. Motors should not be direct geared to crushing machinery on account of excessive and severe vibration. Low-speed motors direct-connected through flexible couplings, or belted, are the proper application. So far as possible, motors and belts should be placed in a separate room for protection from dust and to prevent their agitating the dust in the room where the mills are located.

Type of Motors. Alternating-current preferred; squirrel-cage where starting conditions are fairly easy, and wound-rotor motors where starting is severe. Low- or moderate-speed motors should be used. On account of the unfavorable belt conditions large speed reduction ratios should be avoided. Moreover, low- or moderate-speed motors are more substantial mechanically than high-speed and give less bearing trouble.

Collector rings are not injured by cement dust, which, when dry, does not show the abrasive action that it does in the bearings. It is advisable, however, to protect collectors so that dust cannot settle on them in large quantities.

While the induction motor has been used almost exclusively for cement plant operation, there are indications that synchronous motors will be adapted to this work more in the future. This is specially so in the case of low-speed motors for geared tube mills as mentioned later.

Frequency. Frequencies of 60 cycles and 25 cycles are in common use. Most of the large plants that are operated in connection with steel mills have 25-cycle equipment since they usually get their power from the same source as the steel mill, and 25 cycles has been generally used for steel mill operation.

A few plants are in operation at frequencies other than 25 or 60 cycles but these are on the whole exceptional.

Voltage. 440 volts preferred to 220 on account of copper distribution; 550 volts is satisfactory in a dry process plant. In some large mills 2200 volts has been used for the large motors but this practise is not common.

Load Factor. May be as high as 95 per cent since the machines run at all times under constant full load. The load factor of the average complete cement plant runs from 65 to 75 per cent taken through the entire year and is probably the highest of any industry.

The various kinds of machines that may be employed for the different processes in a cement mill may be grouped as follows:

1. Elevator and conveying machinery for raw material.
2. Crushers.
    Gyratory Jaw Roll.
3. Intermediate crushers
    Roll Hammer mills.
4. Rotary Dryers.
5. Preliminary Grinding Machinery.
    Ball mills Rolls
    Ring-roll mills Huntington mills
    Hammer mills Griffin mills.
    Fuller mills
    Raymond mills.
6. Finish Grinding Machinery.
    Tube mills
    Combination mills.
7. Conveying Machinery for Finished Product.
8. Rotary kilns. Coal feeding machinery.
9. Cement Grinding Machinery.
    (a) Preliminary grinding
        Ball mills
        Rolls
        Griffin mills
        Ring roll mills.
    (b) Finish grinding
        Tube mills
        Ball peb mills
        Combination mills.
10. Coal Grinding and Drying Machinery. Same class of machinery as used for raw material.

ELEVATORS AND CONVEYERS

Every cement plant requires a large number of elevators. and conveyers for handling the material between the various steps in the process. One modern 3000-barrel plant, as an example, has 14 elevators, 2 belt conveyers and 8 screw conveyers in its equipment. It is very important that these auxiliaries be kept operating, as an accident to a conveyer or elevator may shut down an entire department of the plant until repairs can be made.

Various methods of driving this class of machinery have been installed in different mills. The chief difficulties are that the driving shafts run at extremely low speeds and that elevators must be driven from the top. The first necessitates a number of countershafts and belts or gears to give the necessary speed reduction and the second means that the driving motor, if individual drive is used, must often be located in an inaccessible place and in many cases be subjected to considerable vibration due to weak foundations. A very good method of drive which is sometimes applicable, especially in the case of crushers or group drive of grinding machinery, is to belt the elevators and conveyers from a pulley on some part of the mill shafting or from a small pulley on the front end of the motor.

Another method adopted in some plants is to group several elevators and conveyers on one motor, driving through countershafts and belts or chains.

The third possibility is to put an individual motor on each conveyer and elevator. In applying individual motors to this class of work, however, there are certain points which should be borne in mind if trouble is to be avoided. The machinery is rather roughly built, especially bucket elevators and bucket and drag type conveyers. These usually consist of two parallel chains driven by rough cast or forged sprockets, and carrying the buckets between them. It sometimes happens that the chains will climb up on the teeth of the sprockets and throw heavy overload on the driving motor which may be enough to stall it. Screw conveyers occasionally wear their bearings down so far that the screw may rub against the trough for most of its length with consequent overload. These possibilities make it advisable, in the case of individual drive, always to install motors from 25 to 50 per cent larger than necessary to meet the average power demand. This is not true when group drive is used as it is very unlikely that more than one unit of the group would be subjected to overload at the same time and this would probably not affect the operation of the motor to any extent. At the same time, group drive has the disadvantage or requiring more belts and countershafts and of the probability of crippling an entire department if an accident happens to any unit or any part of the transmission.

The lengths and types of elevators and conveyers differ so much with different plants that it is impossible to give a table which will be of much assistance in estimating the power required, but the following test results will probably be of some service in such work.

Screw conveyer on dry clay.

Diameter	16 in.
Pitch	16 in.
Length	120 ft.
Speed of screw	65 rev. per min.
Horse power	5.5

Screw conveyer on pulverized coal.

Diameter	16 in.
Pitch	16 in.
Length	162 ft.
Speed of screw	55 rev. per min.
Horse power	3.7

Belt conveyer on finished cement.

Width	20 in.
Length between centers	230 ft.
Height material is lifted	40 ft.
Speed	200 ft. per min.
Horse power	10–14

Belt conveyer on crushed stone.

Width	20 in.
Length between centers	135 ft.
Height material is lifted	22 ft.
Speed	211 ft. per min.
Horse power about	5

Elevator on crushed stone.

Buckets	18 x 10 x 16 in.
Height material is elevated	52 ft.
Speed	83 ft. per min.
Horse power	3 to 5.5

Elevator same as preceding except elevating to 60 feet instead of 52 feet. Horse power 2 to 3 (probably due to better lubrication.)

Elevator on crushed stone.

Buckets	48 x 8 x 12 in.
Height material is raised	62 ft.
Speed	95 ft. per min.
Horse power	8 to 10 (occasional peaks above 15)

The starting torque of elevators and conveyers is likely to be above normal running torque, but standard squirrel-cage motors, if large enough to take care of the peak loads as mentioned above, should have no difficulty in starting them.

CRUSHERS

All dry process cement mills require extensive crushing plants for their limestone cement rock. Three types of crusher are in common use for this work. Gyratory crushers are used more than any other, although those of the jaw type, or crushing rolls, are preferred in some cases.

Gyratory Crusher. The gyratory crusher has a vertical shaft with a bearing at the top and bottom. The shaft is driven by a bevel gear and pinion at the lower end. The shaft is eccentric with respect to the bottom gear and journal so that the shaft gyrates as the gear revolves, the amount of side movement being a maximum at the bottom and a minimum at the top. Attached to this shaft is a conical crushing head of chilled iron or special steel, sometimes smooth and sometimes corrugated. The shell or mouth of the crusher surrounds the crushing head and is lined with special iron or steel plates. The shell is conical but inverted so that the distance between its surface and the crushing head is a maximum at the top and a minimum at the bottom. As the shaft rotates, its gyratory motion causes the head to alternately approach and recede from the shell so that the large pieces of rock which are fed in at the top are gradually broken up and fall lower and lower in the shell until the pieces are small enough to fall out through the space between the bottom of the head and shell on to a sloping diaphragm between the bottom of the shell and the driving gear, which directs the crushed pieces into the discharge spout.

In all cement plant crushing, the problem is to reduce the rock from the size as it comes from the quarry down to a size small enough to pass an inch or a ¾ inch ring. It has been found that it is better to divide this range into two steps instead of making the entire reduction in one machine. Therefore, in most crushing plants, the rock first passes through a large crusher which reduces it to three- or four-inch cubes, after which it is reduced to the required size (usually ¾ to 1 inch) in smaller crushers. Occasionally, revolving screens are installed to grade the material as it comes from the crusher, all that which will not pass through the screen being returned for further reduction.

There is a considerable difference of opinion regarding the size of the first crusher. The rock as it comes from the quarry is made up of pieces of various sizes ranging from small chips up to pieces three or four feet in diameter, the only limit being what can be handled by a steam shovel. The great majority of the pieces, however, are under 20 inches. Some cement manufacturers install for the first crusher a machine large enough to take the largest pieces, while others use a crusher large enough for the average size and break up the extremely large pieces with dynamite before taking them to the crusher house. The advantage of the first method is that it eliminates this dynamiting, thus saving labor and expense in the quarry. Its disadvantage is that a crusher large enough to take care of the few exceptionally large pieces will probably have a much greater capacity than necessary so that it will be idle a great deal of the time, and will represent considerably more investment than a smaller machine, as well as a greater consumption of power.

The size of the crushers is designated by an arbitrary number. Practically the same system is used by the different manufacturers, so that a No. 8 crusher, for instance, is about the same size capacity, etc., whether one make or another; Table 1 is therefore approximately correct for all makes. As a No. 3 crusher is the smallest which is likely to be used in any cement plant, data are omitted on number 0, 1 and 2.

TABLE 1.
Power Requirements of Gyratory Crushers.

Makers No.	Size of opening on each side of top bearing arms	Size cube which can be crushed	Horse power
3	7 × 28	7	10– 20
4	8 × 34	8	15– 25
5	10 × 40	10	20– 35
6	12 × 44	12	25– 40
7-½	15 × 55	15	50– 70
8	18 × 68	18	65–100
9	21 × 76	21	100–140
10	24 × 99	24	125–175
18		36	150–200
21		42	150–200
24		48	175–225
27		54	200–250

The horse power figures given in Table 1 must be used with discretion because there are a number of factors which affect the power required by crushers. Chief among these are the hardness of the material to be crushed and the method of feeding. On soft rock or shale the power will be lower than when crushing very hard stone, the difference being about as indicated by the two limiting powers in the above table. When crushers are fed intermittently as for example the very large crushers which take the material when it first comes from the quarry and which are of such great capacity that they can run a small carload of rock through in less than a minute, after which they may run light for several minutes, the driving motor can be somewhat smaller than for crushers of the same size fed from bins where the power is more nearly continuous. It is the usual experience that crushers are "over motored", and the horse power data given in the above table are more likely to be high than low.

Belt drive is used almost exclusively, an incidental advantage being that the belts afford some protection against damage in case pieces of iron etc., get into the rock accidentally. Installations seem to be about evenly divided between those using individual motors and those in which the entire crushing plant including possibly three or four crushers with their elevators and conveyers are driven in a group from one large motor.

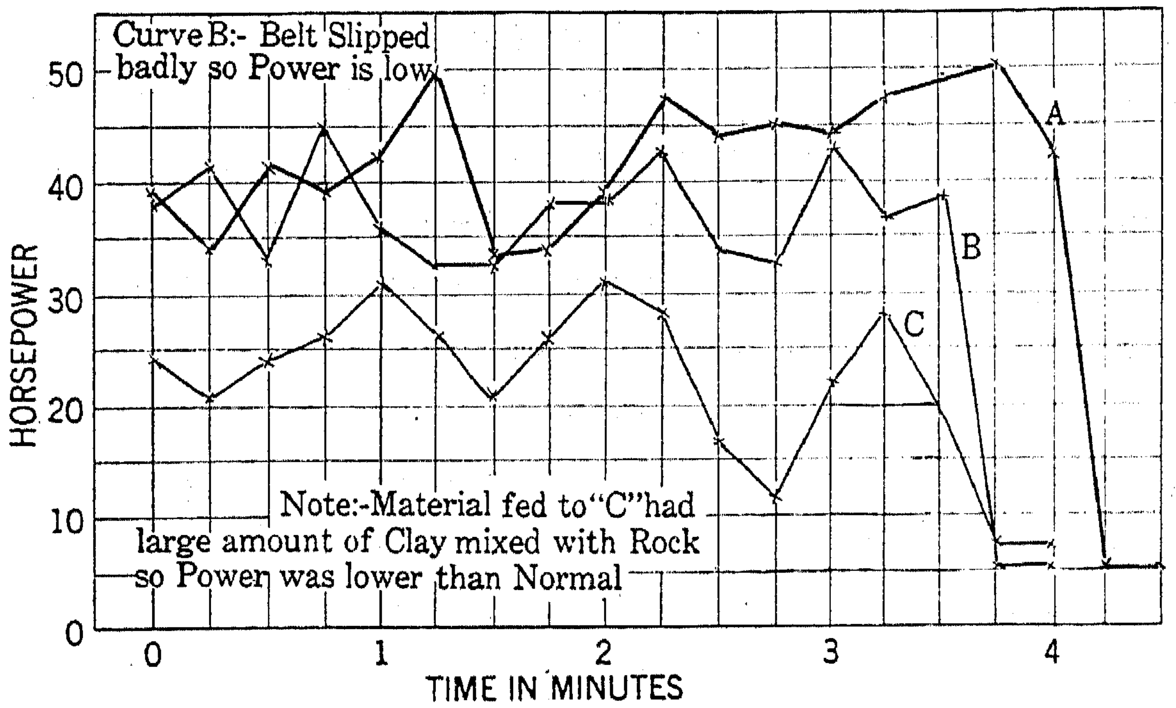

Fig. 3—Tests on Three No. 5 Crushers—Each Driven by 25 Horsepower 600 Rev. per min.
A, Crusher with Corrugated Head—1½ in. Product
B, " " " " —1¼ in. "
C, " " Smooth " —1 in. "

The starting torque required by a gyratory crusher is small unless it is accidentally stopped with material in it, in which case, it is usually necessary to dig it out before it can be started.

Typical load curves on crushers are shown in Figs. 3 and 4. The former illustrates the difference that may exist between the power required by crushers of the same size due to the quality of the material.

JAW CRUSHERS

In jaw crushers of the Blake type the crushing takes place between the two jaws set at an acute vertical angle, one fixed and one movable; the feed opening at the top being greater than the discharge at the bottom, gravity brings the material into the crushing zone and discharges it when crushed. The movable jaw is suspended from the top of the crusher frame and is given a reciprocating motion by means of an eccentrically operated vertical pitman and a pair of toggle plates.

The most favorable condition for jaw crushers is obtained when the ratio of size of feed to product does not exceed 6 to 1. When this ratio is exceeded, the efficiency of the crusher is greatly decreased, and an increase in the power requirement will be noted. With a ratio of 6 to 1, the power requirement remains practically constant whether the reduction is from 18 in. to 3 in., or from 6 in. to 1 in. The amount of material crushed, however, will be in proportion to the width of the discharge opening, *i. e.*, when reducing from 18 in. to 3 in., the crusher will handle approximately three times as much material as when reducing from 6 in. to 1 in., all other things such as size of feed openings, speed, etc. remaining constant. Any increase or decrease within certain limits in the speed of the crusher will increase or decrease the volume of material handled and the power required.

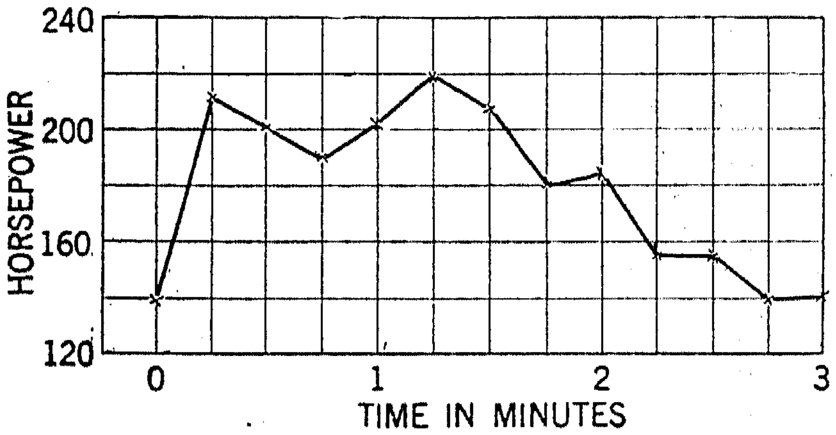

FIG. 4—POWER REQUIRED TO DRIVE MAMMOTH CRUSHERS WHILE CRUSHING ONE CARLOAD OF ROCK—FEED AVERAGE SIZE FROM QUARRY —REDUCES TO 4 IN.-5 IN. SIZE

The power required to operate any given size of jaw crusher, depends on the power required to operate the crusher when running idle and the power required to do the work of crushing the material.

Consider these in the order as given above: The power required to operate a crusher when it is running idle is due principally to friction, and this will vary considerably for different sizes, speeds and makes of machines. When figuring the po requirements of a jaw crusher, it would be preferable to obt the friction of the crusher direct from the manufacturer.

For the Blake crusher, there is one theoretically correct sp based on the law of gravity. The time period of the stroke of movable jaw must not exceed the rate of movement of the mat ial due to gravity. Therefore, there is one theoretical sp for all sizes of crushers, but it is obvious that there is a mechanical limitation when a certain size of crusher is reached that will necessitate the use of a lower speed.

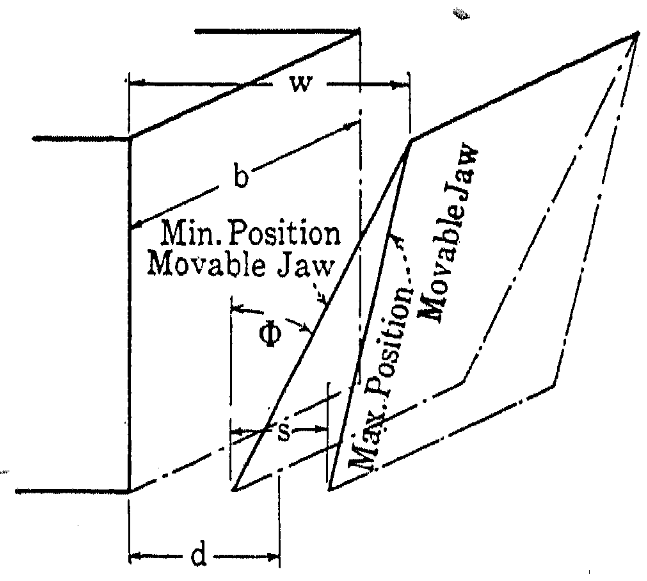

Fig. 5

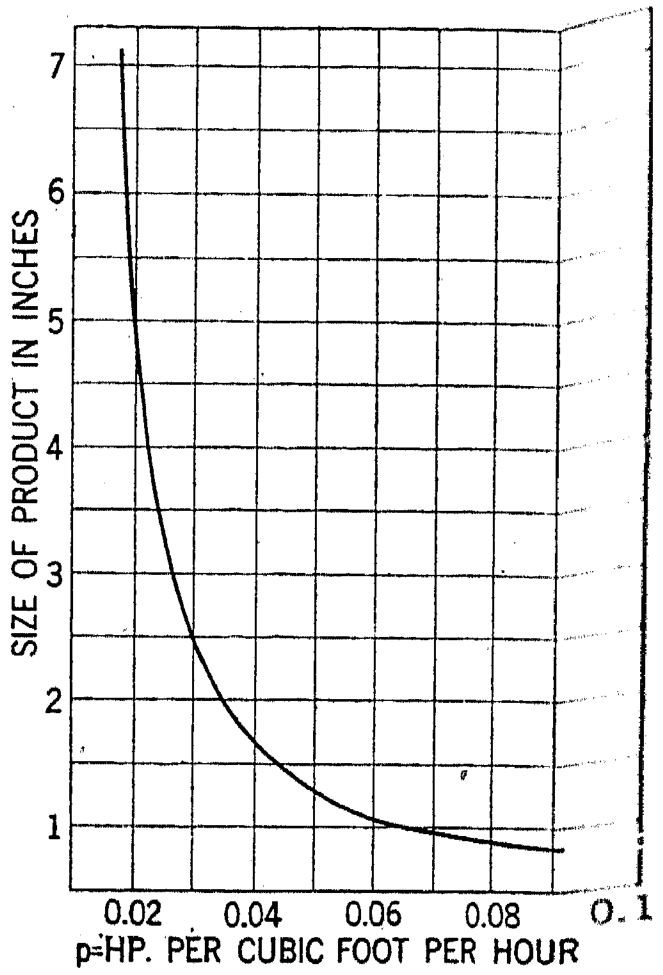

Fig. 6

This curve is drawn on the assumpt that the size of the product will never less than 1 inch and that the reduct will never be more than 6 to 1.

An approximation of the power required to operate t crusher when it is doing no work is

$$p_m = 0.016 \times b \times w$$

where

p_m = horse power

b = breadth of opening of jaws in inches. See Fig. 5

w = width of opening of jaws in inches. See Fig. 5

Power required to crush the material.

The amount of material M passed by the crusher is*

$$M = \frac{sb\ (2d + s)\ R}{57.6 \text{ tan. } \phi}$$

Where M = cubic feet of material per hour.

s = stroke in inches measured at the throat of crushe See Fig. 5

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*Wiard, "Theory and Practise of Ore Dressing."

d = average distance in inches between the fixed and movable jaws and the minimum and maximum position of the swing jaw as measured at the throat of the crusher. See Fig. 5.

b = breadth in inches at opening of jaws. See Fig. 5.

R = revolutions per minute.

The angle ϕ varies for different makes of crushers from 20 deg. to 26 deg.

The above formula gives the theoretical capacity and is the one used in figuring the power requirements. In order to obtain the actual capacity, the theoretical capacity should be divided by 1.6. This is on account of irregularity in feed, hardness of material, size of material, etc.

In the smaller sizes of crushers, where the breadth b is small and the stroke s is large, comparatively, it is found that the theoretical capacity M is as much as 3¼ times the actual capacity. This is due to the fact that these smaller sizes of crushers are very inefficient when compared to the larger sizes; this is found for example to be the case of the 10 in. x 7 in. machine as shown in Table 2. Therefore, the empirical formulas as herein given will not apply to these small crushers.

Power required to crush the material (dry limestone)

$$p_c = p\,M$$

where p_c = horse power.

p = take from curve—Fig. 6.

It is assumed that the size of the product will never be less than one inch and that the reduction will never be more than 6 to 1.

The total power required to operate the crusher when doing work

$$P = p_m + p_c$$

Where P = horse power.

The above formulas have reference only to the crushing of dry limestone. The weight of one cubic foot of broken dry limestone is approximately 100 lb.

As the starting conditions are usually severe, the slip ring type of induction motor is invariably recommended to drive jaw crushers.

FAIRMOUNT CRUSHER

Many of the more recent crushing plants for cement works use the Fairmount type crusher. This machine consists of a single roll working against an anvil plate. The roll is provided

TABLE 2.

Power Requirements of Blake Crushers.
Dry Limestone.

Size or jaw opening b x w	Capacities in tons per hour											Approx. horse power req'd
	Size of product											
	¾ in.	1 in.	1½ in.	2 in.	2½ in.	3 in.	4 in.	5 in.	ϕ	s	R	
10 × 7 in.	1.5	2.5	4	5					26°	$^5/_8$ in.	275	7
15 × 9 "		6	8	10	12				26°	$^7/_{16}$ "	275	10
20 × 10 "			10	15	17.5	20.			26°	$^1/_2$ "	275	15
24 × 12 "				20	25	30.	35		26°	$^9/_{16}$ "	275	25
30 × 18 "					20	37.5	45	50	26°	$^9/_{16}$ "	275	40

with knobs from two in. to three in. in height for crushing against the anvil plate and with sledging knobs from three in. to four in. in height for breaking the large pieces of stone on top of the roll, which are too large to be directly gripped between the roll and the anvil plate. These machines have a very large capacity when operating continuously, but are not objectionable for small capacity on account of their very low friction load when running idle.

The Fairmount type crusher is built in sizes 24 × 36 in., 24 × 48 in. and 24 × 60 in., also 36 × 60 in. and 60 × 84 in., in each case the size being referred to diameter and face of the roll. The size which has so far been used in cement plants is 36 in. diameter by 60 in. face. This size of machine ordinarily requires a 150-h. p. motor to drive it and it has a capacity of from 200 to 500 tons per hour, depending upon the character of the material to be crushed. One of the particular advantages of the Fairmount type crusher is that it makes a very large reduction in one operation so that the number of secondary crushers is reduced and the product from the primary crusher being smaller can be much more easily handled into and out of storage bins.

The starting torque required for the Fairmount crusher when idle, is very low so that squirrel-cage motors are often used. While it is not possible to start the crusher full of stone even with slip-ring type induction motor, the latter will give sufficient torque to start up the crusher without entirely cleaning it out, if it should happen to stop under load. Slip-ring type motors are therefore, recommended for this class of work.

Crushing Rolls

The most favorable condition for crushing rolls is obtained when the ratio of size of feed to product does not exceed 4 to 1. When this ratio is exceeded, the efficiency of the roll is greatly decreased and an increase in the power required will be noted. With the ratio of 4 to 1, the power requirement remains practically constant whether the reduction is from 4 in. to 1 in. or from 1 in. to ¼ in. The amount of material crushed, however, will be in proportion to the size, *i. e.*, when reducing from 4 in. to 1 in. the rolls will handle four times as much material as when reducing from 1 in. to ¼ in; all other things such as size of rolls, speed, etc., remaining constant. Any increase or decrease in the speed of the rolls will increase or decrease the quantity handled and the power required.

Due to irregularity of feed, variation in size and hardness of material, etc., it has been found that the power required is not steady but fluctuates as shown by Fig. 7 which is a power diagram of a 42 × 16-in. roll operating at 60 rev. per min. and reducing dry iron ore from a maximum of two inches to an average of one-half inch, the average power being 13.18 h. p.

Fig. 8 shows the relation existing between horse power per cubic foot per hour and different ratios of reduction assuming maximum size of feed material and average size of product, all friction of the rolls being neglected. This curve has been plotted as an average of points obtained from actual tests. It is to be noted, however, that while dry material has been rolled, most of the points indicate power readings taken while crushing iron ores, and that only one point refers to the rolling of dry limestone such as is used in the cement industry. As the iron ore is a harder substance than the limestone, it is reasonable to assume that the curve as drawn shows values slightly higher than would be obtained had more data been available on dry limestone.

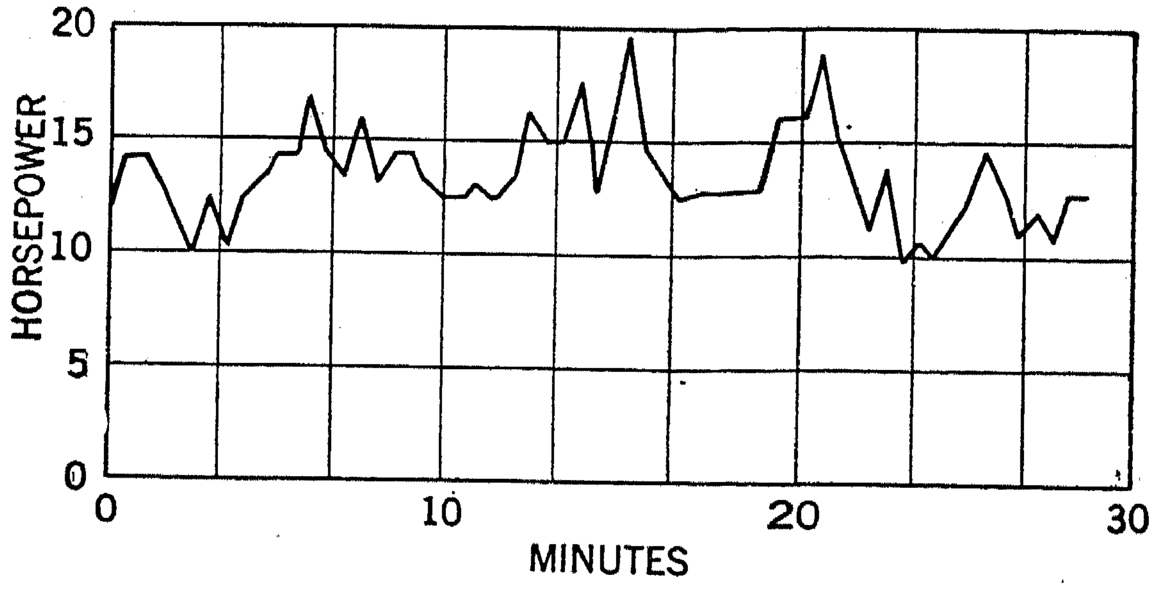

FIG. 7

The friction of crushing rolls, that is, the power required to drive the rolls at their normal speed and without load, varies considerably for different sizes, speeds and makes of machines, but an approximate average is obtained from the following:

$$P_1 = 0.0835\,(D + W)$$

where p_1 = horse power

D = diameter of rolls in inches

W = face of rolls in inches.

When figuring the power requirements of crushing rolls it is preferable to obtain the friction of the rolls direct from the manufacturers.

The amount of material passed by the rolls is

$$M = \frac{d \times W \times S}{144} \times 60$$

where $M =$ cubic feet of material per hour
$d =$ distance in inches from edge to edge of roll (See Fig. 9)
$W =$ face of rolls in inches
$S =$ peripheral speed of rolls in feet per minute.

The above formula gives the theoretical capacity and is the one used in figuring the power requirements. In order, however, to obtain the actual capacity, the theoretical capacity should be divided by 4. This is on account of irregularity in feed, etc.

The total power required to drive rolls for any different rolling condition is

$$P = (p \times M) + p_1$$

where $P =$ total horse power
$p =$ horse power per cubic ft. per hr. (Fig. 8)

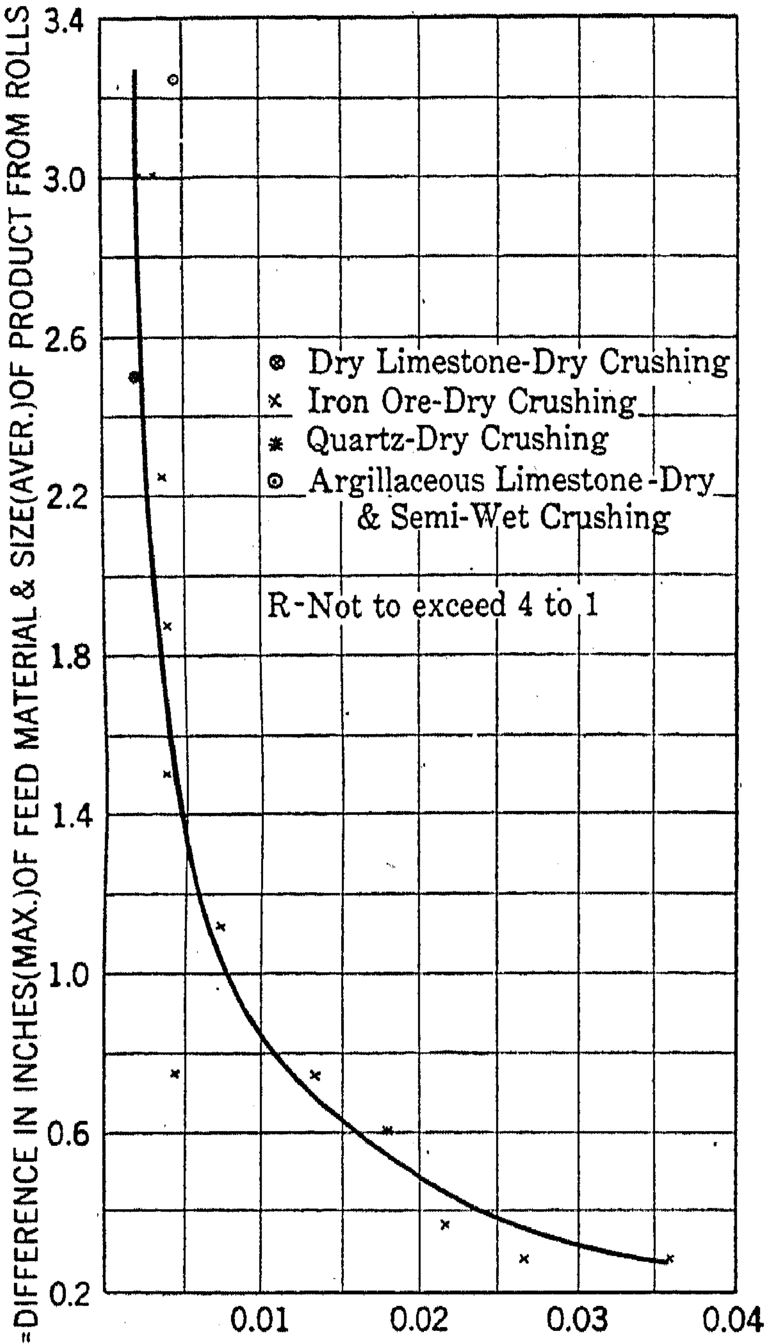

FIG. 8

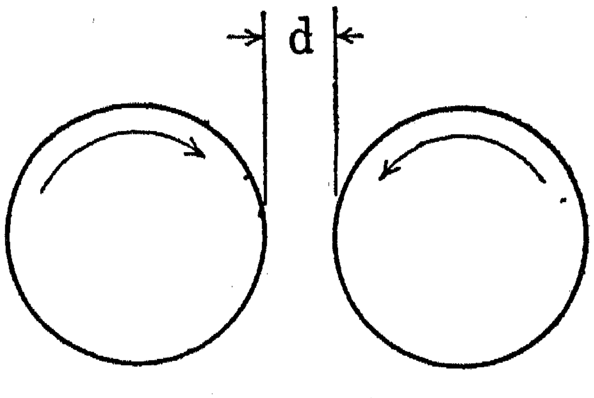

FIG. 9

As the starting conditions are not severe, the squirrel-cage induction motor makes a suitable machine for driving the rolls, provided the conditions are such that this type of motor is connected to a source of power where line disturbances are not objectionable. If line disturbances are objectionable, the slip-ring type of induction motor should be used. When shutting down the mill, the usual procedure is to first cut off the feed to the rolls, allowing same to empty themselves, before closing down the motor. No difficulty, however, is likely to be experienced with the squirrel-cage type of motor, if called upon to start the rolls when full of material and with the feed open, as even under these conditions the starting requirement is not severe.

INTERMEDIATE CRUSHERS

Hammer Mills. These machines are made in several styles but all make use of the same principle, namely, the use of hammers revolving at high speed and abrading the material against a slotted iron casing. Unlike the crushing rolls, where a reduction not exceeding 4 to 1 is most favorable, the hammer mill is used for reducing material of from 2½ to 1 in. to 20 mesh or less, the maximum reduction in this case being 76 to 1. As the power required to drive this type of mill varies considerably, due to irregularity in feed, it is most important that this remain steady and therefore a feed regulator is highly desirable. In addition to this, any variation in the size and hardness of material, provided the feed remains fairly constant, will also cause power fluctuations which will however be within reasonable limits.

There is very little general power data available on hammer mills other than for a specific manufacturer's type, trade size and specific speed or speeds. The latter are supposed to be the most efficient speeds at which the particular mill should operate. A general rule gained by experience indicates that in order to crush one ton of dry limestone in one hour, reducing from a maximum size of 2½ in. to 20 mesh, requires six to seven horse power. This would be the output of a motor and would include the friction load, in addition to the crushing load, of the mill. The actual capacity of any mill is proportional to the size of the product and to some extent to the character of the feed so that for different sizes of product on any given mill, the power requirements will be different.

Sturtevant Hammer Mills are of two types known respectively as hinged hammer and hammer bar mills. The hinged hammer mills are of the general type noted above under this class. The bar hammer mill differs in that there is only one set of hammers instead of six or more and the hammers themselves have an axial length the full width of the rotating spider which carries them. Both mills take material in pieces from 5 to 6 in. wide. The hinged hammer mill reduces from one inch to 20 mesh and finer. It is built in three sizes and operates at a speed of 1000 to 1500 rev. per min. The hammer bar mill is built in two sizes and reduces from ½ in. to 20 mesh. A belted wound-rotor motor is the preferred application. Squirrel-cage motors have, however, proved satisfactory, particularly if used when line disturbances at starting are not objectionable. In any case, if a mill is shut down it will have to be clear of material before started again.

Hammer bar mills are made in two sizes, No. 0 and No. 1. The No. 0 mill will handle from 1½ to 4 tons per hour, depending on material, and requires from 8 to 10 h. p. The No. 1 handles from 4 to 12 tons per hour and takes 20 to 25 h. p.

Hinged hammer mills are made in three sizes 30 by 12 in., 30 by 18 in. and 30 x 24 in. The power varies widely with the material. The 30 x 12-in. machine, for example, might take a maximum of 30 h. p. or a minimum of 10 h. p. The wider machines take increased power in proportion to the width.

TABLE 3.
Power Requirements of Hammer Mills.
(Allis-Chalmers)

Trade size	Rev. per min.	Hammer circle	Face	H. p.
No. 1	1800	24 in.	9 in.	15
" 2	1200	30 "	12 "	30 to 40
" 4	1200	42 "	24 "	75 to 90

The above requirements are based on crushing dry limestone, reducing from a maximum size of 2½ in. to 20 mesh with a constant feed of material best suited to the particular mill.

Williams' Jumbo Crusher. The machine of this make used for intermediate crushing is usually the No. 6 size. It takes stone three inches in size and under and reduces it to ¼ inch and finer. The capacity is 60 to 65 tons per hour, and the motor required is 150 h. p. The mill operates at from 720 to 750 rev. per min. It prepares material for some one of the various finishing mills.

Vulcanite Crushers. (Williams). The Vulcanite crushers are also swing hammer mills. They are built in eight sizes having capacities from two tons per hour at ¼-inch size up to 50 tons per hour at ½-inch size. The standard unit generally employed for cement work is No. 3. This takes material up to three inches and reduces it to ¼-inch for feeding ring roll mills. It handles 15 to 18 tons per hour, operates at a speed of 1000 rev. per min. and requires 50 h. p.

Universal Grinder (Williams). This is termed a tube mill feeder as it will take limestone 2½ inches and under after passing through the dryers and reduce it to a product, 95 per cent of which will pass a 20 mesh sieve. These are built in ten sizes, the two most commonly used in cement plants being the No. 3 and the No. 9. The No. 3 handles from 8 to 10 tons per hour under the above conditions, requires from 50 to 60 h. p., and should operate at a speed of 1100 rev. per min. The No. 9 size, used in plants requiring very large units, handles from 25 to 30 tons per hour, requires 175 to 200 h. p., and operates at a speed of 720 rev. per min. Wound rotor induction motors are the best application on all three mills. The preferred method of drive is by motor direct connected through a flexible coupling.

Sturtevant Rotary Crushers. This type of rotary crusher operates on the principle of the old fashioned coffee mill. It is shaped like an hour-glass or double cone, one cone mounted and reversed on the other. The coarse material is introduced at the top and is acted upon by a rotating cylindrical member which exerts a continual nipping action on the rock since the diameter of the stationary cone decreases as the material falls toward the center. Below the center the material is ground or shredded by the usual mill stone action. The lower portion of the rotating member is cone-shaped and so arranged as to leave a tapering annular space decreasing in size toward the bottom. As the material proceeds by gravity toward the bottom, it is ground and reduced in the annular space by the action of the rotating cone against the stationary one. Both these members are faced with hardened steel burrs or grinding members. These crushers are intended for moderately hard or soft rocks but are not used for hard cutting substances. They crush large rocks

TABLE 4.
Power Requirements of Rotary Crushers.

Machine size	Approx. capacity in tons per hour	Rev. per min.	Approx. h. p.
00	1 to 1½	300	1 to 2
0	½ to 2	250	3 to 4
1	1 to 6	300	6 to 10
1½	4 to 10	200	15
2	8 to 15	250	15 to 20

to approximately ⅛ inch without screens in the smallest sizes and as coarse as 1 inch in other sizes. Their sizes, outputs and power requirements are as shown in Table 4.

A wound-rotor induction motor is the preferred application for these crushers but a squirrel-cage motor can handle them if designed with considerable margin in starting torque.

ROTARY DRYERS

Dryers are used to dry the crushed limestone which contains a considerable amount of moisture that must be driven out before the grinding operation can be performed. They are also used for the same reason, for drying the clay and coal. The dryer consists of a large revolving cylinder which is sufficiently inclined to roll the contents through by gravity. It is heated either by oil, natural gas, or by the waste gases taken from the kilns.

Auxiliary machinery necessary for the dryer, are fans and conveyers, and usually this auxiliary machinery is considered as integral parts of the dryer, and is driven by the same motor. For a fixed rate of feed the load is constant and as a rule the dryers are operated at a given predetermined speed; therefore, squirrel-cage motors are satisfactory for this kind of service. Squirrel-cage motors with 125 per cent to 175 per cent starting torque, are suitable.

Approximate horse power requirements are as follows:

Size of dryer	Horse Power	Recommended speed of motor.
6 ft. x 60 ft.	12	700 to 1200
6½ ft. x 60 ft.	15	700 to 1200

On some particular installations, the following was found. A dryer 6 ft. in diameter 45 ft. length, 9 rev. per min., connected to a 24-in. x 50-ft. horizontal pan conveyer and to a 14 in. x 35-ft. bucket elevator, required a 30 h. p. motor. A 7-ft. x 60-ft. dryer for rock crusher, connected to a 20 in. x 45-ft. inclined conveyer, and 16 in. x 40-ft. elevator, required 40 h. p.

Ordinarily, dryers of 5-ft. diameter and 50- to 60-ft. length, revolving at 3 to 5 rev. per min., were found to require between 10 and 20 h. p.

PRELIMINARY GRINDING MACHINERY

Ball Mills. The ball mill consists principally of a revolving horizontal cylinder lined with overlapping iron plates and containing a charge of steel balls. For the first charge, balls of

3½, 4 and 5 inches diameter are supplied, but under running conditions only the largest size are added from time to time to maintain the charge. These mills will take material up to 6 inches in size. The material as ground falls through perforations in the iron plates to the cylindrical screen or hopper which surrounds the mill, and the oversize rock falls back through the same perforations to the grinding plates. The fine product is removed by a conveyer below the machine and is generally of such fineness that all will pass a 20 to 30 mesh. When pulverizing to pass all through a 20 mesh, from 30 to 40 per cent will pass a 100 mesh sieve.

TABLE 5.
Power Requirements of Ball Mills.

Make	Weight of charge lb.	Capacity on cement clinker bbl. per hr.	H. p. req'd.
Gates No. 7	3,000	12–16	30– 40
" " 8	4,500	18–24	40– 50
Smidth (Kominuter) No. 53–½.....	3,000	.....	30
" " " 66........	6,600	.....	75
" " " 88........	....	.....	100
Krupp No. 7	3,520	12–16	30– 35
" " 8	4,400	16–22	35– 45
Chalmers & Williams..............	10,000	60–70	100–125

Ball mills are low-speed machines generally running from 20 to 25 rev. per min. The charge of balls revolving at this speed causes very severe vibration in the gearing, and motors geared to their countershaft have proved unsatisfactory. Belted motors or low-speed motors for direct connection to the countershaft through flexible couplings give entire satisfaction. When starting, the charge of balls and stone must be rotated through nearly 90 deg. before it begins to roll, and the motor torque required at starting varies from 1¾ to 2 times full-load torque. Squirrel-cage a-c. motors specially designed for this high starting torque have been successfully applied but they have a lower efficiency than wound-rotor motors and make a large current demand at starting. The same starting conditions are found in tube mills and as the size of these has steadily increased of late years, and as the power required has also increased with the use of metallic grinding bodies in the mills, there has been a gradual change to the use of wound-rotor motors in place of the high-torque squirrel-cage motor. Many of the latest mills are using wound-rotor motors for these grinders of large capacity and the saving in power due to increased efficiency, together with better performance at starting, warrants their use in preference to the squirrel-cage type. Table 5 indicates the power requirement for a few typical ball mills.

Hammer Mills. These mills which have been described above are used for preliminary grinding as well as for crushing.

RING-ROLL MILLS

There are a number of different mills that may be classed generally as ring-roll. In these, the material is crushed by means of a ring or large balls rolling inside and against the ring or die. In some types this ring rolls also. There is enough difference in mechanical construction to require a brief explanation of each mill. Under this general type are classified the Kent mill, Griffin mill, Fuller mill, Bonnot mill, Raymond mill, Sturtevant Ring-Roll mill, Huntington mill and others.

These different types of mills are used in many combinations depending on the material and the locality in which they are employed. An analysis of about fifteen mills shows that ball mills are widely used as intermediate grinders taking material from the crushers on the raw side and the kilns on the clinker side and reducing it to a suitable fineness for feeding tube mills. In another combination the ball mills are replaced by ring-roll mills. Another combination uses hammer-mills to reduce the material to ¼ inch and smaller, after which it is crushed in ring-roll mills. In other plants the entire operation of grinding the clinker in the finishing mill is performed by ring-roll mills.

Fuller Mill. Three sizes of this mill are manufactured—33-inch, 42-inch and 57-inch. The smallest mill is best adapted for use on coal or similar materials while the two larger machines are adapted either to coal or harder material such as limestone, rock or cement clinker. This mill accomplishes both intermediate and fine grinding in one operation. It takes material up to ¾ inch in size and reduces it to a fineness of 85 per cent through a 200 mesh sieve. The mill is vertical in construction and the grinding is done by four chilled unattached balls rolling against a circular horizontal die. The balls are propelled by four equi-distant horizontal arms or pushers radiating from the vertical shaft. The shaft is driven at such a speed as to cause the balls to exert a pressure against the die of approximately

1600 pounds. The material is fed in at the top of the mill by a screw and enters the pulverizing zone. By means of fans and screws the material is removed as soon as it has reached the proper degree of fineness. The driving pulley is mounted at the bottom. Squirrel-cage induction motors are used very satisfactorily to drive these mills through belts running in a vertical plane. Special vertical motors have been developed for this application.

The output of these mills and the power required to operate them depends upon the weight and hardness of the material being pulverized and the fineness of finished product desired. On the 33-in. mill the output will vary from two tons per hour when grinding hard material, to four tons per hour when grinding soft material. The power will approximate 45 h. p. for hard material and 25 h. p. for soft material. The above figures are based on grinding to 100 mesh. On the 42-inch mill the output will vary from 4 to 10 tons per hour from hard to soft material, and the horse power will vary from 75 to 45 when grinding to 100 mesh.

The 57-inch mill when grinding raw cement material consisting of limestone and shale or clay or cement rock has a capacity of from 9 to 12 tons per hour and requires from 110 to 125 h. p. The same mill when pulverizing cement clinkers has a capacity of from 500 to 600 barrels per day and requires from 135 to 150 h. p. The performance of the 57-inch mill is based on grinding to 85 per cent through a 200 mesh sieve.

It should be borne in mind that the power required for the different machines as given in this paper represent average performances. It is possible under certain conditions to crowd all these machines thereby increasing their output and also increasing the power consumed so that specific instances are no doubt known to all engineers interested where the figures are somewhat higher than here given. These are matters that take care of themselves in practise.

Kent Mill. The Kent mill consists principally of a revolving ring and three rolls pressing against its inner face. The rolls are convex and the ring is concave and tracks on the rolls. Springs support the rolls yieldingly and the rolls support the ring so that the four crushing parts are free to move. The material falls from the inlets onto the inner face of the ring, and centrifugal force holds it there in a layer an inch deep as it passes under the rolls. The latter are pressed by the springs outwardly against the rock or the ring with a pressure adjustable to 20,000 pounds, the adjustment being accomplished by means of the screws against the springs. As the rolls pass over the rock they crush it against the ring, while the crushed rock falls off each side of the ring into the casing and then into the discharge. It is claimed that 90 per cent of the rock is abraded on itself in crushing so that the wear on the parts of the machine is materially decreased.

The torque required to start this mill does not exceed full-load torque, since the moving parts are comparatively light and there is only a small amount of rock in the machine at a time. The pull-out torque of the motor supplied should be comparatively high as large pieces of rock passing between the rolls and ring will cause severe momentary overloads.

In order to obtain a fine and uniform product, the ground material is usually elevated from the discharge to a screen or separator above the mill and the oversize is returned to the feed. The elevator and separator will add from 7½ to 10 h. p. to the figures given below for the mill alone. The feed should preferably pass through a one-inch ring but anything up to two inches can be handled. The finished product will all pass a 20 mesh.

These mills are made in three sizes known as the Kent mill, the "Maxecon" and the "Big Maxecon" mill. Their capacity depends on the hardness of the material and the fineness to which it is ground. Round approximations might be 25 barrels per hour for the Kent, 35 for Maxecon and 50 for Big Maxecon. These mills operate at 200 rev. per min. and the usual sizes of motors are 30, 40 and 50 h. p. for the Kent, Maxecon and Big Maxecon respectively. When grinding clinker one h. p. per barrel is the average power consumption to pass 20 mesh; also the product will approximate 50 per cent through 100 mesh and 35 per cent through 200 mesh.

Raw rock varies so in hardness that capacities range from 7 to 16 tons per hour, and power estimates from five h. p. per ton for the hardest to three h. p. for the softest.

Sturtevant Ring-Roll Mill. This ring-roll pulverizer consists of a steel anvil ring or die secured in a head supported and revolved by a horizontal shaft. Against the inner face of this ring the three hammer rolls are equally and elastically pressed with great force and revolved by the ring. Substances to be ground are fed to the anvil face of the rotating ring and held thereto by centrifugal force and are crushed by being drawn under the rolls, as the ring revolves. The face of the anvil ring is concave and the rolls convex, thereby crowding the material as it is finished to the outside edges of the ring. These mills may be used with elevator and separator, thus securing very uniform product. They take material up to 1½-inch size and reduce it to a fineness varying from ⅛ inch to 100 mesh or finer. They are usually employed as a preliminary pulverizer rather than a finishing mill for cement work. As is the case in mills of this type, the operation of crushing is largely performed by abrading the rock on itself so that the wear on the mill parts is reduced. These mills are made in five sizes; three sizes of single mills and two of duplex mills which are a double unit. Table 6 gives the sizes, horse power and speed. Regarding the output, the manufacturers furnish this on request, since it varies widely with the nature and conditions of the material handled. As an illustration the following figures are cited for the No. 2 duplex mill. Grinding cement clinker to 20 mesh, 80 to 100 barrels per hour, and to 80 mesh, 30 to 40 barrels per hour. Grinding limestone to 20 mesh, 16 to 20 tons per hour; and to 80 mesh, 8 to 14 tons per hour.

TABLE 6.
Power Requirements—Sturtevant Ring-roll Mills.

Size	Pulley speed rev. per min.	Ring speed rev. per min.	Approx. h. p.
0	125	125	8 to 12
1	320	80	15 to 20
2	300	63	35 to 45
No. 1 Duplex	375	80	30 to 40
No. 2 Duplex	325	64	70 to 90

The elevator and separator if used add about 8 h. p. per single mill. A squirrel-cage induction motor is the preferred type for these mills.

Huntington Mill. This ring-roll mill is a vertical shaft machine with a horizontal spider from the rim of which are suspended four spindles. These spindles carry at their lower end a heavy horizontal crushing roll. The rotation of the main shaft causes these crushing spindles to swing out by centrifugal action and roll against a ring die. The material is fed against this die and crushed by the rolls.

These mills are made in four sizes, but the largest size is suited only for wet grinding. The other three sizes are 3½ ft., 5 ft., and 6 ft. and require 5, 8, and 10 h. p. respectively. This mill is a preliminary pulverizer and not a finishing mill. A belted squirrel-cage motor is a satisfactory application.

Bonnot Mill. This ring-roll type of mill employs air separation of the material. The driving shaft is horizontal and the grinding ring or die is held in a vertical plane in the main frame. The rolls are carried by a head or driver mounted on the main shaft and are free to be thrown outward by centrifugal force, thus furnishing the crushing pressure. The material thrown upward into the screening chamber by the motion of the grinding parts is deflected against the screen by the blades of a fan. The coarse material strikes a baffle and falls fack into the grinding chamber while the fine material passes through a screen and drops through passages into the conveyer. The 36-inch size is used for cement; it operates at 180 rev. per min. and requires 75 h. p. It takes pieces up to two-inch size and when working on clinker will handle 13 to 16 barrels per hour of which 96 per cent will pass 100 mesh and 77 per cent 200 mesh. It will be seen from this that it is a finishing mill as well as a preliminary grinder. When grinding raw material and fed through ⅝-in. grate it will handle 4 to 5 tons per hour, 97 per cent through 100 mesh. A squirrel-cage a-c. motor is the preferred application.

Griffin Mill. This mill employs in its construction the principle of a roll running against a ring or die. Power is received by a pulley running horizontally, from which, by means of a universal joint, a shaft is suspended in pendulum fashion. To the lower extremity of this shaft is rigidly secured the crushing roll which is thus free to swing in any direction within the case. The base or pan contains the ring against which the roll works and upon the inner surface of which the pulverizing is done. On the bottom of the roll are shoes for stirring up the material and throwing it against the ring, and above the roll are attached blowers for blowing the fine material through the screen which surrounds the base. When at rest the roll hangs vertically beneath the driving shaft, but when revolved it flies out to the die ring and travels around in a direction opposite to that of the driving pulley. There is a pressure of approximately 6000 lb. due to centrifugal force brought to bear on the material being pulverized between the roll and the die. These mills are manufactured in three forms known as the "Bradley Hercules," the "Bradley 3-Roll" and the "Giant Griffin" mill. The Bradley Hercules takes material two inches and under and reduces it to a fineness of 53 per cent through a 100 mesh screen at the rate of from 40 to 50 tons per hour, using about 250 h. p. in the operation. When used as a preliminary grinder of cement clinker it has an output of from 150 to 175 barrels per hour and requires 250 h. p. to drive.

The Giant Griffin mill has an output when grinding clinker of from 25 to 30 barrels per hour to a fineness of 60 per cent through a 100 mesh screen. It takes material ¾ inch and under for feed and when operating to full capacity requires about 80 h. p. This mill is a very efficient preliminary machine on account of the high percentage of fineness and has sometimes been used as a finishing mill.

The 3-roll mill in theory is similar to the Griffin, but employs three rolls instead of one. A squirrel-cage a-c. motor is the preferred application for these mills except the Hercules which is better equipped with a phase-wound-rotor motor on account of the starting conditions on so large a unit.

Raymond Mill. This mill performs the operations of preliminary grinding and finishing in one process. The grinding ring is mounted in a horizontal plane and the grinding is performed on its inner surface by means of rolls suspended from a rotating driving head or spider. There are two, three, four, or five of these rollers, depending on the size of the mill. A prominent feature of this mill is the system of air separation employed. When handling raw material the mill will handle about one ton per hour per roll, *e. g.*, the 4-roll and 5-roll mills will handle four and five tons per hour respectively from a size of ¼ inch to 1 ½ inches to a fineness of 95 per cent to 98 per cent through a 100 mesh sieve. There are six sizes in all called two-roller mill, three-roller mill, four-roller mill (low side or high side), and five-roller mill (low side or high side). The power consumption is given as follows:

2 rolls,	45 to 50 h. p.	3 rolls,	55 to 60 h. p.
4 „	75 to 80 h. p.	5 „	85 to 90 h. p.

The driving motor is belted. A squirrel-cage induction motor with good starting characteristics is the preferred application.

Emerick Ball Pulverizer. This is a vertical mill and the crushing is done by four or five 12-inch or 14-inch steel balls which are unattached and roll in and against a grinding race. These balls are driven by a head or spider which keeps them properly separated. The centrifugal force set up in the balls by the rotation of the driving head causes them to exert great pressure against the grinding race, thereby crushing and abrading the material in the mill. The mill is arranged with five 12-inch, four 14-inch, or five 14-inch balls. This mill may be used either as a preliminary grinder or as a finishing mill. When used as a finishing mill it is equipped with an air separator and will reduce from ½ inch and under to 95 per cent to 98 per cent through 100 mesh and 80 per cent to 90 per cent through 200 mesh. On raw material or coal it will handle six to eight tons per hour and clinker 12 to 20 barrels of finished cement per hour. The mill operates at 90 to 130 rev. per min. and requires 35 to 60 h. p. A belted squirrel-cage motor is the proper application either vertical or with a quarter-turn belt.

Symons Disk Crusher. This crusher is of a special type in which the crushing is done by two dish-shaped disks mounted on horizontal shafts which are separate but one of which gyrates slightly with respect to the other. The two disks are set with their concave sides facing each other and are rotated in the same direction at the same speed. This results in the hollow between the two disks having a wider opening between their edges at one part of a revolution than on another. This progressive pinching action admits a piece of coarse material at one side and as the disks rotate they approach one another thus crushing the material which is then thrown out through the opening between the edges of the disks if it is reduced to the proper fineness; if not, the process is repeated.

A wide range of adjustment is provided by changing the distance between the crushing disks. These crushers are made in four sizes as given in Table 7. A squirrel-cage motor with a maximum starting torque of 1 ½ to 1 ¾ full-load torque is satisfactory on this application.

TABLE 7

Power Requirements of Disk Crushers.

Size of crusher	18 inch	24 inch	36 inch	48 inch
Size of material fed	1½ “	2½ “	3½ “	6½ “
Size to which reduced	⅜ “	½ “	¾ “	1 “
Tons per hour	5 to 8	12 to 15	25 to 30	45 to 60
H. p. required	12 to 18	18 to 25	30 to 40	50 to 65

Such material is too coarse to finish in tube mills, but could be handled by some of the types of ring roll finishing mills described in this paper.

FINISHING GRINDING MACHINERY

TUBE MILLS

For the fine grinding of limestone or clinker, the tube mill is used extensively, both wet and dry. The usual form is that of a rotating cylinder and the grinding is done by attrition or by the rubbing together of the grinding medium and the material to be ground. The mill is generally filled to about the center line, and the grinding medium, or charge, as it is usually called, consists of either flint pebbles, cast iron balls, or forged steel balls, etc., depending on the design of the mill and the comparative costs of the various grinding mediums. There are several types of tube mills in commercial use, designated as follows:

TABLE 8

PLAIN TUBE MILLS WITH FLINT PEBBLES.

Operating in Portland Cement Plants with 20 Mesh Feed and Grinding, to 95% through 100 mesh and 80% through 200 mesh.

Size of Mill	Capacity on raw material		Capacity on clinker	Horse power		R.P.M. mill	Initial charge pebbles
	2000-lb. tons per 24 hr.	666-lb. bbl. per 24 hr.	380-lb. bbl. per 24 hr.	to start	to run		
3 ft. 6in. × 20 ft.	50	150	120 to 144	60	35 to 40	40	9,800
4 ft. 0 in. × 20 ft.	64	192	192	70	45 to 48	35	11,600
4 ft. 6 in. × 22 ft.	80	240	240	85	55 to 60	31	14,500
5 ft. 0 in × 22 ft.	117	350	350	120	75 to 80	28	20,000
5 ft. 6 in. × 22 ft.	157	470	480	150	95 to 100	26	24,000
6 ft. 0 in × 22 ft.	192	575	575	185	115 to 125	24	29,000
7 ft. 0 in. × 22 ft.	280	840	815	250	160 to 175	20	39,000

The above figures are based on the charge and material level being carried at the center line of the mill.

1. The plain tube mill using flint pebbles as a grinding medium.
2. The plain tube mill with ball-peb compartment. This type of mill has a partition located near the discharge end. The large compartment on the feed end is charged with flint pebbles, while the small compartment on the discharge end is charged with cast iron or forged steel balls.
3. The ball-peb mill using cast iron or forged steel balls as a grinding medium.

4. The combination or compeb mill with a partition located near the feed end. The small compartment of the feed end is charged with large cast iron or forged steel balls while the large compartment on the discharge end is charged with small cast iron or forged steel balls.

The power requirements of these different types of mills vary not only with the size of the mill, but also with the nature and amount of the charge used. Once the mill is started, the load is practically constant and only slight pulsations in power can be noticed. The power required to drive these mills has been determined from actual tests and refers to dry grinding conditions only. For wet grinding the power required is approximately 30 per cent less than for dry grinding, other conditions remaining the same. The power requirements for the four types of mills, as noted above, are as follows:

1. The plain tube mill using flint pebbles—see Table 8.
2. The plain tube mill with ball-peb compartment; add 20 per cent to the power requirements of the plain tube mill using flint pebbles.
3. The ball-peb mill; see Table 9.
4. The combination or compeb mill; see Table 9.

The most economical speed for any mill should be such that the bulk of the charge is carried to the highest possible point in the mill before falling and this without any of the charge or material being carried around by centrifugal force. This is approximately

$$s = \frac{195}{\sqrt{d}}$$

where s = revolutions per minute.
d = diameter of mill in inches, including the lining.

On this basis of speed and for the dry grinding of limestone and clinker, the power required to drive any of these various types of mills may be approximated by the following:

$$P = \left\{ [0.0034 \ (W + W_m)] \frac{R}{r} + 0.00015\, w \right\} \times \frac{S}{s}$$

where P = Horse power.
W = Weight of charge of pebbles or balls in pounds.
W_m = Weight of the material being ground in mill.
w = Weight of revolving elements of the mill in pounds.

TABLE 9.

BALL-PEB AND COMPEB MILLS.

Operating in Portland Cement Plants with 20 mesh feed and grinding to 95 per per cent through 100 mesh and 80 per cent through 200 mesh.

Dia.	r. p. m.	Periph speed	Length of Mill in feet																	
			5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
			Horse power to run mill																	
5 ft.	24	362	44	50	58	66	74	82	90	98	106	115	123	131	139	147	155	164	172	180
	25	378	46	52	60	68	77	85	94	102	110	119	127	136	144	153	161	170	178	187
	26	392	48	54	63	71	80	89	98	106	115	124	133	142	151	160	168	177	186	195
	27	407	50	56	65	74	83	92	101	110	119	128	137	147	156	165	174	183	192	202
	28	422	52	59	67	77	86	96	105	115	124	134	143	153	162	172	181	191	200	210
5½ ft.	22	368	54	61	71	80	90	100	110	120	130	140	150	160	170	180	190	200	210	220
	23	386	56	64	74	84	94	105	115	125	135	146	156	167	177	187	198	209	219	230
	24	402	58	67	77	88	98	109	120	131	142	153	164	175	185	196	207	218	229	240
	25	418	60	69	80	91	102	114	125	136	147	159	170	182	193	204	215	227	238	250
	26	436	62	72	83	95	107	118	130	142	153	165	177	189	201	213	224	236	248	260
6 ft.	20	367	63	73	84	96	108	119	132	144	155	167	179	191	203	215	227	239	251	263
	21	385	66	76	88	101	113	125	138	150	163	176	188	201	213	226	238	251	263	276
	22	403	69	79	92	105	118	131	145	157	170	184	197	210	223	236	249	262	275	289
	23	422	72	83	96	110	124	138	152	165	178	192	206	220	234	248	261	275	289	303
	24	440	75	86	100	115	129	143	158	172	186	201	215	230	244	258	272	287	301	316
7 ft.	17	366	81	97	113	129	145	161	178	193	209	226	242	258	274	290	306	323	339	355
	18	387	86	102	119	132	154	170	188	204	221	238	250	263	285	307	324	341	358	375
	19	408	91	108	126	144	162	180	198	216	234	252	270	288	306	324	342	360	378	396
	20	428	96	114	133	152	171	190	208	227	246	265	284	303	322	341	360	379	398	417
	21	452	101	119	139	159	179	199	218	238	258	278	298	318	338	358	378	399	418	438
8 ft.	15	369	103	129	150	172	193	205	236	257	278	300	321	343	364	386	407	429	450	472
	16	394	115	137	160	183	206	228	252	274	297	320	343	366	389	412	434	457	480	503
	17	418	122	146	170	195	219	243	268	292	316	340	364	389	413	438	462	487	511	535
	18	443	130	155	180	206	232	257	284	309	334	360	386	412	437	463	489	515	541	567

The above figures are based on the following:

5 ft. 0 in. dia. mill—charge and material level within 6 in. of center line of mill—1800 lb. initial charge per ft. length of mill
5 ft. 6 in. " " " " " " " 7 in " " " " " 2200 " " " " " " " "
6 0 " " " " " " " " 8 " " " " " " 2600 " " " " " " " "
7 0 " " " " " " " " 10 " " " " " " 3600 " " " " " " " "
8 0 " " " " " " " " 11 " " " " " " 4700 " " " " " " " "

$s =$ Theoretically correct speed in rev. per min.
$S =$ Actual speed of mill in rev. per min.
$R =$ Distance in feet from center of mill to center of gravity of mass $W + W_m$.
$r =$ Distance in feet from center of mill to center of gravity of mass $W + W_m$. When mass comes to center line of mill. $r = 0.0177\ d$.

When a metallic grinding medium is used $W_m = 0.304\ W$.

This is based on one cubic foot of charge and material weighing 314 pounds and consisting of 280 pounds of charge and 34 pounds of material. When a flint pebble medium is used $W_m = 0.121\ W$. This is based on one cubic foot of charge and material weighing 150 pounds and consisting of 115 pounds of charge and 35 pounds of material. In the above equation S should not vary more than 15 per cent above or below s.

For operating ball mills both squirrel-cage and wound-rotor type motors are used. The tendency is towards the larger sizes of mills with metallic grinding bodies thus requiring much larger motors than was common a few years ago. These mills are rather difficult to start since in order to get them in motion the charge has to be lifted, and in the case of large squirrel-cage motors this causes a heavy current demand at low power factor even when special high-torque motors are provided. Also in the larger motors the higher efficiency of the wound-rotor type is an item to be considered. Hence for the large mills wound-rotor motors are to be preferred.

The tendency is also towards lower speed motors geared to the mills in place of the higher speed belt driven type previously used. In case of geared drive a flexible coupling should be provided on the motor as there is always considerable vibration present. The low-speed motor has the disadvantage of higher cost and lower power factor, but the saving in space, belting, etc. offsets these disadvantages to a large extent.

There is a trend in some of the latest installations towards the use of synchronous motors for these mills especially where low-speed motors are used. In such applications it is necessary to provide a clutch so that the motor can be started light, but the well known advantages of the synchronous motor are such that it is probable their use for grinding in cement plants will become much more common in the future than it has been heretofore.

ROTARY KILNS

For supplying rotary kilns powdered fuel is required in most cases and the crushing plant for this uses the same class of crushing and grinding machinery as has already been described for handling limestone and cement.

The kiln itself is a long cylinder lined with fire brick, revolving on an inclined axis and fired from the lower end. The material is introduced at the upper end and works its way through the kiln in from one to two hours. A speed variation is necessary on the kiln. Therefore either a squirrel-cage motor with speed change gearing or a wound-rotor motor with resistance controller is required; the latter is preferred. The controller should be such as to permit 50 per cent speed reduction on full-load torque. Since the kiln radiates a large amount of heat it is preferable to place the motor below the floor or to install it in such manner as to protect it as far as possible from the heat radiation of the kiln. For the same reason motors selected for this work must be liberally rated and have low temperature rise.

TABLE 10

Power Requirements of Rotary Kilns.

Size of kiln	Rev. per min.	H. p.
6 × 60 ft.	1 to 3	5 to 7
7 × 80 "	1 to 2	8 to 12
7½ × 100 "	¾ to 1-½	12 to 18
8 × 120 "	½ to 1	15 to 20
9 × 150 "	⅜ to ¾	22 to 30
10 × 170 "	¼ to ½	30 to 40

Kilns vary in size from 6 ft. 6 in. diameter by 100 ft. length to 10 ft. 6 in. by 200 ft. and revolve at speeds ranging from 3 to ½ rev. per min. They require from 10 to 30 h. p. The load is very steady and the horse power requirements are nearly proportional to the speed of the kiln, to the square of the diameter, and to the length. Approximately 125 to 175 per cent starting torque is required.

Table 10 gives approximate power requirements of rotary kilns. The power requirement varies with the speed as shown in the last two columns and the rating of the motor, on account of the severe operating conditions, should be twice the lower value there given. In selecting the resistance for controlling the speed, care must be taken to have it proportioned so as to be capable of reducing the speed to one-half when the load is that actually required to drive the kiln at half speed.

After the material has passed through the kilns it is ground in the finishing mills. For this work the same class of grinding and conveying machinery is employed as that previously described.

ACKNOWLEDGMENTS

This paper has been prepared with the assistance of Mr. A. M. Dudley, Mr. C. A. Kelsey and Mr. H. Weichsel, members of the Committee on Industrial and Domestic Power. Valuable assistance has also been given by Mr. R. C. Newhouse and Mr. Fraser Jeffrey, to whom acknowledgments are due.

DISCUSSION ON "THE APPLICATION OF ELECTRIC DRIVE TO THE CEMENT INDUSTRY" (WILLIAMSON, KELSEY, DUDLEY, AND WEICHSEL). NEW YORK, N. Y., NOVEMBER 8, 1918.

A. M. Dudley: We are really presenting to you two things to-night, and as regards the second we would like to have you feel that you are present at the birth of an idea, if you feel that there is an idea involved.

As regards the paper itself, two years ago Capt. Martindale was appointed Chairman of the Committee on Industrial and Domestic Power. Capt. Martindale, with others on the Committee, felt that the technical committees of the Institute were not entirely discharging their duty.

This Committee had a feeling that the technical committees of the Institute should do active and constructive work every year of their existence, and that succeeding committees should build on the work of the committees that had gone before, so that the knowledge thus accumulated, built up from year to year, would be available to the membership of the Institute at large and become almost a compendium of the progress of the art in that particular line. That idea, if carried out, would not be difficult of application, and it would provide the committees with a working program, which is a very great thing, it having been the case in most instances that the program of the work of the committees depended on the personality of the man in charge of the committee.

With that in view Capt. Martindale drew up a schedule, starting with certain machine tools to which he proposed to apply the proper motor for operating, both alternating current and direct current, and giving its characteristics, its methods of control, its capacity, and all other pertinent facts.

After some discussion the Committee decided such a plan was not sufficiently comprehensive and it was decided to study the entire subject, first from the viewpoint of the industry. It will occur to you that to the man who has made a study of this, to the man who is educated to the application of industrial motors—that the electrical study should be from the standpoint of the characteristics, particularly the speed-torque characteristics of the motor, because the number of different speed-torque characteristics is limited, and these speed-torque characteristics must be applied to all work.

That is true of the specialist, but to the man who is interested in the industry and who is operating the motors, the natural view point is from the standpoint of the industry, so no matter how it is studied, to become available to the membership at large, it must be primarily presented through the industry.

With that in view the committee selected three industries, having in mind one industry of very wide scope, a second industry of somewhat more limited scope, and a third industry, which was very highly specialized. With that classification in view, the machine tool industry was selected as being a very widespread industry, with many ramifications and touching a great many other industries; the manufacture of cement was selected as a more limited industry and passenger elevators as a highly specialized industry.

That brought the work up to the end of the first year. The next year, last year, the entire committee was reappointed because it had shown that it had an idea. We said we had not had time to carry it out. The Board of Directors said—"All right, you can have another year to show what you can do," and the entire committee was reappointed. Unfortunately, for the work of the committee, Capt. Martindale went into the service and we did not do much work last year. But by working along we came through with a paper and it was presented to you to-night.

We would like you to think of this not as a finished presentation. Of necessity all papers prepared by the designing or application engineer, rather than the operating man, have a sort of academic flavor, that is, they look as though the man who is actually up against the job had not prepared the paper, and for that reason perhaps in some details the paper is lacking or not as full as it should be, and it is the hope of the committee that you will criticize the paper fully and freely, and discuss it, and make additions to it, or subtractions from it. The results will be put in form to be available to the entire membership as virtually a study of this industry up to this time, and succeeding committees would find it part of their duty to see that the data are kept up to date, because the industries change rapidly, and there have been great changes in the cement industry in the last five years.

We would like you to consider whether the committee has chosen wisely in making this special study, and whether the idea as a whole is a good one. We feel that eventually this work will be developed to such an extent that, through the PROCEEDINGS or otherwise, the membership at large will be interested in the collection of the data, and as the Committee intends to make a study of certain industries each year, it is hoped that the membership will respond freely and cooperate, and that all who are engaged in the particular industry treated will prepare such information, or furnish such notes as they may have, and send them in to the man compiling the particular data.

As this work grows it may assume the shape of a loose-leaf pamphlet, or a card-index, or something of that nature, so that when you are discussing the paper to-night, we wish it to be discussed, just as the presentation is, as a treatise on the use of electric motors in the manufacture of cement, and we would also like in addition, to have some discussion on the work which the Industrial Power Committee hopes to do, and wishes to do, whether it will be of a constructive nature and valuable to the Institute at large, and whether the Institute can be of service in this particular manner.

Arthur Simon (read by W. C. Kalb): Mr. Williamson's paper covers very thoroughly one part of the work which the Industrial and Domestic Power Committee endeavored to carry out during the past few years, and it is a clear indication how tremendous the amount of work must be if it is intended to cover the application of electric motors in all industries in a like manner.

The writer has thought for a long time of ways and means by which the subject could be most satisfactorily and expeditiously handled and the thought occurred to him that the different industries might be treated in the form of monographs, each one covering a particular industry, or even only a section of that industry. Mr. Williamson's paper could well be made the first one of these monographs as it covers the particular subject under discussion in a very thorough manner.

My reason for suggesting monographs is that each one could be edited by an engineer who is thoroughly familiar with the industry in question and who could get the cooperation of other men equally well qualified. In this manner it would be possible to distribute the work over a larger number of contributions and by publishing the different sections in book form and separately, a market would be created for them outside of the membership of the Institute, sufficient to cover the necessary expenses, which I realize would be rather high.

On the other hand, if the material is gathered and published in the manner indicated, it would be possible to very liberally illustrate the different booklets. Photographs and cuts could be procured from the various manufacturers interested in the particular lines under discussion. Liberal illustrations would be desirable because they would enable the specialist, who has for instance only had experience with the design of electric motors and not with their application, to familiarize himself more with the installation and operating conditions. This as I understand was one of the primary reasons for the work, undertaken originally by the Industrial and Domestic Power Committee.

The idea of publishing monographs to cover the entire field in the manner proposed is, of course, not new. It has been and is being done in other lines of science and engineering, I call attention to certain lines of publications in physics and electrical and mechanical engineering abroad.

Mr. Williamson entitles his paper "Electric Motors in the Cement Industry." I believe that it would have been desirable to change this title to "Electric Motor Applications" and include in the discussion the various forms of controlling devices which are most suitable for the various motor drives.

The tendency in the past in the cement industry has been to put too much emphasis on the desirability of the squirrel-cage motor on account of its simplicity and ruggedness. I am glad to note that Mr. Williamson considers the slip-ring motors, particularly in the larger sizes, equally, or more satisfactory than the squirrel-cage motor. If the control is also taken into consideration, it will probably be found that the slip-ring motor is the preferable type in the majority of cases.

The tendency in the cement as well as in other industries today is to use automatic controlling devices on account of the lack of skilled labor which can be introduced with the operation of hand controllers, and it is no doubt true that the controllers for slip-ring motors in general give better all-around results than squirrel-cage motors in combination with controllers for their operation. Furthermore, by the use of the slip-ring motor the starting currents can be greatly reduced and the risk of burning out the motor due to its being stalled at starting, is greatly reduced.

The writer realizes that this is at the present time to a large extent a matter of opinion and it would be desirable to get the benefit of the experience of operating men who had to deal with the two types of motors and also to get some recording instrument curves, showing the starting conditions for squirrel-cage and slip-ring type motors on various drives.

W. C. Kalb: Mr. Williamson, in the early part of his paper, points out that the tendency in the cement industry has been to over-motor rather than to under-motor the drive on the most of this equipment.

It seems to me in some cases while that may mean that the motor is operating at a lower degree of efficiency than where it is working up to its full rated load, there is still an economical advantage, particularly in types of machinery such as some of the crushers he mentions, where, if the machine is stalled, it is necessary to tear out all the charge before it can be started again. The saving over the loss of time and the labor required to take care of the stalling, with the consequent interruption of production, if a motor was working so near its capacity that such stalling occurred with any frequency, would rapidly pay for the installation of a motor of greater capacity.

On the question of brushes, which is a field in which I am particularly interested, we do not find such an extensive application of brushes in the cement industry as in many others, owing to the predominance of alternating current. However direct current, as has been stated in Mr. Williamson's paper, is used to some extent and with considerable success.

The application of brushes to this industry is very much the same as any other, with perhaps the exception that self-lubricated brushes, so called, or those which are impregnated with the lubricating material are not at all desirable in this industry. The presence of the abrasive material in the form of dust which can be accumulated by any lubricating material on the surface of the brush will cause an unnecessary amount of wear on commutators. Consequently this type of brush should be avoided in the cement industry or in any others where abrasive dust may be present.

As to under-cutting of commutators, in the presence of so much dust of this kind there is not the need for it in all cases that there is in other installations. We see frequent cases where a motor requires the under-cutting of the mica, provided a non-abrasive brush is used, because after a considerable period of operation, perhaps months, the mica will gradually get high. Cases of that kind would not require under-cutting in a cement mill, the abrasion which occurs from the dust present would be sufficient to permit operation with flush mica, and non-abrasive brushes. There will be conditions, however, where undercutting would be advisable, unless abrasive brushes are used. In such cases the undercutting should not be deep—1/32 in. as a maximum will give successful operation, and it will not be deep enough to afford a pocket for the collection of dust, so that it will offer any serious trouble, if the machine is blown out with any reasonable frequency.

Probably more brushes are used on the slip-ring types of motors in cement mills than are used altogether on direct-current motors, due to the predominance of the alternating-current motor.

I would like to put in a word here in urging the greater use of the carbon and graphite brushes as against the metal-graphite composition brush. This, of course, means cutting down the current density in the brush to come within the range of the carbon-graphite brush, but that is not a serious problem as there is always plenty of room for the accommodation of holders.

An objection frequently offered by designing engineers to the use of the carbon brush on the slip-ring type of motor is that it increases the slippage. Again, for this industry, I do not think that would be a serious objection.

My reason for urging the advantage of the carbon brush, however is the tendency for all composition brushes, once abrasion has started, to of themselves continue that abrasion. Once break up the polish, the smooth surface of the brush face and the ring, and it is very slow in forming again. This rough contact affords considerable abrasion, causing short brush life and rapid ring wear. The carbon brush is not open to this objection.

In regard to the types of carbon brush having the necessarily high carrying capacity, there are practically but two from which to choose—these are the all-graphite, which is made as a molded brush, from pure natural or artificial graphite, and the electro-graphitic brush. I have a personal preference for the electro-graphitic brush, due to its greater hardness, greater mechanical strength and ability to stand up under severe mechanical conditions. It can be operated up to a current density of 60 amperes per square inch without difficulty, and where conditions are right a considerably higher current density is permissible, so it will take care of any condition to which a graphite brush can be applied.

Linn Bradley: The portion of cement plant equipment in which I have been interested is the electrical apparatus employed for the collection of dust from the gases that come from the rotary cement kilns. This phase of the subject under discussion may be of interest to members of the Institute, although it is remotely related to the other phases which have been so thoroughly discussed this evening.

The electric circuit that I propose to discuss, consists in part of a source of alternating current, a mechanical rectifier (synchronously driven) which converts the alternating current into a pulsating direct current, and a load which operates as a leaky condenser. This load is what causes so much concern to the operator. It behaves erratically at times and its peculiar operating characteristics are such that the circuit has been aptly termed a "wild indian."

The problems encountered are numerous and difficult and the field is sufficiently developed in cement plants to warrant engineers familiarizing themselves with the technique involved. First, let me say that, instead of following the usual practise, by trying to avoid the formation of arcs, the electrical precipitation engineer is dealing with arcs in the high-tension circuit (at rectifier) as an integral part thereof. It is unnecessary to state that this introduces a number of interesting, even if vexatious problems.

Low-tension alternating current of 25 to 60 cycles, single-phase, is raised to 30,000 or 100,000 volts by means of a static transformer, and this is converted into pulsating direct current by means of the synchronously driven mechanical rectifier previously referred to. This rectifier in its simpler form comprises a disk, about two feet in diameter, made from a good insulating material, such as Micarta, fibre, Dielecto, etc. mounted upon a shaft driven by a small synchronous motor or by a direct connection to the shaft of the low-tension alternator delivering the low-tension alternating current to the step-up transformer. It is thus apparent that the synchronous motor is merely a convenient substitute for the mechanical coupling when the alternator is situated at some distance from the rectifier, *i. e.*, when the alternator is in a central power house and is used for supplying current to other pieces of apparauts. It is desirable to keep this point clearly in mind, that the two methods of driving the rectifier are almost identical, so far as the electric circuit in question is concerned.

Near the periphery of the revolving disk, but separated from it by a small air gap, four shoes or elongated conductors are placed, one for each quadrant. (See Fig. 1) The disk carries four "contactors," these so-called contacts being made through the air gaps by means of arcs. These revolving points might preferably be called "arcing-points" or "arcing-contactors." The general arrangement of the disk and shoes and a wiring diagram are also indicated in Fig. 2. It is not feasible at these high peripheral speeds, to have the shoes and contactors in actual contact,—we wish it were. Two wires connect the transformer high-tension terminals to two shoes spaced apart horizontally, one wire connects the bottom shoe to the ground and another connects the upper shoe to the "discharge electrodes" of the precipitator or leaky condenser. In the sketch the "discharge electrode" is represented as an iron wire (No. 14) placed along the axis of a "collecting electrode", in this case a round iron pipe about 12 in. (30.4 cm.) diameter by 15 ft. long (4.5 m.) These two types of electrodes are separated by suitable insulators, the current flowing from one to the other through the dust-laden gases. There is a corona discharge taking place continuously when the apparatus is in operation. The corona is readily distinguished as negative or positive by visual observation. In practise the negative is used as it permits a higher potential with better operating results. The positive is more brilliant but it is much less effective for precipitation purposes.

The general character of the potential difference between the discharge and collecting electrodes, on the precipitator, or direct-current side of the rectifier, is indicated in Fig. 3. Fig. 4 shows the general character of the potential difference between the two high-tension terminals of the transformer on the a-c. side of the rectifier. *A* represents good conditions and *B* showing poor conditions. An examination of these sketches will point out some of the problems which must be kept in mind in designing, as well as in operating apparatus in conjunction with this process. It is evident that the potential difference does not fall to zero, between the precipitator electrodes, unless a spark or an arc occurs, in the interval between the so-called contact periods at the rectifier, owing to the fact that the rate of leakage decreases rapidly with a reduction in potential, and when the critical corona potential is passed the rate of leakage is too small to permit the condenser to become discharged in the short time which elapses before the transformer is again feeding electric energy into the precipitator (*i. e.*, the leaky condenser). This interval obviously is less with 60-cycle power than with 25-cycle. When the "contact" is renewed, current flows into the precipitator so long as the potential between the transformer terminals is greater than the potential between the electrodes of the precipi-

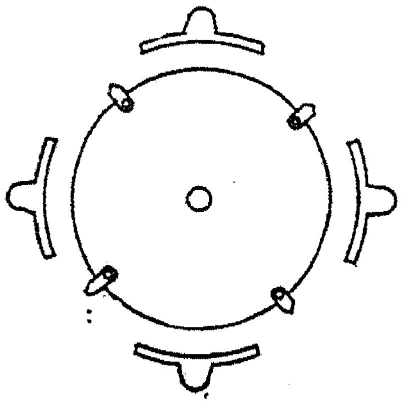

FIG. 1

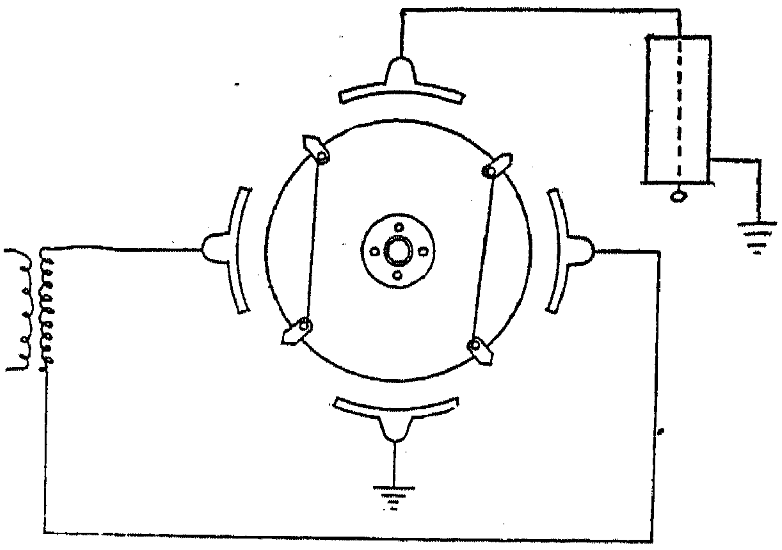

FIG. 2

tator, allowance being made for the resistance of the circuit including the arcs at the rectifier. When the transformer potential decreases to a point just below that of the condenser or precipitator, the contacting period at the rectifier should cease. However, the current and potential are not in phase, and this adds some complications. It would be ideal to end the contact period when the potential difference of the transformer terminals and the precipitator electrodes were equal and the current flow had become zero. Since the current and potential are not in phase, this condition is not obtainable in present practise.

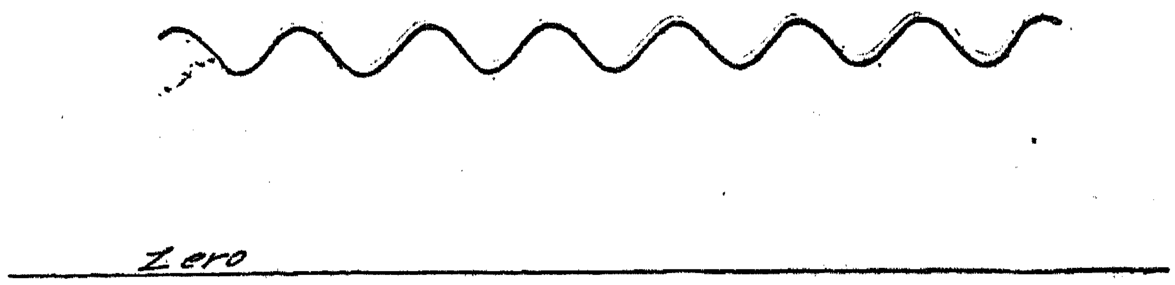

Fig. 3—Voltage Wave (Fluctuation Approximately 20 per cent of Maximum

Current flows into the precipitator for two distinct purposes: First, to charge the condenser and raise its potential; second, to cause ionization of the gas between the electrodes and an electric current to flow between them. These two elements of the current may be divided into one which is purely a capacity current and the other which is purely a load current, the one being out of phase with the potential and the other being in phase with the potential which produces it. These vary relatively when the potential and other conditions change. Until sufficient electric energy has been delivered to the precipitator to raise the condenser to a point where the potential difference between the electrodes is sufficient to produce a corona discharge, the current is almost entirely a capacity current (resistance of the circuit is small). When the critical corona potential has been passed, current is required to increase the capacity charge and raise the potential between the electrodes and also to provide the current which flows through the gas between the electrodes. Since the rate of current flow due to the corona discharge, increases as the square of the increase in potential difference, there will be a change in the ratio of capacity current to load current, and this change of ratio will be taking place throughout the entire period of current flow from the transformer. However, the change in

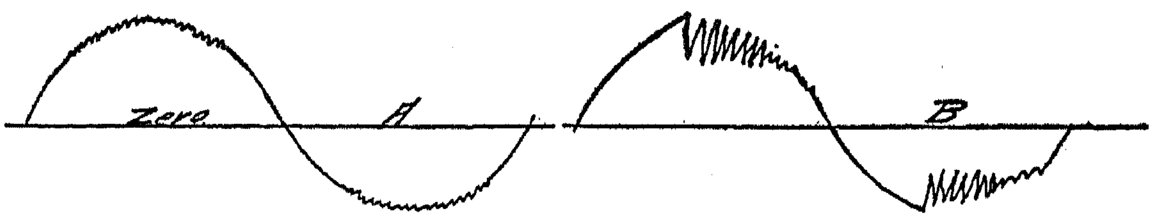

Fig. 4

ratio will not be affected by increasing the length of the pipe, or what is equivalent by adding more pipes of the same kind, provided the wave form, etc. at the transformer remain fixed. The only point I desire to bring out right here is that the amount of lead of the precipitator current over the potential will be practically the same for a precipitator of a few pipes as for one of several pipes. Oscillograms show very plainly the leading characteristic. Oscillograms also show that when a precipitator is operated at near the maximum potential, without spark over, the voltage between the electrodes varies so that the minimum potential is from 70 per cent to 80 per cent of the maximum. When the precipitator is operated at a lower potential and the current flow is small, the variation is even less than the above, in some cases which have been observed. However, it would seem that if the capacity of the precipitator was very small and the alternator and transformer relatively very large, the potential difference between the electrodes of the precipitator would vary with the transformer potential during the time the rectifier was in contact, and that therefore the degree of fluctuation would depend directly upon the number of degrees of the wave which was being rectified. On the other hand, if the alternator and transformer were of small capacity and the precipitator was of relatively large capacity or capacitance, there is a possibility that enough energy might not be delivered to the precipitator during contact periods to materially raise the potential. It has been observed that when the precipitator was large, taking a relatively large load current, and the alternator and transformer were small, the degree of fluctuation of the precipitator potential was quite small, the extent depending considerably upon the number of degrees of the wave being rectified. This condition produces a distortion of the wave form, and, when a heavy resistance is used in the primary or low-tension circuit of the transformer, the potential wave on the high-tension side of the transformer is flattened to a considerable degree. This flattening is greater than would be the case if the precipitator load were a pure resistance load, this difference being due to the character of the corona load.

In practise it is usual to have the rectifier contact period cover about 90 to 120 electrical degrees, the lower portions of the wave being of no value in charging the condenser or providing for the corona load. The potential maintained between the precipitator electrodes does not fall to zero, whereas the potential on the transformer passes through zero during each cycle. The current into the precipitator, on the d-c. side of the rectifier, however, does start at zero, rises to a maximum and again goes to zero. The shape of this current wave depends upon the extent to which the condenser is charged, the amount of load current, the ratio of actual potential to possible maximum potential, the ratio of capacity current to load current, the number of degrees rectified, the portion of the wave used, and to other but more minor factors.

If an arc occurs in the precipitator, the potential between the electrodes falls nearly to zero, and during the next rectifier contact the current flow is heavier than normal and the precipitator becomes charged nearly to its normal, and is again normal in one or two contacting periods. With a single-phase a-c. source, such as described, the arc is broken between contacting periods. With a source of true direct current, the arc continues until broken by special means or until the arc extinguishes itself by becoming elongated to the breaking point. Three-phase alternating current acts in a manner similar to true direct current in this connection. For this reason, single-phase alternating current as a source of the pulsating direct current possesses some practical advantages over non-pulsating direct current.

A study of the arcs which occur at the contacting-points of the rectifier reveals some interesting phenomena. With a rectifier built as described, the four revolving arms and points act as a fan which produces heavy windage in the space between the shoes and the revolving points. This windage tends to blow out the arc and thus produces a very unsteady condition. The more unsteady the conditions the more pronounced are the surges. It has been found that a solid disk produces less windage than a disk built up from arms which project out from the hub, and that smoother operation is obtained. Better results would be obtained if actual contact were possible. At the shoes an arc is established as soon as the point approaches the shoe, and then this arc is drawn out until the resistance is so great that the arc breaks and another arc forms. It is possible that the new arc forms before the old one is broken, but this is not easily proven. Visually these arcs appear to overlap a fraction of an inch. Generally three distinct arcs are observable with the solid disk, each arc being increased in length and decreased in cross section as it is drawn out, indicating an increase in resistance and a decrease in rate of current flow. This condition readily promotes surging. It is difficult, indeed, to get oscillograms which show clearly all of these phenomena, but the phenomena are clearly shown in these arcs.

Furthermore, one can obtain a good view of the so-called static sparks which are shown as little white streamers within the body of the arcs. These are quite close together, thus indicating in some measure, their frequency. Considerable resistance in the high-tension circuit partially clears the arc and eliminates some of these streamers.

It is believed that these white streamers are an indication of the source of the trouble experienced with transformer coils. The end coils of the transformers readily fail unless they are provided with excessive insulation and are otherwise protected against excessive potential strains. The amount of potential difference which can be set up between adjacent turns of a coil depends upon the rate of change, and since the rate of change is very high with these oscillations, a very high potential gradient is readily established when the coil is placed around an iron core. The current flowing in the high-tension circuit consists of the regular charging and load current wave, upon which are superimposed these numerous oscillating current waves. By placing a choke coil on the terminal of each transformer, the load and charging current readily pass through the choke coil while the oscillations are reflected thereby, due to their much higher frequency. They are caused to pass through resistance and thus dissipate themselves in heat. In this way the transformer coils are protected against break-down. A further step is to take advantage of the difference in rate of change, or steepness of wave front, allowing the load and charging current to flow through the choke coil and compelling the oscillations, having higher frequency or steeper wave front, to shunt the coil and pass through a very high but relatively non-inductive resistance, such as a carborundum resistance rod. By utilizing these methods, transformers now operate satisfactorily without frequent break-down, and smooth and fairly steady operation is obtained in this special field of application.

Recently a considerable amount of discussion has taken place and some articles have appeared in the literature relative to the virtues of synchronous motor drive as compared with connecting the mechanical rectifier direct to the shaft of a motor-generator set. From a study of the above, it seems evident that there is no fundamental difference in the two methods, as the synchronous motor merely acts as a substitute for the flexible mechanical coupling used in the other case. The hunting of the synchronous motor or failure to keep in step are no problems in practise, when the power line has satisfactory characteristics. When the line is unsuitable, it is obvious that a motor-generator set is preferable. The other troubles or problems discussed above are much more important and fundamental and point the path for those responsible for designing the electrical equipment, and should be of considerable assistance to those who are charged with the installation and operation of this type of equipment. Designers should remember that the load is due to a corona discharge and that for a given increase in potential the current resulting is greater than if the load were an ordinary resistance with no temperature coefficient. Again, the rectifier is in contact only during a portion of the wave, the current flow taking place during this interval which may be relatively very short. It is not safe to rely entirely upon meter readings placed in the low-tension circuit, as they fail to record the rate of change or rate of current flow. The actual voltage on the high-tension terminals of the transformer can not be correctly read from the low-tension voltmeter reading and the transformer ratio. The error is the greater when the number of degrees rectified is the smaller. For instance, if there was only a point to point contact at the rectifier, the low-tension voltmeter might read say 200, and one might assume that the actual potential delivered to the precipitator electrodes was (200 : 100,000 volts transformer ratio), whereas it might be that during the interval the rectifier was in contact, the transformer potential may have been nearly zero due to the very high rate of current flow during the short interval, which would be the case if there was an arc-over in the precipitator. The peculiar power factor of this circuit may be understood by a consideration of this point. Another point of interest is the flattening effect upon the low-tension and high-tension waves of the transformers when a high non-inductive resistance is placed in the low-tension side of the transformer. This flattening effect is very pronounced when a small alternator is used and the corona current is quite large, and exceeds that which would result if an ordinary resistance load were used. As the voltage increases, the current flow increases rapidly, produces a greater potential drop across the resistance and this in turn makes the potential across the low-tension terminals of the transformer that much lower. The larger the resistance the greater the flattening effect upon the wave shape. The rate of current flow during the time the rectifier is in contact should determine the size of the alternator. The maximum rate of current flow from an alternator for this service is greater than for a lighting load where the entire wave is used and the load is a resistance load. Small alternators should be designed with this in mind. Poor wave shapes from small alternators are due in part to this feature. On the other hand, if a large alternator is used, and a small amount of current is taken from it, the effect upon the wave shape is going to be negligible as compared to the same amount of current taken from a small alternator. This probably accounts for the reason why the synchronous motor operation, taking the power for the precipitator from a large alternator, gives better operating conditions and better wave shape, and enables one to operate with higher potential on the precipitator, than when an undersized or improperly designed small alternator is used. Of course, a power line must not be used if it is subject to excessive voltage variations, since good voltage regulation or control must be had for the precipitator circuit.

One of the principal uses of this process in a cement plant is to recover potash as a by-product from the gases coming from the rotary kilns. Considerable progress is being made in this field, and it is quite probable that the process will be universally adopted in cement plants in time. Therefore engineers should consider some of these special uses and applications and become familiar with the technique and the problems arising from such operation. With this process in cement plants, a considerable tonnage of potash can be recovered as a by-product while cleaning the gases and overcoming a nuisance in the plant and immediate neighborhood, and it is of special interest, that this potash can be recovered at a cost which will permit of competition with the potash which can be supplied by Germany. The experts of the U. S. Bureau of Soils and of the U. S. Bureau of Mines estimate that the amount of potash which can thus be recovered from cement plants per year will be from 50,000 to 100,000 tons.

W. L. Merrill: It has been my experience that in the application of electric drive to various industries that there is a pronounced tendency to complicate, rather than simplify control apparatus. Generally speaking, control apparatus for industrial motors should be as simple as possible to accomplish the desired results and of course it is an obvious fact that the more complicated the duty cycle of the motor is, the more complicated will be the control. We have at the one extreme, the reversing mill motor controls and the reversing planer controls, and at the other extreme we have the squirrel-cage motor on all day service and a simple line switch, or a simple type of starting compensator. Between those two extremes of magnetic control and simple switch control we have the various types of drum, automatic and semi-automatic, and magnetic push button, etc., etc.

As stated in the paper, the cement mill load is long and continuous and fairly uniform, with exceptions of one or two types of machines. Therefore, following the above reasoning it is evident that the control for cement mills should be very simple and most of the successful installations which have been made have followed this policy. This reduces itself to a simple rugged drum-type of controller for collector ring motors, either with primary contacts and some auxiliary overload protection, or simply secondary contacts and primary oil switch with overload and possibly low voltage protection. For the squirrel-cage type of motors, the simple type of hand operated compensator is all that is required and there is at least one successful installation with motors running up to 250 h.p. in which the only control installed is a knife switch; motors being thrown directly on the line.

We must consider the class of operators with which we have to deal in the cement mill and in so doing remember that ruggedness and simple control will give the best results. There have been installations in which very complicated systems of control have been established in order to bring the control of the various motors situated throughout the different parts of the plant to one central station. It is needless to record that the advantages given by this (if there are any) are very much outweighed by first cost, added equipment, many additional lines and circuits not only to keep in repair, but to add to trouble.

There is always a happy medium between simplicity and complication. Many engineers apparently foresee supposed conditions arising which make it necessary to install protective features and the like, so that if the expected happens much danger is not the result. This is often carried to extremes so that the additional protective apparatus and apparatus for the convenience of operation out weighs the original fundamental equipment. The electricians and trouble men devote more of their time to keeping these auxiliaries in working condition than is necessary for the fundamental equipments.

In many industries conditions will warrant magnetic control instead of drum control and of course there is no particular reason why the master controller and the magnetic control panel would not start a collector ring type of motor on a rockcrusher, but it is perfectly possible to do this with a drum controller, and a magnetic control under the atmospheric conditions of a cement mill should be so thoroughly boxed in and protected that the expense of the equipment is not only high but that any claim of advantage over the drum type would be more than outweighed by the complication and cost and probably maintenance, since remote control of cement mill machinery is in no way essential, as the man may as well start and stop his motor by drum controller as by pushing a button.

In conclusion, I wish again to call attention to the advisability of using the simplest and strongest types of control for use in the cement mill industry.

H. D. James: I wish to add a memorandum to Mr. Williamson's paper suggesting suitable control apparatus for use with the principal motors recommended, which are the squirrel-cage induction motor and the wound-secondary induction motor. I have had no experience in the application of control to direct-current motors or synchronous motors for use in cement mills, and must leave comments on this to someone else.

General Features. All control equipment, with a very few exceptions, is for constant speed non-reversing motors. Most of these motors are of the squirrel-cage induction type, except a few applications requiring speed adjustment, or where the wound secondary induction motor is necessary in order to develop the required torque at starting with a reasonable demand on the power circuit. In many cases, the standard squirrel-cage induction motor does not have sufficient starting torque, so that special motors must be designed and the general purpose motor cannot be used, even when installed in a separate room and protected from dust. The use of a transformer type starter for these motors is entirely feasible if the motor will develop the requisite torque at 80 per cent of line voltage or less. Where 90 per cent of line voltage is required to start the load, there is practically no saving in using a current limiting device, and the motor should be thrown directly on the line. This represents very severe starting conditions and may require special bracing of the motor windings to withstand the heavy starting current. The switching mechanism in such cases must be very strong and capable of handling the locked current of the motor without undue wear.

Control apparatus for the wound secondary motor need not differ from standard if it is installed in a separate room and protected from dust.

The atmosphere in these plants is very dusty and some precaution must be taken to protect the electrical apparatus from cement dust and coal dust where powdered coal is used as fuel.

This protection can either take the form of a cabinet or dust-proof enclosure around each piece of control apparatus, or the motors and controllers may be located in an adjacent room from which the dust is excluded. This latter arrangement is preferable as standard equipment can frequently be used and the maintenance charges will be reduced.

All wiring should be in conduit and all controller frames or enclosures grounded. Special care should be taken to ground the framework of push-button stations, master switches, or manual controllers, particularly in the presence of coal dust, in order to protect the operator from accidental shock.

The National Electric Safety Code requires all live parts of control apparatus to be protected from contact by unauthorized persons. This requirement can be met by one of the following arrangements:

(a) Enclosing cases.

(b) Mounting control equipment at least eight feet above the floor.

(c) Placing the control equipment in a separate room not accessible to an unauthorized person.

Usually the first or third arrangement is followed in cement mills.

Mr. Williamson has brought out very clearly the severe starting conditions required for many of the machines. This should not be overlooked in specifying control equipment. Standard general purpose controllers have been classified by manufacturers with reference to the time of starting as "light" and "heavy" duty. The light duty starting should not exceed 15 seconds once every four minutes. Heavy starting should not exceed a 30-second period every four minutes. The heavy starting class will be required for many of the machines in cement mills.

The following features should be embodied in control equipment:

Manual Starters for Squirrel-Cage Induction Motors. Starters for all except the smaller motors should be provided with some form of current limiting device. The preferable form of current limiting device for the larger motors is of the transformer type, known to the trade as auto-starters or compensators. These starters should be equipped with time-element overload relays affected by two legs of the circuit and by low-voltage protection. The handle should be arranged so that it must be held in the starting position. A latch should prevent the accidental throwing of the handle into the running position before it is first thrown in the starting position. Provision should be made for locking the handle in the off position to prevent accidental starting of the motor if work is being done on the machinery. Where the switching member is oil immersed, it is particularly necessary to see that dust is excluded from the case.

Manual Starters and Controllers for Wound-Secondary Motors. These controllers are usually of the drum type, as this design is enclosed and usually very compact. All manual starters and controllers should be provided with a protective panel equipped with a time-limit overload relay, affected by two legs of the circuit and low-voltage protection. The panel should have a magnetic contactor opening at least two legs of the circuit. In addition, a disconnecting switch with a safety lock should be provided. This disconnecting switch may be mounted on the protective panel or separately, depending upon the requirements.

Automatic Starters and Controllers for Wound Secondary Motors. Automatic controllers are preferable for the large size motors, which are generally of the wound-secondary type. These controllers provide automatic acceleration and should be equipped with time-element over-load protection affected by two legs of the circuit, and should be connected for low-voltage protection. A disconnecting switch with a safety lock should be provided. Usually this switch is mounted on the panel with the contactors. Where speed regulation is not required, the master switch can consist of a start and stop push button. Several of these push-button stations may be located around a machine when necessary. Where speed control is required, a drum type of master switch should be used. The starting resistor for an automatic panel should give approximately 150 per cent of full-load current on the first notch. Where speed regulation is required, a resistor of continuous capacity should be used and the starting current may be reduced to 70 per cent of full-load current if the application requires this.

I heartily endorse Mr. Merrill's statement that the control should be as simple as possible. If the motor and line conditions will permit the squirrel-cage motor to be used and thrown directly on the line, that is the preferable arrangement, but the engineer laying out the plant will have to determine the type of motor his line and load will require.

After you determine the motor, the question remains, if it is the slip-ring type, whether you shall use remote control with the larger motors. The principal advantage in remote control is the question of safety. We are giving more and more attention to the question of safety, and it is possible to remove the main switching equipment from the neighborhood of the operator and leave only a small power circuit of the push-button type, and this is in the direction of safety. It adds to the matter of complication, unquestionably, but that must be adjusted and balanced in each particular installation.

It is difficult to lay down a rule to cover every case, and my suggestions are to be taken merely as suggestions along the lines we are discussing.

Francis J. Burd (read by R. H. Goodwillie): I am impressed with the importance of Mr. Williamson's paper and the attempt which is made to put into tangible form, a record concerning the application of motors to the cement industry which is bound to be of mutual benefit to the electrical and cement manufacturers. It should tend to bring the two industries closer together.

It has been the writer's experience in selling electrical equipment that it is generally necessary to thoroughly understand the customer's problems as it relates to the electrical equipment involved before confidence is established to the degree of purchasing the equipment. I feel that papers such as presented by Mr. Williamson will accomplish considerable in bringing about more intimate relations between the electrical manufacturers and more especially the Institute as a whole to many other industries beside the cement industry.

I don't believe that it can be said, that the Institute has heretofore encouraged to any great extent measures to promote closer relations between the electrical and other industries. I believe the tendency has been to drift along with the problems presented by its own members.

Papers such as this one on the cement industry would certainly go a long way toward promoting the relations referred to and I would recommend that the Institute encourage and give publicity to papers of this sort. Such encouragement and support of the Institute as a whole would certainly be of material benefit to all concerned.

It would seem that every member of the Institute could do something in support of this kind of work in the spirit of supporting the Institute as a whole. If the members will only look upon the Institute as an institution of their own and give it their full support, I believe the Institute could easily fulfill its share of the work.

Not very long ago, the Institute members were asked on a questionnaire form to register their training, experience and vocations. It was understood that this data was for the Institute records. On the assumption that the membership responded satisfactorily, those records should form a good basis on which to segregate the membership according to industries.

With committees appointed by the proper authorities to collect data as it relates in an electrical way to certain industries, it would seem that members to obtain the desired information could easily be selected from the questionnaire records by the committee.

I believe this method offers to each member an excellent opportunity to assist the Institute in the manner in which he is best qualified.

I would not recommend that any special file be kept of the collected data, at Institute headquarters. I believe the proceedings would provide ample means for a permanent and continuous record. However, I would recommend that the Institute allow publicity of papers in such a manner that they would appear before the industry concerned.

Practically every industry in the country has one or more associated trade journals. These journals offer an excellent opportunity for the Institute to get before a given industry interesting articles concerning the application of electricity to their processes. It would seem that this is where the Institute could promote closer relations between the electrical and other industries.

E. Friedlaender (read by R. H. Goodwillie): The committee on "Industrial and Domestic Power" is deserving of a great deal of credit for the work it has done in compiling these data. I think the Institute should encourage in every way the presentation of such practical descriptive papers as this one. A great number of our members are practical operating men, who are greatly in need of this kind of data.

A great amount of descriptive matter is gotten out by the manufacturers in regard to the construction of their apparatus and machinery, but very little instruction is available for the customer, for the proper selection, use and installation of the apparatus.

Close co-operation between manufacturers, designers and users is absolutely necessary. The best motor and controller will fail to do the work satisfactorily if it has not been properly installed. Responsibility of the manufacturer of electrical apparatus does not end with the designing and furnishing of same, but should include the additional service of selecting and installing apparatus best suited for the work.

Committees on Power Stations, Transmission and Distribution, Industrial and Domestic Power, Iron and Steel Industry, etc., etc., should compile practical data and present them to the Institute meetings for discussion.

During these trying times, large production rather than new development has been the aim of the electrical industry, in order to help the government to prosecute the war. Wages have about reached their maximum, at the same time efficiency of man power is at its minimum. It is therefore only natural that in the near future great amounts of money will be expended to replace, wherever possible, man power by machinery, and skilled work will be performed by unskilled labor at lower cost with increased production.

The attitude of the public is very favorable towards the use of electric motors. The advantages of motor drives are well known, such as saving in floor space, convenient arrangement and spacing of machinery, safer and cleaner factories, better control and efficiency, greater overload capacity, ability to shut down each individual machine without affecting any other, etc.

Where the output depended on the endurance and physical strength of the operator, controlling machinery manually, the electric motor and automatic controller has greatly increased production. Unexperienced and ignorant operators manipulate large, ponderous machines without the least effort and cannot do any damage to same until control fails. The employment of women in place of men in the iron and steel industry has been possible in many places where machinery is electrically driven.

Naturally electrical installations of such nature are complicated and require experts to look after them. Neither motor nor controller will give sufficient warning of trouble until too late, and regular supervision and inspection is absolutely necessary, especially where continuous operation must be maintained.

In motor applications in general, a number of points must be carefully considered such as: Description of machine and drive; load conditions; location; speed control; voltage and kind of current.

Where a large amount of dust and abrasive material is liable to get in electrical machinery, as in steel mills and cement plants, bearings should be dustproof, all collectors and commutators should be protected, all controllers—especially relays—should be covered. Motors should preferably run at low speeds and be installed in separate, clean, dry and cool rooms, wherever possible, and connected to rough-running machines by means of flexible couplings.

Not over 220-volt current should be used where men are liable to come in contact with same. If higher voltages are to be used electrical machinery must be isolated.

The cement industry is getting its power mostly from steel plants and blast furnaces from valuable by-products and waste gases. Naturally, alternating current is used exclusively on account of transmitting large amounts of power over considerable distances.

Although direct-current motors have ideal speed control and torque characteristics, alternating-current motors are preferable for the cement industry for the following reasons: No commutator troubles, especially on account of great amount of dust; constant speed of motor; stand greater abuse; no transformation losses from alternating current to direct current etc.

Where motors are often started and stopped, especially under load, the proper selection of motor and controller is very important, especially when current is furnished by central stations. Voltage disturbances in line through high current peaks, while starting, must be avoided; also low power factors. In many cases phase-wound motors must be used in place of the simpler squirrel-cage motors. The use of synchronous motors, especially of large size, where starting conditions are not too severe, is advisable for correcting power factor and regulating voltage, especially now when copper is high in price.

Auto-transformers and starting switches must be very strong as the service in a cement plant during starting period is often very severe.

The location of transformers is important. It is often advisable to locate individual transformers as near to the motors as possible in each building, instead of installing a central transformer station and long secondary transmission lines.

W. T. Snyder (read by R. H. Goodwillie): The author makes a reference to out-board bearings; it is considered by many in the steel industry that out-board bearings on motor sizes lower than 75 horse power is good practise, on many applications.

In the steel industry there are many applications, similar to cement-mill drives to which the motor is successfully geared and it has been proven that applications formerly considered suitable only to direct-connected low-speed motors can be successfully driven by high-speed motors rigidly coupled to cut gears at a low initial cost compared to the low-speed motor; and also compare favorably in maintenance cost; and at much lower maintenance and more satisfactory operation than belt drive. The motor and gear-box should be on a common bed-plate or at least on a common foundation.

In the steel industry 220 volts is considered preferable to 440 volts from a safety standpoint. The saving in copper by use of the higher voltage is often over-balanced by the additional safe-guards required. Practically the same safety precautions should be taken against shock with 440 volts as with 2200 volts or higher.

L. E. Underwood: Mr. Williamson says: "Bearings should also be made dust proof by the use of felt and steel washers, as cement has a highly abrasive action and greatly increases bearing wear when present in the bearings." That is undoubtedly true, but there is one application of dust-proofing which has not been mentioned. If in the lip of the housing, a very small clearance, perhaps 0.005 of an inch is obtained, or some amount like that, and circular grooves are put in this lip and filled with a heavy grease, an essentially dust-proof bearing can be made. The grease rubbing on the shaft catches the first particles of dust that come through, and they collect and form their own seal. In addition to that it is very essential that the oil-well opening be permanently covered, that is to say, the cover should be locked in place, so as to make it very difficult to remove it. I think these two features make as nearly a dust-proof bearing as anything I have seen in my experience.

Selby Haar: About nine years ago I had occasion to design some motors, I think, for operating crushers in a cement plant. They were of 150 h. p. capacity at 165 revolutions. They were a-c. motors and it was a rather unusual design of motor. It happened that there were 32 or 33 of them, all alike, which was still more unusual, and on account of the fact that there were so many of them I investigated the results of the tests, and was pleased to find that these motors agreed within two or three per cent of each other in their characteristics.

Since the committee has asked for some suggestions in regard to the construction of this paper, I will say it might possibly be a help to the people who read this paper if it were known that the data on which the paper is based, and which Mr. Dudley, I believe, said were accumulated previous to preparing the paper, are all deposited in the library of the Institute or in the general library of the United Engineering Societies and are available to people who wish to study the subject. I had intended to suggest that the Committee prepare a selected bibliography of the important articles on the subject, but if I am correct in assuming that the underlying facts for this paper are available in the library that would make it unnecessary for the Committee to go to the labor of preparing a bibliography.

J. N. Mahoney: In connection with control problems comes the matter of the degree and character of overload protection. I have in mind the case of a cement mill where the problem of overload protection is a very important one. The power supply to the cement mill electrical apparatus comes through a bank of 4500 kv-a. of transformers at 2200 volts, backed by a very large high-voltage system. It is evident the short-circuit energy which can reach the motor and control apparatus is very great. While this problem is not beyond the application of reasonable apparatus arrangement and cost, I want to call to your attention here that this phase of the situation is usually not well considered in making an electrical apparatus application.

If overload or short-circuit protection is provided at every motor or branch circuit the cost will be large if the power supply units are not properly selected. On the other hand if overload protection is provided only at a few main supply points the cost of shutting down a large portion of the service may be large.

It is well to provide a power limiting arrangement in a large capacity installation, rather than use unusually capable control apparatus to handle the short-circuit energy that may reach the control equipment and probably the motors without such consideration.

W. L. Merrill: In connection with this last point we have had brought to our attention, I believe, the fact that the proper place for control apparatus under these conditions is where the lines go into the cement mill and not at each individual motor.

In any of these industries, particularly in the case of the cement industry, we must bear in mind that what the mill is put up for is to make cement. No motors should go into a cement mill that are not capable of standing up and doing the work. We should not have to surround them with a lot of control apparatus which we have to put off in a separate room in order to keep the operation safe.

If we go to that extreme we will have a high grade power station with a lot of expensive engineers and electricians to keep it running, and not have our minds fixed on the making of cement. That is what the mill is for, and the simpler and stronger the pieces of apparatus we can put in such a mill, the better the output and the less the cost.

R. J. Dearborn: As I glanced over Mr. Williamson's paper before coming here I noticed on the first page a prominent heading, "General Description of Process". I made up my mind that it would be an interesting paper aside from its technical value, and so I came to the meeting.

In listening to the verbal presentation by Mr. Williamson I was disappointed to find that he skipped that part of the paper entirely, and I would like to suggest as a matter of human interest that the speakers in the presentation of papers of this kind give a brief abstract of the general description of the process and try to visualize it to the mind of the listeners in such a way that all may have a good general idea of what the process is, after they leave the meeting, even if little was known about it in advance.

Will it not promote the general interest and increase the attendance of the meetings if the committee on papers assists the writer in the preparation of a digest for verbal presentation, which particularly has the listener in mind. The papers are excellent but the presentation often leaves something to be desired.

James Dixon (abstracted by A. M. Dudley): Direct-current motors of the open type installed in locations where they are exposed to cement dust are noteworthy for the general good condition of the commutator surface. After a comparatively short time in service the commutator face acquires a fine polish and the later wear on commutator and brushes is abnormally slow. In one case emergency repairs were made on a grounded commutator. No facilities for turning it true were available, it was filed to approximate roundness with no sharp irregularities and put in service, the load being about 10 per cent above the motor rating. The sparking which existed at first disappeared after two days and in a week this commutator was as good as any in the plant. The eccentricity of course, still remained but was taken care of by the brush gear.

This tendency of commutators to acquire unusually good surface when exposed to non-abrasive dust has also been noted by the writer in a candy factory and in a chocolate factory. In both of these the motors were exposed to finely pulverized sugar.

As might be expected, long life of commutator and brushes is a result of the highly polished contact face.

A. L. Hadley (abstracted by A. M. Dudley): There is one question which might be brought up for discussion, or for information as to the usual or best practise, and that is relative to the use of commutators with undercut mica for direct-current machines. The author of the paper states: "There is nothing inherently injurious about the action of cement dust on the commutators of d-c. machines, and they are successfully employed in some plants, but, all in all, the a-c. motor is preferred and used."

No doubt the a-c. motor for most new installations is being employed. The fact that cement dust on a commutator does not make it inoperative, would lead to the conclusion that the cement dust is sufficiently abrasive to keep the mica flush. It will be of interest to know whether cement mills have used commutators with undercut mica with any better success than those using commutators with flush mica.

C. H. Sonntag (abstracted by A. M. Dudley): At the time that electric motors were first introduced in cement plants the average mill consisted, at least in its grinding departments, of long lines of shafting carrying many friction clutches, from which the grinding machines were driven by belts. In the cheaper installations the clutches were mounted directly on the line-shaft, and in the better class mills the clutches were of the quill type. In either case, trouble with any clutch meant a shut down of the whole line-shaft until the difficulty could be remedied. The line-shafts were driven by Corliss engines through belts or ropes, or sometimes by direct coupling. This required that the power house be built as an integral part of the mill proper, with the attendant exposure of high-grade machinery to the ever-present dust.

Electric drive was hailed as the best remedy for these conditions. The power house could be removed to a convenient distance, where it could be kept clean, and where it really became a factory in itself, manufacturing kilowatt hours. This put the generation, measurement and use of power on a scientific basis that was practically impossible in the old type of mill. It also permitted the use of turbo-generators, with their attendant economies of cost, space, and steam consumption, a thing out of the question with the old line-shaft arrangement, unless large motors were installed to drive these shafts. This was done in only a few cases, as the advantages of the individual motor drive were too obvious to be overlooked.

Mr. Williamson has given a very good summary of the requirements of motors for this class of service. There are, however, a few points that should be emphasized in selecting additional equipment.

1. *Type.* The a-c. motor is of course the only choice when turbo-generators are used in the power house. They are more desirable in any event, because they are practically fool-proof.

2. *Speed.* To the average man who is putting up the money to build a plant, the cheapest motor is apt to look the best, when the choice lies between the products of reputable builders. A high-speed motor is cheaper than a low-speed one. In the stress of competition that attends the sale of a fair sized list of motors, a salesman of narrow vision who is not familiar with the conditions in a cement plant is apt to recommend to the powers that be, a selection of higher speed motors, because he can thereby underbid his competitors. If this comes to the attention of the financial man of the company, pressure from above may be brought to bear on the designing engineer which it may be very hard to resist. Especially is this so where a plant is being built by a promotor who expects to take his profit and depart for other fields, leaving the enterprise in the hands of some one else. The selection of high-speed motors may handicap the plant throughout the life of the motors. There is a very definite reason for this. The use of high-speed motors involves the use of small pulleys to keep the belt speed within bounds. In the dusty atmosphere of a cement plant the small pulley is a constant

source of trouble. The belt must be kept tight to do its work, and probably doped with sticky belt dressing. Even if made of endless leather its life will be shorter than it should be, and leather is expensive these days. If a textile belt is used, the repeated bending of the splice over the small pulley will soon break the belt at the splice. It will not take very many shut-downs because of belt repairs to pay the difference in price between the high-speed motor and one suited to the work. The writer has gone into this point at length, for the foregoing is the result of actual experience. For general belted service a motor with a speed higher than 900 rev. per min. should not be used, and 720 rev. per min. is better. This applies only to small motors up to say 25 h. p. For larger motors 600 rev. per min. should be the upper limit of speed.

3. *Special Types.* The various sizes of Griffin, Fuller and Raymond mills have pulleys with vertical shafts, and to drive these, as has been mentioned in the original paper, at least three makers have developed vertical motors adapted for belt drive. The design of these motors is entirely satisfactory, and the only object in speaking of them again is to call attention to the rather unfavorable conditions under which they operate, due to the low belt speed necessary.

A 30-in. (76.2 cm.) Griffin mill requires 30 h. p. to operate it, which must be transmitted through a 10-in. (25.4 cm.) belt running at only 1650 ft. (502.9 m.) per min. This means that a 500 rev. per min motor may have only a 12½-in. (31.7 cm.) pulley. A 42-in. (106.6 cm.) Fuller mill requires 75 h. p., and if fitted with a 54-in (137.1 cm.) pulley has a belt speed of 2500 ft. (762 m.) per min., calling for a 19-in. (48.2 cm.) pulley on a 500 rev. per min. motor. These small pulleys mean that the belts must be kept tight, both as to initial tension to prevent slipping and as to working tension to transmit the necessary power. There is some justification for this, in that these machines were originally designed for operation from low-speed line-shafts through quarter-turn drives, so that larger driving pulleys could be used. If there were an insistent demand for it, these machines could be re-designed with larger pulleys, so that a more favorable belt speed could be had.

4. *Ease of Cleaning.* The dust in the air of a cement plant has a tendency to collect in every protected nook and cranny in a motor, and particularly around the ends of the coils. Most of the older types of motors have the end bells of quite open design, so that the natural fanning action of the machine, assisted by the occasional use of compressed air will keep them in fair condition. But there is one late model in which the outer parts of the end bells are solid, and dust accumulations in this type of motor will give serious trouble if not watched. In the plant with which the writer is connected we have had to cut holes in the lower parts of the ends of this type so that the air circulation may carry the dust out.

5. *Dust-proof Control.* The well-known types of auto-transformer starting devices are sufficiently dust-proof as to give no trouble on that score, but where the size or location of the motor calls for remote control through magnetically operated contactors, these may give considerable annoyance by the interference of dust with their operation. Again, in older installations, cartridge fuses on open panels are relied on for overload protection. Dust falling on the upper end of the fuse will work its way between the clips and ferrule, and finally make the contact so poor that heating and destruction of the entire fuse and cartridge occurs when there is no overload. The best way out of this that the writer has seen is the use of inverse-time-limit overload relays which open the circuit of the low-voltage release coil. These may be had in perfectly dust-proof cases, and they make a very satisfactory installation on which there is no up-keep, as there are on fuses.

6. *Motors and Silent Chains.* The use of silent chain is being tried out in two or three plants in connection with kiln, ball and tube mill drives as a substitute for belts. Their makers prefer not to operate them at more than about 1400 ft. (426.7 m.) per min., which calls for a high working tension in the chain to transmit the power. This is of interest to the man installing them in connection with motors because provision must be made for safely caring for the chain tension. The bearings of smaller motors seem to be able to handle this without trouble. We have two 30-h. p., 850-rev. per min. two-bearing motors driving rotary kilns through silent chains, and their operation is perfectly satisfactory, and far superior to that of the belt drives that they displaced. We also have a 375-h.p. silent chain drive on a large tube mill loaded with iron balls. The driving shaft of this chain is mounted in bearings of its own, the motor being direct-connected to the end of this shaft through a flexible coupling. The very evident high tension in this chain when in operation has justified us in arranging to relieve the motor of this strain.

7. *Synchronous Motors.* The outstanding feature of a cement mill load from the power house stand-point is its low power factor, due to the use of induction motors. It is never over 80 per cent, and is usually nearer 70 per cent. We all know the effect of this on the capacity of the generators. The obvious remedy is the use of synchronous motors. They are already in use to a certain extent for driving air compressors, where they can be started light. Their inability to start under a heavy load is all that prevents their use for the larger sizes of crushers, and for ball and tube mills.

The average cement mill crushing and screening plant is usually rather compactly built, giving a good opportunity for the use of group drive with only a few motors, of which one, at least, could be of fair size. If the crushers are always started empty, which is usually the case, this motor could be of the synchronous type, though the writer knows of no case where a synchronous motor is so used.

As stated in the paper, the use of this kind of motor for ball and tube mills would make imperative the use of a friction clutch to start the mill after the motor is up to speed, for these mills always start with a heavy over-load. Cement men were so glad to get rid of friction clutches when the old line-shafts were taken out that they are chary about taking them up again. If there is a ball or tube mill installation on this plan, a discussion of its operation and the cost of up-keep of its clutches would make interesting reading.

D. B. Rushmore (abstracted by A. M. Dudley): The rapid strides in the use of cement for building and construction during the past few years naturally creates an interest in the industry itself, the raw materials, process of manufacture and its magnitude, and before discussing the application of motors to the industry it might not be amiss to refer to the Census of Manufacturers issued by the Bureau of Commerce.

The government statistics report a total number of plants of 133 in 1914 employing 27916 wage earners and give the following data:

Capital	$243,485,000.00
Wages	18,192,000.00
Cost of materials	51,987,000.00
Value of product	101,756,000.00
Value added by manufacture	49,769,000.00

These statistics, however, include other kinds of cement than Portland cement as is indicated by the detailed output for 1914

Puzzolan	68,311.	bbls.	$63,358.00
Natural	751,285.	"	351,370.00
Portland	86,437,956.	"	80,118,475.00
			$80,533,203.00
			21,222,797.00
			$101,756,000.00

The more important manufactured cements in this country are the Natural and the Portland, but the latter is by far the most important from the standpoint of quality and also to the electrical manufacturer because it requires considerably more power per barrel to produce than the Natural cement. The annual production of Natural cement has been steadily decreasing during recent years while the production of Portland cement has increased very rapidly. Owing to this fact it is not necessary to consider the manufacture of Natural cement.

A large number of the cement mills are already electrically operated as is indicated by the power statistics.

Primary Horse Power—1914

Total	490,402.
Owned	
Steam engines and turbines	291,321.
Water wheels and motors	6,346.
Internal combustion engines	28,366.

Rented.	
Electric	164,369.
Electrical horse power	
Owned and rented	336,516.
Fuel Consumption—1914	
Anthracite (long ton)	226,474.
Bituminous (short tons)	6,731,438.
Coke (short tons)	20,072.
Oil including gasoline bbls.	2,502,015.
Gas (cu. ft.)	5,525,894.

Portland cement is of a somewhat uncertain chemical constitution but consists primarily of tri calcium silicate ($3\ CaOSiO_2$) and tri calcium aluminate ($3\ CaOAl_2O_3$) with traces of iron, magnesium and other materials which seems to have small effect on the quality of the cement.

There are two classes of raw materials used in the manufacture, one being rich in calcium in the form of carbonate, such as limestone, marl and chalk and the other containing a large proportion of silica and alumina as clay, shale, slag and natural cement rock, although the latter is sometimes found so rich in calcium that it may be placed in the former class, as clay must be added to bring the silica and alumina up to the proper porportions.

There are two processes used in the manufacture of cement, *i. e.*, the wet process and the dry process. Those plants using the wet process are found principally in Michigan and employ marl (calcium carbonate) and clay. The raw materials are mixed and pulverized while in a semi fluid state, the wet mixture containing about 60 per cent of water which makes it sufficiently fluid to be handled by pumps. It is stored in tanks equipped with agitators to prevent settling and after being properly pulverized and mixed by means of tube or ball mills is pumped into the kilns for burning and from this point on the process is similar to the dry process.

The dry process which is employed by the bulk of the cement plants consists principally of (1) Crushing, (2) Grinding and mixing of raw material, (3) Burning, (4) Grinding the clinker to finished cement. There are various types of machines used in the process of manufacture.

- Crushing
  - Gyratory (from quarry size to 3 in or 4 in.)
  - Gyratory (from 3 in. to 4 in. size to ¾ in or 1 in.)
- Elevators.
- Storage Bins.
- Bins.
- Preliminary Grinding
  - Hammer Mills
  - Kent Mills
  - Rolls
  - Ball Mills
  - Kominuters
- Dryers.
- Mixing Bins.
- Coal Storage.
- Crusher
- Dryer
- Pulverizer Bins.
- Grinding
  - Fuller Mills
  - Griffin Mills
  - Raymond Mills
- Elevator
- Screw Conveyor
- Pulverized Coal Bins.

Final Grinding-Raw
    Fuller Mills
    Griffin Mills
    Raymond Mills
    Tube Mills
Kiln Bins.
Kilns

Coolers.
Clinker Storage.
Gypsum Storage
Pulverizer Bins.
Final Grinding—Finished.
    Fuller Mills
    Griffin Mills
    Tube Mills
Cement Storage
Packing Bins
Packers
Shippers

Both direct and alternating current have been used in the electrification of cement plants and all of the apparatus has proven commercially satisfactory so that it is impossible to say that any one frequency or voltage must be used. However, alternating current is certainly preferable to direct current and 25-cycle equipment is preferable to 60-cycle unless the plant intends to purchase power or wishes to tie in with some existing 60-cycle system, or in cases where the lower cost and better economy of 60-cycle turbines is a determining factor.

The advantages of the 25-cycle equipment are due to the fact that many of the machines run at comparatively low speed and require high starting torque. Furthermore the continuous service in an extremely dusty atmosphere make low-speed motors more desirable from the mechanical standpoint.

Low frequency is best adapted for low-speed motors with reasonably high starting torque and good power factor so that from the motor standpoint 25 cycles is better than 60 cycles. The extreme case of this is shown in the low-speed tube mill drives, where the motor is direct connected to the counter shaft, which would be impracticable with anything higher than 25 or 30 cycles.

Direct-current apparatus operates under the trying conditions of dust much better than would be supposed, but as there is bound to be considerable wear on the brushes and commutators, and as the first cost is higher than that of the a-c. equipment, it is seldom used. Where d-c. motors are used it is customary to enclose them in housings.

The effect of cement dust on bearings is a feature that must be taken care of and it has always been the practise of some manufacturers to furnish dust-proof bearings, but the best insurance against bearing wear is to install low-speed motors and avoid locating them in the dustiest places. The effect of cement dust on motor windings has never been found to be harmful and standard insulation is always used. The dust does collect on the coils and in the air ducts, however, and may interfere seriously with the ventilation if it is not blown off.

In order to eliminate as much as possible the complications in both motors and control and reduce the wearing parts to a minimum it is desirable to use squirrel-cage motors and the tendency has always been to install these in cement plants up to a size somewhat larger than would ordinarily be used in other industries. There are a few machines on which slip-ring motors must be used, such as the kilns on which a variable speed is sometimes required, hammer mills which have a very heavy revolving element and the possibility of being shut down when full, and the larger tube mills; in such cases the motors should be furnished with enclosed collectors and the drum controllers made as nearly dust proof as possible.

The question of the proper voltage to be used in new installations is one that deserves attention on account of the large amount of power involved. Cement plants usually cover considerable ground, and it is often desirable to locate the power house apart from the main plant, so that the feeders may be as long as 1000 ft. In the interest of economy in transmission copper, it has been our practise to recommend 550-volt motors although some of the largest plants, use 440 volts. There are also several 2200-volt plants which have been in successful operation for several years using this voltage for all of the larger motors and there is no objection to this where it is found to be the most economical. It is often questionable however whether the saving in transmission copper over a 550-volt installation is not more than offset by the increased cost of the necessary conduit wiring and the higher price of 2200-volt motors.

The load on the generating station of a cement plant is exceptionally steady as compared with some other industries and there are no special problems in this connection. The principal requirement is that the source of power should be reliable and adapted to continuous operation 24 hours a day, seven days a week. It is customary in new plants to install either two or three generating units, the total capacity being somewhat greater than the average demand of the mill. This gives such a flexible arrangement that interruptions due to generating troubles are very infrequent.

The distribution of load between the different departments of a cement plant is interesting, and records of it may lead to economies in production. However, readings based on a single day are apt to be misleading on account of storage of material in the bins. That is, for a given day the crushing department may crush more or less than the raw grinding department uses, while the latter may grind more or less than the finishing department, etc. The following table gives results in round numbers obtained over an entire year in a typical 2000-barrel dry-process plant using limestone and shale:

Department	Total kw. hr.	Per cent
Crushing	1,800,000	14.7
Raw Grinding	3,400,000	27.7
Coal grinding and kilns	900,000	7.3
Finishing	5,800,000	47.3
Lighting, machine shop and miscellaneous	371,000	3.0
	12,271,000	100.0

We are inclined to believe that in the usual plant there is no such great difference between the power used in the raw and finishing grinding departments as is shown in this table. A 3000-barrel plant, also using limestone and shale showed about the following divisions:

Crushing	15 per cent
Raw grinding	30 " "
Coal grinding and kilns	10 " "
Finishing	40 " "
Lighting and miscellaneous	5 " "

In estimating on cement plants it has been customary to assume that the power required will be about a horse power per barrel per day, *i. e.*, that, for example, a 2000-barrel plant will consume about 2000 h. p. That is fairly accurate, but in actual plants the horse power installed per barrel rated output varies considerably, because the ratings are not always an actual measure of the plant capacity, and because the tendency is always to put in somewhat larger motors than absolutely necessary. The average seems to be about 1¼ h. p. nameplate rating of the motors per barrel per day capacity of the plant.

T. E. Simpers (abstracted by A. M. Dudley): Mr. Williamson mentions certain advantages for the wet process as compared with the dry process. One of these is cleanliness. It is not mentioned that the finishing mill of a wet process plant uses the same operations as that of a dry-process plant, so that this part of the mill is just as dirty in plants using either process. Furthermore, the finishing mill is in most plants by far the dirtiest part of the plant. Also one of the necessities in a wet process plant is a supply of cheap clean water. A number of plants are located so that this is not available.

I would consider the value of the load factor as mentioned by Mr. Williamson, as being too optimistic. It is a fact that all of the apparatus in cement plants is in operation practically all the time, yet considering shut-downs throughout a year on account of operating and labor conditions, shut-downs for repairs, and the usual underloading of motors on account of the severe service, it will be found that the load factors will be less than those values mentioned by Mr. Williamson. A review of complete tests on nine cement plants showed the highest average load factor obtainedto be 66 per cent. A number of these plants operated at values between 55 and 65 per cent, while almost an equal number operated as low as 30 per cent. There is no question however, but that with good engineering judgment and neglecting extensive shut-downs on account of weather or labor conditions that the figure of 65 per cent could be made an average load factor. This is of course, rather higher than obtained in plants of other industries.

Mr. Williamson states that the starting torque required by a gyratory crusher is small, unless it is accidentally stopped with material in it, in which case it is usually necessary to dig out the material before machine can be started. Considering crushers of the smaller capacity, it has been our custom to apply either wound-secondary motors or squirrel-cage motors with starting torque of at least two times full-load torque, which motors are capable of starting the crushers under practically all circumstances, in order to take care of the shut-downs when the material is not run out of the crusher.

Under the sub-subject "Griffin Mill" it is not brought out by Mr. Williamson that this mill requires a motor with a veritcal shaft. Furthermore, as far as the writer knows, a wound-secondary motor has never been applied to this mill, except where it has been driven on a horizontal shaft with a quarter turned belt, and this method of application has practically been discarded. A squirrel-cage motor with a vertical shaft for belt drive is the usual method of connecting the motor application to this mill.

Mr. Williamson mentions the fact that the low-speed motor has the disadvantage of lower torque and lower power factor, but the saving in space, belting, etc. offsets these disadvantages to a large extent. I would call attention to the fact that the maintenance of auxiliary apparatus is an important item in connection with such drives. The Universal Portland Cement Co. use 18-pole, 25-cycle squirrel-cage motors direct connected to the counter-shaft of their tube mills. These motors are naturally much more expensive than the belted motors, but they figure that they represent an eventual saving on account of the fact that by using the low-speed motors, belting or a gear reduction may be omitted, and while the first cost of the apparatus is high, the maintenance of same is low as compared with plants using higher speed motors.

Mr. Williamson mentions the use of synchronous motors in connection with tube mill drive. Naturally the synchronous motor, considering running characteristics, represents an ideal installation in that low-speed motors can be obtained with high efficiency, and with capability of correcting power factor of the remainder of the plant. Since the synchronous motor cannot start the mill, it is necessary to use a clutch in connection with same. It has been the writer's recent experience in connection with somewhat similar applications that the clutch is a source of annoyance on account of excessive maintenance, and furthermore, it is very expensive for the size of application under consideration. Considering such an installation, it would be of interest to know if there have been installations made of motors starting light, and with a magnetic clutch used for connecting the load on the motor, to make a comparison of such installation with installations where no clutch was used.

Furthermore, in respect to the synchronous motors, it would be of interest to know if such installations have been made. It is recognized that the synchronous motor has no slip whatever. Therefore, when a heavy load is thrown on the motor by means of a clutch, either the clutch must slip, thereby causing excessive wear to the clutch, or if it does not slip a very considerable shock will be experienced by the entire driving system.

H. Weichsel (abstracted by A. M. Dudley): Mr. Williamson's paper gives very valuable information regarding the requirements placed on electrical motors when installed in cement mills.

In order to select the most suitable motor for any kind of service, it is absolutely essential to consider the following three factors:

First: The characteristic of the machine to be driven.

Second: Kind of service to which motor and driven machine is going to be subjected.

Third: The kind of supply circuit available.

Mr. Williamson has, in an admirable manner, presented to us the most prominent factors entering in the three items listed above.

From the characteristics of the machine to be driven, it is possible to determine which type of motor should be used, such as for instance, squirrel-cage motor, slip-ring motor, adjustable speed motor.

The kind of service influences only a little the type of motor, but has a large bearing on the most suitable design of the motor, so as to give satisfactory life and service.

Mr. Williamson gives in his paper means for determining the approximate horsepower requirements for the different classes of machines, and also informs us regarding the necessary starting torque. It is quite frequently the starting torque required, which is one of the leading factors in selecting a suitable motor. If no speed adjustment is required, quite frequently either a squirrel-cage motor or a wound-rotor motor can be used for the running period of the machine, but the necessary starting torque might in some cases absolutely prohibit the use of squirrel-cage rotors.

The third factor mentioned above, which is the frequency of the supply circuit, has a great bearing on the decision if a squirrel-cage or a wound-rotor type motor should be used. We assume here that polyphase induction motors are under consideration. Mr. Williamson has pointed out that most of the cement mill machines operate at rather low speed. In order to avoid a large ratio between the pulley of the motor and the pulley of the driven machine, it is therefore essential to make use of low-speed motors. For a given motor speed, however, the 60-cycle motor requires more poles than a 25-cycle motor, and standard squirrel-cage motors designed for 60 cycles have, on account of the higher number of poles the higher frequency, a lower starting torque than 25-cycle motors of equal speed. It might therefore readily occur that, for driving a given machine at 25 cycles, a squirrel-cage motor will have ample starting torque for starting the machine, while a 60-cycle motor, of approximately the same speed, will be unable to perform the starting operation satisfactorily.

If a 25-cycle supply circuit is used, it will make it therefore possible to use a larger number of squirrel-cage motors than if a 60-cycle supply circuit is used.

From the operator's point of view, the squirrel-cage motor, no doubt, has many advantages, on account of its strength and simplicity. The heavy starting current of squirrel-cage motors is practically of no objection in case of cement mills, because an installation of this kind consists invariably of a great number of machines, and as never more than a few squirrel-cage motors will be started simultaneously, it follows that the heavy starting current of squirrel-cage motors cannot influence the power plant to any extent.

If, therefore, the engineer designing the cement mill is free in selecting the frequency of the supply, it seems advisable to use 25 cycles, as this will allow him to make the fullest use of the advantages of squirrel-cage motors. Besides the advantages previously mentioned, it should be considered that the power factor of 25-cycle motors is considerably higher than the power factor of 60-cycle motors with equal speed and horse power capacities. Furthermore, the power factor of squirrel-cage motors is usually slightly superior to the power factor of motors with a wound rotor. Besides this, we must consider that the service conditions in cement mills, as a rule, are extremely severe, partly on account of the low class of labor available, and partly on account of the unavoidable dust. The dust will soon settle inside of the rotor winding of a wound rotor, and is therefore liable to increase the heating of the machine. The squirrel-cage motor, on the other hand, offers little or no opportunity for the dust to settle in the rotor winding, and an increase in temperature rise of a squirrel-cage rotor, has, no influence on the life of the machine. Due to the inferior class of labor available, it will frequently be found that the motors are subject to all kinds of abuses, and not unfrequently, foreign matter, such as small stones, enter the rotor. Such foreign matter is naturally more liable to cause trouble in a wound-rotor than in a squirrel-cage rotor.

The question of dust and foreign matter in the motor, demands further as large an air gap as is consistent with the design of the motor. A large air gap can readily be used on 25-cycle motors without impairing the power factor materially. This, however, is not the case on 60-cycle motors of equal speed.

The bearing construction is another feature which demands careful consideration on account of the dusty atmosphere always present in cement mills. Only high grade babbit should be used, and the bearing housings should be made as dust-proof as possible. Even with the best babbit and dust-proof bearings, it will be necessary to reline the bearings occasionally, and for this reason, it is advisable on larger motors to use split bearings, as these allow removal of the bearing shell without removing the end-plates or rotor.

In case many squirrel-cage motors are used, it is frequently advisable to do away with auto starters, and to install a separate starting circuit, which is fed either from a separate set of transformers, or from taps on the main bank of transformers. In this case, the onlystarting apparatus necessary is a 3-pole double-throw switch, which preferably should be of the oil immersed type.

Many of the cement mill motors are liable to sudden overloads, due to choking of the cement mill machinery. These overloads might be temporary and light, or might be so severe as to stall the motor, or cause the belt to slip. To avoid disastrous results from these unavoidable overload conditions, it is advisable to protect all motors with an overload relay of the inverse-time-element type. Furthermore, it is advisable to install in each motor circuit an ampere meter, which is so located as to be easily observed by the operator.

The information given in Mr. Williamson's paper is of such an important and far reaching nature, that it seems that this information should be made accessible to all parties interested in the subject. This would help the electrical engineer, the cement industry, and the economic life of the nation as a whole.

Mr. Williamson's paper has been presented under the auspices of the Industrial and Domestic Power Committee. No doubt, many other papers will in future be presented by this committee, all of which will be of interest to many people outside of the electrical profession. Papers of this kind will enable the superintendent or engineer of a given industry to familiarize himself with the fundamental principles and characteristics of the various types of motors, and the actual experiences which the electrical engineers had with certain types of motors in certain industries. On the other hand, the discussions will be partly carried on by engineers who are in daily contact with the different industries for which the motors are used. Mr. Williamson's paper, for instance, has influenced several prominent men of the cement industry to contribute valuable information in their discussions.

Let us hope that Mr. Williamson's paper has been the signal for the free cooperation of all parties interested in the progress of the electric drive.

In order to give everybody interested in this subject the fullest opportunity to benefit by such papers as have been presented today, and which, in all probability, will be presented in the near future by the Industrial and Domestic Power Committee, I propose that the Institute devise some means to make such information as available as possible. This, for instance, could be accomplished, by printing these papers in pamphlet form, which the general public could buy for a small amount of money. Furthermore, a card index would be of great value, if it could be arranged according to industries, using one card for each type of machine to be considered. This card could give the fundamental characteristics of the machine to be driven, the most suitable kind of motor, and with possible references to articles published on the electric drive of such machine. If such records could be kept at the headquarters of each institute section, they would probably be of great value.

Naturally, records and investigations of this nature can never be considered as completed, due to the daily progress made. For this reason, it seems that these records should be kept up to date. The Industrial and Domestic Power Committee could probably make arrangements to keep these records up to date, and collect permanently new data, and further, keep up records of all publications on this subject made in home journals, as well as in foreign magazines. If engineers all over the country could be induced to forward to the Industrial and Domestic Power Committee experiences and data on the subject of electric drive of certain industries, it would undoubtedly help to make the work of this committee of the greatest value to all industries interested, and to the economic life of the nation as a whole.

S. H. Harrison (abstracted by A. M. Dudley): I was interested in the formulas given in the paper in connection with the output of crushers, rolls, etc., and thought that perhaps in connection with the subject of rotary kilns, a formula expressing the relationship between the capacity and the diameter and length of the kiln may be of interest to you.

The only formula expressing the above relationship which I have seen heretofore was one suggested by Prof. R. C. Carpenter (See page 73—Powdered Coal as Fuel, by C. F. Herington—D. Van Nostrand Co., N. Y.) Prof. Carpenter gives the following formula for kilns operating on dry material.

$$C = \frac{D^2 L}{24}$$

where C = capacity in 24 hours in barrels of 380 lb.
D = outside diameter of kiln in feet.
L = length of kiln in feet.

From the structure of this formula it is evident that the capacity is assumed to be directly proportional to the volume of the kiln.

From data which I have collected concerning the output of a large number of kilns it appears that the output is not proportional to the volume of the kiln, but to the heating surface or very nearly so. Assuming then that the capacity of the kiln is directly proportional to its diameter and also to some function of its length, the following formula was assumed,

$$C = K\,D\,L^n$$

where K = a constant
D = diameter in feet
L = length in feet
n = constant
C = capacity in barrels of 380 lb. per 24 hours.

From the data which I had covering a large number of kilns in actual operation, fired with powdered coal and calcining dry raw material, I found the following relationship to hold true.

$$C = 0.95\,D\,L^{0.9}$$

In plotting the value $0.95\,L^{0.9}$ against L as abscissas the resulting curve was almost a straight line, so much so that for practical purposes the formula

$$C = 0.6\,D\,L$$

can be used giving results which are a trifle lower for values of L under 100 ft. (30.4 m.) and at trifle higher for values above this number than that given by the exponential formula. In view of the fact that different cement companies obtain different results from kilns of the same size the formula

$$C = 6/10\,D\,L$$

may be depended upon to give results which very closely agree with the outputs obtained in practise.

The table below may be of interest.

Size of kiln	Formula	Formula	Results obtained in practise
	$C = 1/24\,D^2\,L$	$C = 6/10\,D\,L$	
5 ft. x 60 ft.	62.5 bbls.	180	160 to 200
8 ft. x 125 ft.	333 "	600	550 to 650
10 ft. x 200 ft.	833 "	1200	1000 to 1300

The formulas

$$C = 0.95\text{ D. L.}^{0.9},\text{ or } C = 0.6\text{ D. L.}$$

are to be used for kilns of commercial sizes; they do not hold for kilns of unusual lengths or diameters, for instance, 3 ft. in dia. x 100 ft. long. In other words the formula was derived from actual kilns operating under practical conditions.

W. I. Slichter: I would like to say a word of appreciation and approval of the policy which this committee has inaugurated in this meeting, the policy of making an organized survey and study of the various industries included in the scope of its activities. These matters have been discussed at their committee meetings and the conclusions arrived at have been presented to the Institute in the form before us.

This policy might well be followed by many other Institute committees. The committee by arriving at an agreement on a definite statement of the facts of the problem have no difficulty in agreeing upon a solution. The electrical engineers then present a united front in attacking the problem of the electrification of a given industry.

A noticeable feature of the papers this evening is the absence of any plea in justification of the use of electric motors in these industries. Only a few years ago any paper of this character would have given considerable prominence to arguments showing the advantage of electric as compared to other systems of drive but this is no longer disputed. In fact the flexibility of the electric drive is what made it possible to develop the capacity of our factories so rapidly to meet war conditions.

It is stated on reliable information that the applications of electric motors to manufacturing purposes and particularly to factories using units smaller than 50 h. p. has been so standardized that it is only necessary to know the process of manufacture and the type of building in order to place an order for the whole equipment of motors, starters, switches and wiring. This is particularly valuable because when the present war activities are over and the industries resume their peacetime processes these factories built to make war necessities may be easily converted to some reconstruction work.

G. H. Rowe (communicated after adjournment): Mr. Williamson mentions that the use of synchronous motors might work out advantageously for driving certain machines in cement mills.

While the induction motor is undoubtedly best adapted in the majority of applications, it would seem that synchronous motors would find an ideal application for machines running practically continuously and requiring only moderate starting torques. Besides rugged mechanical design, large gap, high efficiency, freedom from dust troubles, they have the added advantage of power factor correction. The saving in line losses during a year due to power factor correction alone would, in the majority of cases, be a very considerable item in plants using so many relatively small induction motors operating at 220 and 440 volts.

Is there any really good reason why induction motors and synchronous motors are not used in combination, even though it becomes necessary to start the latter with a clutch at rather infrequent intervals.

A. L. Hadley (communicated after adjournment): The latest designs of synchronous motors such as used for driving air compressors, etc., are built with starting torque of 30 to 40 per cent on compensator tap of 50 per cent without abnormal current taken from the line. The question of the use of synchronous motors for cement machinery wherever the load is constant and continuous, can well be considered both for low and moderate speeds. I would suggest that this matter be brought up for discussion. A statement of the approximate starting torque in per cent of normal full-load torque needed for each kind of cement machine would be a good indication as to whether a synchronous motor might be used. Doubtless some applications could be made without the use even of a clutch.

R. B. Williamson: The question of squirrel-cage and slip-ring motors has been brought up. I think there is no doubt that in the past there has been more or less prejudice against slip-ring motors in cement mills. Squirrel-cage motors were sometimes forced to do what they were not designed to, and could not do, to the best advantage. The prejudice against the wound-rotor motor was due to the fact that they had collector rings, and cement mill operators were afraid there would be more or less trouble with the brushes. In the later plants slip-ring motors are being used more, and the fact that mills have become so much larger, and must use larger motors, has forced this to a large extent.

There was one point regarding which I hoped to hear more discussion, and that is the possibility of using synchronous motors, to a greater extent in this work. For example, in a ball mill or tube mill, the load is very steady after the mill is once started; the amount of power required is large, and if synchronous motor can be used for that work it would be quite an advantage. The tendency in the newer mills is to cut out belting and use lower speed motors, some of the motors being as large as 400 or 500 h. p.

Most engineers agree that in the future there is going to be a great deal of development in the way of supplying power from large central stations over wide districts. For a number of reasons, not necessary to go into at this time most of this power will be generated at 60 cycles. The 60-cycle frequency, combined with the low-speed induction motor, makes a bad combination from the point of view of power factor, especially with motors having a liberal air gap, such as is necessary for cement mill work.

Synchronous motors on ball mills and tube mills would be a great aid in keeping up the power factor on the plant as a whole, and their successful use hinges very largely on the operation of the clutch between the motor and the mill. There is always more or less prejudice against the use of clutches, but there are some successful installations of this kind in foreign mills and some installations in this country at the present time. These use a magnetic clutch, and it is possible to arrange a magnetic clutch, with an automatic resistance which will gradually increase the exciting current on the clutch and thus giving a gradual gripping effect so as to start the mills. The synchronous motor in addition to its high power factor has a large air gap; it is very simple mechanically, and aside from the clutch and the added complication of providing exciting current for it, it seems to me there is no reason why it may not be adapted to this work in the future.

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ELECTRIC POWER GENERATION IN ONTARIO ON SYSTEMS OF HYDROELECTRIC POWER COMMISSION

BY ARTHUR H. HULL

THE Province of Ontario is the largest and most populous of the Provinces in the Dominion of Canada, having an area of 407,262 square miles and a population (census 1911) of 2,523,274. Ontario's population is about 35 per cent of the total of Canada's population, and its area is 10.9 per cent of the total area of the Dominion. Ten per cent of its area is water.

Ontario has developed into the greatest manufacturing Province in Canada and as there are no coal deposits in the Province, its abundant water power resources are being more and more utilized to furnish the power required for its varied and increasing industrial development. Other fuels, such as natural gas and petroleum are found in the southern part of the Province, gas along the north shore of Lake Erie and petroleum near Sarnia. The natural gas supply, however, is insufficient for present purposes and the quantity of petroleum produced does not begin to meet the needs of the Province. The demand for electric power has increased during the past ten years to such an extent, that, with the war industries in operation, it became necessary to impose restrictions on private and municipal consumers in order to provide the power required by the war industries. The great saving in coal effected by the use of electric power in Ontario's industries is one of the most important aspects of the electric power situation, and further great savings can be, and will be before long, brought about by the electrification of the steam railroads.

Mr. H. G. Acres* in a monograph on Water Powers of the Province of Ontario, written in 1915, gives the following summation of water power capable of development, and of water power already developed in the Province.

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*Hydraulic Engineer, Hydroelectric Power Commission.

Division	Potentiality	Developed.
Ottawa River & Tributaries	688,000 h.p.	71,000 h.p.
Great Lakes Tributaries	446,000 "	137,000 "
Hudson Bay Slope	250,000 "	22,000 "
James Bay Slope	1,500,000 "	70,000 "
International Boundary Rivers	2,045,000 "	462,000 ".
	4,929,000 h.p.	702,000 h.p.

Of the above total for power developed, about 69,000 horse power is used in pulp and paper manufacture, about 59,000 horse power is used as hydraulic power directly applied, and the balance 574,000 horse power is converted into electric energy for light and power.

This paper will deal only with the generation of electric power by the Hydroelectric Power Commission of Ontario which is now the largest producer and distributer of electric energy in the Province.

The Hydroelectric Power Commission of Ontario, which will be referred to hereafter as the Commission, was formed by the Ontario Government in 1906, and first furnished power over its 110,000-volt lines from Niagara Falls in October 1910, the power being purchased from the Ontario Power Company of Niagara Falls, Ontario. It was not until August 1st, 1917, that the Commission obtained direct control of the generation of power at Niagara Falls, through the Ontario Power Company, but in other parts of the Province, the Commission had started to develop its own power by constructing a hydroelectric plant at Eugenia Falls on the Beaver River near Flesherton, which was put into operation in November 1915, and one at Wasdell's Falls on the Severn River which was started in service in October 1914. In 1914 the Commission acquired by purchase the Simcoe Railway and Power Company, whose generating station at Big Chute on the Severn River had previously supplied power under contract to the Commission's Severn System, supplying the district around the southern part of Georgian Bay. In March 1916 the Government of Ontario purchased the entire holdings of the Electric Power Company which, through subsidiary companies, was generating power on the South River near Nipissing, on the Otanabee River

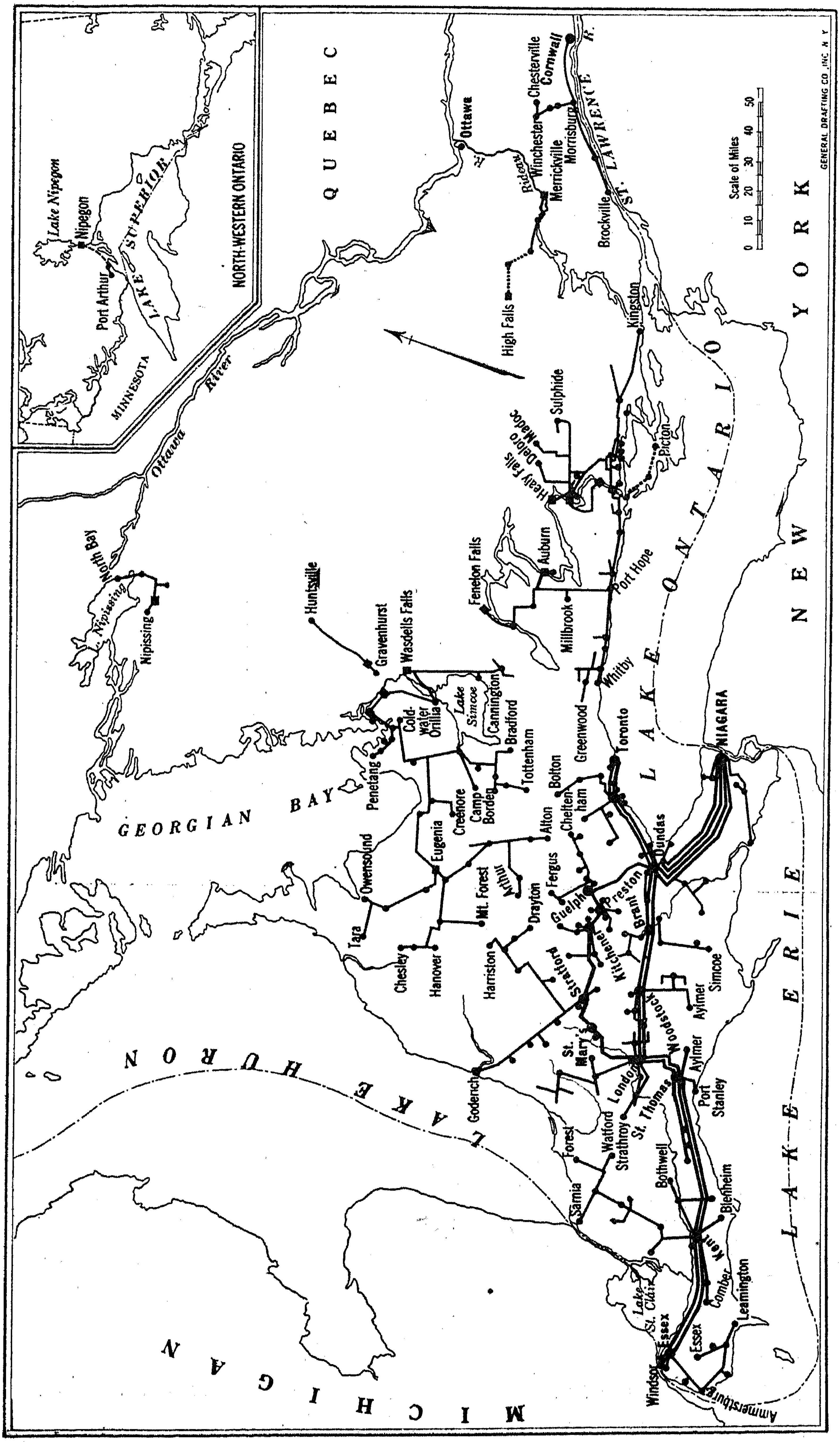

FIG. 1—ELECTRIC GENERATING AND TRANSMISSION SYSTEMS OF THE HYDROELECTRIC POWER COMMISSION OF ONTARIO

at Peterboro, and on Trent River at Healy Falls, Frankford, Campbellford, and Trenton. The operation of the equipments thus acquired was placed in charge of the Commission in June 1916.

The various areas in the Province served by the Commission are designated by Systems as follows

System	Main transmission voltage	Frequency	Phase	Power obtained from
Port Arthur System	22,000	60	3	Kaministiquia Power Company
Nipissing System	22,000	60	3	Commission's Nipissing Station
Muskoka System	22,000	60	3	Comm'n South Falls Station
Wasdell's Falls System	22,000	60	3	Com. Wasdell's Falls Station
Severn System	22,000	60	3	Com. Big Chute Station
Eugenia System	22,000	60	3	Com. Eugenia Station
Central Ontario System	44,000	60	3	Com. Healy Falls Station Com. Auburn Station Com. Campbellford Station Com. Frankford Station Com. Trenton Station
St. Lawrence System	26,400	60	3	M. F. Beach Company
Rideau System	25,700	60	3	Rideau Power Co.
Niagara System	110,000	25	3	Comm's Ontario Power Co's Station Canadian Niagara Co. Electrical Development Co. Queenston Development (future)

PORT ARTHUR SYSTEM

In the Port Arthur System, power is purchased at 22,000 volts, three phase, 60 cycles from the Kaministiquia Power Co., and is delivered to the City of Port Arthur at 22,000 and at 2200 volts, part being transformed in the Commission's Transforming Station in Port Arthur, containing two banks of transformers, each consisting of three 750-kv-a. units connected star-delta with neutral ungrounded.

The first delivery of power was made on December 21st, 1910. The demand has been steadily increasing and in order to take care of the future requirements, the Commission now propose to construct a hydroelectric station on the Nipigon River at Cameron's Pool, about 80 miles from Port Arthur, which will have an ultimate capacity of about 50,000 horse power. The engineering work is now under way for this development. It is proposed to transmit the power at 110,000 volts, three phase, 60 cycles, to Port Arthur. Three other power sites on this same river, when developed, will, together with the Cameron's Pool site, give a total of 150,000 horse power. The needs of the Port Arthur district should, therefore, be amply provided for, but if additional power should be required for this district, another site is capable of development at Silver Falls on the Kaministiquia River about 25 miles from Fort William where with a 350-ft. head a total of 25,000 horse power can be developed. At the present time the electric energy supplied at Port Arthur is used for operation of the street railway, for public and domestic lighting, and for large grain elevators, ship yards, coal docks and miscellaneous industrial purposes.

NIPISSING SYSTEM

The Nipissing System, formerly controlled by the Nipissing Power Company, comprises a generating station on the South River near Nipissing Village, sub-stations in Callander, Powassan and North Bay. The Transmission system comprises 26.5 miles of 22,000-volt, three-phase circuit on wooden poles. The generating station contains two 450-kw. three-phase 2200-volt 60-cycle 450-rev. per min., horizontal generators with 12.5-kw. 125-volt direct-connected exciters, each unit direct coupled to a water wheel of 925 b.h.p. at 86 ft. head. A 37.5-kw. 125-volt motor-driven exciter is also provided. Three 300-kv-a., 2200/22000-volt single-phase water-cooled transformers connected delta-delta are installed, with one 22,000-volt outgoing feeder.

Water for this plant is taken from the South River through an open canal 900 ft. long, then through a wood stave pipe 6 ft. diameter 2300 ft. long to a differential surge tank 72.5 feet high, close to the power house, a steel penstock connecting thence to the turbines. The storage pond has an area of about 100 acres and the drainage area of the river is about 350 square miles.

Provision was made for extension, and by additional storage works, it is possible to increase the capacity to 2500 horse power.

MUSKOKA SYSTEM

The power house on the south branch of the Muskoka River at Muskoka Village was formerly owned by the Municipality of Gravenhurst and was taken over by the Commission in the latter part of 1915, and has been remodelled and enlarged, and now supplies power over a single circuit 22,000-volt, three-phase, 60-cycle line about 26 miles long to Huntsville, and at 6600 volts three phase, 60 cycles, to the Municipality of Gravenhurst over the Municipality's line.

The equipment in this station now comprises one 450-kv-a., three-phase, 6600-volt, 720-rev. per min., 60-cycle generator, direct connected to a 500 b.h.p. turbine; one 750-kv-a. similar 720-rev. per min. generator, direct connected to a 1000 b.h.p. turbine; one bank of three 400-kv-a., 6600/22,000-volt transformers connected delta-delta; double low-tension bus, and single high-tension bus. Four outgoing 6600-volt feeders are provided, two feeding to Gravenhurst and two for future use. One 22,000-volt feeder leaves the station to supply Huntsville. Provision is made in the existing building for a second high-tension feeder.

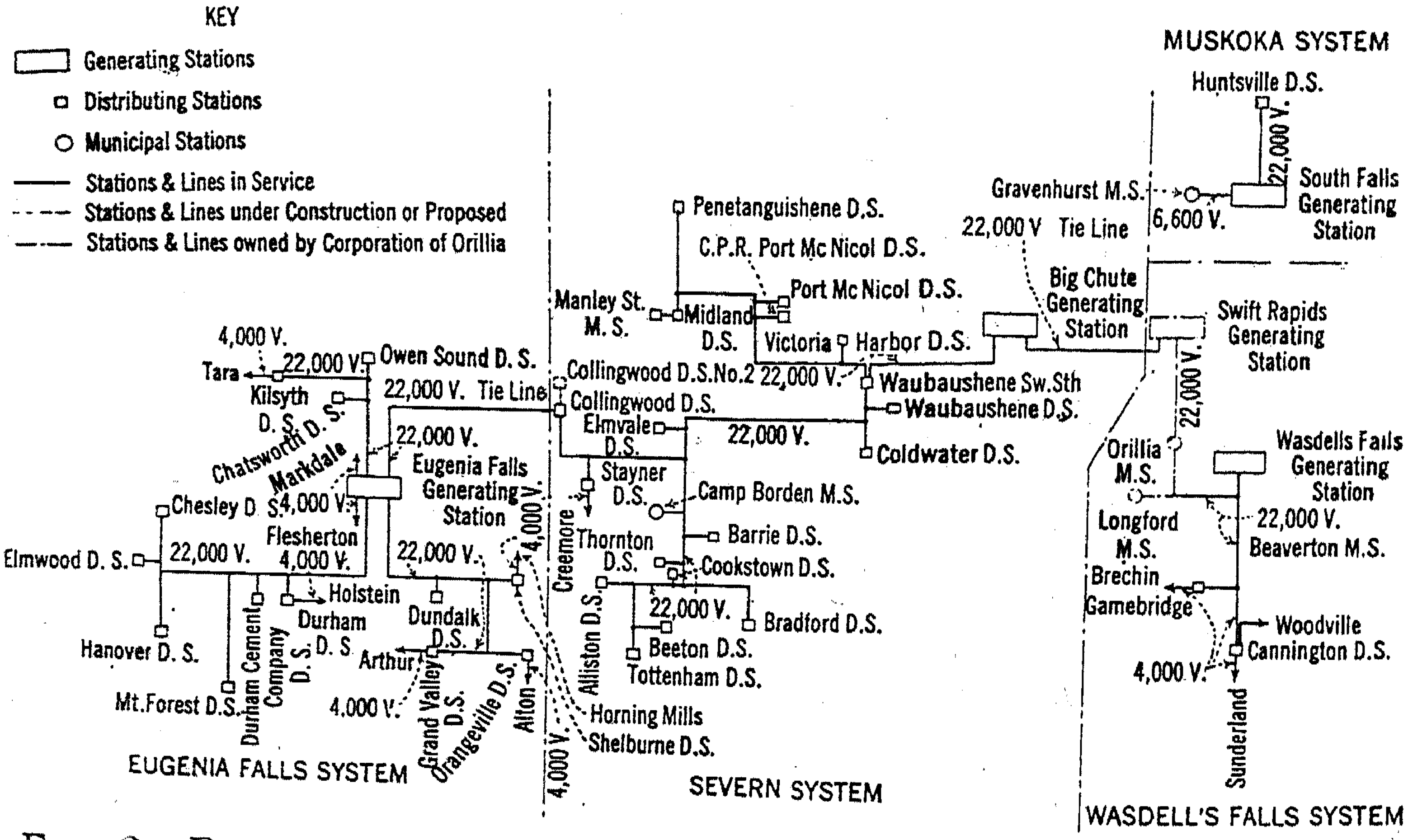

FIG. 2—DIAGRAM OF EUGENIA FALLS, SEVERN, WASDELL'S FALLS, AND MUSKOKA SYSTEMS—60 CYCLES

When this station was taken over by the Commission, only one 450-kv-a. generator was installed. The station was remodelled and enlarged for the larger unit mentioned above, and provision was made for the extension of the building at a future date and for the replacing of existing units so that, when future load conditions require it, there will ultimately be three 1500-kv-a. generators with a turbine capacity of 6000 horse power installed.

The present excitation at this plant is provided by a turbine-driven exciter for the 450-kv-a. unit, a motor-driven exciter for the 750-kv-a. unit, which also has a belted exciter for emergency use, which is so arranged that it may excite either generator.

The hydraulic head at this plant is 102 ft. The water is conducted through one steel and one wood stave pipe, each 946 ft. long to the turbines.

WASDELL'S FALLS SYSTEM

This system comprises a hydraulic generating station on the Severn River at Wasdells' Falls near Severn Bridge, and sub-stations at Beaverton and Cannington. Power is transmitted to Beaverton and Cannington over a single-circuit steel-conductor line at 22,000 volts, three phase 60 cycles, and over a single-circuit aluminum tie line at the same voltage to the Commission's Severn System, connecting at Longford with the Town of Orillia 22,000-volt lines from Longford via Orillia to the Big Chute generating station.

The generating station which was constructed by the Commission in 1914 is interesting on account of the low hydraulic head. The normal head is 12 ft., but variations from 9 to 15 ft. occur. The equipment consists of two 400-kv-a., 90-rev. per min., vertical type 60-cycles, three-phase, 2300-volt generators connected through flexible couplings, to turbines rated at 600 horse power at 100 per cent gate opening with 12-ft. head. Two exciters, one 20-kw. turbine-driven, and one 30-kw. motor-driven are provided. Two banks of transformers, each consisting of three 2200/22,000-volt 150-kv-a. units connected delta-delta are installed. Two outgoing 22,000-volt lines were provided.

There are in this system, 37 miles of 22,000-volt circuit and 23 miles of 4000-volt circuits from the two sub-stations.

SEVERN SYSTEM

This system comprises a hydraulic generating station on Severn River at Big Chute with substations as shown in the diagram.

The generating station as originally built in 1909 by the Simcoe Railway and Power Company contained three 900-kv-a., three-phase, 2200-volt, 60-cycle, 300-rev. per min., horizontal-shaft generators direct connected to turbines each rated at 1300 horse power under 56-ft. head; two 100-kw. 580-rev. per min., 125-volt exciters direct connected to 200-h.p. turbines; two transformer banks each consisting of three 600-kv-a., 2200/25,000-volt water-cooled transformers connected delta-delta, with two outgoing 25,000-volt lines. An extension to the building has been made by the Commission during the past year, and there is now being installed one 1600-kv-a., three-phase, 60-cycle, 2200-volt 300-rev. per min. horizontal generator direct-connected to a 2300-h. p. turbine. The switching equipment is being rearranged to provide a double high-tension bus for greater flexibility in operation. Space is left for a third bank of transformers, and for two future 22,000-volt line equipments. A double low-tension bus is installed. All power is transmitted at 22,000 volts, three phase, over four lines.

An interesting feature of the transmisssion lines of this system is the long spans across Matchedash Bay at Waubaushene, one being 1135 ft. long and one 858 ft. long. The west shore tower is 175 ft. high, the middle and east shore towers are 88 ft. high. No. 00 B. & S. 19-strand copper conductors are used on these spans.

The water is taken from the river through a canal 500 ft. long to the head works and then through two steel penstocks 170 ft. long to the turbines.

EUGENIA SYSTEM

This system obtains power from a hydraulic generating station constructed by the Commission at Eugenia on the Beaver River which was placed in service in November 1915. Extensions are now almost completed to provide double the output. This development is one of the most interesting in Ontario, and has the distinction of having with one exception, the highest hydraulic head of any plant in Canada, being 552 ft. gross. It is also one of the highest heads in the world using reaction wheels.

The first installation consisted of two 2250-h.p. turbines each direct connected to 1410-kv-a., three-phase, 4000-volt, 60-cycle 900-rev. per min., horizontal-shaft generators having neutral grounded without resistance; one bank of three 900-kv-a., 4000/25,000-volt transformers connected delta-delta, single high-tension and low-tension busses, two outgoing 25,000-volt feeders and two 4000-volt feeders, one each to Markdale and Flesherton. The extensions cover enlargement of the building to accommodate two additional generating units, one of which is now being installed, and for double high-tension and low-tension busses, and for six 25,000-volt feeders. The new unit consists of a 2810-kv-a., three-phase, 60-cycle, 720-rev. per min. horizontal maximum-rated generator direct connected to a 4000-h. p. turbine. Each generator has a direct-connected 125-volt exciter of sufficient capacity to excite two generators.

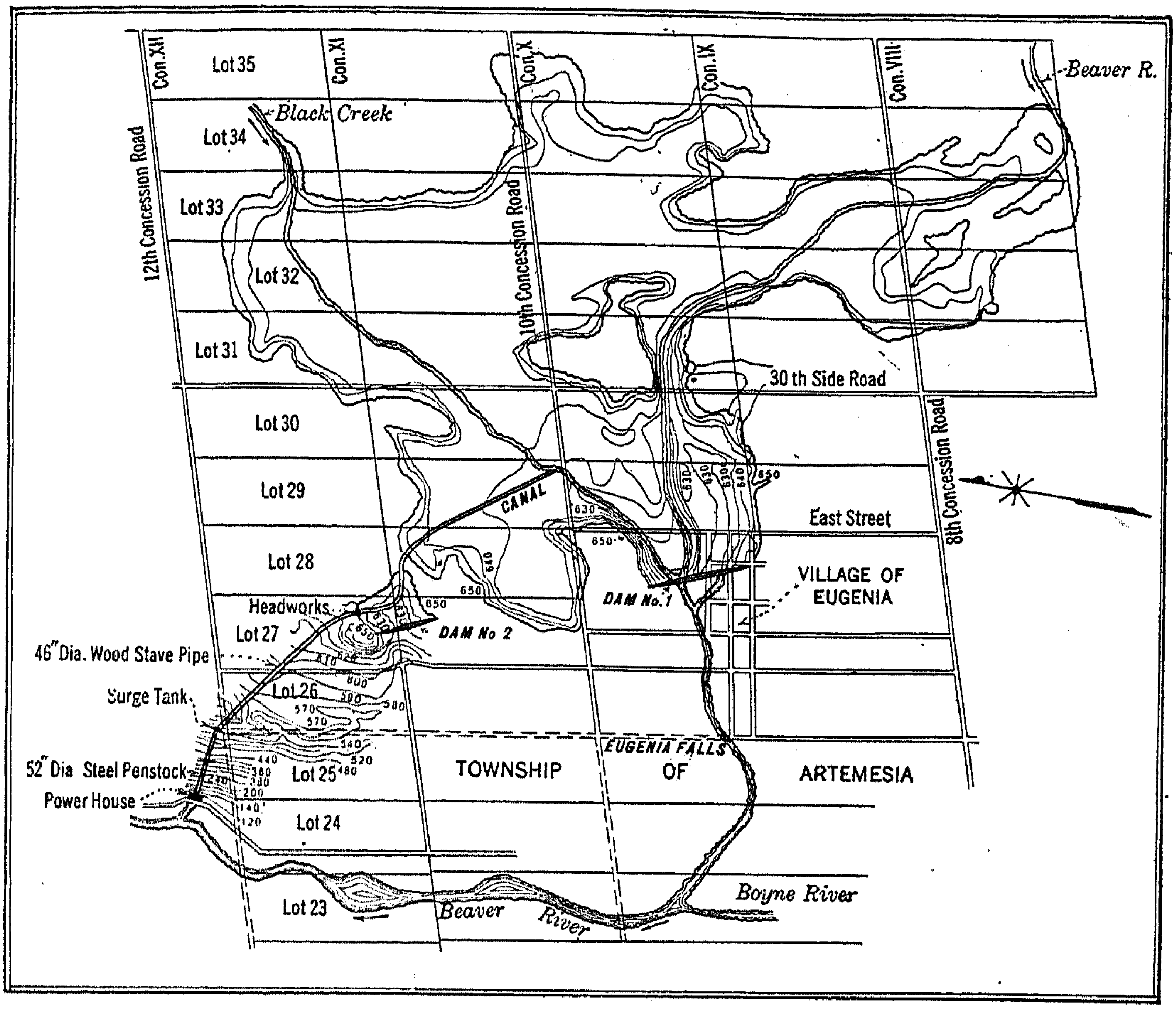

FIG. 4—EUGENIA FALLS—GENERAL PLAN OF POWER DEVELOPMENT ON BEAVER RIVER

The success of the development depended upon the storage of the water of the Beaver River and for this purpose two large storage dams were constructed as shown on accompanying plan. From the head works, a wood stave pipe 46 in. in diameter conducts the water 3400 ft. to the Johnson differential surge tank which is 105 ft. high. From this tank a 52-in. diameter steel pipe is carried 1550 ft. to the power house, which is a brick building 69 ft. wide, 112 ft. long and 34 ft. high above the generator room floor. Actual tests made at this station after the first installation was completed in 1915 gave an overall full-load efficiency of 80 per cent which shows how carefully the design was worked out.

Additional power for the Eugenia System will be obtained, when required, by the construction of further water storage systems and of a second pipe line at the Eugenia Falls development and the installation of a fourth unit, also by the construction of a station on the Saugeen River near Lake Huron which operating in parallel with the Eugenia Station will make available a total output for the system of 15,000 h. p.

The Eugenia System comprises 245 miles of 22,000-volt circuit (176 miles of lines) supplying substations at the points shown on accompanying diagram, and 50 miles of 4000-volt circuits. This system is connected to the Severn System by means of a single-circuit three-phase tie line from the Eugenia generating station to Collingwood, a distance of 24 miles.

By means of this tie line and the tie line from Wasdells' Falls to Big Chute mentioned above, the Eugenia, Severn, and Wasdells' systems are paralleled. The generating station of the Town of Orillia on the Severn River at Swift Rapids near Big Chute is also connected into the Severn system.

CENTRAL ONTARIO SYSTEM

Five main generating stations, one on the Otonabee River and four on the Trent River, all operating in parallel, supply this system. These are now fully loaded and plans are being prepared for new stations near Campbellford to provide additional power. The five exciting stations were built by the subsidiary companies of the Electric Power Company. Immediately after the operation of this system was placed under the Commission by the Ontario Government, arrangements were made to install a new unit in the Healy Falls generating station and work on the same is now nearing completion.

The following table gives the data relative to the existing developments, all generators being three-phase, 60 cycles, with 25 per cent overload guarantees:

In addition to the above stations, a 1000-kv-a., three-phase, 60-cycle, 120-rev. per min. horizontal generator owned by the Town of Campbellford in its generating station, a short distance north of the Campbellford station, delivers its output to the 2400-volt bus in this station. Also at Fenelon Falls, a small generating station is operated, containing two 400-kw., 600-volt,

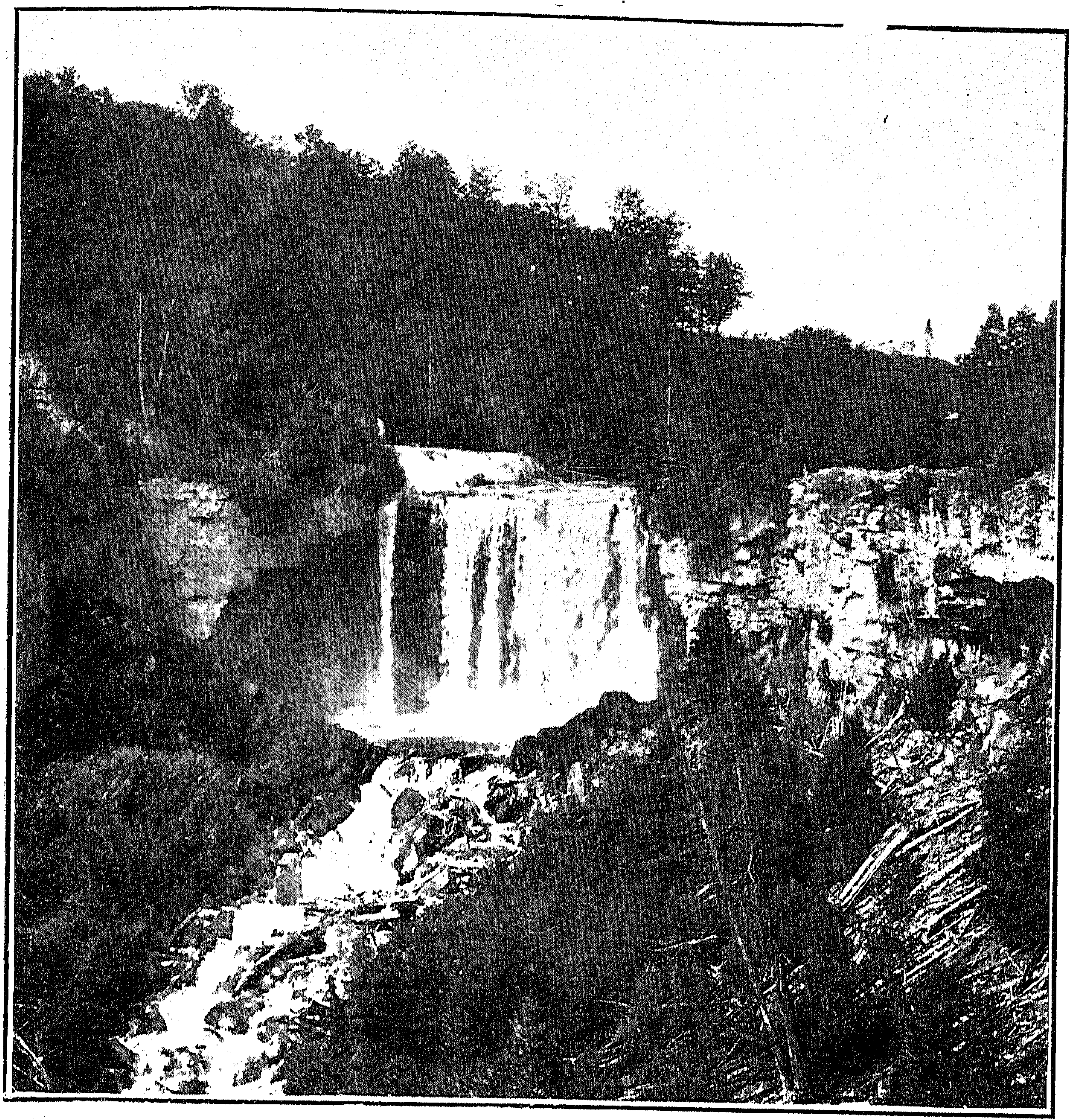

FIG. 3—EUGENIA FALLS [HULL]

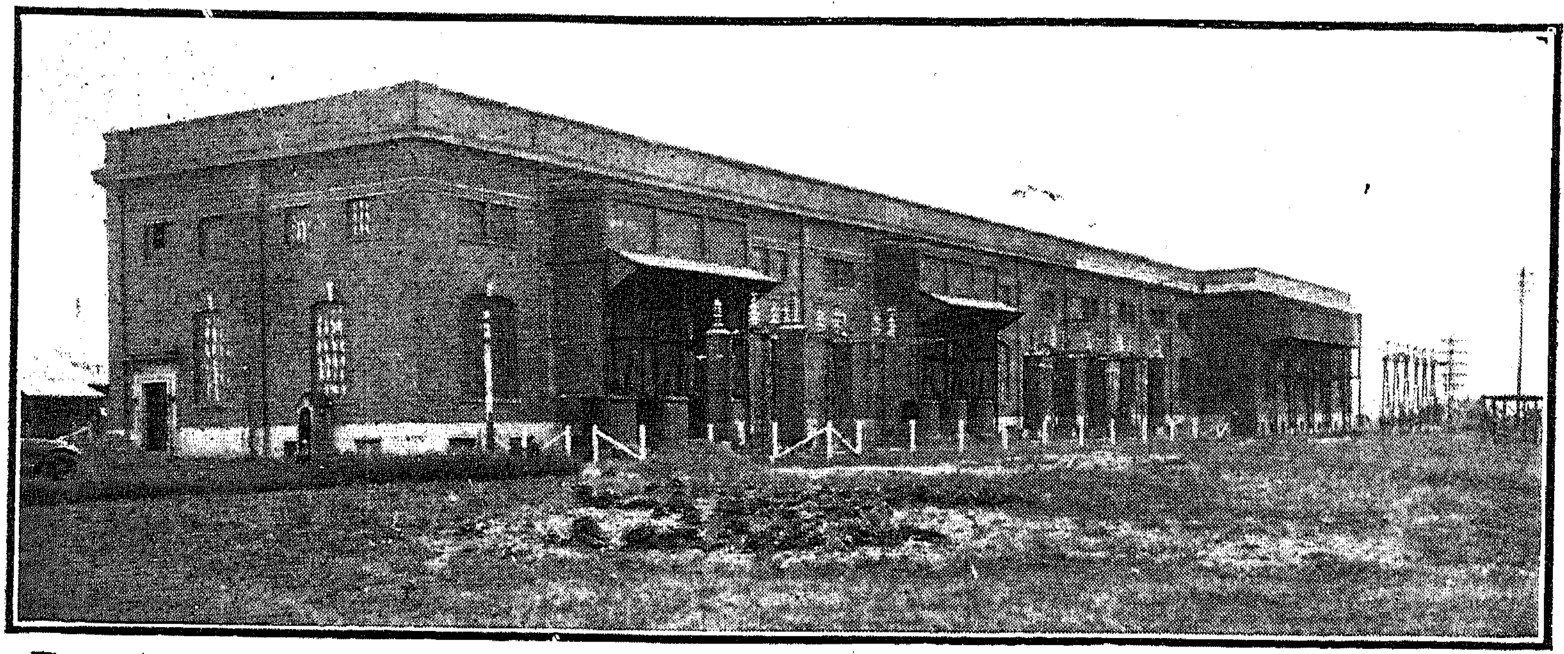

FIG. 7—VIEW OF NIAGARA TRANSFORMER STATION LOOKING SOUTHEAST

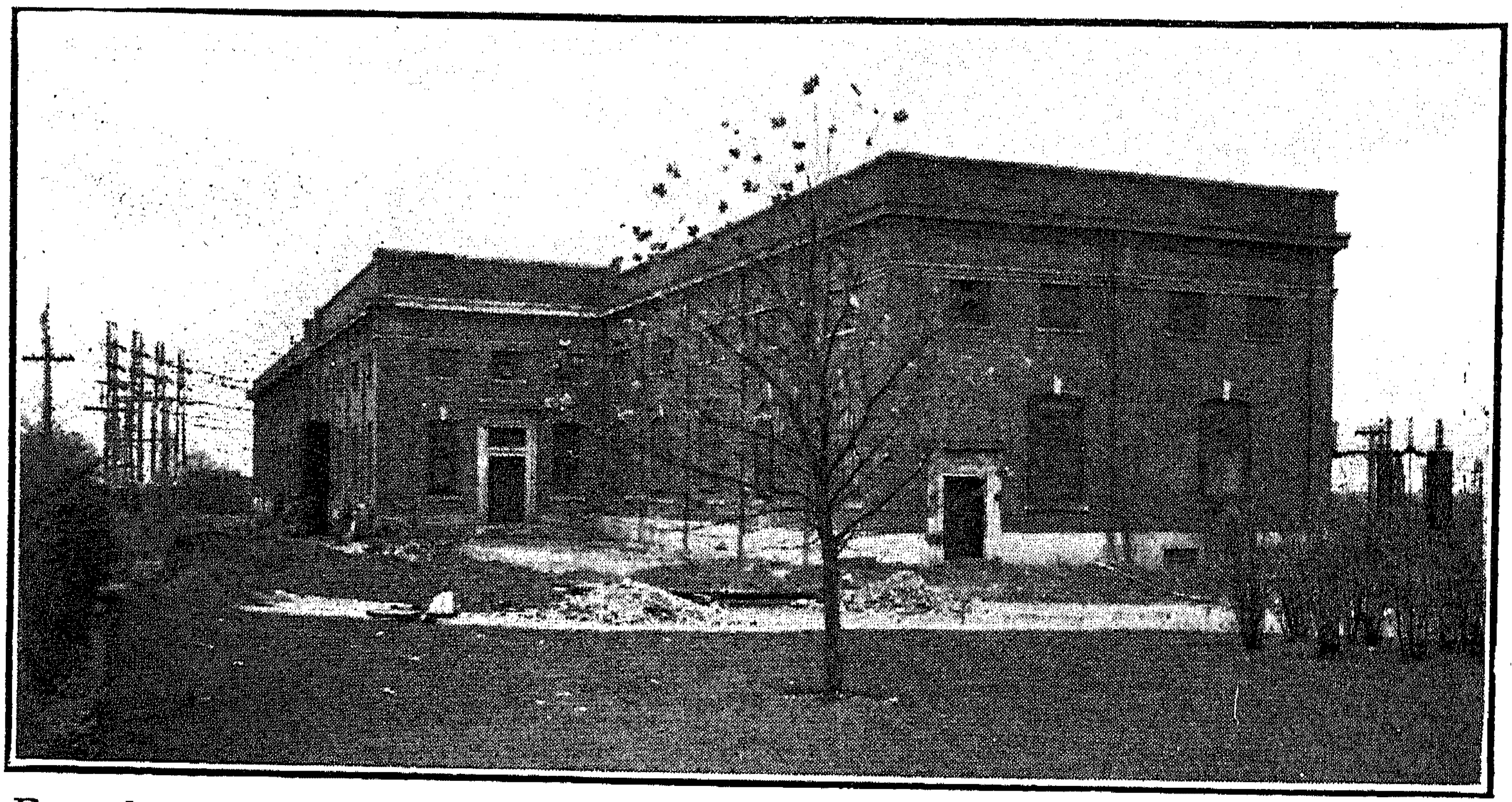

FIG. 8—VIEW OF NIAGARA TRANSFORMER STATION LOOKING SOUTHWEST

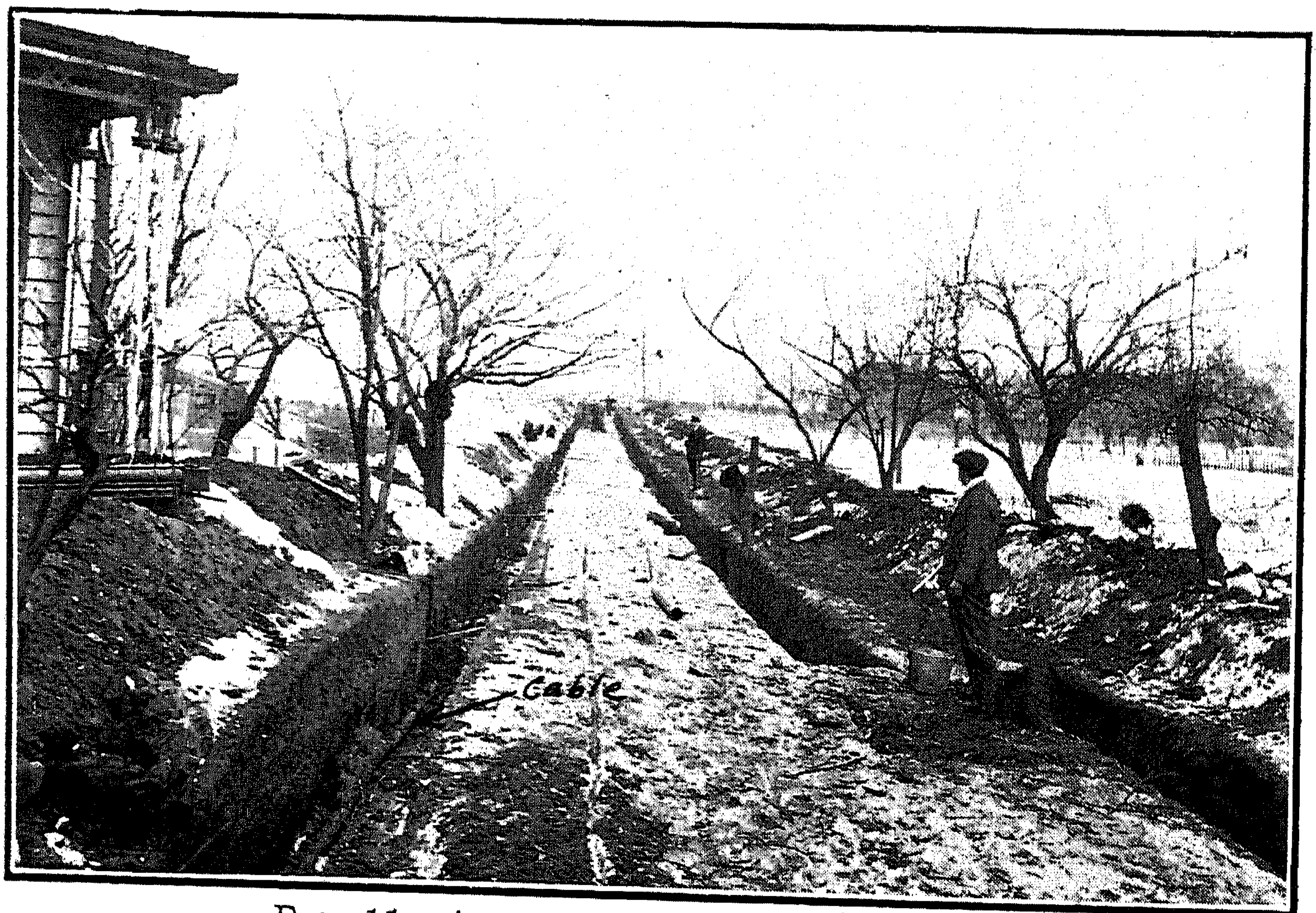

FIG. 11—ARMORED CABLE INSTALLATION

[HULL]

Location	River	Gross head feet	Units				Speed rev. per min.	Type	Generator voltage.
			Number		Rated capacity.				
			Present	Future	Generator kv-a.	Turbine h.p.			
Trenton...........	Trent	20	4	..	937.5	1400	120	Vertical	6600
Frankford	Trent	18	4	..	812.5	1200	112.5	Vertical	6600
Campbellford	Trent	23	5	..	750	1100	150	Vertical	2400
Healy Falls........	Trent	76	3*	1	3750	5600	240	Horizontal	6600
Auburn	Otonabee	18	3	1	625	950	150	Horizontal	2–6600 1–2400

*One unit now being installed.

three-phase 60-cycle, 200 rev. per min. generators connected to two 700-h. p. turbines. These feed into the 44,000-volt net work of the Central Ontario system at Lindsay the voltage being stepped up in the generating station through two banks of transformers to 11,000 volts for transmission to Lindsay.

All these stations are on the route of the Trent Valley Canal, the dams having been constructed by the Government of the Dominion of Canada.

The Trenton station, known as Sidney Dam 2 development, being at Dam 2, section 1 of Trent Valley Canal, about one mile north of the Town of Trenton, was placed in operation by the Sidney Electric Power Co. in August 1911. The four generators feed into a single sectionalized bus, and two 6600-volt feeders conduct the current to a transforming station across the road, which station also receives the output of the Frankford generating station at 6600 volts. Two 75-kw. 125-volt exciter units are installed, one being a vertical turbine-driven unit and the other a motor-driven unit. Each of these exciters is capable of exciting all generators at 25 per cent overload with 65 per cent power factor current lagging.

The Frankford station, known as Sidney Dam 5 development, is located at Dam 5 on Section 1 of Trent Canal, about four miles north of Dam 2. This station was constructed by the Sidney Electric Power Company and placed in operation a short time after the Trenton station. It has a single unsectionalized 6600-volt bus and its entire output is fed over two 6600-volt circuits to the transforming station at Dam 2.

The Trenton transforming station contains three 3000-kv-a., 6600/44,000-volt, three-phase, 60-cycle, shell-type transformers connected delta-star with ungrounded neutral, double high-tension and low-tension buses, switching equipment for three 44,000-volt lines and for six 6600-volt lines.

The Campbellford station, known as the Stephen's Dam station, was first placed in operation in 1909 and is near Dam 1, Section 5 of Trent Valley Canal. This station contains four 1125-kv-a., three-phase, core-type, 2400/44,000-volt transformers and two outgoing 44,000-volt line equipments. Excitation for the generators is obtained from two 60-kw. 125-volt exciters, one turbine-driven and one motor-driven, with two 17½-kw. belted exciters for emergency use. The station is situated some distance below the dam, the water being conducted to it through an open head race.

The station at Healy Falls is situated about seven miles above Campbellford. Water is conducted through steel penstocks from the head gates to the turbines. A long tail race excavation through rock was necessary and the enlargement of this tail race, necessitated by the installation of the third generator, is now being completed. This excavation work has been slow and difficult on account of the nature of the rock.

There are three 3750-kv-a., three-phase, shell-type, 6600/44,000-volt transformers installed, with very flexible switching equipment consisting of double 6600-volt buses, two 44,000-volt outgoing lines, a sectionalized 44,000-volt single transfer bus, with a transformer and an outgoing line forming a unit which may be operated independent of the bus. Provision is made for a fourth generator and transformer and for two additional high-tension line equipments. Excitation is provided by two 160-kw. 125-volt exciters, one being turbine-driven and one motor-driven.

The Auburn generating station is located in the north of Peterboro, on the Otonabee River 1200 ft. below the Auburn Dam. Water is taken to the turbines through an open head race 1200 ft. long paralleling the river. This station was built by the Auburn Power Co. and placed in operation in 1912 and no extensions have been made. The turbines are of the four runner type in open wheel pits 40 ft. long, 16 ft. wide and 14 ft. deep. A travelling steel gate 15 ft. 3⅜ in. by 14 ft. 3 in. is provided with motor-driven lowering and hoisting mechanism. The gate is designed to be lowered against the running water in 2½ minutes. Two 90-kw., 125-volt exciters, one being turbine driven and one motor-driven, furnish the excitation of this plant. One generator is wound for 2400 volts and feeds a section of the bus from which 2400-volt feeders may be taken. This section of bus is connected to the 6600-volt section of the bus through a bank of three 200-kv-a., 2400/6600-volt transformers. Feeders at 6600 volts, three-phase, connect this station to the transformer station adjacent which contains two 1875-kv-a., three-phase, core-type 6600/44,000-volt transformers and one outgoing 44,000-volt line connecting into the 44,000-volt network of the Central Ontario System. Some power is also delivered to Peterboro at 6600 volts from the generating station.

The accompanying map, Fig. 1, and the diagram, Fig. 5, show the location of the generating stations of the Central

Ontario System, and the extent of the transmission net work with substations. Other power sites along the Trent Valley Canal together with those described briefly above will, when developed, provide about 60,000 electrical h. p. with a maximum capacity of 75,000 electrical h.p. The present power output of the system is used for lighting, street railway and manufacturing purposes, a considerable quantity being required at Campbellford for a pulp and paper mill, and near Belleville for cement mills. All transmission lines are constructed on wood poles. The total mileage of 44,000-volt circuits is 372, with 15 miles of 11,000-volt circuit, 16.4 miles of 6600-volt circuits and 52 miles of 4000-volt circuits.

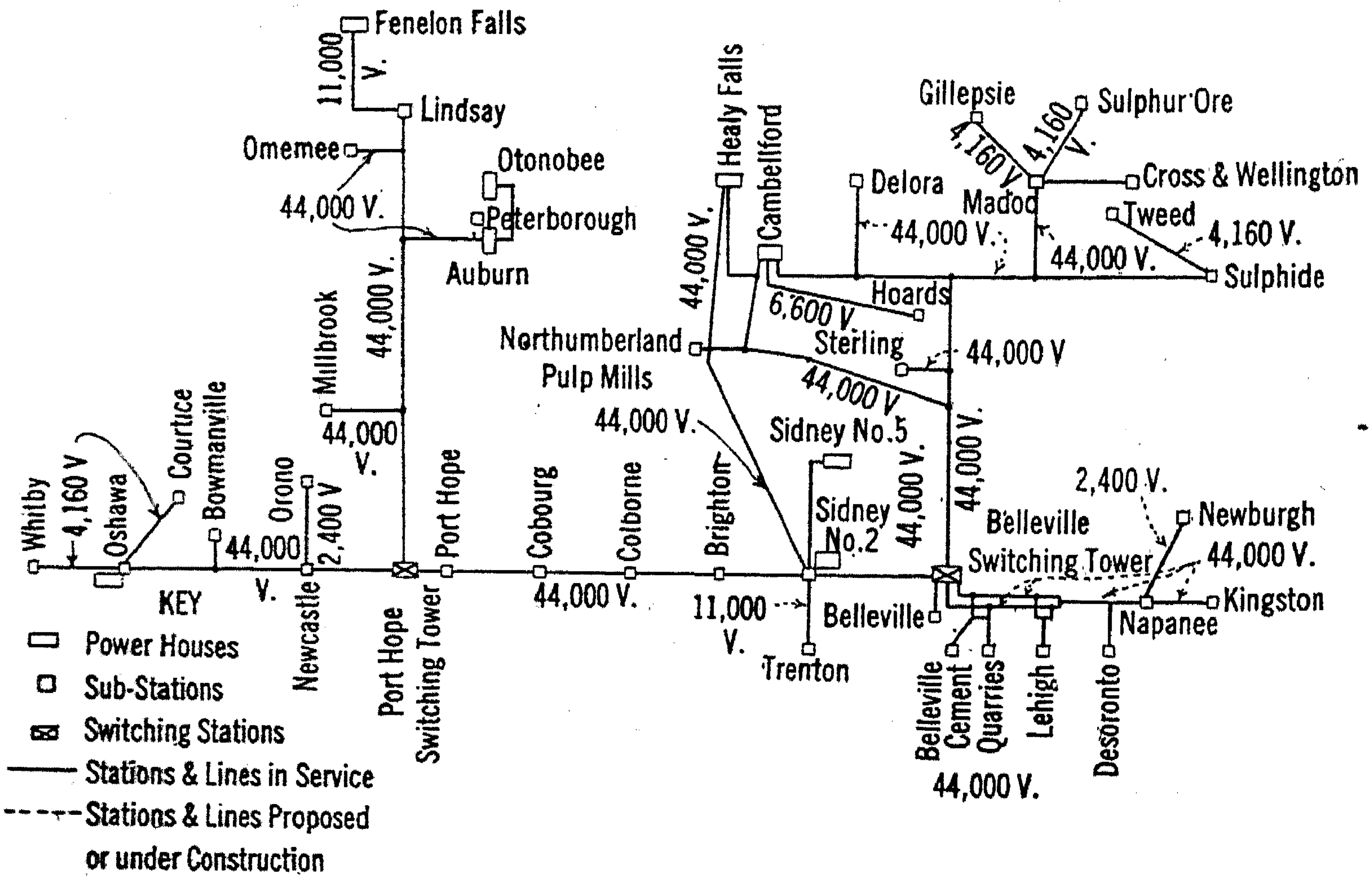

FIG. 5—CENTRAL ONTARIO SYSTEM

ST. LAWRENCE SYSTEM

This system at the present time has no generating station owned by the Commission. Power is secured by contract from the hydraulic station of the M. F. Beach Company at Iroquois, but the amount obtainable proved inadequate and was supplemented by power obtained from the steam generating station of the town of Brockville. As additional power is needed arrangements are now being made to obtain an adequate supply from another source, near Cornwall. To take this power the Commission is now constructing a transforming station near Cornwall which will contain one bank of three 1250-kv-a., single-phase, 63,500/26,400-volt, 60-cycle transformers connected star-delta and switching equipment for two incoming 110,000-volt circuits and for two outgoing 26,400-volt circuits.

This system now supplies power to Brockville, Prescott, Morrisburg, Winchester and Chesterville. Fig. 6 shows this system together with the Rideau system.

Rideau System

A new net work is being developed called the Rideau system and covering a district in the neighborhood of the Rideau River. Plans are now being prepared for a hydraulic generating station at High Falls on the Mississippi River near Clarendon, a point about 50 miles northerly from Kingston, Ontario. The installation at this point will consist of four 350-kw., 2200-volt, three-phase, 60-cycle, 300-rev. per min. horizontal generators connected to two turbines, one generator being at each end of turbine; and one 875-kv-a. 80 per cent power factor, three-phase, 60-cycle, 2200-volt, 300-rev. per min generator direct-connected to its turbine with necessary switching and transformer equipment to transmit the full 2100-kw. output, at 26,400 volts to the Rideau system.

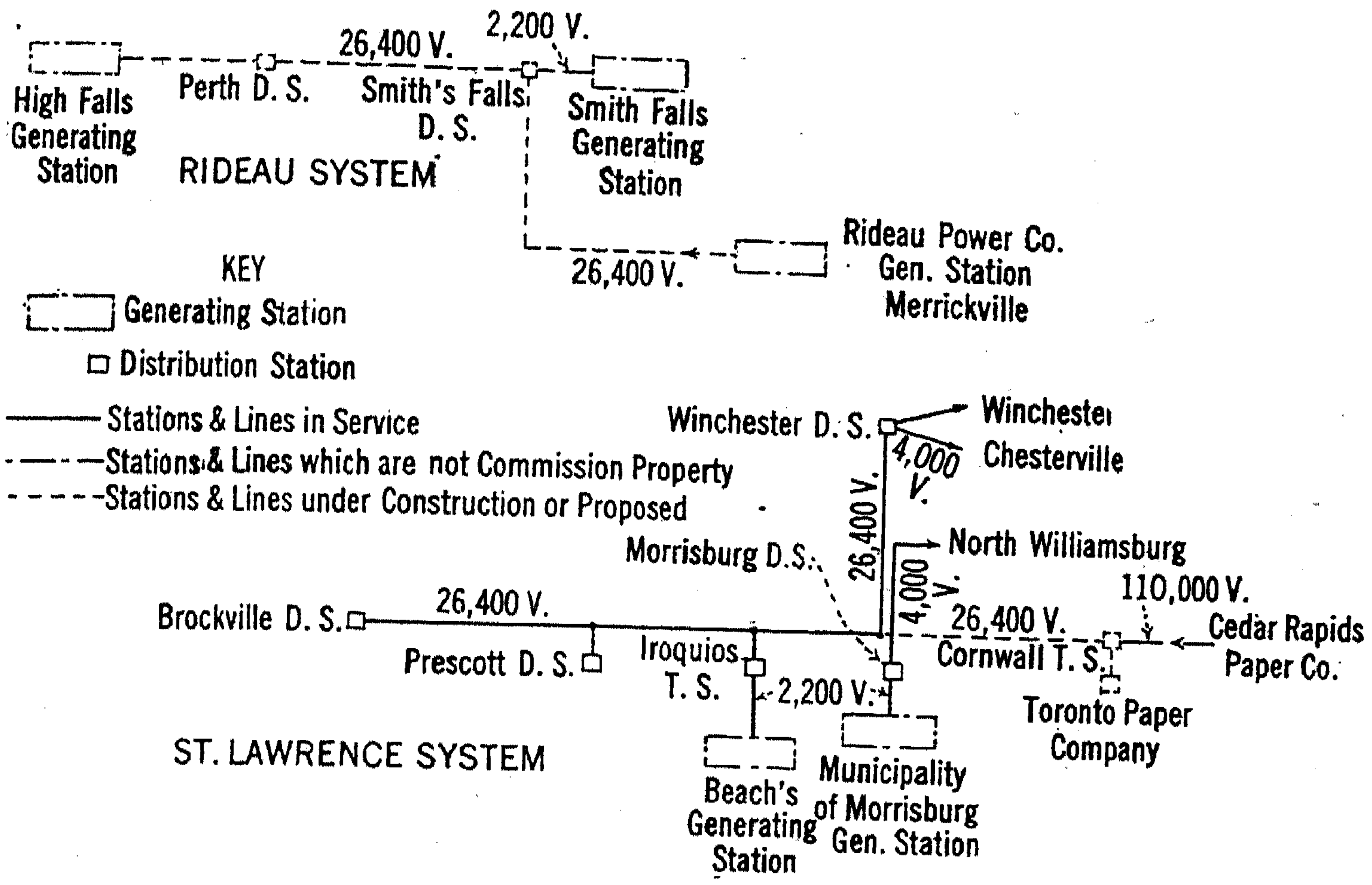

Fig. 6—St. Lawrence and Rideau Systems—60 Cycles.

A portion of this system is now in operation, power being obtained at 26,400 volts, three-phase, 60-cycles, under contract from the Rideau Power Co. at Merrickville, and transmitted to Smith Falls where at the substation the municipality's own generating stations are paralleled with this system on the low-tension bus.

This system may ultimately be connected with the St. Lawrence System.

NIAGARA SYSTEM

The Niagara System comprises all lines and substations that receive power from Niagara Falls, and covers the entire district from Niagara River to the Detroit River and River St. Clair and north to Goderich and Harriston and east to Toronto. Power is received at a transformer station at Niagara Falls from the Ontario Power Co. (now controlled and operated by the Commission) and from the Canadian Niagara Power Company, at 12,000 volts, three-phase, 25 cycles and is transformed to 110,000 volts and to 45,700 volts for transmission.

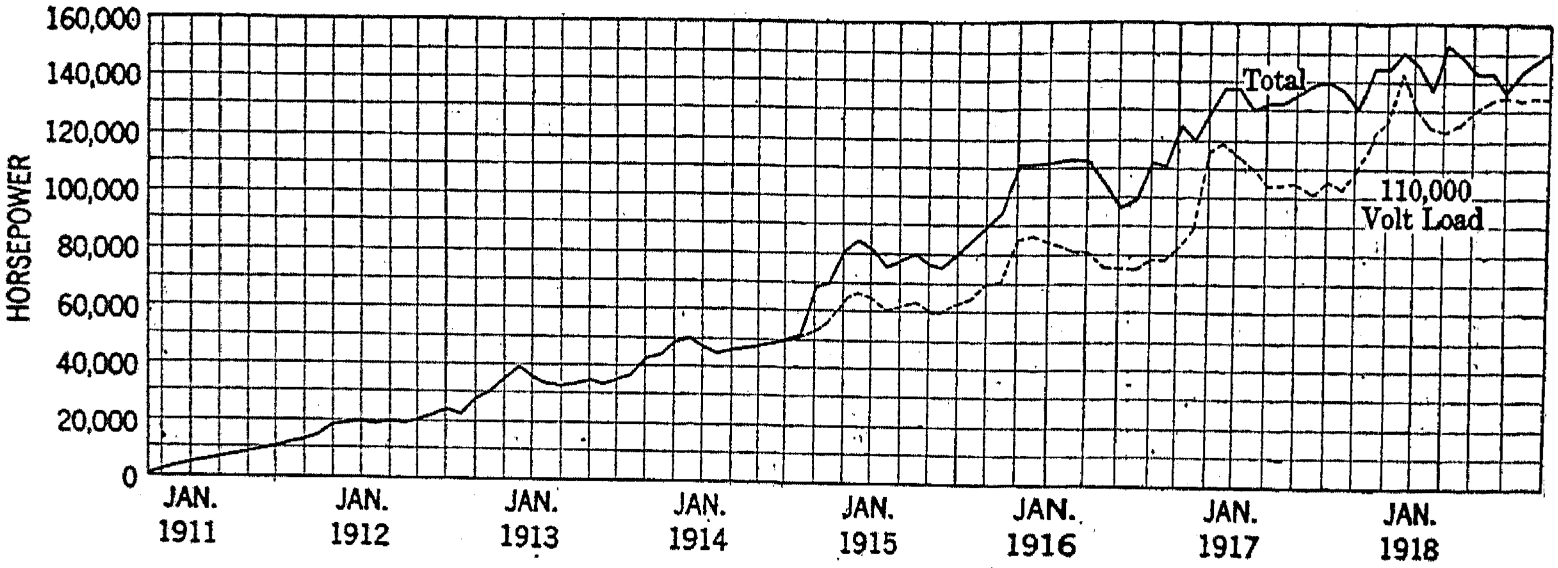

FIG. 9—CURVE SHOWING MONTHLY INCREASE OF POWER LOAD OF MUNICIPALITIES NIAGARA SYSTEM OCTOBER 1910 TO OCTOBER 1918

There is now installed in the Niagara Transformer station 132,000 kv-a. of 110,000-volt transformers in eight banks and 42,000 kv-a. of 45,700-volt transformers in four banks, not including spare units. Four banks of 110,000-volt units are composed of twelve 7500-kv-a., single-phase, shell-type 12,000/63,500-volt transformers, all other transformers being of 3500-kv-a. rating. This station, in point of transformer capacity. is the largest in the world, having a total capacity of 174,000-kv-a, with 7000 kv-a. in spare units additional. Figs. 7 and 8 show two views of this Niagara transformer station.

Four outgoing 110,000-volt circuits feed to Dundas transformer and switching station and from there power is distributed to thirteen 110,000-volt transformer stations. Four 45,700-volt lines feed to Welland to the substations of the Electric Steel & Metals Company, the Union Carbide Co. and the city of Welland. From this latter point a single-circuit, 45,700-volt line runs to Dunnville.

This system was placed in operation in October 1910. The curve in Fig. 9 shows the monthly increase in power load of Municipalities on this System from October 1910 to October 1918. This gives some idea of the growth of the system in this period but the latter portion of the curve cannot be regarded as normal owing to the increase in power required for munitions manufacture and the consequent restrictions on other uses to cut their requirements to a minimum.

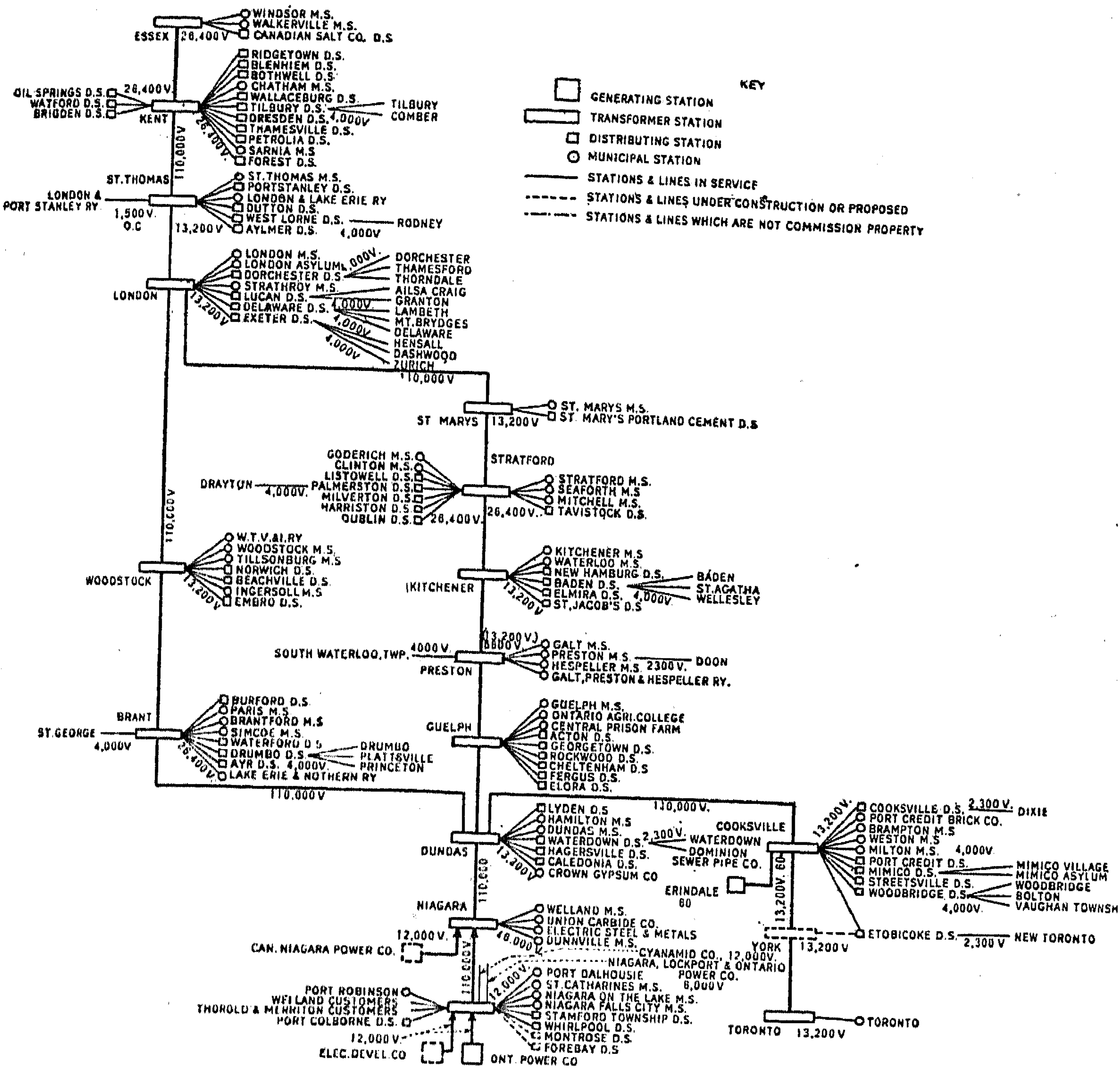

FIG. 10—DIAGRAM OF STATIONS—NIAGARA SYSTEM

The transmission net work, see Fig. 1 and Fig. 10, on this system contains 760 miles of 110,000-volt circuits; 65 miles of 45,700-volt circuits; 529 miles of 26,400-volt circuits; 489 miles of 13,200-volt circuits; 27 miles of 6600-volt circuits; 180 miles of 4000-volt circuits; and 20 miles of 2200-volt circuits. The 110,000-volt and 45,700-volt circuits are carried on steel towers (with exception of one 45,700-volt line from Welland to Dunnville), while the other circuits mentioned above are standard wood pole line construction.

The Commission early in 1917 purchased the Erindale Power Company which had a hydraulic generating station on the Credit River at Erindale, 14 miles west of Toronto, containing two 600-kw. three-phase, 60-cycle, 13,200-volt, 200-rev. per min. generators, each direct-connected to a 1000-h. p. turbine designed for a 60-ft. head. The present operating head is about 50 ft. Water is carried from the dam to the power house through a 12 ft. 6 in. diameter tunnel 900 ft. long. Two 60-kw. belted exciters furnish the excitation.

The output of this generating station is now transmitted to the Cooksville transformer station at 13,200 volts, 60 cycles, and is fed into the 13,200-volt, 25-cycle bus in that station through a 1000-kv-a. 60/25-cycle frequency changer set, augmenting the power supply on the Niagara system, and serving also to raise the power factor at the Cooksville station.

It is not the writer's intention to describe the plant of the Ontario Power Co. as descriptions of this plant have appeared in the A. I. E. E. TRANSACTIONS (See Volume XXIV 1905, p. 807). Extensions to the plant have been made since then until, when the operation was taken over by the Commission in 1917, the generating station contained 14 generator units, generating 12,000-volt, three-phase, 25-cycle power with a total rating of 149,012-kv-a. The original excitation scheme had been changed by the Ontario Power Company and a very complete description of this scheme as it now exists may be found in the *Electric Journal* Vol. 1914 p. 612 in article by Mr. J. A. Johnston. The distributing station of the Company contains the 12,000- 30,000- and 60,000-volt buses, the transformer banks and the outgoing line equipments and the control room for the plant. The switching scheme as originally laid out has been considerably re-arranged the buses being sectionalized and connected through reactors, these changes being due to the great amount of power generated and to the large proportion leaving the station at generator voltage.

The Commission, on behalf of the company, is now making extensions to the generating station. The building is being extended north about 90 ft. and a third pipe line of temporary nature consisting of wood stave pipe is being constructed. Two additional generating units are being installed each rated at 15,000 kv-a. maximum rating at 75 per cent power factor.

These new generators are the same speed as the previous units namely 187.5 rev. per min. and the frames are the same size as the 8775-kv-a. units most recently installed. They have however a higher temperature guarantee and are so designed that they may be operated as synchronous condensers at zero power factor if it ever becomes necessary to remove them from this station. These generators were made in Canada, under war conditions, and all parts, except the laminated steel and the insulation, were produced in this country. This speaks well for Canadian industry in these trying times.

Each of these new units is excited by a 125-kw. 250-volt, motor-driven exciter, thus extending the excitation scheme referred to above.

The Ontario Power Company obtains some power from the Electrical Development Company's generating station at Niagara, this being brought into the distributing station on two incoming 12,000-volt feeders. When the present extensions to the distributing station are completed, there will be six main bus sections connected together through reactors. Three of these bus sections will supply the power delivered to the 12,000-volt bus in the Commission's transformer station to which the feeders from the Canadian Niagara Power Company are also connected through a bus reactor. The concentration of generator capacity on this 12,000-volt bus is consequently very great and has necessitated material changes in the switching equipment and bus construction. The studies covering the installation of reactors on this 12,000-volt system have been most interesting. As these studies are still not fully completed, no diagram of the connections for the Ontario Power Company stations and the Commission's Niagara transformer station can be shown at this time. It will be sufficient to state that there is no 12,000-volt bus in the generating station, and that each generator feeds through an automatic oil circuit breaker in the generating station to its individual cables in the cable tunnels up the hill, thence to its group of circuit breakers in the distributing station where switching arrangements are such that each generator and each feeder may be connected to either of two bus sections. In the Commission's Niagara transformer station there is a single sectionalized 12,000-volt transfer bus system so arranged that feeders are connected through an auxiliary bus to the main bus or direct to a transformer bank.

It is of interest to note that the cables for the two new 15,000-

kv-a. generators were purchased after laboratory tests were made on manufacturers' samples to determine the dielectric losses and tendency of compound to flow, and guarantees on dielectric losses were obtained from the contractor.

Another interesting point in connection with the feeders entering the Commission's transformer station is that eight of these are three-conductor, armored cables laid directly in the earth without ducts. These armored cables all have sector-shaped conductors, with 8/32-in. x 8/32-in. paper insulation, a lead sheath and a double steel tape armor with jute bedding and jute covering. They are placed three feet below the surface and two and three feet apart center to center on private right of way. Where these buried cables cross ducts containing other cables, arrangements are provided to moisten the surrounding earth either by sprays above the surface or by porous tile in the ground. Figs. 11 and 12 show views of this armored cable installation, Fig. 10 showing the ventilating pipes at street crossing, the cables being in 6-inch cast iron pipe across the street.

THE QUEENSTON DEVELOPMENT

When the Commission in March 1908 contracted with the Ontario Power Company for 100,000 horse power, it was thought by many that such amount of power would meet the requirement for many years. This supply was however exhausted in 1915 that is, in 5 years from date of first delivery of power, and the additional power secured by arrangements with the Canadian Niagara Power Company has proved insufficient to meet the demands. It became necessary therefore to look to a new development to secure a further supply and the final decision was to proceed with a development called the Queenston Development which is authorized by an Act passed by Ontario Government in April 1917 called The Ontario Niagara Development Act.

Between Lakes Erie and Ontario there is a difference in elevation of 330 feet. The greatest net head now utilized on the Canadian side at Niagara Falls is about 160 ft. Canada is entitled to divert 36,000 cu. ft. per second from the Niagara River, and of this amount, by an Ontario Government Order in Council in 1915, the Commission were allotted 6610 cu. ft. per second. The Ontario Power Company were allotted 11,180 cu. ft. per second, so that, within the control of the Commission, there is now available 17,799 cu. ft. per second.

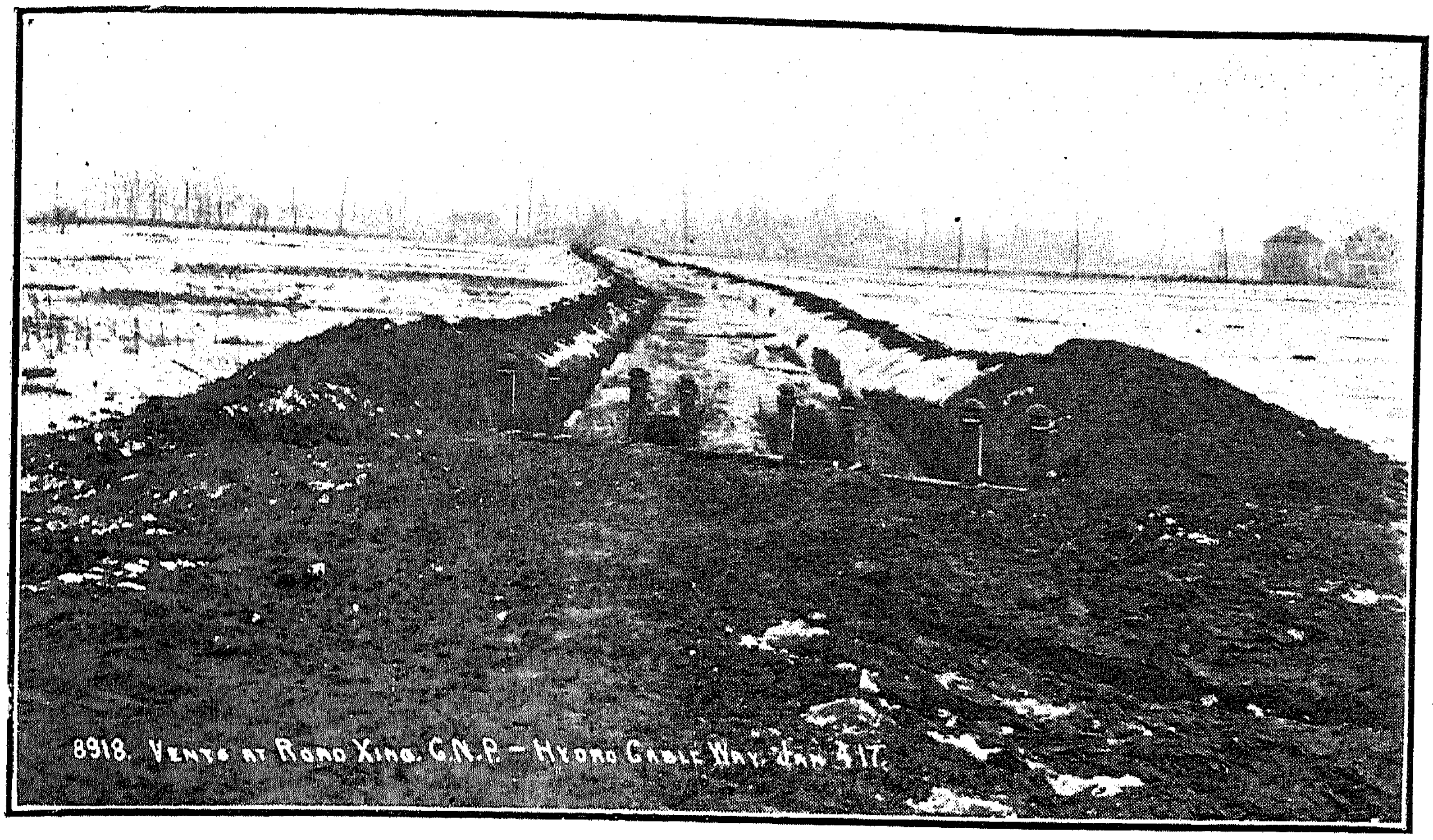
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FIG. 12—ARMORED CABLE INSTALLATION

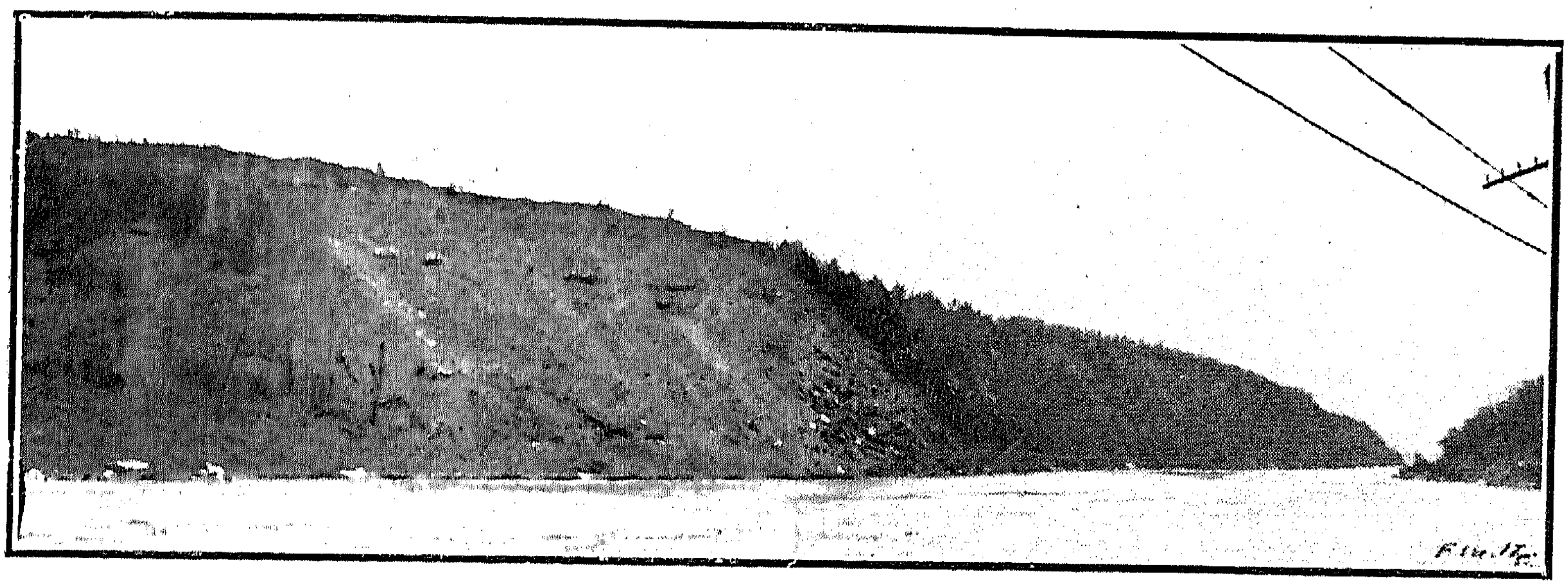

FIG. 14—QUEENSTOWN POWER HOUSE SITE [HULL]

PLATE LXXVII.
A. I. E. E.
VOL. XXXVII, 1918

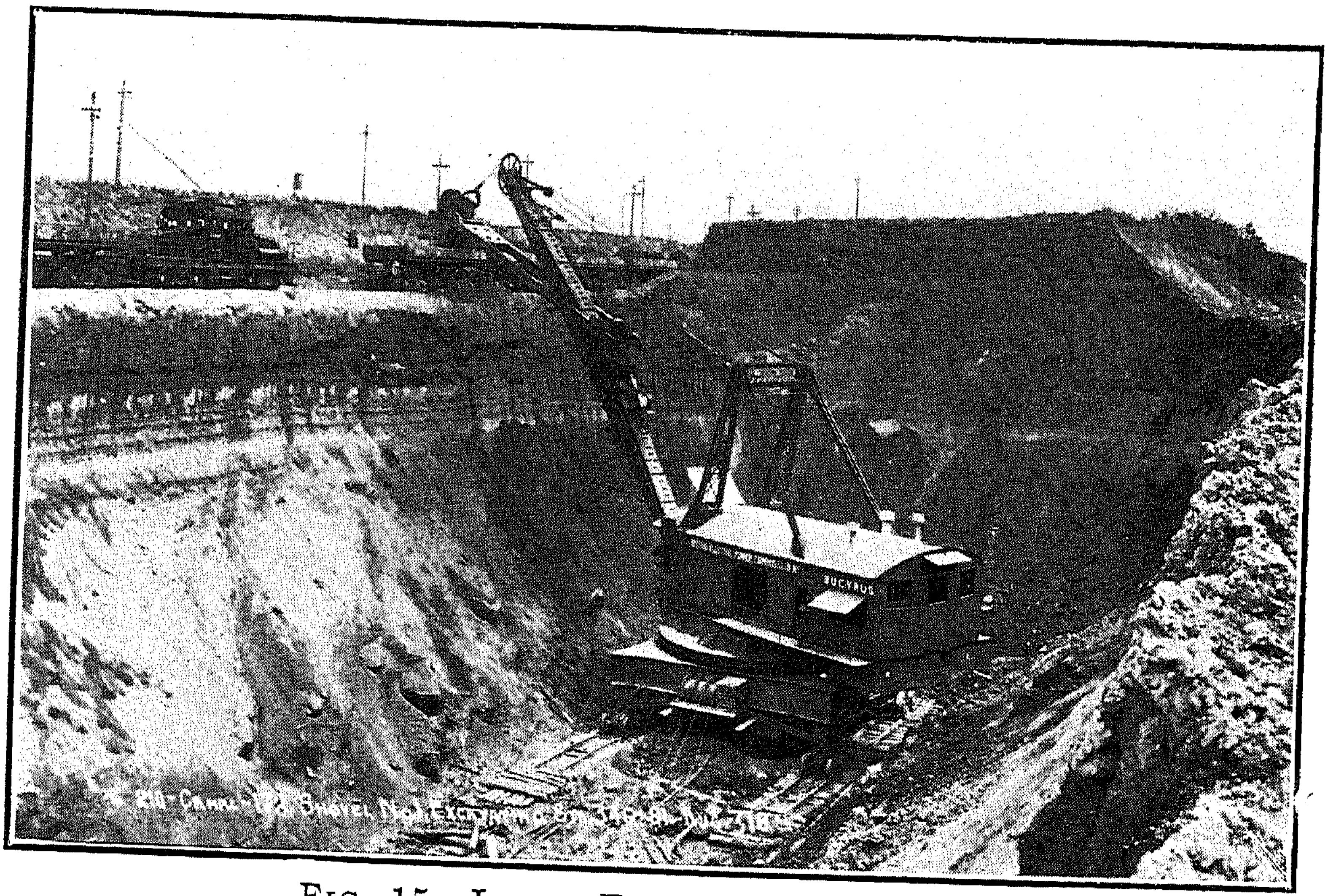

Fig. 15—Large Electric Shovel

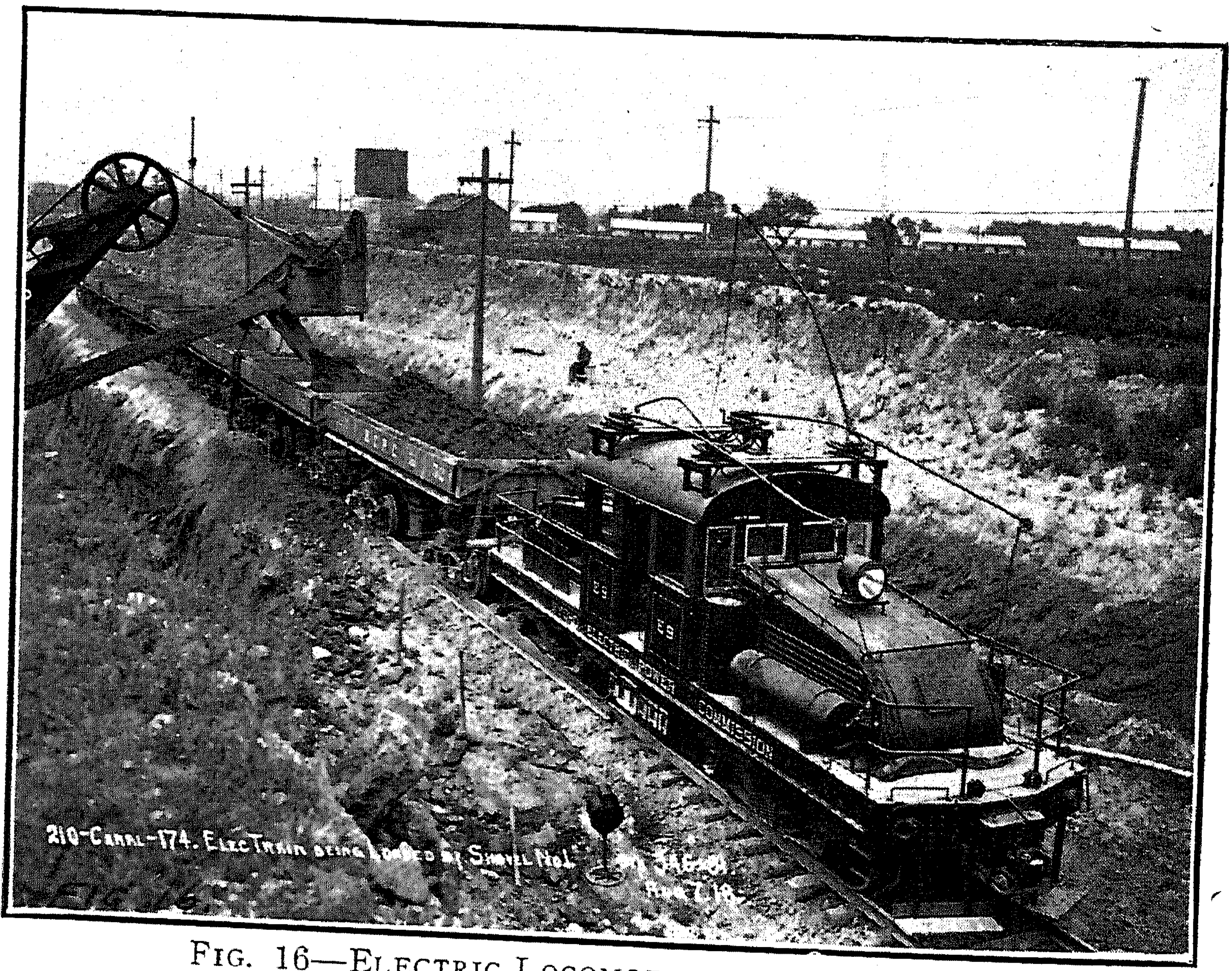

Fig. 16—Electric Locomotive and Dump Cars

[HULL]

To obtain the greatest amount of power from the water available, after careful surveys and studies, it was decided to locate the generating station just above Queenston, and to take water from the Niagara River, through the Welland River (reversing the flow in same) and canals encircling the City of Niagara Falls to Queenston. The first canal is now under construction and the accompanying map of the Niagara District, shows its location. Fig. 13 shows a general plan of this scheme.

The total length of waterway from the Niagara River to the generating station site is about 12½ miles, 4¼ miles of this distance being in the Welland River, and the balance in an excavated canal. The net effective head will be about 305 ft. and the first canal is normally designed for 10,000 cu. ft. per second at minimum low water.

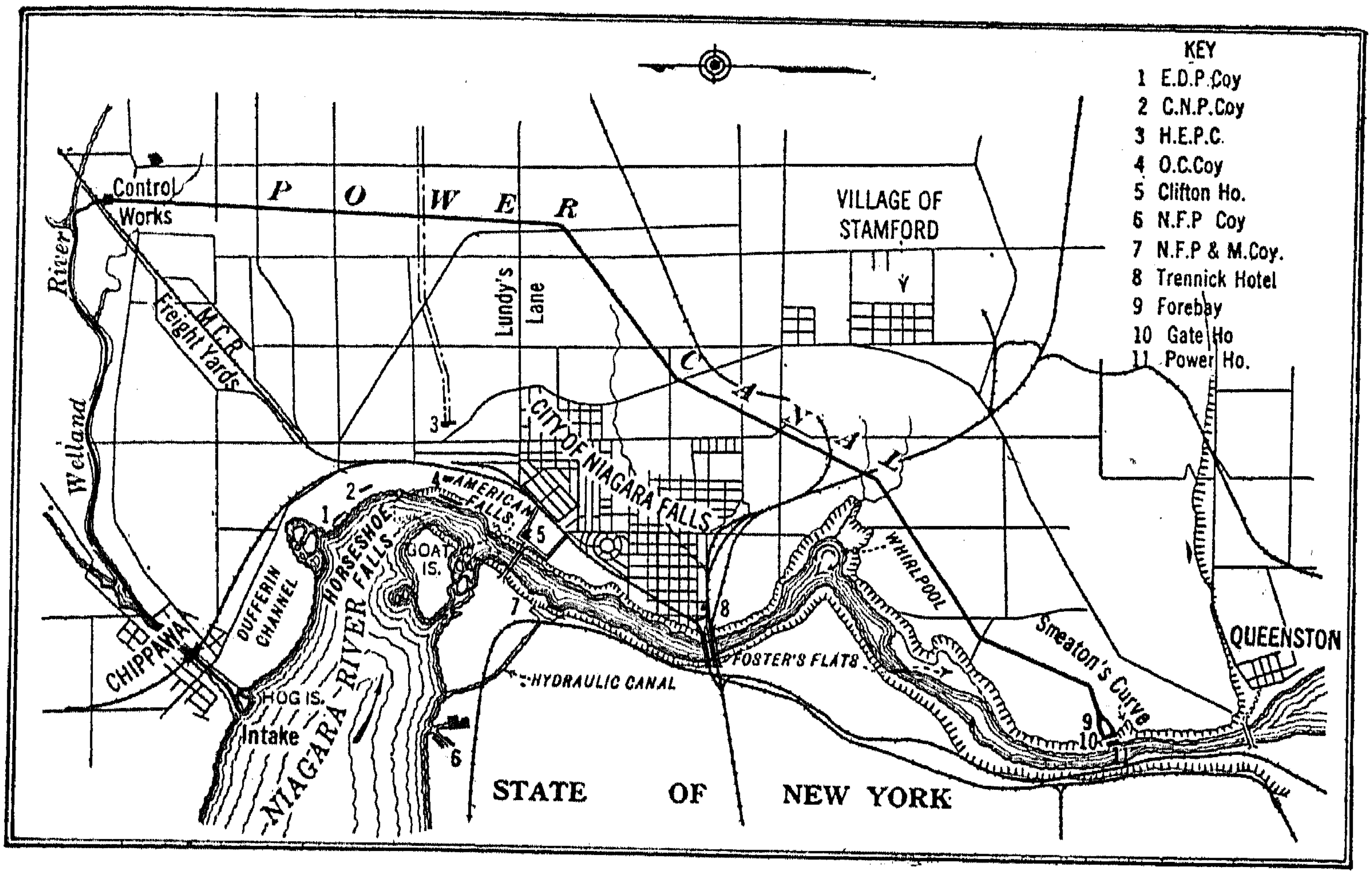

Fig. 13—General Plan of Niagara Power Development—Hydro-Electric Power Commission

The generating station will be located about one mile up stream from Queenston, in the Gorge just at the end of the last rapids in the river. At this point the banks are steep and short penstocks only will be required. Provision is being made for extensions, and right-of-way for two additional canals has been purchased. Fig. 14 shows a view of power house site.

To show the effect of utilizing the greatest possible head of water, it may be pointed out that about 30 h. p. will be developed for each cubic foot per second in this development, whereas about 14 h. p. is all that is obtained in existing plants at Niagara Falls.

The development now under construction is designed as regards canal, forebay, gate house substructure and power house substructure, for a capacity of 300,000 h. p. It is proposed to construct the gatehouse and power house superstructure for an initial installation of 200,000 h. p. in four units. The designs are made so that extensions of power house and gate house can be made to almost any extent. Future plans contemplate the use of 100,000-h. p. units.

The turbine speed has been fixed at 187.5 rev. per min. and specification for the generators are now being issued calling for 45,000-kv-a., 80 per cent power factor, 12,000-volt, three-phase, maximum-rated generators of vertical type equipped with thrust bearings to take the weight of the rotating part of the generator plus the downward thrust of the turbines. Direct-connected exciters mounted above the thrust bearings are proposed. The generators will be liberally equipped with embedded temperature detectors so that close and accurate observations may be kept of operating temperatures. The specified maximum temperature of any part of the generator at rated load is 100 deg. cent. with cooling air at 40 deg. cent. The ventilation problem for such units is of great importance, each unit requiring approximately 125,000 cu. ft. of air per minute. Plans for the switching equipment and transforming equipment are now being developed and as they are not in final form, it is not opportune to give any detail description. It is proposed to install double 12,000-volt sectionalized buses, banks of transformers consisting of three 15,000-kv-a., single-phase units, and to lay out the switching equipment so that one generator, one bank of transformers, and one outgoing high-tension line will be a unit. Bus tie reactors will be provided, and the short-circuit current will be limited to a value that can safely be handled on the circuits and circuit breakers.

It is proposed to install two small service generators to supply power required for the station services such as pumping, cranes, lighting, machine shop, ventilating, etc.

To carry on the construction work on this development it was decided to make as extensive use of electric power as possible. Two large electrically operated revolving shovels each fitted with an eight-cu. yd. bucket for earth excavation and of capacity to handle a five-ton bucket in rock were provided. See Fig. 15. The larger of these two shovels has a boom 90 ft. long and a dipper stick 80 ft. long, while the smaller shovel has a boom 80 ft. long and a dipper stick 58 ft. long. Each shovel has motors of a nominal total rating of 715 h. p. on a half-hour rating. Three shovels weigh over 300 tons each and have a capacity of 5000 cu. yd. of earth per day. Five other electrically operated shovels are also provided having bucket capacities ranging from 7/8 to 4½ cu. yd.

The railway equipment of the construction work includes 150 dump cars of 20 cu. yd. capacity each, six 40-ton steam locomotives and twelve 50-ton electric locomotives, as shown in Fig. 16.

The electric power required for shovels, trains, air compressors etc., is obtained from the Ontario Power Company's station and is transmitted over two overhead feeders to the Whirlpool Substation located near the Whirlpool. A similar substation will shortly be erected near Montrose on the southerly section of the canal. The Whirlpool substation is of semi-permanent construction and contains switching equipment for the two 12,000-volt incoming lines, and for three 1500-kv-a., 12,000/4000-volt transformers, four 500-kw. 600-volt, d-c. synchronous converters with their 12,000-volt transformers. An adjoining room contains eight air compressors of 1000 cu. ft. per minute capacity each against 125 lb. pressure, belt driven from 550-volt motors, supplied through two banks of 200-kv-a., 2300/550-volt transformers. Air is distributed up and down the canal for a distance of three miles from this substation to operate rock drills, channellers and forges, etc.

Power is distributed up and down the canal by 4000-volt, 25-cycle, three-phase, four-wire, grounded-neutral circuits to which the shovels are connected through flexible armored cable. A double track railway has been built for the full length of the canal with a branch to the main dumping point at St. Davids. These railway lines are electrified, the trolley wires being offset to one side of the track so as not to interfere with the shovels.

The substation at Montrose will be identical with that at Whirlpool and it will similarly serve to supply power for the work on the upper end of the canal.

The extensive use of electric power on the construction of works of the magnitude of this development is working out well and is resulting in a great saving of coal.

The enormous amount of power to be transmitted from this Queenston development involves much engineering study to work out a solution particularly when it is borne in mind that a large amount of capital is invested in existing stations and lines operating at 110,000 volts. These studies are now under way and it is to be hoped that at some later date, there will be placed before the Institute a more complete description of this Queenston Development and its transmission lines connecting in to the Commission's Niagara System.

For those who wish further information on the subject of electric power generation in Ontario, reference may be made to the following:

1. Annual Reports of Hydro Electric Power Commission of Ontario from 1908 to date.
2. The Power Commission Act with Amendments—Ontario Government.
3. Report of Commission of Conservation, Canada, on "Water Powers of Canada" 1911, also 8th Annual Report (1917) of this Commission.
4. Report of Dominion Water Power Branch (Canada) on "Water Powers of Canada."
5. Volumes of "Canadian Engineer" Toronto.
6. Volumes of "Electrical News" Toronto.
7. Transactions of Canadian Society of Civil Engineers, (now the Engineering Institute of Canada.

DISCUSSION ON "ELECTRIC POWER DEVELOPMENT IN ONTARIO" (HULL), TORONTO, CANADA, NOVEMBER 22, 1918.

H. U. Hart: The Hydroelectric Power Commission has built up a remarkable system, having in operation 760 (1,223 km.) miles of 110,000-volt transmission lines, and the largest transformer station in the world—174,000 kv-a. installed transformer capacity. The engineers of the Commission have built many switching and transformer stations of all sizes and voltages, and from their experience have made up what they term standard stations, the drawings for the complete building including the location and dimensions of transformers and switching apparatus for various sized stations for 110,000, 46,000, 22,000, and 2200-volt substations. This standardization saves a great amount of engineering work to the manufacturer and in the erection of the station.

It is a matter for congratulation to the engineers of the Commission and to the manufacturers associated in the work to see how successfully the 110,000-volt transformers and other apparatus have performed their work.

I am very sorry Mr. Hull did not have time to tell us of the early operation of these plants, and possibly some time later he may do so. Reactance coils at that date were not developed or used to such an extent as at the present time, and due to line trouble at the start of operation which occurred frequently, trouble developed in the circuit breakers due to the carbonization of the oil from frequent openings, and also to the great amount of power in the arc under short-circuit conditions. The circuit breaker tanks were not strong enough for this abnormal service.

A quick remedy for this trouble in 1910, was to connect two 110,000-volt circuit breakers in series, making one a resistance breaker which tripped first. The 110,000-volt resistance was made up of cast iron grids mounted on pillow-type porcelains. The use of this resistance breaker alone permitted operation at that time and I believe is still giving satisfactory service.

The Queenston development, when completed, will be the largest in size of any of the hydroelectric developments, both as regards the size of units, and the total capacity of station.

W. G. Hewson: I would like to ask Mr. Hull, or any other person who can advise us, regarding the relative efficiency of the various plants in the Niagara district, for instance the DeCew plant. These are days of efficiency and you must make full use of everything you have. I think there is an advertisement running through the papers which asks the question "How many hides has a cow?" which would seem to indicate that all the hides must be used. Those of us who attend our own furnaces know that the coal mine owner is selling us not only his coal, but the rock and earth found along with it.

I noticed an article the other day in a hand book published by the Institute in 1904 describing the developments found in existence at Niagara Falls. Apparently the plants that are now supposed to be inefficient were then regarded as highly efficient, when compared with the three former schemes that had been proposed for utilizing the power at Niagara Falls. The first scheme was a pneumatic one in which air compressors in a central power house were to be driven by turbines and the air then piped to air motors in surrounding plants. Another scheme was to run horizontal radial shafts from the generating station and connect the various plants to these shafts by means of clutches.

Arthur H. Hull: Answering Mr. Hewson's question, I will say we have no efficiency figures available for the machines in the Queenston plant. I do not think the time is opportune to make any comparison. It is hard to say whether the generator efficiency will be 95 or 96, or whether the turbine efficiency will be 90 or 85. The designers calculate on certain figures, but they may or may not get them in these large units. The units are particularly large. I believe it is the largest hydroelectric turbine that has been built for this head and the generators are the largest of the type made for the speed.

Personally, I have no figures on the efficiency of the De Cew Falls plant. If Mr. Acres were here, he could probably give some idea of it.

Thomas A. Worcester: I was in touch with the earlier developments of the hydroelectric transmission schemes, and I marvel when I look at the diagrams which Mr. Hull has shown to see how the whole undertaking has grown. It is many times larger than was anticipated when the original layout was made.

There is one thing which I think is quite noticeable, the original 110,000-volt distribution system was developed for 25 cycles. On all of the smaller systems throughout the northern part of the territory, the development seems to tend toward 60 cycles. There must be a definite reason. We possibly all know the reasons for the early development of the 110,000 volts and 25 cycles, but we may not know why the other developments were made at 60 cycles. I think it would be interesting if Mr. Hull would explain this. It is also noticeable that the new Queenston is to be made at 25 cycles. Furthermore, it is proposed to make the high-tension voltage 110,000 volts. There is no doubt a reason for this. Possibly the load can be picked up within the range of 110,000-volt distribution, but when we notice the extremely large territory to be covered, and the fact that there is a large concentration of power at the Falls, and possibly not so much available in the northern part of the province, one wonders whether 110,000 volts will be sufficient for the ultimate development of the Queenston plant.

I believe Mr. Hull mentioned 300,000 kv-a. as the immediate prospective development. No doubt, this will not be the ultimate, but perhaps it will be a million. Will 110,000 volts be suitable for distribution for 1,000,000 kv-a? I think if Mr.

Hull could answer a few of these points in detail, it would be interesting to all of us.

Arthur H. Hull: In regard to the 60-cycle frequency for the smaller system, I will point out that these systems were acquired by the Commission through purchase, and they had themselves grown from small isolated steam plants, plants where possibly 133 cycles were used, and 60 cycles was the next step.

On the Central Ontario system, I believe, the decision to use 60 cycles was made on account of the feeling at that time that 25 cycles did not give the most beneficial results for incandescent lighting.

In addition, their transmission distances are comparatively short. The Central Ontario system has power houses scattered through its system. The Wasdell's, Muskoka and Eugenia systems are in a district whose adjacent sections were served by the Severn system which operated at 60 cycles and naturally followed the frequency of the Severn system.

As to the system near North Bay, that was served by a steam plant in North Bay, operating at 60 cycles, and the generating station was built for 60 cycles, in order to be able to parallel with the steam plant, and that steam plant is now being used at North Bay as an auxiliary serving as synchronous condensers for voltage regulation or for emergency supply as conditions demand.

As regards the use of 110,000-volts on the transmission lines from Queenston you have asked a big question. It is a problem we are working on at the present time. The possibilities are that the plant will be built for about 190,000 volts, which is the *Y* voltage of 110,000 volts. The transformers may be operated at 110,000 volts delta to start with.

There will be no necessity for paralleling the low-tension side of the Queenston plant with the other generating stations at Niagara Falls, so that they could be paralleled on the high tension with the Queenston low-tension bus out of phase. The present indications are that there will be 110,000 volts for Toronto for many years to come, and there may be 190,000 volts transmission in the western districts by new lines, that is, the district extending to Windsor.

But the whole problem has not been solved yet, and we are making a lot of calculations on voltage regulation of lines, synchronous condenser application, and things of that kind, and until these are actually completed, I would not like to say it will be this or that. I do not see any great difficulty in using 190,000 volts, and the chief trouble at present is possibly in the circuit breakers. I do not think the transformer manufacturers are afraid of the proposition in any way, and I believe that the circuit breaker manufacturers are ready to take it on. The whole question works out on the basis of voltage regulation and line loss.

E. M. Ashworth: There is just one feature which I note has not been emphasized, and which is a feature of interest, particularly no doubt to our visitors from the other side, and that is, that this large, and I think successful, system has been initiated and carried out under public ownership.

H. R. Summerhayes: I would like to ask Mr. Hull what difficulties they have in the matter of regulation of voltage. We usually think of 110,000 volts as a trunk line transmission, and this seems to be a system where it is 110,000-volt distribution with power taken off at various points. Is it necessary to use synchronous condensers to get good voltage regulation? Also, is it contemplated to connect the Niagara Falls and the Toronto district with other parts of the Dominion, by trunk lines of high voltage?

Arthur H. Hull: In answer to Mr. Summerhayes, I might say that we have at the present time 8000 kv-a. in synchronous condensers operating in Toronto. There are some synchronous motors operating at Hamilton, some at London, but there are not very many in this south western section. We have a double circuit line to Windsor, and four circuits from Niagara to Dundas, and we find sometimes during periods of light load, we can cut out a section of line and get some regulation in that way. It is not possible to vary the bus voltage very much at Niagara Falls at the present time, that is, the generating bus voltage, owing to the fact that the bus sections are tied together through reactors and there are other customers of the Ontario Power Company, that have to be looked after. Some of the smaller stations have potential regulators, working on voltages of 4000 or 2300 volts, of capacity sufficient to handle the lighting load of the towns, and we have not had very much trouble with line voltage regulation. It would be better if we could vary the voltage at the generating station to take care of it.

The next step would be to put synchronous condensers at various points in the system. As a start on that, we recently installed 8000 kv-a. in Toronto, which is now being used. I do not know that I understood the second part of the question.

H. R. Summerhayes: It was whether you contemplated to connect the Niagara and Toronto districts, with 110,000-volt distribution, with the other systems in the Province.

Arthur H. Hull: No, not at the present time. These systems (*i.e.* other than the Niagara System) are now generating enough power for the present demands, and by further development in the available power sites, there will be sufficient power to take care of the demand for some years to come. Of course, it is pretty hard to say what is going to happen ten years from now. All this development which we have had here (*i.e.* in the Niagara System), has been made inside of ten years, and I would not undertake to predict so far ahead, as to what will occur within the next ten years.

F. A. Gaby: I will go further and say that the Commission already have prepared estimates on connecting the Niagara System with the St. Lawrence system, by a 220,000-volt line, with frequency changer sets at Toronto to tie the 25-cycle Niagara system with the 60-cycle St. Lawrence system.

Arthur H. Hull: No doubt the Long Sault Rapids on the St. Lawrence will eventually be developed by somebody, and, of course, Ontario will get a lot of that power, which ought to bring about in this district, (eastern Ontario) an enormous development. There is a new system now being developed, called the Rideau System which when completed will pretty well take care of the present needs of the district near the Rideau River. I may point out, however, that there are no water powers of any magnitude in the Niagara System district outside of those at De Cew Falls and Niagara Falls. This part of the country is pretty well cleared up, and there is very little chance of any hydraulic developments other than near Niagara so that there are good prospects of a great amount of additional load in this Niagara section.

Wills MacLachlan: It might be pointed out, for the benefit of the visitors, that in the north country there are large powers at French River, and also in the mining districts of the north there are other power developments. For those that do not know the conditions, they may think that almost all electrical Ontario has been shown on the map put on the screen by Mr. Hull, and these developments all apply to the south end of the Province. There are future developments that will positively obviate the necessity of running a high-tension transmission line to carry the power in the northern part of the province.

Arthur H. Hull: The paper only covers the systems of the Hydroelectric Power Commission. At Fort Francis there is a generating station, there is a development at Sault Saint Marie, and in the Sudbury district, also in the Cobalt district, as well as in the Porcupine districts, and there are some generating stations for pulp and paper manufacture up in this far north country (District of Patricia). There are power sites scattered all through here (north of Lake Superior) and the trans-continental railways run through this district, so if plans of electrification of these railways are carried out there will be an abundance of water power available in those sections to supply them and develop the districts.

Perry A. Borden: There were some articles published some months ago in the local papers dealing with the proposed scheme to divert the Grand River from its natural course, into the Western end of Lake Ontario by way of the Dundas Valley. May I inquire if anyone can give me information as to the authenticity or practicability of this scheme?

Arthur H. Hull: The Grand River starts in Grey County and empties into Lake Erie. There is a valley near the river that runs down to the west end of Lake Ontario. The proposition would be to divert the Grand River into Lake Ontario at this point (Dundas). I cannot say personally, Mr. Borden, what there really is in that scheme. Possibly Mr. Gaby might say something about that.

F. A. Gaby: The only suggestion that has been made is the general suggestion. I do not believe any studies have ever been made on the diversion of the Grand River into the escarpment at Dundas.

The Commission in 1910 and 1911 did make numerous studies on the possibility of using the plateau above the Dundas Valley as a storage basin for an auxiliary plant to be operated in conjunction with the Hydroelectric Power Commission system, as a hydroelectric system, that is, to pump water from Lake Ontario, 300 or 400 feet up to the plateau or storage basin, and then use this during peak load periods to augment the supply of power from the Ontario Power Company, rather than buy its power on the peak load, possibly, and in this way it might be possible to finance the expenditure necessary on such an equipment. This is the method used in the case of large sources of power on low power factor loads in Switzerland and other countries, but I do not know of any studies being made for the diversion of the Grand River, and further, I think it would be very difficult, in view of the uses the Grand River is being put to at the present time.

The Commission had been making a study before the war of possibility of starting to regulate and obtain power on the Grand River. These hydrographic surveys are proceeding at present, and it is hoped several years from now, when we get sufficient information on the characteristics and features of the streams, we will be able to make a comprehensive report of the possibility of regulating the flow and generating the power on this small flow. It is a river with variable flow, very high flood periods, and the minimum flow is very small, and its power possibilities are very small.

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ELECTRIFICATION OF THE MONTREAL TUNNEL ZONE

BY WILLIAM G. GORDON

ABSTRACT OF PAPER

The author describes the electrification of the tunnel through Mt. Royal at Montreal which was built to give the Canadian Northern Railway entrance into the heart of the city. The tunnel emerges from the mountain several feet above the level of the city and it is proposed to extend an elevated line at the same uniform grade to connect with the proposed viaduct on the lines of the Harbor Commission. The tunnel is 3.1 miles long and the method of construction is described in detail.

The power is purchased and delivered to a substation near the west portal of the tunnel. The equipment of the substation is described. At present there are six electric locomotives in operation having a one-hour rating of 1280 h.p., and it is proposed to add multiple unit motor cars for handling the local traffic. Details of the equipment and dimensions of both the locomotives and motor cars are given.

The catenary system, which is described in detail, has a number of unusual features due to special local conditions and the extremely low temperatures which sometimes prevail in Montreal.

THE City of Montreal is divided into two principal levels; the commercial and financial quarter being on a plane only a few feet above high water, and the residential and shopping districts being at a height of about 75 ft. above the river. As the space between Mount Royal and the St. Lawrence River is limited, this district has become very congested. Business has largely forced the residence section up and down the River and around the mountain. The tunnel under Mount Royal was built with the idea of giving the Canadian Northern Railway, which property now belongs to the Dominion Government, an entrance into the heart of the City, and to render available a large area for residential purposes, only a few minutes by train from the main terminal.

The location of the present terminal is about midway between the two levels and it is proposed to extend an elevated line, at the same uniform grade, which will connect up with the proposed viaduct on the lines of the Harbor Commission, thus giving direct access to Trans-Atlantic steamers, and all the harbor facilities.

The tunnel as built is 3.1 miles long and is the shortest line that can be devised to take advantage of the geological formation. The tunnel has a uniform grade of 0.6 per cent toward the city to insure proper drainage.

In order to meet the various physical conditions, different cross sections were used; where hard sound rock, unsound rock and soft ground were encountered respectively.

The twin section type of tunnel was adopted for

1. Economy in construction.
2. Ease and economy in ventilation.
3. Protection and safety in case of derailment or accident.

In addition to working from both ends of the tunnel a shaft was sunk one mile from the West Portal at Maplewood Avenue.

It is of interest to note that when the heading from the West Portal met that being driven from the Maplewood Avenue shaft, the lines checked within 1/16 in. on the alignment, and ¼ in. in grade, and that where the headings from the Maplewood Avenue shaft and that from Dorchester Street met under the highest point of Mount Royal, the error was ¾ in. in alignment, and ¼ in. in grade.

The method employed in driving the tunnel was to drive a bottom center heading about 8 ft. high by 12 ft. wide, as this heading could be driven ahead rapidly without much regard to the character of the ground, and from which the full size excavation could be developed at as many places, simultaneously, as desired.

Four drills were used in each heading, supported on a horizontal bar; the drills being operated by compressed air at a pressure of about 100 lb. per sq. in. The break-ups, where the upper part of the tunnel section was excavated to its full width and height, were opened at intervals of from 500 ft. to 800 ft. along the center bottom heading, the practise being to open up as many of these as necessary to keep up with the heading progress.

The compressed air used for operating the drills and other pneumatic machinery was obtained from two plants, one at each end of the tunnel, with an aggregate capacity of 11,000 cu.ft. of free air per minute, compressed to 110 lb. per sq. in.

The muck from the tunnel was handled by two 10-ton and one 8-ton trolley locomotives, and six 5-ton storage battery locomotives.

The load curve was worked up from the following data:

TRAIN WEIGHTS AND SPEEDS ASSUMED FOR MONTREAL TERMINAL ELECTRIFICATION

Class.	Trailing tons.	Speed level.	0.6 per cent up-grade.	Schedule.
Transcontinental	1130	37.0	26.5	21.2
Express and local	550	37.5	27.1	21.6
One motor coach	60	50.0	41.5	22.2
Three motor coaches	180	50.0	41.5	22.2
Three motor coaches and two trail coaches	260	47.8	34.8	21.8
Freight	1000	32.5	23.5	....

The substation is a handsome building, and will harmonize with the buildings which will be erected in the neighborhood.

Power is purchased from the Montreal, Light, Heat & Power Company at 63 cycles, 11,000 volts, three-phase. It is delivered to the substation by a lead-covered, three-conductor cable carried in a duct through the tunnel and also by an overhead line to insure continuity of service. The general arrangement and capacity of the switching equipment provides for the later addition of a steam auxiliary plant at the Back River near the Cartierville Yards for extension of the electrification of the main line to Ottawa.

There are two motor-generator sets with provision for a third, later. Each of these sets consists of a synchronous motor direct coupled to and on a common bedplate with two 750-kw., 1200-volt, d-c. generators, the set running at 600 rev. per min. The generators are connected in series giving 1500 kw. at 2400 volts per unit.

The sets have an overload capacity of 200 per cent for five minutes. The heavy overload capacity of these machines is obtained by the use of a pole-face winding. This winding of tubes and rods through holes near the pole faces is so connected as to directly oppose the armature reaction, thus insuring satisfactory operation up to the heavy overload mentioned. The pole-face windings and the series and commutating field windings are all connected on the ground side of these machines.

The shunt fields of the d-c. generators and the synchronous motor fields are arranged for 125-volt excitation. Each of the synchronous motors is started by a three-phase, 11,000-volt compensator. This auto-transformer has one coil per phase with suitable starting taps brought out.

The three exciter sets each consist of a 50-kw., 125-volt d-c. generator driven by an induction motor. . The generators are commutating pole type, flat compounded for the specified voltage and are especially adapted for exciter work and voltage regulator control. A bank of six 100-kw. single-phase transformers supply the induction motors of the exciter sets and miscellaneous station requirements.

All oil switches on the 11,000-volt circuits, except the synchronous motor magnetizing and starting switches, are enclosed in masonry cells and have two breaks per pole, each break in a separate tank. These switches have a rupturing capacity of 16,000 arc amperes at 11,000 volts. They are motor operated and will open automatically on overload, the incoming line switches excepted, either instantaneously or with a time-limit action, as desired. The incoming line switches operate automatically on the reversal of power only.

The synchronous motor starting switches are remote-control, solenoid-operated, mounted in cells, and have a rupturing capacity of 2000 arc amperes at 11,000 volts.

The main switchboard is of three-section panels of natural black slate, 90 in. high. The 2400-volt direct-current circuit breakers and lever switches are mounted on a panel back of and above the main switchboard. They are operated by insulated handles on the front of the main board so as to eliminate any possibility of the operator coming in contact with the 2400-volt circuit.

The circuit breakers are mounted between fireproof barriers and are equipped with powerful magnetic blowouts. The field switches are mounted on a base back of the panels with the operating handles on the front of the main board.

There are six locomotives in operation. Each locomotive has four axels with all the weight of the locomotive upon the eight driving wheels. The running gear consists of two four-wheel trucks, articulated with a heavy hinge. The equalization of the trucks is accomplished by a semi-elliptic leaf spring over each journal box, connected through spring hangers to the frame and to the equalizer bars. The equivalent of a three-point suspension is thus obtained through the side equalization of one of the trucks and both side and cross equalization of the other truck.

The friction draft gear is mounted in the end frame casting of

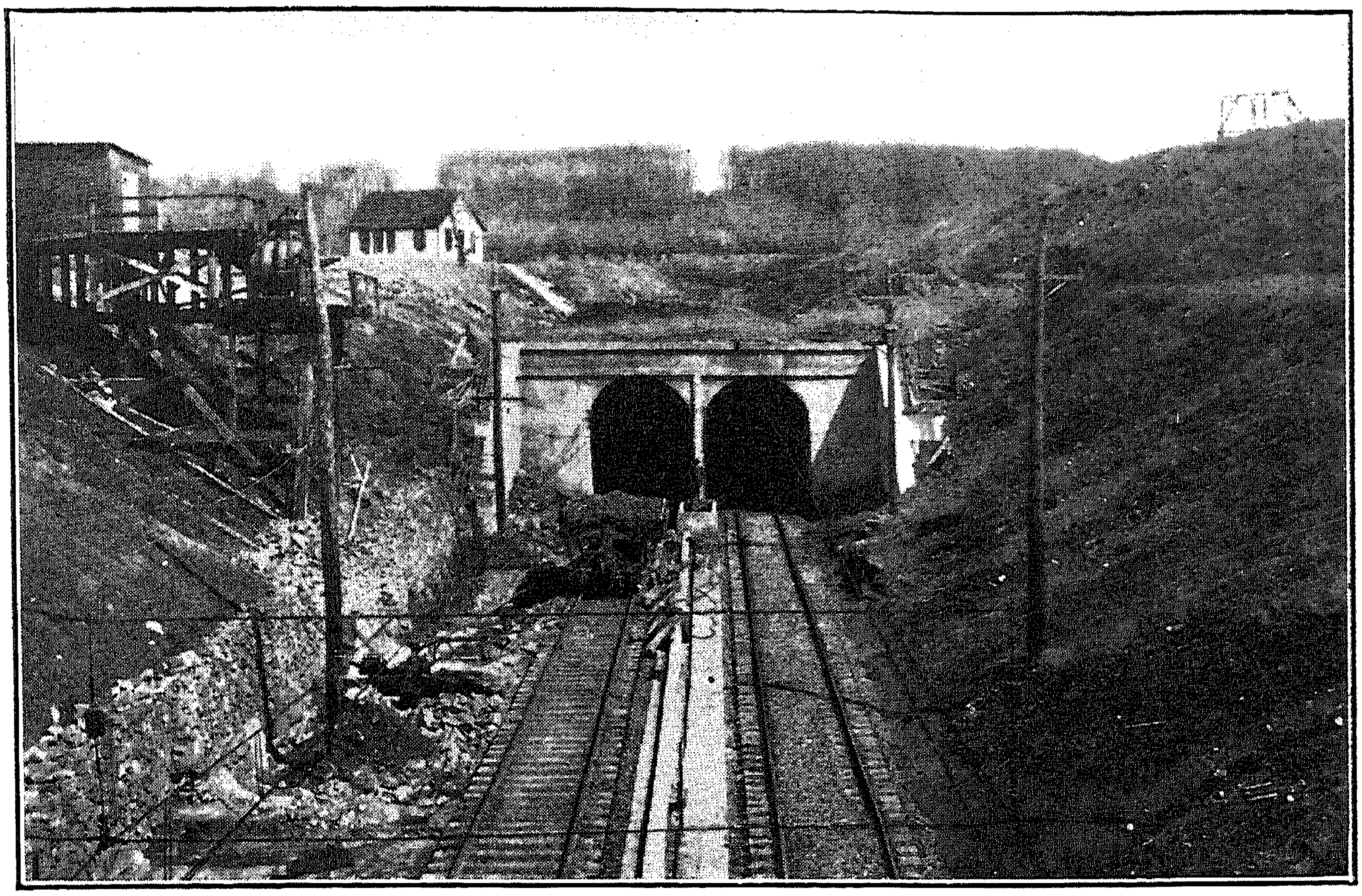

FIG. 1—APPROACH TO WEST PORTAL OF MOUNT ROYAL TUNNEL

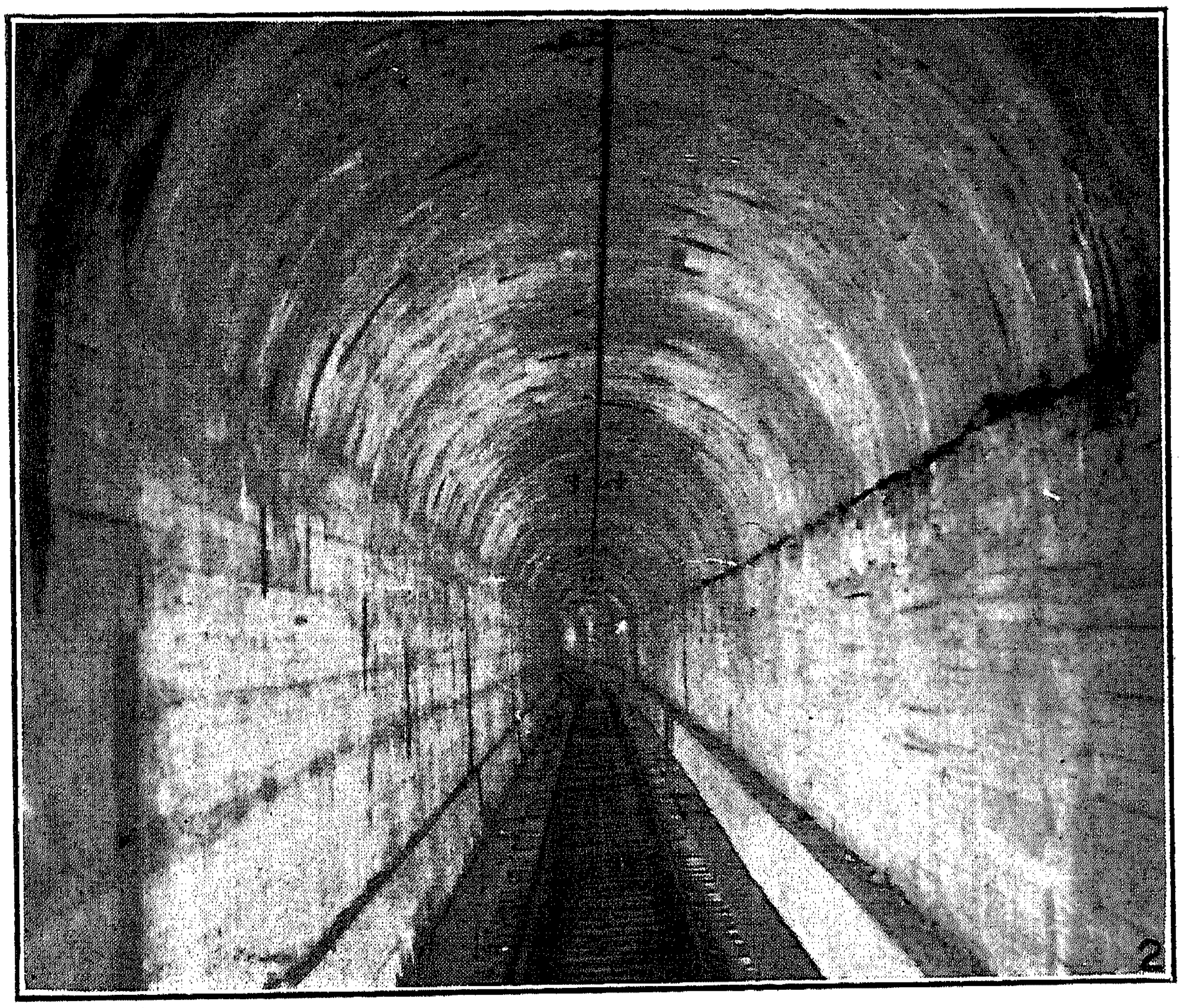

FIG. 2—TWO-DEGREE CURVE IN THE TUNNEL [GORDON]

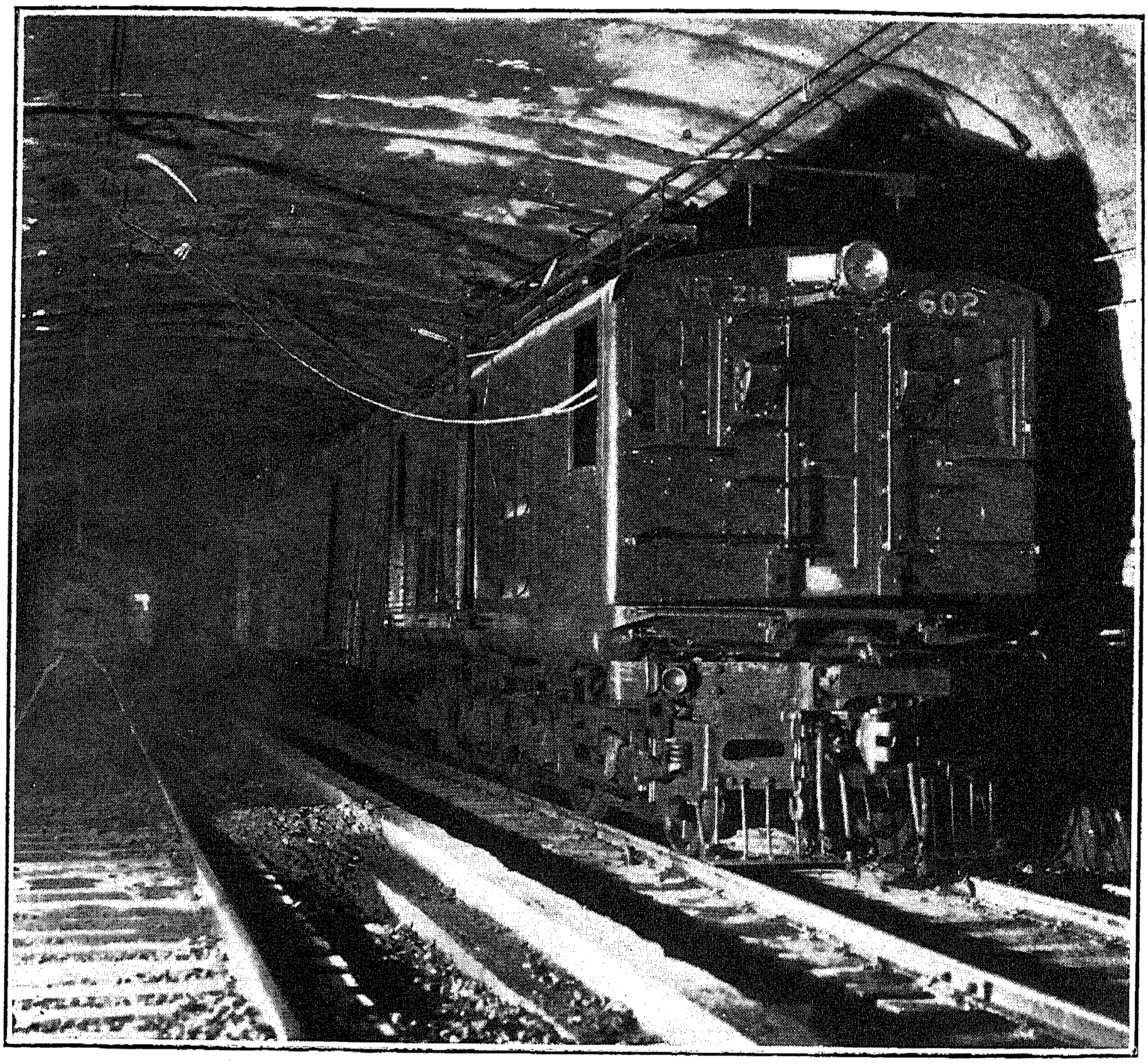

FIG. 3—LOCOMOTIVE PULLING IN MESSENGER AND TAKING CURRENT FROM OPPOSITE TRACK

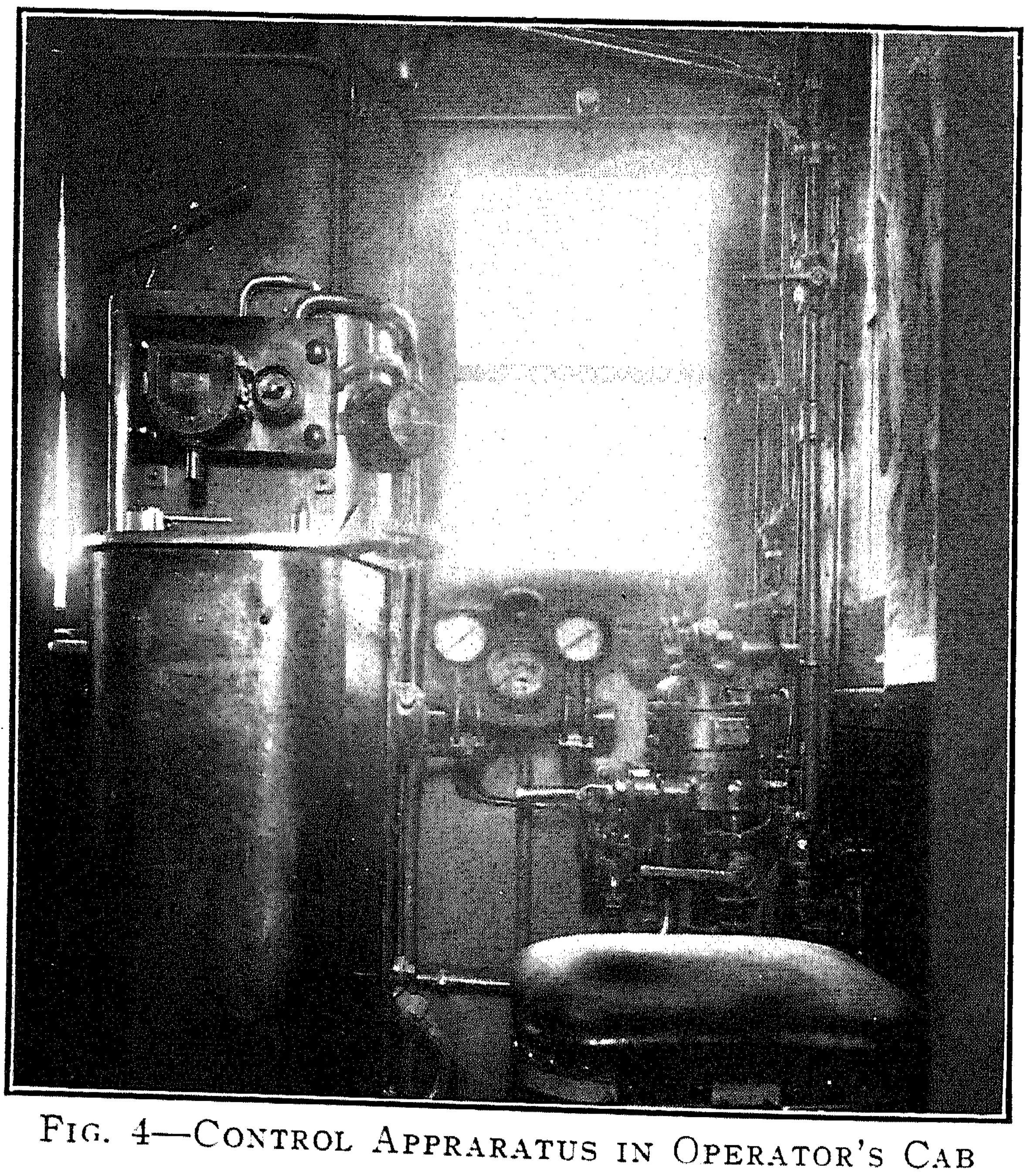

FIG. 4—CONTROL APPRARATUS IN OPERATOR'S CAB [GORDON]

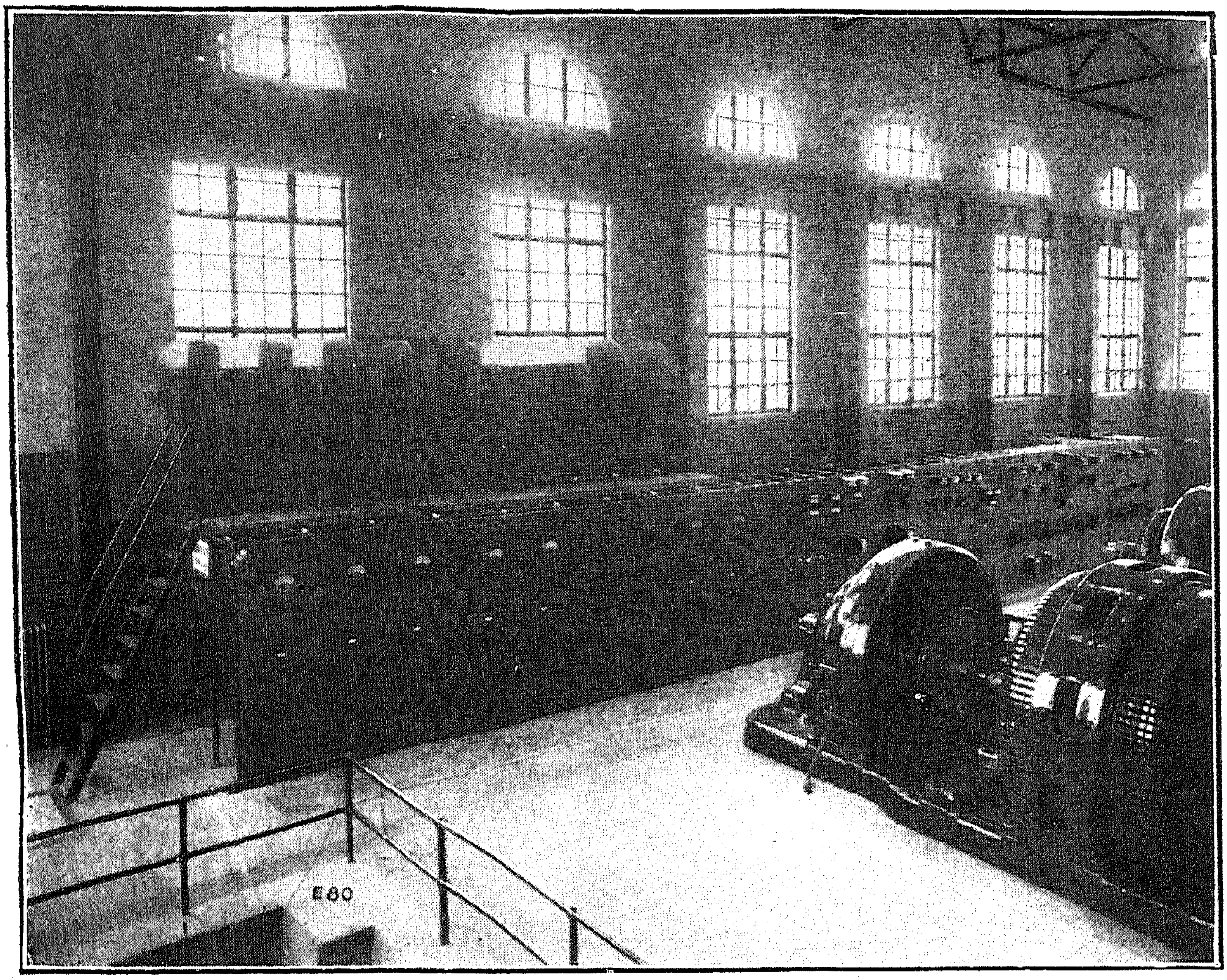

FIG. 5—VIEW OF SWITCHBOARD IN SUBSTATION

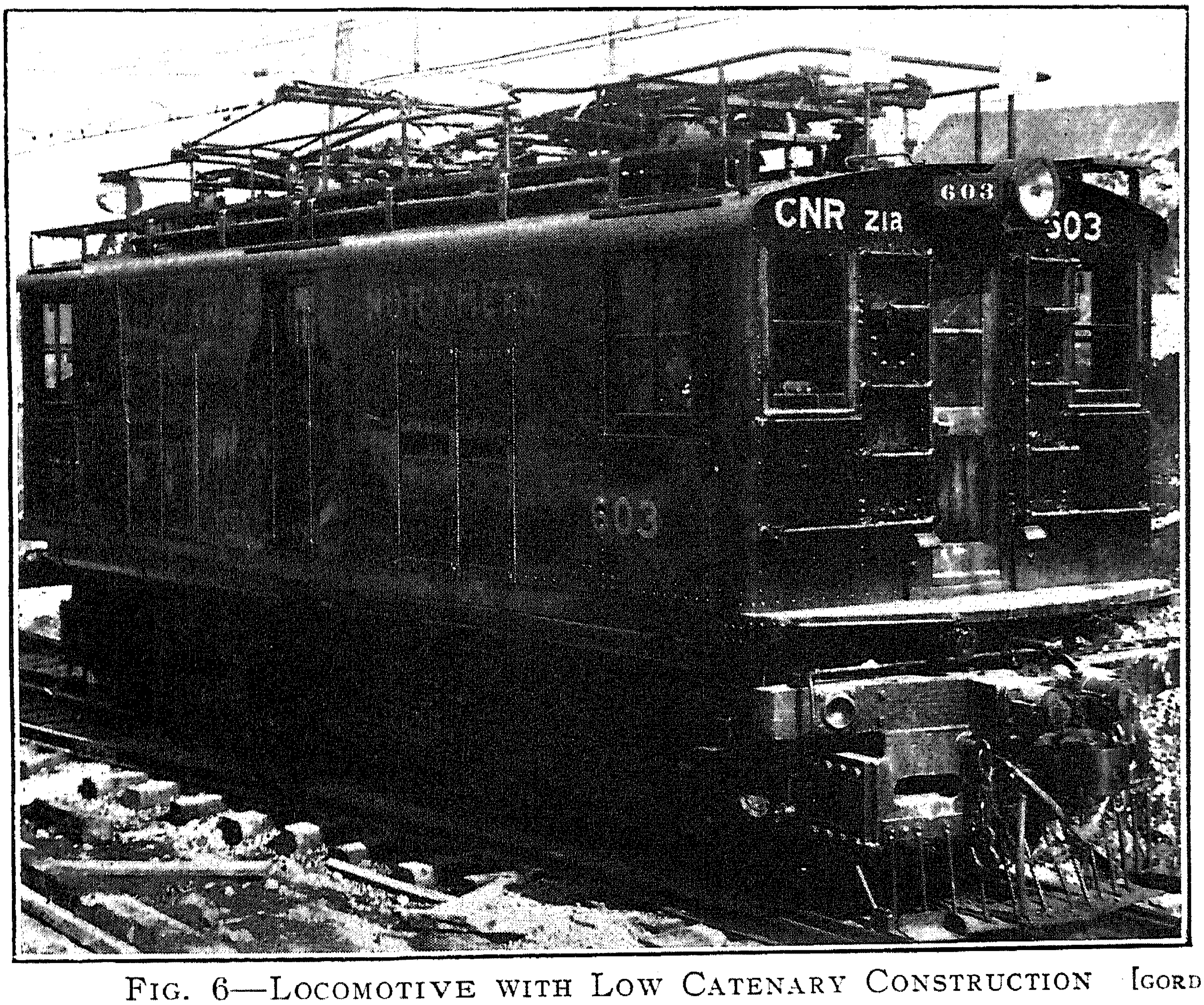

FIG. 6—LOCOMOTIVE WITH LOW CATENARY CONSTRUCTION [GORDON]

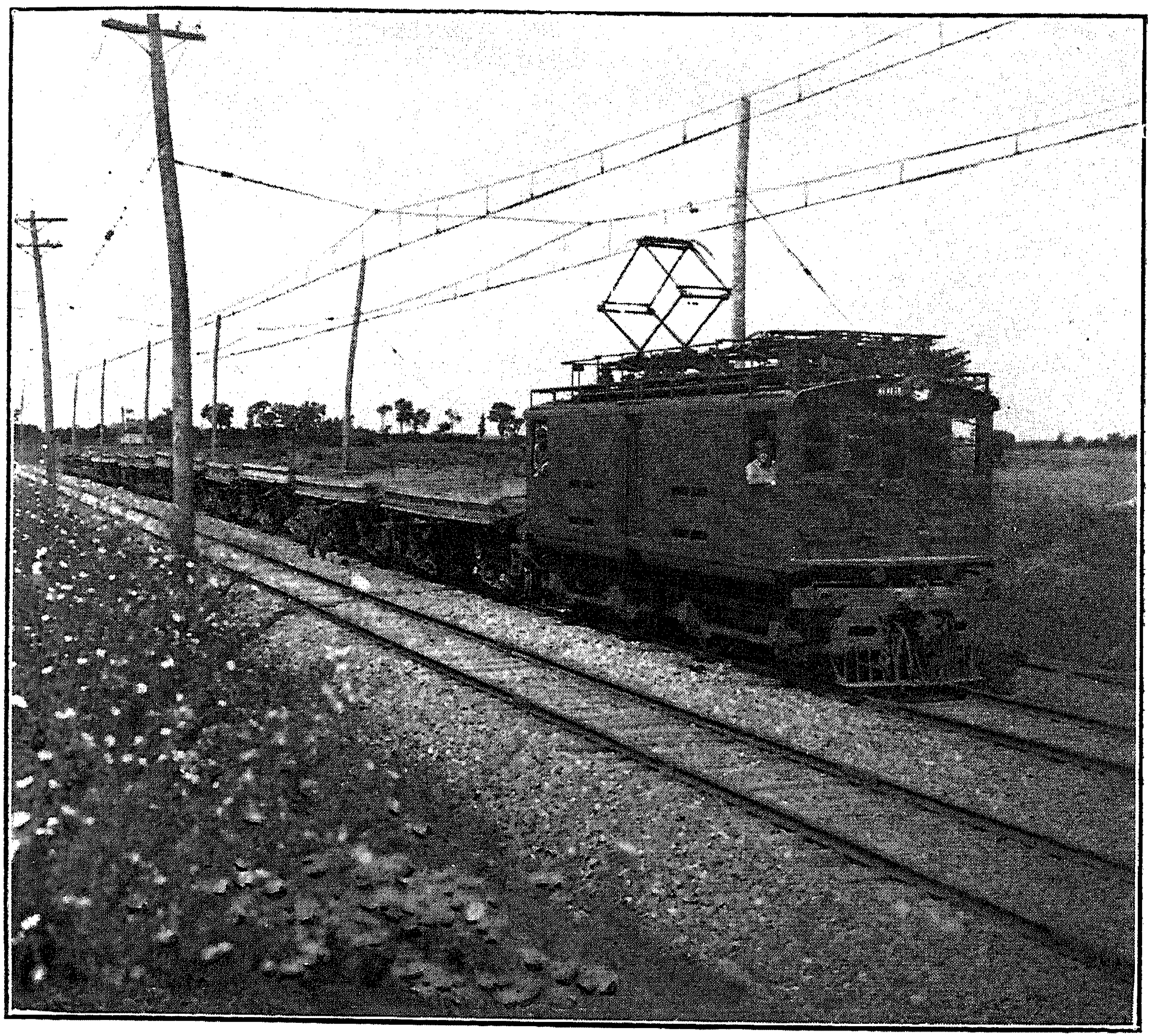

FIG. 7—CATENARY CONSTRUCTION ON TANGENTS [GORDON]

the truck. This type of construction restricts the hauling and buffing stresses to the truck side frames and articulated joint, thus relieving the cab and apparatus from the effects of severe shocks. The cab, which is of the box type, is divided into three compartments, the center compartment for the apparatus, and the two end compartments for the operator. Each operator's compartment is supplied with controller, control switches, ammeter, air brake and pantograph control, air gages, 2400-volt cab heater, bell rope, and control for the whistle and sanders, thus providing the locomotive with complete double end control. The motors are nose-supported in the usual way and geared to the axle by means of twin gears, each of four-inch face.

The motor equipment consists of four commutating pole motors wound for 1200 volts and insulated for 2400 volts, two of these motors being permanently connected in series for operating on the 2400-volt trolley circuit. The one-hour rating of each motor is 320 h.p. at 1200 volts. The motors are designed for forced ventilation which is obtained by means of a blower in the locomotive cab. Either pair of motors may be cut out by a special handle on the change-over switch. The locomotives are geared for a free running speed on tangent level track of approximately 45 mi. per hr. with ten points in series and nine points in series-parallel. The master controller used is of the non-automatic type and has two handles, one regulating the applied voltage at the motors and the other controlling the direction of rotation of the motors. The rheostats which form the external motor resistance are placed near the roof of the cab and provided with ample natural ventilation.

The master controller and contactor energizing circuits are designed for 125 volts. Each contactor is easily accessible without any disturbance to adjacent contactors. A special electropneumatic change-over switch is used for making the transition between series and series parallel connection of the pair of motors.

The 125-volt current for operating the contactors and for lighting the cab and headlights is obtained from a motor-generator set, the motor of which has two 1200-volt windings and two 1200-volt commutators in series for operation on 2400 volts. This set is mounted in the center cab and also drives the blower for providing forced ventilation to the main motors.

Fuses of the copper ribbon type placed in the fuse boxes provide protection for each individual circuit as well as the main circuit from the trolley. These fuse boxes are all arranged to blow into a common chamber designed to take care of the arc. In addition to the fuse on the main circuit, a main switch is also provided. This is of the knife-blade type, being opened and closed by a handle in a position for easy operation in case of emergency, or when it might be necessary to open the circuit while carrying current. This main switch blows into the chamber provided for the fuses and has a powerful magnetic blowout.

The trolleys are of the slider pantograph type, pneumatically operated and mounted on insulated bases. Two pantographs are used per locomotive.

A speedometer, similar to the type largely used on automobiles, but especially designed for locomotives, is located in each operating cab. These are connected to the driving wheels of the locomotive by means of flexible shaft and gearing.

A combined straight and automatic airbrake equipment is provided on each locomotive. This equipment includes a 2400-volt, motor-driven air compressor, the set consisting of two 1200-volt motors operating in series on 2400 volts and direct-connected to an air compressor having a displacement of 100 cu. ft. of free air per minute. The approximate total weight of each locomotive is 83 tons. Some of its principal dimensions and characteristics are given in the following table:

Length inside knuckles	37 ft.	4 in.
Length over cab	31 "	0 "
Overall height, pantograph down	15 "	6 "
Height over cab	12 "	10 "
Overall width	10 "	0 "
Total wheelbase	26 "	0 "
Rigid wheelbase	8 "	8 "
Total weight, all on drivers	83 tons	
Wheel diameter	46 in.	
Tractive effort at 30 per cent tractive coefficient	49,800 lb.	
Tractive effort at one-hour rating	20,300 lb.	
Tractive effort at continuous rating	16,200 lb.	
Speed at rated amperes, one-hour rating	23.4 mi. per hr.	
Total horse power, one-hour rating	1280 h.p.	
Speed at rated amperes, continuous rating	24.6 mi. per hr.	
Total horse power, continuous rating	1090 h.p.	
Gearing 80–25	Reduction 3.2	

The multiple unit motor cars for handling local traffic are not yet in operation. The principal dimensions of these cars are given in the following table:

Length over buffers	67 ft.	5¾ in.
Length over body corner posts	57 "	6¼ "
Truck centers	42 "	9 "
Width over side sill angles	9 "	10½ "
Width over eaves	10 "	2¾ "
Height top of rail over roof	13 "	0 "
Height top of rail to underside of side sill	3 "	7½ "
Center to center of body side bearings	4 "	10 "
Center to center deck sills	5 "	6 "

The approximate weight of the car loaded and equipped is 160,000 lb. The electric hot air system of car heating is used. One complete heater is placed underneath each car and receives its energy direct from the 2400-volt supply. The heater has a capacity of approximately 25 kw. and is constructed for two heat combinations so as to provide for the changes in temperature conveniently and economically.

The complete heating equipment consists of the heating unit, blower and regulating mechanism, the controlling switch and thermostat of the regulating mechanism being arranged for operation from the 600-volt supply. Air is forced over the heating unit by means of the blower and distributed to the car through the air ducts along the sides of the car. The blower used for the circulation of the air is operated by a motor which is connected in series with the heating unit on the ground side. The capacity of the blower is approximately 1000 cu ft. of air per minute.

The motor equipment consists of four fully ventilated, 125 h.p. 1200-volt, commutating-pole motors insulated for 2400 volts. Two of these motors are permanently connected in series for 2400-volts operation. Ventilation of the motor is accomplished by drawing air into the armature at the pinion end by means of the fan on the armature shaft. The air passes longitudinally through the whole interior of the motor and is expelled through an opening in the frame at the commutator end, protected by wire mesh.

The control is of the non-automatic type for multiple-unit operation. The equipment includes a motor-generator set for furnishing 600-volt current for the control circuits, the air compressor and lighting circuits. This set consists of two 1200-volt motors, operating in series on 2400 volts, direct connected to a 600-volt generator. The master controller, contactors, switches, reversers and pantograph are essentially the same construction and appearance as those already described for the locomotives. The controller has five steps in series and four steps in parallel. It differs from the locomotive controller in having the usual motorman's operating handle instead of a lever. This handle is provided with the so called "dead man's" feature for cutting off power and applying the air brakes in case the motorman removes his hand.

Copper ribbon fuses similar to those on the locomotive are used, and an aluminum-cell lightning arrester is installed on each car.

Special local conditions and extremely low temperatures introduced features making the design of the catenary system for this electrification somewhat out of the ordinary. The electrified track at present is about 10 miles long, and in this distance there is a passenger terminal station and coach yard in the city, a double track tunnel, double tracks in a cut with low clearances under highway bridges, a long stretch of single track, both tangent and curve, and a large freight yard with repair shops and storage tracks. The temperature in the coldest winter weather reaches 35 deg. below zero, while in the hottest summer weather it will go as high as 110 in the sun. In the early spring severe sleet storms sometimes occur.

The poles are of Eastern white cedar. The specifications for these poles and also for the creosote oil used as a preservative, were based upon those of the National Electric Light Association. Steel poles are used in the terminal yard in the city on account of their more sightly appearance. The wood poles are set seven feet in the ground and are all back-guyed. They are long enough to carry two cross-arms for feeders, signal circuit and a three-phase transmission line for supplying the shops in the Cartierville yard with electric power. On top of the poles there is a No. 000 copper ground wire which serves both as a protection against lightning for the circuits on the poles and also as a preventive against any trouble that might be caused by breakage of the rail bonds, which latter are of the welded U type.

The poles throughout the single track construction are spaced 150 ft. on tangents and 20 ft. on the two-deg. curve. On the double-track portion, where the overhead clearance is limited, the spacing is reduced to 105 ft. on tangents.

The messenger for the electrification outside the tunnel consists of a ½ in. seven-strand Siemens Martin steel cable with an ultimate strength of 11,000 lb. and an elastic limit of 6600 lb.

Two No. 0000 copper feeders are installed, on the full length of the electrification outside the tunnel and the other for about one mile west of the substation.

The messenger is anchored every half mile. This is accomplished by running the end of one half-mile length past the end of the next for a distance of one span. It is then made fast to an anchor eye on the bracket through an insulator and turnbuckle, and the same point of the bracket guyed back to the next pole, which in turn is guyed against this strain. The two messengers where they pass each other, are kept from eight to ten inches apart. By anchoring the trolley wire on the same bracket the anchorage becomes a section insulation, the air space between the messenger and trolley wire forming the insulation. Where a section insulator is not required a copper jumper is placed between the messenger and trolley wires.

For the double-track portion of the line cross-span construction, is used, the cross-span being a 3/8-in. seven-strand Siemens Martin steel cable. The messenger is fastened to this by means of a small malleable clamp. This cross-span is made up with a turnbuckle, strain insulator, and wedge grip in each end, and fastened to the poles by means of eyebolts.

In yard work spanning more than two tracks the construction is similar, but with the addition of a cross messenger of 1/2-in. cable above the 3/8-in. cable. This cross messenger is made fast to the poles directly, without insulators or turn-buckles, and carries the weight of the spans below through lengths of 1/4-in. steel cable. These fasten to eyes in the tops of the messenger hangers and to the cross messenger by means of Crosby clips, There is a strain insulator in each of these lengths.

Pull-offs are used on curves for holding the contact wire and messenger in the correct position over the track and at intervals on long tangents for steadying the contact wire. The pull-offs are made of sherardized steel tubing bent to avoid fouling the pantograph. Each pull-off is fitted with a clamp ear at one end and an eye at the other. Adjustable links are sometimes required with the pull-offs to keep the trolley wires the right distance apart at certain points, such as where the trolley wire for a turnout approaches the main trolley wire at an angle. Each link is composed of two malleable iron brackets, with clamp ears, connected by means of a 1/2-in. pipe, the length of which is adjusted between the brackets and held by set screws.

Porcelain strain insulators are used in two sizes. The larger, used with ½-in. and ⅜-inch. steel cable, withstands a wet flash-over test of 14,000 volts and has a breaking strength of 22,000 lb. The smaller, used with ⅜-in. and ¼-in. steel cable has a breaking stength of 12,000 lb.

The insulator used on the bracket construction is of the ordinary glazed porcelain, double petticoat, pin type, 4½-in. in diameter. It has a wet flash-over test of 20,000 volts. The messenger rests in the groove in the top of this insulator and is not tied except on curves.

The contact wire is of special bronze composition, size No. 0000, with a breaking stength of 65,000 lb. per sq. in. and an elastic limit of 39,000 lb. per square inch. Its section is the standard of the American Electric Railway Association for No. 0000 grooved trolley wire. The use of this wire instead of hard-drawn copper was thought advisable both because of its longer life when subjected to the wear caused by sliding pantographs, and also because it could be pulled up tighter than copper on account of its greater strength. This latter reason was considered of special importance because of the wide variation in temperature in Montreal, with the consequent great variation in the sag of ordinary copper trolley wire between winter and summer.

The trolley wire is hung straight over the center of the track as the natural side sway of the pantograph is sufficient to prevent wearing grooves in the contact strips.

The height of the trolley wire above the top of rail is ordinarily 23 ft. except along the double track construction and in the tunnel, where it is 16 ft. In this section two wires are used over each track. They hang side by side, supported from the same messenger, the hangers of one wire being staggered with those of the other. These double wires do not raise the hanger loops as high as would a single wire when a pantograph passes along, which is an obvious advantage where the head room is limited. Sparking and consequent wear both of the contact shoes and contact wires is reduced to a minimum, as there is always good contact between the slider strips and one of the contact wires.

The hangers are all of the long-loop type, having a malleable iron, single-bolt clamp ear, and a strap varying in length to suit its position in the span. All parts are sherardized. In spans of all lengths from 150 ft. to 90 ft. the hangers are spaced 15 ft. apart.

Lightning arresters of the magnetic blowout type are installed at half-mile intervals. The arrester is placed near the top of the pole and the ground wire runs down the pole to a $\frac{3}{4}$-in. iron pipe driven about ten feet into the ground. Before driving this pipe, a 2-in. pipe was driven down about 5 ft. then withdrawn and the hole filled with rock salt. The $3/4$-in. pipe was driven down through the salt.

In addition to these arresters on the poles, aluminum cell arresters are installed in the substation on the positive busbars and on each feeder.

In order to string the messenger cable with the proper tension a dynamometer was used. It was therefore necessary for the foreman of the line gang to know what the tension should be at different atmospheric temperatures. The right sag at any given temperature was also of importance as a check on the tension. This information was supplied in the form of tables to which the line gang worked, the sags and tensions being given at 5 deg. intervals.

In the tunnel the overhead clearance was so limited that the catenary had to be very flat. This meant pulling the messenger up very tight for spans of reasonable length. A cable of phosphor bronze was decided upon,-composed of nineteen wires, and having an overall diameter of 0.888 in. This cable has an ultimate breaking strength of 22,000 lb. and an elastic limit of 18,600 lb.

This messenger is supported every 90 ft. from the roof of the tunnel by a combination of iron yokes held in the concrete by four one-in. bolts. The cross yoke carries the messenger insulator and is supported on two insulators carried on the two end yokes, so that there are two insulators between the messenger and the ground. The insulators are of glazed porcelain and have a wet flashover test of 20,000 volts. All clamps and small parts of the messenger supports are of malleable iron sherardized. The yokes are of 2-in. by $5/8$-in. and $1\frac{1}{2}$-in. by $7/8$-in. mild steel, painted with an asphaltum compound as a protection against rust. Two No. 0000 phosphor-bronze contact wires hang side by side from the messenger. The hangers for each contact wire are spaced 15 ft., or 7 ft. 6 in. between adjacent hangers. The hanger lengths vary from 6 in. to $13\frac{3}{4}$ in. with a 90-ft. span. The two hangers nearest the messenger support, namely those $11\frac{1}{4}$ and $13\frac{3}{4}$ in. in length, are made with two loops one sliding inside the other, where the clearance to the roof is small. The remaining hangers are similar to those used outside the tunnel, except that the loop is wider in order to take the larger messenger. It was found that the two messenger cables and the four contact wires over the two tracks in the tunnel would give ample conductivity, so that no feeders through the tunnel were required. Both the messenger and contact wires are anchored every half mile. Two "bridles" of ½-in. steel cable are fastened to the messenger by means of six ⅞-in. Crosby clips, and the ends of these bridles fastened each way, through two cemented-type strain insulators in series, a turnbuckle and wedge grip to roof plates. The contact wire is anchored by lapping the ends for a distance of one span and then carrying each end up and slightly to one side of the center, making fast to a roof plate through two insulators, a turnbuckle and a wedge grip.

At the only curve in the tunnel, one of two deg. two pull-offs are placed in each span, over each track, one for each of the contact wires. The pull-offs are fastened to the tunnel arch through two strain insulators in series by means of an expansion bolt. The two pull-offs are placed 7 ft. 6 in. apart and this arrangement prevents hard spots and at the same time keeps the two contact wires close enough together for satisfactory operation.

DISCUSSION ON "ELECTRIFICATION OF THE MONTREAL TUNNEL ZONE" (GORDON), TORONTO, CANADA, NOVEMBER 22, 1918.

W. G. Hewson: I would like to ask Mr. Gordon if he knows the reason for putting the substation so close to the end of the line. Undoubtedly the 2400-volt system was not adapted for pioneer work—it must have been with the idea of extending the system in the future, and it seems as if the substation is very close to the end of the line.

W. G. Gordon: I think I covered that, Mr Hewson, in my presentation of the paper. I mentioned that the general layout was with the idea of extending the electrification later on, as far as Ottawa. The substation was laid out to tie in with a turbine plant later on to be installed at Back River. The selection of the location was not just for the electrification of the present length of line.

W. G. Hewson: Is that installation going to be extended to Ottawa?

W. G. Gordon: That was the original intention.

W. G. Hewson: Why was not the substation put farther out on the line?

W. G. Gordon: It was more convenient in connection with the residential district scheme they had in mind, in opening up, to locate the substation where it is, and the few miles that would be gained in regard to the spacing to the next substation location was not worth considering at the time.

W. A. Bucke: There is one point that has nothing to do with the electrification of railways, which I would like to bring forward. I wondered how the matter was working out in actual practise. That is, the question of grounding the lightning arresters along the line, which is an important part of the system of protection. I understood Mr. Gordon to say they drove in a 2 in. pipe, filled the hole with salt, and then drove in a smaller pipe. I wondered how it had worked out in practise. It seemed to me it would not be many years before the pipe would be eaten away leaving a mass of iron oxide, etc. This would shake loose, and you would not have any ground. I would like to know how that arrangement will work out.

W. G. Gordon: The idea is to start the ground with a thoroughly saturated saline condition; but as far as it will actually work out in time, I have not seen it tried out exactly in this way. So far they have had remarkably good results. Last summer there were some very severe lightning storms, and they did not get by the arresters into the station.

W. A. Bucke: I should think it would be effective as long as it lasted.

Arthur H. Hull: That brings up a point to be emphasized—the importance of maintaining good grounds. The ordinary practise in the past seems to have been to drive in a pipe any old place, and whether they had a good ground or not seemed to be immaterial, as long as it looked like one. It seems to me that it is very important to test the ground connections and see that the grounds are maintained.

W. G. Hewson: If it is not switching the discussion from that point, may I ask Mr. Gordon about the motor cars. I think he mentioned that they weighed 160,000 lb. or 80 tons. That is heavier than what is in ordinary use for suburban work, but I presume the operation in the tunnel has something to do with it.

Another point—he might tell us something about the air brake arrangements. I did not understand the exact location of the single compressor. I should think it would be very necessary with the locomotives on a 0.6 per cent grade, leading down into conditions like they have on that line, that the air brake should be very reliable, and I was wondering whether these compressors were put on the locomotives or on the cars, and whether they used one or two compressor units.

W. G. Gordon: They have a single air compressor on the locomotives, with a capacity of 100 cu. ft. per min., but there is nothing more to be done, as regards operating down that 0.6 per cent grade, than to see that the air is in proper operating condition. There is no question of the reliability of the operation of the air brake, if the pressure is up on the reservoirs. If a man is taking his locomotive through the tunnel by itself, he is working on straight air, and not taking any chances on the triple sticking. In taking a train through in the normal way, he operates on the triple, with the automatic braking feature. The men who operate the locomotives are all ex-steam men, familiar with air. They watch the gauges, to see that they have proper pressure, and naturally would not take any chances on starting off on the down grade without having tested the brakes. There is no chance of them getting into trouble, provided the proper supply of air is there. Regarding the weight of the motor cars, as you say, they are heavy. The standard steam coach design was taken and the equipment mounted on that, and that equipment runs into a good deal of weight, as you know. The figures I have given represent the weight loaded and completely equipped.

Harry U. Hart: I was wondering if the circuit breakers are the type that open in a few thousandths of a second. They must get some heavy short circuits, like all railways, and I would like to know if they are like any of those recently shown in the technical journals.

Another point in the paper which raised an inquiry in my mind was why a synchronous motor and two generators were run as a unit, instead of using synchronous converters.

W. G. Gordon: In regard to the breakers they have the standard type—the ones shown were the standard type of breaker—no special quick-acting feature. As a matter of fact, they have no quick-acting negative breakers installed, and up to date they have not felt the need of them. They have some pretty severe short circuits, and one or two have caused considerable damage to the brush holders, but not sufficient to put them out of commission, and it is exceptional to have a bad flash-over.

With 60 cycles however, the use of rotaries was out of the question as each rotary would only be good for from 600 to 700 volts.

With regard to the use of synchronous motor-generator sets as against rotaries, of course, that is a question on which where either is suitable you will find arguments on both sides. Had it been 25 cycles instead of 60 cycles by putting two in series we could have used 2400 or 3000 volts.

Arthur H. Hull: I ask Mr. Gordon if the feeders in the substation are tapped on to the trolley wires in the vicinity of the substation, or are they carried out along the lines some distance?

W. G. Gordon: They carried the feeder out a little over a quarter of a mile, but decided there was nothing gained over tapping directly in, as far as mitigating in any way the severity of flash-overs.

Arthur H. Hull: Do you think to carry the feeder out a considerable distance is impracticable?

W. G. Gordon: Against carrying out the feeder is the constant loss in the feeder with the questionable insurance you are getting, as to how far this will cut down the serious effects of the flash-overs. In other words, the saving in the seriousness of the flash-overs might not begin to meet the investment in the copper or the steady losses over that extra length of feeder.

W. H. Mulligan: I ask Mr. Gordon if the tunnel is lighted.

W. G. Gordon: They carry a lighting service all along one side of the tunnel. There are regular incandescent lights for the purpose of lighting the tunnel.

Wills MacLachlan: In connection with these high-tension generators, is there any provision for the protection of the operators in the case of a flash-over? There have been cases of men badly pitted with copper in going in front of one of the generators at the time of a flash-over.

Another point—in connection with the design of the switchboard, back and above the main switchboard, there is a platform, and running on the back of the switchboard there is a bus, and there is no protection to prevent forming a short circuit across the bus.

W. G. Gordon: There is no special provision taken for providing against the conditions you speak of, in case a man happens to be right over a commutator at the time of flash-over. It is a rare occurrence that you do actually find, in this class of work, that a man is burned by being right over the commutator, or passing the commutator, in pursuance of his work, at the time of flash-over.

On these generator sets, there is a considerable distance from the commutator to the outboard bearing, and the probabilities are that a man would not get anything beyond a scare even if there was quite a bad flash-over.

With regard to the small platform shown on the back and top of the tie rods of the main board, that is never used, except when examining these 2400-volt switches or breakers, with the current off, and when they are dead, the corresponding busses behind them are also dead.

W. M. Gifford: I would like to ask Mr. Gordon what the mileage obtained with the pantagraph is.

W. G. Gordon: I cannot speak definitely for these locomotives, but we are operating exactly the same type of pantagraph on the London & Port Stanley System. The life they obtain from the two copper wearing strips is from 10,000 to 12,000 miles. We find once wear is established, once a minute film of oil is established on the contact wire, and it does become so established, the wear on the contact wire is almost negligible. We have found that, after operating sliders on just the ordinary hard drawn copper trolley wire for a number of years.

R. R. Stevenson: In a case where a heavy electric trunk road is taking power from a central station system, I ask Mr. Gordon if there are any special features in the power supply contact which will affect the layout of the switching and transformer equipment. Were there any special provisions made in the power supply contact which would affect the overload protection which would have to be supplied?

W. G. Gordon: In practically every contract—while I am not at liberty to speak freely in regard to this one in question—they have certain provisions as regards the power factor below which they must not operate, and also the contract is made on a maximum demand basis on peaks of definite duration, but the switching in general in this case is laid out to take care of the maximum pull on the substation.

Presented at the 344th meeting of the American Institute of Electrical Engineers, Toronto, Canada, November 22, 1918.

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110,000-VOLT TRANSMISSION LINE OVER THE ST. LAWRENCE RIVER

BY S. SVENNINGSON

ABSTRACT OF PAPER

The paper deals with some remarkable construction recently completed by the Shawinigan Water & Power Company near Three Rivers, Quebec. The St. Lawrence river is crossed by transmission line wires on a span of 4800 feet, being the longest span in the world. Due to necessities of navigation clearance, the towers are 350 feet high.

The preliminary investigation leading to the adoption of this construction is outlined, and a general description is given of the design and construction of the towers, insulators and cables. The provisions for protection from ice and the method of sag calculations are also given.

THE Shawinigan Water & Power Company's principal hydroelectric development is located at Shawinigan Falls on the St. Maurice River about 20 miles north of Three Rivers, where the St. Maurice empties into the St. Lawrence River, and about 85 miles north east of Montreal.

The Company has for a number of years been transmitting power to Towns south of the St. Lawrence River. Two lines carry the current to a switching station at Victoriaville, 35 miles south of the St. Lawrence River. At Victoriaville the lines branch, one branch 50 miles long running to Sherbrooke and supplying various towns and industries between, the other branch 40 miles in length feeding the asbestos mines and other industries in the Thetford district. The current has been transmitted at 50,000 volts from Shawinigan Falls to the St. Lawrence where the voltage was stepped down to 25,000 for transmission across the river over Submarine Cables, then stepped up again to 50,000 and transmitted at this voltage to Thetford and Sherbrooke.

The first submarine cable was installed in 1906. At this time the alternative of putting in an overhead crossing was considered, but the amount of power to be transmitted was so small that it was decided that the expense of an overhead crossing was not warranted. However, the demand for power on the south shore steadily increased until by the beginning of 1916 five submarine cables were in operation, two three-conductor cables and three single-conductor cables, and the capacity of the transformer house, 10,000 kw. had been reached. The company was then faced with the necessity of increasing the capacity of the crossing and the question of an overhead crossing to replace the submarine cables again came up for consideration.

Submarine cables have always been a weak point in this part of the system and a source of more or less trouble and expense. The river bed is hardly suitable for a submarine crossing. It is sandy but with a generous sprinkling of rocks and boulders. The bottom slopes out very gradually from both shores to the steamboat channel, the sides of which are rather steep. Owing to this abrupt change in slope the cable in the channel is partly suspended and kept constantly swinging by the current in the river so that the armouring quickly wears through and the cable burns out at the points of suspension. The current in the channel carries the cable down stream and has even been strong enough to pull them loose from their moorings and break their connections in the cable house. In the winter the ice usually puts at least one of the cables out of commission, and it has been found necessary at times to erect temporary pole lines across the ice to maintain the service to the south shore. When, therefore, in the Fall of 1916 the demand came for more power for the south shore partly for war work and it became a question of putting in an additional submarine or an overhead crossing the Company decided in favor of the latter.

In order to increase the capacity of the submarine crossing it would have been necessary, in addition to purchasing and installing new cables, to build extensions to our cable houses and install new transformers with their necessary switches lightning arresters and other equipment. This would have involved an expenditure of at least $150,000. and at the same time the weak point in the line would not have been improved.

The overhead crossing was estimated to cost $200,000. the difference between the two being offset, in the opinion of the company, by the elimination of the weak link, by obtaining greater security from interruptions to the service and by a gain of from 2 per cent to 3 per cent in regulation by cutting out the transformers. A considerable amount of operation and maintenance expense would also be eliminated and the transformers, cables and other equipment tied up in the submarine crossing were needed and could be used to advantage in other parts of the system.

PRELIMINARY INVESTIGATION

The two shores of the St. Lawrence River upstream, as well as downstream of the cable houses were carefully surveyed in order to find the most advantageous point of crossing. As a result of this preliminary survey it was finally decided to investigate in detail two alternatives:

a. A three span crossing at Point-du-Lac, each span approximately 2200 ft. long.

b. A single span crossing between the cable houses 4800 ft. long.

Point-du-Lac is situated at the eastern end of Lake St. Peter about six miles up the river from the cable crossing. From a construction point of view the site at this point appeared at first to be the most favorable for an overhead crossing. The St. Lawrence River at this point is about 7000 ft. wide, but as the water is very shallow except for a distance of 2000 ft. in the center a crossing could have been built using three spans of approximately 2200 ft. each. The towers on either side of the main channel would have been about 205 ft. high, while the other two towers would have been about 110 ft. high. The crossing at this point probably would have been somewhat cheaper than the other alternative but it would have necessitated the construction of about 15 miles of double circuit high-tension pole lines in order to connect up with the main transmission lines. The cost of these connecting lines would have brought the total cost up to that of the single span scheme. A fairly strong point against the three-span crossing was the inaccessibility of the towers during certain periods in the spring and fall when the river is full of floating ice. The single span scheme was finally decided on as being the most advantageous, although it was fully realized that there were many difficult problems to solve in connection with the design and construction.

GENERAL DESCRIPTION

The crossing as completed consists of a central span 4801 ft. long and two anchor spans, the north shore span being 571 ft. long and the south shore span 951 ft. long.

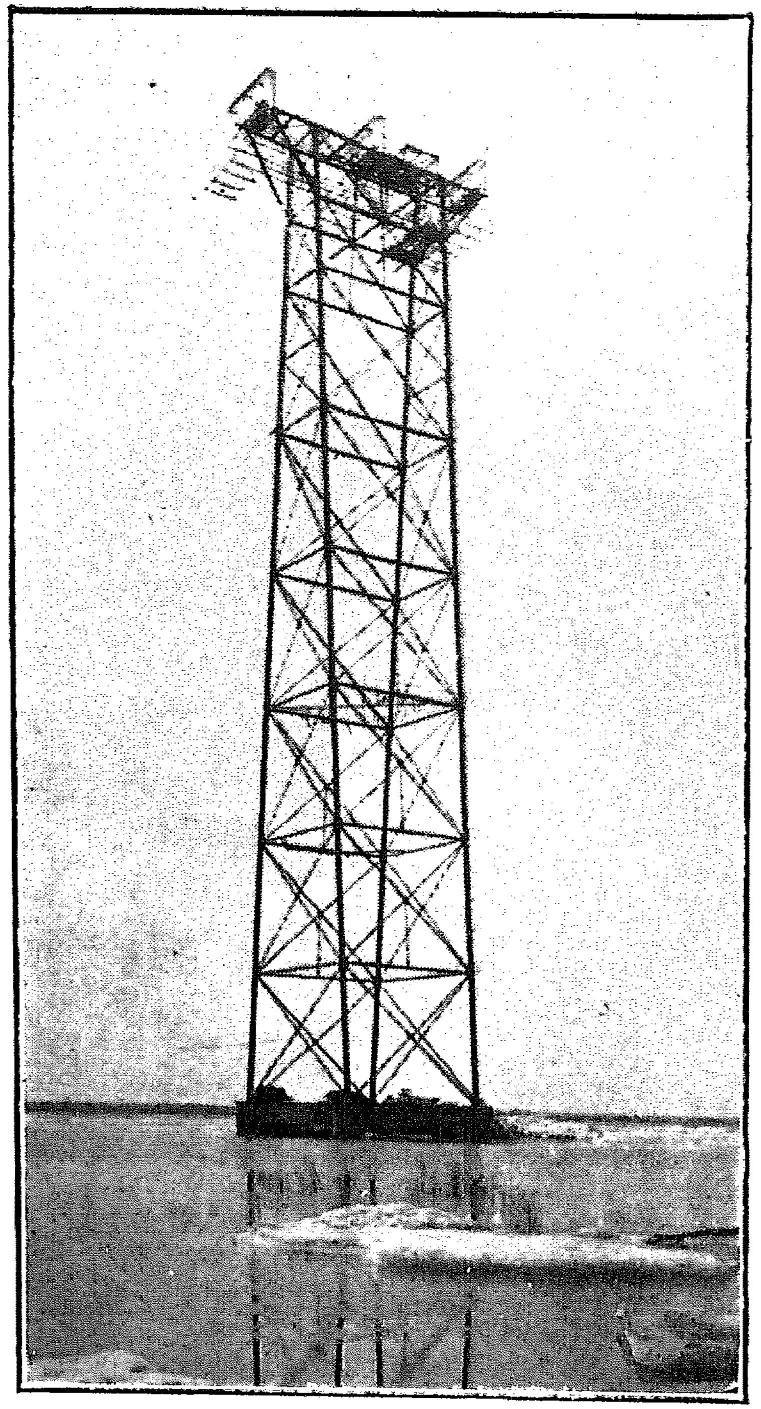
There are two towers 350 ft. high and 60 ft. square at the base, the upstream and downstream faces tapering to a width of 14 ft. at the top. A crossarm at the top, 14 ft. wide by 100 ft. long, carries three double-groove sheaves 50 ft. apart over which the anchor cables pass. The tower foundation is made up of four circular reinforced concrete piers 11 ft. in diameter placed on the corners of a 60 ft. square. These piers are connected by heavily reinforced concrete beams 4 ft. wide by 8 ft. deep. The tip of the foundation is 14 ft. above the normal water level.

Three lines of 1 3/8 in. diameter steel cables 50 ft. apart span the river between the two towers. To each end of each center span cable is yoked two anchor span cables. These are carried over the tower on the main sheaves and then down to a point about 20 ft. from the anchors. At this point equalizing beams are cut into the lines and the load is transmitted from them to the anchor piers by means of short straps of 1¾ in. diameter cable. The cables are gripped at the ends by means of heavy steel bridge sockets in accordance with the usual practice for suspension bridge cables and other structures of this type.

It was originally intended to use the main cables as conductors and to insulate them from the towers by means of specially designed insulators. Unfortunately these insulators were not completed in time for erection and for the present the main cables are used as messengers from which No. 1/0 stranded copper conductors are suspended. These suspended lines are supported every 250 ft. by suspension insulators of eight units to a string.

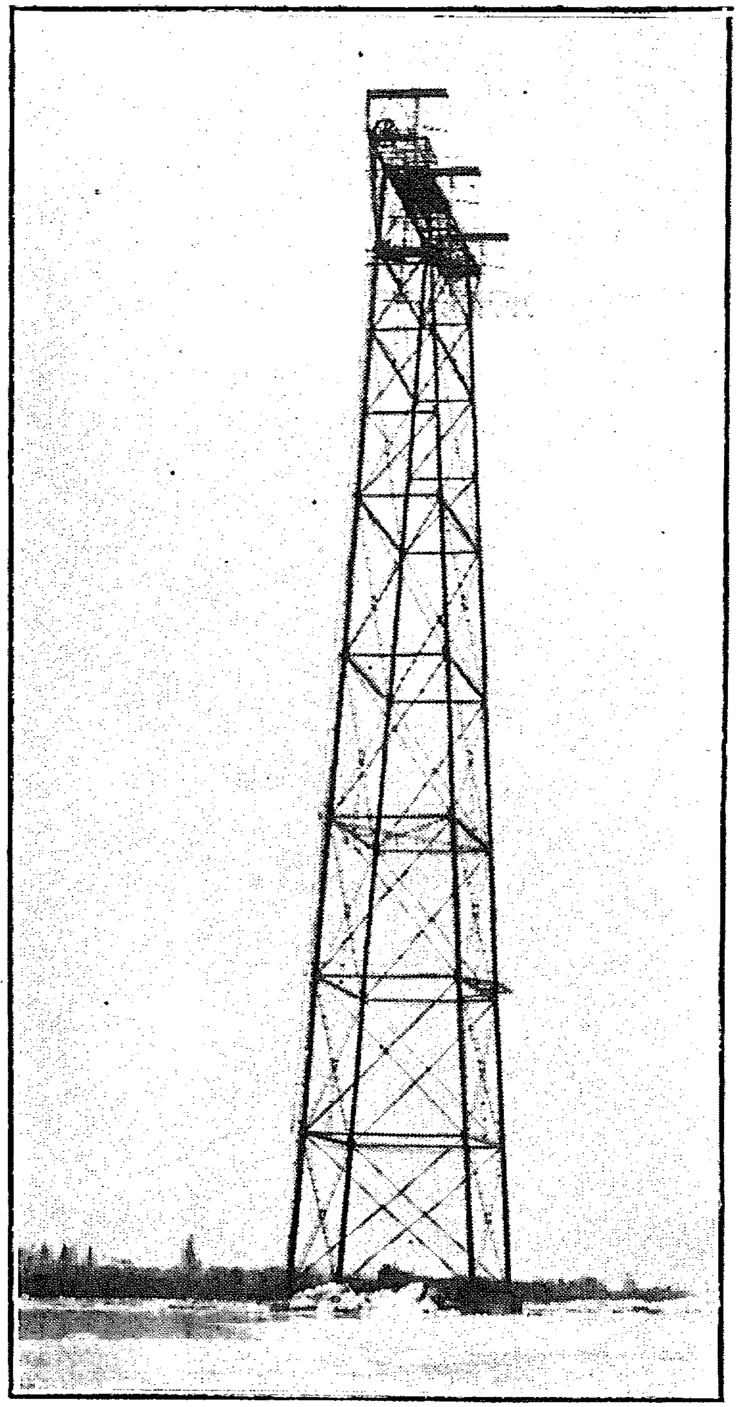
The original scheme contemplated the use of part of the submarine crossing as a standby until some future date when an additional overhead crossing identical with the present one would be installed. However, the success that has attended the operation of the steel cables as messengers has led us to the conclusion that it is possible to use the present crossing for two circuits and thus to eliminate most of the expense of an additional overhead crossing. We have at present under consideration two alternatives for the accomplishment of this object. One alternative provides for the use of the steel cables for one circuit and the suspended conductors for the other circuit. The other alternative contemplates the carrying of two conductors on each steel cable. This would be accomplished by suspending the two conductors from the ends of light triangular steel or aluminum frames attached to the steel cables at intervals of 200 to 500 ft.

The anchor piers are large mass-concrete "dead men" each anchor being designed to take the full overturning moment when submerged.



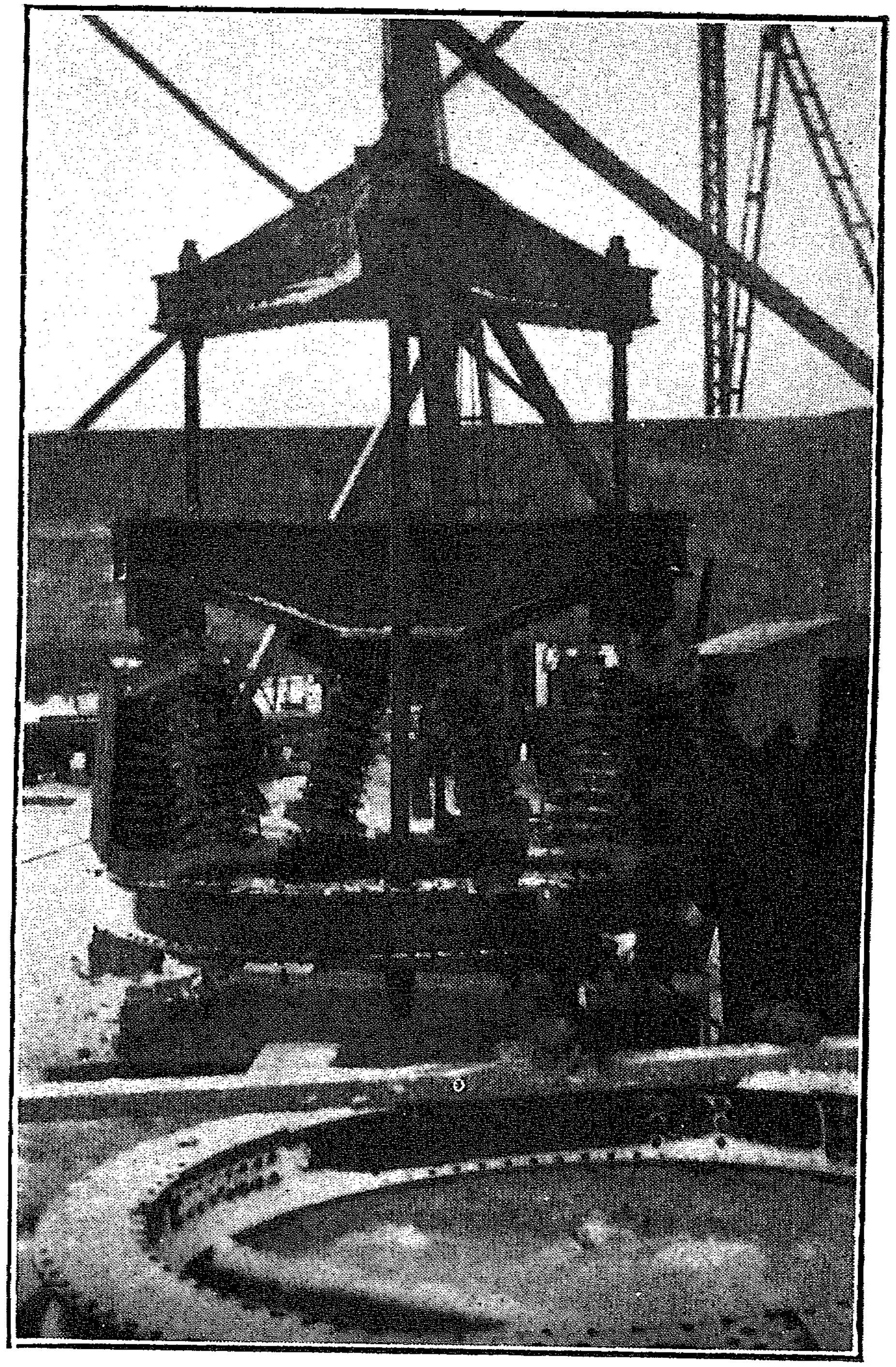
VIEW OF CROSSING LOOKING UPSTREAM

Towers are 350 ft. high and 4,801 ft. center to center.



VIEW OF NORTH SHORE TOWER

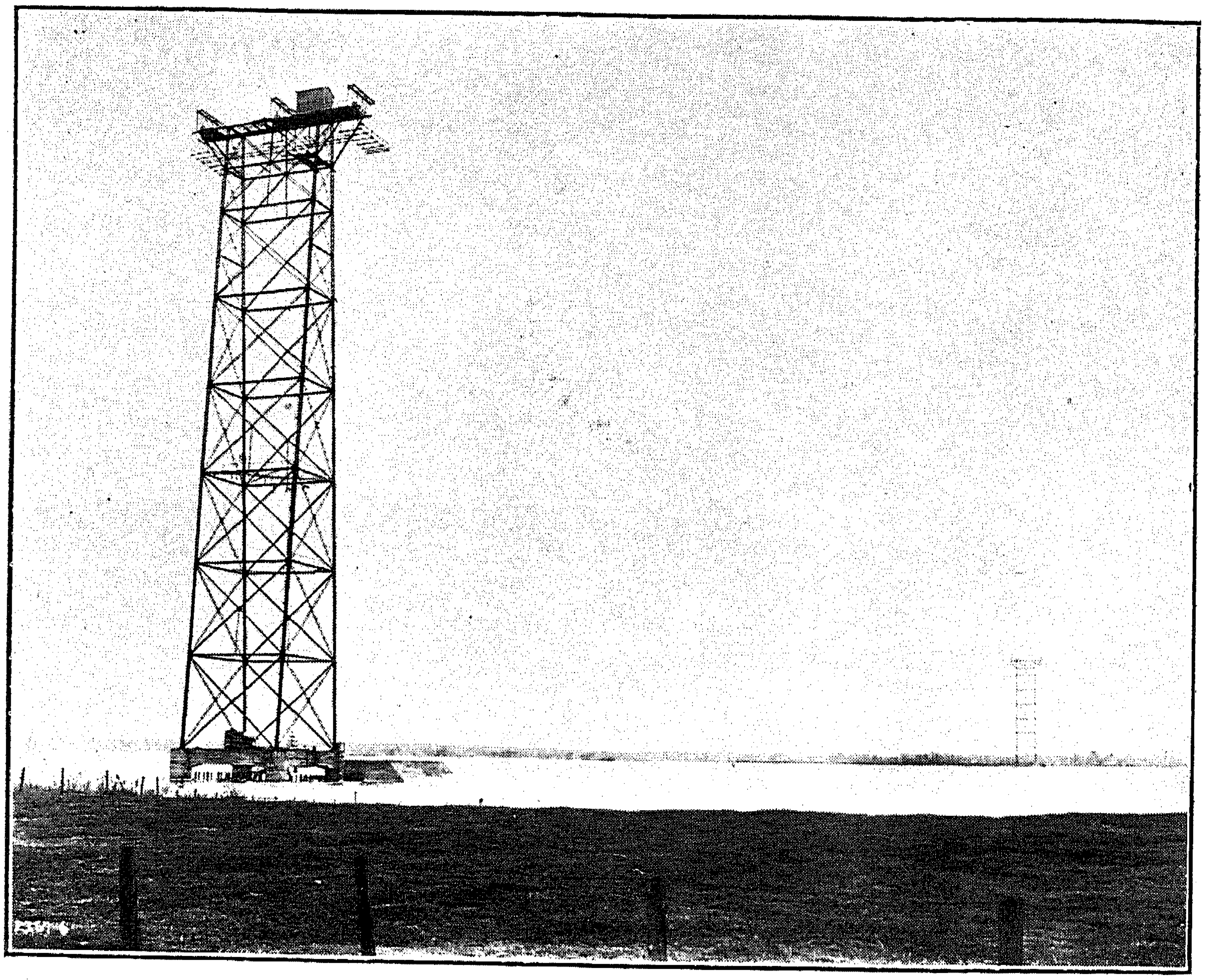
Showing ice piled up around the foundation after one of the ice shoves.



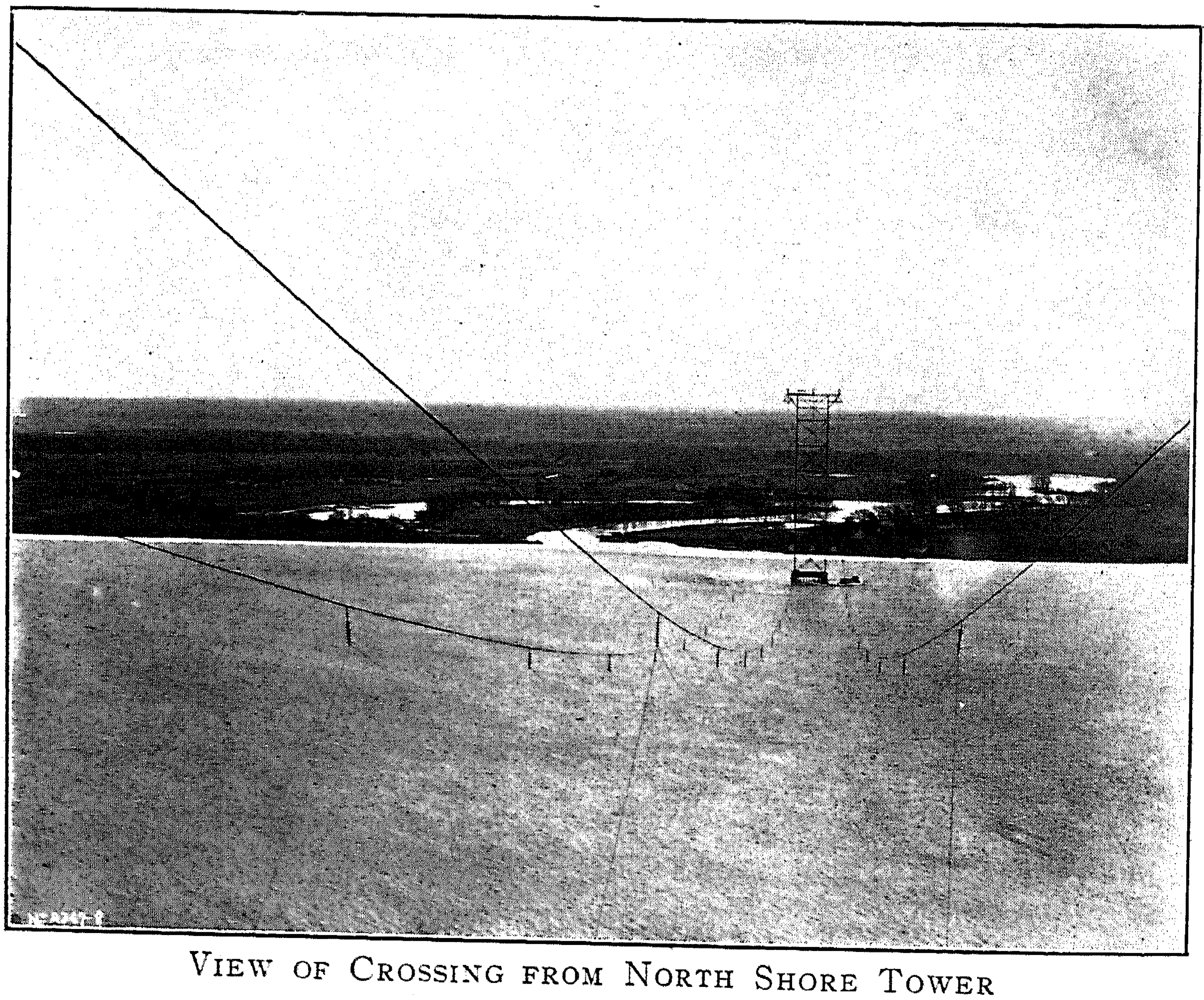
VIEW OF INSULATOR ERECTED

Parts of crates on insulators not removed—Insulator is 8 ft. in diameter, 13 ft. high, weight 6 tons—Anchor cables attached to the upper spider—Center span cable passes through center of the ring girder and is attached to middle spider.

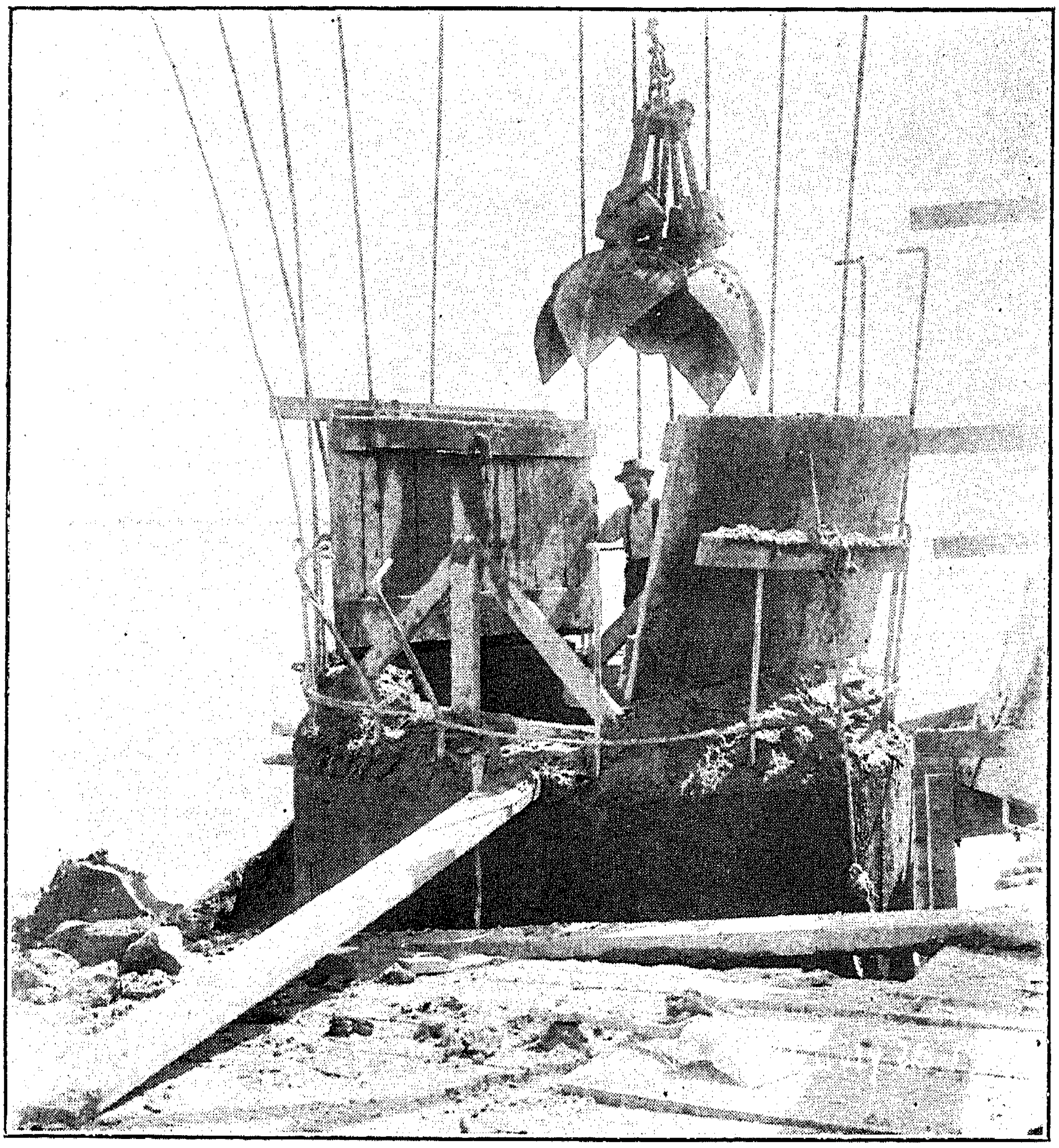
[SVENNINGSON]



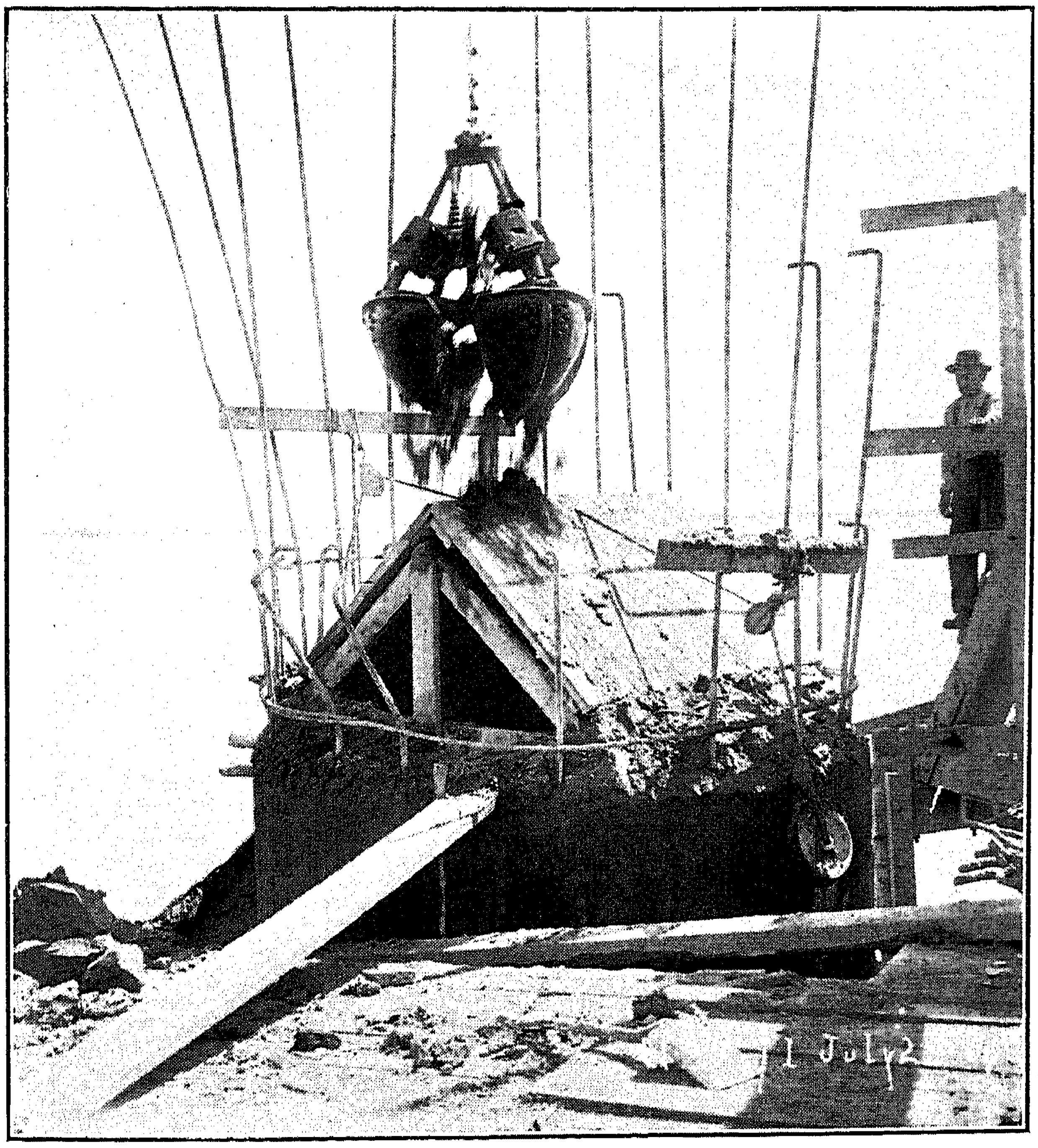
SHAWINIGAN WATER AND POWER COMPANY'S ST. LAWRENCE RIVER CROSSING. VIEW FROM NORTH SHORE.



VIEW OF CROSSING FROM NORTH SHORE TOWER



Cassion Construction—View Showing Method of Excavation with Orange Peel Bucket—Trap Door Open to Lower Bucket



Caisson Construction—View Showing Method of Excavation with Orange Peel Bucket—Trap Door Closed to Dump Bucket

FOUNDATIONS

During February 1917 a number of borings were taken about the site of the towers to determine the nature of the river bottom. These borings penetrated to a depth of 100 ft. and we found that the foundation on which we would have to build our towers consisted for the full depth of these borings of very fine white sand with occasional strata in which a little clay was mixed with the sand. The difficulty of obtaining a secure pile foundation in this kind of soil and the uncertainty as well as the cost of placing a mat foundation in the dry led us to adopt the form of pier foundation which we used.

The piers were constructed in the form of hollow cylinders of reinforced concrete with an outside diameter of 11 ft. and an inside diameter of 7 ft. These cylinders or caissions were poured in 6 ft. lifts, the first lift tapering on the inside towards the bottom to a diameter of 10 ft. and being shod with a 6 in. by 6 in. angle cutting edge. This lift was poured on a working platform and lowered into the water by means of four 2-in. screws. The second lift was then poured and after the concrete had set the bottom was excavated by means of an orange peel bucket rigged up on a derrick. As the caissons gradually settled successive lifts were poured until they had penetrated to a depth of about 40 ft.

Little trouble was experienced on the north side, but on the south side we encountered large numbers of boulders, some of which were so large that they could not be picked up by the bucket, so that we had to drill and shoot them. In order to do this the caissons had to be unwatered, a tedious process which delayed the work considerably. When a caisson had reached its penetration of 40 ft. a plug of rich concrete was poured in the conical section at the bottom and the inside of the caisson was then filled with mass-concrete. The four piers forming one foundation were finally connected by reinforced concrete beams.

This work was begun early in the year and we expected to have it finished by mid summer, but high water, high winds, rain and labor troubles delayed us so much that it was not completed until about the middle of September.

TOWER DESIGN AND ERECTION

The towers were designed for the following loads:

A vertical load of 530,000 lb. made up of 350,000 lb. due to the weight of the tower itself and 180,000 lb. due to the vertical component of the tension in the cable.

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A vertical load of 530,000 lb. made up of 350,000 lb. due to the weight of the tower itself and 180,000 lb. due to the vertical component of the tension in the cable.

A horizontal load of 42,000 lb. at the top of the tower parallel to the line due to the horizontal component of the tension in the cable.

A horizontal load of 26,000 lb. at the top of the tower normal to the line due to wind load on the line.

A wind load on the towers of 400 lb. per foot height or 140,000 lb. total.

The maximum compression on each of the front legs was estimated to be 575,000 lb. and the maximum uplift on the rear legs 233,000 lb. per leg.

The calculated deflection of the towers under maximum load is 4½ in. The wind and dead load stresses in the tower members do not exceed 20,000 lb. per square inch.

The bottom sections of the tower legs are composed of two 18-in. I beams weighing 70 lbs. per foot. These sections get lighter towards the top, the top sections being made up of two 12 in. channels weighing 20½ lbs. to the foot. The two members composing each leg are laced together with 2 in. by 2 in. angles.

Access to the top of the towers is obtained by means of Otis-Fensom Automatic Elevators, operated by electric hoists at the top and controlled by push buttons at top and bottom. The elevated baskets are composed of angle frames with plank floors 3 ft. wide by 4 ft. long. The elevator guides consist of four ⅝ in. diameter cables one at each corner of the basket, suspended from the top and weighted at the bottom to provide a constant tension of 1000 lbs. A steel ladder along one of the legs also provides access to the top and to the various levels at which insulators are strung. The ladder is enclosed from the 50 ft. level to the top with a wire cage and provided with trap doors and seats every 50 feet.

The towers were erected as follows:—

A timber erection tower 10 feet square was built up to a height of about 20 feet on piles which had been driven in the center of the working platform. A steel derrick with a 60-ft. boom was hoisted to the center of the erection tower by means of a jin-pole. The material for the towers was unloaded from the cars on the wharf at Three Rivers and transported to the site in barges. The bottom section, with its cross-bracing and girts was erected in place by means of the steel derrick. The erection tower was then raised another 50 feet, the derrick hoisted to the top as before and the next section of tower erected and so on to the top. The erection tower was guyed to the legs of the steel tower as it went up. Rivetting gangs followed close on the heels of the erection gang. As soon as the north shore tower was completed work was begun on the south shore tower. The erection of the north shore tower was begun early in September. The winter set in rather early and working conditions were so adverse that, in spite of the fact that the Shawinigan Company paid the men a substantial bonus, the south shore tower was not fully completed until after the first cable was erected in March.

Each of the towers is grounded by means of a copper plate four feet square which is buried in the river bed and connected to one of the tower legs.

The three sheaves at the top of each tower are eight ft. in diameter and weigh about 2.5 tons each. The rims and hubs are of cast steel and were machined before the sheaves were assembled. The rims are in four sections bolted together. The spokes are built up of steel plates and angles rivetted together and bolted to the rims and hubs. The shafts are 5 in. in diameter of medium open-hearth steel. After the sheaves were assembled the hubs were shrunk on the shafts. The sheave bearings are of cast steel bushed with bronze. Lubrication is provided by means of wicks.

INSULATORS

The insulators which we proposed using in the steel lines were devised by our Engineering Department in conjunction with that of the Canadian Porcelain Company. They consist of a large ring-girder and two spiders.

The ring-girder is 8 ft. in diameter and made up of two 9-in. channels 12 in. apart with ⅜-in. cover plates. The spiders are built up of plates and angles with a heavy steel casting in the center. The upper spider is connected to the ring-girder by means of three 2½-in. bolts 10 ft. long, one at the end of each spider arm. The center spider is supported on the ring-girder by six porcelain insulators of eight skirts each, two insulators at the end of each spider arm. The clear distance between the spiders is about 36 in.

The porcelain insulators used are special compression insulators having a tested breaking strength of 60 tons each, this is about four times the estimated maximum load. Electrical tests showed a dry flash-over of 302,000 volts and a wet flash-over of 262,000 volts.

The completed insulator has a net weight of about six tons.

CABLES

The cables are 1⅜ in. in diameter, of galvanized plough steel, made up of six strands of 19 wires each and a stranded steel core of 30 wires with a small hemp centre. Tests made at McGill University showed that the individual wires had an average yield point of 221,000 lb. per square inch, and an average breaking strength of 258,000 lb. per square inch.

The completed cable was tested, the yield point being found to be 158,500 lb. and the ultimate strength 186,400 lb. or 193,000 lb. per square inch and 227,000 lb. per square inch respectively.

The test of the completed cable indicated a modulus of elasticity of 7,250,000 lb. or 8,800,000 lb. per square inch. We are in doubt as to the correctness of our test in this regard on account of the fact that the usually accepted value for the modulus of elasticity for stranded steel cables is about 21,000,000 lb. per square inch. However, the behavior of the cable during erection bore out the results of the test.

No electrical tests were made on these cables but their sectional area would indicate a conductivity equivalent to that of a 100,000-cir. mil copper cable.

The bridge sockets used for connecting the cables were machined out of solid blocks of steel. Those attached to the center span cables are 15 in. long, taper in width from 5½ in. at the front to 8 in. at the back and are 9 in. thick, providing a grip of this length on the cable. The sockets attached to the anchor span cables are slightly smaller. A conical hole, tapering in diameter from 1 9/16 to 5 in. is bored through the center of the block from front to back and on each side of this hole a 2 5/8-in. diameter hole is provided for the connecting bolts.

The cable was passed through the tapered hole in the center of the bridge socket and broomed out on the end for a length of 15 to 18 inches. The wires were then cleaned with gasoline and held in place by means of a templet made of ⅛-in. steel plate which fitted over the back of the bridge socket. The bridge socket was then suspended with the back up and heated by gasoline torches for about half an hour when spelter was poured into the conical hole through a one-inch diameter hole in the center of the templet. After the bridge socket had cooled the ends of the wires projecting from the templet were cut and the templet was removed.

Before adopting this form of connection tests were run under our direction at McGill University to determine the depth of socket required. We found that if the spelter was heated to just the right temperature *i. e.* just hot enough to ignite a sliver of wood thrust into it, that the full breaking strength of the wire was in the majority of cases developed in a length of six inches.

Shortly after the bridge sockets were poured we found it necessary to shorten two of the cables and the speltered end was cut off. We had one of these cones of spelter cut in the machine shop and found that the spelter adhered so firmly to the wires that the section could be machined without lifting the wires out.

Erecting Cables

Owing to a constant succession of delays that occurred in the construction of the foundations and in the erection of the towers we had to abandon our original plan of stringing the cables in the fall of 1917 before the ice formed in the river and so decided to do this part of the work after the ice had become thick enough to support the weight of the heavy reels of cable.

Throughout the heavy snows of January and February we managed by constant rolling and scraping to keep a road open between the two towers. Early in March the center span cables were laid out along this road. The anchor cables were then laid out, measured, and cut, and their bridge sockets attached.

The three lines were erected one at a time, the middle line first and then the down stream and the upstream lines in succession. The ends of the anchor cables were hoisted over the towers, the south shore cables made fast to the center span cable, drawn over the tower until the bridge sockets touched the main sheave, tied to the top of the tower and attached to the anchor pier. The north shore cables were next attached to the center cable, the suspension insulators and copper line fastened to this and the cable hoisted into place. The proper location of the suspension insulators was determined beforehand for both the steel and copper lines, due allowance being made for stretch in the cables and these distances were chained off on the cables as they lay on the ice. After the cables had been hoisted into place the end spans of the copper lines were attached to strain insulators on the towers and drawn up until the suspension insulators hung vertically.

The hoisting was done by a steam hoist braced against the center anchor pier on the north shore. Two ⅝-in. steel hoisting lines reeved through two pairs of three-sheave blocks were used to draw the end of the cable up to within 40 ft. of the anchor pier, the final 40 feet being taken up by means of two ¾-in steel cables reeved through two pairs of six-sheave blocks.

This method of pulling up the cables appealed to us as being quite simple and easy of accomplishment. However, we found that it was a nerve racking process the first time we tried it. The cable seemed to come up easily to within a short distance of the anchor pier when the hoisting engine began to show signs of distress. When the end of the cable was within three ft. of the anchor pier the hoist suddenly coughed and quit. Those of us who had calculated the length of the cable and measured it as it lay stretched out on the ice wondered if we had slipped up on our figures or our measurements for at the calculated tension in the cable, allowing generously for friction in the blocks the hoist should have been capable of pulling the cable right home through twelve parts of line without any trouble. The riggers didn't believe that we were able to calculate the tension in the cable and were wondering if their pins and lashings would hold. A wait of ten minutes for the hoist to recuperate and for an inspection of the tackle and connections, a little sand in the clutch, a few more revolutions of the drum and the cable came in another foot. Another ten minutes for inspection and recuperation, another puff or two, a few more inches gained and finally, by dint of nursing the hoist and pulling for it on the part of all concerned the cable was hooked up and made fast. A slight change in the tackle made it possible to pull up the other two lines without any mental effort whatever.

The copper conductor in each line is supported by 17 suspension insulators spaced about 250 ft. apart, the end insulators being about 400 ft. from the towers. The copper lines drop from the end insulators to strain insulators on the towers at the 150-ft. level, pass through the towers to the back where they are connected to another set of strain insulators. On the north shore the lines pass directly from the main tower to a transmission line tower on the shore a distance of about 600 feet. On the south shore a light structural steel truss, 50 ft. long, hung from two sets of the anchor cables, provides an intermediate point of suspension, forming two spans of 500 ft. each. Access to the insulators attached to the truss is provided by a foot bridge running up from the anchor pier and suspended from the anchor cables.

After the cables were erected we noticed an almost constant vibration in them, varying in intensity and somewhat similar to that in a violin string, with definite nodes 12 to 15 ft. apart as nearly as could be judged. About a month after the line was put in service this vibration shook loose the bolts connecting two of the suspension insulators to the cable and they dropped and hung suspended on the copper line.

This condition brought us face to face with a problem which we had considered with misgiving and which had led us to discard the idea of suspended conductors when the crossing first came up for consideration. However, the difficulty was solved for us by two of our riggers who volunteered to go out on the steel cable, for a consideration, and to fish up and re-attach the insulators. A trolley was devised and constructed of two sheaves on which a small platform was hung and they had little difficulty in getting out to the point from which the insulators had fallen, about 1000 feet from the tower. By means of a small tackle line they hauled the insulators back into place and started back towards the tower. Unfortunately the sheaves on the trolley jammed and they could not pull themselves up. At this point one of the riggers lost his nerve and the episode might have had a tragic ending but for the fact that the other kept his head. He lowered his mate to a boat waiting in the river 250 ft. below, then lowered the trolley and finally passed the line over the steel cable, slid down to the boat and pulled the line down after him. A short time later an insulator on one of the other lines broke loose and it was similarly reconnected. This time, however, we profited by our former experience and provided a tail line by means of which the riggers were pulled back to the tower. Since then we have experienced no trouble from this source.

So far as we have been able to judge the suspension insulators had not been damaged in any way by the vibration in the line but there is a possibility that it may cause the cement in which the caps and bolts are set to disintegrate. We are at present carrying on investigations with the object of devising some means of absorbing this vibration between the steel cables and the insulators.

The cables as originally strung allowed the following clearances between the copper conductors and the average water level during the season of navigation: Downstream, 172.5 ft.; Center, 178.8 ft.; Upstream 180.6 ft.

The temperature at the time of erection was about 20 deg. fahr. As there is a change in sag of approximately one foot for each 10 deg. change in temperature the above would correspond roughly to clearances at 110 deg. fahr. of 163.5, 169.8, and 171.6 feet respectively.

At the time these cables were erected we naturally expected the sag to increase as the cables stretched under the load until the strands were drawn tightly together. There was no data available with regard to the amount of stretch to be expected so that it was impossible to allow for this in sagging the cables. The hoist, therefore, was left in position so that we could pull up the cables when the sag became too great.

In May of this year we found that the sag in the cables had increased by from 24 to 27½ ft. and that in order to obtain the necessary clearance over the channel we would have to take up 24 ft. in the sag of the downstream cable and 13 ft. and 14 ft. in that of the center and upstream cables respectively. The amount by which a cable will stretch in taking up a given amount in the sag varies inversely as the modulus of elasticity of the cable. Owing to the low modulus which we worked out for the cable from the results of the tests made at McGill University we were in doubt as to the amount of take-up required. We found that in order to take up 24 ft. in the sag we would need to pull the cable in between 7.2 and 10.4 ft. depending on the value of this modulus. This cable was taken in about eight ft. with a consequent reduction in the sag of about 25 ft. This corresponds to the result that would be obtained if the modulus of elasticity of the cable were 17,000,000 lbs. In other words it would appear that from the time of the original sagging of the cable to the time the cable was re-sagged the modulus of elasticity had increased from 7,250,000 lb. to 17,000,000 lb. This change in modulus is no doubt due to the gradual stretching of the cable causing the wires and strands to draw more closely together under the constantly applied tension of the span.

Ice Protection

Ice conditions in the St. Lawrence River at this point are at times very troublesome and we considered it advisable to construct some kind of guard piers outside the towers to obviate the possibility of damage from this source. During the winter we deposited about 3000 tons of field stone on the river bed on each side about 75 ft. from the upstream and river faces of the towers, carrying the rock up to an elevation about three ft. above the level of the ice. The ice usually goes out about this level, but last year conditions were exceptional, and before the ice moved it had risen above the tops of our ice-breakers and passed clear over them piling up around the tower foundations to a height of 25 or 30 feet. Fortunately no damage was done. We are at present completing the guard piers, by means of reinforced concrete cribs filled with rock and carried to about the level of the maximum recorded high water.

SAG CALCULATIONS

In our calculations for sag, tension, length of cable, etc. under various conditions, we used the parabolic formulas in preference to the hyperbolic formulas for the catenary, on account of the greater simplicity of the former. Comparison was made however between the two sets of formulas and we found, as we had expected, that at working tensions the difference was negligible. The formulas for the parabola gave us about six in. more sag and about one ft. less length of cable than the catenary formulas for the same conditions of tension and temperature.

We assumed the maximum load on the cable to be three-fourths of an inch of ice all around and 10 lb. of wind per square foot of projected area, for both the steel and copper lines, at a temperature of 0 deg. fahr. Under these conditions the calculated tension in the cable is about 106,000 lb. with a sag of 228 ft. The normal tension at summer temperatures is about 61,000 lb. with a sag of 185 ft.

CONCLUSION

There are still a few details to be worked out in connection with the crossing before we can consider the undertakings completed. The questions of providing two circuits on the crossing and of devising some means of absorbing the vibration are still under consideration. The problem of inspection and renewal of the suspension insulators has been only partially solved.

The crossing has been in uninterrupted service now for about nine months. It has not yet weathered a winter with its low temperatures, gales, and sleet storms, so that we still have something to learn about its action under these conditions. The allowable stresses, however, have been kept within reasonable limits and we hardly expect serious trouble from this source. The success that has attended the operation of the crossing since it was put into service gives us reason to expect a satisfactory solution to the problems still remaining.

Discussion on "110,000-Volt Transmission Line Over the St. Lawrence River" (Svenningson), Toronto, Canada, November 22, 1918.

C. W. Baker: There are two questions I would like to ask. First, in the crossing with the submarine cables, why, suspend them in the steamer channel, why not let them lie right on the bottom? Would that obviate the trouble you had with the swinging of the cable in the steamer channel, breaking it off at the points of suspension?

Second, in blasting out the boulders you encountered in sinking the caissons, at the south shore, how did you prevent destroying the caissons at the same time?

S. Svenningson: With regard to the submarine cables, the reason for the cables hanging loose was on account of the very steep slopes at certain points of the river and at the same time the very strong current. Of the five submarine cables we have in place, there has really been one cable which has given absolutely satisfactory service, and that happened to be a very heavy cable, three-core conductor with a very heavy armoring feature. The river bottom in the St. Lawrence changes from time to time on account of shifting of fine sand and light stones which form the river bed. We tried very hard to weight down the cables and then put weights on them, but did not succeed.

With regard to blasting of the stones in the caisson, we had to take particular care that the caissons were emptied before the shot was fired. For that reason, we had a steam pump that would pump the caissons dry, and then we went down and drilled the stone and placed the charge. In spite of that we did have an accident. One caisson cracked for a distance of five feet, but fortunately it held together and no damage was done.

Ernest V. Pannell: I do not think sitting in this room we can appreciate the stupendous character of the work described by Mr. Svenningson's paper. It would be necessary for us to take a trip to Three Rivers. If we imagine the Excelsior Life Building, 100 ft., placed on top of the Royal Bank Building, 250 ft. high, we shall have some conception of the height of the towers employed in this crossing.

I wish to ask Mr. Svenningson about a point which I do not think was covered in the paper. That is the formula he used for the compression legs of the towers. I examined a few of these formulas lately and find that the results vary as much as 100 per cent, according to the different expressions used, and it would make the paper more complete if, when he revises it, Mr. Svenningson would express the formula used for obtaining the compression in the legs, and also the value of L/r for these legs.

The point about the modulus of elasticity of the cables is very interesting. I have tested out wire myself and expected to get a modulus of nine million, and found that it came out three and one-half million; that is, the actual modulus is about one-third of the theoretical modulus, and I cannot account for it, and I think that Mr. Svenningson may have some theory, as to whether the difference is brought about in the limited lengths of cable tested between grips, or whether there is something about a stranded cable which there is not about solid material. I do not know, but has Mr. Svenningson any theory why, when the theoretical elasticity limit of the cable is 21,000,000 it should test out at 8,000,000. That is not only true of steel cables, but it is also true of other forms of cables.

S. Svenningson: In regard to the formula for the compression of legs of the tower, it is a standard formula used by the Canadian Bridge Company. I do not remember exactly that formula. L/r was somewhere around 80.

In regard to the modulus of elasticity, I believe that our observation at Three Rivers has shown that the difference in the modulus of elasticity, the theoretical from the practical, is due to the effect of a stretching of the cable the first month, or six weeks and, as I said in the paper, that might introduce a great deal of vibration, very violent at times, which I believe was due to the center core taking relatively more load than the outside strands. In making up the cores, the center was made of a softer material than the outside. It was made of soft steel to allow for a good deal of stretching, and, as I remarked, the modulus of elasticity that we found is about 7,000,000 lb., while after the cable had been up for six weeks or two months, it had apparently increased to about 17,000,000 lb., which I believe is due to all the wires taking their proportion of the load. In other words, when you test a small piece of cable in a testing machine, possibly 25 per cent of the wires will take 75 per cent of the load.

H. B. Dwight: What precautions are taken to protect the steel cable against corrosion as the years go on? I presume the galvanizing is good for a few years, but probably after that time something will have to be done for a structure of such importance. Its life, to be worth while, must necessarily be a good many years.

S. Svenningson: When the time comes the galvanizing will flake off, and, of course, we have to paint the wire and use what we call "cable dope." We send a man out on the cables to paint them.

H. B. Dwight: How often would that have to be done?

S. Svenningson: I believe it would have to be done about once a year, and it may be, after the cable has been up ten or fifteen years, the cable will have to be replaced.

H. B. Dwight: While you are answering that question, may I ask if you have had any experience with the ground cables on your transmission lines; have you had any necessity for replacing the steel ground cables?

S. Svenningson: No, we have not.

H. C. Don Carlos: Mr. Svenningson mentions as one of the deciding features in determining the long span construction as against the short span construction, the inaccessibility of the towers in a short span construction. It seems to me that would probably be more than offset by the inaccessibility of the insulators in the long span construction, and the suspended conductor from the messenger. I think, undoubtedly, the insulators will require more attention than the towers would, and will have to be gotten at more often. In the short span construction, it would have been a very simple matter to have provided a messenger cable for getting at the towers under all conditions.

In reading over the paper and noting the very wide difference of tension between the summer conditions and the winter conditions with the ice load and heavy wind pressure, it occurs that it might have cut down the cost of the construction considerably to have counter-weighted these cables and provided uniform tension rather than approximating a uniform sag. During the winter, when you have a heavy ice load, it would obviously have been unnecessary to provide navigation clearances. It might be, with the ice loading and the wind loading, they would not have sufficient clearance from ground for safe operation; but if it did work out that the sags were not too great, it seems to me that it would have been a much safer construction and probably much cheaper construction, to have counter-weighted the cables. I would ask if they considered that proposition at all.

Another question I would like to ask is this—I notice there is considerable difference between the clearances of the different cables, but I could not figure out just why that was, and I was wondering whether that would be accounted for by the grade in the river.

Another thing that struck me as being a rather dangerous proposition in this construction is the possibility of damage to the messenger cable through an insulator breakdown or flashover, and the arcing to the grounded messenger cable might pit the messenger cable, and if anything should happen to the protective relays to prevent the line dropping out quickly, it might even seriously damage the cable and weaken it by the arc hanging on. I believe the Shawinigan Company operated with a dead ground neutral, and it probably would not be possible to hang on to an arcing ground for any length of time, but it struck me there was that possibility of an arcing ground damaging the cable and burning it to the point of failure.

What is the horizontal displacement of the cable under the maximum wind conditions that they have experienced. Have any observations been made along that line?

Also, what is the length of the next longest span? I believe up to the time of this construction the longest span in existence was across San Francisco Bay, but I do not recall what the length of that span is, and probably the author of the paper will tell us.

S. Svenningson: As regards the inaccessibility of the towers at Point du Lac of course, it should be borne in mind to start with that we did not intend to have any suspension insulators on the main cable. That is one reason. To get out on the main cable, we use the present crossing cables, and in that case the insulator which we have to replace will would only be of light weight, while if one of the heaviest pieces, in a series of insulators, on the Point-du-Lac scheme, should break down, it would be hard to replace them.

H. C. Don Carlos: I suggest that in a case of the 2000-ft. span which you had for the short span construction, you would have only used standard insulators?

S. Svenningson: We would have to use standard insulators in parallel.

H. C. Don Carlos: Yes.

S. Svenningson: Assuming you adopt a similar type, the same as you start with on the cable line, if a series of insulators, or one insulator breaks, the whole thing falls down practically, and that means that you must pull back the cable.

In regard to counter-weighting, we considered that, but, of course, the heavy windstorms and the sleet will occur at times of the year when the river is still open to navigation,—for instance, we had at the beginning of this week a very heavy sleet storm and a great rise in the tide, so that the water in Quebec rose about fifteen feet. Under such conditions, if we had employed counter-weighting, and a steamer should come up the channel, something might happen.

Another thing is that on account of that slight angle where the cable leaves the tower, the horizontal pull on the tower is slight, due to the difference in tension. The maximum tension on the tower from each cable is only 14,000 lb., and the weight of the tower would be practically the same, due to its height and the wind pressure which is exerted on the tower.

You asked a question about the displacement of the cable. Do you mean the horizontal deflection?

H. C. Don Carlos: Yes.

S. Svenningson: No, we have not seen much of that.

Another question was the difference in sag of different cables. There is a difference of sag of about 10 ft.—I think that was the maximum—and that was undoubtedly due to the difference in the stretching of the cable during erection. The cables were measured, and they were exactly of the same length, but during erection one cable was stretched, probably, a little more than the other.

With regard to the arcing over of the insulators, the line is now operating at 50,000 volts, and we have eight skirt insulators in use, so that the possibility of arcing over is rather remote.

In regard to the next longest span, as Mr. Don Carlos mentions, that is the span over San Francisco Bay, which is 4400 ft. in length. I believe the United States Aluminum Company is erecting a span in the South somewhere, which is over 5000 ft. in length. That is a span between two mountains and the cable only consists of a 7-in. cable.

H. C. Don Carlos: Steel core aluminum?

S. Svenningson: I do not know.

T. A. Worcester: May I ask Mr. Svenningson if it is contemplated to abandon the use of the special insulator and continue to use the suspension insulator with copper conductors?

S. Svenningson: That has not finally been decided. We have the two schemes. One scheme is to put in the heavy insulator, which means we have to take down the lines, and that will have to be done in the winter time when we can build a pole line across the ice. With the other scheme we also find it necessary to have a pole line, but that can be coincident with taking down the wires. That has not been finally decided on.

F. J. Wyman: I think it occurs, perhaps quite frequently, where we have a combination of submarine or underground cable and overhead cable, and apparatus at each end, that the combination of inductance and capacity causes more or less disturbance in the lines in the nature of surges and static disturbances, consequently I would like to ask Mr. Svenningson if, in changing this line over, there has been any diminution of any kind in the disturbances, that is, if they were noticable before to any extent, and whether, since the change over, there has been any diminution. Were there any oscillograms taken before the change, and has there been any since?

S. Svenningson: We have two transmission lines leading down to the St. Lawrence River, and two transmission lines beginning from the South Shore down to the Sherbrooke district. As soon as the overhead crossing was built one line, the Thetford line, went over the overhead crossing, and the submarine crossing took care of the Sherbrooke line; but, so far as I know, they were not operated in parallel.

No tests have been made of the crossing, and so far as disturbances are concerned, I really do not know that I have noticed any difference.

T. A. Worcester: Another question—in the original plan, when it was contemplated to use the special strain insulator, was the problem of sleet considered, and were any arrangements made for melting the sleet from the conductors by heating them, either by the normal current, or by special short-circuit arrangements at times of light load? Would such an arrangement materially reduce the size of conductors which were required?

S. Svenningson: No, that is not considered, because very often we have disturbances at a time when we have no current to melt the sleet.

W. P. Dobson: I was interested in the two types of formulas for calculating spans, especially in connection with shorter spans, and I think this work Mr. Svenningson has done shows that the simpler formulas and simple methods of calculation are quite suitable for spans of practically any commercial limits.

I ask Mr. Svenningson what the stress in the porcelain insulators at the ends of the span was.

S. Svenningson: The stress per square inch?

W. P. Dobson: Yes.

S. Svenningson: The total area was 300 sq. in. and the maximum tension is 106,000 lb. That would be about 300 lb. per sq. in.

Presented at the 342nd meeting of the American Institute of Electrical Engineers, Philadelphia, Pa., December 13, 1918.

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THE USE OF ELECTRIC POWER IN THE MINING OF ANTHRACITE COAL

BY J. B. CRANE

ABSTRACT OF PAPER

This paper gives figures as to the power cost and current consumption of anthracite mines and the reasons for these being in excess of the requirements for bituminous mines.

Estimates are also given as to the additional coal that will be released by the electrification of the anthracite mines.

Illustrations are included showing representative installations of electric drive.

THE production of anthracite coal in the United States for the past five years was as follows:

Years	Total Gross Tons
1913	81,809,782
1914	81,580,479
1915	79,803,374
1916	78,406,387
1917	89,720,982

The undeveloped beds are to such a large extent owned or controlled by the large producers that no extensive opening of new mines has taken place in recent years nor is to be expected in the near future.

In the bituminous fields there are many independent holdings, and when the coal business was flourishing many new companies were formed and opened up new mines, using electric drive, mainly on account of lower first cost for installation of machinery. The resulting economies were such as to force the former steam-operated mines to electrify even where it was necessary to throw away expensive steam equipment.

Coal producers are conservative and not having the many examples of successfully operated electrified mines before them, the anthracite operators have been slower in seeing the benefits to be derived and the economies to be effected by throwing out their steam-operated and replacing it with electrically-operated equipment. The margin of profit, due to the better organization of the producers, was also higher and the necessity of saving in the cost of production had not been brought home to them with the same force. For these reasons the possibilities of making a greater progress, in the decrease in amount of coal burned for power, in increased production of coal, in decrease in amount of labor and in the general living conditions for the employees, exists to a greater extent in the anthracite than in the bituminous coal regions.

The anthracite operators, are at the present time keenly alive to this fact and are making every effort to take advantage of the improvements to be made by electrification of their mines, but are handicapped by the difficulty of securing electrical equipment and by the impossibility of securing sufficient power from the central stations supplying power to the anthracite regions.

The mines that have been electrified show some surprising results. The following figures are taken from a paper read before the A. I. M. E. (Economy of Electricity over steam for Power Purposes in and about Mines, by R. E. Hobart, Feb. 18th, 1918.)

	Steam Operation April 1914 to April 1915	Electrical Operation Nov. 1916 to Nov. 1917
Cost of Power	$46,992	$21,590
Cost of Heating	Included in above	8,700
Total	$46,992	$30,290
Tons of coal mined	343,665	435,073
Cost per ton	$0.137	$0.0696

When it is remembered that the value of the coal burned with steam operation has more than doubled and that the output of coal per man per year employed has increased from 540 tons to 647 tons it will be seen the figures above, large as they are, represent only part of the saving to be effected.

Some other figures obtained are interesting as showing costs of electrification, power used, etc.

Item	Mine No. 1	Mine No. 2
Gross tons-yearly	641,533	670,000
Kw-hr. consumption	2,312,195	3,477,876
Kw. demand	870	1,135
Annual L. F.	30.4	35

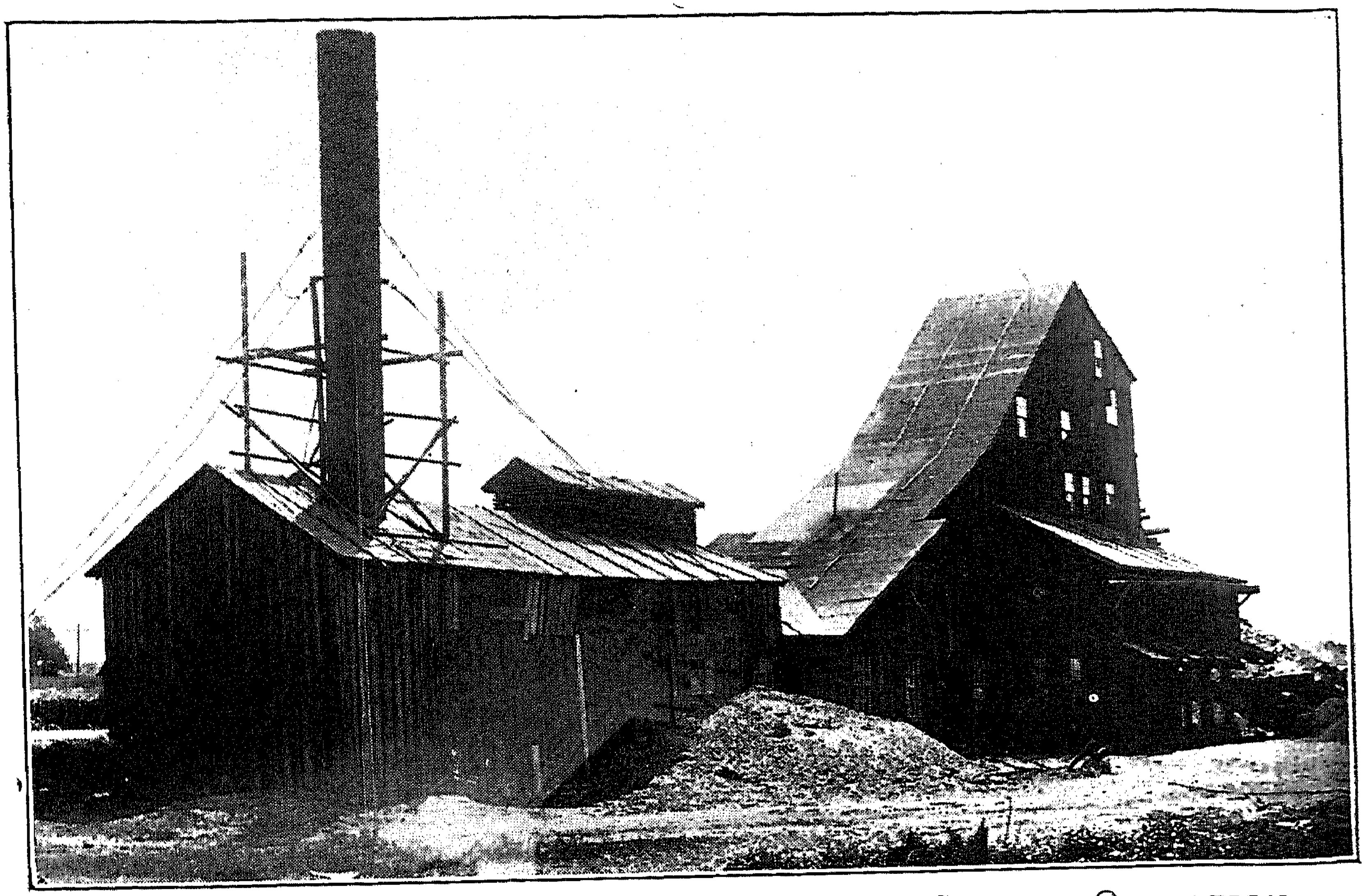
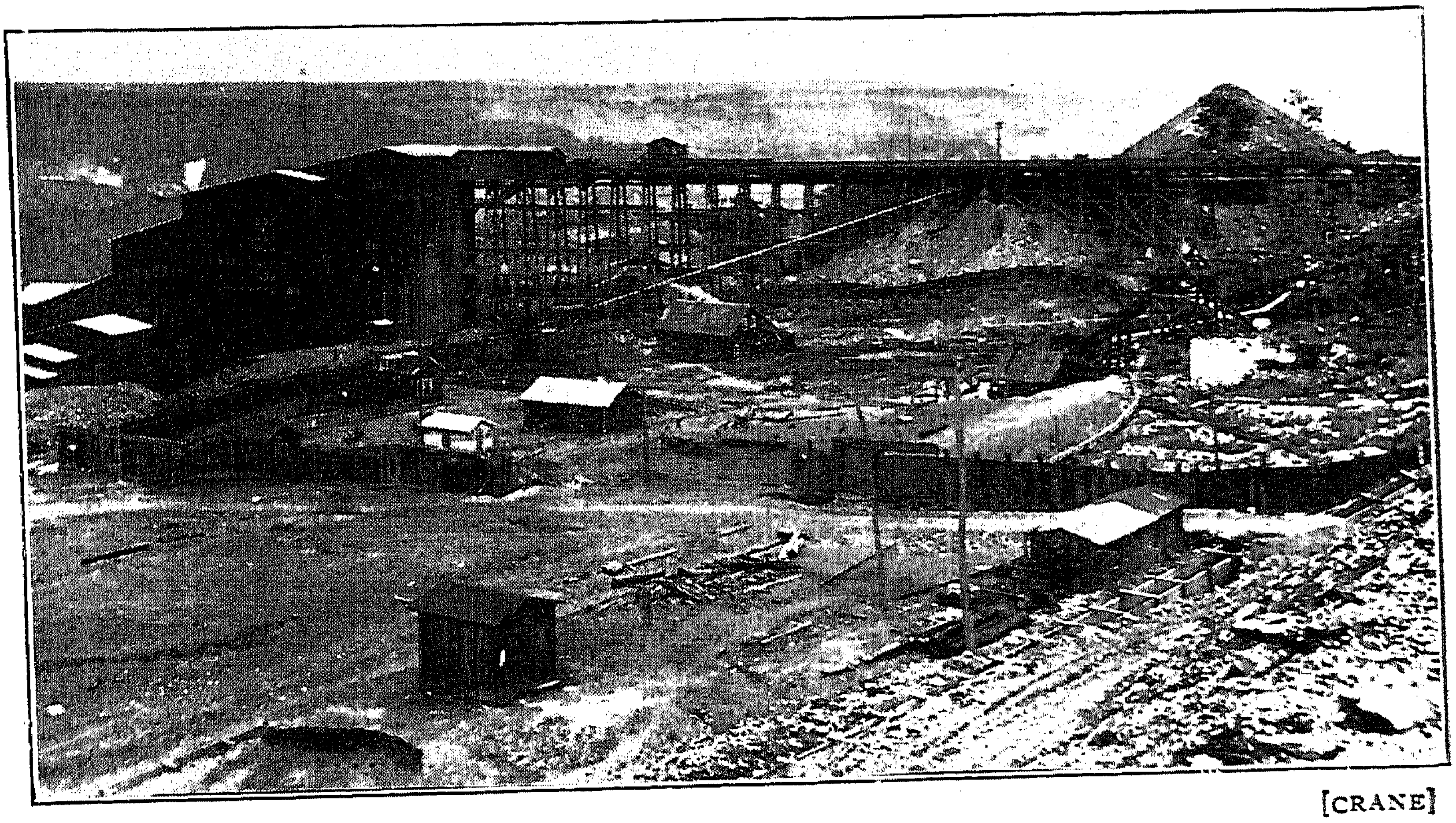


FIG. 1—AN EARLY ANTHRACITE BREAKER—STILL IN OPERATION



[CRANE]

FIG. 2—A MODERN ANTHRACITE BREAKER—ELECTRICALLY EQUIPPED

Kw-hr. per ton mined....	3.6	5.2
Kv-a. trans. capacity....	2,000	3,500
H. P. connected.........	2,400	2,535
Cost of electrification....	$134,500	$325,000
Cost per h. p. connected.	56.04	128.20
Per cent kw-hr. per pumping..................		50
Average depth of mine...	650 ft.(198.1m.)	800 ft.(243.8m.)

Mine No. 2 had a boiler plant of 800 h. p., used 900 tons of coal per month and 11 men have been put to other work about the mine. The output will be largely increased this year as the above investment provides for additional equipment, not yet in operation.

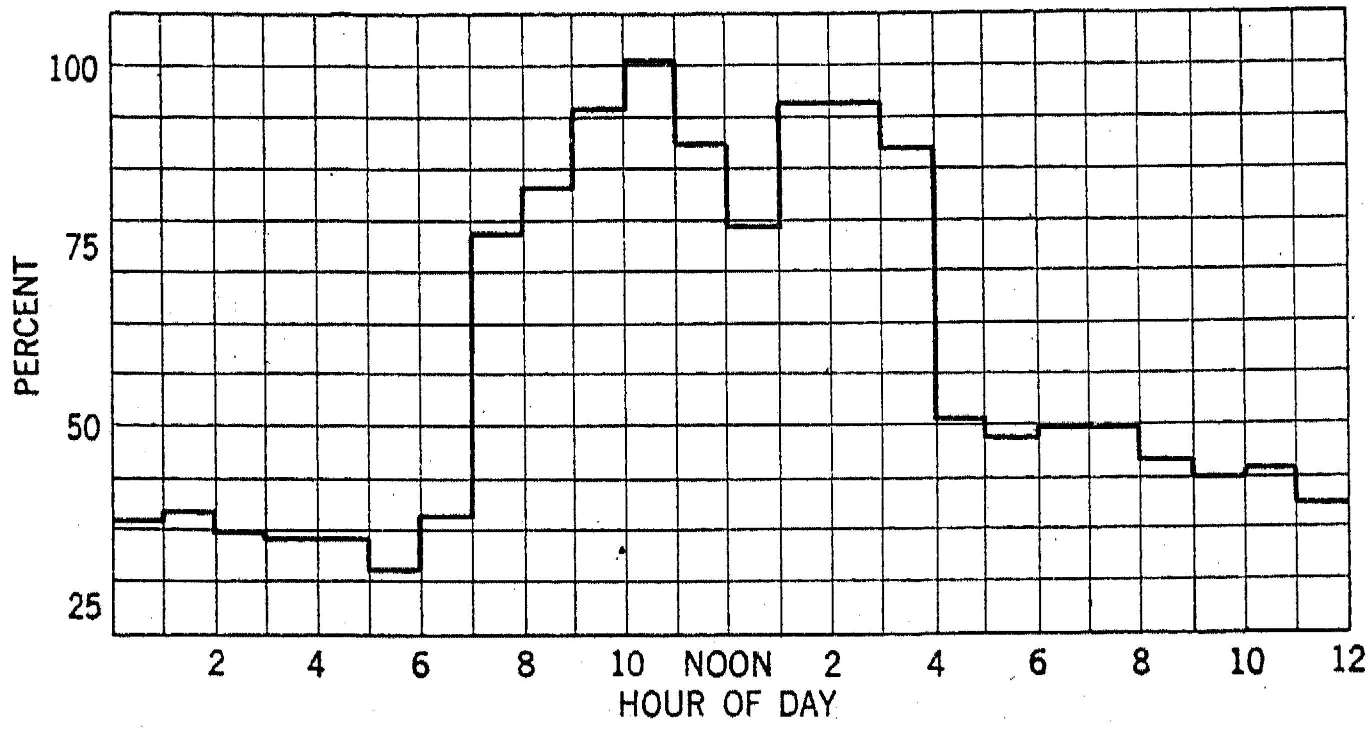


FIG. 3—TYPICAL ANTHRACITE MINE LOAD

Mine No. 1 has no pumping, hence the low kw-hr. per ton mined.

The best figures I have yet been able to obtain indicate that 12 kw-hr. per gross ton of coal mined is the average figure for an anthracite mine. This is of course subject to wide variations in individual cases on account of depth of mine, amount of air required for ventilation, quantity of water to be pumped as well as the head pumped against, and amount of work necessary to prepare coal for market. The writer, three years ago secured figures from over fifty bituminous mines and it is interesting to note the average kw-hr. per gross ton mined was 3.57.

The less power required for mining bituminous coal is due to various causes, among which may be mentioned:

Depth of Mine. Bituminous mines in many cases are situated above tipple, so that coal is brought to the surface on level tracks, and from this point is loaded into the cars by gravity, while the anthracite mines are from 300 ft. (91.4 m.) to 1000 ft. (304.8 m.) in depth and all of the coal has to be raised to the surface and from there to the breaker.

Pumping of Water. The anthracite mines have more water to be taken care of and this water has to be elevated from the lowest level.

Ventilation. In spite of the fact that there is less volatile in the anthracite coal there is generally more gas to be taken from the mine and a larger amount of ventilation has to be provided for and the additional space used for shafts, etc. has to be ventilated.

Preparation. The breaker of the anthracite mine requires more power as the coal is sized to a greater extent than is bituminous coal.

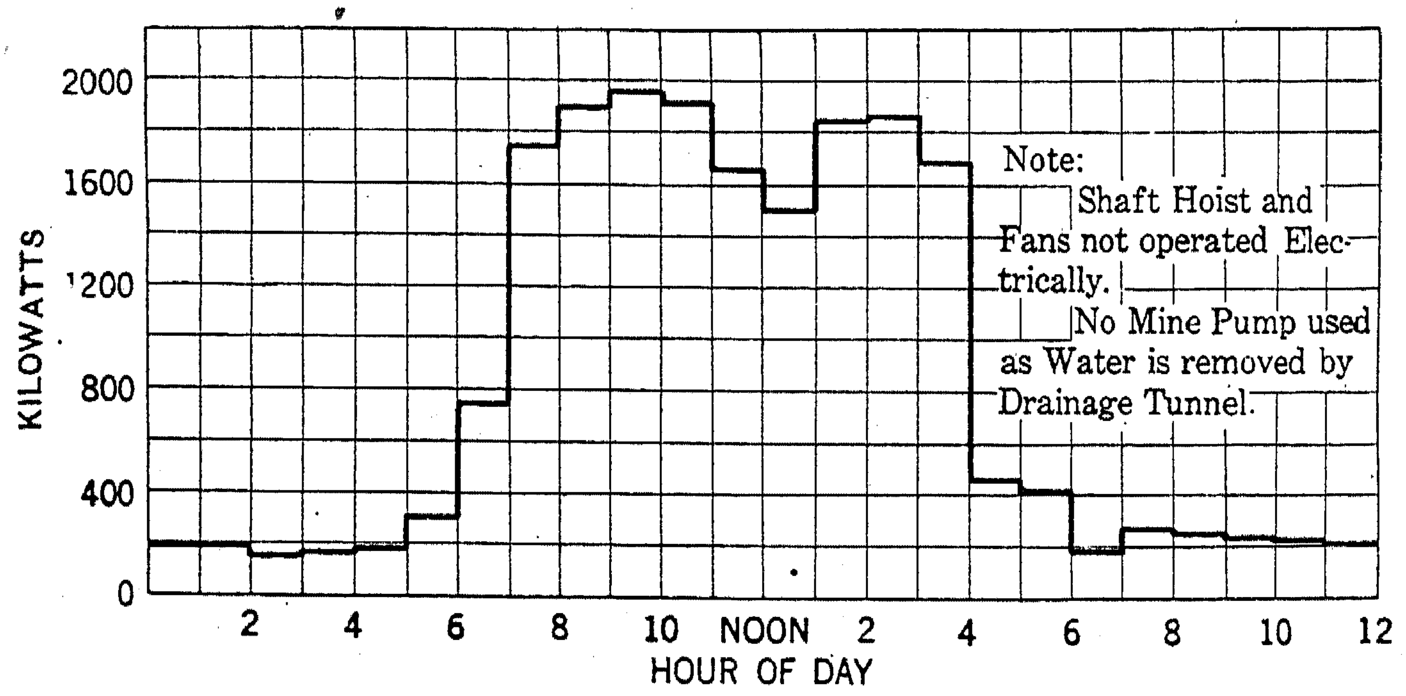


FIG. 4—TYPICAL BREAKER LOAD AND MINE HAULAGE

One anthracite mine had meters installed for the different services for one year and the following average figures were obtained.

Operation	Kw-hr. per gross ton of coal produced
Haulage	1.73
Ventilation	1.62
Drainage	1.30
Lighting (inc. charging station)	0.12
Hoisting	1.02
Air compressor	2.11
Breaker	4.75
	12.65

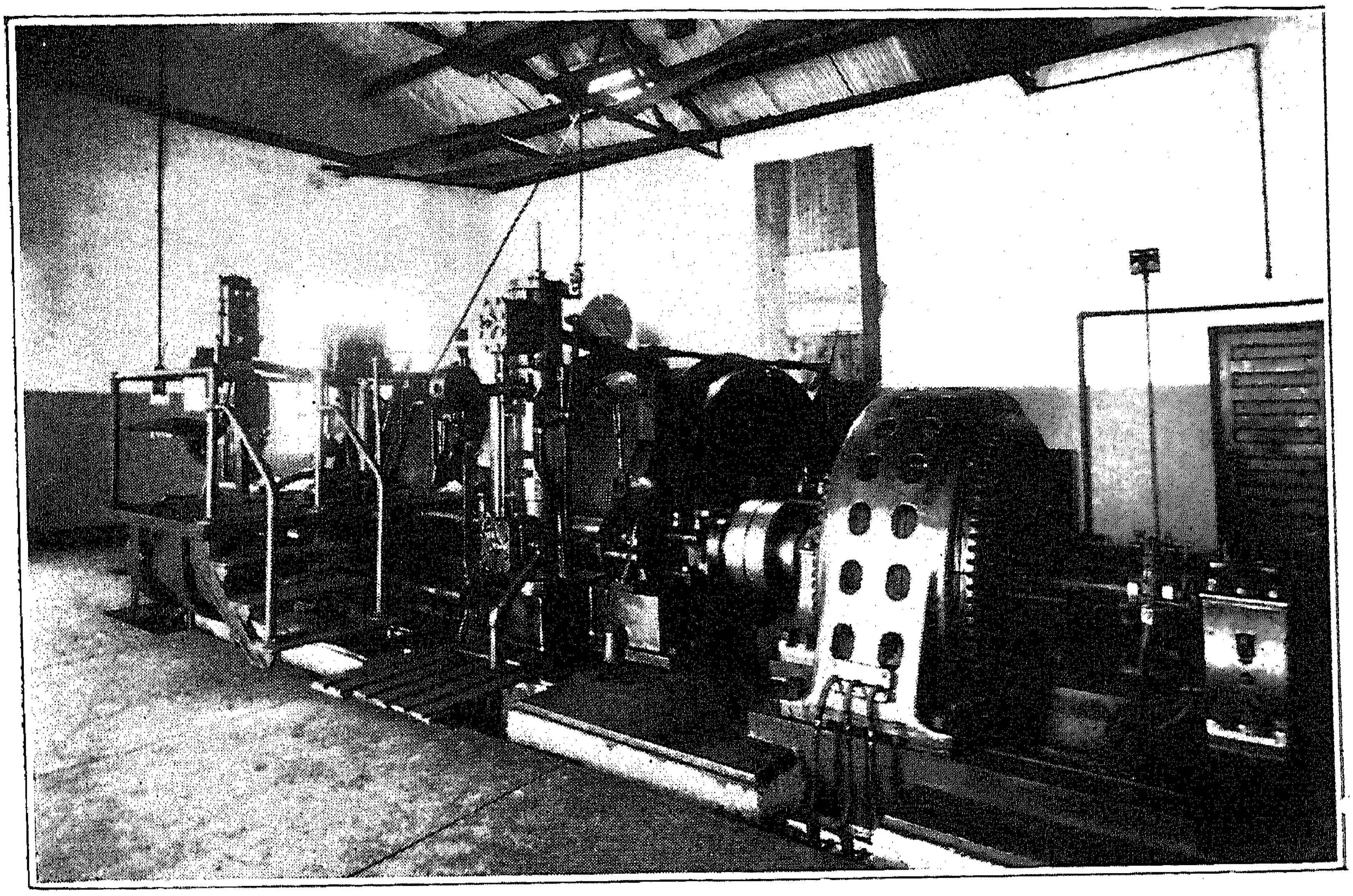


FIG. 5—AN ELECTRICALLY DRIVEN HOIST

60-Cycle—2200-volt—675 h. p.—450 rev. per min.—wound-rotor motor with liquid rheostat—double drum hoist running 1500 feet per minute with two 3-ton cars—slope 1100 feet long—95 cars per hour.

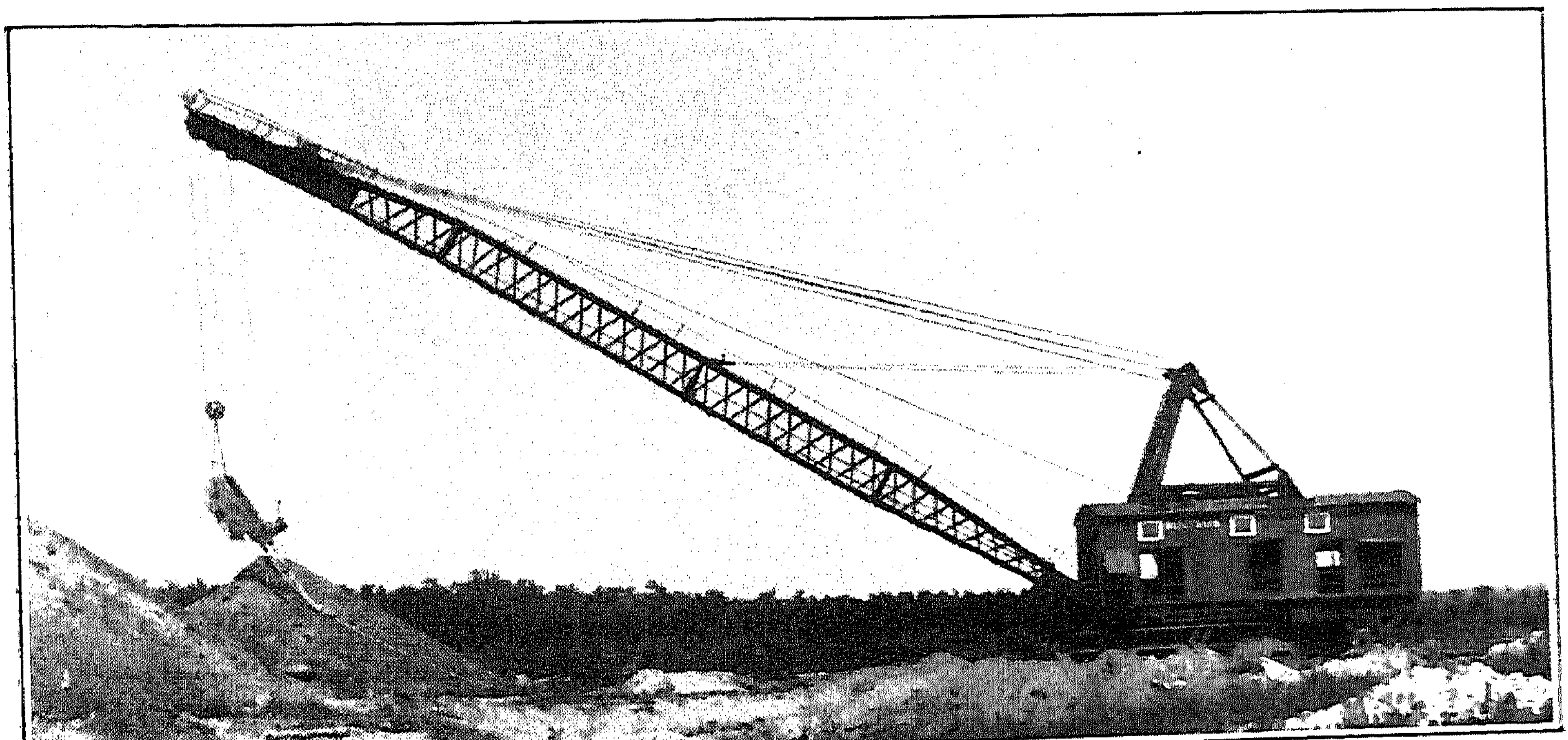


FIG. 6—AN ELECTRIC SHOVEL

[CRANE]

Current supplied through a bank of three 100-kw-4000/400 volt transformers.

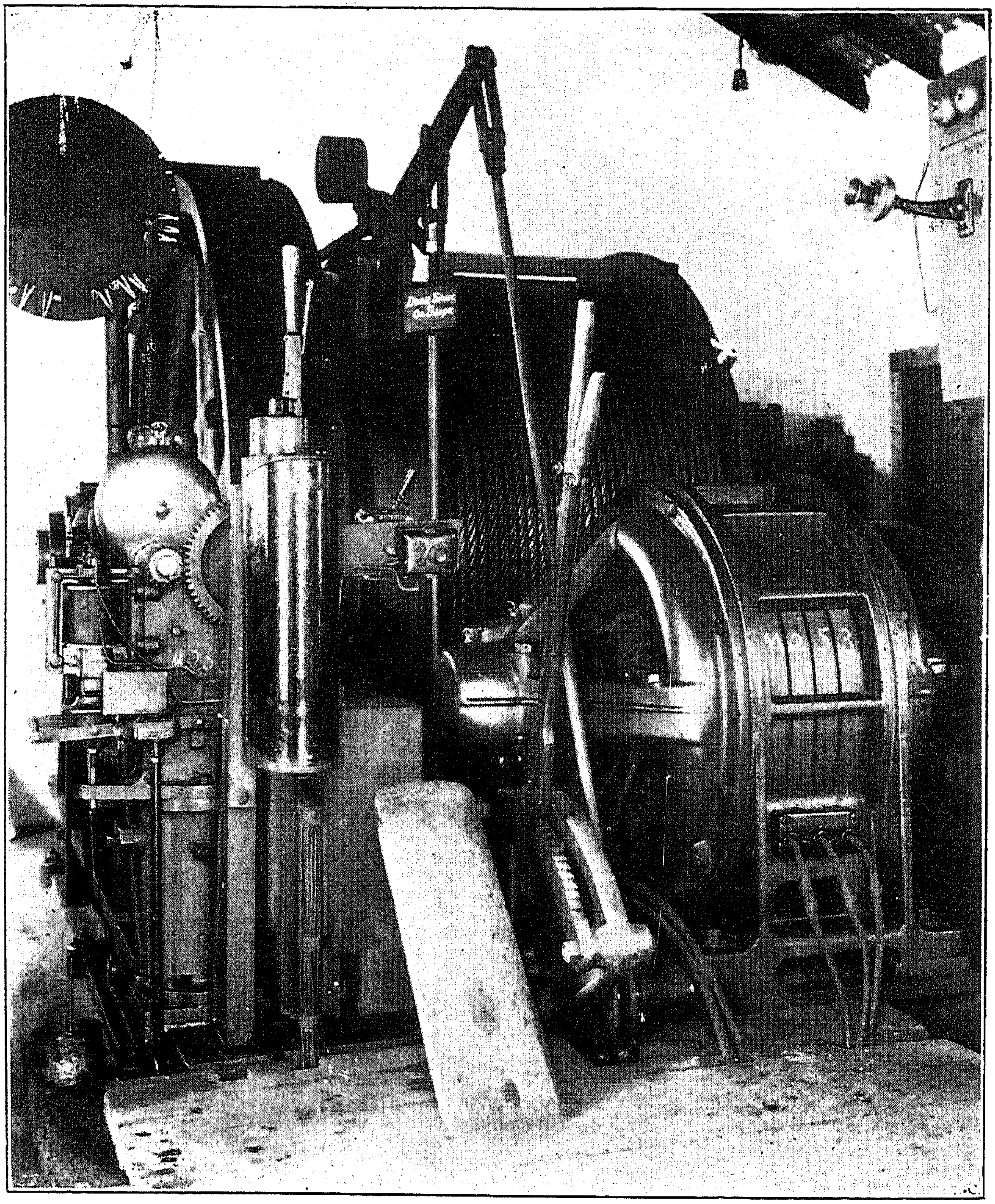


FIG. 7—AN ELECTRICALLY DRIVEN HOIST [CRANE]

60-cycle—440-volt—125-h. p.—500-rev per min.—wound-rotor motor—single drum hoist 900 feet per minute—one 3-ton car—slope 1000 feet long 52 degree pitch—18 cars per hour.

The above mine produced 558,394 tons for the year.

If we assume 12 kw-hr. per ton of coal produced there would have been required in 1917 if all the coal had been produced electrically........................... 1,076,652,000 kw-hr.

There was actually used in 1917 in electrically operated anthracite mines from central station and mine plants.......... 215,000,000

Total kw-hr. necessary to produce remainder of coal electrically.............. 861,652,000

At the present time 10 per cent of the coal produced is used to provide power to mine the remainder. There are some very bad cases, at one colliery 400 tons of coal is burned in the boiler plant for every 1000 tons of coal shipped. The above 10 per cent does not include the coal used at central stations so that to produce the 89,720,982 tons in 1917 there was burned under boilers................................ 8,972,000 tons

In large stations we can produce 1 kw-hr. for 2.5 lb. of small anthracite coal.......... 965,000
(2.5 × 861,650 divided by 2240)

There would be released for sale............ 8,007,000 tons

At one colliery 2.5 per cent of the men were released by electrification. There are employed in the anthracite region 150,000 men. If 2.5 per cent are released by electrification this makes a total of 3,750 men additional, which would be put to mining coal. Each man produces 550 tons per year so we should get additional (550 × 3750).................. 2,062,500

Additional coal produced by total electrification.................................. 10,069,500

The cost of providing for this is estimated as follows:

Additional kw-hr. required................ 861,652,000

At 40 per cent load factor this represents a station capacity of...................... 394,000 kw.

Cost of plant at $75 per kw................ $29,550,000

Transmission and distribution lines 250 miles at $5,000.............................. 1,250,000

$30,700,000

Substation and mine installations 900,000 h. p. at $40 per h. p.		$36,000,000
The savings to be effected would be 10,069,500 tons of coal at $2.00 per ton		20,139,000
Reduced mining cost, 700,000,000 tons at 7c. per ton		4,900,000
		$25,039,000
870,000,000 kw-hr. (861,652,000 plus losses) at 11 mills for current delivered at mine	$9,570,000	
Depreciation 5 per cent on $36,000,000	1,800,000	11,370,000
		$13,669,000

This represents 38 per cent on an investment of $36,000,000.

At the present time the mining companies are financially able and are installing as rapidly as possible electrically driven machinery for increasing production and cutting down the requirements for labor. They cannot at this time secure the material for power houses and the central stations are prevented from supplying this power by the impossibility of securing the necessary capital to add to their facilities. It is suggested that means should be found for supplying the amount required to finance additions to these plants so that the vital needs of this industry for power can be met.

Presented at the 342nd meeting of the American Institute of Electrical Engineers, Philadelphia, Pa., December 13, 1918.

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DRUM SHAPES AS AFFECTING THE MINE HOIST DUTY CYCLE AND MOTOR RATING

BY F. L. STONE

Abstract of Paper

The standardization of mine hoists, from an engineering standpoint is considered impossible on account of the wide variation in the conditions and methods under which anthracite coal is mined.

The problem of drum shape consists in varying the diameter of different parts of the winding drum so that the load may be accelerated and retarded at the beginning and end of its travel with the minimum consumption of power.

Numerical examples of the performance of various drum shapes under assumed conditions are given.

In order to make the subject clear I propose to describe in a very general way why there is a problem connected with drum shape.

Coal in this country lies at a depth of from 40 ft. (12.1 m.) to 1800 ft. (548.6 m.) or more below the surface. There may be only one vein or there may be a number of veins lying over each other. In some instances the parent vein will split and form two or more veins. The pitch of these veins in any given direction may vary from horizontal to vertical.

In nearly all cases the veins outcrop on the surface somewhere though they may be so thin and pinched at the outcrop that it would be unprofitable to attempt mining operations at that point.

Possibly the most violent upheavals in coal measures have occurred in the anthracite mines in the Panther Creek Valley section. An approximate cross section of this locality is shown in Fig. 1. From this you will see how the measures twist and turn and what a different mining problem is found as compared with mining flat coal.

Fig. 2 shows a hard coal mine in which all conditions of mining are encountered. This picture tells in plain language the various methods involved. This model is 9 ft. (2.7m.) by 16 ft. (4.8m.). The front portion about two thirds the area of the model shows the workings of an 8-ft. (2.4m.) vein with all the pitches from flat to 80 deg. Back of this the overlying measures are shown in section with a shaft and a skip hoist or gun boat by means of which the coal is taken to two breakers. A conveyor is shown taking the coal from the shaft to a breaker. A stripping operation is also shown with two steam shovels in operation. The back ground is formed by a painting of characteristic coal region scenery.

The coal is mined in the various rooms and brought to the foot of the shaft or slope in mine cars which hold from one and one-half to five tons. It is usually gathered with small locomotives and made up into trips and taken to the foot by a large haulage locomotive. This locomotive returns with the empties.

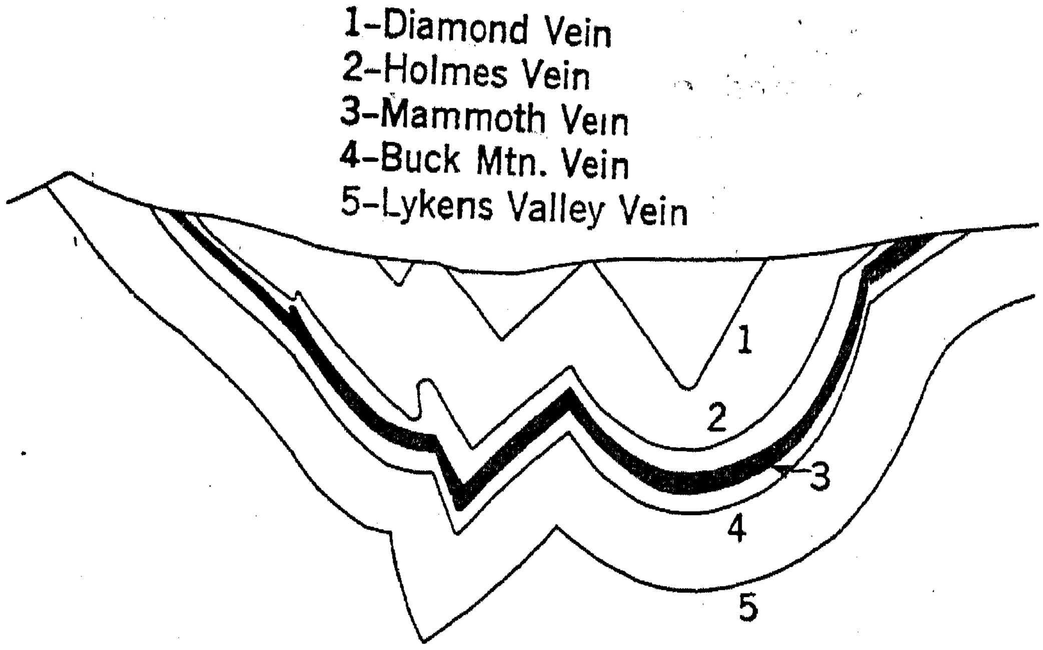
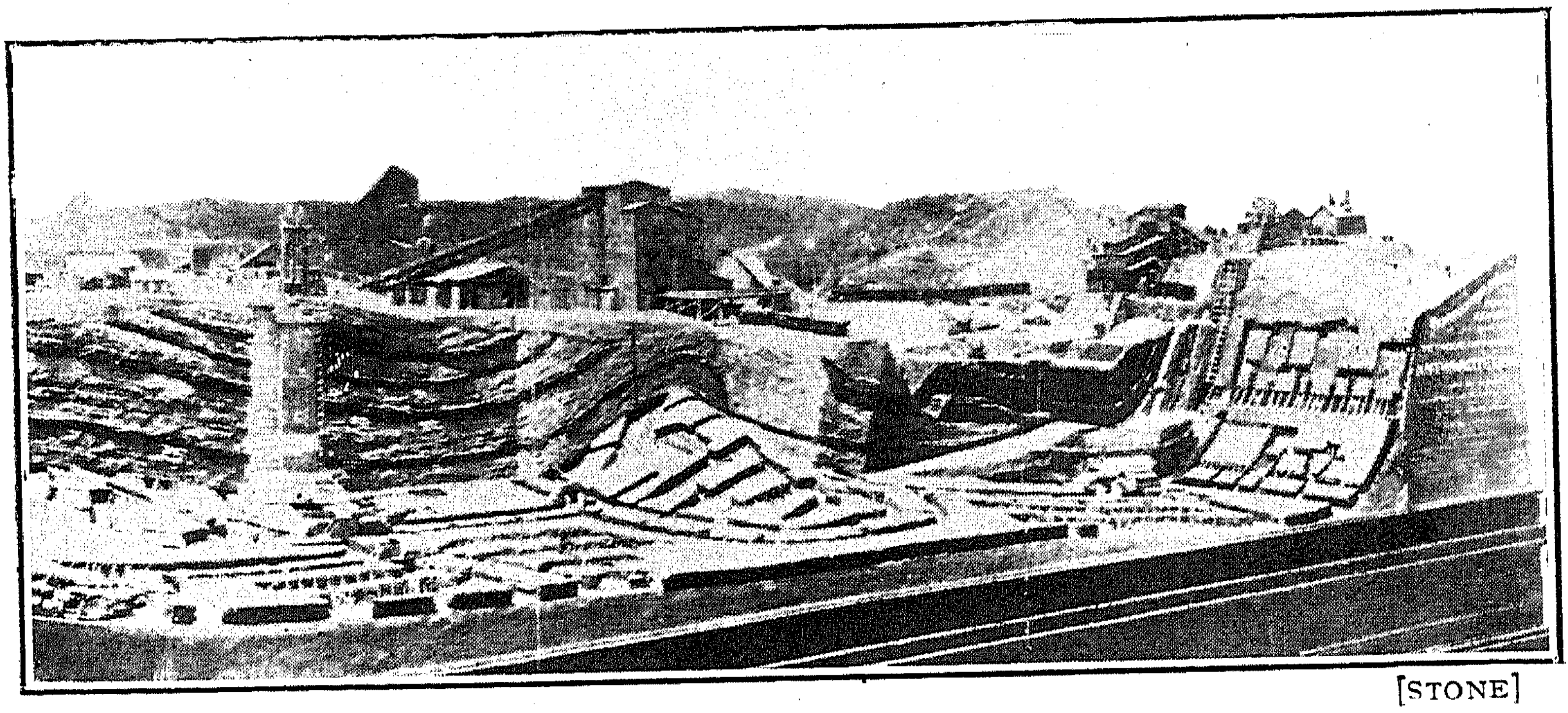


FIG. 1—SECTION OF COAL MEASURES IN PANTHER CREEK VALLEY

The coal is brought to the surface in a number of different ways. First the loaded car is put on the cage and hoisted to the landing while an empty is being lowered. At the landing the car is taken off the cage and sent to the breaker or tipple over the surface. The second method is that of using a dumping cage, that is to say the loaded car is taken to the surface and then the cage tilted and the car door opened and the contents dumped into a chute, the car never leaving the cage.

The third method is that of dumping the coal directly into a skip, the skip holding one or two mine car loads, the car never leaving the mine. This method involves an increase in the breakage of coal but not necessarily to any great extent. The fourth method is somewhat similar to the third except that the coal is dumped from the cars into a hopper and then loaded into the skip.



[STONE]

FIG. 2—MODEL SHOWING METHODS OF MINING ANTHRACITE COAL UNDER THE SEVERAL CONDITIONS OF ITS OCCURRENCE, ALSO TYPES OF MINING PLANTS

This method has many features to recommend it. Heavy loads can be lifted per trip at a comparatively low speed—for example if a given tonnage is required and it takes three trips per minute to get this output by the first or second method, a motor or engine of a h. p. X would be required. If the output could be obtained by the fourth method carrying heavy loads at low speed, the capacity of the drive would be reduced to $X/2$ or more. The mechanical parts, would, of course have to be stiffened up to take care of the increased rope stress.

From the foregoing remarks you will realize that standardization of mine hoists from the engineers standpoint is out of the question, since there are so many factors entering into the calculations for determining the proper size of motor and a change of any one may change the result very materially.

The following are the important factors which enter into the calculations:

Tonnage per hour
Weight of material per trip.
Total lift.
Rest period between trips.
Amount of material to be accelerated and retarded per trip.

You can readily see how at least one of these values will vary in every problem.

The problem is usually stated as follows:

Mine hoist is required to hoist—
Tons per day of eight hours *a*
Total lift..................*b*
Weight of material per trip *c*
Weight of cages and cars...*d*
Rest between trips.........*e*

If the average rope speed is comparatively high, say 20 ft. (6m.) per sec. or greater and the time of a trip comparatively short, say 20 sec. including rest period, then one may be fairly sure that the horse power required to accelerate the mass will be a large factor in determining the motor rating.

The ordinary horse power—time curve of a hoist using a cylindrical drum follows the general shape shown in Fig. 3. Motor starting from rest accelerates the total mass to running speed. This is held until the cage approaches a landing; then the speed is reduced to rest. The cars are then changed and the cycle repeated.

The heating of the motor is, of course, proportional to the square of the current and the current is proportional to the torque, and therefore the r. m. s. value of the area from start to start (making proper allowances for the acceleration, retardation and rest periods) will be the rating of the motor.

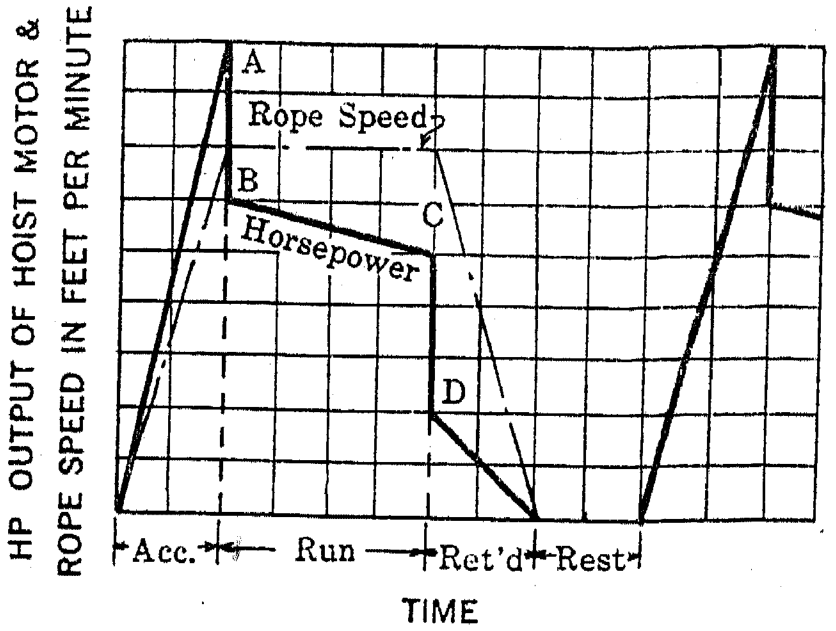


FIG. 3—TYPICAL MINE HOIST DUTY CYCLE

You can appreciate the important part played by the accelerating area in the motor rating. It is this peak which, by the shape of the drum, we are trying to reduce.

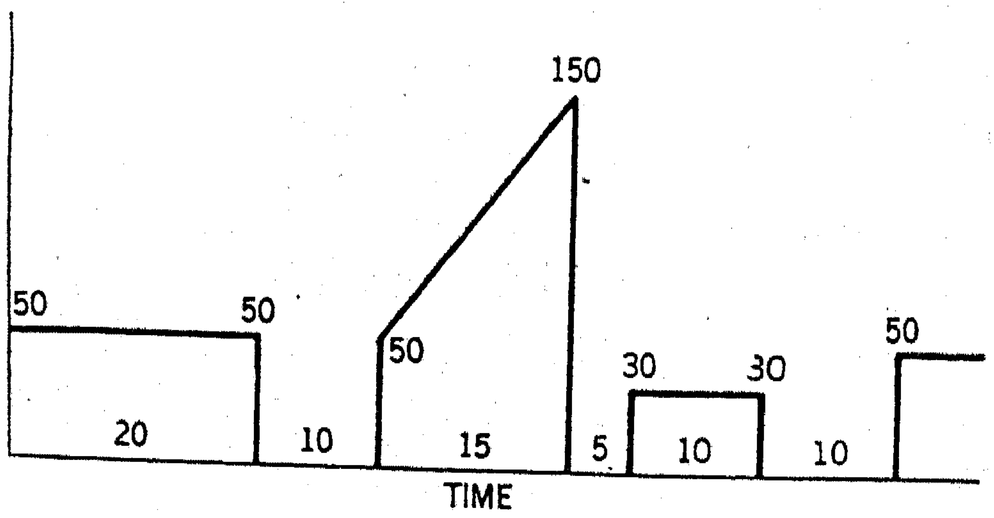


FIG. 4

It is obviously impossible to reduce the $\frac{m v^2}{2}$ but we can reduce the point *B* to which the h. p. of acceleration is added.

The depths I have selected are 300 (91.4m.) and 500 ft. (152.4m.). I have assumed the same tonnage output in each case.

In none of the examples chosen has the armature acceleration or retardation been included as this would only complicate matters and would not change the relative result to any great extent.

The method of arriving at this motor rating is the usually accepted one of r. m. s. By this is meant the square root of the average squared ordinate of the entire area under the duty cycle. This is not strictly correct but sufficiently accurate for the normal hoist cycle.

I will illustrate the method by the following simple examples:

1st. A motor running at constant speed and the load varying rapidly. Fig. 4.

$$\begin{aligned}
50^2 \times 20 &= 50{,}000 \\
\frac{(50^2 + 150^2 + 50 \times 150)}{3}\,15 &= 162{,}500 \\
30^2 \times 10 &= 9{,}000 \\
\hline
&\ 221{,}500 = \text{Total squared area.}
\end{aligned}$$

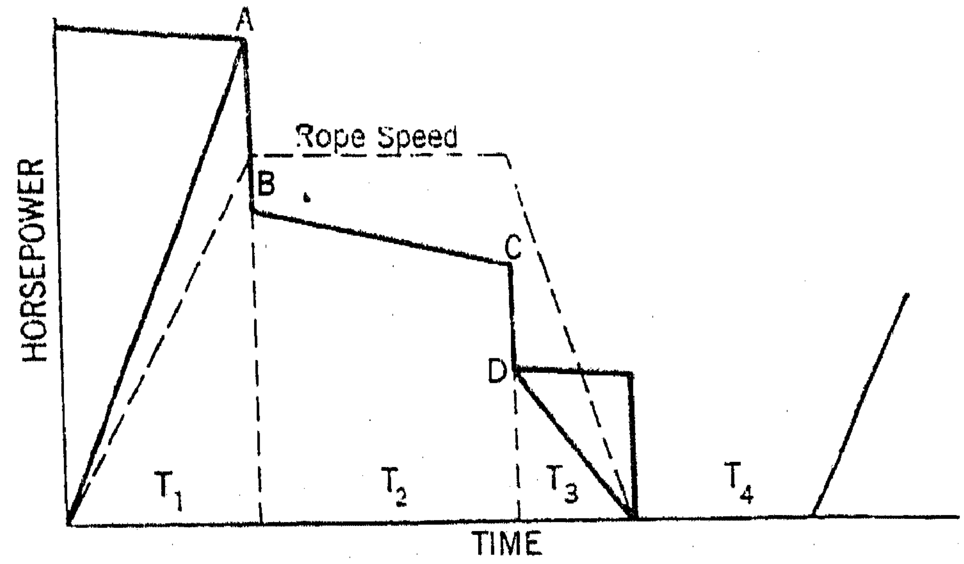


FIG. 5

$$\frac{221{,}500}{70} = 3164 = \text{mean squared ordinate.}$$

$$\sqrt{3164} = 56.2 \text{ r. m. s.}$$

If the values are amperes then the same heating would be obtained in the motor as if it were running continuously at 56.2 amperes. The same is equally true if the values are in horse power or kw. direct current, or kv-a. alternating current.

Now take the case of an intermittent running motor such as a hoist drive. Assume a cycle similar to Fig. 5.

$$\sqrt{\frac{(A^2 \times T_1) + \left(\frac{B^2 + C^2 + B\,C}{3}\right) T_2 + D^2 \times T_3}{\frac{T_1}{K} + T_2 + \frac{T_3}{K} + \frac{T_4}{L}}} = \text{r. m. s.}$$

Since the torque must be held constant or nearly so during acceleration the current corresponding to A horse power is flowing in the motor during the time T_1, but since the speed is O at start the horse power must be O also. For the slightly cumbersome expression $\left(\frac{B^2 + C^2 + B\,C}{3}\right) T_2$ may be substituted the expression $\left(\frac{B + C}{2}\right)^2 \times T_2$ if the difference between B and C is not greater than 10 or 15 per cent. The line joining B^2 and C^2 is in reality a curve and the area under this curve is represented actually by the expression above. The same remarks regarding A apply to D, the torque must be held constant from the time retardation begins until the hoist comes to rest. In getting the average square ordinate it would be incorrect to use the full value of T_1, T_3 or T_4 as in the first two cases the motor starts from rest and only reaches full speed at end of T_1 and is at rest at end of T_3. Therefore the ventilation is cut down quite materially. Experience has shown that for a-c. motors the value 2 may be used for K; and for d-c. machines which are more exposed the value 1.33 may be used, while in the case of L_1 for a-c. machines 4 is used and for d-c. 2.

In the calculation of all the cases, the moment method has been used. By that I mean that the moment of all the forces that require power such as the up-cage car and coal, up-rope, acceleration and friction are working against the motor and considered as positive moments while the down-cage and car, down-rope, and retardation are assisting the motor and therefore considered negative. The total negative values are subtracted turn for turn from the total positive values and the net moment found. From the shape of the drum and the depth, the total turns may be readily found.

From the total turns may be calculated the r. p. s. as follows:

$$\text{R. P. S.} = \frac{\text{Turns}}{T - \frac{t_a + t_r}{2}} = V$$

Where T = total running time, t_a = time of acceleration, t_r = time of retardation.

In regard to the friction value which has been assumed as 25 per cent of the useful work done in all cases. This value covers the mechanical losses in hoist from motor coupling to the actual work done in the shaft, including gear and bearing losses, rope bending, guide friction and cage windage. There is, of course, no absolute method of predetermining this value and an intelligent guess is about the best method for practical cases. Many formulas have been evolved for the calculation of this value but they are based on assumptions which may or may not be correct.

Having the total moment including friction the horse power can be found by substituting in the following simple formula:

$$\text{H. P.} = \frac{2\,\pi\,r\,M}{550} \quad (r = \text{r. p. s.}, M = \text{Moment.})$$

This formula is derived as follows:

at any point $M = W \times \text{radius}$

Distance traveled in one second $= 2\,\pi \times \text{radius} \times \text{r. p. s.} = V$.

$$V \times W = \text{ft. lb. per sec.}$$

$$\frac{V \times W}{550} = \text{h. p.} = \frac{2\,\pi \times \text{Radius} \times W \times \text{r. p. s.}}{550}$$

$$= \frac{2 \times M \times \text{r. p. s.}}{550}$$

We now come to the consideration of the cases selected. Only in the case of the cylindro-conical drum (Case 4) are the actual figures of the calculation shown. This case will serve to illustrate the general method used.

Weight of coal hoisted	5,000	lb.
Total lift	300	ft.
Weight of cage	11,000	lb.
Weight of car	4,000	lb.
Rope	1.25	in.
Number of trips per min.	3	
Rest between trips	4	sec.

Case 1.—Cylindrical drum.
Case 2.—Conical drum.
Case 3.—Cylindro-conical with long cone.
Case 4.—Cylindro-conical with short cone.

One of the most important points to be determined in the calculation of these cycles is the weight and radius of gyration of the rotating parts of the hoist. The figures used have been secured from various hoist builders at various times and are believed to be fairly accurate.

Calculation of Case 4:—

Acceleration in 5 sec., run 6 sec., retard 5 sec., rest 4 sec.

To find number of turns on small drum to take up acceleration.

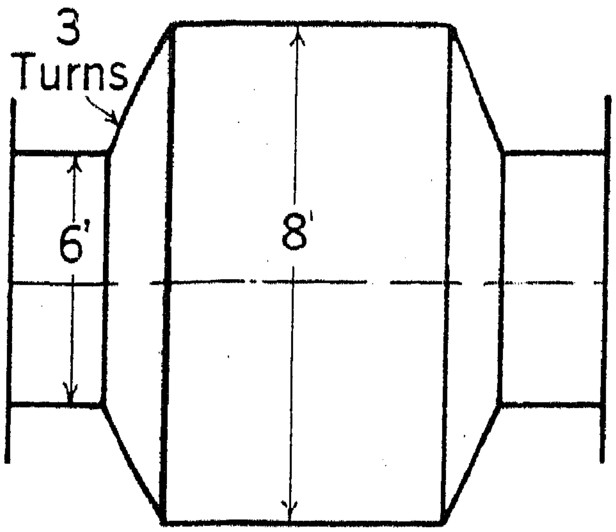


Fig. 6

$x = \text{r. p. s.}$

$\frac{5x}{2}$ = Turns during acceleration.

$\frac{5x}{2} \times \pi \times 6$ = length of rope wound on during acceleration. (1)

$\frac{5x}{2} \times \pi \times 8$ = length of rope wound on during retardation. (2)

$3 \times 7\pi$ = length of ropewound on cone. (3)

$6x$ = turns during constant running.

$6x - 3$ = turns on large diameter during constant run.

$(6x - 3)\,8\pi$ = Rope wound on. (4)

$$\frac{5x}{2} \times 6\pi + \frac{5x}{2} \times 8\pi + 21\pi + (6x - 3)\,8\pi = 300$$

$$30\pi x + 40\pi x + 42\pi + 96\pi x - 48\pi = 600$$

$$166\pi x - 6\pi = 600$$

$$166\pi x = 618.9$$

$$x = \frac{618.9}{166.\pi} = 1.187 \text{ r. p. s.}$$

Turns acceleration	$\frac{1.187 \times 5}{2}$	2.97
Turns on cone		3
Turns on large diam.	$6 \times 1.187 - 3$	4.11
Turns retard large diam.	$\frac{1.187 \times 5}{2}$	2.97
Total Turns		13.05

Distance passed over.

Acceleration	$2.97 \times 6\pi$	56 ft.
Up cone	$3 \times 7\pi$	66 ft.
Large diam. constant speed	$4.11 \times 8\pi$	103.3 ft.
Large diam. retard	$2.97 \times 8\pi$	74.7 ft.
Total Lift		300.0

Moments.

Up load

11,000	cage
5,000	coal
4,000	car
20,000	total.

Turns	Weight		Radius		Moment.
0	20,000	×	3	=	60,000
2.97	20,000	×	3	=	60,000
5.97	20,000	×	4	=	80,000
10.08	20,000	×	4	=	80,000
13.05	20,000	×	4	=	80,000

Down Load

11,000	cage
4,000	car
15,000	total.

Turns	Weight		Radius		Moment.
0	15,000	×	4	=	60,000
2.97	15,000	×	4	=	60,000
7.08	15,000	×	4	=	60,000
10.08	15,000	×	3	=	45,000
13.05	15,000	×	3	=	45,000

Up-rope (1.25 in. diam. weight 2.45 lb. per foot)

Turns	Weight		Radius		Moment.
0	300 × 2.45	×	3	=	2,205
2.97	244 × 2.45	×	3	=	1,795
5.97	178 × 2.45	×	4	=	1,743
10.08	74.7 × 2.45	×	4	=	732
13.05	0				0

Down-rope

Turns							
0	0						0
2.97	74.7	×	2.45	×	4	=	732
7.08	178.	×	2.45	×	4	=	1,743
10.08	244.	×	2.45	×	3	=	1,795
13.05	300	×	2.45	×	3	=	2,205

Plot all curves on coordinate paper—Moments against turns.

Sum of Moments.

			UP			
Turns......	0	2.97	5.97	7.08	10.08	13.05
Up-load....	60,000	60,000	80,000	80,000	80,000	80,000
Rope	2,200	1,790	1,740	1,500	730	0
Total up ...	62,200	61,790	81,740	81,500	80,730	80,000
			DOWN			
Down-load .	60,000	60,000	60,000	60,000	45,000	45,000
Rope	0	730	1,500	1,740	1,790	2,200
Total	60,000	60,730	61,500	61,740	46,790	47,200
Total up ...	62,200	61,790	81,740	81,500	80,730	80,000
Total down.	60,000	60,730	61,500	61,740	46,790	47,200
Net	2,200	1,060	20,240	19,760	33,940	32,800
Friction....	4,600	4,600	4,600	4,600	4,600	4,600
Gross M ...	6,800	5,660	24,840	24,360	38,540	37,400

Friction Determination.

$$\frac{2200 + 1060}{2} \times 2.97 = 4{,}830$$

$$\frac{1060 + 20{,}240}{2} \times 3 = 31{,}950$$

$$\frac{20{,}240 + 19{,}760}{2} \times 1.11 = 22{,}200$$

$$\frac{19{,}760 + 33{,}940}{4} \times 3 = 80{,}550$$

$$\frac{33{,}940 + 32{,}800}{2} \times 2.97 = 99{,}000$$

Area in moment turns 238,530

Average ordinate

$$\frac{238{,}530}{13.050} = 18{,}300$$

$$\frac{18{,}300}{0.80} = 22{,}900$$

Friction ordinate

$$22{,}900 - 18{,}300 = 4600$$

Acceleration and retardation of loads in shaft due to cone.

$$a = \frac{v_1 - v_0}{T}$$

Acceleration up-load $v_0 = 6\,\pi \times 1.187 = 22.4$

$$v_1 = 8\,\pi \times 1.187 = 29.85$$

$$T = \frac{3}{1.187} = 2.53 \text{ sec.}$$

$$a = \frac{7.45}{2.53} = 2.93$$

Allowing 750 lb. for rope in shaft.

$$F \text{ of acceleration } = \frac{20{,}750 \times 2.93}{32.2} = 1887.$$

Moment at 2.97 turn = 1,887 × 3 = 5661.
Moment at 5.97 turn = 1,887 × 4 = 7548.

Retard down-load.

$$F \text{ of retardation } = \frac{15{,}750 \times 2.93}{32.2} = 1435$$

Moment at 7.08 turn = 1435 × 4 = 5740
Moment at 10.08 turn = 1435 × 3 = 4305

Acceleration of up-load from start to full speed of drum.

$$v = 6\,\pi \times 1.187 = 22.4 \qquad a = \frac{22.4}{5} = 4.48.$$

$w = 20{,}000 +$ Rope $750.$

$$F = \frac{20{,}750 \times 4.48}{32.2} = 2870 \qquad M = 2870 \times 3 = 8600.$$

Acceleration down-load.

$$v = 8\,\pi \times 1.187 = 29.85 \qquad a = 5.97$$

$$w = 15{,}000$$

$$F = \frac{15{,}000 \times 5.97}{32.2} = 2780$$

$$M = 2780 \times 4 = 11{,}120$$

Drum and 1/2 rope.

$WR^2 = 350{,}000,$ W at 3 ft. $= 38{,}900$

$$V = 22.4 \qquad a = 4.48$$

$$F = \frac{38{,}900 \times 4.48}{32.2} = 5410$$

$$M = 5410 \times 3 = 16{,}200$$

Total acceleration moment

Up-cage, car, coal and rope	8,600
Down-cage and car	11,120
Drum	16,200
	35,920

Retardation

Up-load $V = 29.85 \qquad a = 5.97$

$$M = \frac{20{,}000 \times 5.97}{32.2} \times 4 = 14{,}900$$

Down-load and rope $V = 22.4 \qquad a = 4.48$

$$M = \frac{15{,}750 \times 4.48}{32.2} \times 3 = 6560$$

Drums as before 16,200

Total retard moment up	14,900
Down	6,560
Drums	16,200
	37,660

SUM OF MOMENTS—FINAL

Turns	0	2.97	2.97	5.97	5.97	7.08	7.08	10.08	10.08	13.05
Gross	6,800	5,660	5,660	24,840	24,840	24,360	24,360	38,540	38,540	37,400
Acc.	35,920	35,920	...	...	...	...	...	...	...	...
Acc. on cone	...	...	5,661	7,548	...	...	...	...	...	...
Retd. on cone	...	...	...	...	...	...	−5,740	−4,305	...	...
Retd.	...	...	...	...	...	...	...	...	−37,660	−37,660
Total	42,720	41,580	11,321	32,388	24,840	24,360	18,620	34,235	880	−260
Time	0	5	5	7.53	7.53	8.47	8.47	11	11	16
H. P.	580	564	154	438	338	330	253	466	11.9	3.5

R. M. S.

$$\left(\frac{580 + 564}{2}\right)^2 \times 5 = 1{,}640{,}000$$

$$\left(\frac{154^2 + 438^2 + 154 \times 438}{3}\right) \times 2.53 = 239{,}000$$

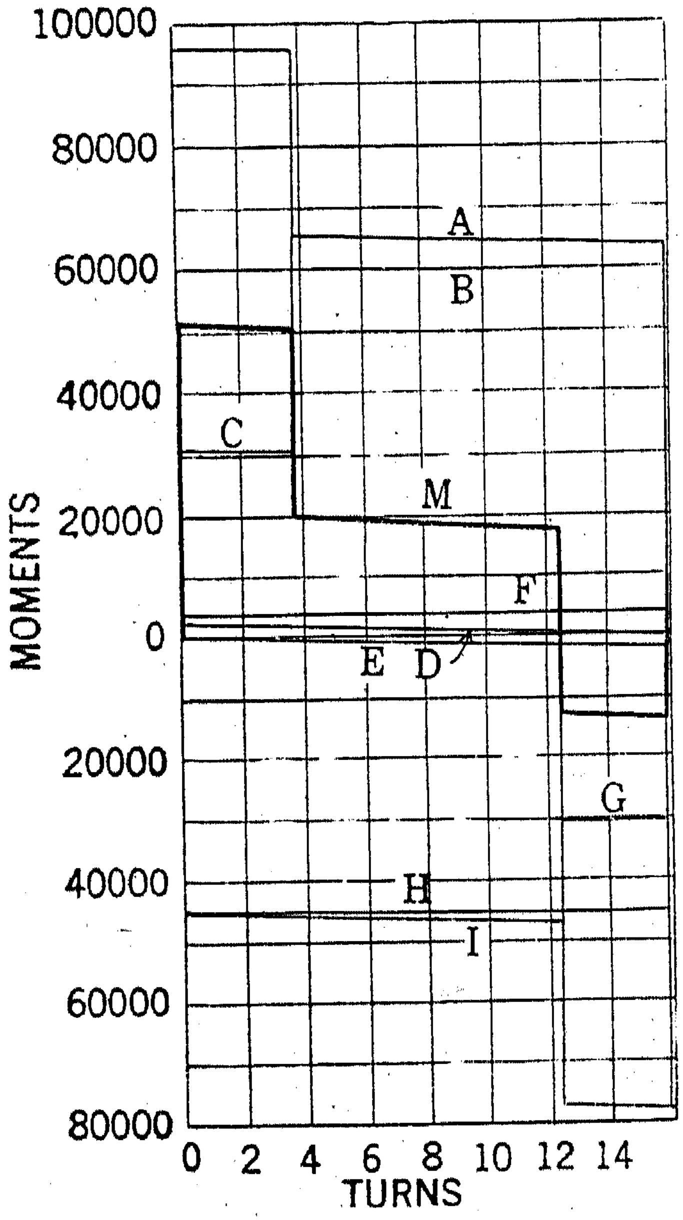
$$\left(\frac{338 + 330}{2}\right)^2 \times .94 = 110{,}500$$

$$\left(\frac{253^2 + 466^2 + 253 \times 466}{3}\right) \times 2.53 = 338{,}000$$

$$\left(\frac{11.9 - 3.5}{2}\right)^2 \times 5 = 88$$

2,327,588

$T_1 = 5$	$K = 2$	Equiv. Time =	2.5
$T_2 = 6$		" "	6.
$T_3 = 5$	$K = 2$	" "	2.5
$T_4 = 4$	$L = 4$	" "	1.
Total Equiv. Time..........			12.0



FGI. 7—MOMENT DIAGRAM—CASE I—SINGLE CYLINDRICAL DRUM—$W R^2$ ROTATING PARTS 200,000

A = Total monent up load
B = Moment up cage car and ore
C = Acceleration moment
D = Up rope moment
E = Down rope moment
F = Friction moment
G = Retardation moment
H = Down cage and car moment
I = Total down moment
M = Total net moment

$$\frac{2,327,588}{12} = 194,000$$

$$\sqrt{194,000} = 440.$$

Looking at Case I, Figs. 7 and 8, you will note the acceleration peak goes to 825 h. p. and the retardation goes below the zero line which indicate the motor must be plugged or the mechanical brakes applied to stop the hoist. This in the case of an induction motor drive is not conducive to good operation as the operator will invariably plug, which is expensive from a power standpoint, or he will take longer to retard which reduces the output. The r. m. s. value of this cycle is 600 h. p.

Case II is that of a conical drum 6 to 8 ft. in diameter. It will be noted the acceleration peak is considerably less in this than in Case I. This is brought about by the fact that the point *B* in the master diagram, has been reduced to approximately 200 h.p. Here, again during retardation the value becomes negative and as in Case I the motor must be plugged or stopped by means of mechanical brakes, though not to any such great extent as in Case I. The r. m. s. value of this cycle is 482 h. p. This is shown in Figs. 9 and 10.

Case III is that of a cyclindro-conical drum in which the cylindrical parts are just large enough to take the turns during acceleration and retardation. The turns on the large diameter are common to both ropes, the drum being symmetrical around the center line. Here we have reduced the accelerating h. p. to approximately 600, and the retarding h. p. above the line which indicates that power must be kept on to a very small extent during retardation. The r. m. s. value in this instance is 452 h. p. Shown in Figs. 11 and 12.

Case IV, the calculation of which has been given in the previous pages is that of a cylindro-conical drum with a very steep pitch or spiral cone. This gives a good many of the characteristics found in Case III and a r.m.s. value of 440, so that

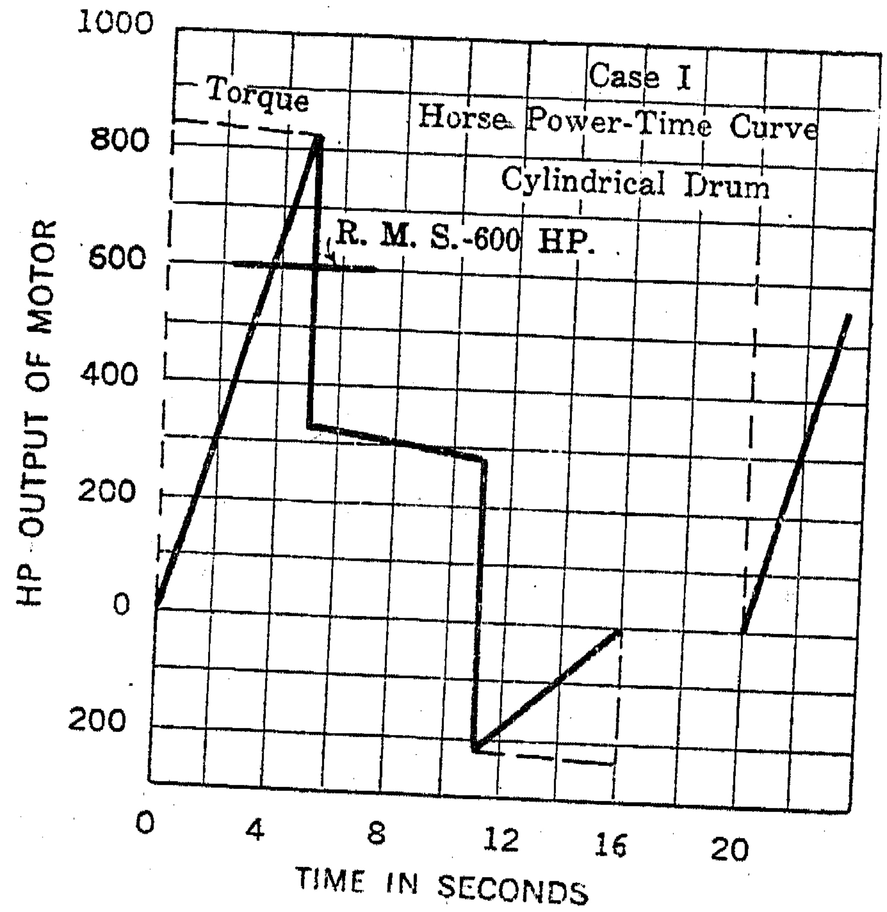


FIG. 8

apparently little is gained in the use of the steep pitch cone over the medium pitch cone of Case III and there is always some question in the minds of some operators, whether or not the pitch cone is a very desirable design. Personally I am not experienced enough with their operation and life to express an opinion, but feel that if the same result can be obtained with a moderate pitched cone, I personally would prefer the latter. Figs. 13 and 14 show results clearly.

In Cases VI, VII, VIII and IX the same tonnage output has been used, but the depth of the shaft increased to 500 ft. This, of course, means a very much more rapid speed of hoisting.

Case VI is that of a cylindrical drum which shows a very undesirable cycle from an induction motor standpoint. It has

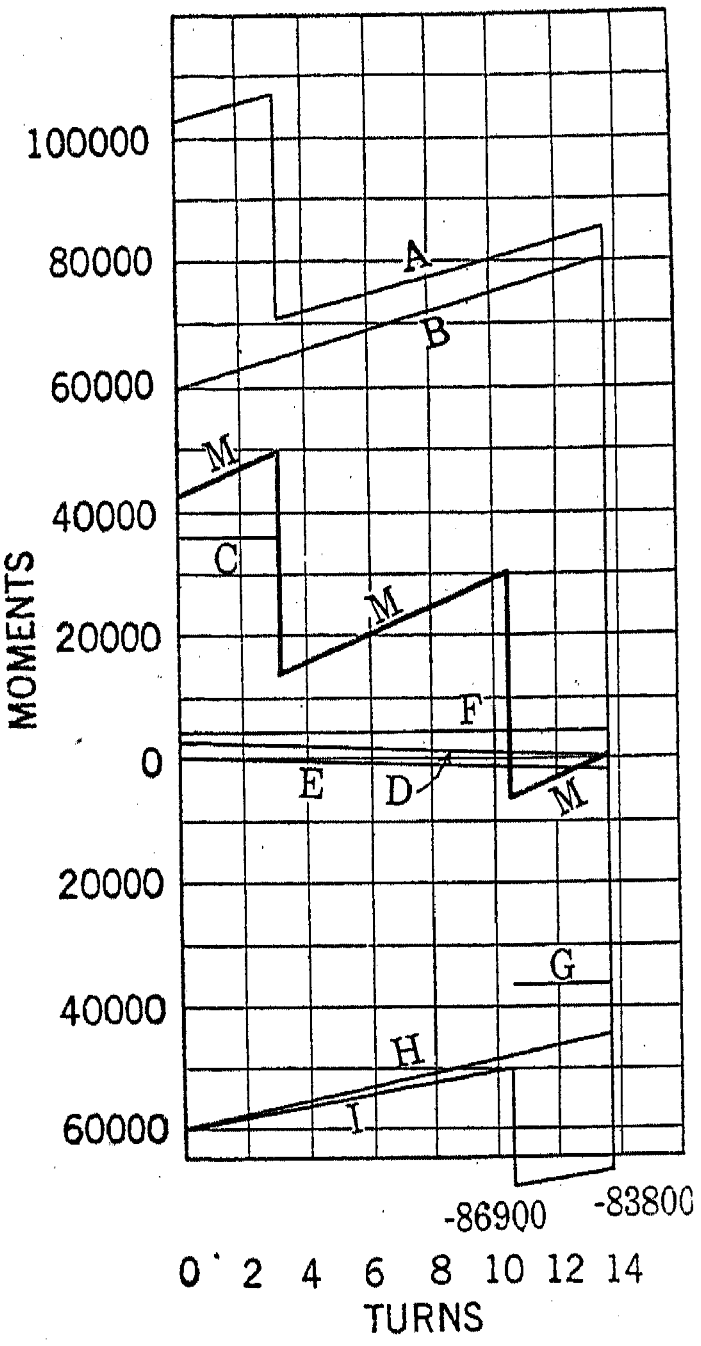


FIG. 9—MOMENT DIAGRAM—CASE II—CONICAL DRUM 6 FT. TO 8 FT.—WR^2 ROTATING PARTS 300,000

A = Moment up load, total
B = Moment up car, cage and ore
C = Moment acceleration
D = Moment up rope
E = Moment down rope
F = Moment friction
G = Moment retardation
H = Moment down car and cage
I = Moment total down side
M = Moment total net

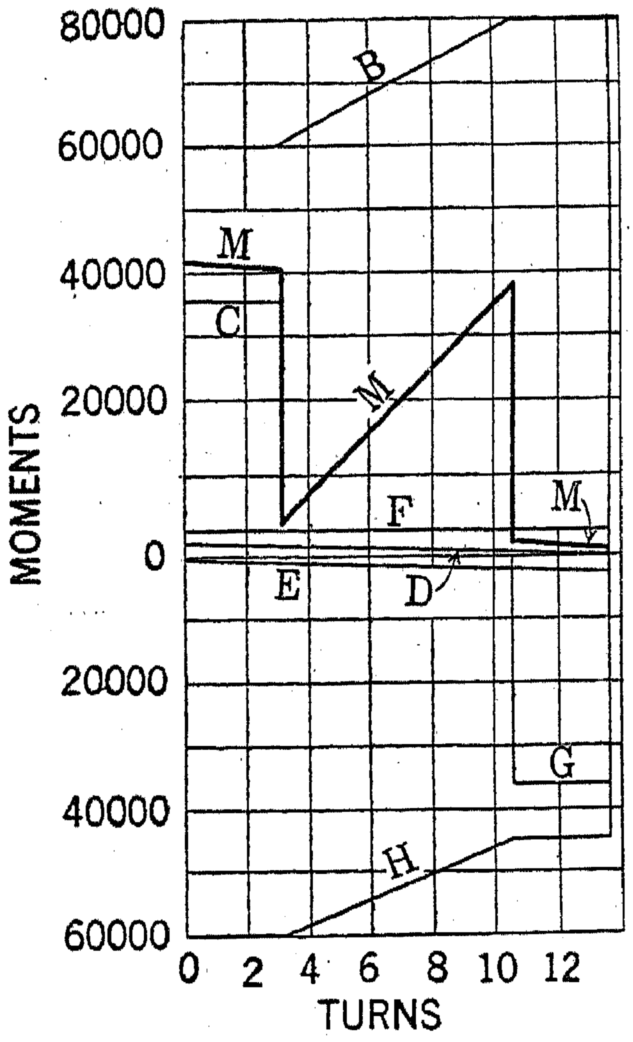


FIG. 11—MOMENT DIAGRAM—CASE III — CYLINDRO-CONICAL DRUMS 6 FT. TO 8 FT.—WR^2 ROTATING PARTS 300,000

B = Moment up car, cage and ore
C = Moment acceleration
D = Moment up rope
E = Moment down rope
F = Moment friction
G = Moment retardation
H = Moment down cage and car
M = Total net moment

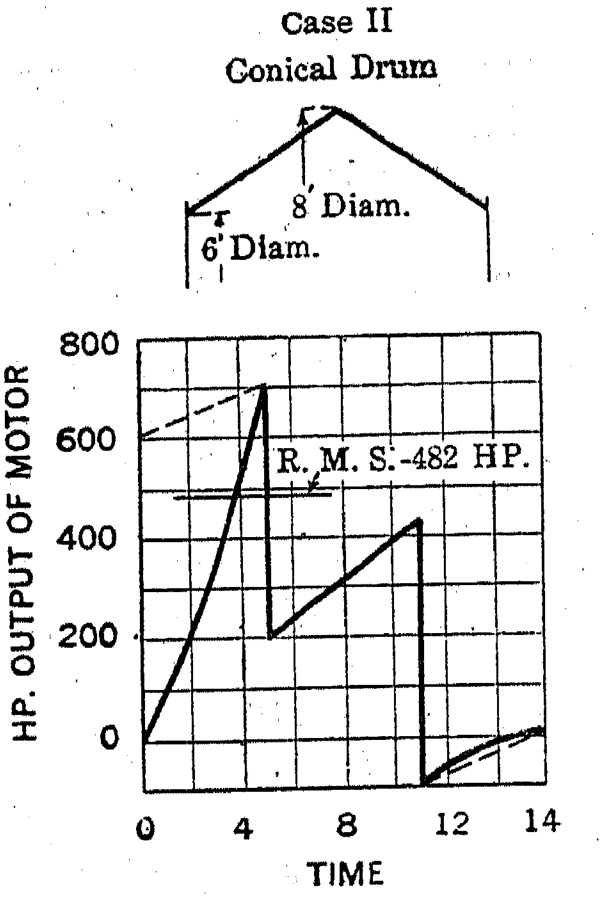


FIG. 10

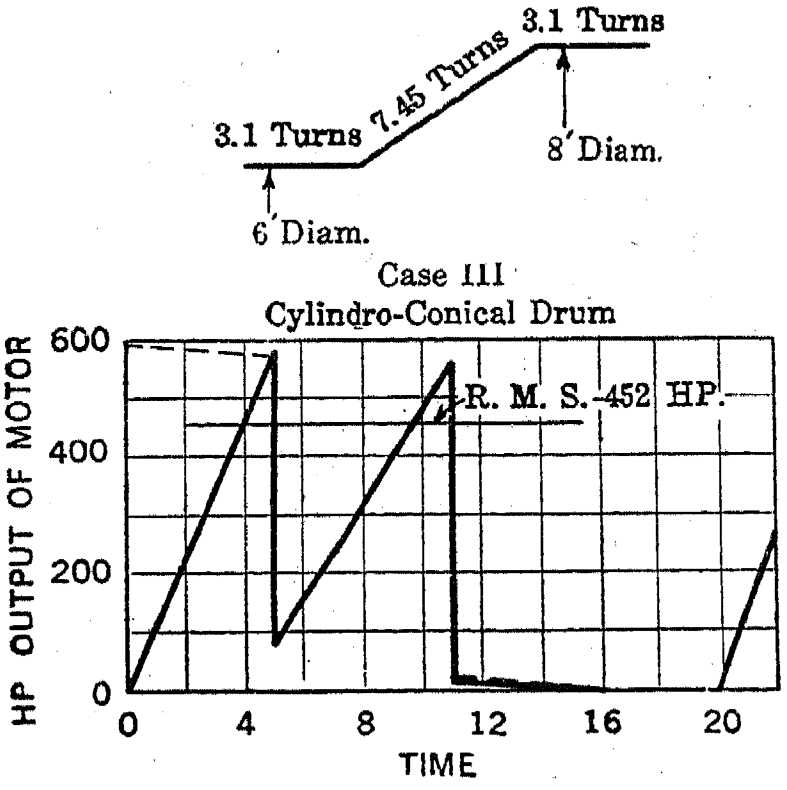


FIG. 12

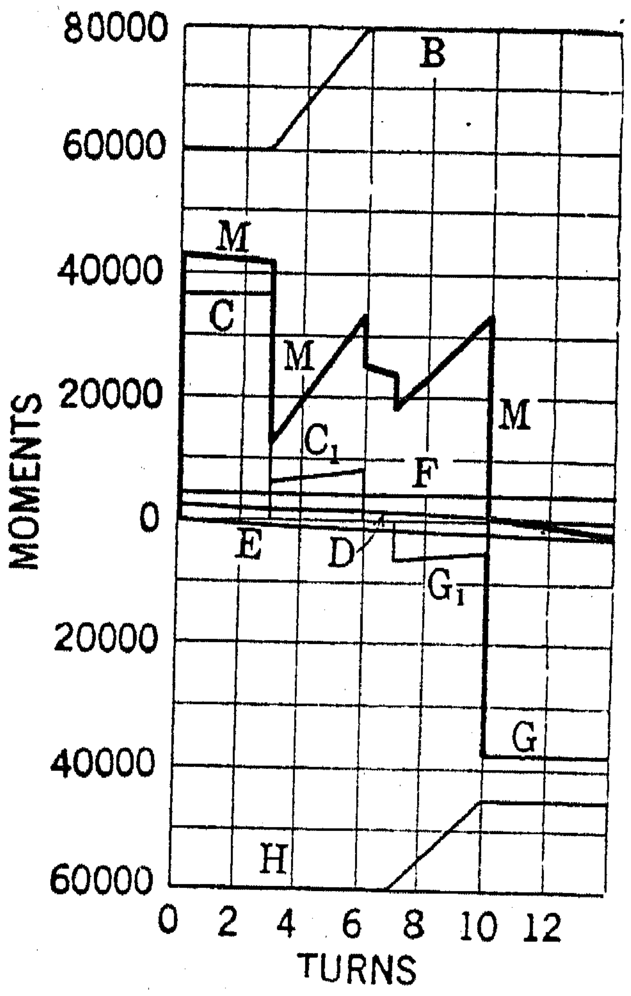


FIG. 13—MOMENT DIAGRAM—CASE IV—CYLINDRICAL CONICAL DRUM—3 TURNS ON CONE—WR^2 ROTATING PARTS 350,000

B = Moment up cage, car and ore
C = Acceleration
C_1 = Acceleration up load on cone
D = Moment up rope
E = Moment down rope
F = Moment friction
G = Moment retardation
G_1 = Retardation down load on cone
H = Moment down cage and car
M = Moment total net

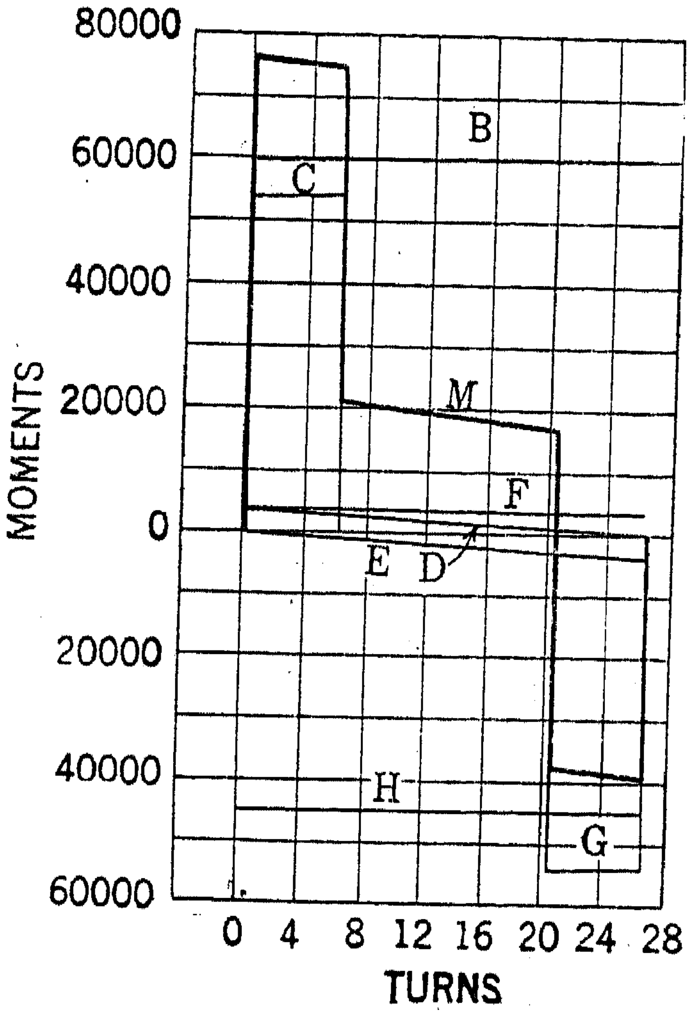


FIG. 15—CASE VI—MOMENT DIAGRAM—CYLINDRICAL DRUM—$WR^2 = 250{,}000$—DEPTH 500 FT.

B = Moment up car cage and load
C = Moment acceleration
D = Moment up rope
E = Moment down rope
F = Moment mechanical friction
G = Moment retardation
H = Moment down cage and car
M = Total net moments

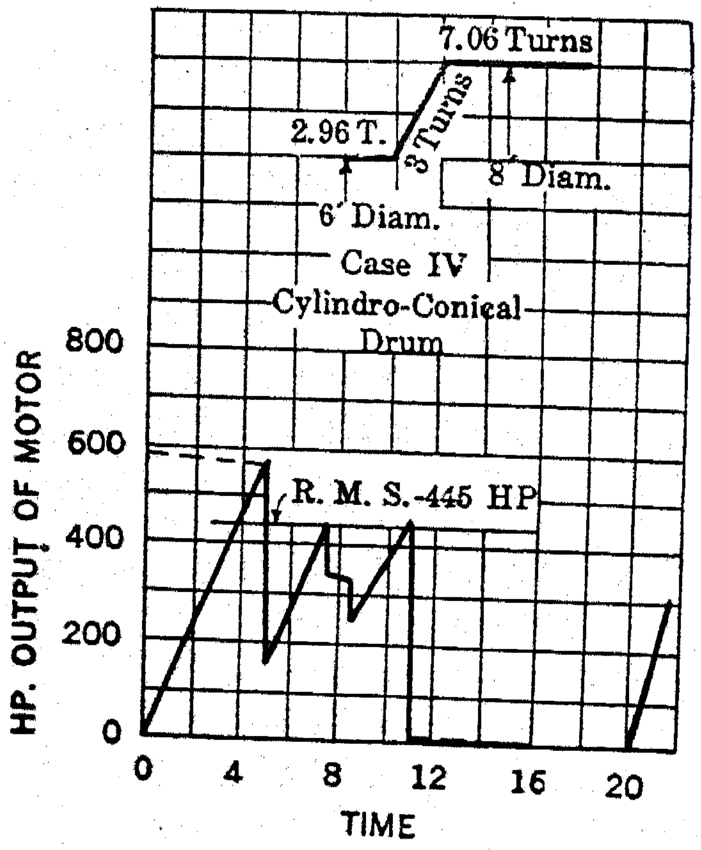


FIG. 14

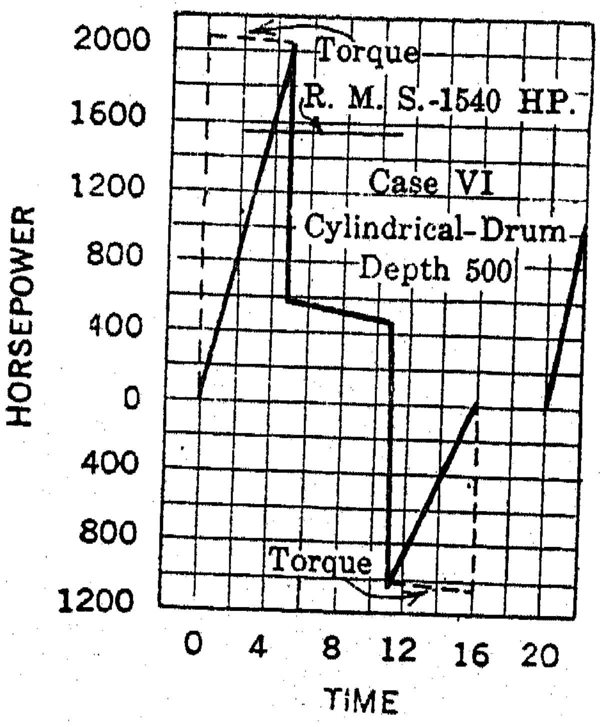


FIG. 16

an acceleration peak of over 2000 h. p. and r. m. s. value of approximately 1540 h. p. with a large quantity of braking to be done to bring the hoist to rest. Figs. 15 and 16.

Case VII is that of a conical drum 6 to 9 feet. It shows a r. m. s. value of 1210 h. p. and a considerable amount of area below the line and an acceleration of approximately 1800 h. p. Figs. 17 and 18.

Case VIII, which is a cylindro-conical drum 6 to 9 ft. with only sufficient cylindrical portions to take care of the active turns during acceleration and retardation. In this the peak is reduced still further and is the case of the conical drum, namely to approximately 1460 h. p. The r. m. s. value of 1128 h. p. and incidently the negative area has been considerably reduced. Figs. 19 and 20.

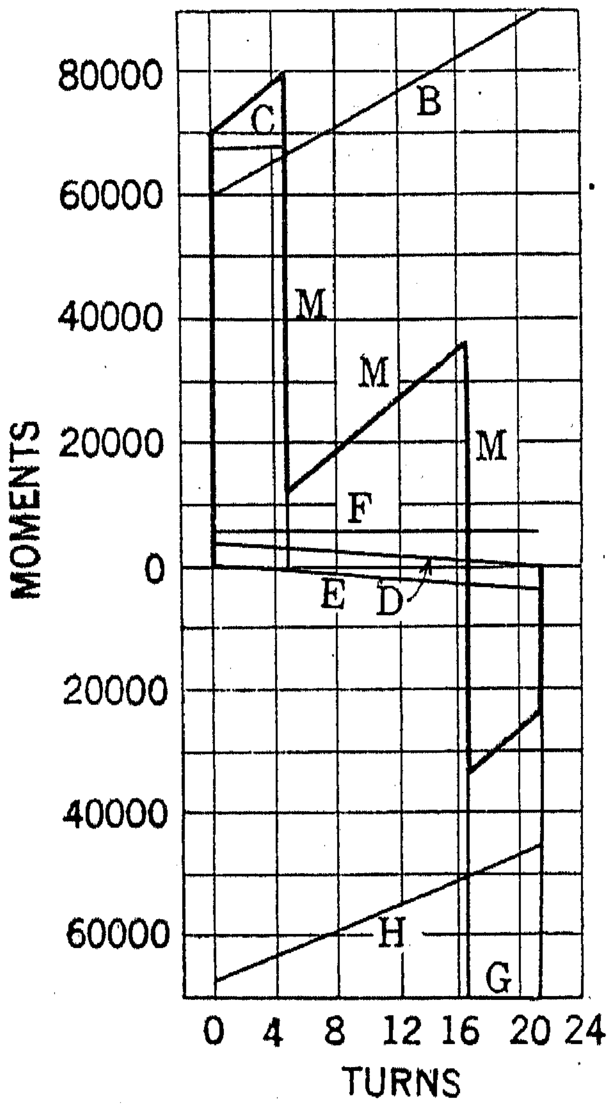


FIG. 17—CASE VII—CONICAL DRUM 6 FT. TO 9 FT.—DEPTH 500 FT.—$W R^2 = 400,000$

B = Moment up cage car and load
C = Moment acceleration
D = Moment up rope
E = Moment down rope
F = Moment mechanical friction
G = Moment retardation
H = Moment down cage and car
M = Total net moment

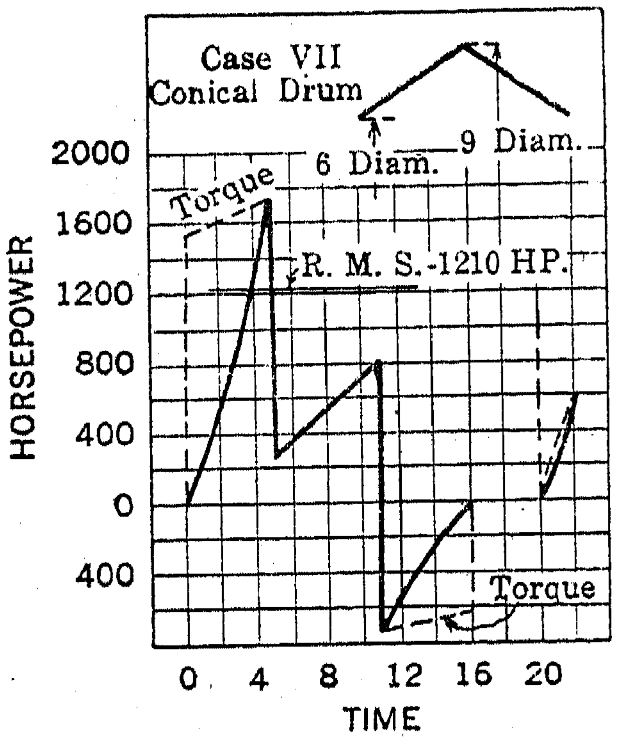


FIG. 18

Case IX is a cylindro-conical drum with steep pitched cone. This shows approximately the same peak as the drum with the long cone, in Case VIII. but has a slightly less r. m. s. value of 1070 h. p. Figs. 21 and 22.

None of the latter cases, I believe, are really suitable cycles for induction motor operation and should be driven by Ward-Leonard equipments.

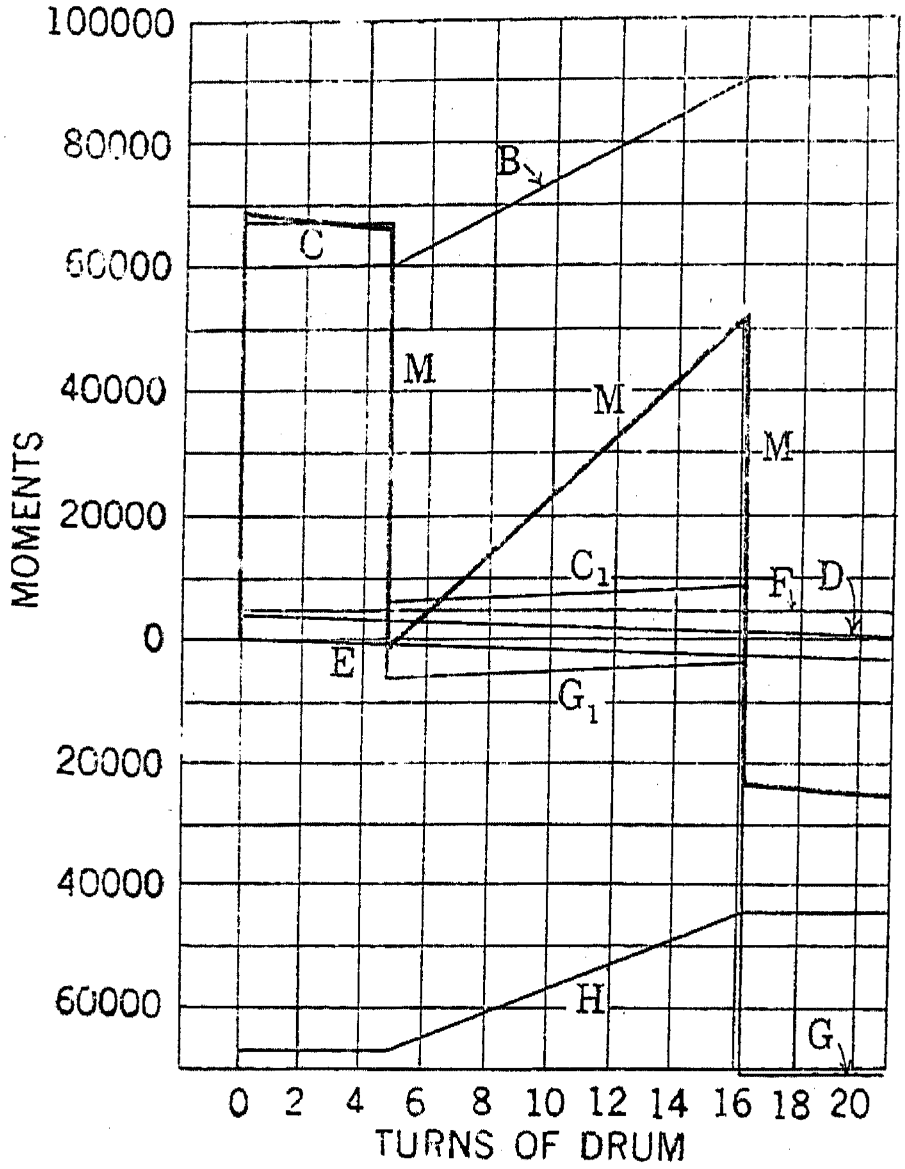


FIG. 19—MOMENT DIAGRAM—CASE VIII — CYLINDRICAL-CONICAL DRUM — $W R^2$ 400,000

B = Up cage car and load
C = Total acceleration
C_1 = Acceleration up cone
D = Up rope
E = Down rope
F = Mechanical friction
G = Retardation
G_1 = Retardation down cone
H = Down cage and car
M = Total net moments

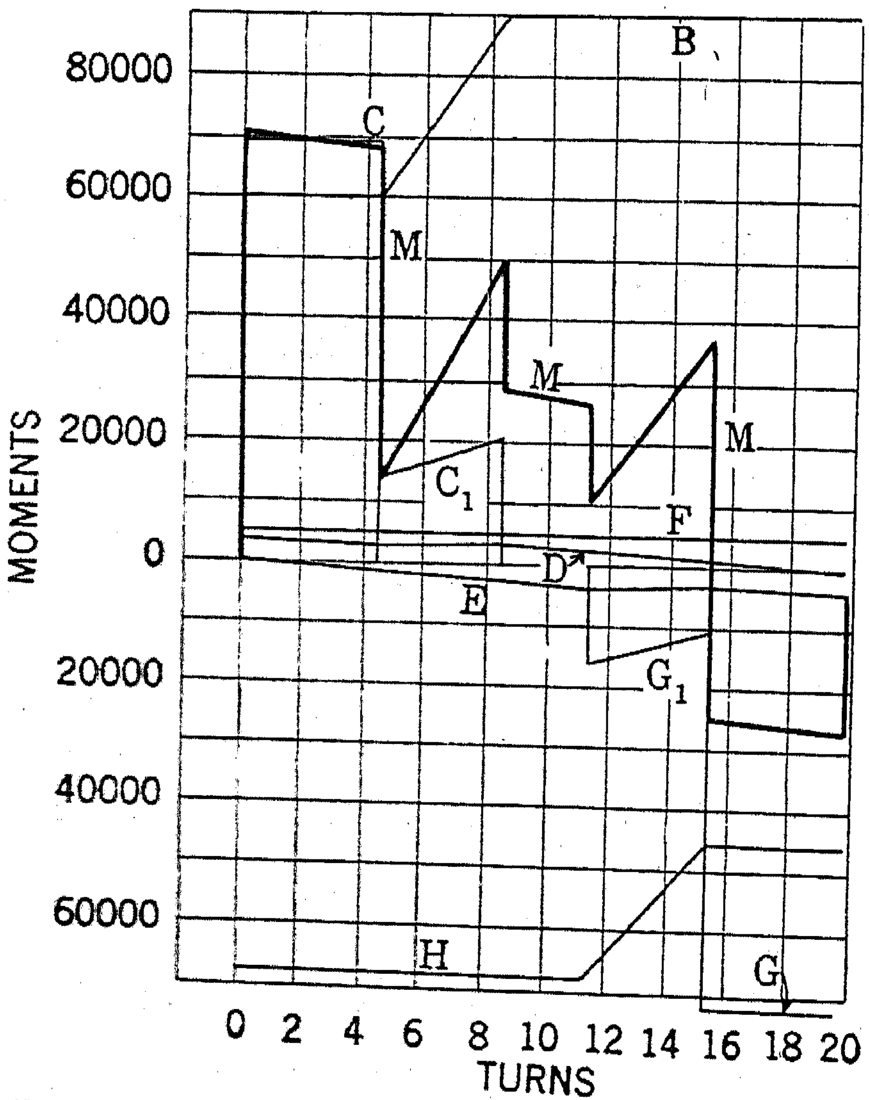


FIG. 21—MOMENT DIAGRAM—CASE IX—CYLINDRICAL-CONICAL DRUM

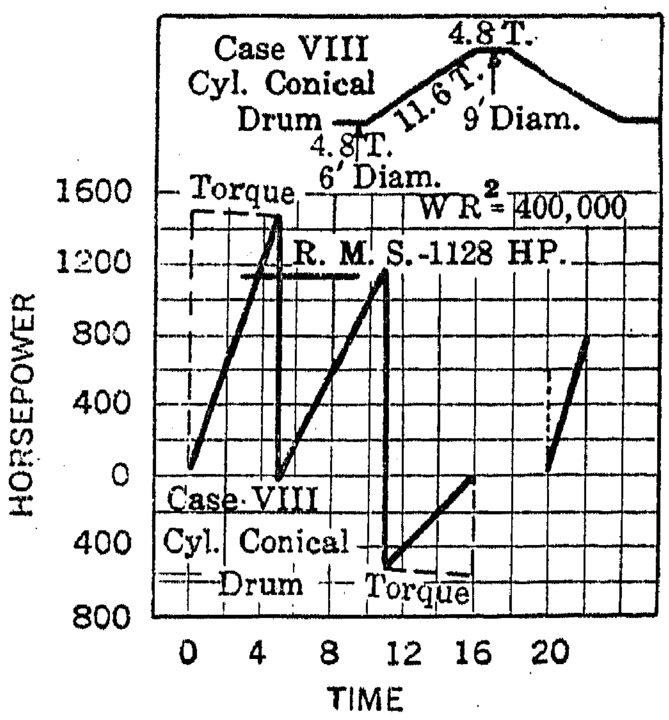


FIG. 20

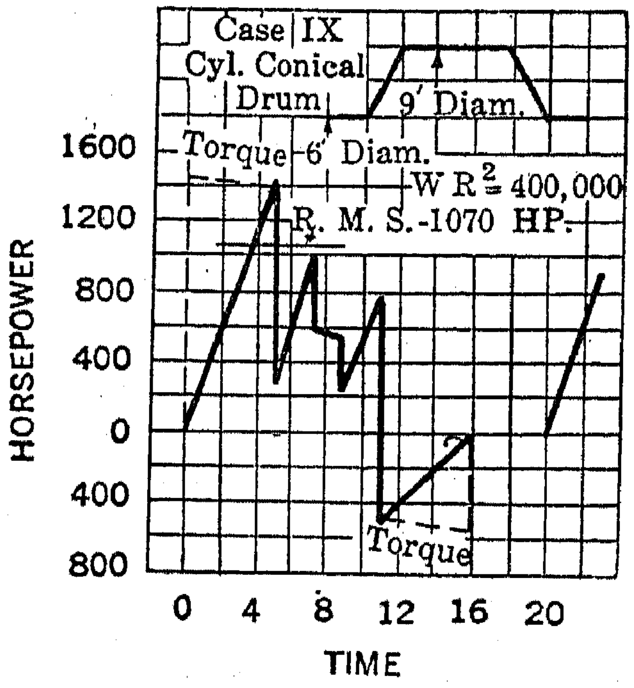


FIG. 22

I realize, of course, that in this short paper I have only touched the high spots in the problem of drum shapes, but I have, I hope selected sufficient examples to show that there is in the proper selection of drum an opportunity for some real engineering.

I have kept away from the problem of drum shapes connected with hoisting from great depths as in this class of problem the rope weight is usually the reason for any departure from the cylindrical drum. The reduction of the acceleration peak in deep hoisting is, of course, an advantageous point which comes automatically with the coning of the drums. The weight of the hoisting cable in some of the deep shafts exceeds by a considerable amount the weight of the active material hoisted. As before stated this is a problem by itself.

DISCUSSION ON "THE USE OF ELECTRIC POWER IN THE MINING OF ANTHRACITE COAL" (CRANE) AND "DRUM SHAPE AS AFFECTING THE MINE HOIST DUTY CYCLE AND MOTOR RATING" (STONE), PHILADELPHIA, PA., DECEMBER 13, 1918.

Graham Bright: In regard to Mr. Crane's paper, I think that one of the real reasons why the change to electrical operation has been more general in bituminous mines than in anthracite mines has been due to the fact that the electrification for the average bituminous mine costs less money and takes less apparatus than the average anthracite mine. This is due to the fact that many bituminous mines are drift mines and the apparatus generally consists of a small rotary or motor-generator set, two or three locomotives, a few small electric pumps and a fan. This equipment does not run into a great deal of money while in the case of the anthracite mines, the coal is taken out from several levels requiring a fairly large hoist and heavy pumping. The fan and breaker also require considerable power and a large amount of electrical equipment.

Mr. Crane mentions that the margin of profit is a little more in the case of the anthracite mine than for the average bituminous mine, in fact the profit has been so low in normal times in the bituminous mines that in order to compete and make any money at all, it was necessary to take every advantage available, and since it has been well proven that electrification cuts down the cost of operation to quite an extent, this accounts for the fact that many bituminous mines have electrified their operation during the last few years.

The difficulty of obtaining electrical equipment has been rather a bar to electrification in the last two or three years. This situation should, however, be greatly improved in the near future so that little difficulty should be experienced in obtaining equipment with fairly good delivery.

Mr. Crane mentions some figures taken from a paper of Mr. R. E. Hobart presented before a meeting of the American Institute of Mining Engineers last February. The results shown in that paper were taken from actual tests and are certainly startling. In discussing this paper when delivered, I made the statement that if the electrical manufacturer had gone to the mine operator and made any such claims as are substantiated in Mr. Hobart's paper, he would have probably been thrown out of the office and not given any consideration at all.

Mr. Crane mentions the fact that the value of coal is about double what it was a few years ago. When the price of coal was low, it was generally conceded that the electrification of a mine, especially by the use of central station power, would show considerable saving over steam operation. The price of coal has now been doubled but the price of power from a central station has not, so that if there was a decided advantage before, there should be a greater advantage at present with the high price of coal and central station power at a price not greatly in excess of what it was formerly.

There is certainly a marked difference between the amount of power used per ton of coal mined between the bituminous and anthracite mines. In a number of bituminous mines in Ohio and West Virginia, the kilowatt-hour consumption per ton of coal mined will average from 2 ½ to 3, while in the average anthracite mine the kilowatt-hours per ton are about 12. It is true that some of the bituminous mines in central Pennsylvania require from 5 to 8 kilowatt-hours per ton due to heavy pumping, heavy fan loads and long hauls. Mr. Crane points out that the breaker load alone on the average anthracite mine requires 4 kilowatt-hours per ton, so that the preparation of the coal after it is brought to the surface requires more power than the total amount required in the average bituminous mine.

In some of the anthracite mines, it is necessary to pump from 12 to 15 tons of water to the surface for every ton of coal hoisted. The result is that the load factor in an anthracite mine should average very much higher than in a bituminous mine. In some of our bituminous mines, the load factor will range from 15 to 25 per cent while in the average anthracite mine a load factor of over 50 per cent is frequently obtained. For this reason the cost of power per kilowatt-hour should be less in an anthraceit mine than for the average bituminous mine where the cost of producing the power is about the same.

The average mine operator is an experienced man in the mining and shipping of coal and as a rule does not know much about the production of power. This results occasionally in such condition as shown by Mr. Crane where for every 1000 tons mined, 400 tons were burned under the boiler. A condition of this kind can frequently be greatly improved by the use of central station power where the mine operator can secure the advice of the engineers of the power company who can frequently show him a great many ways of economically using his power. We have all been led to believe that there is considerable coal in Alaska, but I was talking to an engineer today who has spent considerable time in Alaska and he stated that he knew of a mine in Alaska where they had to burn wood under the boilers because they could not get sufficient coal from the mine to run their own plant.

The figures given by Mr. Crane regarding the quantity of coal and labor released, provided the anthracite mines should electrify, are certainly interesting. If by electrifying the anthracite mines we could save a total of 10,000,000 tons per year, this would mean considerably over 10 per cent of the total amount of anthracite coal mined in this country, and this would be available for other uses.

In regard to Mr. Stone's paper, the question of hoisting coal by means of skips is one about which there has been a wrong impression, especially in regard to the breakage of coal. It has always been thought that handling coal in skips would greatly increase the breakage. In talking with some of the operators who have tried the use of skips, I find that the general impression is that there would be less breakage provided the skips are of fairly large capacity. It seems to be the experience that the coal moving in large masses protects the individual lumps so that there is very much less breakage than would be anticipated. It is my opinion that in the future we will see more hoisting by means of skips which will eliminate the high-speed cycles.

Mr. Stone mentioned that his method of figuring the r. m. s. as given in his paper would not be exactly correct for large motors using forced ventilation. The formula will be correct if the total rest period is used instead of a fraction of the rest period as shown. It has been found that with large low-speed motors, we cannot get sufficient ventilation by the ordinary fan blades which are placed on the rotating part. A considerable increase in capacity can be obtained by means of forced ventilation and it is probable that in the future all low-speed motors of large capacity will be equipped with forced ventilation. A comparatively small amount of power is required for this ventilation, considering the large advantage obtained. Forced ventilation also permits the customer to put in an air conditioning plant, which will allow clean air to be used in ventilating motors, and this will in turn cut down the maintenance and lessen the repairs.

If we calculate the capacity of motor and generator required for a large hoist using the Ward-Leonard system of control by formula as shown by Mr. Stone, we will find that if the rest period is very long and natural ventilation is used, the hoist motor must have a larger capacity than the generator which supplies power to the hoist motor. This is on account of the generator operating continuously and getting full ventilation during the entire period, while the motor receives only part ventilation during a considerable portion of the entire cycle.

In regard to the shape of the drums, it will be found that a great many operators still favor the old plain cylindrical drum which, after all, has some advantages over the conical and cylindro-conical drums which are some times called "freak" shaped drums. The advantage of the cylindro-conical drum is shown clearly in the curves submitted, but as stated by Mr. Stone these curves are based on certain inertia values given by the hoist builders. The straight cylindrical drum is a very simple piece of apparatus and its weight and inertia can be accurately predetermined. In building such drums as the cylindro-conical type, there is considerable risk due to cracking caused by shrinkage strains, so that the designer is apt to make these drums considerably heavier than necessary. This extra weight tends to counteract the good effects obtained by the shape of the drum. There is a particular case I have in mind of a hoist furnished in the Middle West which gave considerable trouble with the resistance of the controller when operation was started. It was found that the peak loads were much heavier than had been calculated and it was necessary to furnish heavier resistance to take care of these loads. We then started to locate just what the trouble was and found that the weight of the drums was just twice what the hoist manufacturer had said it was going to be. For this reason, it is best not to take too low a value for the inertia of a "freak" shape drum.

A second disadvantage of the special drum is that the minimum diameter should be the same as the diameter of the cylindrical drum that would be used. This means that the average diameter of the special shaped drum is much greater than the cylindrical drum, which in turn means that its speed and revolutions per minute will be lower. This in turn means a lower speed motor which not only costs more, but also has a poorer efficiency and power factor than the higher speed motor.

The third disadvantage of the cylindro-conical drum is especially true when a short cone is used. The rope travels up the cone in the shape of a spiral and the rope wear on this spiral is much greater than on the cylindrical portions of the drum. This means that the rope is worn out and ready to scrap when a large portion of it is still in very good shape.

The real advantage which we obtain by the use of a cylindro-conical drum is lower peak loads and a slightly smaller capacity of equipment, but we do not save much on power consumption unless as shown we have a very fast cycle where we have high retarding rates which require plugging the motor or using up the power in brakes.

There are some cases where the cylindro-conical drum will be an advantage, but I do not think that they should be used indiscriminately. There are many cases where the straight cylindrical drum is really the best drum to use.

The question of friction is one which, as Mr. Stone says, we could argue on all day, as it is difficult to test just what the friction of a hoist is. I think that 25 per cent of the actual work done is rather low value for the friction. The total efficiency of a hoist generally runs about 50 per cent. This means that we are putting in twice as much power as we are getting out and taking 25 per cent of the actual work as friction means that one-eighth of the power going into the hoist motor is allowed for friction of the hoist. However, if this value of 25 per cent was actually doubled, it would practically make no difference in the capacity of equipment required to meet a certain set of conditions.

The cycles that Mr. Stone shows for a depth of 500 ft. are certainly not suitable for induction motors, and I should hardly want to use them even with a direct-current motor using the Ward-Leonard system. The average rope speed is 2740 ft. per min. which is not a practical operating speed for a depth of 500 ft. The accelerating and retarding rates are about 9 ft. per sec. per sec., and it will be found that for good practise we should not go much above 5 or 6 ft. per sec. per sec. In fact, the cycle although possible, shows that we are getting on dangerous ground. It means that the operator must be on his toes at all times and the slightest trouble in the safety device or error in judgment of the operator may bring about disastrous results, and while it might be physically possible to operate for a short time under such a cycle, with the Ward-Leonard system, it is certainly not good practise. Where the output requires any such cycle, I agree with Mr. Stone that the only safe method is the installation of skips handling a much heavier quantity of coal for each trip, which results in a much lower rate of speed and a considerable lengthening of the cycle.

C. W. Parkhurst: Mr. Crane's paper clearly brings out the need for more complete electrification of anthracite mines. I notice in the paper, he has given 3.57 kw-hr. per ton mined as an average of 50 bituminous mines.

It has never been my good fortune to be connected with any bituminous mine where the coal came out of the ground so easily. The lowest power consumption per ton mined that I have ever seen, was 4.4 kw-hr. per ton, and the highest well over 20 and the average around 9 kw-hr.

I know of a great number of mines where the power used for ventilating the mine is over 5 kw-hr. I hastily gathered some figures together of three big bituminous mines, averaging about 300,000 tons per month, and for five years back the kw-hr. consumption was as follows: 5.5, 6.6, 7.9, 8.9, and 8.2.

Except for the last year, there was a continuous increase, and that is accounted for by more complete electrification each succeeding year, and because of the fact that as the mine was getting bigger, ventilation became a more serious problem each year, more water had to be pumped and the haulage distance was increased. It shows that the kw-hr. per ton is going to increase as the mine is developed.

I know of a big mine, with a consumption of 10.15 kw-hr. per ton, of which 5.74 kw-hr. is for ventilation. That mine, when first started, had a 15 h. p. motor on the fan, that was changed to 30 h. p. to 75 h. p., to 125 h. p., to 300 h. p., and the last change was to a 750 h. p. motor, with a 300 h. p. motor in reserve for Sunday and night work. The big motor shuts down at night.

I have some power consumption per ton figures for bituminous mines as follows:

A group of five drift mines, averaging about 51,000 tons per month, has a consumption of 6.8 kw-hr. per ton; a shaft mine, 9.2 kw-hr. per ton; a group of four drift mines averaging about 14,000 tons per month 9.08 kw-hr per ton; a group of six drift mines, mining together about 50,000 tons a month, 8.99 kw-hr. per ton.

F. L. Stone: I will say that this matter is fairly well covered in the paper, where it says: The coal is brought to the surface in a number of different ways. First the loaded car is put on the cage and hoisted to the landing while an empty is being lowered. At the landing the car is taken off the cage and sent to the breaker or tipple over the surface. The second method is that of using a dumping cage, that is to say the loaded car is taken to the surface and then the cage tilted and the car door opened and the contents dumped into a chute, the car never leaving the cage.

The third method is that of dumping the coal directly into a skip, the skip holding one or two mine car loads, the car never leaving the mine. This method involves an increase in the breakage of coal but not necessarily to any great extent. The fourth method is somewhat similar to the third except that the coal is dumped from the cars into a hopper and then loaded into the skip.

E. J. Cheney: In addition to Mr. Stone's illumination of the subject, I think some people may not understand the general system of mine hoisting, which may be explained by means of Fig. 1.

There are two drums, on which the ropes are wound, these drums being driven by engines or motors. The ropes go from the drums to the head sheaves at the top of the shaft and then down the shaft. At the end of each rope is suspended a cage or skip, as the case may be. Nearly all mine hoists are of this double type, ordinarily known as balanced hoists.

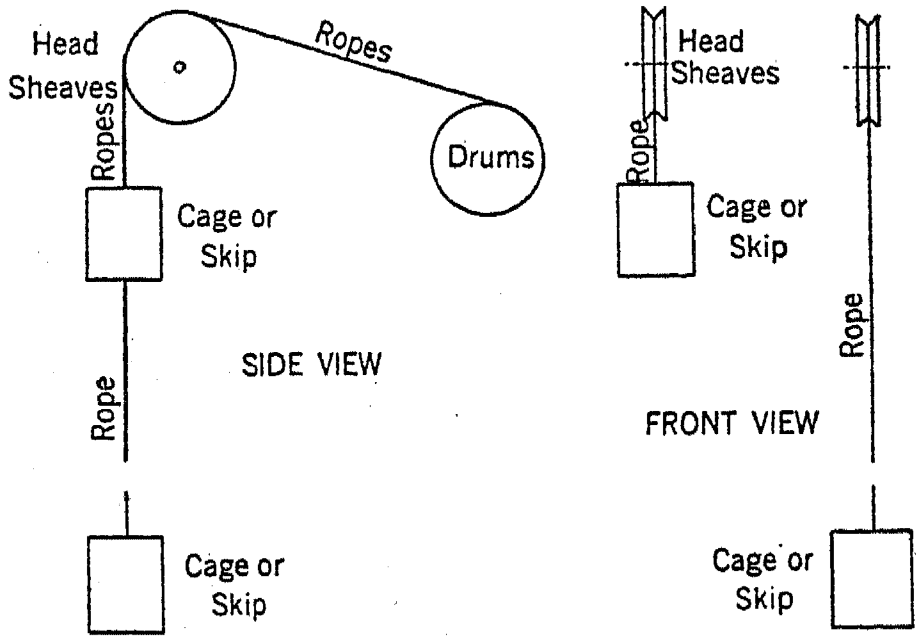


FIG. 1

When one skip or cage is at the bottom, the other is at the top. The two balance each other, and the only unbalanced load is that due to the coal or ore, or whatever it is you are hoisting. As the empty skip, or the cage with the empty car goes down, the loaded one comes up. The difference due to the unbalancing of the load is constant, but the weight of the rope becomes a very serious factor in the case of a deep mine, so that actually, when you go to depths of 1000 ft. (304.8 m.), or more, the light side, so-called, may be the heavier; in other words, the weight of rope hanging in the shaft may exceed the coal or ore you have on the loaded side. The curves in Mr. Stone's paper will be more readily understood if you have in mind the fact that there are those two systems, one going up and the other going down, and that you have an enormous amount of dead-weight, in drums, sheaves, skips, and ropes, to start and stop every time, whereas you are actually handling a comparatively small amount of live load.

W. I. Slichter: I would like to ask whether the fundamental principles governing the design of the shape of the drum are: that for the most economical use of power in accelerating any mass, a large force should be applied at the beginning, at standstill and at low speeds, and then as the speed increases this force should decrease, thereby limiting the power, the product of force and speed, to some definite reasonable value. It appears to me that the object of giving the drum a particular shape is to accomplish what the inherent characteristics of the series motor accomplishes in railway work.

Peter Kain: I would like to know if the figures of kw-hr. per gross ton of coal under "haulage" include the two types of locomotives used. The figure given is 1.73 kw-hr. gross ton charge for hauling. Does that include all the power used for hauling, both storage battery locomotives and trolly locomotives.

J. B. Crane: Yes.

L. H. Rittenhouse: We who are associated with the Fuel Administration at Washington recognize and endorse a great many of the conclusions which Mr. Crane has presented in his paper. In regard to the conservation of coal, that one glaring case of 400 tons of coal consumed per thousand tons mined is a case which caught our eye early as one that should be remedied. The inefficiency might appeal to the electrical engineer more forcibly if you transfer it to terms of coal per kilowatt-hour. That is, taking 400 tons of coal to mine 1000 tons of coal, would be 70 to 75 lb. of coal per kw-hr. That is the equivalent of what they were using.

Another advantage, conservation of men, also appealed to us very strongly, when men were the all-important article, are today, and will be in the future; the saving in men, or man-power, appealed to us very strongly.

There is another saving which the author mentioned in his paper, only from the investment point of view, and that is the iron and steel conservation. The War Industries Board was especially interested in the conservation of every pound of iron possible, and as the iron and steel go much further in the central station power plant than in the isolated plant, this was an additional argument in favor of the large station.

The ordinary advantages of the central station apply in the coal field, as in any other application of the central station, but, of course, there are some peculiar advantages which are brought out in the paper, among these being the smaller amount of coal used.

Still another point which the author might properly have indicated, and which I wish especially to emphasize, is that the central stations are truly a national resource. We, at the Fuel

Administration, felt very strongly about it at the time, because we realized the stations are really an asset of the country, and we felt the lack of these stations when we were looking around for them. The speaker was associated with the Production Bureau, in that section which concerned itself with the continuity of electric power for coal production, and when we were getting telegrams every day—"Why don't we get power for digging coal."—we looked around for the central stations and found they were loaded to the limit, and there was really no way by which these coal operators who wanted the additional power could get it.

As I say, we felt that it would be a national resource to have a few more central stations available, and it appealed to us from that angle. There were a few breakdowns, and we looked up the question of supplying power, and in the meantime we took certain steps to anticipate some of these breakdowns, if possible; in other words we started a census of coal mines purchasing power from central stations, so that we could anticipate some of these power demands and avoid the difficulties during the coming winter, compared with those we had last winter. However, we had to drop the census and assist and cooperate with the many cases that were reported to us.

I will tell you of some of our troubles, just to give an idea of their range. At one extreme, we simply sent a letter to the judge of the Circuit Court, asking him to be lenient with a central station company in the western part of Pennsylvania, which was being sued for breaking the law against the emission of cinders, and the smoke nuisance and asked him to be lenient and to put the brakes on, as the company was doing all it could to get coal out of the mines. That was comparatively simple.

On the other extreme, there was a case in Illinois where we assisted the superintendent of a large plant in operating a train of 30 tank cars, operating those tank cars every day, carrying water for condensing purposes to a station 20 miles away, located alongside a stream which happened to go dry. Incidentally that misfortune curtailed production by tens of thousands of tons of coal.

The point I am making is that these central stations are really a national asset, and it was brought home to us, who were trying to cooperate with them, very forcibly this summer, and I think an organization like this cannot be better employed than to engage in missionary work of this kind and help the central stations along.

There was a case in which one central station did have extra power. The New York Edison Company had about 20,000 kw. We had our eye on that, and I believe the Fuel Administration of the City of Philadelphia also had its eye upon it. But Newark "beat us to it" and they started to put cables under the Hudson River to connect up with the New York Edison Co. to help out the Newark situation, there being a lack of power facilities in that important manufacturing district.

That indicates the great advantage of the resources of these stations, and I think this institute should take this matter up in a proper way to bring it before the authorities. A number of well designed central stations with ample reserve, efficiently managed and strategically located provides a national asset in times of peace as well as of emergencies, that should not be overlooked. We think that the Government should be advised of the possibilities of such situations. We were doing our part. The natural question arises—Why did not the Government help? As Mr. Crane mentioned, they started to do something, and they were cooperating in regard to the financial questions he raised in his paper, and we were doing as much missionary work on Capitol Hill as possible. Of course, very properly, the members of Congress were watching out for the interests of the Government's purse strings—that is proper, and their function—, but the money was not going to be given away, on the contrary it would have been of great assistance, and in passing, I may mention that every dollar of it was being taken care of to the extent of the ability of the different government departments that were connected with the project.

Supplementing Mr. Crane's paper, I have a few figures in regard to the census of mines purchasing power. After the armistice was signed, we took time to add up the returns we had received. As far as we could discover, there were 107 central stations supplying approximately 1000 coal mines with electric power.

In regard to the question of the kw-hrs. per ton, we were vitally interested in that, because we wanted to conserve power in the mining operations and make it go further than it had in the past. Mr. Crane gave a figure in his paper of an average which was 3.57 kw-hr. per ton of coal mined, which figure was criticized later in the discussion.

I picked up some figures we had available from one of these central stations, and it is interesting to note that the average I obtained was 3.58. The figures I have given are for one station supplying 56 mines, and he said his was for a station supplying a little over 50. I wonder if both figures related to the same group. This particular group to which I refer was located west of the Mississippi.

J. B. Crane: My figures related to a group of mines in West Virginia.

L. H. Rittenhouse: An entirely different set of mines. But as was pointed out by a speaker, in the discussion a few moments ago, the conditions differ and I think possibly that 3.58 kw-hr. per ton is low, in other words, as far as my investigation carried me, the limit of the 56 mines ran from a fraction of a kw-hr., showing the power was used exclusively for electric lights, up to 53 kw-hr. per ton, with a foot-note calling attention to the fact that this mine had recently shut down. No wonder! These figures, however, account for the averages. We found it very difficult to get exact figures, since in many mines connected to central station service some of the power used in developing coal was not electric. In other words, some operators may still retain a few mules, as well as auxiliary steam, and these do not figure in the kw-hr. unit.

I would like to ask Mr. Crane, in connection with mine No. 1 and Mine No. 3, 3.6 kw-hr. and 5.2 kw-hr., were those figures for an anthracite or a bituminous mine.

J. B. Crane: That was for an anthracite mine, fully electrified.

C. A. Adams: This is one of those numerous cases where the emergency situation produced by the war has forced us to the adoption of methods of cooperation which have proved so immensely effective and economical.

I do hope that this institute and the other engineering societies, and all other such bodies, will use their best efforts to help the nation perpetuate these advantages which have been forced upon us by the war during this reconstruction period and afterwards.

F. L. Stone: Since scarcity of power seems to be one of the limiting features in connection with the electrification of the anthracite field, I would like to ask Mr. Crane if everything has been done that is possible to secure the advantage of the diversity factor that exists between many of the smaller plants located in the region. I know that in some instances plants have tied together and thereby made available, for the use of both, several thousand kilowatts. It would seem to me that a great deal could be accomplished along this line.

L. S. Randolph: There is one phase of electrical operation which it is almost impossible to put in dollars and cents, viz., the convenience. Anyone who has put a 4 in. to 8 in. steam-pipe with exhaust pipe into a mine, and then puts a centrifugal pump with two small wires, sees where this comes in.

There is no chance of coming in the morning and finding a drunken fireman has burned a crown sheet the night before, and as a consequence shut the mine down for three or four days. One does not have any of that trouble.

Those are factors which cannot be put in dollars and cents, and yet they are tremendously valuable.

E. J. Cheney: There is one point these papers bring out, and that is the tremendous advantage which electrification produces in the case of mining work, as in almost every other instance, in that you are able to tell, in the first place what you want to do, and afterwards what you are doing.

So far as I know, in the case of steam-driven hoists, the hoist is simply designed according to the ideas of the man who gets up the drum part, and then a steam engine is put on that is big enough to drive it, and the hoist is put in and operated.

When you come to install an electric hoist, you have people like Mr. Stone, and the others who are figuring on electrical equipment, to calculate the effect of weight and inertia, and drum size, and drum shape, and they go back to the hoist makers and say:—"See here, if you do so and so, that means a big difference in the result." A steam hoist is operated without much attention being paid to economy. It is hard to find out whether the inefficiency is in the boiler plant, or in the engine, or in the shaft, or somewhere else, and so it is usually not done; but in the electrical hoist, a few indicating and recording wattmeters tell the story all the time. You can look at them and see what is going on, and you can do it day after day, checking up watthours against tons of coal mined, and you can find out whether there is anything wrong and tell which type of installation is giving the best results. You are working with your eyes open all the time, and are not in the dark about anything.

Graham Bright: A few years ago I used to indicate steam hoists in order to find out just what work was being done, so that the proper size of motor could be determined where it was desirable to change a steam hoist to electric drive. I soon found out that these results were of very little value because in every case a square steam card was obtained which showed that no cut off whatever was used. In practically all cases the operator who would open up the throttle wide, used no cut off, and when the accelerating period was over, would throw off and drift so that at no time during the operation was there any cut off, so that you can see that there would be very little use in taking indicator cards from such an operation.

We can determine much more regarding the electrical equipment necessary if the operator will tell us how much material he wants to hoist each trip, how many tons he wants to get out in a given time, what his rest periods are and the weights and diameters of his drums, cages or skips, cars and size of rope. By having this information, we can figure out closely the electrical equipment necessary, but the steam engine itself is no guide whatever in determining the proper electrical equipment to put on a hoist. Regarding the steam engine, it is necessary that either side of the engine be capable of lifting the maximum weight on account of the fact that one side may be on dead center and unable to exert any torque at the start. This means, as a rule, that the engine is considerably larger than necessary and the result is that the operator uses very high rates of acceleration when a considerably lower rate would be ample to obtain the output required. The average operator thinks that he requires this high rate of acceleration and also thinks it necessary to have high rope speed. At the average mine, a trip of cars comes to the bottom of the shaft and the operator hoists this to the surface as rapidly as possible and then takes a rest. In a large majority of the cases, by using slower rates of acceleration and slower rope speeds, he could get out as much coal as the mine could produce except that he would not have so much time to loaf. In many cases, if we were to attempt to exactly duplicate the steam operation of a hoist, it would require an unnecessarily large and expensive electrical equipment which is not justified when a smaller and less expensive equipment would be ample to hoist all of the coal that the mine can produce in a day's time.

If the operator wants to produce a certain amount of coal in eight hours, it is not safe to figure out a cycle which will require the full eight hours. It is much better to estimate a cycle which would take the coal out in from six and one-half to seven hours if a mine works an eight hour day, which allows an hour to an hour and a half for the usual delays which are bound to occur. It is practically impossible to operate on a given cycle for a period of eight hours without delay. There are so many things about a mine to cause delays that in order to be safe, this margin should be anywhere from one to two hours per day.

J. B. Crane: I would like to bring out a few points. Mr. Bright mentioned the fact that the central station men were a little modest in their claims for the advantages of the use of central station power. That fact was brought home to me forcibly about three years ago. I had the pleasure of addressing the West Virginia Mining Institute, and they asked me to prepare a paper on "The Use of Central Station Power in Mines from the Central Station Standpoint." The man who was asked to prepare a paper to be presented at the same session, from the mine user's standpoint, was a man we had worked with for quite some time, to get him to see the advantages of electric power, etc., and I remember at one time one of our men went into his office and he asked him who he was with, and he told him he was with the power company, this man said to him, "Well, you might as well be on your way, no use stopping here." We naturally expected, when he read the paper we would get many knocks, but instead of that, his paper was strongly in favor of the use of central station power, and we were most surprised at his change of attitude.

Mr. Parkhurst brought out the question of the kilowatt hours per ton of coal mined, and his figures for the bituminous coal were larger than those I had given. These figures will vary with different districts. But I never heard of any bituminous mine which used as much power per ton of coal mined as those which he mentions, and I secured figures from several mines, and they were around the ones I secured in West Virginia as indicating the amount of power it takes to operate the mine, and the deeper the mine becomes, the more power it takes.

I had the pleasure last year of visiting the Maror Velho mine in Brazil. They went down 7000 feet, and intended to go another 1000 feet. They were using 4000 h. p. to operate the mine for the first 7000 feet. In going down the additional 1000 feet, they would require 4000 h. p. additional power for ventilation.

The conditions which they would then encounter with regard to ventilation, etc., would prevent them from going any deeper, because the cost of power would run into such large figures that it would not pay.

Mr. Rittenhouse brought out the fact that power stations have been greatly overloaded during the period just passed, and, that this condition still exists.

Regarding the diversity factor which Mr. Stone mentioned, we have in our territory two power stations in the coal field, one good for about 16,000 kilowatts, and another one good for 30,000 kilowatts, and at a nearby point we have another power station good for 20,000 kilowatts, and one small station, or rather several small stations, good for 3000 or 4000 kilowatts.

The peak which we were getting from all these stations was 55,000 kilowatts. We connected the ones in the coal field with the station at Allentown, which is the one I have just referred to as being nearby, with a capacity of 20,000 kilowatts, and we have been going out and taking power from these three stations, amounting to 69,000 kilowatts.

In burning small size anthracite fuel, we are getting beyond an economic boiler rate. With stokers this rate is 175 per cent. We have been running them as high as 300 per cent, and averaged for the month over 200 per cent boiler rating. As a consequence, we have had to replace a boiler arch every other day, and have all kinds of trouble, but we have been able to carry the load in our particular territory.

The thing which Mr. Stone mentions, and has brought out forcibly, is the fact that in the coal fields the peak load is from 7 o'clock in the morning until 4 o'clock in the afternoon, and then we have about 10,000 kilowatts to dispose of between that time and 7 o'clock the next morning.

We are only 90 miles from Philadelphia, and they have a peak in the evening of something like 20,000 kilowatts above the daily load. You can see what that would mean if a transmission line could be run from our central power system to Philadelphia. The same equipment which takes care of the coal mines in the day time would take care of the lighting load in Philadelphia at night.

I know of one case in Texas where they connected in several stations, and in that case they were able to sell 10,000 kilowatts additional by connecting the central stations together and getting the benefit of the diversity factor.

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AMERICAN ENGINEERING RESEARCH

BY W. R. WHITNEY

THE purpose of this paper is to call attention to our national condition in relation to engineering research, in the hope that the desire among engineers to advance the sciences may be discussed and crystallized into the most acceptable form. The subject must have some sharpness of definition, and so I will give the meanings I attach to the title.

I am using the word research, to cover painstaking investigations which *add* to knowledge. Research alone may be too general a term to interest all engineers, so I limit my use of the term by the adjective "engineering", and mean by that, the study of matter. Much research may be done in fields not connected with engineering, but I am not referring now to archaeological, geographical, psychological research, or other similar fields where study and experiment also add to human knowledge. For the sake of a simple and clear field I am omitting such sciences. I use the prefix American, to call attention to one of the most burning questions which confronts us today. We have been so keen in engineering that American engineering is unsurpassed in the world, but we never made a fair start in American engineering research. Moreover, we have no plan for producing research men, though we have plenty of plans for consuming them.

Research is the foundation of engineering, whether it be electrical, physical, chemical, biological, or what not. It sometimes seems as though much of our best engineering is only very slightly removed from the original research in time or space. If the research forms the foundation, then engineers quickly add the first story. When the investigators showed the value of typhus vaccination, the Army doctors used it on a million soldiers. General Gorgas recently wrote Dr. Keen that of over 1½ million soldiers, there were only 18 deaths from this disease in 1917. At the Civil War rate there would have been 7200 deaths. That is good American engineering research. But who is responsible for the research men? How may we procure and support them? A part of this responsibility is your own.

During a certain period of our American history there was a Puritan atmosphere in research. It hung like a fog over the territory of the Pilgrims, and even now it clouds the issues between engineering and research. It meant purity to the extent of sterility. It was with interest that I recently read, with the above geographical thought in mind, one of Emerson's Essays. He puts our duty in such a clear light that, before going into problems of detail, I want to quote him. His use of exactly the right word to give clearness to his thought seems very wonderful and his words will bear a more careful analysis than anything I can say.

> The destiny of organized nature is amelioration and who can tell its limits? It is for man to tame the chaos. On every side while he lives, to scatter the seeds of science and of song, that climate, corn, animals and men may be milder and the germs of love and benefit may be multiplied.

If we could all learn that by heart, I could well stop my talk at this point.

The experience of the past few months developed in the mind of practically every intelligent man a desire to do something to advance the country's interest. This was not confined to scientists and engineers, but was marked in those cases. The number of scientific and engineering organizations which evinced this desirable spirit is too long to read. Our leading societies united in committees representing all the sciences, and they in turn formed many sub-committees. The National Research Council enlisted in one committee or another almost all the scientists of the country. The Council of National Defense established scores of committees on engineering and allied lines. Inventories of everything conceivable were taken, including most of the material and personnel of our colleges and engineering schools.

Now there is one defect in this system of rapid utilization of our resources. It does not produce any new material, though it may organize the old. We needed organized engineering research long before the war. We needed it more during the war, and we shall certainly need it after the war. It is the duty of engineers to study this matter and take active steps to correct it. The problem is: how shall we insure the preparation of plenty of American men of science by some system which makes us independent of foreign assistance? My belief is that it can well be done in our existing colleges by greater attention to new material truth. Experimental work in the sciences best prepares and preserves those desirable qualities in teachers: interest and enthusiasm. Any plan which would insure properly supported investigators whose vocation was experiment and whose avocation was teaching, where students could *work* with them, would be a most natural development. It is a way already thoroughly tested abroad and would be a step of the greatest value to this country.

It is unfortunate that we have no generally acceptable plan for insuring our scientific work and our engineering research. There has been little better than chaos on this subject thus far in America: No one's life work was research, though to a few it was a by-product. Within the past few years, and in a few lines in certain universities, investigations into the laws or properties of matter have been under way. This has been poorly appreciated. The *exceptions* are those successes which led to rapid physical amelioration, as of death or pain (naturally the first places for such appreciation). The value of research on the hook worm is readily understood. Heredity-studies in corn or cattle are recognized as worth while, but there are many researches undertaken whose value is not so apparent on the surface, and yet whose ultimate potency is warrant for their consummation. But for us at present, while the product of research work is desirable, the preparation of the men is of greater importance.

Because our condition has been described before and remedies suggested, is no reason why attention should not be called to it again. Something should be done. Important steps in a democracy call for repeated attempts to inform the interested people. Perhaps scores of addresses on general preparedness delivered before the war were wasted, but it was safer to assume that the reiteration was necessary.

It is owing to our Yankee inventiveness that I am interested in seeing much greater attention given to the problems of our engineering research. We are inclined to be so elated by our apparent successes in industry, that we imagine that there is some way to build without foundation, to practise without experience, to reap where we have never sown. Hundreds of thousands of inventors are choking our patent offices with ideas which can have but one end, the unhappy end of the inventor. This condition is wrong. We cannot wish to perpetuate it. It may be greatly ameliorated by careful planning of engineering and research education in America. This planning falls as a duty on the engineers themselves.

There is a general opinion that discovery of truth is very prevalent here, and that somewhere in our national archives there lie buried a lot of wonderful but undeveloped possibilities. You have probably received notices from different groups of people who want to bring to light and life some such desirable inventions. As a matter of fact, there are probably very few such things in existence. One reason is that there is actually more capital to develop new ideas and more capitalists who see possibilities which do not exist, than are justified by works. In fact, while we lack training and knowledge as inventors, we more sorely lack men who are sufficiently expert to pass with certainty on suggestions of abnormal minds. The reason the steam engine and the submarine were not earlier recognized was due to the scarcity of men who knew science. When we had one Langley on aeronautics we needed 100. Paucity of knowledge is our present trouble, not superabundance of inventions. The point in this illustration is that we badly need more highly trained quantitative men of science. Our National planning for research, except for research in agriculture (which planning, strange to say, was done during the Civil War), has been entirely neglected. Agriculture is not our only interest, but even in that field, the future lies in the hands of the young research chemists and engineers who should do the work on potash and fixed nitrogen.

It is on this point of the special need for planning that I want to lay stress. Ways to accomplish the end have been tested and proved. Particularly during the past century there have appeared in other countries men who devoted their entire time to study in fields of new knowledge which by contact had attracted them. We might possibly have called them specialists, high brows, hobbyists, or men of large interrogation. They were very valuable.

These men were found in many walks of life, but by far the greater part of them were, or soon became, attached to foreign universities. They gave fully of their products to one another and took extreme and careful steps to give to the world accurate and detailed reports of their researches. Davy and Faraday were of this type, Liebig and Wöhler were like them, and it is safe to say that such benefactors of the race could be cited by the hundred. In most cases their entire lives were spent in their laboratories and all that they learned was freely published. Still more important; in the laboratories of these men, particularly promising students were allowed to work, and these teachers illustrated for them scientific research by example. They did not confine their teaching to talks about the work of others. The men referred to, and others, such as Helmholtz, Kelvin, Berthelot, and more recent prototypes, such as Ramsay, Moissan, and Haber, were alike in these respects. We have produced altogether too few of them in America, and supported none. They became unusual men, but they started much like others. They were developed by a system or plan. As they developed in interest, in knowledge of new things, and grew abnormal in this respect, the opportunity for further development was provided, usually by the state. There is no reason to assume that the product they represent cannot be grown on American soil. They were the result of a policy almost identical in all cases. Each was freed by some planned system to evolve himself and follow his studies along the lines he chose, and he was given the assistance of students who were at the same time learning the value and rare power of careful thought and painstaking investigation. They created atmospheres of scientific pioneering. The teacher was the leader. He was exploring. His interest was transferred to others by that most direct way—close contact. The lines of scientific endeavor were not chosen by some remote director, or committee of experiment, but the subject itself led the investigator along, and new truth was the sole aim. The professor then became the leader in the university, because he was following a new line of action and not because, at the request of a faculty committee, he was perhaps trying to solve some particular indigenous problem, or reducing the cost of some existing factory product. He was being led by Nature, instead of trying, as the industrial laboratories so often do, to force Nature along lines which we think most desirable.

Someone has suggested that possibly our industrial research laboratories could supply the country's deficiency in scientific research and its teaching. Many industrial laboratories have been established recently, but, to my mind, there does not seem the slightest possibility of thus meeting the real need of the country. It is neither natural, nor desirable, and most recent writers on the subject have been clear on this point. The interesting papers by Colonel Carty, Dr. Jewett, and Mr.

Skinner, in the TRANSACTIONS of the A. I. E. E., are in good accord on this point.

It may be asked, do you consider it a good plan to arrange our colleges so that every teacher with a hobby may develop it by the aid of the students? If all our teachers had enjoyed good training in science and were good scientific investigators, the answer would certainly be "yes", but this is not always the case. Therefore we must lend our aid in helping to establish a condition which will insure a greater number of scientific teachers who are also trained as productive scientists. This is in reality the method employed in those countries where scientific education has been most advanced.

In other words, our great American engineering problem today is not, how shall we make use of our trained investigators? Shall we organize a few large centers of research and solve there the industrial problems of the country, or, shall we encourage and assist the industries to give more attention to research work? But it is rather: how may we produce annually active young students of matter at a greater rate but of higher quality than any other country? We must think in terms of thousands per year of the type of trained men represented, say by the doctor's degree, or by two or more years of individual experimental work.

The direct object of this paper is not to ask for the appointment of some committee to cooperate in planning American engineering training and knowledge. We do not need more committees for this work, but more constructive activity on a dozen existing committees. In other words, enough machinery already exists to do the work, and we need but a clear view of the necessary steps. Probably no one person has completed a perfect plan, but many have advanced suggestions, all of which seem to have failed to meet sufficient acceptance to move the wheels. My own ideas are not presented as new and original. They are plans of others, but they seem most natural and desirable because they accept higher education as our common right. They seem democratic.

Several years ago the American Association for the Advancement of Science appointed a Committee of One Hundred to bring about cooperation between universities and industries along the line of improving our work in science, its growth, and its use. Other organizations undertook similar studies, but at that time most of the effort was expended in arranging for the use of, rather than for the production of facts, or of useful men. It was a shortsighted plan, helpful but not broad enough. In several colleges, research fellowships have been established, but these were generally for the solution of definite manufacturing problems where private interests seemed justified in paying for the work. This was done because scientific research which was aimed solely towards new physical truth could not be thus supported. Such methods are doubtless useful, but their aim is low. They advance single industries but are not intended to establish and make known new truths. Such organizations will only rarely become research centers where new leads in science will be persistently followed by groups of intensely interested investigators. There is greater value and attraction in disclosure of new physical phenomena than in any probable financial returns, and the atmosphere in a laboratory of scientific research is a thing by itself. But it cannot evolve of itself, and it is for practical beneficiaries, such as engineers and the public, to find the way.

Central bureaus of science and research have been repeatedly advocated and many are still being planned. The plans vary from separate departmental government laboratories like those of the Department of Agriculture, the Bureau of Mines, and the Bureau of Standards, to those of separate groups of colleges, groups of industries, societies, and finally, groups of men. One such is the most recent plan of the National Institute of Inventors. None of these is probably perfect. Most of them would consume (possibly ossify), but not produce men. Perhaps all of them should be studied by engineers and some of them developed, but if many were carried out, it would immediately become evident that our crying need is for general college research work to prepare trained investigators.

A few years ago a bill was prepared for Congress which had for its object the establishment of engineering research work in the federally assisted colleges of the country, and this called for additional federal support. One of the criticisms of the plan was that it did not include all colleges. It had been thought that the precedent having been established in 1862 of giving governmental aid to these institutions for their research and teaching work in agriculture, it might not be difficult to extend this line of activity to other scientific research work. One of the reasons this bill was not sufficiently supported was that a large number of struggling state universities and other colleges objected to the distinction which would be made. This finally led to other proposals being drafted, and one, known as the Smith-Howard Bill, seems to be the most satisfactory suggestion yet made. Others are also appearing. It is not my intention to go into the details of this bill here. But its existence and its fundamental concept deserves your careful attention and your active interest. It called for funds for research in the colleges; $30,000. for a college in each state. Fifteen minutes of our present year's war cost would care for that plan a whole year. The plan seems a good experiment, but needs support. This line of thought raises the question whether our democracy should pay for its education and its standing in science and engineering, or whether it should be left to accidental philanthropy, to the begging ability of college presidents, or be neglected altogether.

President Shurman, of Cornell, said a few months ago:

> Germany has the advantage over all her opponents, especially Great Britain and the United States, in the solid fabric she has constructed during the last generation by means of generous government encouragement of, and appreciation for, scientific research and instruction. Our Government today, instructed by the war, is ready to spend billions of dollars for new scientific truths in which, prior to the war, it felt no concern and for which it would make no provisions.

This same thought has been expressed by others in many ways. The facts are pretty well known, but are still not entirely digested. We are such creatures of habit that we may never awaken to the fact that we are a slow but sanguine people. Slow, because our rate of advance as measured over the last century is less than Japan's; sanguine, because we hope for the best without being willing to provide for it.

If you consult the university president or engineering educator who is well acquainted with higher education in America and ask him why we have been so backward in the highest class of scientific teaching, he will generally reply, "lack of money". We are forced to train so many men to reach a certain stage of preparedness for standard vocations that we cannot afford to encourage advanced work nor can we obtain teachers who are primarily workers in their science. Our conditions in America are still in that stage of teaching where our energies must be exclusively used to repeat to students annually certain knowledge already acquired, but we cannot provide for acquirement of new knowledge nor for the training of men to obtain it. As far as our collective powers and plans go, the sciences would remain stationary.

Apparently persons of highest intelligence believe that while our schools should be supported by taxation, this should apply only up to a certain grade. Above that grade we should beg private interests for our advance in knowledge and for our most highly trained men, for our studies of disease and our search for cures. This fact accounts for the difficulty of the college president. It is merely a bad habit. Certainly no one really approves of it. Everyone who thinks carefully can find plenty of fault with it, but the engineering of a better plan is left undone. That is an engineer's job. In the past few decades intelligent men of wealth have fortunately recognized some of the needs of higher education in our country where our pioneer activities have not developed any satisfactory system. By this means, great good has certainly been accomplished, but the philanthropic method can scarcely be a strengthening one. It cannot lead to proper appreciation of the value of the advancement of knowledge by the people.

One of our greatest dangers is that, knowing how Germany arranged her higher education, quickly advanced and reaped those returns which were so harmful to us, we may spurn the adoption of some similar scheme because of the degraded utility she finally made of it.

What do we want? We want our democracy to do as well in science as German autocracy did, until we learn to do better. I do not care to compare the scientific work of different countries, but I want to call attention to the peculiar way science was treated in Germany for the half century preceding the war. I want to show that some of her ablest men were life-long workers in and teachers of science; that they were leaders in their civil groups, that they were appreciated by the country and given the widest support and honor. We have too little to compare with it, and we collectively, or our Government, are not making sufficient effort to support new scientific work.

Let me once more warn you against the fear that material truth and experimental knowledge are a menace. Only indolent people think this, but it is frequently expressed. Professor Richards, of Harvard, says on this point (1916):

> Science has recently been blamed by superficial critics, but she is not at fault if her great potentialities are distorted to serve malignant ends. Is not this calamity (war) due rather to the fact that the spiritual enlightenment of humanity has not kept pace with the progress of science? The study of nature can lead an upright and humane civilization ever higher

and higher to greater health and comfort and a sounder philosophy, but that same study can teach the ruthless and selfish how to destroy more efficiently than to create.

There are many European cities which owe their reputations to their scientific work. Most of them are university towns. Their renown is not based on the nine, nor on the eleven, nor on the local glee club. Freiburg, Bonn, Heidelberg, Jena, Charlottenburg, Leipzig, Halle, Karlsruhe, Gottingen, are such places. They recall scientific accomplishments. Some of our corresponding towns are Columbus, Ithaca, New Haven, Urbana, Madison, Princeton, Lawrence, Pittsburg, Burlington, Ames, and Lincoln. Does each remind you of scientific work? We are not yet doing our duty by them. Can we not help the scientific work in such towns?

Good scientists grow in all countries, but the support which investigators received in Germany is a point I want to emphasize. To the average physicist, chemist, or biologist, most of the following teachers recall fundamental advances in the sciences. I refer to men like Pettenkeffer, Liebig, Wöhler, Voit, Ludwig, Kühne, Hoppe-Seyler, Weber, Helmholtz, du Bois Reymond, Fechner, Engelmann, Pfluger, Hertwig, Leuckart, Ehrlich, Virchow, Hering, Emil Fisher. Buchner, Pfeffer, Weissman, Koch, Quincke, Hofmeister, Pringsheim, Boveri, Richter, Kekulé, Wislicenus, Kolbe, Hofman, Bunsen, Victor, Lothar and J. R. Meyer, Bayer, Clausius, Ostwald, Kolrausch, Planck, Nernst, Hertz, Boltzmann, Roentgen, Wien, and Haber.

Are we to suppose that America could not do as well? In my opinion, we should try the method already tested, and modify it from time to time as we see indicative developments.

The German universities operate under charters granted by the Government, which supports them. The faculty practically manages the institution, though the Government is represented by the curator, who manages the finances. The full professors regulate the internal affairs by their votes, and they elect their various officers, the rector, deans, etc.

On the other hand, we in America ask certain unprepared civilians to serve as our college trustees. They soon find that the condition of finances calls for help, and, together with the president, they must solve the almost insoluble problem of the year's budget. There is little chance for new work. Practically every college has all it can possibly do to meet the demands of large classes of undergraduate students, and the trustees, despite their sympathy with scientific developments, cannot possibly care for demands which are less dominant. We have about 500 universities or colleges in this country. Many of them are equipped with everything necessary for good scientific work except the encouragement. Many laboratories are better equipped than those abroad. Would it not be worth while to arrange for the modest addition to our educational expenses which a fair trial of American university research would entail?

The Students' Army Training Corps has recently thrown all our colleges into an unsatisfactory and an unstable condition. Had that work continued, it is possible that permanent improvements in education would have resulted. It was too radical to continue, and the pressing need has disappeared, but we have all seen the possibilities of quickly effecting such changes, and we ought now to work and hope for some justifiable improvements. Is it good engineering to force the presumed leaders of our highest education to spend their best energies in perpetual scrambles for funds, instead of directing education? Probably no one thinks so, but everyone waits for the change to be forced by someone else.

The recent report on Engineering Education, published through the Carnegie Foundation, to which we are greatly indebted, leaves me with the thought that the past aim of our engineering education was to teach carefully selected men to merely repeat certain standard processes and thereby earn a livelihood. This is not enough. Watch good, experienced engineers at work and see them plan. Is their work repetition of old steps they were taught? It is nearer right to say that they were always trying experiments. We read daily of the newest and largest building, the most novel or the longest bridge, new types of tunnels or subways, longer range guns and new armor plates, lighter aero engines, larger turbines, heavier locomotives, Armco iron, Tantiron, Duriron, Monel, Stellite, etc., etc. There is certainly no limit to this line. Therefore I would not teach fixity, but right change, and do it by precept and *example.* I would increase for the engineer, during his college years, the facility with which he could begin to learn the elasticity and nobility of his calling, and that by contact with men interested in experiment.

What will be done to continue more men in advanced scientific lines in college? Can you engineers not help here? Is it not a good plan to arrange for the highest possible training on the part of a selected few? Is it not the duty of organized engineers and scientists to take part in activities which influence legislation and to make our country the foremost for new finds in nature and in the development of men? Engineers will then be co-products with investigators.

I would be better satisfied with this suggestion if I did not know that very few realize how much the strength of a whole people depends on its scientific novelties. The facts are frequently something like this: some searcher for new truth in natural phenomena uncovers a fact. He gets a few dollars or a little mental satisfaction out of it. A few industries then base the work and the living of many people upon it, and its value, there, may be expressed in, say, many thousands of dollars. But it is not so readily seen that the people at large get most of the return and are therefore most highly interested in the advance. Their earnings in such cases are in millions and their advantages incalculable. The telegraph and telephone, the gas engine, the aeroplane, wireless, artificial lights, and all engineering improvements have this in common. The ratios may be 1 to 1000 to 1,000,000 for the three groups, but the largest group does least planning. It is not organized for it. Engineering societies might do this.

Large central laboratories of research recently suggested for industrial chemistry are worth good trial, and are being tried in England, but to be most effective they would have to follow somewhat the lines of other smaller laboratories. We have knowledge in such cases which may be of value. Several now exist in which scores of experimenters are employed. A group of one hundred working on miscellaneous research would probably represent between one-half and one million dollars a year outlay. The return would be problematical and cannot be estimated, but would seem very promising in various lines of federal interest. One result is, however, certain. Manned at once with any hundred investigators now available, it would be very much less useful than if manned with well trained research men, of whom we have too few. Our already extensive use of trained men such as electricians, chemists, and physicists is exceeding the supply.

That the young men of our colleges and engineering schools early quit school and went into war work is not a cause for complaint. That in itself was a splendid thing and accomplished wonders. Over a thousand young chemists have done the work of our gas mask development, our poisonous gases, our smoke screens, our new explosives, etc., and the physicists went into the wireless, the submarine defense work, and similar problems in large numbers and with marked success. They have done things which older men would not have undertaken. Therefore this is rather a complaint that we had so few young engineers, and is a petition that in the future the supply be insured and the energies directed to peace time needs.

I do not see any possible way of insuring our future commercial and engineering supremacy that can compare with utilizing our colleges more perfectly. When barbarous war methods are forever stopped, science and engineering should be the main factors recognized by the League of Nations as the legitimate ground for a nation's supremacy. How can we better insure our position than by taking care of that amelioration which tames the chaos for all the people?

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RESEARCH IN AMERICA AFTER THE WAR

BY R. A. MILLIKAN

IT takes no statement from the chairman nor admission from myself, to apprise most of you of the fact that I belong to the genus "Damn professor," as we are affectionately called in Washington.

It may interest you, however, and perhaps be a little diverting too, to know how nearly we professors have to come to acting like real soldiers once in a while when we get on suitable clothes. About four weeks ago I chanced to go from Waterbury, Conn., to New Haven, on a trolley, starting about half past eleven o'clock at night. At a small station between the two towns, there boarded the trolley a couple of well-filled soldiers. They took seats on opposite sides of the car and began quarreling loudly. The conductor was quite unable to cope with the situation; finally the men arose and began hammering each other back and forth across the car. The passengers were in an uproar, when someone called "Here's an officer, let him attend to it." So with the aid of a couple of privates, I had to take part in getting these men into their seats, a task which we succeeded in accomplishing, thanks to the respect inspired by my uniform, and a certain muscular efficiency which the privates showed. We later succeeded in getting the drunks off the train.

Another incident, which has elements almost as heroic, occurred one night last summer when I was on the way from Washington to New York. I stopped off in Baltimore, for no ulterior or interior reasons, although it might have looked as if I had such motives from the fact that I missed the train at Baltimore and had to sit down in the station and wait for another train. In order to utilize my time, I took out my brief case and began writing. In doing so, I pulled my fountain pen from my pocket, and the cap fell off on the floor. Before I had time to recover it, an elderly lady, about sixty years old, rushed from her seat and brought it back to me saying, "Oh, let me do something for a soldier."

To experiences like these should be added the sweet delight which a soldier hero feels as he passes through Indiana and Illinois (as I did yesterday) of not only being permitted, but of actually being forced to take from fair young hands reached up to the windows of the train, sandwiches made of thin slices of bread and thick slices of chicken, with a dill pickle added, all free to the man in uniform.

But the tragedies and the comedies of the war are ended. To use the memorable words of the official announcement of the British War Office—words so brief, so modest, so tactful that they contrast strangely with the vulgar boastings of both our press and our statesmen. "In the fifty-second month of a war without precedent in history, the French Army, aided by the Allies, has achieved the defeat of the enemy," And with the ending of the war all of our war activities either have ceased already or will soon do so.

But there are things that the war has done which should not, which fortunately will not, cease. Men like Dr. Whitney and like myself have been pulled out of the ruts in which our lives had been running and have been thrown into completely new surroundings; men from England have come to this country to teach our young soldiers, and to participate in our war researches; millions of our own men have gone to France and Italy, and to England, so that, if ever there was a time in the history of the world, when men were in a position to begin to see things as the other fellow sees them, to get a little larger outlook than they normally have, that time is surely the present. The country is just now in a state of fluidity, and before the crystallization process has set in great things may be accomplished. Movements which the inertia of mankind make it normally impossible to bring to fruition in any reasonable time, can now be put across with relative ease; for we are all seeing how things look from some other angle than that from which we have been in the habit of looking at them. So that the next few months, so it seems to me, are going to be pregnant months for the history of the United States and of the world.

I wish, then, to call attention to a few of the lessons which from my point of view, the war has taught, and I trust I shall be able to do something more than to make a mere academic recital of them, for I hope that I may bring something before you which will inspire to action.

In the research activities in the aid of the war in which I have been associated, one thing has stood out exceedingly strikingly; namely, that the distinctions between the man whom you commonly call the pure scientist and the man whom you call the applied scientist have absolutely disappeared. In every one of the eighty or ninety researches with which I have had connection there have been associated physicists from the colleges, engineers from the General Electric Company, the Westinghouse Electric and Manufacturing Company, the Western Electric Company, and other companies, and they have all worked as one, and you could no more make one of these men say, "This problem belongs to physicists," or, "This belongs to engineers," than you could say that a man's head belongs to one being and his body to another. The pure and applied scientists have functioned as one single group as they have never done before, and I hope very much that we may be able to do something right now to keep bridged the chasm between the academic group and the industrial group. This chasm is unquestionably a detriment to both groups and an effective bar to progress—a bar which Germany had removed in some fields, at least, much better than we have. If the process of forming societies has already gone so far in this country that for the purposes of organization and classification—which are to be looked upon always as necessary evils—the Physical Society and the Engineering Societies must continue to exist as distinct units, at any rate I certainly feel that it is possible to bring about such an interlocking of committees within these societies that the two organizations may act in many respects as one, and I hope that this possibility will soon become a reality.

You noticed in Dr. Whitney's address that engineering research was defined as "anything which had to do with matter." That is fairly broad, isn't it, and it wipes out completely all of these distinctions about which I have been speaking. I am quite willing to accept his definition, and I would not object to being called an engineer if that is what it means to be one. As a matter of fact, I presume I should have had more satisfaction in my war work if I could have been called an engineer instead of a "damn scientist." You certainly cannot draw any distinction between scientists and engineers on the basis of the utilities of their work, and you all know that you cannot.

I know of hardly an investigation in so-called pure science which has not sooner or later found its utility, nor do I know of any so-called pure scientist whose motives are not utilitarian, in the sense that he believes that he is doing something which will sooner or later contribute to human good. But what I am aiming at is after all not mere words. A change of name alone is of no importance. What I want is rather an actual fusion of groups of men who are now further apart than they ought to be.

My second lesson is one of even greater importance. If there is anything which has been demonstrated by the experiences not only of America, but of all other countries in this war, it is that one cannot safely depend upon the so-called undirected inventive genius of a people to obtain large results. Advances have been made in this war almost exclusively by one method, namely that of bringing together groups of well trained thoroughly competent men and setting them upon the problems needing solution. In other words large and rapid progress has been made in this war and in my judgment it can in general be made only by adopting the strictly research method of approach. In this war every one of the major belligerent countries has had its Board of Inventions, and it has been the uniform experience in all of them, in Italy, France, England, and the United States, that not one suggestion in ten thousand which is of any value comes in through these bodies. Further, the chance is about 100 to 1 that the good suggestion which does occasionally come in this way has already been thought of, and worked upon, by the groups of trained men who have been giving their attention and skill exclusively to these research matters.

But how much in the past have we used this method? How much have we done in this country during the last hundred years in organizing such research groups? Dr. Remsen told me the other day that in 1872, when he was first called to Johns Hopkins, there was not a place in the United States where there was any research atmosphere or where any serious research work was being done. Perhaps some one may wish to raise a question as to the literal accuracy of that statement, but on the whole it gives a correct picture of the research development and the research interest in this country at the time of which Dr. Remsen spoke. I would not cast any dimness over the luster of names like those of Franklin and Henry and men of that type whom this country had produced before that time, but, after all, these were individual geniuses. They were not the leaders of research groups and yet they accomplished wonderful things. But we have passed from the stage of the world's development in which we could depend solely on individual geniuses for advancing the wheels of progress. If these wheels are to move forward rapidly we must do in peace exactly as we have done in this war, namely, call together groups of research men in our industries, in our educational institutions—I do not care where—give them the problems, and let them push them forward. In this way these wheels will move forward ten times as rapidly, yes, a hundred times as rapidly, as will be the case if we depend upon the exceptional inventive genius of the country, however valuable that may have been to us in the past.

These are the two lessons which I would draw from the war.

Let me next ask you to consider how we may profit by them, and perhaps the path which we ought to take may be made more clear if we analyze ourselves somewhat objectively, and try to look, for the time being at least, not at our virtues but at our weaknesses, however disagreeable such introspection may be. Let me begin by asserting that in spite of our boasted inventiveness, we are as yet, from the scientific standpoint, a second rate people, whether we look at our accomplishments in war or in peace. I am certainly well within the bounds of the truth when I say that the contributions of the United States to this war in research and development lines have been less, far less, than is proportionate to our resources and our population. I think Dr. Whitney will agree that that is not an overstatement, and he knows fairly well what we have been doing, for he has been associated with much of it. England and France have both done far, far more effective war research than we have done. True, they have had more time, but they have not had either the men or the facilities which we have had. With almost every available man in France at the front, with most of her manufacturing facilities in the hands of the enemy, is it not an extraordinary tribute to the French scientific mind that the two most novel and most promising weapons which we had at the close of the war for combating the submarine came out of France. We have done some creditable work in this country on submarine detection, but in comparison with the number of men engaged on the work and the facilities at command, both France and England did vastly more than we. The other day an officer who had

had exceptional opportunity to appraise our contribution to the war was in my office and I was bewailing the fact that a particular device which we had been developing had not been completed in time to be of actual use on the front. But said I—"I do not know that I ought to complain, because I presume that it is in the same boat with approximately 96 per cent. of all American effort." Said he—"I think you overestimate the amount that got across."

Now that is an impression which one does not gain from the newspapers. I would not unduly belittle the American effort, but I would like to do something to combat the lethal poison which is being introduced into the American system by the vain-glorious boasting in which our newspapers and politicians are indulging. We did indeed cast the deciding vote. It is true that save for us the Kaiser might not perhaps have been so easily persuaded to call out "Don't shoot Boss, I'll come down," but let us at least remember that 20 Frenchmen gave up their lives for every American boy who died, that more than ten Englishmen "died for France" to one American. These figures show the real pressure of our contribution, and let us not forget them.

Again I state that whether one measures our position by our contributions to the war, or, what is more to the point to-night, by our accomplishments in science in times of peace, I think that every American who is perfectly honest with himself will admit that we have no claim as yet to being anything more than a second rate people. Second rate whether you take as the criterion of first rate-ness either the number of men of outstanding ability and reputation which we possess, or the total output of the country in proportion to our population, or the general appreciation and respect in which science is held in the community at large. Look at the first criterion; who would for a moment claim that we can show a list of names comparable with these? Faraday, Maxwell, Kelvin, Stokes, Rayleigh, Thomson, Larmor, Schuster, Rutherford, Jeans. Or going to the field of chemistry, in which we have done as well perhaps as in any science, who would undertake to present such a list of illustrious American names as that which Dr. Whitney has just presented for Germany?

When we look at the second criterion and reflect that we have 100,000,000 population against England's 45,000,000 and Holland's 4,000,000, no man who is at all familiar with our relative outputs would claim for a moment our equality with either of them. It will be generally admitted that as measured by this criterion we cannot think of claiming to be in the first rank. Such a statement may not be palatable to our pride, but it is true in spite of the fact that you see so much in our newspapers and elsewhere which might lead you to think otherwise. Here is something fairly typical. There is a tablet erected in Dayton, Ohio, which gives a list of twenty of the world's greatest inventions, and attributes thirteen of them to the United States, just one to England, the locomotive, one to Scotland, the steam engine, two to Germany, the Fahrenheit thermometer and the X-Ray, only one to France, the daguerreotype, but two to Italy, the pendulum clock and wireless, and two to Dayton, the cash register and the airplane. Such claims need no comment. Unfortunately they are more or less typical. I wish they were not typical, and perhaps we may do something to make them less so.

As for the third criterion, the public appreciation of science in any country may be gaged first by the general respect and honor in which the calling of the scientist is held, and second by the character of the general scientific journals which that country supports. Now it is a matter of common knowledge that in all European countries the scientist is held in greater respect than in this country, and as for journals, let me just mention the names of three English publications in different fields and see what kind of comparisons they bring up in your minds. The *Philosophical Magazine, The London Electrician and Nature.* Have we journals which are of a full equality with any of these?

I am presenting these unpleasant truths not because I think our situation is hopeless, but just exactly because I think these conditions are remediable, and because I think that you and I can assist in changing them. I say them because I believe in all my heart in the effectiveness in the long run of our American democracy. I say them because I believe with all my heart that when you once get the public opinion of the United States aroused it will set about changing bad conditions and will do it rapidly. You and I have already seen it doing certain things which a few years ago nobody believed a democracy could do. We have seen it handling the liquor problem for example in a better way, as it seems to me, than that in which it is being handled in any country in the world, and this is the last problem which you would expect that a democracy could handle. I am pointing out conditions which ought not to exist because I believe at the bottom of my heart that our method, the democratic method, is capable of rectifying bad conditions more rapidly on the whole than is any other method, if only we can once get the real situation and the real need before the consciousness of the American people.

What we need then if we would become a first-rate rather than a second-rate people from the standpoint of our scientific development is, first, the development of more highly trained and more able scientific personnel in this country than we now have. Second, we need the development of a new public opinion on scientific matters. We need appreciation of what science is and what it can do. We do not need any more so-called "Popular Science Monthlys," but we do need a better educated, scientific intelligence than we have at the present time.

How are we going to get these things? Dr. Whitney has said that we have enough organizations. Maybe we have, at any rate we have not enough of concerted action among existing organizations. This is precisely what the National Research Council is endeavoring now to bring about, and if the Research Council cannot succeed in this work which it has been organized to do, then some other organization must arise which will do it. In other words, we must in some way get the scientific men of the country pulling together. I think this can be done and that it is the biggest, most worth while enterprise which any of us have ever had an opportunity to take part in. The problem is to persuade the engineering societies, and the physical societies, and the biological societies, and the medical fraternities, no longer to work all by themselves, but instead to pull all together in one common effort, first to educate the American public to a larger appreciation of the possibilities of science, second, to make the present American scientist more effective than he now is, and third and most important, to find means by which we may develop more and better scientific men.

The education of the public is of course a slow process, but it can be accomplished by directed effort and concerted action such as the Council is now attempting to bring about. The stimulation to greater effectiveness of existing scientific men also cannot fail to result from all of these new activities. But the producing of a continuous new crop of able scientific men is a problem to which I should like to give a little more detailed consideration. *The result aimed at can be attained only through the agency of the American Universities.* These however cannot possibly make the kind of men Dr. Whitney wants out of poor material. Something can be done to a man by education alone, and yet there is no one who knows as well as the college professor how little merely going through the routine of getting an education amounts to. Our primary need is to find a way of getting a better *selection* of men than we now have, that is, a way of diverting the brains that now exist in the country into research channels. How is that to be done? The answer is to my mind very direct though perhaps not so simple, since it involves funds. It is to deliberately and consciously set about the task of creating more opportunities for research careers than we now have, and giving greater dignity to such careers. In connection with American Universities, I see ways in which this can be done, first by persuading philanthropic men and institutions to endow a considerable number of scholarships similar to the so-called 1851 Exhibition Scholarship of the British Universities, second, by the creation through private means of many more partial research professorships than we now have. I say partial research professorships because I think that the research man needs the broadening influence of a small amount of advanced teaching even for the greatest effectiveness in his research work. The creation of more pure research institutes like the Rockefeller Institute for Medical Research or the various research laboratories of the Carnegie Institution of Washington, may also be encouraged though I am myself a strong believer in building up research enterprises in connection with educational institutions so that the atmosphere of research may be inhaled by the young blood which is continually passing through these institutions. Create a real research atmosphere in a score of American Universities and a new generation of research men will quickly be created which will place America in the front rank in the fields of science. I should regard it as a calamity if the research funds available in America went exclusively or even primarily into the founding of central research laboratories. This country already has scores of magnificently equipped laboratories which produce one tenth as much as European laboratories one fourth as well equipped. We do not want more brick and mortar, more physical facilities, *we want more and better men.* As Dr. Whitney has so well said, *we want a man-producing program, not merely a man-consuming program. The man-producing agencies are primarily the universities.*

Funds which are donated to colleges or universities for research work should be devoted by the terms of gift to research purposes and made altogether undivertable into the purely educational budget. One of the chief aims of the Research Council has been from the start to stimulate this sort of giving. The finest outgrowth of our democratic, individualistic civilization has been the development of a great multitude of public spirited men who devote their means to public ends and it will probably continue to be true in the future as it has in the past that our biggest and most important public enterprises will be inaugurated not by public but by private initiative. I see no reason therefore for not using the method which has been most successful in the past in many fields and which is most characteristic of the genius of Anglo-Saxon institutions —viz., the method of the private foundation of research scholarships and professorships.

But I agree with Dr Whitney that in a country like this, there is no reason why the federal government, as well as private persons and institutions, should not promote the development of research. The whole world has been aroused by the war to the importance of this sort of activity. England has appropriated governmental funds for scientific and industrial research, and Canada has done the same, and so has Australia, and Italy, and I think Japan also. The United States is almost the only large country which has not thus far done anything in this direction. But there is now before Congress a bill—the so-called Smith-Howard Bill—which in its original form had most unfortunate features, but which is apparently now in process of being changed into a really valuable measure. I am permitted to say that the authors and sponsors of this bill recently asked the National Research Council to appoint a committee to give it thorough study and make possible recommendations. A fairly representative committee of some ten men, including Dr. Whitney and the Director of the Bureau of Standards and the author of the bill, Mr. P. V. Stephens, has met and reported unanimously upon certain types of modifications in that bill. The most essential of these changes consist in the setting up of the principle, different from that incorporated in the original bill, that the funds appropriated by the federal government to each state shall be allotted to institutions within the state *by a board composed of at least five of the most eminent available scientists and engineers of the state, upon the basis of programs of research submitted annually by these institutions to this scientific board.* It was the judgment of the Committee, and also of the executive board of the National Research Council which unanimously approved this report, that if the allottment of research funds within each state can be placed in the hands of men who know what research is, and if the institutions of a state can be stimulated to a rivalry in the development of research programs there will develop in existing laboratories of which we have already an abundance, an atmosphere of research which is now wanting. *It is only through the creation of such atmospheres that research men can be developed.* This sort of a plan seems to make it possible to plant men all over the country who will be creating such atmospheres. It provides for the stimulation of research throughout the country in both industrial and pure science lines much better than any scheme for federal aid which I have seen discussed.

There is very grave danger, however, that the desire of public officials, legislators and private individuals to leave brick and mortar *monuments* to themselves will divert funds which ought to go to the development of men to the creation of more elaborately equipped laboratories without men, such as already constitute a reflection upon American judgment and American brains. If the Smith-Howard Bill modified in some such vital way as I have indicated does not pass then I have little doubt that the National Research Council will in time itself establish within the states research boards which may then obtain state funds to be used in some such way. California has already such a State Board, working in cooperation with the National Research Council and this board has within the past year received and appropriated more than $50,000 for research within California.

I have said enough to suggest many ways in which research in industrial and pure science lines is likely to be stimulated in the near future. Let me close with a brief word on the organization of the National Research Council which is occupying itself with these large problems of the promotion of research in the United States by a method which is compatible with the genius of our Anglo-Saxon institutions, namely, by the democratic method of the voluntary associations of research men, societies and institutions into a great body which can effectively create public sentiment and stimulate in a large way research enterprises.

The Council is, in a word, simply a federation of the research organizations and activities of the United States for the purpose of obtaining concerted action, mapping out new lines of progress, establishing the cross connections between different fields, acting as a central bureau of information and coordination, and promoting research in general. It has for the past year had a so-called war organization, because one of the most important problems which confronted our country was to get the scientific and engineering brains of the country effectively at work upon its war problems. The National Research Council has been one of the influential and effective agencies which has been doing this and it has thrown all its energies into the task. Now the time has come when it must reorganize so as to adapt itself as completely as possible to the problems of peace, which are more important even than those of war. The present tentative plan is to have the Council a thoroughly representative body in that at least a majority of its membership is essentially elected by the constitutent societies for a period of say three years. Each of the seven or eight divisions, namely, the division of Engineering, the division of Physics, Geophysics, Astronomy, Mathematics, the division of Chemistry and Chemical Technology, the division of Medicine and related sciences, the division of Biology, the division of Geology and related sciences, etc., to consist of 25 to 35 men presided over by a chairman elected by the group. The most essential feature of the proposed organization is the Executive Board which consists of these eight chairmen, eight vice-chairman and say ten members at large. The Board would meet at least once a month in Washington. The eight chairmen are to give up, in general, all other activities for their term of office, of probably a year's duration, to be paid suitable salaries, say $6,000 a year, are to be resident in Washington, and to devote their whole time while in office to the promotion of research in the United States. With eight of the best and most influential scientific men in the country devoting themselves each year to the large problems of surveying, stimulating, and coordinating the research activities of the country, the United States can I believe very soon be placed in the front of the nations of the earth in scientific discovery and its application to the arts and industries.

The National Research Council bespeaks your active coöperation and support in this great new democratic and characteristically American program.

DISCUSSION ON "AMERICAN ENGINEERING RESEARCH" (WHITNEY) AND "RESEARCH IN AMERICA AFTER THE WAR" (MILLIKAN), PHILADELPHIA, PA., DECEMBER 13, 1918.

Chas. E. Skinner: I made a plea from this very platform, not a great while ago, for the training of research men, and if iteration and re-iteration of the necessity of the case will help any, I am glad to lend my voice to a further plea along the lines that have been so ably indicated here this evening by Dr. Whitney and Col. Millikan.

I made a plea for the training of research men in the universities. I did not have at that time a well-organized plan in my mind for carrying out such training but plans are now well under way by the National Research council.

I have been asked a number of times recently to lend my support to the Smith-Howard bill. I have not been entirely satisfied with the bill, as presented in the early drafts because I felt that there would be a very considerable waste of money in arbitrarily selecting certain places and giving those places a fixed amount of money without any program to work on. I am more than pleased to hear Col. Millikan outline certain modifications which have been proposed to the bill which completely remove any objections along that line which I may have had in the past.

I have felt throughout the war that our program of research after the war was possibly of more importance to us than our program during the war, if we are to hold our place among the nations of the world. To carry out that program we need, above all things, a knowledge on the part of our people as to what properly directed research can accomplish. We need trained research men in all phases of our work, and particularly in connection with engineering research. If the desired results can be brought about by the plans proposed here this evening and I believe they can, these plans should have our heartiest support. In closing I wish to express my admiration at the way this subject has been presented, and to say that I feel we are, well on the road to establishing a plan of action for caring for this very important matter.

E. E. F. Creighton: There is nothing so valuable to our country at the present time as the consummation of the plans involved in the presentations of our eminent speakers this evening. Both Dr. Whitney and Col. Millikan have been able by their own efforts to show the enormous value and wealth to the country that accrue from research. So much can be obtained in this line of endeavor that the proposed appropriations of Congress seem far too small. Dr. Whitney has shown in more than a few instances that pure research will lead to the most valuable results for the whole of civilization. The mention of even one of them argues completely the justification of research: Langmuir's gas-filled lamp and the resulting saving in coal, or Coolidge's X-Ray tube and its benefits to the medical profession, to say nothing of its uses in other fields. In some cases, I understand, years of pure research were carried on before a utilitarian result was reached.

The subject on which I wish to make my appeal I surmise has been characterized in Col. Millikan's great contribution to the electron theory in the mental processes preceding the counting of these tiny particles one by one. It appears to me like an invention to conceive of the mechanism of a floating particle of oil which, by the natural movements of electrons, would finally be struck and electrified. The presence of the electron was made known by observing the particle of floating oil between two plates which could be electrically changed in direction and intensity as desired. How much research preceded this invention we do not know, but it would seem that the valuable part of the research succeeded it. At any rate, the assumption that the invention could precede all of the research, the invention being founded on well-known principles of attraction and repulsion in the electrostatic field, illustrates the point I have in mind.

My interest for many years was in the development of invention as an activity distinct from research, although not separated from it. One may approach a problem from either the purely research standpoint or from an inventive standpoint. Research is seldom useless in that, if carried on properly, it gives more information on the laws of nature or on the characteristic of some material, but it is liable to be mostly lost unless it leads to some immediate useful end. On the other hand, invention is, in general, futile unless accompanied by a considerable amount of research. This would easily account for the low percentage of suggestions by inventors which proved valuable to the Naval Consulting Board. If one approaches the problem with an inclination toward the process of mind of invention before any great amount of research, one invention in ten would be a high percentage of success, but since the nine may be discarded with brief analysis, rapid advancement is possible. On one hand, in the extreme, there may be a blind research for whatever results may come from the investigation. On the other hand, research may be carried along specific lines to discover materials or characteristics of materials or laws which will fulfil the conditions of the invention. The difference between the two is the efficiency of directed force as compared to more or less chaotic force.

While America's list of workers in the pure research is comparatively small and our inventors have lost much by not harnessing research to their inventions, still we have much to be proud of in the line of invention. It is necessary only to mention the name of the greatest of all our inventors—Mr. Edison. Mr. Edison has done much research but as I understand his work, his research was usually preceded by an actual invention. It is customary to think and speak of Mr. Edison as a genius, and the methods used by him unavailable to the rest of us. While it may be a long time before society is fortunate enough to find again the combination of ingenuity, interest, perseverance, enthusiasm, and assiduous application that has made it possible for Mr. Edison to so enrich the world, I can state by actual experience there is no reason why the methods cannot be taught to any student with the taste and the desire to become an inventor, with the results as good, on the average, I think, as obtained in the other professions. In brief, I believe inventing is just as much a profession as medicine, or law, or engineering. Invention results exactly and naturally from a particular view point or cultivated attitude of mind. There is much more which should be said of details.

The remarks of the speakers of the evening bring forth the plea for the desirability of drawing into the proposed research laboratories that body of chaotic American inventors who need the stabilizing effect of the exact knowledge of research; furthermore, to cultivate among the research students the utilitarian spirit which leads to invention.

P. G. Agnew: Coming from a National Laboratory, I can only say I hope some mechanism can be evolved for closer cooperation between the research agencies and these National Laboratories.

C. H. Sharp: I want to emphasize the point which has been brought out by the two eminent speakers of the evening, that research is more a matter of men than it is of material means. A great research worker can get results with inferior means that an inferior research worker cannot get with the best means in the world at his disposal. As Dr. Whitney said, we have in this country laboratories more splendid than can be found elsewhere, but we are not turning out men to get the results out of these laboratories.

An effort towards education of research men and towards stimulation of research work is being made in the Smith-Howard bill. The amendments which Dr. Millikan tells us have been proposed by the National Research Council to this bill have removed some of the very serious objections to that bill. We should bear in mind that while this bill calls for an appropriation of $15,000 per year to each state and territory, that figure obtains only for one year. It is increased year by year until in the fourth year, it becomes $30,000 per state and territory, and remains at that figure.

The proposition is, then, to subsidize each of the states and territories in the amount of $30,000 per year. This amounts to something over one million and a half dollars for the country.

Now, that is really not an excessive amount for the Federal Government to appropriate to this purpose, if there is a reasonable prospect that it will be wisely and economically expended. The British Government has recently appropriated One Million Pounds, Sterling, for the stimulation of research during the year. However, the one million pounds includes the support of the National Physical Laboratory which the British Government has recently taken over, and the proposition in England is in other respects not parallel with the one in this country. It is proposed over there to subsidize individual research workers, men who show competency and talent for research, and thus give them an adequate training and a chance to develop their abilities. It is also proposed to subsidize Research Associations organized by various industries and chartered by the Board of Trade for the purpose of carrying out in cooperation, researches covering the great problems which are common to the industry as a whole.

Moreover the British Government is appropriating this money on top of a huge war debt and in the face of enormous taxes, such as we do not know and probably never shall know, in this country. A million and a half dollars therefore while by no means excessive is enough money to make it worth our while to see that it is expended in the way which will yield the best results.

A feature of this bill is that it denominates the Bureau of Standards as the co-ordinating agency for the direction of the research work in these various institutions which are so subsidized. This throws upon the Bureau of Standards an enormous responsibility, a responsibility which was never anticipated when the Bureau of Standards was organized, and which comes to it on top of the other greatly increased labors which have accrued to it as a result of the development during the years which have elapsed since the time of its organization.

Of course, this proposal at once suggests the thought that the Bureau of Standards thereby becomes the all-powerful agency in directing research in this country. This being the case, it is necessary that we should inquire whether this agency has directed its researches along such lines, and will probably direct the research contemplated on the Smith-Howard bill along such lines, as to yield the most useful results for the public which it is intended to benefit. This is a very serious question.

Now, we are all very strong supporters of the Bureau of Standards. We believe that it is an institution of inestimable value in this country as a scientific agency. As such we want to see it prosper and grow great, and we rejoice in the added prosperity which has come to it in recent years. I think, however, some of us would feel a little better about it, if we had a little more assurance that the lines of progress of the Bureau, that is to say, the lines which it attacks in its endeavors, the research problems which it takes up, are problems which are such, as it, of all agencies, is the most competent to tackle, problems which it alone of all agencies is able to solve, and that it is never wasting its time or its endeavor in assailing problems which can be assailed, perhaps, almost as well by other agencies; that it is maintaining its original status as a scientific institution, and is not diverting its energies into other channels which, however important they may be in themselves, cannot be considered to lie in the domain of the physical sciences to which, as we conceive it, the activities of a Bureau of Standards should be confined. Certainly, if this added responsibility is to be put upon the Bureau of Standards, we ought to have some assurance along these lines, such as we have not had in the past.

To state the matter in general terms, when the future of this country as far as the guidance of industrial research is concerned is entrusted, to any agency, we who represent engineering and industry must make sure that safeguards are established such that there is no chance for this agency to fail to perform its functions to the most advantageous result for the public good.

E. P. Hyde: I was very delighted to hear Col. Millikan speak of the closer cooperation between the physical societies and the engineering societies. That has been a hobby of mine. It was not always a hobby. I grew up in an institution, the Johns Hopkins University, to which Col. Millikan referred in his address, in which pure science was placed on a very sacred pedestal, and the first two or three years of my connection with the Bureau of Standards were a veritable hell, because I was called upon to do something that was of some use to somebody, and I was prompted to go off and do something for the sake of pure science. I felt for a long while that the physical society was a group of moss-backs, who gathered periodically in some cobwebby room, in some University to discuss academic scientific subjects, and I think in a small way I was partly responsible for having the Physical Society break away from that custom, and meet once at an industrial institution, and that proved so satisfactory that they planned to do it each year, and I am only too sorry that some situation prevented the program which had been planned for a joint meeting of the American Physical Society with the American Institute of Electrical Engineers last October.

The second thing to which I would like to refer is getting the proper men to develop into research men. I do not think we would expect to draw strong men into a line of work which offers neither remuneration nor appreciation. This may not be the place to say this, but I am not at present connected with an educational institution, and therefore I am free to speak.

I think a thing we can all do,—and I am talking to men who are not in the universities now—is to help place scientific research in our institutions on a higher plane, and if we cannot induce the strongest men to go into research work, in the first place, as leaders, as guides, as pioneers, to inspire young men to follow in their footsteps, we cannot expect to get strong men to engage in research undertakings.

The process is cumulative. If we have no appreciation and no remuneration that is commensurate with the work that is done, we will get comparatively weak men in our institutions, and the prevalence of these weak men in the institutions will tend to degrade the men who enter upon research work as a life work. We must increase the appreciation, and I think we must increase the income, incident to this work. As I say, it is not our place to do that, it is not our responsibility, possibly, but I have felt some of the responsibility, and I think, if more of us felt some of that responsibility as individuals, if we saw to it that there was more remuneration for men who do that work, and if we saw to it that there was great appreciation of what they did, we would only be performing our duty.

Can you imagine this situation occurring in this country. It happened to me in Karlsruhe. I had made an engagement with Herr Gehimrat Professor Doctor Bunte, Technische Hochschule, at Karlsruhe. I had made no provision for hotel accommodations, and when I reached the town in the evening I found there was some fete in progress and the hotel was filled. I secured an attic room that must have been used for the lowest grade of menial. It had absolutely no conveniences. I had to call over the banister for a maid if I wanted a glass of water. The next morning after breakfast I went to the laboratory of the Hochschule. I was met by a rather austere gentleman who wanted to know my business. I told him that Doctor Bunte had made an engagement to see me, and as I was at the hotel I would see him there if that was agreeable to him. I went back to the hotel and I said to the manager, Herr Professor Doctor Bunte is going to meet me here, is there some place where I can receive him, and he showed me into a private sitting room with a great deal of deference, and when in a few moments Doctor Bunte came, the proprietor at the hotel was at the entrance, to receive the doctor, and was making obeisance to the doctor all the way coming in, and the doctor was poking him aside with his cane, and after Doctor Bunte left, the proprietor of the hotel found that he had a very good room on the *premiere etage.* for me. Can you imagine anything like that happening in any of our small towns to which Dr. Whitney referred to, such as Columbus or Princeton, or anywhere else. It indicates a great deal, I think, and we must in this country establish that appreciation, and with that must come reasonable remuneration for our men who are training the young men, a reasonable remuneration for those men who are to inspire the young men in research work, if we are to hope to accomplish what has been accomplished in Germany, and, of course, it is our hope to go far beyond that.

L. T. Robinson: The Smith-Howard Bill which has been referred to seems to me to be a good move in the right direction. I say this because I did not know until tonight that it had been modified or that a somewhat similar bill had been prepared by others. It is the general idea that appeals to me as being good.

Such an arrangement will attract the right kind of professors and give them opportunities to carry on much research work that could not be supported by present methods of financing. The arrangement will also be of benefit to students.

The only fear seems to be that the bill contemplates giving the Bureau of Standards too much responsibility.

F. A. Millikan: That feature was eliminated at the request of Dr. Stratton.

L. T. Robinson: I am rather sorry for that because I have confidence in the Bureau. I think all of us should have confidence in it. There must be some directing and coordinating force somewhere.

However, I do appreciate the fact that when you make the Bureau responsible for coordinating all this proposed work, that you are placing a tremendous burden of responsibility on the organization.

It seems to me that if the Bureau is to be the agency to do this thing—and I can think of no better one—we ought to make an effort to help them.

The arrangement that suggests itself to my mind is that an advisory board consisting of representatives from the four National Engineering Societies, The American Physical Society (which has tonight become an engineering society), The American Chemical Society (which is also an engineering society as defined tonight), together with representatives of the industries and business—covering the whole field that is and should be served by the Bureau of Standards—be created in some way. Probably a good way would be for the President or Secretary of Commerce to appoint representatives nominated by the various societies.

I am assuming that some such arrangement would be agreeable to all parties concerned. Such a board might be a branch of the National Research Council or perhaps it might act in the same capacity to the National Research Council and to the Bureau. The details of the arrangement have not occurred to me and are, at this time, relatively unimportant. I can at this time only express my belief that a non-partisan, and non-political advisory board created from engineering and scientific societies from the industries, public utilities, etc., could be formed that would be productive of great benefit to the Bureau and to the country as a whole.

We have seen recently in this country good results, accomplished by boards appointed almost haphazard. If you can get results with boards appointed in that way, how much better results could you get with a board created by nomination from the national engineering, technical and business associations, to help this great movement along and to support and advise the Bureau whose work we all regard so highly.

M. W. Franklin: There are three points in the addresses of Dr. Whitney and Prof. Millikan which have interested me particularly. In the first place, I refer to Dr. Whitney's disclaimer of any desire to form a board, a new committee, to take hold of this matter, and the assumption is made that there are enough committees already in existence.

In the second place, I want to refer to the premise of Prof. Millikan that the research of the present day is becoming pragmatic—practical—that is, it is having a utilitarian purpose, and that this is dominating the present-day research, and to a good end, and the other remark of Prof. Millikan, that he prefers the democratic method as against the autocratic method, and thinks it will produce better results.

I happen to be in a position to say something very cogent on the subject of research in this country at this time, and because of the interest that these three points have had for me, I am saying it. There is an engineering society in this country, if not the most obscure, perhaps one of the most obscure, a society with about 750 members, a society which until within the last five years, has been scarcely an engineering society in anything but name a society which had its inception in an aggregation of salesmen of a utilitarian article, and which has been struggling along but which has done something in connection with this matter of research recently, that merits your attention and your commendation, because I think it is a model for all the greater societies on which they can base their endeavors.

I refer to the American Society of Heating and Ventilating Engineers. I happen to be a member of that society. I say that advisedly—there was a time in my ill-starred past when I had some vicarious connection with the ventilating business—I do not boast about it—and I got into the society at that time and in spite of several resignations presented to the society, I am still a member of it. In fact, I have just been nominated as one of the vice-presidents, and in the course of events, in the irony of fate, I will be the president of the society if I do not die or get fired. I think it was due to another activity, equally unfortunate, in which I was interested, the institution of the research movement.

There was a group of boiler salesmen in the first place, who made up that society and who did not know anything about research matters. They could measure a house and guess how much radiation surface was needed. They carried a two-foot rule, and they carried some pipe and some radiator units, and one thing and the other, and they were great engineers, or thought they were.

About five years ago I got very busy in connection with the subject of research, I do not know why, and they took it seriously and got started on the research proposition.

Recently, since the war, on account of the different bureaus constructing battleships and barracks, and other things, in which human beings have to live for a certain length of time, they discovered that they needed something like precise knowledge on the subject of ventilation. They found that there was an enormous amount of data on the subject of ventilation, and that most of it was incorrect. That was discovered by accident when they began to ask different experts how to construct the living spaces in battleships and barracks, and they found also that there were plenty of opinions, no two of which coincided in any respect whatever.

So then the American Society of Heating and Ventilating Engineers found itself in a very unenviable position—you might say in an equivocal position, they were looked up to and bowed down to, but in the secret recesses of their own hearts, they realized that it was camouflage in the most fully developed sense of the term.

They got together this research data, which had died a natural death sometime before, and got busy on the subject of getting something that was reliable in the way of research, and they have organized a series of sections, one of which I have been called upon to organize myself, and they have appointed a committee in each one of the twenty-nine sections or local chapters, as they call them. I will repeat, that the whole membership in the aggregate amounts to fewer than 800 persons.

They had organized in each of the 29 chapters a Local Research Committee, and each Local Research Committee went out to get the money and to get something done in the way of research, that will put on the records for all time reliable information about ventilating subjects, which are important subjects, and in connection with which this research study is especially valuable.

I was much interested in what Mr. Louis Robinson just said and I cannot help but admire his practical good sense.

These committees to which I have referred got together, in the first place, on the pragmatic basis, the practical basis, and they said:—"Let us do something that will be of value to somebody, and let us do it in a practical way. We will get together and make a list of all the manufacturers of apparatus in the United States, who can even remotely be interested in processes of manufacturing ventilating equipment, and in precise knowledge on the subject of heating and ventilating. We will get together, all the engineers in the United States, who remotely, at least, can be interested in this subject, always financially interested, don't forget that. That is pragmatic, that is higher philosophy according to Ole Yonson.

Then they said:—"Let us get together all the contracting engineers who can be even remotely interested in operations in which heating and ventilating play a part. We will make a list and divide them up according to locality. We will 'sic' the members of these societies on these men, and make them sign something which looks like a liberty bond voucher, and make them sign it to be good for a period of five years, to support this research bureau."

We made arrangements at the Bureau of Mines at Pittsburg, which has put at the disposal of this little society its beautiful research laboratory and facilities free of charge, and they are delighted to have the members of the society come in and do something. The members of this society are live wires. They get on the job. We call all the members in, and hold a meeting, and 80 per cent of our membership comes to that meeting. I will admit that the methods of getting the members to the meetings are sometimes questionable. On one occasion they promised the members that I was going to read a paper on "The Control of Fumes and Noxious Gases,"—a subject I know little about—and the members came, and we sprung this on them. I really intended to speak on the subject but when we got on Research there was no time for the paper. We took our apportionment cards with a list of some 400 firms on them, and we read off the names and made the members volunteer to take some of these cards and go to the firms and "get it," the idea being to: "get it," which is again pragmatic, the means being subsidiary in every case.

We scorned the easier method of getting these names and going to two or three of the larger firms and having them subsidize this project. We could have done that. I could have picked out five firms in the United States that would have given us $35,000 or $40,000 a year for five years to have the work done.

We preferred to choose the method of democratizing the scheme, and get everybody into it. Almost every member contributed ten dollars a year for five years, and some contributed more, and everybody did his bit, and even those who are remotely interested, from monetary points of view, if they happen to have knowledge of the subject, in a general way, they put something in, and we are going to appoint two research men, and we will be consumers of research men, and incidentally producers of research men, and we will put these men at the Bureau of Mines at Pittsburgh and they will do the research. Every one of these manufacturers will have, without fear or favor, the right in a democratic way to say:—"This is a great problem, something of great importance, the transmission of heat through buildings and material, the transmission of heat through covered pipes, the efficiencies of certain types of boilers, certain types of grates." Without fear or favor, they will present their views and preferences for research subjects, and these will be carried out to a hard-boiled finish, and the results will be printed by the American Society of Heating and Ventilating Engineers, and when these things are published, in the course of five years, this research bureau will be a big thing and a live thing, and the Government will be ashamed not to be behind it, and will subsidize it, and pay for all the great work it is going to do, and the American Society of Heating and Ventilating Engineers will get the credit for this work. "Out of the mouths of babes and sucklings,"—not meaning me—but the great American Society of Heating and Ventilating Engineers, you can get something.

C. A. Adams: Dr. Franklin's emphasis on the pragmatic or dollar and cents side of this subject recalls what I have said on many similar occasions in the past, namely that whereas some manufacturers are inclined to look askance at cooperative research since they cannot control the results but must share them with the other cooperators, the gains certainly far outweigh any loss due to sharing the results. After the results have been obtained there still remains ample opportunity for competition of intelligence and resourcefulness in their application. The best minds will win out in any case. Moreover cooperative industrial research means not only a great saving to the individual manufacturer but also the possibility of getting together the foremost experts in each field and a vastly greater probability of successful outcome.

John B. Taylor: Since the war came on, there has been much talk of research, much more than before, and through the newspapers and popular magazines, the public have at least learned the word, even if they do not know what research is. They are asking now, "What is Research." and they are receiving many replies.

To my mind research is not a college degree, or a laboratory, nor an equipment. It is an attitude of mind. You can find that attitude of mind, of course, in the colleges and in the laboratories, where there is equipment, and you can also find it on the farm, and in other places.

G. Francis Gray (communicated after adjournment): The notable addresses to which we have listened will leave a profound impression on all who are interested in research either in pure science or in the industries, and the closeness of these two to each other has been strikingly indicated by the fact that in these addresses the pure-science representative has advocated the union of science and engineering while the director of a great industrial laboratory has plead the cause of pure science.

In the discussion of the Smith-Howard bill it seems to me there is danger of confusion as to the exact purpose of this particular legislation. The four things generally spoken of are

a. Benefit to the industries from the results of research.

b. The training of advanced college students to be research experts.

c. Stimulating popular appreciation of the value of research work.

d. Developing college professors who are primarily leaders in science rather than college executives or professional educators.

Obviously a scheme that is best for one of these is likely to be somewhat unsuitable for the others, and unless the issue is clearly defined, discussion is likely to be at cross purposes. It is my opinion that the practicable plan for the present is to concentrate on a program designed to definitely and promptly benefit the industries of the several States, for the reason that this can readily be done, and well done, with people already available. Moreover the process will at the same time serve to train men by experience in productive work, and will provide the material for the most effective kind of publicity, since every business man definitely helped by the results of research work will become its strong advocate. As for developing the real leaders in science, that will be possible only when our colleges awake to their responsibility and when the positions they have to offer carry adequate salaries, freedom from academic duties and the social and business prestige they deserve. This bill alone can not hope to accomplish such a revolution in our present conditions. It seems much better therefor not to expect too much of it and not to hamper it by too ambitious plans, but to recognize its limitations, while appreciating its very real possibilities, and by concentration on a small and definite part of the big program really make a beginning at once along lines in which success is practically certain.

STANDARDIZATION RULES

OF THE

AMERICAN INSTITUTE OF ELECTRICAL ENGINEERS

Approved by the Board of Directors, November 8, 1918

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OTHER APPROVED STANDARDIZATION RULES

The following resolution, adopted by the Standards Committee, was approved by the Board of Directors on April 14, 1916:

" The Standards Committee, with the approval of the Board of Directors, recommends the use of the following rules and standards as adopted by other societies. These have been formally presented to the Standards Committee by the societies concerned and are found not to be incompatible with the Standardization Rules."

Standardization of Service Requirements for Motors, as printed in the 1915 report of the National Electric Light Association.

Standardization of Sizes, Voltages and Taps for Transformers, as printed in the 1916 report of the Electrical Apparatus Committee of the National Electrical Light Association.

Standard Specifications for Magnetic Tests of Iron and Steel, of the American Society for Testing Materials.

Report of the Joint Rubber Insulation Committee, published in the April, 1917, PROCEEDINGS of the American Institute of Electrical Engineers.

Accuracy Specifications in Sections IV and V of the **Joint Meter Code** of the Association of Edison Illuminating Companies and of the National Electric Light Association.

Accuracy Specifications in Section II of Circular 56 of the Bureau of Standards entitled **Standards for Electric Service.**

Report of the **Boiler Code Committee** of the American Society of Mechanical Engineers.

Suggested Safety Rules for Installing and Using Electrical Equipment in Bituminous Coal Mines, issued as Technical Paper 138 by the Bureau of Mines.

PREFACE

In framing these rules, the chief purpose has been to define the terms and conditions which characterize the rating and behavior of electrical apparatus, with special reference to the conditions of acceptance tests.

It has not been the purpose of the rules to standardize the dimensions or details of construction of any apparatus, lest the progress of design and production should be hampered.

NOTE.

The Standards Committee takes this occasion to draw the attention of the membership to the value of suggestions based upon experience gained in the application of the Rules to general practise.

Any suggestions looking toward improvement in the Rules should be communicated to the Secretary of the Institute, for the guidance of the Standards Committee in the preparation of future editions.

COOPERATING SOCIETIES

The following societies directly and through the committees named, have given helpful cooperation in the preparation of these Rules:

American Society for Testing Materials,
Committee B-1.

Association of Edison Illuminating Companies,
Committee on Meters.

Illuminating Engineering Society,
Committee on Nomenclature and Standards.

Electric Power Club,
Committee on Engineering Recommendations; Standardization Committee.

National Electric Light Association
Committee on Meters.
Committee on Apparatus.

Association of Railway Electrical Engineers
Committee on Wires and Cables.

American Electric Railway Engineering Association,
Committees on Equipment and Distribution.

Institute of Radio Engineers,
Committee on Standardization.

Society of Automobile Engineers,
Standards Committee

Railway Signal Association.

SCOPE OF THE 1918 REVISION

This edition of the Standardization Rules does not differ radically from the 1916 edition, although a number of changes, deletions and additions have been made.

Historical matter which previously had not been carried beyond 1916 is omitted from this printing. There is also omitted the By-Laws of the Standards Committee of the A. I. E. E. as these are undergoing revision.

The chapter on **Transformer Connections** and **Markings of Leads,** (sections **600-617**) has been modified to accord with current American practise.

The chapter on **Controllers, Circuit Breakers, Switches, Fuses,** and **Accessories** (sections **700** *to* **744**) has been extended, rearranged, and considerably altered. The chapter on **Telephony** and **Telegraphy** (sections **910** *to* **963**) has been extended and revised. And, in addition to minor typographical corrections, there have been the following:

Revisions: Sections **17, 50, 52, 152, 164, 171, 172, 301, 307, 320, 348** (footnote), **405, 406, 502, 504, 765** (footnote), Table **III.**

Deletions: Sections **169, 170, 657, 1017.**

Additions: Sections **17a, 84, 85, 142** *to* **147c, 237** *to* **244, 407** *to* **409, 1034** *to* **1036.**

STANDARDIZATION RULES

OF THE

AMERICAN INSTITUTE OF ELECTRICAL ENGINEERS

DEFINITIONS

NOTE. The following definitions are intended to be practically descriptive, rather than scientifically rigid.

CURRENT, E.M.F. and POWER.

(The definitions of currents given below apply also, in most cases, to electromotive force, potential difference, magnetic flux, etc.)

1 **Direct Current.** A unidirectional current. As ordinarily used, the term designates a practically non-pulsating current.

2 **Pulsating Current.** A current which pulsates regularly in magnitude. As ordinarily employed, the term refers to unidirectional current.

3 **Continuous Current.** A practically non-pulsating direct current.

4 **Alternating Current.** A current which alternates regularly in direction. Unless distinctly otherwise specified, the term "alternating current" refers to a periodic current with successive half waves of the same shape and area.

5 **Oscillating Current.** A periodic current whose frequency is determined by the constants of the circuit or circuits.

6 **Cycle.** One complete set of positive and negative values of an alternating current.

7 **Electrical Degree.** The 360th part of a cycle.

8 **Period.** The time required for the current to pass through one cycle.

9 **Frequency.** The number of cycles or periods per second. The product of 2π by the frequency is called the *angular velocity* of the current.

10 **Root-Mean-Square** or **Effective Value.** The square root of the mean of the squares of the instantaneous values for one complete cycle. It is usually abbreviated r.m.s. Unless otherwise specified, the numerical value of an alternating current refers to its r.m.s. value. The r.m.s. value of a sinusoidal wave is equal to its maximum, or crest value, divided by $\sqrt{2}$. The word "virtual" is sometimes used in place of r.m.s., particularly in Great Britain.

11 **Wave-Form** or **Wave-Shape.** The shape of the curve obtained when the instantaneous values of an alternating current are plotted against time in rectangular co-ordinates. The distance along

the time axis corresponding to one complete cycle of values is taken as 2π radians, or 360 degrees. Two alternating quantities are said to have the same wave-form when their ordinates of corresponding phase (see § **13**) bear a constant ratio to each other. The wave-shape, as thus understood, is therefore independent of the frequency of the current and of the scale to which the curve is plotted.

12 **Simple Alternating** or **Sinusoidal Current**. One whose wave-shape is sinusoidal.

Alternating-current calculations are commonly based upon the assumption of sinusoidal currents and voltages.

13 **Phase.** The distance, usually in angular measure, of the base of any ordinate of an alternating wave from any chosen point on the time axis, is called the phase of this ordinate with respect to this point. In the case of a sinusoidal alternating quantity, the phase at any instant may be represented by the corresponding position of a line or *vector* revolving about a point with such an angular velocity ($\omega = 2\pi f$), that its projection at each instant upon a convenient reference line is proportional to the value of the quantity at that instant.

14 **Non-Sinusoidal Quantities.** Quantities that cannot be represented by vectors of constant length in a plane. The following definitions of phase, active component, reactive component, etc., are not in general applicable thereto. Certain "equivalent" values, as defined below, may, however, be used in many instances, for the purpose of approximate representation and calculation.

15 **Crest-Factor** or **Peak-Factor.** The ratio of the crest or maximum value to the r.m.s. value. The crest factor of a sine-wave is $\sqrt{2}$.

16 **Form Factor.** The ratio of the r.m.s. to the algebraic mean ordinate taken over a half-cycle beginning with the zero value. If the wave passes through zero more than twice during a single cycle, that zero shall be taken which gives the largest algebraic mean for the succeeding half-cycle. The form factor of a sine-wave is **1.11**.

17 **Deviation Factor of a Wave.**—The deviation factor of a wave is the ratio of the maximum difference between corresponding ordinates of the wave and of the equivalent sine wave of equal length to the maximum ordinate of the equivalent sine wave when the waves are superposed in such a way as to make this maximum difference as small as possible. (See §**406**)

17a **Interference Factor of a Wave.**—The interference factor of a wave is a measure of its departure from the sinusoidal shape based upon its disturbing effects when acting by induction on other circuits. (See §§**407** to **409**).

18 **Equivalent Sine Wave.** A sine wave which has the same frequency and the same r.m.s. value as the actual wave.

***19** **Phase Difference: Lead and Lag.** When corresponding cyclic values of two sinusoidal alternating quantities of the same frequency occur at different instants, the two quantities are said to differ in phase by the angle between their nearest corresponding

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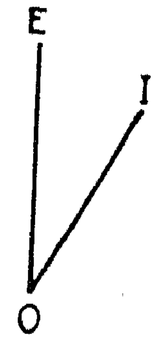
***See note next page.**

values; *e.g.*, the phase angle between their nearest ascending zeros or between their nearest positive maxima. That quantity whose maximum value occurs first in time is said to lead the other, and the latter is said to lag behind the former.



*20 **Counter-Clockwise Convention.** It is recommended that in any vector diagram, the leading vector be drawn counter-clockwise with respect to the lagging vector, † as in the accompanying diagram, where OI represents the vector of a current in a simple alternating-current circuit, lagging behind the vector OE of impressed e.m.f.

*21 **The Active or In-Phase Component** of the current in a circuit is that component which is in phase with the voltage across the circuit; similarly the active component of the voltage across a circuit is that component which is in phase with the current. The use of the term *energy component* for this quantity is disapproved.

*22 **Reactive or Quadrature Component of the current in a circuit.** That component which is in quadrature with the voltage across the circuit; similarly, the reactive component of the voltage across the circuit is that component which is in quadrature with the current. The use of the term *wattless component* for this quantity is disapproved.

*23 **Reactive Factor.** The sine of the angular phase difference between voltage and current; *i. e.*, the ratio of the reactive current or voltage to the total current or voltage.

*24 **Reactive Volt-Amperes.** The product of the reactive component of the voltage by the total current, or of the reactive component of the current by the total voltage.

*25 **Non-Inductive Load and Inductive Load.** A *non-inductive* load is a load in which the current is in phase with the voltage across the load. An *inductive* load is a load in which the current lags behind the voltage across the load. A *condensive* or *anti-inductive* load is one in which the current leads the voltage across the load.

26 **Power in an Alternating-Current Circuit.** The average value of the products of the coincident instantaneous values of the current and voltage for a complete cycle, as indicated by a wattmeter.

27 **Volt-Amperes or Apparent Power.** The product of the r.m.s. value of the voltage across a circuit by the r.m.s. value of the current in the circuit. This is ordinarily expressed in kv-a.

28 **Power Factor.** The ratio of the power (cyclic average as defined in §26) to the volt-amperes. In the case of sinusoidal current and voltage, the power factor is equal to the cosine of their difference in phase.

29 **Equivalent Phase Difference.** When the current and e.m.f. in a given circuit are non-sinusoidal, it is customary, for pur-

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*Note: Definitions 19, 20, 21, 22, 23, 24, 25 refer strictly only to cases where the voltage and current are both sinusoidal (see §11 and 12).

†See Publication of 12 the International Electrotechnical Commission (Report of Turin meeting, Sept. 1911, p. 78).

poses of calculation, to take as the "equivalent" phase difference, the angle whose cosine is the power factor (see §28) of the circuit. There are cases, however, where this equivalent phase difference is misleading, since the presence of harmonics in the voltage wave, current wave, or in both, may reduce the power factor without producing a corresponding displacement of the two wave forms with respect to each other; *e.g.*, the case of an a-c. arc. In such cases, the components of the equivalent sine waves, the equivalent reactive factor and the equivalent reactive volt-amperes may have no physical significance.

30 **Single-Phase.** A term characterizing a circuit energized by a single alternating e.m.f. Such a circuit is usually supplied through two wires. The currents in these two wires, counted positively outwards from the source, differ in phase by 180 degrees or a half-cycle.

31 **Three-Phase.** A term characterizing the combination of three circuits energized by alternating e.m.f.'s. which differ in phase by one-third of a cycle; *i.e.*, 120 degrees.

32 **Quarter-Phase, also called Two-Phase.** A term characterizing the combination of two circuits energized by alternating e.m.f's. which differ in phase by a quarter of a cycle; *i.e.*, 90 degrees.

33 **Six-Phase.** A term characterizing the combination of six circuits energized by alternating e.m.f's. which differ in phase by one sixth of a cycle; *i.e.*, 60 degrees.

34 **Polyphase.** A general term applied to any system of more than a single phase. This term is ordinarily applied to symmetrical systems.

Per Cent Drop.

50 In electrical machinery, the ratio of the **internal resistance drop** at 75°C. to the terminal voltage, expressed in percent, is called the "**percent resistance drop.**"

51 Similarly the ratio of the **internal reactance drop** to the terminal voltage, expressed in per cent, is called the "**per cent reactance drop.**"

52 Similarly the ratio of the **internal impedance drop** at 75°C. to the terminal voltage, expressed in percent is called the "**percent impedance drop.**"

Unless otherwise specified, these percent drops shall be referred to rated load and rated power factor.

53 **In the case of transformers,** the per cent drop will be the sum of the primary drop (reduced to secondary turns) and the secondary drop, in per cent of secondary terminal voltage.

54 **In the case of induction motors,** it is advantageous to express the drops in per cent of the internally induced e.m.f.

55 **The Load Factor of a machine, plant or system.** The ratio of the average power to the maximum power during a certain period of time. The average power is taken over a certain period of time, such as a day, a month, or a year, and the maximum is taken as the average over a short interval of the maximum load within that period.

In each case, the interval of maximum load and the period over which the average is taken should be definitely specified, such as a "half-hour monthly" load-factor. The proper interval and period are usually dependent upon local conditions and upon the purpose for which the load factor is to be used.

56 **Plant Factor.** The ratio of the average load to the rated capacity of the power plant, *i.e.*, to the aggregate ratings of the generators.

57 **The Demand of an Installation or System** is the load which is drawn from the source of supply at the receiving terminals averaged over a suitable and specified interval of time. Demand is expressed in kilowatts, kilovolt-amperes, amperes, or other suitable units.

58 **The Maximum Demand of an Installation or System** is the greatest of all the demands which have occurred during a given period It is determined by measurement, according to specifications, over a prescribed time interval.

59 **Demand Factor.** The ratio of the maximum demand of any system or part of a system, to the total connected load of the system, or of the part of system under consideration.

60 **Diversity Factor.** The ratio of the sum of the maximum power demands of the subdivisions of any system or parts of a system to the maximum demand of the whole system or of the part of the system under consideration, measured at the point of supply.

61 **Connected Load.** The combined continuous rating of all the receiving apparatus on consumers' premises, connected to the system or part of the system under consideration.

62 **The Saturation Factor of a machine.** The ratio of a small percentage increase in field excitation to the corresponding percentage increase in voltage thereby produced. Unless otherwise specified, the saturation factor of a machine refers to the no-load excitation required at normal rated speed and voltage. It is determined from measurements of saturation made on open circuit at rated speed.

63 **The Percentage Saturation** of a machine at any excitation may be found from its saturation curve (generated voltage as ordinates, against excitation as abscissas), by drawing a tangent to the curve at the ordinate corresponding to the assigned excitation, and extending the tangent to intercept the axis of ordinates drawn through the origin. The ratio of the intercept on this axis to the ordinate at the assigned excitation, when expressed in percent, is the percentage saturation, and is independent of the scales selected for excitation and voltage. This ratio, as a fraction, is equal to the reciprocal of the saturation-factor at the same excitation, deducted from unity; or, if f be the saturation factor and p the percentage saturation,

$$p = 100\left(1 - \frac{1}{f}\right)$$

64 **Magnetic Degree.** The 360th part of the angle subtended, at the axis of a machine, by a pair of its field poles. One **mechanical degree** is thus equal to as many magnetic degrees as there are pairs of poles in the machine.

65 **The Variation in Prime Movers** which do not give an absolutely uniform rate of rotation or speed, as in reciprocating steam engines, is the maximum angular displacement in position of the revolving member expressed in degrees, from the position it would occupy with uniform rotation, and with one revolution taken as 360 degrees.

66 **The Variation in Alternators** or alternating-current circuits in general, is the maximum angular displacement, expressed in electrical degrees, (one cycle = 360 deg.) of corresponding ordinates of the voltage wave and of a wave of absolutely constant frequency equal to the average frequency of the alternator or circuit in question, and may be due to the variation of the prime mover.

67 **Relations of Variations in Prime Mover and Alternator.** If p is the number of pairs of poles, the variation of an alternator is p times the variation of its prime mover, if direct-connected, and pn times the variation of the prime mover if rigidly connected thereto in such a manner that the angular speed of the alternator is n times that of the prime mover.

68 **The Pulsation in a Prime Mover, or in the alternator connected thereto.** The ratio of the difference between the maximum and minimum velocities in an engine-cycle to the average velocity.

80 **Capacity.** The two different senses in which this word is used sometimes lead to ambiguity. It is therefore recommended that whenever such ambiguity is likely to arise, the descriptive term *power capacity* or *current capacity* be used, when referring to the power or current which a device can safely carry, and that the term "*Capacitance*" be used when referring to the electrostatic capacity of a device.

81 **Resistor.** A device, heretofore commonly known as a resistance, used for the operation, protection, or control of a circuit or circuits See **§ 740.**

82 **Reactor.** A coil, winding or conductor, heretofore commonly known as a reactance coil or choke coil, possessing inductance the reactance of which is used for the operation, protection or control of a circuit or circuits. See also **§§ 214 and 736.**

83 **Efficiency.** The efficiency of an electrical machine or apparatus is the ratio of its useful output to its total input.

84 **Base Speed of an Adjustable-Speed Motor.**—That speed of the motor obtained with full field under full load with no resistor in the armature circuit.

85 **Electric Circuit.**—An electric circuit is a path in which an electric current may flow. Strictly speaking, an electric circuit is a complete circulatory path, but the term circuit is commonly employed to designate a specific part of a complete path. When part of a complete path is referred to, such as a branch circuit, a derived circuit, or a conductor, the terminals of that path should be specified in order to avoid ambiguity: e. g., the circuit *a-b-c*. When the whole circuit is referred to, it may be designated as a complete or closed circuit.

90 SYMBOLS AND ABBREVIATIONS

TABLE I.

Name of Quantity.	Symbol for the Quantity.	Unit.	Abbreviation for the Unit.
Electromotive force, abbreviated e.m.f.	E, e	volt	....
Potential difference, abbreviated p.d.	V, v or E, e	"	....
Voltage	E, e or V, v	"	....
Current	I, i	ampere	....
Quantity of electricity	Q, q	coulomb, ampere-hour	....
Power	P, p	watt	....
Electrostatic flux	Ψ	....	....
Electrostatic flux density	D	....	....
Electrostatic field intensity	F		
Magnetic flux	Φ, φ	maxwell	....
Magnetic flux density	B, $\mathfrak{B}$	gauss	....
Magnetic field intensity	H, $\mathcal{H}$	gilbert per centimeter or gauss†	gilbert per cm. ....
Magnetomotive force, abbreviated m.m.f.	$\mathfrak{F}$	gilbert*	....
Intensity of magnetization	J	....	....
Susceptibility	$\kappa = J/H$	....	....
Permeability	$\mu = B/H$	....	....
Resistance	R, r	ohm	....
Reactance	X, x	"	....
Impedance	Z, z	"	....
Conductance	g	mho	....
Susceptance	b	"	....
Admittance	Y, y	"	....
Resistivity	ρ	‡ohm-centimeter	ohm-cm

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†The gauss is provisionally accepted for the present as the name of both the unit of field intensity and flux density, on the assumption that permeability is a simple numeric.

*An additional unit for m. m. f. is the "ampere-turn," for flux the "line," for magnetic flux-density "maxwells per sq. in."

‡*Note.* The numerical values of these quantities are *ohms resistance* and *mhos conductance* between two opposite faces of a cm. cube of the material in question, but the correct names are as given, not ohms and mhos per cm. cube, as commonly stated.

Conductivity	γ	‡mho per centimeter	mho per cm.
Dielectric constant	ϵ or k	....	....
Reluctance	$\mathcal{R}$	....	....
Capacitance (Electrostatic capacity)	C	farad	....
Inductance (or coefficient of self induction)	L	henry	....
Mutual Inductance (or coefficient of mutual induction)	M	henry	....
Phase displacement	θ, φ	degree or radian	°
Frequency	f	cycle per second	~
Angular velocity	ω	radian per second	....
Velocity of rotation	n	revolution per second	rev. per sec.
Number of conductors or turns	N	convolution or turns of wire	
Temperature	T, t, θ	degree centigrade	C
Energy, in general	U or W	joule, watt-hour	....
Mechanical work	W or A	joule, watt-hour	....
Efficiency	η	per cent	....
Length	l	centimeter	cm.
Mass	m	gram	g
Time	t	second	sec.
Acceleration due to gravity	g	centimeter per second per second	cm. per sec. per sec.
Standard acceleration due to gravity (at about 45 deg. latitude and sea level) equals 980.665†	g_0	centimeter per second per second	cm. per sec. per sec.

91 E_m, I_m and P_m should be used for maximum cyclic values, e, i and p for instantaneous values, E and I for r.m.s. values (see **§10**) and P for the average value of the power, or the active power. These distinctions are not necessary in dealing with continuous-current circuits. In print, vector quantities should be represented by boldface capitals.

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‡See note preceding page

†This has been the accepted standard value for many years and was formerly considered to correspond accurately to 45° Latitude and sea level. Later researches, however, have shown that the most reliable value for 45° and sea-level is slightly different; but this does not affect the standard value given above.

CLASSIFICATION OF MACHINERY

100 The machinery under consideration in these rules may be classified in various ways, these various classifications overlapping or interlocking in considerable degree. Briefly, they are Direct-Current or Alternating-Current, Rotating or Stationary. Under Rotating Apparatus there are two principal classifications: *First*, according to the function of the machines; Motors, Generators. Boosters, Motor-Generators, Dynamotors, Double-Current Generators, Converters and Phase Advancers; *Second*, according to the type of construction or principle of operation; Commutating, Synchronous, Induction, Unipolar, Rectifying. Obviously, some of these machines could be rationally included in either classification, *e.g.*, Motor-Generators and Rectifying Machines.

In the following, self-evident definitions have for the most part, been omitted.

ROTATING MACHINES.

FUNCTIONAL CLASSIFICATION OF ROTATING MACHINES.

101 **Generator.** A machine which transforms mechanical power into electrical power.

102 **Motor.** A machine which transforms electrical power into mechanical power.

103 **Booster.** A generator inserted in series in a circuit to change its voltage. It may be driven by an electric motor (in which case it is termed a motor-booster) or otherwise.

104 **Motor–Generator Set.** A transforming device consisting of a motor mechanically coupled to one or more generators.

105 **Dynamotor.** A transforming device combining both motor and generator action in one magnetic field, either with two armatures, or with one armature having two separate windings and independent commutators.

106 **A Direct-Current Compensator** or **Balancer** comprises two or more similar direct-current machines (usually with shunt or compound excitation) directly coupled to each other and connected in series across the outer conductors of a multiple-wire system of distribution, for the purpose of maintaining the potentials of the intermediate wires of the system, which are connected to the junction points between the machines.

107 **A Double-Current Generator** supplies both direct and alternating currents from the same armature-winding.

108 **A Converter** is a machine employing mechanical rotation in changing electrical energy from one form into another. There are several types of converters as follow:

109 A **Direct-Current Converter** converts from a direct current to a direct current, usually with a change of voltage. Such a machine may be either a motor-generator set or a dynamotor.

110 A **Synchronous Converter** (sometimes called a Rotary Converter) converts from an alternating to a direct current, or vice-versa. It is a synchronous machine with a single closed-coil armature winding, a commutator and slip rings.

111 A **Cascade Converter,** also called a **Motor Converter,** is a combination of an induction motor with a synchronous converter, the secondary circuit of the former feeding directly into the armature of the latter; *i.e.*, it is a synchronous converter concatenated with an induction motor.

112 A **Frequency Converter** converts the power of an alternating-current system from one frequency to another, with or without a change in the number of phases, or in the voltage.

113 A **Rotary Phase-Converter** converts from an alternating-current system of one or more phases to an alternating-current system of a different number of phases, but of the same frequency.

114 A **Phase Advancer** is a machine which supplies reactive volt-amperes to the system to which it is connected. Phase advancers may be either synchronous or asynchronous.

115 A **Synchronous Condenser** or **Synchronous Phase Advancer** is a synchronous machine, running either idle or with load, the field excitation of which may be varied so as to modify the power-factor of the system, or through such modification to influence the load voltage.

CONSTRUCTIONAL CLASSIFICATION OF ROTATING MACHINES

Commutating Machines

130 **Direct-Current Commutating Machines** comprise a magnetic field of constant polarity, an armature, and a commutator connected therewith. These include: Direct-Current Generators; Direct-Current Motors; Direct-Current Boosters; Direct-Current Motor-Generator Sets and Dynamotors; Direct-Current Compensators or Balancers; and Arc Machines.

131 **Alternating-Current Commutating Machines** comprise a magnetic field of alternating polarity, an armature, and commutator connected therewith.

132 **Synchronous Commutating Machines** include synchronous converters, cascade-converters, and double-current generators.

133 **Synchronous Machines** comprise a constant magnetic field and an armature receiving or delivering alternating-currents in synchronism with the motion of the machine; *i.e.*, having a frequency strictly proportional to the speed of the machine. They may be sub-divided as follow:

134 **An Alternator** is a synchronous alternating-current generator, either single-phase or polyphase.

135 **A Polyphase Alternator** is a polyphase synchronous alternating-current generator, as distinguished from a **single-phase alternator.**

136 **An Inductor Alternator** is an **Alternator** in which both field and armature windings are stationary, and in which masses of iron or inductors. by moving past the coils, alter the magnetic flux through them. It may be either single-phase or polyphase.

137 **A Synchronous Motor** is a machine structurally identical with an alternator, but operated as a motor.

138 **Induction Machines** include apparatus wherein primary and secondary windings rotate with respect to each other; *i.e.*, induction motors, induction generators, certain types of frequency converters and certain types of rotary phase-converters.

139 **An Induction Motor** is an alternating-current motor, either singlephase or polyphase, comprising independent primary and secondary windings, one of which, usually the secondary, is on the rotating member. The secondary winding receives power from the primary by electromagnetic induction.

140 **An Induction Generator** is a machine structurally identical with an induction motor, but driven above synchronous speed as an alternating-current generator.

141 **Unipolar or Acyclic Machines** are direct-current machines, in which the voltage generated in the active conductors maintains the same direction with respect to those conductors.

DEFINITIONS OF TERMS USED IN DESCRIBING ALTERNATING-CURRENT COMMUTATOR MOTORS.

142 It is not intended that the use of any one term will exclude, or render unnecessary, the use of any one or more of the other terms. Each term is intended to refer to a certain group of alternating-current commutator-motors. An alternating-current commutator motor logically may be classified under a number of different, but non-conflicting groups.

Classification by Phases of Energy Supply:

143 **Single-Phase Commutator Motor.**—A single-phase commutator motor is one that receives the whole of its energy from only one phase of an alternating-current supply system, without requiring external phase-converting apparatus.

143a **Polyphase Commutator Motor.**—A polyphase commutator motor is one that receives its energy from a plurality of phases of an alternating-current supply system, or from a single-phase system through phase-converting apparatus external to the motor.

144 Classification by Speed Characteristics.—For convenience, alternating-current commutator motors may be classified with reference to their speed characteristics as (1) constant-speed motors, (2) multispeed motors, (3) adjustable-speed motors, and (4) varying-speed motors. Definitions of these terms as given in §§**151** to **154** for motors in general, should be adopted for alternating-current commutator motors, in so far as they are applicable.

Classification by Excitation:

145 Stator-Excited Commutator Motor.—A stator-excited commutator motor is one in which the torque-producing field* is due to a current in a winding located on the stator.

145a Rotor-Excited Commutator Motor.—A rotor-excited commutator motor is one in which the torque-producing field* is due to a current in a winding located on the rotor.

145b Stator- and Rotor-Excited Commutator Motor.—A stator- and rotor-excited commutator motor is one in which the torque-producing field* is due to currents in windings located on the stator and on the rotor.

145c Constant-Field Commutator Motor.—A constant-field commutator motor is one in which the torque-producing field* remains practically constant, independent of the load.

Alternating-current commutator motors of this class will in general have load-speed characteristics similar to those of the direct-current shunt motor, but not all alternating-current commutator motors having such load-speed characteristics are constant-field machines.

145d Varying-Field Commutator Motor.—A varying-field commutator motor is one in which the torque-producing field* varies in some proportion with the current in the armature (which latter is generally the rotor).

Such a motor will in general have load-speed characteristics similar to those of the direct-current series motor.

Classification by Neutralization and Compensation:

146 Neutralized Commutator Motor.—A neutralized commutator motor is one in which use is made of a winding for producing a magnetizing force which at each instant and at each point in the air-gap under the pole face is practically equal and opposite to the magnetizing force due to the armature current.

146a Compensated Commutator Motor.—A compensated commutator motor is one in which means, other than a neutralizing winding, are provided within the motor for improving the power-factor.

Classification by Energy Reception:

147 Conduction Commutator Motor.—A conduction commutator motor is one in which the working energy† is supplied to only one of the members, and is conveyed to it by conduction.

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*The term "torque-producing field" is intended to describe that component of the magnetic field which, with the in-phase component of current, produces the torque of the motor.

†See note next page.

147a Transformer Commutator Motor.—A transformer commutator motor is one in which the working energy† is transmitted from one member to the other by transformer action.

A motor in which the energy required by its armature (which is generally the rotor) is conveyed to it by electromagnetic induction or transformer action, may properly be referred to either as an "induction motor" or as a "transformer motor". Although it is equally applicable to a motor having a commutator, the term "induction motor" is usually applied to a motor without a commutator. The term "transformer commutator motor" is therefore recommended for use with motors of the induction, or transformer type, having commutators.

147b Transformer-Conduction Commutator Motor.—A transformer-conduction commutator motor is one in which the energy required by its armature (which is generally the rotor) is conveyed to it by both conduction and electromagnetic induction.

147c Repulsion Commutator Motor.—A repulsion commutator motor is a transformer commutator motor in which use is made of brushes for short-circuiting a number of coils of the commutated winding.

SPEED CLASSIFICATION OF MOTORS.

150 **Motors** may, for convenience, be classified with reference to their speed characteristics as follow:

151 **Constant-Speed Motors,** whose speed is either constant or does not materially vary; such as synchronous motors, induction motors with small slip, and ordinary direct-current shunt motors.

152 **Multispeed Motor (Change Speed Motor).**—A multispeed motor (change speed motor) is a motor which can be operated at any one of several distinct speeds (these speeds being practically independent of the load), but which cannot be operated at intermediate speeds.

153 **Adjustable-Speed Motors,** in which the speed can be varied gradually over a considerable range, but when once adjusted remains practically unaffected by the load; such as shunt motors designed for a considerable range of speed variation.

154 **Varying-Speed Motors,** or motors in which the speed varies with the load, ordinarily decreasing when the load increases; such as series motors, compound-wound motors, and series-shunt motors. As a sub-class of varying-speed motors, may be cited, **adjustable varying-speed motors,** or motors in which the speed can be varied over a considerable range at any given load, but when once adjusted, varies with the load; such as compound-wound motors arranged for adjustment of speed by varying the strength of the shunt field.

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†The term "working energy" is intended to describe the energy which is directly converted into mechanical energy, and which includes the shaft energy output plus core losses and friction.

CLASSIFICATION OF ROTATING MACHINES RELATIVE TO THE DEGREE OF ENCLOSURE

160 The following types are recognized:

Open
Protected
Semi-enclosed
Enclosed
Separately ventilated
Water-cooled
Self-ventilated
Drip-proof
Moisture-resisting
Submersible
Explosion-proof
Explosion-proof slip-ring enclosure

161 An **"open" machine** is of either the pedestal-bearing or end-bracket type where there is no restriction to ventilation, other than that necessitated by good mechanical construction.

162 A **"protected" machine** is one in which the armature, field coils, and other live parts are protected mechanically from accidental or careless contact, while free ventilation is not materially obstructed.

163 A **"semi-enclosed" machine** is one in which the ventilating openings in the frame are protected with wire screen, expanded metal, or other suitable perforated covers, having apertures not exceeding ½ of a square inch (3.2 sq. cm.) in area.

164 A **"Totally Enclosed" machine** is one so enclosed as to prevent circulation of air between the inside and the outside of the case, but not sufficiently to be termed "air-tight."

165 A **" separately ventilated " machine** has its ventilating air supplied by an independent fan or blower external to the machine.

166 A **" water-cooled " machine** is one which mainly depends on water circulation for the removal of its heat.

167 A **" self-ventilated " machine** differs from a **separately** ventilated machine only in having its ventilating air circulated by a fan, blower, or centrifugal device integral with the machine.

If the heated air expelled from the machine is conveyed away through a pipe attached to the machine, this should be so stated

168 A **"drip-proof " machine** is one so protected as to exclude falling moisture or dirt. A "drip proof" machine may be either "open" or "semi-enclosed", if it is provided with suitable protection integral with the machine, or so enclosed as to exclude effectively falling solid or liquid material.

171 An **"explosion-proof" machine** (or **"flame-proof" machine**) is a machine in which the enclosing case can withstand, without injury, any explosion of gas that may occur within it, and will not transmit the flame to any inflammable gas outside it.

172 A machine in which slip rings and brushes alone are included within an explosion-proof case should not be described as an explosion proof machine, but as a machine "with **explosion-proof slip-ring enclosure.**"

STATIONARY INDUCTION APPARATUS

200 **Stationary Induction Apparatus** changes electric energy to electric energy, through the medium of magnetic energy, without mechanical motion. It comprises several forms, distinguished as follow:

201 **Transformers**, in which the primary and secondary windings are ordinarily insulated one from another.

202 The terms **"high-voltage"** and **"low-voltage"** are used to distinguish the winding having the greater from that having the lesser number of turns. The terms **" primary "** and **" secondary "** serve to distinguish the windings in regard to energy flow, the primary being that which receives the energy from the supply circuit, and the secondary that which receives the energy by induction from the primary.

203 The **rated current** of a **constant-potential transformer** is that secondary current which, multiplied by the rated-load secondary voltage, gives the kv-a. rated output. That is, a transformer of given kv-a. rating must be capable of delivering the rated output at rated secondary voltage, while the primary impressed voltage is increased to whatever value is necessary to give rated secondary voltage.

The **Rated Primary Voltage** of a constant-potential transformer is the rated secondary voltage multiplied by the turn ratio.

204 The **ratio of a transformer,** unless otherwise specified, shall be the ratio of the number of turns in the high-voltage winding to that in the low-voltage winding; *i.e.*, the **" turn-ratio "**

205 The **voltage ratio** of a transformer is the ratio of the r.m.s. primary terminal voltage to the r.m.s. secondary terminal voltage, under specified conditions of load.

206 The **" current ratio "** of a current-transformer is the ratio of r.m.s. primary current to r.m.s. secondary current, under specified conditions of load.

207 The **" marked ratio "** of an instrument transformer is the ratio which the apparatus is designed to give under average conditions of use. When a precise ratio is required, it is necessary to specify the voltage, frequency, load and power factor of the load.

208 **Volt-Ampere Ratio of Transformers.**

The volt-ampere ratio, which should not be confused with real efficiency, is the ratio of the volt-ampere output to the volt-ampere input of a transformer, at any given power factor.

209 **Auto-transformers** have a part of their turns common to both primary and secondary circuits,

210 **Voltage Regulators** have turns in shunt and turns in series with the circuit, so arranged that the voltage ratio of the transformation or the phase relation between the circuit-voltages is variable at will. They are of the following three classes:

211 **Contact Voltage Regulators,** in which the number of turns in one or both of the coils is adjustable.

212 **Induction Voltage Regulators,** in which the relative positions of the primary and secondary coils are adjustable.

213 **Magneto Voltage Regulators,** in which the direction of the magnetic flux with respect to the coils is adjustable.

214 **Reactors,** heretofore commonly called **Reactance Coils,** also called **Choke Coils**; a form of stationary induction apparatus used to supply reactance or to produce phase displacement. See also §§**82** and **736.**

METERS AND INSTRUMENTS

225 Although the terms **Instruments** and **Meters** are frequently used synonymously in referring to electrical measuring devices, the meter departments of manufacturing and operating companies commonly use the word "meters" in the collective sense to designate only those devices which register the total energy or quantity of electricity consumed in or supplied to a circuit, and reserve the term "instruments," in the collective sense, for all other electrical measuring or indicating devices.

226 In general, the names of meters and instruments are self-defining, particularly when considered in connection with existing definitions. The following terms are preferred to other terms sometimes used for the same devices: **Reactive-Factor Meter, Power-Factor Meter, Watt-hour Meter,** etc.

227 **Crest Voltmeter.** A voltmeter depending for its indications upon the crest, that is the maximum value of the voltage of the system to which it is connected. These instruments are so calibrated that they indicate the r. m. s. value of the sinusoidal voltage having the same crest value.

228 **Synchronoscope** (also called a Synchroscope or Synchronism Indicator). A device which in addition to indicating synchronism between two machines, shows whether the speed of the incoming machine is fast or slow.

229 **Reactive-Volt-Ammeter** (also called a Reactive-Volt-Ampere Indicator). An instrument which indicates the reactive volt-amperes of the circuit to which it is connected.

230 **Line Drop Voltmeter Compensator.** A device used in connection with a voltmeter which causes it to indicate the voltage at some distant point of the circuit.

231 **Recording Ammeters, Voltmeters, Wattmeters, etc.,** are instruments which record graphically upon time-charts the values of the quantities they measure.

232 A **Demand Meter** is a device which indicates or records the demand or maximum demand (See §§**57** and **58**). In practice two types are recognized:

233 An **Integrated-Demand Meter** is one which indicates or records the maximum demand obtained through integration.

234 A **Lagged-Demand Meter** is one in which the indication of maximum demand is subject to a characteristic time lag.

235 **Errors of Indicating Instruments.** In specifying the accuracy of an indicating instrument, the error at any point on the scale shall be expressed as a percentage of the full scale reading.

236 **Torque** of meters and instruments shall be **expressed in millimeter-grams.**

237 Period of an Instrument.—The period of an instrument, sometimes called the "periodic time," is the time taken for the pointer to make one complete oscillation (two consecutive swings). A swing is a complete movement in either direction.

In strongly damped instruments, the period is influenced by the amplitude of the movement.

238 Damping.—The damping of an instrument shall be expressed in terms of the following quantities, all three of which are essential to a complete description of the behavior:

a. The number of swings taken by the pointer in coming to rest.
b. The time, in seconds, required for the pointer to come to rest.
c. The overshooting, in per cent of the angular displacement due to the disturbance.

These quantities shall be measured by suddenly applying and maintaining a load which will give a steady deflection of one-half full angular scale. The pointer shall be at zero when the load is applied.

239 Rating Limitation of the Circuits in Meters and Instruments.—The rating of any circuit in a meter or instrument shall be no greater than that corresponding to the load to which it may be continuously subjected.

The full scale marking of an instrument does not necessarily correspond to its rating. Where the rating differs from the full scale marking, the rating shall be marked on the instrument.

240 Standard Temperature of Reference for Meter and Instrument Characteristics.—The standard temperature of reference for meter and instrument characteristics shall be 20° C. For purposes of rating, the standard ambient temperature shall be 40°C.

241 Temperature Rise and Ultimate Temperature of Shunts.—Shunts shall be so rated that the rise of temperature at the hottest spot shall not be greater than 80°C., and their use so restricted that the ultimate temperature (hottest spot) shall not exceed 120°C.

Exception is made of shunts having no soldered joint and made of material which is not permanently changed in resistance if continuously subjected to higher temperature. Hottest spot temperature should be measured in such a manner as not to cause undue local rise of temperature.

243 Marking of Switchboard Shunts.—Marking of switchboard shunts shall include the rating in amperes, the drop in volts at that rating, and, if the shunt is calibrated in connection with a particular instrument, the serial number of such instrument.

When shunts are designed to be used with devices taking sufficient current to be an appreciable portion of the whole, this fact shall be indicated. For example, a 100-ampere device having a drop of 0.050 volts, will have its shunt marked:

Volts 0.050
Amperes 100—10,

indicating that with 90 amperes in the shunt and 10 amperes in the measuring circuit, the drop across the shunt will be 0.050 volts.

244 **Grounding of Meters and Instruments.**—To avoid errors due to electrostatic action, the covers of meters and instruments which are used with current and potential transformers shall be connected to the secondary circuits of the transformers and grounded.

INSTRUMENT TRANSFORMERS

See §§**741** to **744**.

STANDARDS FOR ELECTRICAL MACHINERY

250 *The expressions "machinery" and "machines" are here employed in a general sense, in order to obviate the constant repetition of the words "machinery or induction apparatus."*

251 *All temperatures are to be understood as centigrade.*

252 *The expression "capacity" is to be understood as indicating "capability", except where specifically qualified, as, for instance, in the case of allusions to electrostatic capacity, i. e., capacitance.*

253 *Wherever special rules are given for any particular type of machinery or apparatus (such as switches, railway motors, railway substation machinery, etc., these special rules shall be followed, notwithstanding any apparent conflict with the provisions of the more general sections. In the absence of special rules on any particular point, the general rules on this point shall be followed.*

260 **Objects of Standardization.** To ensure satisfactory results, electrical machinery should be specified to conform to the Institute Standardization Rules, in order that it shall comply, in operation, with approved limitations in the following respects, so far as they are applicable.

Operating temperature
Mechanical strength
Commutation
Dielectric strength
Insulation resistance
Efficiency
Power factor
Wave shape
Regulation

261 **Capacity or Available Output of an Electrical Machine.** So far as relates to the purposes of these Standardization Rules, the Institute defines the Capacity of an Electrical Machine as the load which it is capable of carrying for a specified time (or continuously), without exceeding in any respect the limitations herein set forth.

Except where otherwise specified, the capacity of an electrical machine shall be expressed in terms of its available *output*. For exceptions see §§**277** and **802**.

262 **Rating of an Electrical Machine.** Capacity should be distinguished from Rating. The Rating of a machine is the output marked on the Rating Plate, and shall be based on, but shall not exceed, the maximum* load which can be taken from the machine under prescribed conditions of test. This is also called the rated output.

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*The term "maximum Load" does not refer to loads applied solely for mechanical commutation, or similar tests.

263 **The Principle upon which Machine Ratings** are based, so far as relates to thermal characteristics, is that the rated load, applied continuously or for a stated period, shall produce a temperature rise which, superimposed upon a standard ambient temperature, will not exceed the maximum safe operating temperature of the insulation.

264 **A. I. E. E. and I. E. C. Ratings.** When the prescribed conditions of test are those of the A. I. E. E. Standardization Rules, the rating of the machine is the Institute Rating. (See **§620**). When the prescribed conditions of the test are those of the I. E. C.† Rules, the rating of the machine is the I. E. C. rating. A machine so rated in either case may bear a distinctive sign upon its rating plate.

265 **Standard Temperature and Barometric Pressure for Institute Rating.** The Institute Rating (See §262) of a machine shall be its capacity when operating with a cooling medium of the ambient temperature of reference (40° for air or 25° for water, see §§**305** and **309**) and with barometric conditions within the range given in §**308**. See §§**305A, 307, 320** and **321.**

266 **The Temperature Rises Specified in these Rules** apply to all ambient temperatures up to and including, but not exceeding, 40°C. for air and 25°C. for water. (For definition of ambient temperature see §**303**.)

267 **Any Machinery Destined for Use with Higher Ambient** temperatures of cooling mediums, and also any machinery for operation at altitudes for which no provision is made in §**308**, should be the subject of special guarantee by the manufacturer. The methods of test and performance set forth in these Rules will, however, afford guidance in such cases.

UNITS IN WHICH RATING SHALL BE EXPRESSED

274 The rating of **Direct-Current Generators,** shall be expressed in kilowatts (kw.) available at the terminals at a specified voltage.

275 The rating of **Alternators** and **Transformers,** shall be expressed in kilovolt-amperes (kv-a.) available at the output terminals, at a specified voltage and power factor.

276 It is strongly recommended that the **rating of motors** shall be expressed in kilowatts* (kw.) available at the shaft. (An ex-

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†I.E.C. stands for "International Electrotechnical Commission This rating has not yet been established.

*Since the input of machinery of this class is measured in electrical units and since the output has a definite relation to the input, it is logical and desirable to measure the delivered power in the same units as are employed for the received power. Therefore, the output of motors should be expressed in kilowatts instead of in horse power. However, on account of the hitherto prevailing practice of expressing mechanical output in horse power, it is recommended that for machinery of this class the rating should, for the present, be expressed both in kilowatts and in horse power; as follows:

kw.————approx. equiv. h.p.————

For the purposes of these Rules the horse-power shall be taken as 746.0 watts. In order to lay stress upon the preferred future basis, it is desirable that on Rating plates, the Rating in kilowatts shall be shown in larger and more prominent characters than the rating in horse power.

ception to this rule is made in the case of Railway motors, which, for some purposes, are also rated by their *input*, see **§802.**)

277 **Auxiliary machinery,** such as regulators, resistors, reactors, balancer sets, stationary and synchronous condensers, etc., shall have their ratings appropriately expressed. It is essential to specify also the voltage (and frequency, if a-c.,), of the circuits on which the machinery may appropriately be used.

KINDS OF RATING

There are various kinds of rating such as:

281 **Continuous Rating.** A machine rated for continuous service shall be able to operate continuously at its rated output, without exceeding any of the limitations referred to in **§260.**

282 **Short-Time Rating.** A machine rated for short-time service; (*i.e.* service including runs alternating with stops of sufficient duration to ensure substantial cooling), shall be able to operate at its rated output during a limited period, to be specified in each case, without exceeding any of the limitations referred to in **§260.** Such a rating is a **short-time rating.**

283 **Nominal Ratings.** For railway motors and sometimes for railway substation machinery, certain nominal ratings are employed. See §§**765** and **800.** Nominal ratings for automobile propulsion motors and generators are not recommended; see **§837.**

284 **Duty-Cycle Operation.** Many machines are operated on a cycle of duty which repeats itself with more or less regularity. For purposes of rating, either a continuous or a short-time equivalent load, may be selected, which shall simulate as nearly as possible the thermal conditions of the actual duty cycle.

285 Standard durations of equivalent tests shall be for machines operating under specified duty-cycles as follow (see also **§836**):

5 minutes
10 "
15 "
30 "
60 "
120 "
and continuous.

Of these the first six are short-time ratings, selected as being thermally equivalent to the specified duty cycle.

When, for example, a short-time rating of 10 minutes duration is adopted, and the thermally equivalent load is 25 kw. for that period, then such a machine shall be stated to have a 10-minute rating of 25 kw.

286 In every case the equivalent short-time test shall commence only when the windings and other parts of the machine are **within 5°C of the ambient temperature** at the time of starting the test.

287 In the absence of any specification as to the kind of rating, the continuous rating shall be understood.*

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*An exception is made in the case of motors for railway service, where in the absence of any specification as to the kind of rating, the "**nominal rating**" as defined in **§319** and **415** shall be understood.

288 Machines marked in accordance with **§264** shall be understood to have a continuous rating, unless otherwise marked in accordance with **§285**

HEATING AND TEMPERATURE

300 **Temperature Limitations of the Capacity of Electrical Machinery.** The capacity, so far as relates to temperature, is usually limited by the maximum temperature at which the materials in the machine, especially those employed for insulation, may be operated for long periods without deterioration. When the safe limits are exceeded, deterioration is rapid. The insulating material becomes permanently damaged by excessive temperature, the damage increasing with the length of time that the excessive temperature is maintained, and with the amount of excess temperature, until finally the insulation breaks down.

301 The result of operating at temperatures in excess of the safe limit is to shorten the life of the insulating material. This shortening of life is, in certain special cases, warranted, when necessary for obtaining some other desirable result, as, for example, in some instances of railway and other motors for propelling vehicles, in providing greater power within a limited space. See **§804.** The design of coils used in control and switching apparatus, such as contactors, controllers, induction starters, and in other operating mechanisms as arc lamps, magnet windings, etc., is such that the effect of higher temperature in the coils is less serious owing to the fact that these devices usually have shorter life than electrical machinery in general. Consequently, higher temperatures for the windings are recommended, in accordance with §**717a.**

302 There does not appear to be any advantage in operating at lower temperatures than the safe limits, so far as the life of the insulation is concerned. Insulation may break down from various causes, and when these breakdowns occur, it is not usually due to the temperature at which the insulation has been operated, provided the safe limits have not been exceeded.

303 **The Ambient Temperature** is the temperature of the air or water which, coming into contact with the heated parts of a machine, carries off its heat. See §§**309, 310** and **314.**

304 **The cooling fluid may** either be led to the machine through ducts, or through pipes, or merely surround the machine freely. In the former case the ambient temperature is to be measured at the intake of the machine itself. In the latter case see **§314.**

305 **Ambient Temperature of Reference for Air.** The standard ambient temperature of reference, when the cooling medium is **air,** shall be 40°C.

305A. **Whatever may be the Ambient Temperature** when the machine is in service, the limits of the maximum observable temperature or of temperature rise specified in the rules should not be exceeded in service; for, if the maximum temperature be exceeded, the insulation may be endangered, and if the rise be exceeded, the excess load may lead to injury, by exceeding limits other than those of temperature; such as commutation, stalling load and mechanical strength. For similar reasons, loads in excess of the rating should not be taken from a machine.

306 The permissible rises in temperature given in column 2 of table III page 38 have been calculated on the basis of the standard ambient temperature of reference, by subtracting 40° from the highest temperatures permissible, which are given in column 1 of the same table.

307 A machine may be tested at any convenient ambient temperature, preferably not below 10°C., but *whatever be the value of this ambient temperature, the permissible rises of temperature must not exceed those given in column 2 of table III page* 38.

308 **Altitude.** Increased altitude has the effect of increasing the temperature rise of some types of machinery. In the absence of information in regard to the height above sea level at which a machine is intended to work in ordinary service, this height is assumed not to exceed 1000 meters (3300 feet.) For machinery operating at an altitude of 1000 meters or less, a test at any altitude less than 1000 meters is satisfactory, and no correction shall be applied to the observed temperatures. Machines intended for operation at higher altitudes shall be regarded as special. See **§267.** It is recommended that when a machine is intended for service at altitudes above 1000 meters (3300 ft.) the permissible temperature rise at sea level, until more nearly accurate information is available, shall be reduced by 1 per cent for each 100 meters (330 ft.) by which the altitude exceeds 1000 meters. Water cooled oil transformers are exempt from this reduction.

309 **Ambient Temperature of Reference for Water-Cooled Machinery.** For water-cooled machinery, the standard temperature of reference for incoming cooling water shall be 25° C, measured at the intake of the machine.

310 **In the testing of water-cooled transformers,** it is not necessary to take into account the surrounding-air temperature, except where the cooling effect of the air is 15 per cent or more of the total cooling effect, referred to the standard ambient temperature of reference of 25°C. for water and 40°C. for air. When the effect of the cooling air is 15 per cent. or more of the total, the temperature of the cooling water should be maintained within 5°C. of the surrounding air. Where this is impractical, the ambient temperature should be determined from the change in the resistance of the windings, using a disconnected transformer, supplied with the normal amount of cooling water, until the temperature of the windings has become constant.

311 **In the case of rotating machines,** cooled by forced draught, a conventional weighted mean shall be employed, a weight of four being given to the temperature of the circulating air supplied through ducts (see **§304**), and a weight of one to the surrounding room air. **In the case of air-cooled transformers,** see " exception " **§321.**

312 **Machines Cooled by Other Means.** Machines cooled by means other than air or water shall receive special consideration.

313 **Outdoor Machinery Exposed to Sun's Rays.** Outdoor machinery not protected from the sun's rays at times of heavy load, shall receive special consideration

314 **Measurement of the Ambient Temperature During Tests of Machinery.** The ambient temperature is to be measured by means of several thermometers placed at different points around and half-way up the machine, at a distance of 1 to 2 meters (3 to 6 feet), and protected from drafts, and abnormal heat radiation, preferably as in **§316**.

315 The value to be adopted for the ambient temperature during a test, is the mean of the readings of the thermometers (placed as above), taken at equal intervals of time during the last quarter of the duration of the test.

316 Inorder to avoid **errors due to the time lag** between the temperature of large machines and the variations in the ambient air, all reasonable precautions must be taken to reduce these variations and the errors arising therefrom. Thus, the thermometer for determining the ambient temperature shall be immersed in a suitable liquid, such as oil, in a suitable heavy metal cup. This can be made to respond to various rates of change, by proportioning the amount of oil to the metal in the containing cup. A convenient form for such an oil-cup consists of a massive metal cylinder, with a hole drilled partly through it. This hole is filled with oil and the thermometer is placed therein with its bulb well immersed. The larger the machine under test, the larger should be the metal cylinder employed as an oil-cup in the determination of the ambient temperature. The smallest size of oil cup employed in any case shall consist of a metal cylinder 25 mm. in diameter and 50 mm. high (1 in. in diameter and 2 in. high).

317 **Thermometers used for taking temperatures of Machinery** shall be covered by felt pads 3 mm. (⅛ inch) thick and 4 x 5 cm. wide (1½ "x 2"), cemented on; oil putty may be used for stationary and small apparatus.

318 **In Transformer Testing,** and sometimes in testing other machines, it may be desirable to avoid errors due to time lag in temperature changes, by employing an idle unit of the same size and subjected to the same conditions of cooling as the unit under test, for obtaining the ambient temperature as described in **§310.**

319 Where **machines are partly below the floor line** in pits, the temperature of the rotor shall be referred to a weighted mean of the pit and room temperatures, the weight of each being based on the relative proportions of the rotor in and above the pit. Parts of the stator constantly in the pit shall be referred to the ambient temperature in the pit.

320 **Correction for the Deviation of the Ambient Temperature of the Cooling Medium, at the Time of the Heat Test, from the Standard Ambient Temperature of Reference.** Numerous experiments have shown that deviation of the temperature of the cooling medium

from that of the standard of reference, at the time of the heat run, has a negligible effect upon the temperature rise of the apparatus; therefore, no correction shall be applied for this deviation. It is, however, desirable that tests should be conducted at ambient temperatures not lower than 10°C.

321 *Exception*—A **Correction shall be applied** to the observed temperature rise of the windings of *Air-blast transformers*, due to difference in resistance, when the temperature of the ingoing cooling air differs from that of the standard of reference. This correction shall be the ratio of the inferred absolute ambient temperature of reference to the inferred absolute temperature of the ingoing cooling air, *i. e.* the ratio $274.5/(234.5 + t)$; where t is the ingoing cooling-air temperature.

Thus, a cooling-air room temperature of 30°C. would correspond to an inferred absolute temperature of 264.5° on the scale of copper resistivity, and the correction to 40°C. (274.5° inferred absolute temperature) would be $274.5 / 264.5 = 1.04$, making the correction factor 1.04; so that an observed temperature rise of say 50°C. at the testing ambient temperature of 30°C. would be corrected to $50 \times 1.04 = 52$°C. this being the temperature rise which would have occurred had the test been made with the standard ingoing cooling-air temperature of 40°C.

322 **Duration of Temperature Test of Machine for Continuous Service.** The temperature test shall be continued until sufficient evidence is available to show that the maximum temperature and temperature rise would not exceed the requirements of the rules, if the test were prolonged until a steady final temperature was reached.

323 **Duration of Temperature Test of Machine with a Short-Time Rating.** The duration of the temperature test of a machine with a short-time rating shall be the time required by the rating. (See §§**285** and **286**).

324 **Duration of Temperature Test for Machine having more than One Rating.** The duration of the temperature test for a machine with more than one rating shall be the time required by that rating which produces the greatest temperature rise. In cases where this cannot be determined beforehand, the machine shall be tested separately under each rating

325 **Temperature Measurements during Heat Run.** Temperature measurements, when possible, shall be taken during operation, as well as when the machine is stopped. The highest figures thus obtained shall be adopted. In order to abridge the long heating period, in the case of large machines, reasonable overloads of current, during the preliminary period, are suggested for them.

TEMPERATURE MEASUREMENTS

340 **The Life of the Insulation of a Machine** depends in great measure upon the actual temperatures attained by the different parts, rather than on the rises of temperature in those parts.

341 **The Temperatures in the Different Parts of a Machine** which it would be desirable to ascertain, are the maximum temperatures reached in those parts.

343 **As it is Usually Impossible to Determine the Maximum Temperature** attained in insulated windings, it is convenient to apply a correction to the observable temperature, which approximates the difference between the actual maximum temperature and the observable temperature by the method used. This correction, or margin of security, is provided to cover the errors due to fallibility in the location of the measuring devices, as well as inherent inaccuracies in measurement and methods.

344 **In Determining the Temperature of Different Parts of a Machine** three methods are provided.

345 **Method No 1. Thermometer Method.**

This method consists in the determination of the temperature, by mercury or alcohol thermometers, by resistance thermometers, or by thermocouples, any of these instruments being applied to the hottest accessible part of the *completed* machine, as distinguished from the thermocouples or resistance coils embedded in the machine as described under Method No. 3.

346 **When Method No. 1 is Used,** the hottest-spot temperature for windings shall be estimated by adding a hottest-spot correction of 15°C to the highest temperature observed, in order to allow for the practical impossibility of locating any of the thermometers at the hottest spot.

347 *Exception.* When the thermometers are applied directly to the surfaces of bare windings, such as an edgewise strip conductor, or a cast copper winding, a hottest-spot correction of 5°C., instead of 15°C, shall be made. For commutators, collector rings, bare metallic surfaces not forming part of a winding, or for oil in which apparatus is immersed, no correction is to be applied.

348 **Method No. 2. Resistance Method.**

This method consists in the measurement of the temperature of windings by their increase in resistance, corrected* to the instant of shut-down when necessary. In the application of this method, *thermometer measurements shall also be made* whenever practicable without disassembling the machine† in order to increase the probability of revealing the highest observable temperature. Whichever measurement yields the higher temperature, that temperature shall be taken as the "highest observable" temperature and a hot-test-spot correction of 10°C added thereto.

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*Whenever a sufficient time has elapsed between the instant of shut-down and the time of the final temperature measurement to permit the temperature to fall, suitable corrections shall be applied, so as to obtain as nearly as practicable the temperature at the instant of shut-down. This can sometimes be approximately effected by plotting a curve, with temperature readings as ordinates and times as abscissas, and extrapolating back to the instant of shut-down. In other instances, acceptable correction factors can be applied. In transformers of 200 kv-a. and less the measured temperature shall be increased one degree for every minute between the instant of shut-down and the time of the final temperature measurement, provided this time does not exceed three minutes.

In cases where successive measurements show *increasing* temperatures after shut-down, the highest value shall be taken.

In testing air blast transformers the air supply should be shut off immediately at the end of the heat run and the air duct closed to prevent a further circulation of air. In checking the temperature ascertained by the Resistance Method, the reading of all the thermometers in contact with coils should be noted carefully and the maximum temperature reached by any of them, if higher than that determined by resistance, should be taken as the maximum observable temperature of the windings. When the above procedure has been followed, a hottest spot correction of 5° shall be applied.

349 **The Temperature Coefficient of Copper** shall be deduced from the formula $1/(234.5 + t)$. Thus, at an initial temperature $t = 40$°C., the temperature co-efficient of increase in resistance per degree centigrade rise, is $1/(274.5) = 0.00364$. The following table, deduced from the formula, is given for convenience of reference.

TABLE II.

Temperature Coefficients of Copper Resistance.

Temperature of the winding, in degrees C. at which the initial resistance is measured.	Increase in resistance of copper per °C., per ohm of initial resistance.
0	0.00 427
5	0.00 418
10	0.00 409
15	0.00 401
20	0.00 393
25	0.00 385
30	0.00 378
35	0.00 371
40	0.00 364

350 **In Circuits of Low Resistance,** other than transformer windings (see §**351**), where joints and connections form a considerable part of the total resistance, the measurement of temperature by the resistance method shall not be used.

351 **The Temperature of the Windings of Transformers** is always to be ascertained by Method 2. In the case of air-blast transformers, it is especially important to place thermometers on the coils near the air outlet. See §348.

352 **Method No. 3. Embedded Temperature-Detector Method.** This method consists in the use of thermo-couples or resistance temperature detectors, located as nearly as possible at the estimated hottest spot. When method No. 3 is used, it shall, when required, be checked by method No. 2; the hottest spot shall

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†As one of the few instances in which the thermometer check cannot be applied in Method No. 2, the rotor of a turbo-alternator may be cited

then be taken to be the highest value by either method, the required correction factors (§**348** and §**356**) being applied in each case.

353 **By Building into the Machine** suitably placed temperature detectors, a temperature not much less than that of the hottest spot will probably be disclosed. When these devices are adopted for such temperature determinations, a liberal number shall be employed, and all reasonable efforts, consistent with safety, shall be made to locate them at the various places where the highest temperatures are likely to occur.

354 **Temperature-Detectors** should be placed in at least two sets of locations. One of these should be between a coil-side* and the core, and one between the top and bottom coil-sides where two coil-sides per slot are used. Where only one coil-side per slot is used, one set of detectors shall be placed between coil-side and core, and one set between coil-side and wedge.

355 Method No. **3** should be applied to all stators of machines with cores having a width of 50 cm. (20 in.) and over. It should also be applied to all machines of 5000 volts and over, if of over **500** kv-a., regardless of core width. This method is not required for induction-regulators, which shall be tested as transformers.

356 **Correction Factor for Method No. 3.**—In the case of two-layer windings, with detectors between coil-sides, and between coil-side and core, add 5° C to the highest reading. In single-layer windings, with detectors between coil-side and core and between coil-side and wedge, add to the highest reading 10° C. plus 1° C. per 1000 volts above 5000 volts of terminal pressure.

TEMPERATURE LIMITS

375 Table III, page 38, gives the limits for the hottest-spot temperatures of insulations. The permissible limits are indicated in column 1 of the Table. The limits of temperature rise permitted under rated-load conditions are given in column 2, and are found by subtracting 40° C. from the figures in column 1. Whatever be the ambient temperature at the time of the test, the rise of temperature must never exceed the limits in column 2 of the table. The highest temperatures, and temperature rises, attained in any machine at the output for which it is rated, must not exceed the values indicated in the Table and clauses following.

376 **Permissible Temperatures and Temperature Rises For Insulating Materials.** Table III (see next page) gives the highest temperatures and temperature rises to which various classes of insulating materials may be subjected, based on a standard ambient temperature of reference of 40°C.

377 Note. The Institute recognizes the ability of manufacturers to employ Class B insulation successfully at maximum temperatures of 150° C. and even higher. However, as sufficient data covering experience over a period of years at such temperatures are at present unavailable, the Institute adopts 125° C as a conservative limit for this class of insulation, and any increase above this figure should be the subject of special guarantee by the manufacturer.

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*A coil side is one of the two active sides of the coil lying in a slot.

TABLE III.

Permissible Temperatures and Temperature Rises for Insulating Materials.

Class	Description of Material	1 Maximum Temperature to which the material may be subjected	2 Maximum Temperature Rise
A.	Cotton, silk, paper and similar materials, when so treated or impregnated as to increase the thermal limit, or when permanently immersed in oil; also enamelled wire*	105°C	65°C
B.	Mica, asbestos and other materials capable of resisting high temperatures, in which any Class A. material or binder is used for structural purposes only, and may be destroyed without impairing† the insulating or mechanical qualities of the insulation	125°C	85°C
C.	Materials capable of resisting higher temperatures than Class B, such as pure mica, porcelain, quartz, etc.	No limits specified.	

*For cotton, silk, paper and similar materials, when neither treated, impregnated nor immersed in oil, the highest temperatures and temperature rises shall be 10°C below the limits fixed for Class A, in Table III.

†The word impair is here used in the sense of causing any change which would disqualify the insulation for continuous service.

378 When a lower-temperature class material is comprised in a completed product to such an extent, or in such ways, that its subjection to the temperature limits allowed for the higher-temperature class material, with which it is associated, would affect the integrity of the insulation either mechanically or electrically, the permissible temperature shall be fixed at such a value as shall afford ample assurance that no part of the lower-temperature class material shall be subjected to temperatures higher than those approved by the Institute and set forth above.

379

TABLE IV

Permissible Hottest Spot Temperatures and Limiting Observable Temperature Rises in Other than Water-Cooled Machinery‡

		Class	A*	B
		Permissible Hottest-Spot Temperature	105°	125°
METHOD I THERMOMETER ONLY See §§345 to 347		Hottest Spot Correction	15°	15°
		Limiting Observable Temperature	90°	110°
		Limiting Observable Temperature Rise	50°	70°
METHOD II RESISTANCE See §348		Hottest Spot Correction	10°	10°
		Limiting Observable Temperature	95°	115°
		Limiting Observable Temperature Rise	55°	75°
METHOD III EMBEDDED TEMPERATURE DETECTORS See §§ 352 to 356	Double-Layer Windings. For all Voltages	Hottest Spot Correction	5°	5°
		Limiting Observable Temperature	100°	120°
		Limiting Observable Temperature Rise	60°	80°
	Single-Layer Windings. For 5000 volts or less	Hottest Spot Correction	10°	10°
		Limiting Observable Temperature	95°	115°
		Limiting Observable Temperature Rise	55°	75°
	Single-Layer Windings. For more than 5000 Volts	Hottest Spot Correction	$10° + (E-5)$†	$10° + (E-5)$
		Limiting Observable Temperature	$95° - (E-5)$	$115° - (E-5)$
		Limiting Observable Temperature Rise	$55° - (E-5)$	$75° - (E-5)$

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*For cotton, silk, paper and similar materials, when neither treated, impregnated nor immersed in oil, the highest temperatures and temperature rises shall be 10° C. below the limits fixed for class A.

†In these formulas, E represents the rated pressure between terminals in kilovolts. Thus for a three-phase machine with single-layer winding, and with 11 kilovolts between terminals, the hottest-spot correction to be added to the maximum observable temperature will be 16° C.

‡For exceptions to these limitations see §§301, 717a, 721, 726, 729, 731, 740a, and Special Cases §§385 to 392.

Special Cases of Temperature limits.

385 **Temperature of Oil.** The oil in which apparatus is permanently immersed shall in no part have a temperature, observable by thermometer, in excess of 90°C.

386 **Water-Cooled Transformers.** In these the hottest-spot temperature shall not exceed 90°C.

386A **Enclosed Motors and Generators.** In an enclosed machine (see §164) the limiting observable temperature and the limiting observable temperature rise shall be taken as 5°C. higher than in Table IV. This is not to be interpreted as an increase in the permissible hottest spot temperature, but is in recognition of the lesser difference between the hottest spot temperature and the observable temperature within an enclosed machine. This rule does not apply to those types of machines defined in §§163, 165 and 167.

387 **Railway Motor Temperature Limits,** see §804 and 805

387A. **Automobile Propulsion Motors and Generators,** see §838.

388 **Squirrel-Cage and Amortisseur Windings.** In many cases the insulation of such windings is largely for the purpose of making the conductors fit tightly in their slots, and the slightest effective insulation is ample. In other cases, there is practically no insulating material on the windings. Consequently, the temperature rise may be of any value such as will not occasion mechanical injury to the machine.

389 **Collector Rings.** The temperature of collector rings shall not be permitted to exceed the "hottest-spot" values set forth in §376 and 379 for the insulations employed either in the collector rings themselves, or in adjacent insulations whose life would be affected by the heat from the collector rings.

390 **Commutators.** The observable temperature shall in no case be permitted to exceed the values given in §376 and 379 for the insulation employed, either in the commutator or in any insulation whose life would be affected by the heat of the commutator. These temperature limits are intended only to protect the insulation of the commutator and of the adjacent parts, and are not intended as a criterion of successful commutation. See §402.

391 **Cores.** The temperature of those parts of the iron core in contact with insulating materials must not be such as to occasion in those insulating materials temperatures or temperature rises in excess of those set forth in §376 and 379.

392 **Other parts,** (such as brush-holders, brushes, bearings, pole-tips, cores, etc.) All parts of electrical machinery other than those whose temperature affects the temperature of the insulating material, may be operated at such temperatures as shall not be injurious in any other respect.

METHODS OF LOADING TRANSFORMERS FOR TEMPERATURE TESTS

393 Whenever practicable, transformers should be tested under conditions that will give losses approximating as nearly as possible to those obtained under normal or specified load conditions, maintained for the required time (See §**322** to **324**). The maximum temperature rises measured during this test should be considered as the observable temperature rises for the given load.

An approved method of making these tests is the "**loading-back**" method. The principal variations of this method are:

394 **With duplicate single-phase transformers.**

Duplicate single-phase transformers may be tested in banks of two, with both primary and secondary windings connected in parallel. Normal magnetizing voltage should then be applied and the required current circulated from an auxiliary source. One transformer can be held under normal voltage and current conditions, while the other may be operating under slightly abnormal conditions.

395 **With one three-phase transformer.**

One three-phase transformer may be tested in a manner similar to §**394** provided the primary and secondary windings are each connected in delta for the test. Normal three-phase magnetizing voltage should be applied and the required current circulated from an auxiliary single-phase source.

396 **With three single-phase transformers.**

Duplicate single-phase transformers may be tested in banks of three, in a manner similar to §**395** by connecting both primary and secondary windings in delta, and applying normal three-phase magnetizing voltage and circulating the required current from an auxiliary single-phase source.

397 NOTE:— Among other methods that have a limited application and can be used only under special conditions may be mentioned—

(1) Applying dead load by means of some form of rheostat.

(2) Running alternately for certain short intervals of time on open circuit and then on short circuit, alternating in this way until the transformer reaches steady temperature. In this test, the voltage for the open-circuit interval and the current for the short-circuit interval shall be such as to give the same integrated core loss, and the same integrated copper loss, as in normal operation.

ADDITIONAL REQUIREMENTS

398 **Short-Circuit Stresses.**

The Institute recognizes the self-destructibility, both mechanical and thermal, of certain sizes and types of machines, when subjected to severe short-circuits, and recommends that ample protection be provided in such cases, external to the machine if necessary.

Over-Speeds.

399 **All Types of Rotating Machines** shall be so constructed that they will safely withstand an over-speed of 25 per cent, except in the case of steam turbines, which, when equipped with emergency governors, shall be constructed to withstand 20 per cent over-speed.

400 **In the case of Series Motors,** it is impracticable to specify percentage values for the guaranteed over-speed, on account of the varying service conditions.

401 **Water-wheel Generators** shall be constructed for the maximum runaway speed which can be attained by the combined unit.

Momentary Loads.

402 **Continuously Rated Machines** shall be required to carry momentary loads of 150 per cent of the amperes corresponding to the continuous rating, keeping the rheostat set for rated load excitation, (See §§**281**, **764** and **803**.) and commutating machinery shall commutate successfully under this condition. Successful commutation is such that neither brushes nor commutator are injured by the test. In the case of direct-connected generators, this clause is not to be interpreted as requiring the prime mover to drive the generator at this overload.

403 **Machines for duty-cycle operation** shall be rated according to their equivalent load, either on the short-time or continuous basis, but if intended for operation with widely fluctuating loads, shall commutate successfully under their specified operating conditions. See §§**284** and **285.**

404 **Stalling Torque of Motors**

Motors for continuous service shall, except when otherwise specified, be required to develop a running torque at least 175 per cent of that corresponding to the running torque at their rated load, without stalling.

Obviously, duty-cycle machines must carry their peak loads without stalling.

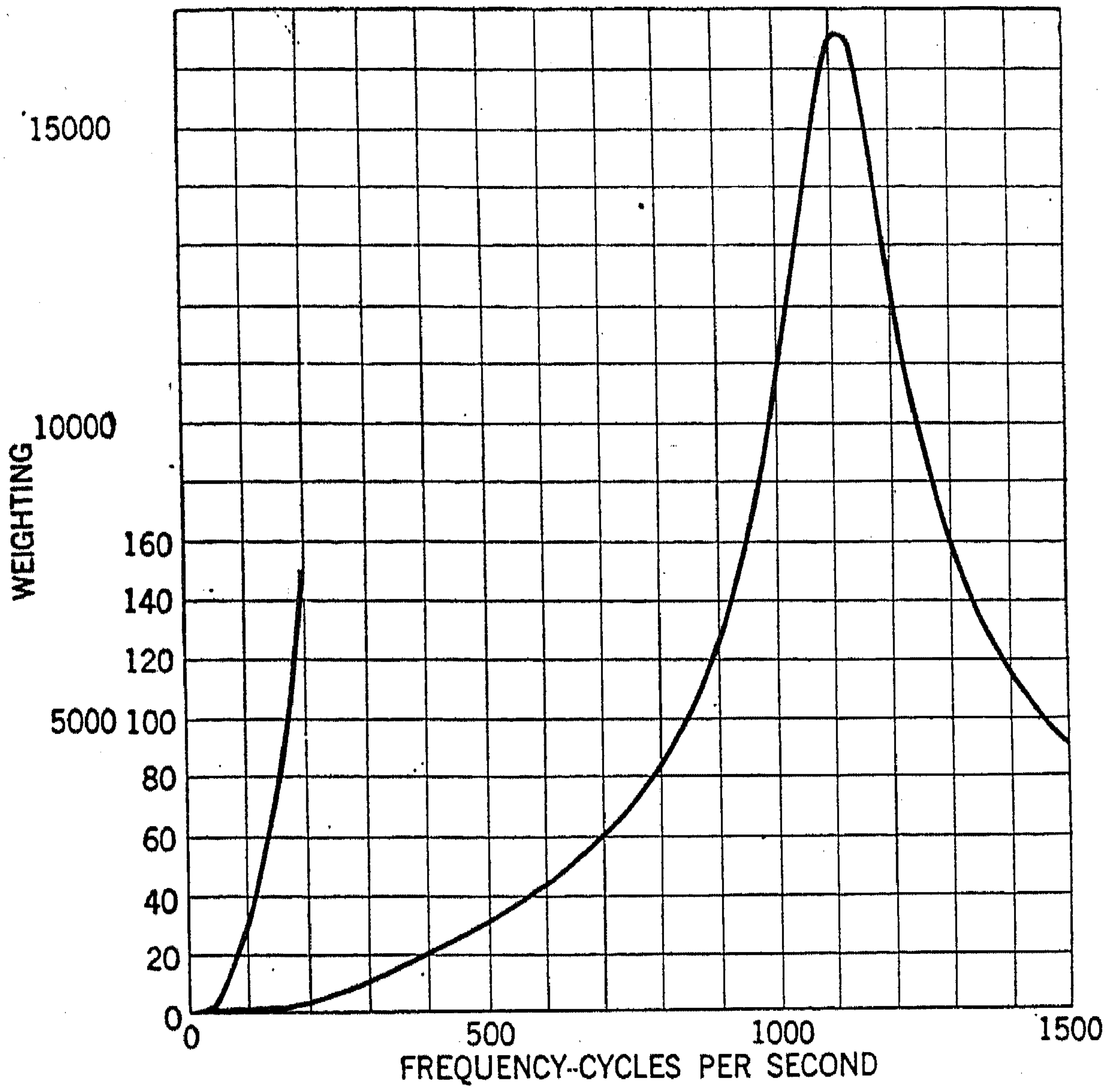
WAVE SHAPE

405 **The Sine Wave** shall be considered as standard, except where the departure therefrom is inherent in the operation of the system of which the machine forms a part.

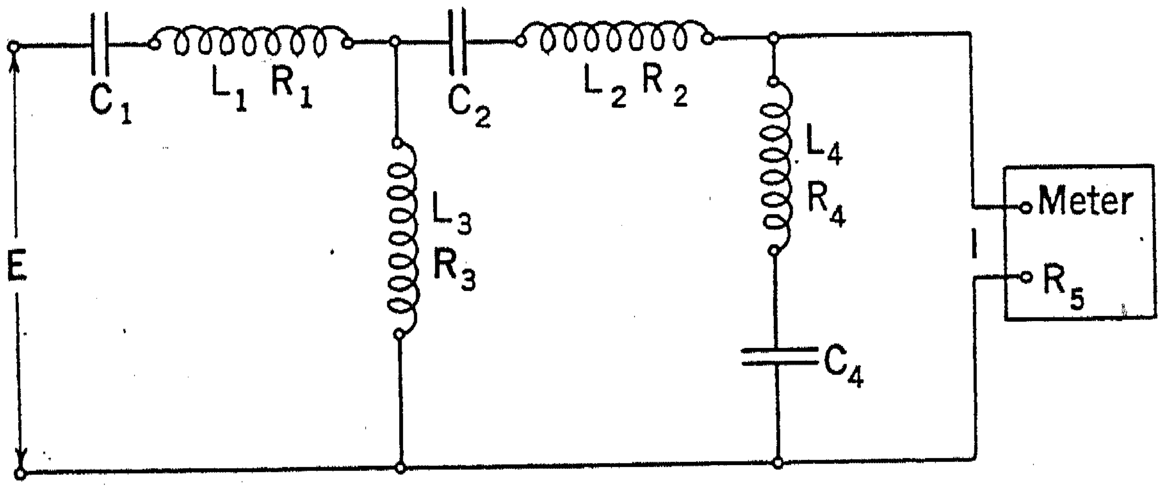
406 **Deviation Factor.**—A deviation factor (See §17) of the open-circuit terminal voltage wave of all synchronous machinery not exceeding 10% (0.10) is permissible except when otherwise specified.

407 **Telephone Interference Factor.**—The telephone interference factor is the ratio of the square root of the sum of the squares of the weighted values of all the sine-wave components (including in alternating waves both fundamental and harmonics) to the r.m.s. value of the wave. (See §**17a.**)

408 Relative Weighting of Components.—The weighting of the sine-wave components of different frequencies is as given in the following curve. (For trial only.)



The telephone interference factor of a voltage wave, corresponding to this weighting, may be measured by the use of the network shown in the following diagram.



Network Constants

$C_1=0.9$ mf.
$C_2=0.9$ "
$C_4=7.5$ "
±0.5%

$L_1=0.023$ henry
$L_2=0.0205$ "
$L_3=0.068$ "
$L_4=0.019$ "
±0.5%

$R_1=5$ ohms ±2%
$R_2=12$ " ±2%
$R_3=73$ " ±1%
$R_4=22.5$ " ±2%
$R_5=43$ " ±1%

With this network the telephone interference factor is the ratio of the current I in micro-amperes in the meter branch of the network to the voltage *E* applied to the external terminals of the network. The measurement may be made on the low tension side of a potential transformer. A sensitive vacuum thermocouple provided with a shunt, and a direct current mil-ammeter have been found convenient for measuring the current.

409 **Limiting Value of Telephone Interference Factor.**—A telephone interference factor (See §§**17a** and **407**) of the open-circuit terminal voltage wave not exceeding*———— is permissible except when otherwise specified. This rule shall apply in cases where synchronous machinery is required for use in connection with open-wire transmission or distribution circuits and shall be supplementary to §**406**. The purchaser shall in each case indicate whether or not the conditions are such that this section applies.

EFFICIENCY AND LOSSES

420 **Machine Efficiency** is the ratio of the power delivered by the machine to the power received by it.

421 **Plant or System Efficiency** is the ratio of the energy delivered from the plant or system to the energy received by it in a specified period of time.† In calculating plant or system efficiency it may be desirable to calculate the losses in each individual machine or part of the system at the actual temperature of that machine or part during the specified interval. These losses may be appreciably different from the losses at 75°C., which latter shall be the standard temperature of reference for all efficiency guarantees. See §**432**.

422 **In the Case of Machinery two Efficiencies are Recognized,** conventional efficiency (see §**423**) and directly measured efficiency. Unless otherwise specified, the conventional efficiency is to be employed. When the efficiency of a machine is stated without specific reference to the load conditions, rated load is always to be understood whether the efficiency be the conventional or directly measured efficiency.

423 **Conventional Efficiency** of machinery is the ratio of the output to the sum of the output and the losses; or of the input minus the losses to the input; when, in either case, conventional values are assigned to one or more of these losses. The need for assigning conventional values to certain losses, arises from the fact that some of the losses in electrical machinery are practically indeterminable, and must, in many cases, either be approximated by an approved method of test, or else values recommended by the Institute and designated "conventional" values shall be employed for them in arriving at the "conventional efficiency."

425 **Directly-Measured Efficiency.** Input and output determinations of efficiency may be made directly, measuring the output by brake, or equivalent, where applicable. Within the limits of practical application, the circulating-power method, sometimes described as the Hopkinson or "loading-back" method, may be used.

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*The appropriate limiting value of this factor has not yet been determined, but will be recommended after further experience shall have been obtained.

†An exception should be noted in the case of the efficiency of storage batteries.

426 **Values of the Indeterminate Losses** may also be obtained by brake or other direct test, and used in estimating actual efficiencies of similar machines, by the separate-loss method.

427 **Normal Conditions.** The efficiency shall correspond to, or be corrected to, the normal conditions herein set forth, which shall be regarded as standard. These conditions include voltage, current, power-factor, frequency, wave-shape, speed, temperature, or such of them as may apply in each particular case.

428 **Measurement of Efficiency.** Electric power shall be measured at the terminals of the apparatus. In polyphase machines, sufficient measurements shall be made on all phases to avoid errors of unbalance.

429 **Point at Which Mechanical Power Shall be Measured.** Mechanical power delivered by machines, shall be measured at the pulley, gearing, or coupling, on the rotor shaft, thus excluding the loss of power in the belt or gear friction. See, however, an **exception in §800.**

430 **The Efficiency Specified for Alternators and Transformers** shall be of the ratio of the kilowatt output to the kilowatt input at the rated kv-a. and power factor.

431 **Efficiency of Alternating-Current Machinery in regard to WaveShape.** In determining the efficiency of alternating-current machinery, the sine wave is to be considered as standard, unless a different wave form is inherent in the operation of the system. See **§405.**

432 **Temperature of Reference for Machine Efficiency.** The efficiency, at all loads, of all apparatus, shall be corrected to a reference temperature of 75°C, but tests may be made at any convenient ambient temperature, preferably not less than 15°C. See §§**348** and **445.**

433 **The losses in constant-potential machinery,** either of the stationary type, or of the constant-speed rotary type, are of two classes; namely, those which remain substantially constant at all loads, and those which vary with the load. The former include iron losses, windage and friction, also I^2R losses in any shunt windings. The latter include I^2R losses in series windings. The constant losses may be determined by measuring the power required to operate the machine at no load, deducting any series I^2R losses. The variable loss at any load may be computed from the measured resistance of the series windings and the given load current.

434 **Stray Load Losses.** The above simple method of determining the losses and hence the efficiency is only approximate, since the losses which are assumed to be constant do actually vary to some extent with the load, and also because the actual loss in the copper windings is sometimes appreciably greater than the calculated I^2R loss. The difference between the approximate losses, as above de-

termined, and the actual losses, is termed the "stray load losses"*. These latter are due to distortions in electric or magnetic fluxes from their no-load distributions or values, brought about by the load current. They are usually only approximately measurable, or may be indeterminable.

TABLE V

Classification of Losses in Machinery

435 Losses in machinery may be classified as follows:

Accurately Measurable or Determinable	*Approximately Measurable or Determinable*	*Indeterminable*
a. No-Load Core Losses including eddy-current losses in conductors at no-load	*c.* Brush Friction Loss	*h.* Iron Loss due to flux distortion.
b. Load I^2R losses in windings No-Load I^2R losses in windings	*d.* Brush-Contact Loss	*i.* Eddy-Current losses in conductors due to transverse fluxes occasioned by the load currents.
	e. Losses due to windage and to bearing friction	*k.* Eddy-Current losses in conductors due to tooth saturation resulting from distortion of the main flux.
	f. Extra copper loss in transformer windings, due to stray fluxes caused by load currents	*l.* Tooth-frequency losses due to flux distortion under load.
	g. Dielectric Losses.	*m.* Short-Circuit Loss of Commutation.

436 **Evaluation of Losses.** The larger individual losses are either accurately or approximately determinable, but certain of the indeterminable losses reach values in various kinds of machinery which require that they should be taken into account.

LOSSES TO BE TAKEN INTO ACCOUNT IN VARIOUS TYPES OF MACHINES

440 **Direct-Current Commutating Motors and Generators.**

Core losses. See §452.

I^2R loss in all windings.

Brush contact I^2R loss. Unless otherwise specified, use the Institute Standard of 1 volt for contact drop per brush; *i. e.*, 2 volts for total brush drop, either carbon or graphite brushes. (See §§**454** and **819**). In the case of low voltage automobile propulsion motors and generators this loss should be determined experimentally; see §**839.**

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*In Table V, the stray load losses include f, h, i, k, l and m; but do not include increased core losses due to increased excitation for compensating internal drop under load.

Friction of bearings and windage.

Rheostat losses, when present.

Brush friction.

All indeterminable load losses (including stray-load iron losses) which may be important, which vary with the design, and for which no satisfactory method of determination has been found, shall be included as zero per cent in estimating conventional efficiency.

441 Synchronous Motors and Generators.

Core losses. See §**452.**

I^2R loss in all windings, based upon rated k-va. and power factor.

Stray load-losses. In approximating these losses, the method described in §**458** shall be employed.

Friction of bearings and windage.

Brush friction and brush-contact loss is negligible, except in the case of revolving armature machines.

Rheostat losses, when present, corresponding to rated kv-a. and power factor.

442 Induction Machines.

Core losses. See §**452.**

I^2R losses in all windings.

Stray load-losses. In approximating these losses, the method described in §**459** shall be employed

Brush friction when collector rings are present.

Brush-contact loss. Unless otherwise specified, use the Institute Standard of 1 volt for contact drop per brush, for either carbon or graphite brushes. See §**454.**

Friction of bearings and windage.

443 Commutating A-C. Machines

Core losses. See §**452.**

I^2R losses in all windings.

Brush friction.

Brush-contact loss. Unless otherwise specified, use the Institute Standard of 1 volt for contact drop per brush, for either carbon or graphite brushes. See §**454** and **819.**

Friction of bearings and windage.

Short-circuit loss of commutation. Iron loss due to flux distortion. Eddy-current losses due to fluxes varying with load and saturation.	The Institute is not at this time prepared to make recommendations for approximating these losses.

444 **Synchronous Converters.**

Core losses. See §**452**.

I^2R losses in all windings, based on rated kw. and unity power factor. The I^2R losses in the armature winding shall be derived from those corresponding to its use as a direct-current generator, by using suitable factors.

Brush friction.

Rheostat losses when present, corresponding to rated kw. and unity power factor.

Brush-contact loss. Unless otherwise specified, use the Institute Standard of 1 volt for contact drop per brush, for either carbon or graphite brushes. See §**454**.

Short-circuit loss of commutation. Iron loss due to flux distortion when present. Eddy-current losses due to fluxes varying with load and saturation.	These losses, while usually of low magnitude, are erratic, and the Institute is not at this time prepared to make recommendations for approximating them.

Friction of bearings and windage.

For the booster type of synchronous converter, where the booster forms an integral part of the unit, its losses shall be included in the total converter losses in estimating the efficiency.

445 **Transformers**

No-load losses. These include the core loss, and the I^2R loss due to the exciting current, also the dielectric loss in the insulation. (See §**470**).

Load losses. These include I^2R losses, and stray load-losses due to eddy currents caused by fluxes varying with load. (See §**471**).

DETERMINATION OR APPROXIMATION OF LOSSES IN ROTATING MACHINERY

450 **Bearing Friction and Windage** may be determined as follows. Drive the machine from an independent motor. the output of which shall be suitably determined. The machine under test shall have its brushes removed and shall not be excited. This output represents the bearing friction and windage of the machine under test.

The bearing friction and windage of induction motors may be measured by running motors free at the lowest voltage at which they will rotate continuously at approximately rated speed; the watts input, minus I^2R loss, under these conditions being taken as the friction and windage.

In the case of engine-type generators, the windage and bearing friction loss is ordinarily very small,amounting to a fraction of one per cent of the output. In these rules this loss is neglected owing to its small value and the difficulty of measuring it.

451 **Brush Friction of Commutator and Collector Rings.** Follow the test of **§450,** taking an additional reading with the brushes in contact with the commutator or collector rings. The difference between the output obtained in the test in **§450** and this output shall be taken as the brush friction. Note: The surfaces of the commutator and brushes should already be smooth and glazed from running when this test is made.

452 **Core Loss.** Follow the test of §**451** with an additional reading taken in a similar manner, except that the machine is to be excited so as to produce at the terminals a voltage corresponding to the calculated internal voltage for the load under consideration. The difference between the output obtained by this test and that obtained by test under Sec. **451** shall be taken as the core loss.

> **For Synchronous Machines,** the internal voltage shall be determined by correcting the terminal voltage for the resistance drop only.
>
> **The Core Loss of Induction Motors** may be determined by measuring the watts input to the motor when running free at rated voltage and frequency and subtracting therefrom the no-load copper loss, bearing friction and windage.

454 **The Brush-Contact I^2R Loss** depends largely upon the material of which the brush is composed. As indicating the range of variation the following table will be of interest:

TABLE VI.

Brush-Contact Drop.

Grade of Brush	Volts drop across one brush-contact. (Average of positive and negative brushes)
Hard Carbon	1.1
Soft Carbon	0.9
Graphite	0.5 to 0.8
Metal-Graphite types	0.15 to 0.5 (The former for largest proportion of metal)

One volt drop per brush shall be considered as the Institute Standard drop corresponding to the I^2R brush-contact loss, for carbon and graphite brushes. Metal-graphite brushes shall be considered as special. See **§819.**

455 **Field-Rheostat Losses** shall be included in the generator losses where there is a field rheostat in series with the field magnets of the generator, even when the machine is separately excited.

456 **Ventilating Blower.** When a blower is supplied as part of a machine set, the power required to drive it shall be charged against the complete unit; but not against the machine alone.

457 **Losses in Other Auxiliary Apparatus.** Auxiliary apparatus, such as a separate exciter for a generator or motor, shall have its losses charged against the plant of which the generator and exciter are a part, and not against the generator. An exception should be noted in the case of turbo-generator sets with direct-connected exciters, in which case the losses in the exciter shall be charged against the generator. The actual energy of excitation and the field-rheostat losses, if any, (see §455) shall be charged against the generator.

458 **Stray Load-Losses in Synchronous Generators and Motors.** These include iron losses, and eddy-current losses in the copper, due to fluxes varying with load and also to saturation.

Stray load-losses are to be determined by operating the machine on short circuit and at rated-load current. This, after deducting the windage and friction and I^2R loss, gives the stray load-loss for polyphase generators and motors. These losses in single-phase machines are large; but the Institute is not yet prepared to specify a method for measuring them.

459 **Stray Load-Losses in Induction Machines.**

These include eddy-current losses in the stator copper, and other eddy-current losses due to fluxes varying with the load. In windings consisting of relatively small conductors, these eddy-current losses are usually negligible.

With rotor removed, measure the power input to the stator with different values of current at the rated frequency. The curve plotted with these values gives the combined I^2R and stray load-losses due to eddy currents in the stator copper. Deduct the I^2R loss determined from the resistance, and the difference will represent the stray load-losses corresponding to the various currents. While this method is not accurate for some types of motors it usually represents a sufficiently good approximation.

460 **Polyphase Induction-Motor Rotor I^2R Loss.** This should be determined from the slip, whenever the latter is accurately determinable, using the following equation:

$$\text{Rotor } I^2R \text{ loss} = \frac{\text{Output} \times \text{slip}}{1 - \text{slip}}$$

In large slip-ring motors, in which the slip cannot be directly measured by loading, the rotor I^2R loss shall be determined by direct resistance measurement; the rotor full-load current to be calculated by the following equation:

$$\text{Current per ring} = \frac{\text{watts output}}{\text{rotor voltage at stand-still} \times \sqrt{3} \times K}$$

This equation applies to three-phase rotors. For rotors wound for two phase, use 2 instead of the $\sqrt{3}$. K may be taken as 0.95 for motors of 150 kw. or larger. The factor K usually decreases as the size of motor is reduced, but no specific value can be stated for smaller sizes.

DETERMINATION OR APPROXIMATION OF LOSSES IN TRANSFORMERS

470 **No-Load Losses.** These shall be measured with open secondary circuit at the rated frequency, and with an applied primary voltage giving the rated secondary voltage plus the IR drop which occurs in the secondary under rated load conditions.

471 **Load Losses.** These include I^2R and stray load-losses. They shall be measured by applying a primary voltage, preferably at rated frequency, sufficient to produce rated load current in the windings, with the secondary windings short circuited.

DIELECTRIC STRENGTH TESTS OF MACHINERY

480 **Basis for Determining Test Voltages.** The test voltage which shall be applied to determine the suitability of insulation for commercial operation is dependent upon the kind and size of the machinery, and its normal operating voltage, upon the nature of the service in which it is to be used, and upon the severity of the mechanical and electrical stresses to which it may be subjected. The voltages and other conditions of test which are recommended, have been determined as reasonable and proper for the great majority of cases, and are proposed for general adoption, except when specific reasons make a modification desirable.

481 **Condition of Machinery to be Tested.** Commercial tests shall, in general, be made with the completely assembled machinery and not with individual parts. The machinery shall be in good condition, and high-voltage tests, unless otherwise specified, shall be applied before the machine is put into commercial service, and shall not be applied when the insulation resistance is low owing to dirt or moisture. High-voltage tests shall be made at the temperature assumed under normal operation or at the temperature attained under the conditions of commercial testing. High-voltage tests to determine whether specifications are fulfilled, are admissible on new machines only. Unless otherwise agreed upon, high-voltage tests of a machine shall be understood as being made at the factory.

482 **Points of Application of Voltage.** The test voltage shall be successively applied between each electric circuit and all other electric circuits and metal parts grounded.

483 **Interconnected Polyphase Windings** are considered as one circuit. All windings of a machine except that under test, shall be connected to ground.

484 **Frequency, Wave Form and Test Voltage.** The frequency of the testing circuit shall not be less than the rated frequency of the apparatus tested. A sine-wave form is recommended. See §405. The test shall be made with alternating voltage having a crest value equal to $\sqrt{2}$ times the specified test voltage. In d.c. machines, and in the general commercial application of a.c. machines, the testing frequency of 60 cycles per second is recommended.

485 **Duration of Application of Test Voltage.** The testing voltage for all classes of apparatus shall be applied continuously for a period of 60 seconds. See exception, **485A.**

485A *Exception*—**For All Standard Devices Produced in Large Quantities** and with a standard test pressure of 2500 volts or less, a test pressure applied for one second 20 per cent higher than the one-minute test pressure will be satisfactory.

486 **Apparatus for Use on Single-Phase, 3-Phase-Delta or 3-Phase-Star Circuits.** Apparatus, such as transformers, which may be used in star connection on three-phase circuits, shall have the delta voltage of the circuits on which they may be used indicated on the rating plate and the test shall be based on such delta voltage

VALUES OF A-C. TEST VOLTAGES

500 **The Standard Test for All Classes of Apparatus, Except‡ as Otherwise Specified, Shall be Twice the Normal Voltage of the Circuit to Which the Apparatus is Connected, Plus 1000 Volts.**

501 *Exception*—**Alternating-Current Apparatus connected to Permanently Grounded Single-Phase Systems, for use on Permanently Grounded Circuits of more than 300 Volts,** shall be tested with 2.73 times the voltage of the circuit to ground + 1000 volts. **This** does not refer to three-phase apparatus with grounded star neutral.

502 *Exception*—**Distributing Transformers.** Transformers for primary pressures from 550 to 4500 volts, the secondaries of which are directly connected to consumers' circuits and commonly known as distributing transformers, shall be tested with 10,000 volts from primary to core and secondary combined. The secondary windings shall be tested with twice their normal voltage plus 1,000 volts.

503 *Exception*—**Auto-Transformers** used for starting purposes, shall be tested with the same voltage as the test voltage of the apparatus to which they are connected.

504 *Exception*—**Household Devices.**—Apparatus taking not over 660 watts† and fractional horsepower a-c. and d-c. motors in capacities of less than ½ h. p., or 373 watts, intended solely for operation on supply circuits not exceeding 275 volts, shall be tested with 900 volts, except in the case of heating devices, which latter shall be tested with 500 volts at operating temperature.

505 *Exception*—**Apparatus for use on Circuits of 25 Volts or Lower,** such as bell-ringing apparatus,* electrical apparatus used in automobiles, apparatus used on low-voltage battery circuits, etc., shall be tested with 500 volts.

506 *Exception*—**Field Windings of Alternating-Current Generators** shall be tested with 10 times the exciter voltage, but in no case with less than 1500 volts nor more than 3500 volts.

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†The present National Electric Code power limit for a single outlet.

*This rule does not include bell-ringing transformers of ratio 125 to 6 volts. See National Electric Code.

‡Exceptiocs §§501 to 512.

507 *Exception*—Field Windings‡ of Synchronous Machines, including motors and converters which are to be started from alternating-current circuits, shall be tested as follows:

a. When machines are started with fields short-circuited they shall be tested as specified in §**506**.

b. When machines are started with fields open-circuited and sectionalized while starting, they shall be tested with 5000 volts.

c. When machines are started with fields open-circuited and connected all in series while starting, they shall be tested with 5000 volts for less than 275-volt excitation and 8000 volts for excitation of 275 volts to 750 volts.

508 *Exception*. **Phase-Wound Rotors of Induction Motors.** The secondary windings of wound rotors of induction motors shall be tested with twice their normal induced voltage, plus 1000 volts. By normal induced voltage is here meant the voltage between slip rings on open circuit at standstill with normal voltage impressed on the primary.

When induction motors with phase-wound rotors are reversed, while running at approximately normal speed, by reversing the primary connections, the test shall be four times the normal induced voltage, plus 1000 volts.

509 *Exception*—**Switches and Circuit Control Apparatus** above **600** volts, shall be tested with 2½ times rated voltage, plus 2000 volts. See §§**720** to **759b**.

510 *Exception*—**Assembled Apparatus.** Where a number of pieces of apparatus are assembled together and tested as an electrical unit, they shall be tested with 15 per cent lower voltage than the lowest required on any of the individual pieces of apparatus.

510A *Exception*. **Meters and Instruments.** The Institute is not at present in a position to make a recommendation in regard to the dielectric tests of meters and instruments.

511 **Testing Transformers by Induced Voltage.** Under certain conditions it is permissible to test transformers by inducing the required voltage in their windings, in place of using a separate testing transformer. By "required voltage", is meant a voltage such that the line end of the windings shall receive a test to ground equal to that required by the general rules.

512 **Transformers with Graded Insulation** shall be so marked. They shall be tested by inducing the required test-voltage in the transformer and connecting the successive line leads to ground.

Transformer windings permanently grounded within the transformer shall be tested by inducing the required test voltage in such windings. (See §**500**).

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‡Series field coils should be regarded as part of the armature circuit and tested as such.

MEASUREMENT OF VOLTAGE IN DIELECTRIC TESTS OF MACHINERY

530 **Use of Voltmeters and Spark-Gaps in Insulation Tests.** When making insulation tests on electrical machinery. every precaution must be taken against the occurrence of any spark-gap discharges in the circuits from which the machinery is being tested. A non-inductive resistance of about one ohm per volt shall be inserted in series with one terminal of the spark gap. If the test is made with one electrode grounded, this resistance shall be inserted directly in series with the non-grounded electrode. If neither terminal is grounded, one-half shall be inserted directly in series with each electrode. In any case this resistance shall be as near the measuring gap as possible and not in series with the tested apparatus. The resistance will damp high-frequency oscillations at the time of breakdown and limit the current which will flow. A water tube is the most reliable form of resistor. Carbon resistors should not be used because their resistance may become very low at high voltages.

531 **For Machinery of Low Capacitance.** When the machinery under test does not require sufficient charging current to distort the high-voltage wave shape, or change the ratio of transformation, the spark gap should be set for the required test voltage and the testing apparatus adjusted to give a voltage at which this spark gap just breaks down. This adjustment should be made with the apparatus under test disconnected. The apparatus should then be connected, and with the spark gap about 20 per cent longer, the testing apparatus is again adjusted to give the voltage of the former breakdown, which is the assumed voltage of test. This voltage is to be maintained for the required interval.

532 **For Machinery of High Capacitance.** When the charging current of the machinery under test may appreciably distort the voltage wave or change the effective ratio of the testing transformer, the first adjustment of voltage with the gap set for the test voltage should be made with the apparatus under test connected to the circuit and in parallel with the spark gap.

When making arc-over tests of large insulators, leads, etc. partial arc-over of the tested apparatus may produce oscillations which will cause the measuring gap to discharge prematurely. The measured voltage will then appear too high. In such tests the "equivalent" ratio" of the testing transformer should be measured by gap to within 20% of the arc-over voltage of the tested apparatus with the tested apparatus in circuit. The measuring gap should then be greatly lengthened out and the voltage increased until the tested apparatus arcs over. This arc-over voltage should then be determined by multiplying the voltmeter reading by the equivalent ratio found above. Direct measurement of the spark-over voltage over one gap by another gap should always be avoided.

533 **Measurements with Voltmeter.** In measuring the voltage with a voltmeter, the instrument should preferably derive its

voltage from the high-tension circuit, either directly, or by means of a voltmeter coil placed in the testing transformer, or through an auxiliary *ratio transformer.* It is permissible to measure the voltage at other places, such as the transformer primary provided corrections can be made for the variations in ratio caused by the charging current of the machinery under test, or provided there is no material variation of this ratio. In any case, when the capacitance of the apparatus to be tested is such as to cause wave distortion, the testing voltage must be checked by a spark gap as set forth in §**538**, or by a crest voltage meter. If the crest-voltage meter is calibrated in crest volts, its readings must be reduced to the corresponding r. m. s. sinusoidal value by dividing with $\sqrt{2}$.

534 **Measurements with Spark Gaps.** If proper precautions are observed, spark gaps may be used to advantage in checking the calibration of voltmeters when set up for the purposes of high-voltage tests of the insulation of machinery.

535 **Ranges of Voltages.** For the calibrating purposes set forth in §**534** the sphere-gap shall be used for voltages above 50 kv., and is to be preferred down to 30 kv. The needle spark-gap may, however, be used for voltages from 10 to 50 kv.

536 **The Needle Spark Gap.** The needle spark gap shall consist of new sewing needles, supported axially at the ends of linear conductors which are at least twice the length of the gap. There must be a clear space around the gap for a radius of at least twice the gap length.

537 The sparking distances in air between No. 00 sewing needle points for various root-mean-square sinusoidal voltages are as follows:

TABLE VII.

Needle-Gap Spark-Over Voltages

(At 25°C and 760 mm. barometer).

R M S Kilovolts	Millimeters	R.M.S. Kilovolts	Millimeters
10	11.9	35	51
15	18.4	40	62
20	25.4	45	75
25	33	50	90
30	41		

The above values refer to a relative humidity of 80 per cent. Variations from this humidity may involve appreciable variations in the sparking distance.

538 **The Sphere Spark-Gap.** The standard sphere spark-gap shall consist of two suitably mounted metal spheres. When used as specified below, the accuracy obtainable should be approximately 2 per cent.

No extraneous body, or external part of the circuit, shall be nearer the gap than twice the diameter of the spheres. By the "gap" is meant the shortest path between the two spheres.

The shanks should not be greater in diameter than 1/5th the sphere diameter. Metal collars, etc., through which the shanks extend, should be as small as practicable and should not, during any measurement, come closer to the sphere than the maximum gap length used in that measurement.

The sphere diameter should not vary more than 0.1 per cent and the curvature, measured by a spherometer, should not vary more than 1 per cent from that of a true sphere of the required diameter.

539 **In using the spherometer** to measure the curvature, the distance between the points of contact of the spherometer feet should be within the following limits:

TABLE VIII

Spherometer Specifications

Diameter of Sphere in m.m.	Distance between contact points in mm.	
	Maximum	Minimum
62.5	35	25
125	45	35
250	65	45
500	100	65

539A **In using Sphere Gaps** constructed as above, it is assumed that the apparatus will be set up for use in a space comparatively free from external dielectric fields. Care should be taken that conducting bodies forming part of the circuit, or at circuit potential, are not so located with reference to the gap that their dielectric fields are superposed on the gap; *e.g.*, the protecting resistance should not be arranged so as to present large masses or surfaces near the gap, even at a distance of two sphere diameters.

In case the sphere is grounded, the spark point of the grounded sphere should be approximately five diameters above the floor or ground

540 **The sparking distances between different spheres** for various r.m.s. sinusoidal voltages shall be assumed to be as follows:

TABLE IX.

Sphere-Gap Spark-Over Voltages

(At 25°C and 760 mm. barometric pressure)

Kilo-volts	Sparking Distance in Millimeters.							
	62.5 mm. spheres		125 mm. spheres		250 mm. spheres		500 mm. spheres	
	One sphere grounded	Both spheres insulated	One sphere grounded	Both spheres insulated	One sphere grounded	Both spheres insulated	One sphere grounded	Both spheres insulated
10	4.2	4.2	..	..	..	..	..	..
20	8.6	8.6	..	..	..	..	..	..
30	14.1	14.1	14.1	14.1	..	..	..	..
40	19.2	19.2	19.1	19.1	..	..	..	..
50	25.5	25.0	24.4	24.4	..	..	..	..
60	34.5	32.0	30.	30.	29	29	..	..
70	46.0	39.5	36	36	35	35	..	..
80	62.0	49.0	42	42	41	41	41	41
90		60.5	49	49	46	45	46	45
100			56	55	52	51	52	51
120			79.7	71	64	63	63	62
140			108	88	78	77	74	73
160			150	110	92	90	85	83
180				138	109	106	97	95
200					128	123	108	106
220					150	141	120	117
240					177	160	133	130
260					210	180	148	144
280					250	203	163	158
300						231	177	171
320						265	194	187
340							214	204
360							234	221
380							255	239
400							276	257

The sphere gap is more sensitive than the needle gap to momentary rises of voltage and the voltage required to spark over the gap should be obtained by slowly closing the gap under constant voltage, or by slowly raising the voltage with a fixed setting of the gap. Open arcs should not be permitted in proximity to the gap during its operation, as they may affect its calibration.

AIR-DENSITY CORRECTION-FACTORS FOR SPHERE GAPS

541 **The Spark-Over Voltage,** for a given gap, decreases with decreasing barometric pressure and increasing temperature. This correction may be considerable at high altitudes.

The spacing at which it is necessary to set a gap to spark over at some required voltage, is found as follows: Divide tne required **voltage by the** correction factor given below in Table X. A new voltage is thus obtained. The spacing on the standard curves obtained from Table IX, corresponding to this new voltage, is the required spacing.

The voltage at which a given gap sparks over is found by taking the voltage corresponding to the spacing from the standard curves of Table IX, and multiplying by the correction factor.

When the variation from sea level is not great, the relative air density may be used as the correction factor; when the variation is great, or greater accuracy is desired, the correction factor corresponding to the relative air density should be taken from Table X below, in which

$$\text{Relative air density} = \frac{0.392\, b}{273 + t}$$

b = barometric pressure in mm.
t = temperature in deg. C.

Corrected curves may be plotted for any given altitude, if desired.

Values of relative air density and corresponding values of the correction factor are tabulated below. It will be seen that for values above .9, the correction factor does not differ greatly from the relative air density.

TABLE X.

Air-Density Correction Factors for Sphere Gaps

Relative air density	Diameter of standard spheres in mm.			
	62.5	125	250	500
0.50	0.547	0.535	0.527	0.519
0.55	0.594	0.583	0.575	0.567
0.60	0.640	0.630	0.623	0.615
0.65	0.686	0.677	0.670	0.663
0.70	0.732	0.724	0.718	0.711
0.75	0.777	0.771	0.766	0.759
0.80	0.821	0.816	0.812	0.807
0.85	0.866	0.862	0.859	0.855
0.90	0.910	0.908	0.906	0.904
0.95	0.956	0.955	0.954	0.952
1.00	1.000	1.000	1.000	1.000
1.05	1.044	1.045	1.046	1.048
1.10	1.090	1.092	1.094	1.096

INSULATION RESISTANCE OF MACHINERY

550 **The insulation resistance of a machine** at its operating temperature shall be not less than that given by the following formula:

$$\text{Insulation Resistance in megohms} = \frac{\text{voltage at terminals}}{\text{rated capacity in kv-a.} + 1000}$$

The formula only applies to dry apparatus. Such high values are not attainable in oil-immersed apparatus.

Insulation resistance tests shall, if possible, be made at a d.c. pressure of 500 volts. Since the insulation resistance varies with the pressure, it is necessary that, if a pressure other than 500 volts is to be employed in any case, this other pressure shall be clearly specified.

The order of magnitude of the values obtained by this rule is shown in the following table:

TABLE XI.

Insulation Resistance of Machinery

Rated Voltage of machine	Megohms		
	100 kv-a.	1000 kv-a.	10,000 kv-a.
100	0.091	0.05	—
1,000	0.91	0.50	0.091
10,000	9.1	5.0	0.91
100,000	—	50	9.1

551 It should be noted that the insulation resistance of machinery is of doubtful significance by comparison with the dielectric strength. The insulation resistance is subject to wide variation with temperature humidity and cleanliness of the parts. When the insulation resistance falls below that corresponding to the above rule, it can, in most cases of good design and where no defect exists, be brought up to the required standard by cleaning and drying out the machine. The insulation-resistance test may therefore afford a useful indication as to whether the machine is in suitable condition for the application of the dielectric test.

REGULATION

DEFINITIONS

560 **Regulation.** The regulation of a machine in regard to some characteristic quantity (such as terminal voltage or speed) is the change in that quantity occurring between any two loads. Unless otherwise specified, the two loads considered shall be zero load and rated load, and at the temperature attained under normal operation. The regulation may be expressed by stating the numerical values of the quantity at the two loads, or it may be ex-

pressed by the "percentage regulation", which is the percentage ratio of the change in the quantity occurring between the two loads, to the value of the quantity at either one or the other load, taken as the normal value. It is assumed that all parts of the machine affecting the regulation maintain constant temperature between the two loads, and where the influence of temperature is of consequence, a reference temperature of 75°C shall be considered as standard. If change of temperature should occur during the tests, the results shall be corrected to the reference temperature of 75°C.

The normal value may be either the no-load value, as the no-load speed of induction motors; or it may be the rated-load value, as in the voltage of a.c. generators.

It is usual to state the regulation of d-c. generators by giving the numerical values of the voltage at no load and rated load, and in some cases it is advisable to state regulation at intermediate loads.

561 The **Regulation of d-c. Generators** refers to changes in voltage corresponding to gradual changes in load and does not relate to the comparatively large momentary fluctuations in voltage that frequently accompany instantaneous changes in load.

In determining the regulation of a compound-wound d-c. generator, two tests shall be made, one bringing the load down and the other bringing the load up, between no-load and rated load. These may differ somewhat, owing to residual magnetism. The mean of the two results shall be used.

562 In **constant-potential a-c. generators,** the regulation is the rise in voltage (when the specified load at specified power factor is reduced to zero) expressed in per cent of normal rated-load voltage.

563 In **constant-current machines,** the regulation is the ratio of the maximum difference of current from the rated-load value (occurring in the range from rated-load to short-circuit, or minimum limit of operation), to the rated-load current.

564 In **constant-speed direct-current motors,** and induction motors, the regulation is the ratio of the difference between full-load and no-load speeds to the no-load speed.

565 In **constant-potential transformers,** the regulation is the difference between the no-load and rated-load values of the secondary terminal voltage at the specified power factor (with constant primary impressed terminal voltage) expressed in per cent of the rated-load secondary voltage, the primary voltage being adjusted to such a value that the apparatus delivers rated output at rated secondary voltage.

566 In **converters, dynamotors, motor-generators and frequency converters,** the regulation is the change in the terminal voltage of the output side between the two specified loads. This may be expressed by giving the numerical values, or as the percentage of the terminal voltage at rated load.

567 In **transmission lines, feeders etc.,** the regulation is the change in the voltage at the receiving and between rated non-inductive load and no load, with constant impressed voltage upon the sending

end. The percentage regulation is the percentage change in voltage to the normal rated voltage at the receiving end.

568 In **steam engines, steam turbines and internal combustion engines,** the percentage speed regulation is usually expressed as the percentage ratio of the maximum variation of speed, to the rated-load speed in passing slowly from rated load to no load (with constant conditions at the supply.)

569 If the test is made by passing suddenly from rated load to no load, the immediate percentage speed regulation so derived shall be termed the **fluctuation.**

570 In a **hydraulic turbine,** or other water motor, the percentage speed regulation is expressed as the percentage ratio of the maximum variation in speed in passing slowly from rated load to no load (at constant head of water), to the rated-load speed.

571 In a **generator unit,** consisting of a generator combined with a prime mover, the speed or voltage regulation should be determined at constant conditions of the prime mover; *i.e.* constant steam-pressure, head, etc. It includes the inherent speed variations of the prime mover. For this reason, the regulation of a generator unit is to be distinguished from the regulation of either the prime mover, or of the generator combined with it, when taken separately.

CONDITIONS FOR TESTS OF REGULATION

580 **Speed and Frequency.** The regulation of generators is to be determined at constant speed, and of alternating-current apparatus at constant frequency.

581 **Power Factor.** In apparatus generating, transforming or transmitting alternating currents, the power factor of the load to which the regulation refers should be specified. Unless otherwise specified, it shall be understood as referring to non-inductive load, that is to a load in which the current is in phase with the *e.m f.* at the output side of the apparatus.

582 **Wave Form.** In the regulation of alternating-current machinery receiving electric power, a sine wave of voltage is assumed, except where expressly specified otherwise. See **§405.**

583 **Excitation.** In commutating machines, rectifying machines, and synchronous machines, such as direct-current generators and motors, as well as in alternating-current generators, the regulation is to be determined under such conditions as to maintain the field adjustment constant at that which gives rated-load voltage at rated-load current, as follows:

(1) In the case of separately-excited field magnets—constant excitation.

(2) In the case of shunt machines, constant resistance in the shunt-field circuit.

(3) In the case of series or compound machines, constant resistance shunting the series-field windings.

584 **Tests and Computation of Regulation of A-C. Generators.**

Any one of the three following methods may be used. They are given in the order of preference.

Method a.

The regulation can be measured directly, by loading the generator at the specified load and power factor, then reducing the load to zero, and measuring the terminal voltage, with speed and excitation adjusted to the same values as before the change. This method is not generally applicable for shop tests, particularly on large generators, and it becomes necessary to determine the regulation from such other tests as can be readily made.

585 **Method b.**

This consists in computing the regulation from experimental data of the open-circuit saturation curve and the zero-power-factor saturation curve. The latter curve, or one approximating very closely to it, can be obtained by running the generator with over excitation on a load of idle-running under-excited synchronous motors. The power factor under these conditions is very low and the load saturation curve approximates very closely the zero power factor saturation curve. From this curve and the open circuit curve, points for the load saturation curve, for any power factor, can be obtained by means of vector diagrams.

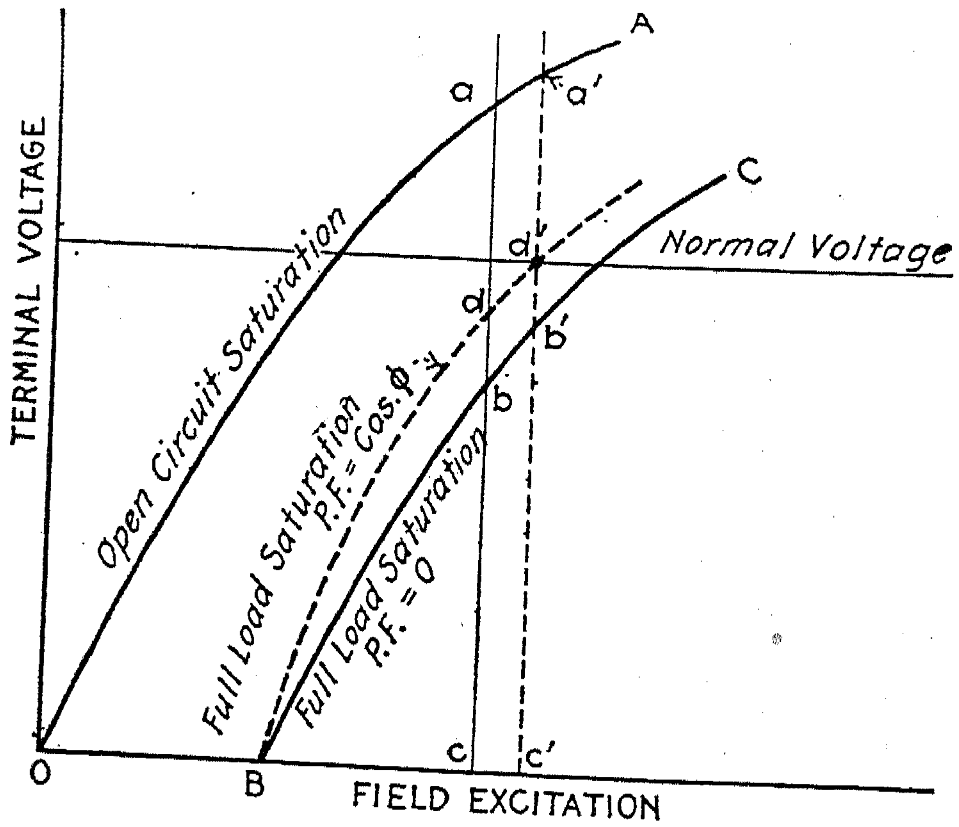

FIG. 1

To apply Method b, it is necessary to obtain from test, the open-circuit saturation curve *OA*, Fig. 1, and the saturation curve *BC* at zero power factor and rated-load current. At any given excitation *Oc*, the voltage that would be induced on open circuit is *ac*, the terminal voltage at zero power factor is *bc*, and the apparent internal drop is *ab*. The terminal voltage *dc* at any other power factor can then be found by drawing an e.m.f. diagram* as in Fig. 2, where ϕ is an angle such that *cos* ϕ is the power factor of the load, *b e* the resistance drop (IR) in the stator winding, *ba* the total internal drop, and *ac* the total induced voltage; *ba* and *ac* being laid off to correspond with the values obtained from Fig. 1. The terminal voltage at power factor *cos* ϕ, is then *cb* of Fig. **2,** which, laid off in Fig. 1, gives point *d.* By finding a number of such points, the curve *Bdd'* for power factor *cos* ϕ is obtained and the regulation at this power factor (expressed in per cent) is $\frac{100\times a'd'}{d'\,c'}$, since $a'\ d'$ is the rise in voltage when the load at power factor *cos* ϕ is thrown off at normal voltage $c'\ d'$.

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*Method b, for deducing the load saturation curve, at any assigned power factor, from no-load and zero power-factor saturation curves obtained by test, must be regarded as empirical. Its value depends upon the fact that experience has demonstrated the reasonable correctness of the results obtained by it.

Generally, the ohmic drop can be neglected, as it has very little influence on the regulation, except in very low-speed machines where the armature resistance is relatively high, or in some cases where regulation at unity power factor is being estimated. For low power factors, its effect is negligible in practically all cases. If resistance is neglected, the simpler e.m.f. diagram, Fig. 3, may be used to obtain points on the load saturation curve for the power factor under consideration.

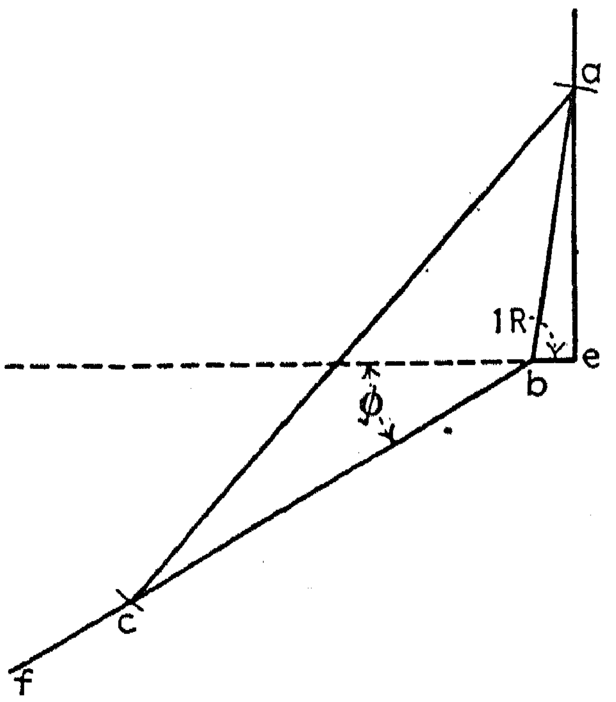


FIG. 2

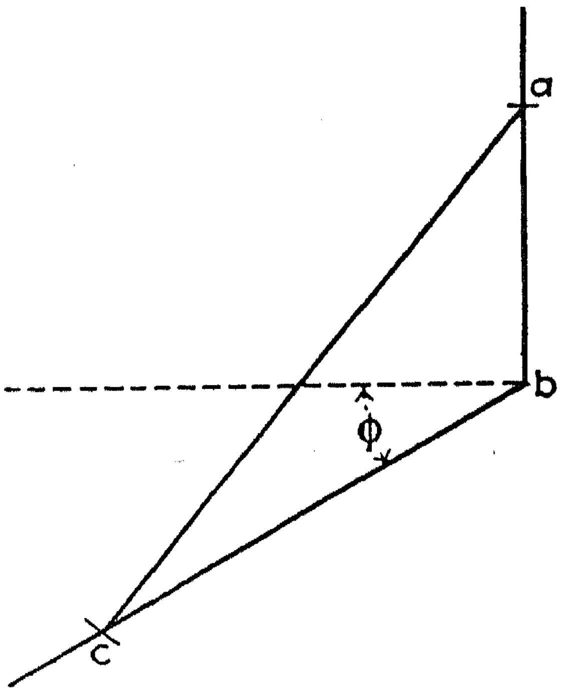


FIG. 3

586 **Method c.**

Where it is not possible to obtain by test a zero-power-factor saturation curve as in Method b, this curve can be estimated closely from open-circuit and short-circuit curves, by reference to tests at zero power factor on other machines of similar magnetic circuit. Having obtained the estimated zero-power-factor curve, the load saturation for any other power factor is obtained as in method b.

Thus Method c is the same as Method b; except that the zero-power-factor curve must be estimated. This may be done as follows. In Fig. 4, *OA* is the open-circuit saturation curve and *OE* the short-circuit line as shown by test. The zero-power-factor curve corresponding to any given current *BF* will start from point *B*, and for machines designed with low saturation and low reactance, will follow parallel to *OA* as shown by the dotted curve *BD*, which is *OA* shifted horizontally parallel to itself by the distance *OB*. In high-speed machines, or in others having low reactance and a low degree of saturation in the magnetic circuit, the zero-power-factor curve will lie quite close to *BD*, particularly in those parts that are used for determining the regulation. This is the case with many turbo-generators and high-speed water-wheel generators. In many cases, however, the zero-power-factor curve will deviate from *BD*, as shown by *BC*, and the deviation will be most pronounced in machines of high reactance, high saturation, and large magnetic leakage. The position of the actual curve *BC* with relation to *BD*, can be approximated with sufficient exactness by investigating the corresponding relation as obtained by test at zero power factor on machines of similar characteristics and magnetic circuit. Or curve *BC* can be calculated by methods based on the results of tests at zero power factor. After *BC* has been obtained, the saturation curve and regulation for any other power factor can be derived as in Method (b).

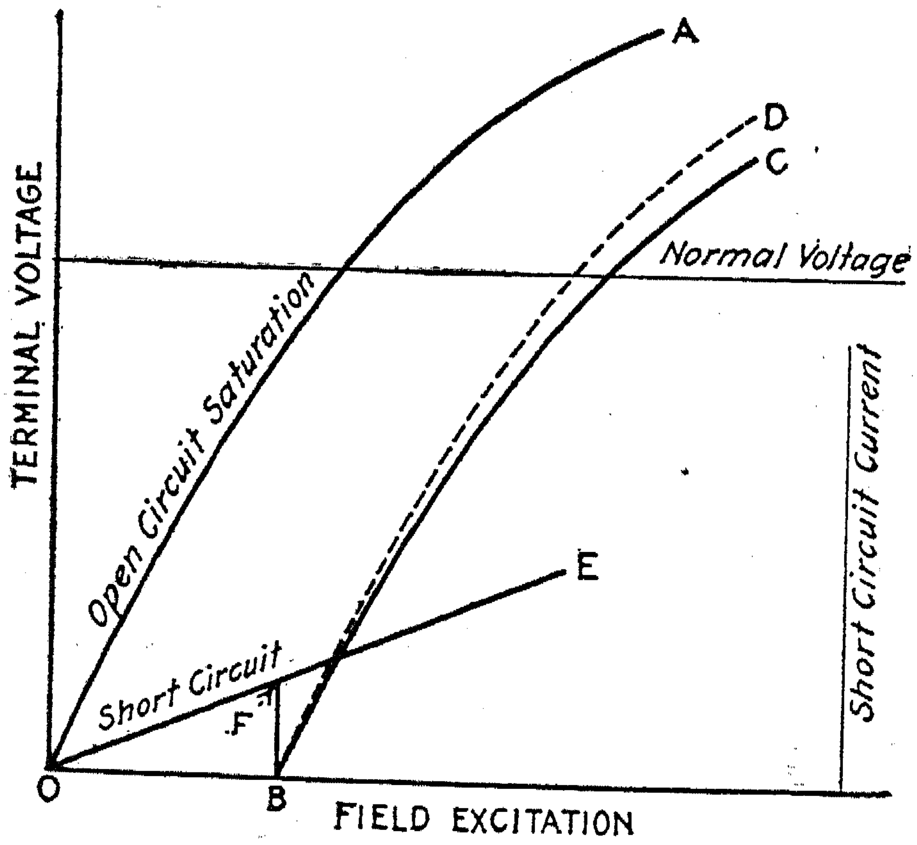


FIG. 4

587 **Tests and Computation of Regulation for Constant-Potential Transformers.**

The regulation can be determined by loading the transformer and measuring the change in voltage with change in load, at the specified power factor. This method is not generally applicable for shop tests, particularly on large transformers.

The regulation for any specified load and power factor can be computed from the measured impedance watts and impedance volts, as follows:

Let:

P = impedance watts, as measured in the short-circuit test and corrected to 75°C.
E_z = impedance volts, as measured in the short-circuit test.
IX = Reactance Drop in Volts.
I = Rated Primary Current.
E = Rated Primary Voltage.

q_r = percent drop in phase with current.
q_x = percent drop in quadrature with current

$$IX = \sqrt{E_x^2 - \left(\frac{P}{I}\right)^2}$$

$$q_r = 100\frac{P}{EI}$$

$$q_x = 100\frac{IX}{E}$$

588 Then—

1. For **unity power factor**, we have approximately:-

$$\text{Per cent regulation} = q_r + \frac{q^2_x}{200}$$

589 2. For **inductive loads** of power-factor m and reactive-factor n,

$$\text{Per cent regulation} = mq_r + nq_x + \frac{(mq_x - nq_r)^2}{200}$$

TRANSFORMER CONNECTIONS

(These rules do not apply to auto-transformers)

600 **Scope.** These rules specify the markings of leads brought out of the case but not the markings of winding terminals inside of the case, except that these terminals shall be marked with numbers in any manner that will permit of convenient reference and that cannot be confused with the markings of the leads brought out of the case.

NOTE—It is recognized that special cases will arise from time to time that these rules will not cover and that it would be very difficult to cover by any set of general rules.

601 **Markings of Leads.** (a) **In General.** The leads shall be distinguished from one another by marking each lead with a capital letter followed by a number. The letters to be used are: *H* for high-voltage leads, *X* for low-voltage leads and *Y* for tertiary-winding leads. The numbers to be used are **1, 2, 3,** etc.

NOTE.—By "tertiary winding" is meant a third winding that, compared with both of the other two windings, has smaller kv-a. rating than either or, if the kv-a. rating is the same as one or both of the other two, has lower voltage.

E. g., if a transformer has three separate windings, one for 1000 kv-a., 33,000 volts, one for 600 kv-a., 550 volts and one for 400 kv-a. 6,600 volts, the 400-kv-a. winding is the tertiary winding.

Or, if a transformer has three separate windings each with a capacity of 1,000 kv-a., and with voltages or 33,000, 6,600 and 550 respectively, the 550 volt winding is the tertiary winding.

According to this definition neither one of two similar windings arranged for series-parallel connection is to be classed as a tertiary winding.

(b) **A neutral Lead** shall be marked with the proper letter followed by *O*, *e.g.*, *HO*, *XO*.

EXCEPTION.—A lead brought out from the middle of a winding for some other use, than that of neutral lead, *e.g.*, a 50 per cent starting tap, shall be marked as a tap lead.

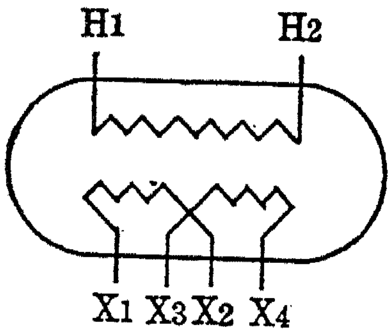
602 **Diagrammatic Sketch of Connections.** The manufacturer shall furnish with each transformer a complete diagrammatic sketch showing the leads and internal connections and their markings and the voltages obtainable with the various connections.

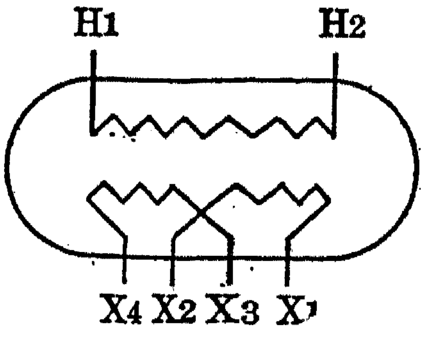
This sketch should preferably be on a metal plate attached to the transformer case.

SINGLE PHASE TRANSFORMERS

603 **Order of Numbering Leads in any Winding.** The leads of any winding (high-voltage, low-voltage or tertiary) brought out of case shall be numbered 1, 2, 3, 4, 5, etc., the lowest and highest numbers marking the full winding and the intermediate numbers marking fractions of winding or taps. All numbers shall be so applied that the potential difference from any lead having a lower number toward any lead having a higher number shall have the same sign at any instant.

If a winding is divided into two or more parts for series parallel connections, and the leads of these parts are brought out of case, the above rule shall apply for the series connection with the addition that the leads of each portion of winding shall be given consecutive numbers.





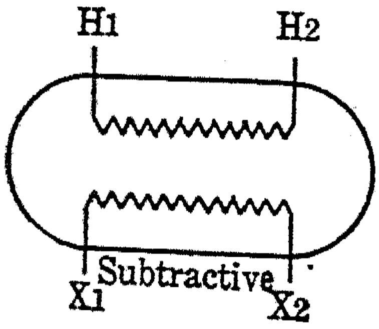
604 **Relation of Order of Numbering Leads of Different Windings.** The numbering of the high-voltage and low-voltage leads shall be so applied that when H_1 and X_1 are connected together and voltage applied to the transformer, the voltage between the highest numbered H lead and the highest numbered X lead shall be less than the voltage of the full high-voltage winding.

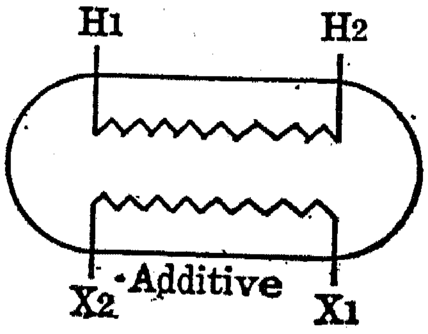
The same relation shall apply between high-voltage and tertiary and low-voltage and tertiary winding.

605 **Polarity.** When leads are marked in accordance with the above rules, the polarity of a transformer is

Subtractive when H_1 and X_1 are adjacent.

Additive when H_1 is diagonally located with respect to X_1.





606 **Location of H_1 Lead.** To simplify the work of connecting transformers in parallel it is recommended that the H_1 lead shall be brought out on the right hand side of the case, facing high-voltage side of the case.

607 **Parallel Operation.** Transformers having leads marked in accordance with these rules may be operated in parallel by connecting

similarly marked leads together, provided their ratio, voltages, resistances and reactances are such as to permit parallel operation.

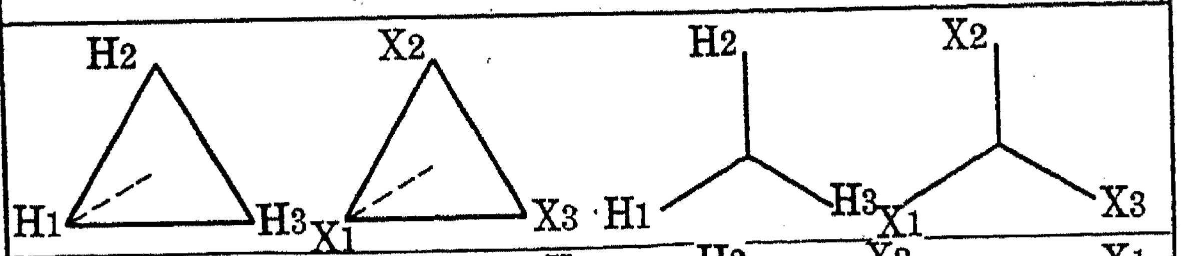
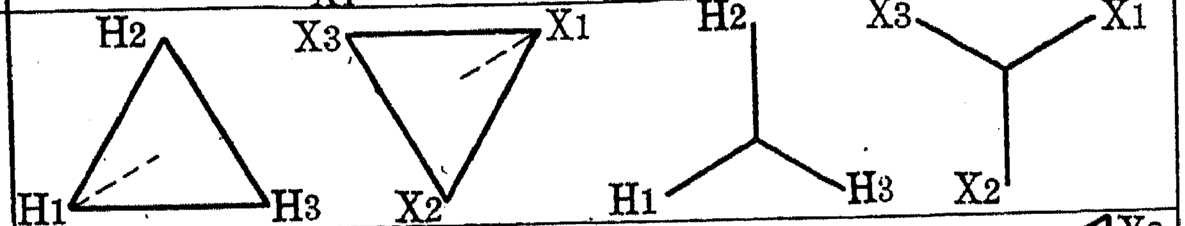
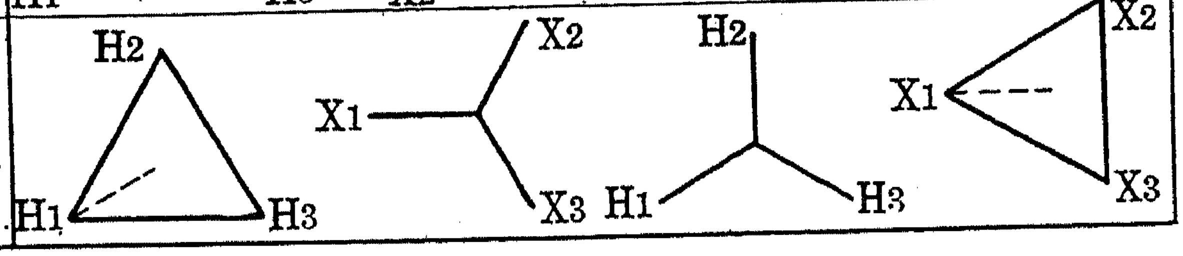
In some cases design may be such as to permit parallel operation, although due to the difference in the number of tap leads, the leads to be connected together may not have the same number.

THREE PHASE TRANSFORMERS

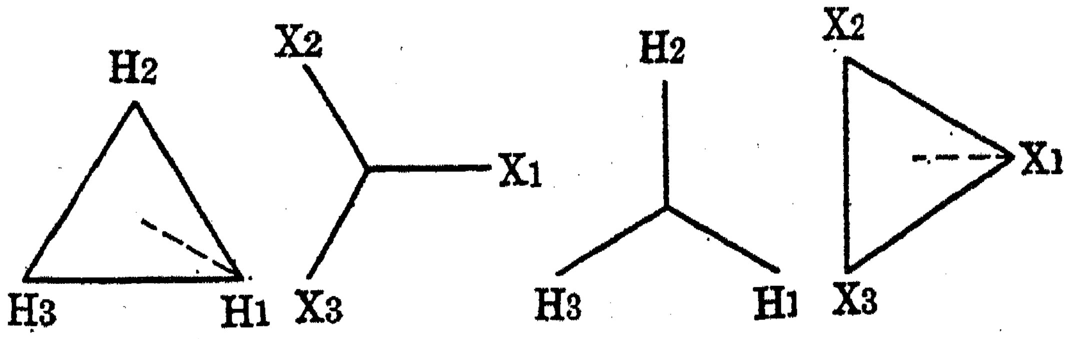
608 **Marking of Full Winding Leads.** The (3) high-voltage leads and the (3) low-voltage leads which connect to the full-phase windings, shall be marked H_1, H_2, H_3, and X_1, X_2, X_3. The full-phase winding of a tertiary winding shall be marked Y_1, Y_2, Y_3.

609 **Relation between High- and Low-Voltage Windings.** (a) The markings shall be so applied that if the phase sequence of voltage on the high voltage side is in the time order H_1, H_2, H_3 it is in the time order of, X_1, X_2 X_3 on the low-voltage side and, Y_1, Y_2 Y_3 for a tertiary winding.

(b) **Angular Displacement.** In order that the markings of lead connections between phases shall indicate definite phase relations, they shall be made in accordance with one of the three three-phase groups as shown. The angular displacement between the high-voltage and low-voltage windings is the angle in each of the voltage vector diagrams between the lines passing from its neutral point through H1 and X1 respectively.

	Three Phase Transformers
Group-1 Angular Displacement 0°	 H2 H1 H3 X2 X1 X3 H2 H1 H3 X2 X1 X3
Group-2 Angular Displacement 180°	 H2 H1 H3 X3 X1 X2 H2 H1 H3 X3 X1 X2
Group-3 Angular Displacement 30°	 H2 H1 H3 X2 X1 X3 H2 H1 H3 X2 X1 X3

Group (3) may also be represented by the diagrams herewith and transformers marked according to them will parallel with those given above by connecting similarly marked leads. They are, therefore, superfluous but are given to simplify the application of the rule.



610 **Tap Leads.** (a) Where tap leads are brought out of the case (neutral lead excepted) they shall be marked with the proper letter followed by the figures 4, 7, etc., for one phase, 5, 8, etc., for another phase and 6, 9, etc., for the third phase.

(b) **Star Connection.** The order of numbering tap leads shall be as follows: 4, 7, etc., from lead 1 towards neutral; 5, 8, etc., from lead 2 towards neutral; and 6, 9, etc., from lead 3 towards neutral.

611 Interphase Connection made Outside of Case. Where the interphase connections are made outside of case, the leads will be marked with the proper letter followed by the numbers 1, 4, 7, 10, etc., for one phase; 2, 5, 8, 11, etc., for the second phase; and 3, 6, 9, 12,etc., for the third phase.

The markings shall be so applied that when a star connection is made by joining together the highest numbered leads of each phase, all rules here given, excepting rule (**601b**) apply.

612 Parallel Operation. Transformers having leads marked in accordance with these rules may be operated in parallel by connecting similarly marked leads together provided their angular displacements are the same and provided also their ratios, voltages, resistances, and reactances are such as to permit parallel operation.

Note.—In some cases designs may be such as to permit parallel operation although, due to a difference in the number of tap leads, the leads to be connected together are not similarly marked.

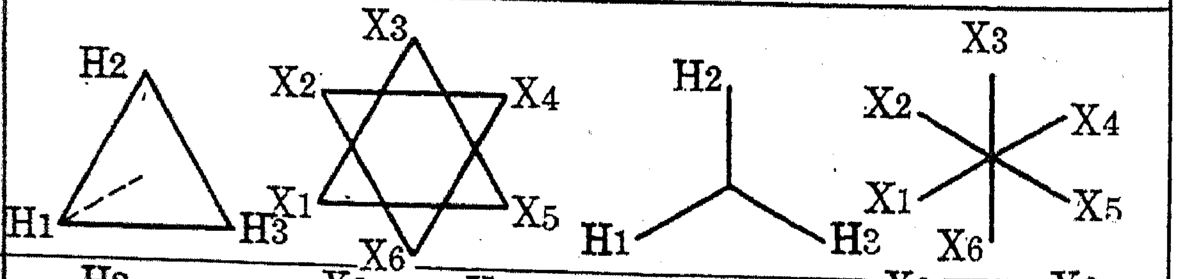
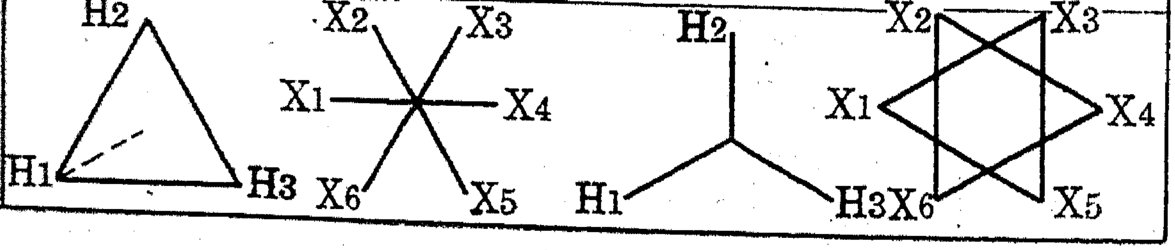
613 Location of H1 Lead. To simplify the work of connecting transformers in parallel it is recommended that the *H1* lead shall be brought out on the right hand side of the case, facing the high-voltage side of the case.

THREE-PHASE TO SIX-PHASE TRANSFORMERS

614 Rules for Three-Phase Transformers that are Applicable. Rules **609b** and **611** shall apply to three-phase to six-phase transformers. Rules **608** and **610** shall apply to three-phase windings but not to six-phase windings.

615 Markings of Six-Phase Leads. The six leads which connect to the full-phase windings shall be marked *X1*, *X2*, *X3*, *X4*, *X5*, *X6*.

616 Relation between Three-Phase and Six-Phase Windings. (a) The markings shall be so applied that if the phase sequence of voltage on the three-phase side is in the time order *H1*, *H2*, *H3*, it is in the time order of *X1*, *X2*, *X3*, *X4*, *X5*, *X6* on the six-phase side.

	Six Phase Transformers
Group-4 Angular Displacement 0°	
Group-5 Angular Displacement 30°	

(b) **Angular Displacement.** In order that the markings of lead connections between phases shall indicate definite phase relations, they shall be made in accordance with one of the two six-phase groups shown. The angular displacement between the high-voltage and low-voltage windings is the angle in each of the voltage vector diagrams from its neutral through *H1* and *X1* respectively.

617 Tap Leads. Where tap leads from low-voltage windings are brought out of the case (neutral lead excepted), they shall be marked as follows:

(a) **Diametrical Connection** tap leads shall be marked from the two ends of each phase winding towards the middle or neutral point in the following order; *X7*, *X13*, etc., from *X1* towards neutral; *X8*, *X14*, etc., from *X2* towards neutral; *X9*, *X15*, etc., from *X3* towards neutral; *X10*, *X16*, etc., from *X4* towards neutral; *X11*, *X17*, etc., from *X5* towards neutral; *X12*, *X18*, etc., from *X6* towards neutral.

A tap from the middle point of any phase winding, not intended as a neutral, shall be given a number determined by counting from *X1*, *X2* or *X3* and not from *X4*, *X5*, or *X6*; *e.g.*, if the only taps brought out are 50 per cent starting taps, they shall be numbered *X7*, *X8*, and *X9*.

(b) **Double Delta Connection.** Tap leads shall be marked in the following order; *X7*, *X13*, etc., from *X1* towards *X3*; *X8*, *X14*, etc., from *X2* towards *X4*; *X9*, *X15*, etc., from *X3* towards *X5*; *X10*, *X16*, etc., from *X4* towards *X6*; *X11*, *X17*, etc., from *X5* towards *X1*; *X12*, *X18*, etc., from *X6* towards *X2*.

NOTE.—For starting purposes it is generally customary to bring out only two taps from one delta and start three-phase.

INFORMATION ON THE RATING PLATE OF A MACHINE

620 It is recommended that the rating plate of machines which comply with the Institute rules shall carry a distinctive special sign, such as "A.I.E.E. 1918 Rating" or "A18" Rating.

621 The absence of any statement to the contrary on the rating plate of a machine implies that it is intended for continuous service and for the standard altitude and ambient temperature of reference. See **§§287, 305, 308** and **309.**

622 The rating plate of a machine intended to work under various kinds of rating must carry the necessary information in regard to those kinds of ratings.

STANDARDS FOR WIRES AND CABLES

TERMINOLOGY*

635 **Wire.**—A slender rod or filament of drawn metal.

The definition restricts the term to what would ordinarily be understood by the term "solid wire." In the definition, the word "slender" is used in the sense that the length is great in comparison with the diameter. If a wire is covered with insulation, it is properly called an insulated wire; while primarily the term "wire" refers to the metal, nevertheless when the context shows that the wire is insulated, the term "wire" will be understood to include the insulation.

636 **Conductor.**—A wire or combination of wires not insulated from one another, suitable for carrying a single electric current.

The term "conductor" is not to include a combination of conductors insulated from one another, which would be suitable for carrying several different electric currents.

Rolled conductors (such as busbars) are, of course, conductors, but are not considered under the terminology here given.

637 **Stranded Conductor.**—A conductor composed of a group of wires, or of any combination of groups of wires.

The wires in a stranded conductor are usually twisted or braided together.

638 **Cable.**—(1) A stranded conductor (single-conductor cable); or (2) a combination of conductors insulated from one another (multiple-conductor cable).

The component conductors of the second kind of cable may be either solid or stranded, and this kind of cable may or may not have a common insulating covering. The first kind of cable is a single conductor, while the second kind is a group of several conductors. The term "cable" is applied by some manufacturers to a solid wire heavily insulated and lead-covered; this usage arises from the manner of the insulation, but such a conductor is not included under this definition of "cable." The term "cable" is a general one, and, in practise, it is usually applied only to the larger sizes. A small cable is called a "stranded wire" or a "cord," both of which are defined below. Cables may be bare or insulated, and the latter may be armored with lead, or with steel wires or bands.

639 **Strand.**—One of the wires, or groups of wires, of any stranded conductor.

640 **Stranded Wire.**—A group of small wires, used as a single wire.

A wire has been defined as a slender rod or filament of drawn metal. If such a filament is subdivided into several smaller filaments or strands, and is used as a single wire, it is called a "stranded wire." There is no sharp dividing line of size between a "stranded wire" and a "cable." If used as a wire, for example in winding inductance coils or magnets, it is called a stranded wire and not a cable. If it is substantially insulated, it is called a "cord," defined below.

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*From Circular No. 37 of the Bureau of Standards.

641 **Cord.**—A small cable, very flexible and substantially insulated to withstand wear.

There is no sharp dividing line in respect to size between a "cord" and a "cable," and likewise no sharp dividing line in respect to the character of insulation between a "cord" and a "stranded wire." Rubber is used as the insulating material for many classes of cords.

642 **Concentric Strand.**—A strand composed of a central core surrounded by one or more layers of helically-laid wires or groups of wires.

643 **Concentric-Lay Cable.**—A single-conductor cable composed of a central core surrounded by one or more layers of helically-laid wires.

644 **Rope-Lay Cable.**—A single-conductor cable composed of a central core surrounded by one or more layers of helically-laid groups of wires.

This kind of cable differs from the preceding in that the main strands are themselves stranded.

645 **N-Conductor Cable.**—A combination of N conductors insulated from one another.

It is not intended that the name as here given be actually used. One would instead speak of a "3-conductor cable," a "12-conductor cable" etc. In referring to the general case, one may speak of a "multiple-conductor cable" (as in definition §638 above.)

646 **N-Conductor Concentric Cable.**—A cable composed of an insulated central conducting core with (N — 1) tubular stranded conductors laid over it concentrically and separated by layers of insulation.

This kind of cable usually has only two or three conductors. Such cables are used in carrying alternating currents. The remark on the expression "N-conductor" given for the preceding definition applies here also.

647 **Duplex Cable.**—Two insulated single-conductor cables, twisted together.

They may or may not have a common insulating covering.

648 **Twin Cable.**—Two insulated single-conductor cables laid parallel, having a common covering.

649 **Triplex Cable.**—Three insulated single-conductor cables twisted together.

They may or may not have a common insulating covering.

650 **Twisted Pair.**—Two small insulated conductors, twisted together, without a common covering.

The two conductors of a "twisted pair" are usually substantially insulated, so that the combination is a special case of a "cord."

651 **Twin Wire.**—Two small insulated conductors laid parallel, having a common covering.

SPECIFICATION OF SIZES OF CONDUCTORS

652 The sizes of solid wires shall be stated by their diameter in mils, the American Wire Gage (Brown and Sharpe) sizes being taken as standard. The sizes of stranded conductors shall be stated by their cross-sectional area in circular mils. For brevity, in cases where the most

careful specification is not required, the sizes of solid wires may be stated by the gage number in the American Wire Gage, and the sizes of stranded conductors smaller than 250,000 circular mils (*i.e.*, No. 0000 A.W.G. or smaller) may likewise be stated by means of the gage number in the American Wire Gage of a solid wire having the same cross-sectional area. Futhermore, an exception is made in the case of "Flexible Stranded Conductors," for which see **§655** below. In stating large cross-sections, it is sometimes convenient to use a circular inch (507 sq. mm.) instead of 1,000,000 circular mils.

STRANDING

653 Cables not requiring special flexibility shall be stranded in accordance with the following table.

TABLE XII

Standard Stranding of Concentric Lay-Cables

SIZE (See note 1.)	Number of Wires (See note 2)	
	A Bare, insulated or weather-proof cables for aerial use.	B Insulated cables for other than aerial use.
2.0 Cir. Inches	91	127
1.5 "	61	91
1.0 "	61	61
0.6 "	37	61
0.5 "	37	37
0.4 "	19	37
0000 A. W. G.	19 or 7 (See note 3.)	19
00 "	7	19
2 "	7	7
7 and smaller	..	7

1. For intermediate sizes, use stranding for next larger size.
2. Conductors of 0000 A. W. G. and smaller are often made solid and this table of stranding should not be interpreted as excluding this practice.
3. Class A cable, sizes 0000 and 000 A. W. G., is usually made of 7 strands when bare and 19 strands when insulated or weatherproof.

654 **Sectional Area of Cables.** The cross-sectional area of a cable shall be considered to be the sum of the cross-sectional areas of its component wires, when measured perpendicular to their axes.

655 **Flexible Stranding.** Conductors of special flexibility should ordinarily be made with wires of regular A.W.G. sizes,* the number of wires and size being given. The approximate gage number or approximate circular mils of such flexible stranded conductors may be stated. The stranding of standard flexible cables is given in Table XIII and a tentative stranding for apparatus cable in Table XIII-A.

TABLE XIII

Stranding of Flexible Cables

Nearest A.W.G. size (see Note 1)	Circular mils (see Note 3)	Diam. of cable. Mils	No. of wires	Size of each wire		Make-up (see Note 2)
				A.W.G.	Diam. Mils	
..	2039000	1885.	703	15.5	53.9	37 × 19
..	1816000	1779.	"	16.0	50.8	"
..	1617000	1679.	"	16.5	48.0	"
..	1440000	1584.	"	17.0	45.3	"
..	1284000	1496.	"	17.5	42.7	"
..	1103000	1372.	427	16.0	50.8	61 × 7
..	874600	1222.	"	17.0	45.3	"
..	693600	1088.	"	18.0	40.3	"
..	550000	969.	"	19.0	35.9	"
..	436200	863.	"	20.0	32.0	"
..	345900	768.	"	21.0	28.5	"
..	274300	684.	"	22.0	25.3	"
..	264600	671.	259	20.0	32.0	37 × 7
0000	209800	598.	"	21.0	28.5	"
000	171300	538.	133	19.0	35.9	19 × 7
00	135900	479.	"	20.0	32.0	"
0	107700	427.	"	21.0	28.5	"
1	82780	332.	91	20.5	30.2	Concentric
2	65650	295.	"	21.5	26.9	"
3	52060	263.	"	22.5	23.9	"
4	39190	228.	61	22.0	25.3	"
5	31080	203.	"	23.0	22.6	"
6	24650	181.	"	24.0	20.1	"
8	17410	152.	"	25.5	16.9	"
10	10560	118.	37	25.5	16.9	"
12	6642	94.	"	27.5	13.4	"
14	4176	74.	"	29.5	10.6	"
Smaller	....	....	To equal Required Size	30.0	....	Bunched

NOTE 1. The A.W.G. sizes except for 61 strands are approximated within 2 per cent. In the case of 61 strand cables the approximation is 6 per cent.

NOTE 2. "61 × 7" in the description of a rope-lay cable signifies 61 strands of 7 wires each.

NOTE 3. Circular mils are based on theoretical diameters of A.W.G. sizes which vary above or below values given in table by less than 0.1 mil.

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*Where necessary to closely approximate a regular size cable, the strands may be made of half-size wires from No. 15 to No. 30 A. W. G.

TABLE XIII-A

Proposed Standard Stranding of Apparatus Cables

(This table is offered for consideration but will not be recommended for final adoption until ratified by other societies interested.)

Nearest A.W.G. size (see Note 1)	Circular mils (see Note 3)	Diameter of cable Mils	No. of wires	Size of each wire		Make-up
				A.W.G	Diam. Mils	
..	2053000	1900.	2257	20.5	30.2	61 × 37
..	1828000	1793.	"	21.0	28.5	"
..	1628000	1692.	"	21.5	26.9	"
..	1450000	1597.	"	22.0	25.3	"
..	1291000	1500.	"	22.5	23.9	"
..	1150000	1422.	"	23.0	22.6	"
..	1054000	1357.	1159	20.5	30.2	61 × 19
..	938900	1282.	"	21.0	28.5	"
..	836100	1209.	"	21.5	26.9	"
..	744600	1141.	"	22.0	25.3	"
..	663100	1076.	"	22.5	23.9	"
..	590500	1016.	"	23.0	22.6	"
..	525900	959.	"	23.5	21.3	"
..	451900	887.	703	22.0	25.3	37 × 19
..	402200	837.	"	22.5	23.9	"
..	358200	790.	"	23.0	22.6	"
..	319000	746.	"	23.5	21.3	"
..	284000	704.	"	24.0	20.1	"
..	253000	664.	"	24.5	19.0	"
0000	217600	609.	427	23.0	22.6	61 × 7
000	172500	543.	"	24.0	20.1	"
00	136800	483.	"	25.0	17.9	"
0	104600	422.	259	24.0	20.1	37 × 7
1	82990	376.	"	25.0	17.9	"
2	65820	335.	"	26.0	15.9	"
3	52200	298.	"	27.0	14.2	"
4	42610	268.	133	25.0	17.9	19 × 7
5	33800	239.	"	26.0	15.9	"
6	26800	213.	"	27.0	14.2	"
see Note 2.						

NOTE 1. The A.W.G. sizes are approximated within 3 per cent.
NOTE 2. For sizes smaller than No. 6 see table XIII.
NOTE 3. Circular mils are based on theoretical diameters of A.W.G. diameters of the component wires.

656 **Correction for Lay.** The resistance and mass of a stranded conductor are greater than in a solid conductor of the same cross-sectional area, depending on the lay (*i.e.*, the pitch of the twist of the wires). Two per cent shall be taken as the standard increment of resistance and of mass. In cases where the lay is definitely known, the increment should be calculated and not assumed.

The direction of lay is the lateral direction in which the strands of a cable run over the top of the cable as they recede from an observer looking along the axis of the cable.

CONDUCTIVITY OF COPPER.

675 The following I. E. C. rules are adopted:*

The following shall be taken as normal values for standard annealed copper:

(1) At a temperature of 20°C., the resistance of a wire of standard annealed copper one meter in length and of a uniform section of 1 square millimeter is 1/58 ohm = 0.017241....ohm.

(2) At a temperature of 20°C., the density of standard annealed copper is 8.89 grams per cubic centimeter.

(3) At a temperature of 20°C, the "constant mass" temperature coefficient of resistance of standard annealed copper, measured between two potential points rigidly fixed to the wire, is 0.00393 = 1/254.45....per degree centigrade.

(4) As a consequence, it follows from (1) and (2) that, at a temperature of 20 °C. the resistance of a wire of standard annealed copper of uniform section, one meter in length and weighing one gram, is (1/58) × 8.89 = 0.15328......ohm.†§

676 **Copper Wire Tables.** The copper-wire Tables published by the Bureau of Standards in Circular No. 31 are adopted. These Tables are based upon the I. E. C. rules stated in §**675**.

HEATING AND TEMPERATURE OF CABLES.

677 **Maximum Safe Limiting Temperatures.**

The maximum safe limiting temperature in degrees C. at the surface of the conductor in a cable shall be:—

For impregnated paper insulation	(85—E)
" varnished cambric	(75—E)
" rubber insulation	(60—0.25E)

where E represents the r.m.s. operating e.m.f. in kilovolts between conductors.

Thus, at a working pressure of 3.3 kv., the maximum safe limiting temperature at the surface of the conductor, or conductors, in a cable would be:—

For impregnated paper	81.7°C.
" varnished cambric	71.7°C.
" rubber insulation	59.2°C.

ELECTRICAL TESTS.

678 **Lengths Tested.** Electrical tests of insulation on wires and cables shall be made on the entire lengths to be shipped.

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*See I. E. C. Publication No. 28 "International Standard of Resistance for Copper" March 1914.

†Paragraphs (1) and (4) of § 675 define what are sometimes called "volume resistivity," and "Mass resistivity" respectively. This may be expressed in other units as follows:—volume resistivity = 1.7241 microhms-cm. (or microhms in a cm. cube) at 20°C. = 0.67879 microhm-inch at 20°C., and mass resistivity = 875.20 ohms (mile, pound) at 20°C.

§For detailed specifications of commercial copper, see the "Standard Specifications" of the American Society for Testing Materials.

679 **Immersion in Water.** Electrical tests of insulated conductors not enclosed in a lead sheath, shall be made while immersed in water after an immersion of twelve (12) consecutive hours, if insulated with rubber compound, or if insulated with varnished cambric. It is not necessary to immerse in water insulated conductors enclosed in a lead sheath.

In multiple-conductor cables, without waterproof overall jacket of insulation, no immersion test should be made on finished cables, but only on the individual conductors before assembling.

680 **Dielectric-Strength Tests. Object of Tests.** Dielectric tests are intended to detect weak spots in the insulation and to determine whether the dielectric strength of the insulation is sufficient for enabling it to withstand the voltage to which it is likely to be subjected in service, with a suitable factor of assurance.

The initially-applied voltage must not be greater than the working voltage, and the rate of increase shall not be over 100 per cent in 10 seconds.

681 **Factor of Assurance.** The factor of assurance of wire or cable insulation shall be the ratio of the voltage at which it is tested to that at which it is used.

682 **Test Voltage.** The dielectric strength of wire and cable insulation shall be tested at the factory, by applying an alternating test voltage between the conductor and sheath or water.

683 **The Magnitude and Duration of the Test Voltage** should depend upon the dielectric strength and thickness of the insulation, the length and diameter of the wire or cable, and the assurance factor required, the latter in turn depending upon the importance of the service in which the wire or cable is employed

684 **The following test voltages** shall apply unless a departure is considered necessary, in view of the above circumstances. Rubber covered wires or cable for voltages up to 7 kv. shall be tested in accordance with the National Electric Code. Standardization for higher voltages for rubber insulated cables is not considered possible at the present time.

Varnished cambric and impregnated paper insulated wires or cables shall be tested at the place of manufacture for five (5) minutes in accordance with the Table XIV below.

TABLE XIV

Recommended Test Kilovolts Corresponding to Operating Kilovolts

Operating kv.	Test kv.	Operating kv.	Test kv.
Below 0.5	2.5*	5	14
0.5	3	10	25
1	4	15	35
2	6.5	20	44
3	9	25	53
4	11.5		

*The minimum thickness of insulation shallbe $\frac{1}{16}$ in. (1.6 mm.)

Different engineers specify different thickness of insulation for the same working voltages. Therefore, at the present time the test kv. corresponding to working kv. given in Table XIV are based on the **minimum** thickness of insulation specified by engineers and operating companies.†

685 **The Frequency of the Test Voltage** shall not exceed 100 cycles per second, and should approximate as closely as possible to a sine wave. The source of energy should be of ample capacity.

686 **Where Ultimate Break-Down Tests** are required, these shall be made on samples not more than 6 meters (20 ft.) long. The maximum allowable temperature at which the test is made for the particular type of insulation and the particular working pressure, shall not be greater than the temperature limits given in § **677**.

687 **Multiple-Conductor Cables.** Each conductor of a multiple-conductor cable shall be tested against the other conductors connected together with the sheath or water.

INSULATION RESISTANCE

688 **Definition.** The insulation resistance of an insulated conductor is the electrical resistance offered by its insulation, to an impressed voltage tending to produce a leakage of current through the same.

689 **Insulation Resistance** shall be expressed in megohms for a specified length (as for a kilometer, or a mile, or one thousand feet), and shall be corrected to a temperature of 15.5° C.. using a temperature coefficient determined experimentally for the insulation under consideration.

690 **Linear Insulation Resistance,** or the insulation resistance of Unit Length, shall be expressed in terms of the megohm-kilometer, or the megohm-mile, or the megohm-thousand-feet.

691 **Megohms Constant.** The Megohms Constant of an insulated conductor shall **be** the factor " K " in the equation

$$R = K \log_{10} \frac{D}{d}$$

where R = The insulation resistance, in megohms, for a specified unit length.

D = Outside diameter of insulation.

d = Diameter of conductor.

Unless otherwise stated, K will be assumed to correspond to the mile unit of length.

692 **Test.** The apparent insulation resistance should be measured after the dielectric-strength test, measuring the leakage current after a one-minute electrification, with a continuous e.m.f. of from 100 to 500 volts, the conductor being maintained negative to the sheath or water.

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†The Standards Committee does not commit itself to the principle of basing test voltages on working voltages, but it is not yet in possession of sufficient data to base them upon the dimensions and physical properties of the insulation.

693 **Multiple-Conductor Cables.** The insulation resistance of each conductor of a multiple-conductor cable shall be the insulation resistance measured from such conductor to all the other conductors in multiple with the sheath or water.

CAPACITANCE OR ELECTROSTATIC CAPACITY

694 Capacitance is ordinarily expressed in microfarads. Linear Capacitance, or Capacitance per unit length, shall be expressed in Microfarads per unit length (kilometer, or mile, or one thousand feet) and shall be corrected to a temperature of 15.5° C.

695 **Microfarads Constant.** The Microfarads Constant of an insulated conductor shall be the factor " K " in the equation

$$C = \frac{K}{\text{Log}_{10}\frac{D}{d}}$$

where C = the capacitance in microfarads per unit length.

D = the outside diameter of insulation.

d = the diameter of conductor.

Unless otherwise stated, K will be assumed to refer to the mile unit of length.

696 **Measurement of Capacitance.** The Capacitance of low-voltage cable, shall be measured by comparison with a standard condenser. For long lengths of high-voltage cables, where it is necessary to know the true capacitance, the measurement should be made at a frequency approximating the frequency of operation.

697 **Paired Cables.** The capacitance shall be measured between the two conductors of any pair, the other wires being connected to the sheath or ground.

698 **Electric Light and Power Cables.** The capacitance of low-voltage cables is generally of but little importance. The capacitance of high-voltage cables should be measured between the conductors, and also between each conductor and the other conductors connected to the lead sheath or ground.

699 **Multiple-Conductor Cables** (not paired). The capacitance of each conductor of a multiple-conductor cable shall be the capacitance measured from such conductor to all of the other conductors in multiple with the sheath or the ground.

STANDARDS FOR CONTROLLERS, CIRCUIT-BREAKERS. SWITCHES, FUSES AND ACCESSORIES*

GENERAL DEFINITIONS

The following definitions are tentative. Criticisms and suggestions, addressed to the Secretary of the Standards Committee, will be welcomed.

700 **Conducting Parts.**—Those parts designed to carry current or which are conductively connected therewith.

701 **Contact.**—The surface common to two conducting parts, united by pressure, for the purpose of carrying current.

702 **Grounded Parts.**—Those parts which may be considered to have the same potential as the earth.

703 **"Air" as a Prefix.**—The prefix "air" applied to a device which interrupts an electric circuit indicates that the interruption occurs in air.

704 **"Oil" as a Prefix.**—The prefix "oil" applied to a device which interrupts an electric circuit indicates that the interruption occurs in oil.

705 **Fume-Resisting.**—Apparatus is designated as fume-resisting when so constructed that it will not be readily injured by the specified fumes.

706 **Drip-Proof.**—Apparatus is designated as drip-proof when so protected as to exclude falling moisture or dirt. Drip-proof apparatus may be either open or semi-enclosed, if it is provided with suitable protection integral with the apparatus, or so enclosed as to exclude effectively falling solid or liquid material.

707 **Dust-Proof.**—Apparatus is designated as dust-proof when so constructed or protected that the accumulation of dust within or without the device will not interfere with its successful operation.

707a **Dust-Tight.**—Apparatus is designated as dust-tight when so constructed that the dust will not enter the enclosing case.

708 **Explosion-Proof.**—Apparatus is designated as explosion-proof when so constructed that explosions of gas within the casing will not injure it or ignite inflammable gas outside it.

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*The "National Electrical Code" of the National Fire Protection Association deals with certain capacities of circuit-breakers up to 550 volts and of switches and fuses up to 600 volts.

The question of establishing uniform rules for apparatus coming under this classification is under consideration by the A. I. E. E. Standards Committee and the Electrical Committee of the N. F. P. A.

709 **Gas-Proof.**—Apparatus is designated as gas-proof when so constructed or protected that the specified gas will not interfere with its successful operation.

709a **Gas-Tight.**—Apparatus is designated as gas-tight when so constructed that the specified gas will not enter the enclosing case.

710 **Moisture-Resisting.**—Apparatus is designated as moisture-resisting when so constructed or treated that it will not be readily injured by moisture. (Such apparatus shall be capable of operating in a very humid atmosphere, such as found in mines, evaporating rooms, etc.)

711 **Splash-Proof.**—Apparatus is designated as splash-proof when so constructed or protected that external splashing will not interfere with its successful operation.

712 **Submersible.**—Apparatus is designated as submersible when so constructed that it will operate successfully when submerged in water under specified conditions of pressure and time.

713 **Sleet-Proof.**—Apparatus is designated as sleet-proof when so constructed or protected that the accumulation of sleet will not interfere with its successful operation.

714 **Under-Voltage or Low-Voltage Release.**—A term applied to a device which on the reduction or failure of voltage operates to cause the interruption of power to the main circuit, but which does not prevent the re-establishment of the main circuit on return of voltage.

714a **Under-Voltage or Low-Voltage Protection.**—A device which, on the reduction or failure of voltage, operates to cause and maintain the interruption of power to the main circuit, is said to provide under-voltage or low-voltage protection.

715 **Phase-Failure Protection.**—A device which, on the failure of power in one wire of a polyphase circuit, operates to cause and maintain the interruption of power on the remaining circuits, is said to provide phase-failure protection.

715a **Phase-Reversal Protection.**—A device which, on the reversal of the phase relations in a polyphase circuit, operates to cause and maintain the interruption of power in all of the circuits, is said to provide phase-reversal protection.

CONTROLLERS

716 **Electric Controller.**—A device, or group of devices, which serves to control in some predetermined manner the operation of the apparatus to which it is connected.

717 **Contactor.**—A device for repeatedly establishing and interrupting an electric circuit under **normal** conditions.

717a **Temperature of Contactors.**—

Operating Coils for Contactors.—The limiting observable temperature rise of operating coils for magnetic contactors shall be 70°C. when measured by thermometer.

Temperature of Contacts.—The limiting observable temperature rise for the contacts of magnetic contactors shall be the following:

65° C. rise for laminated contacts
100° C. " " solid "

Current Carrying Parts Insulated With Asbestos or Other Fire Proof Insulation.—The limiting observable temperature rise for current carrying parts insulated with asbestos or other fire proof material shall be 150° C.

CIRCUIT-BREAKERS.

718 **Circuit-Breaker.**—A circuit-breaker is a device (other than a fuse) constructed primarily for the interruption of a circuit under **infrequent abnormal** conditions.

719 **Rating.**—The rating of a circuit-breaker or switch includes (1) the normal r. m. s. current which it is designed to carry; (2) the normal r. m. s. pressure (voltage) of the circuit on which it is intended to operate; (3) the normal frequency of the current; (4) the interrupting capacity of the device (see §**720**).

720 **Interrupting Capacity.** By interrupting (breaking or rupturing) capacity is meant the highest r. m. s. current at normal voltage which the device can interrupt under prescribed conditions at stated intervals a specified number of times.

721 **Temperature Tests.**—Rated current at rated frequency shall be applied continuously until the temperature becomes constant. The maximum temperatures of the various parts shall not exceed the following when the ambient temperature of reference is 40° C.:

Contacts in air when clean and bright.. 70° C.
Oil and contacts therein............. 70° C.
Coils (see §§**376** to **379** incl.)
Other parts (see §**392**)

The Institute calls attention to the inherent decrease in capacity of switch and circuit-breaker contacts in air, due to oxidization of the contact surfaces. The rating of air switches and circuit-breakers is, therefore, based on sufficient maintenance to keep the temperature within the specified limits.

722 **Dielectric Tests.**—For apparatus of 600 volts or less see §§**482, 484, 485,** and **500.** Apparatus above 600 volts shall be tested at 2¼ times rated voltage, plus 2,000 volts, at a specified altitude.

As a supplementary test, devices for outdoor use should be capable of withstanding for 10 seconds a dielectric wet test at twice rated voltage plus 1,000 volts. This assumes a precipitation downward of 0. 1 inches per minute at an angle of 45° from the perpendicular, with water having a resistivity as low as 7,000 ohm-centimeters.

SWITCHES

723 **Switch.**—A switch is a device for making, breaking or changing the connections in an electric circuit.

724 **Master-Switch.**—A master-switch is a device which serves to govern the operation of contactors and auxiliary devices of an electric controller.

24a **Control Switch.**—A control switch is a switch for controlling electrically-operated switches and circuit breakers.

724b **Auxiliary Switch.**—An auxiliary switch is a switch actuated by some main device, for signalling, interlocking, etc.

725 **Rating.**—Same as for circuit-breakers; see §§**719** and **720**.

726 **Tests.**—Same as for circuit-breakers; see §§**721** and **722**.

FUSES

727 **Fuse.** A fuse is an element designed to melt or dissipate at a predetermined current value, and intended to protect against abnormal conditions of current.

Any terminals, tubes, etc., integral with this element are included as part of the fuse.

Fuses may be divided into two classes:

(A) Those designed to protect the circuit and apparatus both against short-circuit and against definite amounts of overload (*e.g* fuses of the National Electric Code which open on 25 per cent overload.)

(B) Those designed to protect the system only against short-circuits; (*e.g.* expulsion fuses, which blow at several times the current which they are designed to carry continuously). The line separating these two classes is not definitely fixed.

728 **Continuous Current Carrying-Capacity of a Fuse.**—Fuses shall be so constructed that they will carry continuously **110** per cent of their rated current.

729 **Temperature.**—The temperature of coils or windings (such as accompany fuses of the magnetic blow-out type) shall not exceed the limits set for machine coils having the same character of insulation. (See §§**376** to **379**). The highest temperature for the fuse proper should not exceed the safe limit for the material employed.

RELAYS

730 **Relay.**—A relay is a device by means of which contacts in one circuit are operated by change in conditions in the same or other circuits.

731 **Temperature Tests.**—Same as § **721**.

732 **Dielectric Tests.**—Same as § **722**.

LIGHTNING ARRESTERS

733 **Definition.** A lightning arrester is a device for protecting circuits and apparatus against lightning or other abnormal potential rises of short duration.

734 **Rating.** Arresters shall be rated by the voltage of the circuit on which they are to be used.

Lightning arresters may be divided into two classes:

(a) Those intended to discharge for a very short time.

(b) Those intended to discharge for a period of several minutes.

735 **Performance and Tests.** Dielectric Test same as **§722**.

The resistance of the arrester at double potential and also at normal potential, shall be determined by observing the discharge currents through the arrester.

(c) In the case of any arrester using a gap, a test shall be made of the spark potential on either direct-current or 60-cycle a-c. excitation.

(d) The equivalent sphere gap under disruptive discharge shall also be measured, using a considerable quantity of electricity.

(e) The endurance of the arrester to continuous surges shall be tested.

PROTECTIVE REACTORS

736 **Definition.** A reactor (See §§**82** and **214**) is a device for protecting circuits by limiting the current flow and localizing the disturbance under short-circuit conditions.

737 **Rating.**

(a) In kilovolt-amperes absorbed by normal current.

(b) By the normal current, frequency and line (delta) voltage for which the reactor is designed.

(c) By the current which the device is required to stand under short-circuit conditions.

738 **Performance and Tests.**

The Heat Test shall be made with normal current and frequency applied until the temperature is constant. The temperature should not exceed the safe limits for the materials employed. See **§§376** to **379.**

739 **Dielectric Test.** 2¼ times line voltage plus 2000, for one minute, from conductor to ground.

Note. The reactor shall be so designed as to be capable of withstanding, without mechanical injury, rated current at normal frequency, suddenly applied.

RESISTOR OR RHEOSTAT

740 **Definition.** Any device heretofore commonly known as a resistance, used for operation or control. (See **§81**) See National Electric Code.

740a **Temperature of Resistors.—**

Cast Grid Resistors.—The limiting observable temperature rise for cast grids used as resistors shall be 350° C.

INSTRUMENT TRANSFORMERS†

[For **Meters and Instruments** see §§**225** to **244.**]

741 **Instrument Transformer.**—An instrument transformer, is a transformer suitable for use with measuring instruments; that is, one in which the conditions of phase and of current or potential in the primary circuit, are represented with acceptable accuracy in the secondary circuit. An instrument transformer may be either an instrument current transformer or an instrument potential (voltage) transformer.

742 **Instrument Current Transformers on Open Secondary Circuit.**—Under conditions of open secondary circuit, current transformers shall be capable of carrying continuously rated primary current without damage to the primary insulation and without interruption of service.

743 **Instrument Current Transformers on Closed Secondary Circuit.**—Under conditions of closed secondary circuit, current transformers shall withstand 40 times rated current applied for 1 second, without injury.

744 For further definitions relative to instrument transformers, see §§ **205** to **207**. For dielectric tests of instrument potential (voltage) transformers see § **500**. The dielectric tests of instrument current transformers shall be the same as specified in § **509.**

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†The terms "load," "secondary load" and "secondary connected load" have been used in connection with the constants of the circuit connected to the secondary of a current transformer. It is suggested that the term "secondary burden" be used as a general term to designate this external resistance and inductance and shall be expressed quantitatively in terms of ohms and henrys.

STANDARDS FOR ELECTRIC RAILWAYS

DEFINITIONS

760 **Transmission System:** When the current generated for an electric railway is changed in kind or voltage, between the generator and the cars or locomotives, that portion of the conductor system carrying current of a kind or voltage substantially different from that received by the cars or locomotives, constitutes the *transmission system.**

761 **Distribution System:** That portion of the conductor system of an electric railway which carries current of the kind and voltage received by the cars or locomotives, constitutes the *distribution system.**

762 **Substation:** A substation is a group of apparatus or machinery which receives current from a transmission system, changes its kind or voltage, and delivers it to a distribution system.

RATING OF RAILWAY SUBSTATION MACHINERY

763 **Continuous Rating.** The rating of a substation machine shall be the kv-a. output at a stated power factor input, which it will deliver continuously with temperatures or temperature rises not exceeding the limiting values given in Sections **376** and **379** and also fulfilling the other requirements set forth in these rules and summarized in Section **260.**

764 **Momentary Loads.** These machines should be capable of carrying a load of twice their rating for one minute, after a continuous run at rated load, without disqualifying them for continuous service.

765† **Nominal Rating.** Where the continuous rating is inconvenient, the following nominal rating may be used. The nominal rating of a substation machine shall be the kv-a. output at a stated power factor input, which, having produced a constant temperature in the machine may be increased 50 per cent for two hours, without producing temperatures or temperature rises exceeding by more than 5°C. the limiting values given in §**376** and **379.** These machines should be capable of carrying a load of twice their nominal rating for a period of one minute, without disqualifying them for continuous service. The name plate should be marked " nominal rating."

CONDUCTOR AND RAIL SYSTEMS.

766 **Contact Conductors.** That part of the distribution system other than the traffic rails, which is in immediate electrical contact with

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*These definitions are identical in sense, although not in words, with those of the Interstate Commerce Commission, as given in their Classification of Accounts for Electric Railways.

†The nominal rating should be applied only to machinery and apparatus carrying traction loads.

the circuits of the cars or locomotives, constitutes the contact conductors.

767 **Contact Rail:** A rigid contact conductor.

768 Overhead Contact Rail: A contact rail above the elevation of the maximum equipment line.†

769 Third Rail: A contact conductor placed at either side of the track, the contact surface of which is a few inches above the level of the top of the track rails.

770 Center Contact Rail: A contact conductor placed between the track rails, having its contact surface above the ground level.

771 Underground Contact Rail: A contact conductor placed beneath the ground level.

772 Gage of Third Rail: The distance, measured parallel to the plane of running rails, between the gage line of the nearer track rail and the inside gage line of the *contact surface* of the third rail.

773 Elevation of Third Rail: The elevation of the contact-surface of the third rail, with respect to the plane of the tops of running rails.

774 Standard Gage of Third Rails: The gage of third rails shall be not less than 26 inches (66 cm.) and not more than 27 inches (68.6 cm.).

775 Standard Elevation of Third Rails: The elevation of third rails shall be not less than 2¾ inches (70 mm.), and not more than 3½ inches (89 mm.).

776 Third Rail Protection: A guard for the purpose of preventing accidental contact with the third rail.

777 **Trolley Wire:** A flexible contact conductor, customarily supported above the cars.

778 **Messenger Wire or Cable:** A wire or cable running along with and supporting other wires, cables or contact conductors.

A primary messenger is directly attached to the supporting system. A secondary messenger is intermediate between a primary messenger and the wires, cables or contact conductors.

779 **Classes of Construction:** Overhead trolley construction will be classed as *Direct Suspension* and *Messenger or Catenary Suspension*.

780 Direct Suspension: All forms of overhead trolley construction in which the trolley wires are attached, by insulating devices, directly to the main supporting system.

781 Messenger or Catenary Suspension: All forms of overhead trolley construction in which the trolley wires are attached, by suitable devices, to one or more messenger cables, which in turn may be carried either in *Simple Catenary, i.e.*, by primary messengers, or in *Compound Catenary, i.e.*, by secondary messengers.

782 Supporting Systems shall be classed as follows:

783 Simple Cross-Span Systems: Those systems having at each support a single flexible span across the track or tracks.

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†The contour which embraces cross-sections of all rolling stock under all normal operating conditions.

784 MESSENGER CROSS-SPAN SYSTEMS: Those systems having at each support two or more flexible spans across the track or tracks, the upper span carrying part or all of the vertical load of the lower span.

785 BRACKET SYSTEMS: Those systems having at each support an arm or similar rigid member, supported at only one side of the track or tracks.

786 BRIDGE SYSTEMS: Those systems having at each support a rigid member, supported at both sides of the track or tracks.

787 STANDARD HEIGHT OF TROLLEY WIRE ON STREET AND INTERURBAN RAILWAYS: It is recommended that supporting structures shall be of such height that the lowest point of the trolley wire shall be at a height of 18 feet (5.5m.) above the top of rail under conditions of maximum sag, unless local conditions prevent. On trackage operating electric and steam road equipment and at crossings over steam roads, it is recommended that the trolley wire shall be not less than 21 feet (6.4m.) above the top of rail, under conditions of maximum sag.

RAILWAY MOTORS†

RATING

800 **Nominal Rating**: The nominal rating of a railway motor shall be the mechanical output at the car or locomotive axle, measured in kilowatts, which causes a rise of temperature above the surrounding air, by thermometer, not exceeding 90 °C. at the commutator, and 75 °C. at any other normally accessible part after one hour's continuous run at its rated voltage (and frequency in the case of an alternating-current motor) on a stand with the motor covers arranged to secure maximum ventilation without external blower. The rise in temperature as measured by resistance, shall not exceed 100 °C.*

801 The statement of the nominal rating shall also include the corresponding voltage and armature speed.

802 **Continuous Rating:** The continuous ratings of a railway motor shall be the *inputs* in amperes at which it may be operated continuously at ½, ¾ and full voltage respectively, without exceeding the specified temperature rises (see §805), when operated on stand test with motor covers and cooling system, if any, arranged as in service. Inasmuch as the same motor may be operated under different conditions as regards ventilation, it will be necessary in each case to

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* This definition differs from that in the 1911 edition of the Rules, principally by the substitution of a kilowatt rating for the horse-power rating and the omission of a reference to a room temperature of 25°C. For the purposes of these Rules the horse-power shall be taken as 746.0 watts. On account of the hitherto prevailing practise of expressing mechanical output in horse-power, it is recommended that, for the present the capacity be expressed both in kilowatts and in horse-power, a double rating, namely,

kw.————————approx. equiv. h.p.————————

In order to lay stress upon the preferred future basis, it is desirable that on rating plates, the rating in kilowatts shall be shown in larger and more prominent characters than the capacity in horse power.

†See also Appendix II.

define the system of ventilation which is used. In case motors are cooled by external blowers, the flow of air on which the rating is based shall be given.

803 **Maximum Input.** The subject of momentary loads for railway motors is under investigation.

TEMPERATURE LIMITATIONS

804 The allowable **temperature** in any part of a motor in service will be governed by the kind of material with which that part is insulated. In view of space limitations, and the cost of carrying dead weight on cars, it is considered good practice to operate railway motors for short periods at higher temperatures than would be advisable in stationary motors. The following temperatures are permissible:

TABLE XV
Operating Temperatures of Railway Motors

Class of Material See §376 to 379.	Maximum Observable Temperature of windings when in continuous service. By Thermometer See §345	By Resistance
A	85	110
B	100	130

For infrequent occasions, due to extreme ambient temperatures, it is permissible to operate at 15° higher temperature.

805 With a view to not exceeding the above temperature limitations, the continuous ratings shall be based upon the **temperature rises** tabulated below:

TABLE XVI
Stand-Test Temperature Rises of Railway Motors*

Class of Material See §376 to 379	Temperature Rises of windings By Thermometer See §345	By Resistance
A	65	85
B	80	105

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*The temperature rise in service may be very different from that on stand test. See § **1104** for relation between stand test and service temperatures, as affected by ventilation.

806 **Field-Control Motors.** The nominal and continuous ratings of field-control motors shall relate to their performance with the operating field which gives the maximum motor rating. Each section of the field windings shall be adequate to perform the service required of it, without exceeding the specified temperature rises.

CHARACTERISTIC CURVES

810 **The Characteristic Curves** of railway motors shall be plotted with the current as abscissas and the tractive effort, speed and efficiency as ordinates. In the case of a-c. motors, the power factor shall also be plotted as ordinates.

811 **Characteristic curves of direct-current motors** shall be based upon full voltage, which shall be taken as 600 volts, or a multiple thereof.

812 **In the case of field-control motors,** characteristic curves shall be given for all operating field connections.

EFFICIENCY AND LOSSES

815 **The efficiency** of railway motors shall be deduced from a determination of the losses enumerated in **§816** to **820.** (See also **§ 1100** and **1101.**)

816 **The copper loss** shall be determined from resistance measurements corrected to 75° C.

817 **The no-load** core loss, **brush friction, armature-bearing friction and windage** shall be determined as a total under the following conditions:

In making the test, the motor shall be run without gears. The kind of brushes and the brush pressure shall be the same as in commercial service. With the field separately excited, such a voltage shall be applied to the armature terminals as will give the same speed for any given field current as is obtained with that field current when operating at normal voltage under load. The sum of the losses above-mentioned, is equal to the product of the counter-electromotive force and the armature current.

818 The **core loss** in d-c. motors shall be separated from the friction and windage losses above described by measuring the power required to drive the motor at any given speed without gears, by running it as a series motor on low voltage and deducting this loss from the sum of the no-load losses at corresponding speed. (See **§1101** for alternative method).

The friction and windage losses under load shall be assumed to be the same as without load, at the same speed.

The core loss under load shall be assumed as follows:

TABLE XVII

Core Loss in D-C. Railway Motors at Various Loads.

Per cent of Input at Nominal Rating	Loss as Per cent of No-load Core Loss
200	165
150	145
100	130
75	125
50	123
25 and under	122

Note:—With motors designed for field control the core losses shall be assumed as the same for both full and permanent field. It shall be the mean between the no-load losses at full and permanent field, increased by the percentages given in the above Table.

819 **The brush-contact resistance loss** to be used in determining the efficiency, may be obtained by assuming that the sum of the drops at the contact surfaces of the positive and negative brushes is three volts.

820 The losses in gearing and axle bearings for single-reduction single-geared motors, varies with type, mechanical finish, age and lubrication. The following values, based on accumulated tests, shall be used in the comparison of single-reduction single-geared motors.

TABLE XVIII

Losses in Axle Bearings and Single-Reduction Gearing of Railway Motors.

Per Cent of Input at Nominal Rating	Losses as Per Cent of Input
200	3.5
150	3.0
125	2.7
100	2.5
75	2.5
60	2.7
50	3.2
40	4.4
30	6.7
25	8.5

Note:—Further investigation may indicate the desirability of giving separate values of the losses for full and tapped fields, or low- and high-speed motors.

ELECTRIC LOCOMOTIVES

830 **Rating.** Locomotives shall be rated in terms of the weight on drivers, nominal one-hour tractive effort, continuous tractive effort and corresponding speeds.

831 **Weight on Drivers.** The weight on drivers, expressed in pounds, shall be the sum of the weights carried by the drivers and of the drivers themselves.

832 **Nominal Tractive Effort:** The nominal tractive effort, expressed in pounds, shall be that exerted at the rims of the drivers, when the motors are operating at their nominal (one-hour) rating.

833 **Continuous Tractive Effort.** The continuous tractive effort, expressed in pounds, shall be that exerted at the rims of the drivers when the motors are operating at their full-voltage continuous rating, as indicated in **§802.**

In the case of locomotives operating on intermittent service, the continuous tractive effort may be given for ½ or ¾ voltage, but in such cases the voltage shall be clearly specified.

834 **Speed:** The rated speed, expressed in miles per hour, shall be that at which the continuous tractive effort is exerted.

See also Appendix II on Additional Standards for Railway Motors.

RATING OF AUTOMOBILE PROPULSION MOTORS AND GENERATORS

(ROAD VEHICLES)

835 **Continuous Rating.** Automobile propulsion motors and generators shall be given a continuous rating, expressed in kilowatts output available at the shaft at specified speed. The machines shall be able to operate continuously at their rated outputs without exceeding any of the limitations referred to in **§260.**

836 **Short-Time Rating.** Owing to the variety of services which road vehicles are called upon to perform, no single standard period for short-time ratings is recommended.

837 **Nominal Rating.** No special nominal rating is required for automobile propulsion motors or generators.

838 **Temperature Rises.** Owing to space limitations and the cost of carrying dead weight on automobiles, it is considered good practice to operate the propulsion machinery at higher temperatures than would be advisable in stationary machines. The rating of automobiles motors and generators shall be based upon temperature rise, on a stand test and with motor covers arranged as in service, fifteen degrees by thermometer or twenty-five degrees by resistance, above those of **§379.**

839 **Efficiency and Losses.** Unless otherwise specified the efficiency of automobile propulsion machines shall be based upon the output at the shaft, using conventional losses as tabulated in **§440.** When such machines are of low voltage, the great influence of brush-contact losses on the efficiency requires that these losses be determined experimentally for the type of brush used.

DEFINITIONS and UNITS for ILLUMINATION AND PHOTOMETRY

The following Sections, **850** to **895**, are abstracts from the rules of the Nomenclature and Standards Committee of the Illuminating Engineering Society. They are here included by permission.

850 **Luminous Flux** is radiant power evaluated according to its capacity to produce the sensation of light.

851 **The stimulus coefficient** K_λ for radiation of a particular wavelength, is the ratio of the luminous flux to the radiant power producing it.

852 **The mean value of the stimulus coefficient,** K_m, over any range of wave-lengths, or for the whole visible spectrum of any source, is the ratio of the total luminous flux (in lumens) to the total radiant power (in ergs per second, but more commonly in watts).

853 **The luminous intensity** of a point source of light is the solid angular density of the luminous flux emitted by the source in the direction considered; or it is the flux per unit solid angle from that source.

Defining equation:

Let I be the intensity, F the flux and ω the solid angle.

Then if the intensity is uniform,

$$I = \frac{F}{\omega},$$

854 **Illumination,** on a surface, is the luminous flux-density over that surface, or the flux per unit of intercepting area.

Defining equation:

Let E be the illumination and S the area of the intercepting surface.

Then when uniform,

$$E = \frac{F}{S},$$

855 **Candle,** the unit of luminous intensity maintained by the National Laboratories of France, Great Britain, and the United States.[1]

856 **Candle-power,** luminous intensity expressed in candles.

857 **Lumen,** the unit of luminous flux, equal to the flux emitted in a unit solid angle (steradian) by a point source of one candle-power.[2]

858 **Lux,** a unit of illumination equal to one lumen per square meter. The C. G. S. unit of illumination is one lumen per square centimeter. For this unit Blondel has proposed the name "Phot." One millilumen per square centimeter (milliphot) is a practical derivative of the C. G. S. system. One foot-candle is one lumen per square foot and is equal to 1.0764 milliphots.

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[1] This unit, which is used also by many other countries, is frequently referred to as the international candle.

[2] A uniform source of one candle emits 4π lumens.

860 **Specific luminous radiation,** the luminous flux-density emitted by a surface, or the flux emitted per unit of emissive area. It is expressed in lumens per square centimeter.

Defining equation:

Let E' be the specific luminous radiation.

For surfaces obeying Lambert's cosine law of emission.

$$E' = \pi b_0.$$

863 **The Lambert, the C. G. S. Unit of Brightness,** the brightness of a perfectly diffusing surface radiating or reflecting one lumen per square centimeter. This is equivalent to the brightness of a perfectly diffusing surface having a coefficient of reflection equal to unity and illuminated by one phot.

864 **For most purposes, the millilambert** (0.001 lambert) is the preferable practical unit. A perfectly diffusing surface emitting one lumen per square foot will have a brightness of 1.076 millilamberts.

865 **Brightness expressed in candles per square centimeter** may be reduced to Lamberts by multiplying by π.

Brightness expressed in candles per square inch may be reduced to foot-candle brightness, by multiplying by the factor $144\,\pi = 452$.

Brightness expressed in candles per square inch may be reduced to lamberts by multiplying by $\pi/6.45 = 0.4868$.

In practise, no surface obeys exactly Lambert's cosine law of emission; hence the brightness of a surface in lamberts is, in general not numerically equal to its specific luminous radiation in lumens per square centimeter.

866 **Coefficient of reflection,** the ratio of the total luminous flux reflected by a surface to the total luminous flux incident upon it. It is a simple numeric. The reflection from a surface may be regular, diffuse or mixed. In perfect regular reflection, all of the flux is reflected from the surface at an angle of reflection equal to the angle of incidence. In perfect diffuse reflection, the flux is reflected from the surface in all directions, in accordance with Lambert's cosine law. In most practical cases, there is a superposition of regular and diffuse reflection.

867 **Coefficient of regular reflection** is the ratio of the luminous flux reflected regularly to the total incident flux.

868 **Coefficient of diffuse reflection** is the ratio of the luminous flux reflected diffusely to the total incident flux.

Defining equation:

Let m be the coefficient of reflection (regular or diffuse).

Then, for any given portion of the surface,

$$m = \frac{E'}{E}$$

869 **Lamp,** a generic term for an artificial source of light.

870 **Primary luminous standard,** a recognized standard luminous source reproducible from specifications.

871 **Representative luminous standard,** a standard of luminous intensity adopted as the authoritative custodian of the accepted value of the unit.

872 **Reference standard,** a standard calibrated in terms of the unit from either a primary or representative standard and used for the calibration of working standards.

873 **Working standard,** any standardized luminous source for daily use in photometry.

874 **Comparison lamp,** a lamp of constant but not necessarily known candle-power, against which a working standard and test lamps are successively compared in a photometer.

875 **Test lamp,** in a photometer,—a lamp to be tested.

876 **Performance curve,** a curve representing the behavior of a lamp in any particular (candle-power, consumption, etc.) at different periods during its life.

877 **Characteristic curve,** a curve expressing a relation between two variable properties of a luminous source, as candle-power and volts, candle-power and rate of fuel consumption, etc.

878 **Horizontal Distribution Curve.** A polar curve representing the luminous intensity of a lamp, or lighting unit, in a plane perpendicular to the axis of the unit, and with the unit at the origin.

879 **Vertical Distribution Curve.** A polar curve representing the luminous intensity of a lamp, or lighting unit, in a plane passing through the axis of the unit, and with the unit at the origin. Unless otherwise specified, a vertical distribution curve is assumed to be an *average* vertical distribution curve, such as may in many cases be obtained by rotating the unit about its axis and measuring the average intensities at the different elevations. It is recommended that in vertical distribution curves, angles of elevation shall be counted positively from the nadir as zero, to the zenith as 180 degrees. In the case of incandescent lamps, it is assumed that the vertical distribution curve is taken with the tip downward.

880 **Mean horizontal candle-power** of a lamp,—the average candle-power in the horizontal plane passing through the luminous center of the lamp.

It is here assumed that the lamp (or other light source) is mounted in the usual manner, or, as in the case of an incandescent lamp, with its axis of symmetry vertical.

881 **Mean spherical candle-power** of a lamp,—the average candle-power of a lamp in all directions in space. It is equal to the total luminous flux of the lamp, in lumens, divided by 4π.

882 **Mean hemispherical candle-power** of a lamp (upper or lower),—the average candle-power of a lamp in the hemisphere considered. It is equal to the total luminous flux emitted by the lamp, in that hemisphere, divided by 2π.

883 **Mean zonal candle-power** of a lamp,—the average candle-power of a lamp over the given zone. It is equal to the total luminous flux emitted by the lamp in that zone, divided by the solid angle of the zone.

884 **Spherical reduction factor** of a lamp,—the ratio of the mean spherical to the mean horizontal candle-power of the lamp.[3]

885 **Photometric Tests** in which the results are stated in candle-power should be made at such a distance from the source of light that the latter may be regarded as practically a point. Where tests are made in the measurement of lamps with reflectors, the results should always be given as "apparent candle-power" at the distance employed, which distance should always be specifically stated.

886 **The output of all illuminants** should be expressed in lumens.

887 **Illuminants should be rated** upon a lumen basis instead of a candle-power basis.

888 **The specific output of electric lamps** should be stated in lumens per watt; and the specific output of illuminants depending upon combustion should be stated in lumens per b.t.u. per hour. The use of the term "efficiency" in this connection should be discouraged.

When auxiliary devices are necessarily employed in circuit with a lamp, the input should be taken to include both that in the lamp and that in the auxiliary devices. For example, the watts lost in the ballast resistance of an arc lamp are properly chargeable to the lamp.

889 **The Specific Consumption** of an electric lamp is its watt consumption per lumen. "Watts per candle" is a term used commercially in connection with electric incandescent lamps, and denotes watts per mean horizontal candle-power.

890 **Life Tests. Electric Incandescent Lamps** of a given type may be assumed to operate under comparable conditions only when their lumens per watt consumed are the same. Life-test results, in order to be compared, must be either conducted under, or reduced to, comparable conditions of operation.

891 **In Comparing Different Luminous Sources,** not only should their candle-power be compared, but also their relative form, brightness, distribution of illumination and character of light.

892 **Lamp Accessories. A reflector** is an appliance, the chief use of which is to redirect the luminous flux of a lamp in a desired direction or directions.

893 **A Shade** is an appliance, the chief use of which is to diminish or to interrupt the flux of a lamp in certain directions, where such flux is not desirable. The function of a shade is commonly combined with that of a reflector.

894 **A Globe** is an enclosing appliance of clear or diffusing materials, the chief use of which is either to protect the lamp, or to diffuse its light.

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[3] In the case of a uniform point-source, this factor would be unity, and for a straight cylindrical filament obeying the cosine law it would be $\pi/4$.

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TABLE XIX.
Photometric Units and Abbreviations.

Photometric quantity	Name of unit	Abbreviations, Symbols and defining equations
1. Luminous flux	Lumen	$F.\ \Psi$
2. Luminous intensity	Candle	$I = \frac{dF}{d\omega},\ \Gamma = \frac{d\Psi}{d\omega}$, cp.
3. Illumination	Phot., foot-candle, lux	$E = \frac{dF}{dS} = \frac{I}{r^2}\cos\theta.\beta$
4. Exposure	Phot-second	$t\ \ E$
5. Brightness	Apparent candles per sq. cm. Apparent candles per sq. in.	$b = \frac{dI}{dS\ \cos\ \theta}$
	Lambert	$L = \frac{dF}{dS}$
6. Normal brightness	Candles per sq. cm. Candles per sq. in.	$b_0 = \frac{dI}{dS}$
7. Specific luminous radiation	Lumens per sq. cm. Lumens per sq. in.	$E' = \pi b_0 \beta'$
8. Coefficient of reflection	—	$m = \frac{E'}{E}$
9. Mean spherical candlepower		scp
10. Mean lower hemispherical candlepower		lcp
11. Mean upper hemispherical candlepower		ucp
12. Mean zonal candlepower		zcp

13. 1 lumen is emitted by 0.07958 spherical cp.
14. 1 spherical candlepower emits 12.57 lumens.
15. 1 lux = 1 lumen incident per square meter = 0,0001 phot = 0.1 milliphot.
16. 1 phot = 1 lumen incident per sq. cm. = 10.000 lux = 1000 milliphot.
17. 1 milliphot = 0.001 phot = 0.929 foot-candle.
18. 1 foot-candle = 1 lumen incident per square foot = 1.076 milliphot = 10.76 lux.
19. 1 lambert = 1 lumen emitted per square centimeter.*
20 1 millilambert = 0.001 lambert.
21. 1 lumen, emitted, per square foot* = 1.076 millilambert.
22. 1 millilambert = 0.929 lumen, emitted, per square foot*.
23. 1 lambert = 0.3183 candle per sq. cm. = 2.054 candles per sq. in.
24. 1 candle per sq. cm. = 3.1416 lamberts.
25. 1 candle per sq. in. = 0.4868 lamberts = 486.8 millilamberts.

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*Perfect diffusion assumed.

DEFINITIONS FOR TELEPHONY AND TELEGRAPHY

(See also §§**407** to **409**.)

910 Many of the following definitions are tentative and not yet fully established. Criticisms and suggestions, addressed to the Secretary of the Standards Committee, will be welcomed. Some of the definitions are specific to telephony, and differ in detail from similar definitions appearing in other parts of the rules.

911 **Damping of a Circuit**. The damping at a given point in a circuit from which the source of energy has been withdrawn, is the progressive diminution in the effective value of electromotive force and current at that point resulting from the withdrawal of electrical energy.

912 **Damping Constant.**—The damping constant of a circuit is a measure of the ratio of the dissipative to the reactive component of its admittance or impedance.

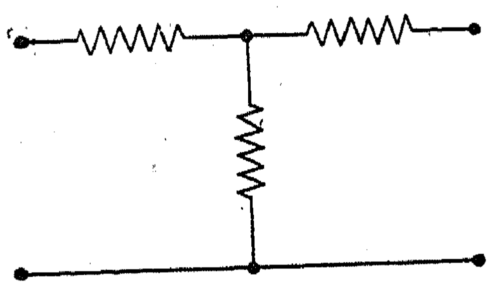
Applied to the admittance of a condenser or other simple circuit having capacity reactance, the damping constant for a harmonic electromotive force of given frequency is the ratio of the conductance G, of the condenser or simple circuit at that frequency to twice the capacitance, C, of the condenser at the same frequency, $(G/2C)$.

Applied to the reactance of a coil or other simple circuit having inductive reactance, the damping constant for a harmonic current of a given frequency is the ratio of the resistance, R, of the coil or circuit at that frequency to twice the inductance, L, at the same frequency $(R/2\,L)$.

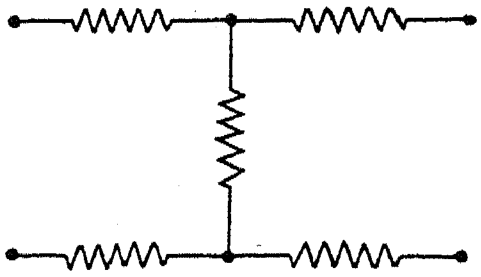
913 **Equivalent Circuit.**—An equivalent circuit is a simple network of series and shunt impedances, which, is the approximate electrical equivalent of a complex network.

914 **"*T*" Equivalent Circuit.**—A "*T*" equivalent circuit is a triple-star or "*Y*" connection of three impedances externally equivalent to a complex network.

Symbol:

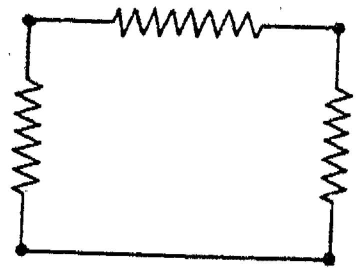


914a **"*I* Equivalent Circuit."** An "*I*" equivalent circuit is a connection of five impedances in the form shown below, which is externally equivalent to a complex network. It differs from the "*T*" equivalent circuit in that the impedances are arranged symmetrically on the two sides of the circuit which is often desirable in connection with practical problems, as indicating that the circuit is balanced with respect to ground.

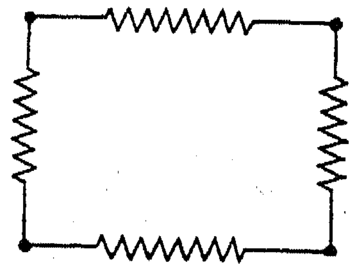


915 **"π" Equivalent Circuit.**—A "π" equivalent circuit is a delta connection of three impedances externally equivalent to a complex network. It is also called a "*U*" equivalent circuit.

Symbol:



915a **"*O*" Equivalent Circuit.**—An "*O*" equivalent circuit is a connection of four impedances in the form shown below, externally equivalent to a complex network. It differs from the π equivalent circuit in that the impedances are arranged symmetrically on the two sides of the circuit, which is often desirable in connection with practical problems, as indicating that the circuit is balanced with respect to ground.



Note: As ordinarily considered, the simple networks as defined, are the electrical equivalents of complex networks only with respect to definite pairs of terminals.

IMPEDANCE

916 **Mutual Impedance.**—The mutual impedance for single frequency alternating currents, between a pair of terminals and a second pair of terminals of a network, under any given condition, is the negative ratio* of the electromotive force produced between either pair of terminals on open circuit to the current flowing between the other pair of terminals. A receiving-end impedance is an example of a mutual impedance.

917 **Self-Impedance.**—The self-impedance between a pair of terminals of a network, under any given condition, is the ratio* of the electromotive force applied across the terminals to the entering current.

LINE CHARACTERISTICS

918 **Characteristic Impedance.**—The characteristic impedance of a line is the ratio* of the applied electromotive force to the resulting steady-

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*Single frequency voltages and currents are here supposed to be represented by complex numbers. Their ratio is therefore a complex number.

state current upon a line of infinite length and uniform structure, or of periodic recurrent structure.

NOTE: In practice, the terms (1) line impedance, (2) surge impedance, (3) iterative impedance, (4) sending-end impedance, (5) initial sending-end impedance, (6) final sending-end impedance, (7) natural impedance and, (8) free impedance, have apparently been more or less indefinitely and indiscriminately used as synonyms with what is here defined as "characteristic impedance."

919 **Sending-End Impedance.**—The sending-end impedance of a line is the ratio* of the applied electromotive force to the resulting steady-state current at the point where the electromotive force is applied.

NOTE: See note under "Characteristic Impedance". In case the line is of infinite length of uniform structure or of periodic recurrent structure, the sending-end impedance and the characteristic impedance are the same.

920 **Propagation Constant.**—The propagation constant of a uniform line, or section of a line of periodic recurrent structure, is the natural logarithm of the ratio* of the steady-state currents at various points separated by unit length in a uniform line of infinite length, or at successive corresponding points in a line of recurrent structure of infinite length. The ratio is determined by dividing the value of the current at the point nearer the transmitting end by the value of the current at the point more remote.

921 **Attenuation Constant.**—The attenutation constant for a single frequency is the real part of the propagation constant taken at that frequency.

922 **Wave-Length Constant.** The wave-length constant is the **imaginary** part of the propagation constant.

923 **Standard Cable.**—A standard cable is an ideal uniform line in terms of which the attenuation of a line or network may be specified. It is characterized by the following constants: Linear resistance, 88 ohms per loop mile (54.7 ohms per loop km.). Linear capacitance between wires 0.054 microfarad per loop mile (0.03355 microfarad per loop km.). Linear inductance and linear leakance, 0.

LINE CIRCUITS

930 **Ground-Return Circuit.** A ground-return circuit is a circuit consisting of one or more metallic conductors in parallel, with the circuit completed through the earth.

931 **Metallic Circuit.** A metallic circuit is a circuit of which the earth forms no part.

932 **Two-Wire Circuit.**—A two-wire circuit is a metallic circuit formed by two parallel conductors insulated from each other.

933 **Superposed Circuit.** A superposed circuit is an additional circuit obtained from a circuit normally required for another service, and in such a manner that the two services can be given simultaneously without mutual interference.

934 **Phantom Circuit.** A phantom circuit is a superposed circuit, each side of which consists of the two conductors of a two-wire circuit in parallel.

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*See note preceding page.

935 **Side Circuit.** A side circuit is a two-wire circuit forming one side of a phantom circuit.

936 **Non-Phantomed Circuit.** A non-phantomed circuit is a two-wire circuit, which is not arranged for use as the side of a phantom circuit.

937 **Simplexed Circuit.**—A simplexed circuit is a two-wire telephone circuit, arranged for the superposition of a single ground-return signalling circuit operating over the wires in parallel.

938 **Composited Circuit.** A composited circuit is a two-wire telephone circuit, arranged for the superposition on each of its component metallic conductors, of a single independent ground-return signalling circuit.

939 **Quadded or Phantomed Cable.**—A quadded or phantomed cable is a cable adapted for the use of phantom circuits.

940 **Simplex Circuit.**—A simplex circuit in telegraphy is one arranged for operation in one direction at one time.

941 **Duplex Circuit.**—A duplex circuit in telegraphy is one arranged for simultaneous operation in opposite directions.

942 **Diplex Circuit.**—A diplex circuit in telegraphy is one arranged for the simultaneous transmission of two messages in the same direction.

943 **Quadruplex Circuit.**—A quadruplex circuit in telegraphy is one arranged for the simultaneous transmission of two messages in each direction.

944 **Multiplex Circuit.**—A multiplex circuit in telegraphy is one arranged for the simultaneous transmission of several messages in one direction.

945 **Interference.**—Interference (electrical) is an electrical disturbance introduced into a communication system by an extraneous source. Examples of interference are lightning interference. interference between radio stations and interference from power systems into telegraph and telephone systems. (See §§**407** to **409**). Interference due to the effect of one telephone line on another is usually referred to as "cross-talk."

946 **Linear Electrical Constants.**—The linear electrical constants of a line are the electrical constants per unit length of the line, e. g. linear resistance, linear inductance etc.

947 **Smooth Line.**—A smooth line is a line whose electric elements are all continuously and uniformly distributed throughout its length.

948 **Periodic Line.**—A periodic line is a line consisting of successive similar sections in each of which one or more electric elements are not distributed uniformly. As examples of periodic lines are (1) loaded lines and (2) artificial lines consisting of successive similar sections of lumped constants.*

949 **Equivalent Smooth Line.**—An equivalent Smooth Line of a periodic line is a smooth line having the same electrical behavior as the periodic line, at a given single frequency, when measured at terminals or at corresponding section junctions. Similarly,

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*The term periodic in this definition refers to the line constants and not to time relations.

949a **Equivalent Periodic Line.**—An equivalent periodic line of a smooth line is a periodic line having the same electrical behavior, for an assumed single frequency, as the smooth line, when measured at terminals or at corresponding section junctions. The terms conjugate smooth line and conjugate periodic line are also sometimes used.

949b **Composite Line.**—A composite line is a line consisting of a plurality of successive sections having different linear electrical constants, as in the case where an underground cable section is joined to an overhead open-wire section.

949c **Amplifier.**—See **§1034.**

LOADING

950 **Loaded Line.**—A loaded line is one in which the normal reactance of the circuit has been altered for the purpose of increasing its transmission efficiency.

951 **Series Loaded Line.**—A series loaded line is one in which the normal reactance has been altered by reactance serially applied.

952 **Shunt Loaded Line.**—A shunt loaded line is one in which the normal reactance of the circuit has been altered by reactance applied in shunt across the circuit.

953 **Continuous Loading.** A continuous loading is a series loading in which the added inductance is uniformly distributed along the conductors.

954 **Coil Loading.** A coil loading is one in which the normal inductance is altered by the insertion of lumped inductance in the circuit at intervals. This lumped inductance may be applied either in series or in shunt.

NOTE: As commonly understood, coil loading is a series loading, in which the lumped inductance is applied at uniformly spaced recurring intervals

MISCELLANEOUS

955 **Microphone.** A contact device designed to have its electrical resistance directly and materially altered by slight differences in mechanical pressure.

957 **Resonance.**—Resonance of a harmonic alternating current of given frequency, in a simple series circuit, containing resistance, inductance and capacity, is the condition in which the positive reactance of the inductance is numerically equal to the negative reactance of the capacity. Under these conditions, the current flow in the circuit with a given electromotive force has its highest r.m.s. value.

958 **Retardation Coil.** A retardation coil is a reactor (reactance coil) used in a circuit for the purpose of selectively reacting on currents which vary at different rates.

NOTE: In telephone and telegraph usage, the terms "impedance coil," "inductance coil," "choke coil" and "reactance coil" are sometimes used in place of the term "retardation coil."

959 **Skin Effect.**—Skin effect is the phenomenon of the non-uniform distribution of alternating current throughout the cross-section of a linear conductor, occasioned by variations in the intensity of the magnetic field due to the current in the conductor.

960 **Telephone Receiver.** A telephone receiver is an electrically operated device designed to produce sound waves or vibrations which correspond in form to the electromagnetic waves or vibrations actuating it.

961 **Telephone Transmitter.** A telephone transmitter is a sound-wave or vibration-operated device designed to produce electromagnetic waves or vibrations which correspond in form to the sound waves or vibrations actuating it.

962 **Coefficient of Coupling of a Transformer.**—The coefficient of coupling of a transformer at a given frequency is the ratio* of the mutual impedance between the primary and secondary of the transformer, to the square root of the product of the self-impedances of the primary and of the secondary.

963 **Repeating Coil.** A term used in telephone practice meaning the same as transformer, and ordinarily a transformer of unity ratio.

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*Single frequency voltages and currents are here supposed to be represented by complex numbers. Their ratio is therefore a complex number.

APPENDIX I.

DEFINITIONS FOR RADIO COMMUNICATION

The following Sections **1000** to **1033** have been abstracted from the report of the Standardization Committee of the Institute of Radio Engineers, and are here included by permission as an Appendix, until further revised. For full particulars, see the I.R.E. Standardization Committee report.

1000 **Acoustic Resonance Device.** One which utilizes, in its operation, resonance to the audio frequency of the received signals.

1001 **Antenna.** A system of conductors designed for radiating or absorbing the energy of electromagnetic waves.

1002 **Atmospheric Absorption.** That portion of the total loss of radiated energy due to atmospheric conductivity.

1003 **Audio Frequencies.** The frequencies corresponding to the normally audible vibrations. These are assumed to lie below 10,000 cycles per second.

1004 **Capacitive Coupler.** An apparatus which, by electric fields, joins portions of two radio frequency circuits, and which is used to transfer electrical energy between these circuits through the action of electric forces.

1005 **Coefficient of Coupling (Inductive).** The ratio of the effective mutual inductance of two circuits to the square root of the product of the effective self-inductances of each of these circuits.

1006 **Direct Coupler.** A coupler which magnetically joins two circuits having a common conductive portion.

1007 **Counterpoise.** A system of electrical conductors forming one portion of a radiating oscillator, the other portion of which is the antenna. In land stations a counterpoise forms a capacitive connection to ground.

1008 **A Damped Alternating Current** is an alternating current whose amplitude progressively diminishes.

1009 **The Damping Factor** of an exponentially damped alternating current is the product of the logarithmic decrement and the frequency.

Let I_0 = initial amplitude
I_t = amplitude at the time t
ϵ = base of Napierian logarithms
a = damping factor
Then: $I_t = I_0 \epsilon^{-at}$

1010 **Detector.** That portion of the receiving apparatus which, connected to a circuit carrying currents of radio frequency, and in conjunction with a self-contained or separate indicator, translates the radio frequency energy into a form suitable for operation of the indicator. This translation may be effected either by the conversion of the radio frequency energy, or by means of the control of local energy by the energy received.

1011 **Electromagnetic Wave.** A periodic electromagnetic disturbance progressing through space.

1012 **Forced Alternating Current.** A current, the frequency and damping of which are equal to the frequency and damping of the exciting electromotive force.

1013 **Free Alternating Current.** The current following any electromagnetic disturbance in a circuit having capacity, inductance, and *less* than the critical resistance.

1014 **Critical Resistance of a Circuit.** That resistance which determines the limiting condition at which the oscillatory discharge of a circuit passes into an aperiodic discharge.

1015 **Group Frequency.** The number per second of periodic changes in amplitude or frequency of an alternating current.

NOTE 1. Where there is more than one periodically recurrent change of amplitude or frequency, there is more than one group frequency present.

NOTE 2. The term " group frequency" replaces the term " spark frequency."

1016 **Inductive Coupler.** An apparatus which, by magnetic forces, joins portions of two radio frequency circuits and is used to transfer electrical energy between these circuits, through the action of these magnetic forces.

1018 **Logarithmic Decrement** of an exponentially damped alternating current is the logarithm of the ratio of successive current amplitudes in the same direction.

NOTE: Logarithmic decrements are standard for a complete period or cycle.

Let: I_n and I_{n+1} be successive current amplitudes in the same direction.

d = logarithmic decrement

Then: $$d = \log_e \frac{I_n}{I_{n+1}}$$

1019 **Radio Frequencies.** The frequencies higher than those corresponding to the normally audible vibrations, which are generally taken as 10,000 cycles per second. See also Audio Frequencies.

NOTE: It is not implied that radiation cannot be secured at lower frequencies and the distinction from audio frequencies is merely one of definition based on convenience

1020 **Resonance** of a circuit to a given exciting alternating e.m.f. is that condition due to variation of the inductance or capacity in which the resulting effective current (or voltage) in that circuit is a maximum.

1024 **A Standard Resonance Curve** is a curve the ordinates of which are the ratios of the square of the current at any frequency to the square of the resonant current, and the abscissas are the ratios of the corresopnding wave length to the resonant wave length; the abscissas and ordinates having the same scale.

1026 **Sustained Radiation** consists of waves radiated from a conductor in which an alternating current flows.)

1027 **Tuning.** The process of securing the maximum indication by adjusting the time period of a driven element. (See Resonance.)

1028 **A Wave-Meter,** is a radio frequency measuring instrument, calibrated to read wave lengths.

1030 **Decremeter.** An instrument for measuring the logarithmic decrement of a circuit or of a train of electromagnetic waves.

1031 **Attenuation, Radio.** The decrease with distance from the radiating source, of the amplitude of the electric and magnetic forces accompanying (and constituting) an electromagnetic wave.

1032 **Attenuation, Coefficient of (Radio).** The coefficient, which, when multiplied by the distance of transmission through a uniform medium, gives the natural logarithm of the ratio of the amplitude of the electric or magnetic forces at that distance, to the initial value of the corresponding quantities.

1033 **Coupler.** An apparatus which is used to transfer radio-frequency energy from one circuit to another by associating portions of these circuits.

1034 **Amplifier.**—An amplifier is an instrument which modifies the effect of a local source of energy in substantial accordance with the waveform of the received energy, and gives out a wave of greater amplitude than that which it receives.

1035 **Interference.**—See §§17a, 407, 408, 409 and 945.

1036 **Phase Angle Defect.**—The phase angle defect of a condenser is the departure from quadrature of the phase difference between potential and current at terminals. This is sometimes called the phase angle of a condenser: although strictly speaking the phase angle of a condenser is 90° less the phase angle defect, and is therefore exactly 90° when the phase angle defect is zero.

APPENDIX II.

ADDITIONAL STANDARDS FOR RAILWAY MOTORS*

1100 In comparing projected motors, and in case it is not possible or desirable to make tests to determine mechanical losses, the following values of these losses, determined from the averages of many tests over a wide range of sizes of single-reduction single-geared motors, will be found useful, as approximations. They include axle-bearing, gear, armature-bearing, brush-friction, windage, and stray-load losses.

TABLE XX
Approximate Losses in D-C. Railway Motors.

Input in per cent of that at nominal rating	Losses as per cent of input
100 or over	5.0
75	5.0
60	5.3
50	6.5
40	8.8
30	13.3
25	17.0

1101 The core loss of railway motors is sometimes determined by separately exciting the field, and driving the armature of the motor to be tested, by a separate motor having known losses and noting the differences in losses between driving the motor light at various speeds and driving it with various field excitations.

1102 **Selection of Motor For Specified Service**

The following information relative to the service to be performed, is required, in order that an appropriate motor may be selected.

(a) Weight of total number of cars in train (in tons of 2000 lb.) exclusive of electrical equipment and load.

(b) Average weight of load and durations of same, and maximum weight of load and durations of same.

(c) Number of motor cars or locomotives in train, and number of trailer cars in train.

(d) Diameter of driving wheels.

(e) Weight on driving wheels, exclusive of electrical equipment.

(f) Number of motors per motor car.

(g) Voltage at train with power on the motors—average, maximum and minimum.

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*See also Sections **800** et seq.

(h) Rate of acceleration in mi. per. hr. per second.
(i) Rate of braking (retardation in m. per hr. per second).
(j) Speed limitations, if any (including slowdowns).
(k) Distances between stations.
(l) Duration of station stops.
(m) Schedule speed including station stops in m.p.h.
(n) Train resistance in pounds per ton of 2000 pounds at stated speeds.
(o) Moment of inertia of revolving parts, exclusive of electrical equipment.
(p) Profile and alignment of track.
(q) Distance coasted as a per cent of the distance between station stops.
(r) Time of layover at end of run, if any.

1103 **Stand-Test Method of Comparing Motor Capacity with Service Requirements:** When it is not convenient to test motors under actual specific service conditions, recourse may be had to the following method of determining temperature rise.

1104 The essential motor losses affecting temperatures in service are those in the motor windings, core and commutator. The mean service conditions may be expressed, as a close approximation, in terms of that continuous current and core loss which will produce the same losses and distribution of losses as the average in service.

A stand test with the current and voltage which will give losses equal to those in service, will determine whether the motor has sufficient capacity to meet the service requirements. In service, the temperature rise of an enclosed motor (**§164**), well exposed to the draught of air incident to a moving car or locomotive, will be from 75 to 90 per cent (depending upon the character of the service) of the temperature rise obtained on a stand test with the motor completely enclosed and with the same losses. With a ventilated motor (**§165 and §167**), the temperature rise in service will be 90 to 100 per cent of the temperature rise obtained on a stand test with the same losses.

1105 In making a stand test to determine the temperature rise in a specific service, it is essential in the case of a self-ventilated motor (**§ 167**), to run the armature at a speed which corresponds to the schedule speed in service. In order to obtain this speed it may be necessary, while maintaining the same total armature losses, to change somewhat the ratio between the I^2R and core-loss components.

1106 **Calculation for Comparing Motor Capacity with Service Requirements.** The heating of a motor should be determined, wherever possible, by testing it in service, or with an equivalent duty cycle. When the service or equivalent duty-cycle tests are not practicable, the ratings of the motor may be utilized as follows to determine its temperature rise.

1107 The motor losses which affect the heating of the windings are as stated above, those in the windings and in the core. The former are proportional to the square of the current. The latter vary with the voltage and current, according to curves which can be supplied by the manufacturers. The procedure is therefore as follows:

1108 (a) Plot a time-current curve, a time-voltage curve, and a time-core loss curve for the duty cycle which the motor is to perform, and calculate from these the root-mean-square current and the average core loss.

1109 (b) If the calculated r.m.s. service current exceeds the continuous rating, when run with average service core loss and speed, the motor is not sufficiently powerful for the duty cycle contemplated.

1110 (c) If the calculated r.m.s. service current does not exceed the continuous rating, when run with average service core loss and speed, the motor is ordinarily suitable for the service. In some cases, however, it may not have sufficient thermal capacity to avoid excessive temperature rises during the periods of heavy load. In such cases a further calculation is required, the first step of which is to compute the equivalent voltage which, with the r.m.s. current, will produce the average core-loss. Having obtained this, determine, as follows, the temperature rise due to the r.m.s. service current and equivalent voltage.

Let t = temperature rise, $p_0 = I^2R$ loss, kw., p_c = core loss, kw., with r.m.s. service current, and equivalent service voltage.

T = temperature rise, $P_0 = I^2R$ loss, kw., P_c = core loss, kw., with continuous load current corresponding to the equivalent service voltage.

Then

$$t = T\frac{p_0 + p_c}{P_0 + P_c}, \text{ approximately.}$$

1111 (d) The thermal capacity of a motor is approximately measured by the ratio of the electrical loss in kw. at its nominal (one-hour) capacity, to the corresponding maximum observable temperature rise during a one hour test starting at ambient temperature.

1112 (e) Consider any period of peak load and determine the electrical losses in kilowatt-hours during that period from the *electrical* efficiency curve. Find the excess of the above losses over the losses with r.m.s. service current and equivalent voltage. The excess loss, divided by the co-efficient of thermal capacity, will equal the extra temperature rise due to the peak load. This temperature rise added to that due to the r.m.s. service current, and equivalent voltage, gives the total temperature rise. If the total temperature rise in any such period exceeds the safe limit, the motor is not sufficiently powerful for the service.

1113 (f) If the temperature reached, due to the peak loads, does not exceed the safe limit, the motor may yet be unsuitable for the service, as the peak loads may cause excessive sparking and dangerous mechanical stresses. It is, therefore, necessary to compare the peak loads with the short-period overload capacity. If the peaks are also within the capacity of the motor, it may be considered suitable for the given duty cycle.

APPENDIX III.

BIBLIOGRAPHY OF LITERATURE RELATING TO ELECTRICAL ENGINEERING STANDARDIZATION

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REPORT OF THE BOARD OF DIRECTORS

FOR FISCAL YEAR ENDING APRIL 30, 1918

The Board of Directors of the American Institute of Electrical Engineers presents herewith to the membership its Thirty-fourth Annual Report, for the fiscal year ending April 30, 1918. A General Balance Sheet showing the condition of the Institute's finances on April 30, 1918, together with other detailed financial statements, is included herein.

Directors' Meetings.—The Board of Directors held ten regular meetings during the year and one adjourned meeting. Eight meetings were held in New York, one in Philadelphia in October, and one in Cleveland in March. The adjourned meeting was held in New York in March.

The Executive Committee held two meetings in New York, on January 5 and January 22, 1918. Both of these meetings were held for the purpose of considering matters relating to the Engineering Council.

In accordance with the practise established years ago, the Board has endeavored to keep the membership informed of its proceedings by the monthly publication of a résumé of the business transacted at each meeting. These notices have appeared in each issue of the PROCEEDINGS, but they are not, however, a complete report of the work done by the Board at any one meeting, for the reason that many important matters are referred to committees for further consideration, and publicity in such cases must generally be deferred pending their final disposition. Information relating to such matters is usually published in subsequent issues.

Annual Meeting.—The Annual Business Meeting was held at Institute headquarters, New York, on May 18, 1917. The Annual Report of the Board of Directors for the fiscal year ending May 1, 1917, was presented as published in full in the June 1917 issue of the PROCEEDINGS. The Tellers Committee presented its report upon the election of officers for the year beginning August 1, 1917.

The principal feature of the meeting was the presentation of the Edison Medal to Nikola Tesla; this ceremony following the business meeting.

Annual Convention.—The Thirty-fourth Annual Convention was originally scheduled to be held at Hot Springs, Va., June 26-29, 1917. In view of the national crisis which developed in April it was decided by the Board of Directors at the May meeting to cancel the Convention and to hold instead a special meeting in New York City, for the presentation of the papers originally scheduled for presentation at the Convention. This special meeting was held on June 27-28.

Pacific Coast Convention.—The Pacific Coast Convention for 1917 was to have been held under the auspices of the Los Angeles Section in September 1917. At the May meeting of the Board of Directors, upon the recommendation of the Los Angeles Section and the Meetings and Papers Committee, the Convention was cancelled.

Philadelphia Meeting.—The first Institute meeting of the administrative year of 1917-18 was held in Philadelphia on October 8, 1917. There were two technical sessions, at which three papers were presented.

Three-City Meeting.—A Three-City Meeting was held in Boston, January 8, New York, January 11, and Chicago January 14, 1918, the same paper being presented at all three meetings. This innovation was tried for the purpose of providing an opportunity for the local members in all three cities to participate in the discussion, all of which will be published in the TRANSACTIONS.

Midwinter Convention, New York.—The Sixth Midwinter Convention was held in New York on February 15 and 16, 1918. Four technical sessions were held. The attendance was about 400. On the evening of February 15 an informal dinner was held which was attended by 225 members and guests.

Cleveland Meeting.—An inter-section meeting in which the Toledo, Toronto, Detroit, Pittsburgh and Cleveland Sections participated, was held in Cleveland, Ohio, on March 8, 1918. Four technical papers were presented at two technical sessions. An informal dinner was held between the afternoon and evening sessions. The latter session being held jointly with the Association of Iron and Steel Electrical Engineers, one of the four papers was presented on behalf of the A. I and S. E. E. The meeting was attended by Presidents E. W. Rice, Jr., of the A. I. E. E., and C. A. Mink, of the A. I. and S. E. E., and members were present from all the sections mentioned above. The attendance at this meeting was about 150.

Pittsburgh-New York Meeting.—The Institute meeting for April was held in Pittsburgh on April 9 and in New York on April 12, under an arrangement similar to the meetings held in Boston, New York and Chicago in January 1918.. The same two papers were presented at both the New York and Pittsburgh meetings.

New York Meetings.—In addition to the meetings referred to above, Institute meetings were also held in New York in November and April. The attendance at the November meeting was 175 and one paper was presented; the attendance at the April meeting was 250 and two papers were presented. The April meeting was preceded by an informal buffet supper under the auspices of the New York Membership Acquaintance committee.

National Defense.—In the annual reports for 1916 and 1917 the published references to the Institute's connection with matters pertaining to National Defense were limited to the various actions taken during the year covered by each report. It has been thought, in view of the entry of the United States into the war, that the membership will gain a better perspective of the services actually rendered by the Institute if in preparing the present report there be included not only a record of the developments of the past year, but a summary of all of the defense work done during the three years from the time when the co-operation of the engineering societies was first suggested in connection with national preparedness. It is felt that such a statement will also be useful as a record for future reference, and the following has been prepared with these objects in view.

Engineer Officers' Reserve Corps.—Early in 1915, when the subject of an Officers' Reserve Corps in connection with the Army reorganization plan was under consideration in Congress, the suggestion was made that the National Engineering Societies tender their assistance to the War Department in the formation of an Engineer Reserve, there being no adequate provision at the time for any large body of engineers in the Army. Acting on this suggestion the Institute and other national engineering societies formed a joint committee for the purpose of taking such steps as seemed desirable to assist in the organization of such a corps as a part of the regular Army. Conferences were held with the Secretary of War, officers of the General Staff and the War College, and the chairmen of the House and Senate Committees on Military Affairs. Largely as a result of the work of this joint committee, the National Defense Act of 1916, which became effective on July 1st of that year, embodied a provision for an Engineer Reserve Corps as part of the Officers' Reserve Corps. A circular containing an abstract of the engineer section of this Act was mailed to all members of the Institute on July 5, 1916.

Upon completion by the War Department of the details of the requirements and qualifications for commissions in the Officers' Reserve Corps as provided in the new law, a second circular was issued by the Institute to the membership under date of September 12, 1916, giving complete information regarding the qualifications for commissions in the Reserve, thus enabling interested members to apply for commissions promptly. Many applications were filed at that time by members of the Institute who are now in active service.

Naval Consulting Board.—In July 1915 the Institute was invited by the Honorable Josephus Daniels, Secretary of the Navy, to select two members for appointment by him upon a proposed Naval Advisory Board, to be presided over by Mr. Thomas A. Edison, and to be composed of men recognized throughout the country for their inventive genius and engineering achievements, to assist the Navy Department, instructively and critically, in the development of such new ideas for naval advance as might be found worthy of consideration. The underlying idea was to make available to the Navy Department the latent inventive and engineering genius of the country to improve the Navy and to bring the officers of the service into more intimate contact with the industrial resources of the country. Similar invitations were extended to ten other scientific and engineering organizations.

Industrial Iuventory.—The good work of the Institute's representatives upon the Naval Consulting Board is attested to in the following letter and invitation received by the Institute from the President of the United States, under date of January 13, 1916:

"The White House
Washington.

January 13, 1916.

My dear Sir:

The work which the American Institute of Electrical Engineers has done through its members on the Naval Consulting Board is a patriotic service which is deeply appreciated. It has been so valuable that I am tempted to ask that you will request the Institute to enlarge its usefulness to the Government still further by nominating for the approval of the Secretary of the Navy a representative from its membership for each state in the Union to act in conjunction with representatives from the American Society of Mechanical Engineers, the American Society of Civil Engineers, the American Chemical Society, and the American Institute of Mining Engineers, for the purpose of assisting the Naval Consulting Board in the work of collecting data for use in organizing the manufacturing resources of the country for the public service in case of emergency. I am sure that I may count upon your cordial cooperation.

With sincere regard,

Cordially yours,
(Signed) Woodrow Wilson.

Mr. John J. Carty, President,
American Institute of Electrical Engineers,
New York City."

This invitation was accepted by the Board of Directors of the Institute at a special meeting called on January 21, 1916, and the nominees selected by the Institute were subsequently appointed by the Secretary of the Navy. The resulting organization became officially known as the *State Directors of the Organization for Industrial Preparedness and Associate Members of the Naval Consulting Board of the United States.* These State Directors made a canvass of all the industrial establishments in their respective states by means of a confidential industrial inventory form giving in great detail data regarding their manufacturing and producing resources. The statistics obtained by the Government as the result of this work fully justified the confidence shown by the President of the United States in entrusting to engineers a task of such magnitude and detail.

General Co-operation with Government.—On February 5, 1917, during the diplomatic exchanges between the American and German Governments, resulting from the announced policy of the latter to sink merchant shipping in certain prescribed zones without warning, the following joint telegram, signed by the Presidents of the Civil, Mining, Mechanical and Electrical Engineers, was sent to the President of the United States:

"To the President,
Executive Mansion,
Washington, D. C.

We, the presidents of the National Societies of Civil, Mining, Mechanical and Electrical Engineers, and of the United Engineering Society, with a membership of thirty thousand, cordially unite in supporting Congress and the Administration in the stand for freedom and safety on the seas, and we are confident that we represent the membership of the four societies in offering to assist toward the organization of engineers for service to our country in case of war."

This was followed by a special meeting of the Executive Committee on February 6, at which it was decided to issue immediately, on behalf of the Board of Directors, to all Institute members in the United States, a circular letter calling attention to this telegram and to the opportunities existing for patriotic service to the nation in case of emergency. This circular letter was issued under date of February 8, 1917, and was accompanied by a simple classification sheet which members were requested to fill out and return to Institute headquarters, indicating whether or not they would be available for military or naval service if required. Over two thousand of these classification sheets were filled out and returned to Institute headquarters and subsequently were placed at the disposal of the War Department. A considerable number of the members who sent in data sheets was communicated with by the branches most urgently in need of their services.

On April 2, 1917, in co-operation with several other engineering societies, the Institute issued to the membership two pamphlets containing instructions and methods of procedure for engineers who desired to offer their services in the Army or Navy of the United States. The pamphlets also contained all information available regarding all of the branches of the Army and Navy in which the services of electrical engineers could be utilized to good advantage. The circulation of these pamphlets aroused great interest and resulted in the filing of a great many more applications by Institute members for commissions in the Officers' Reserve Corps and in the Naval Reserve, many of which were favorably acted upon.

During the past year the Institute has maintained its connection with the Government through a number of agencies, among which may be mentioned: the Naval Consulting Board, the Subcommittee on Wires and Cables of the Standards Committee, the National Research Council, the Engineering Council, and the General Engineering Committee of the Advisory Commission of the Council of National Defense.

Personnel.—In the work of obtaining technically trained men and men of special qualifications for Government service, the Institute has been able to render considerable assistance to the Government. It has been called upon repeatedly by various branches of the Army, and the Navy for technical men for special services, also by industrial corporations and various Government bureaus, to recommend individuals or small groups of men for special work.

On September 20, 1917, Rear-Admiral L. C. Palmer, Chief of the Bureau of Navigation, Navy Department, in a letter addressed to the President of the Institute, informed him that the Secretary of the Navy had authorized the commissioning of one hundred graduate electrical engineers as Lieutenants, junior grade, in the U. S. Naval Reserve Force, that three organizations, namely, the Naval Consulting Board, the National Research Council and the American Institute of Electrical Engineers, were each requested to nominate eighty-five men possessing certain specified qualifications, and that from the total of two hundred and fifty-five candidates thus nominated, a Board of Naval Examiners would select the one hundred men to be commissioned. A circular containing this information was mailed to the entire membership of the Institute in the United States, so that every member might have an opportunity to apply for one of these commissions or to recommend desirable candidates.

At the Directors meeting held on October 8, 1918, a special committee was appointed to examine the applications received by the Secretary and to select therefrom the Institute's eighty-five nominees. This committee, consisting of five members of the Board of Directors, met at Institute headquarters on October 16 and 17, and from the applications received up to that date, selected, solely upon their merits, eighty-five nominees whose names were transmitted to Rear-Admiral Palmer on October 18. The Naval Consulting Board and the National Research Council transmitted their nominations upon the same date.

Of the eighty-five men nominated by the Institute, thirty-eight were included in the first hundred commissioned. It was learned later from reports received from individual members of the Institute that in addition to the one hundred men who were originally commissioned a considerable number of other applicants received commissions.

Under date of March 22, 1918, the Institute was requested by Rear-Admiral Palmer to nominate, in groups of twenty-five each, specially qualified electrical engineers, for training as submarine officers in the Naval Reserve Force. The successful candidates are to be commissioned as Ensigns, and after satisfactorily completing a special technical course of instructions they will become part of the active Submarine Officer Complement of the Navy. The first group of nominations was submitted to the Navy Department, early in April.

This request was supplemented several weeks later with a call for an additional 50 nominees for the rank of Ensign in general service in the Naval Reserve Force. Work on the selection of these nominees is now progressing.

Other requests received by the Institute for co-operation in obtaining the services of technical men include the following:

Navy Department: Bureau of Navigation, candidates for aviation inspection duty for training as Ensigns in the Naval Reserve Flying Corps; Bureau of Yards and Docks, candidates for appointment in the Civil Engineer Corps of the Naval Reserve Force, for the ranks of Ensign, Lieutenant (junior grade) and Lieutenant.

Bureau of Chemistry: electrical engineers for special investigations in grain-dust explosions in mills throughout the country.

War Department: Signal Corps, electrical men for radio division, radio operators and mechanicians for Air Service, and graduate electrical engineers for the aviation section; Coast Artillery Corps, electrical graduates for training school at Ft. Monroe, Va. with opportunity for commissions; Ordnance Department, Trench Warfare Section, technical men for commissions in Ordnance Officers Reserve Corps; Corps of Engineers, enlisted men for engineering and railway units and replacement regiments of engineers.

Eight hundred and twenty-four members of the Institute are now serving with the uniformed forces in the Army or Navy of the United States, and in addition, a large number are serving the Government in various civilian capacities. A great many members are also giving their services as members of various committees engaged in war activities.

General Engineering Committee.—Shortly after the entry of the United States into the war and at the suggestion of President Wilson, each of the four National Engineering Societies designated two representatives upon a committee of the Advisory Commission of the Council of National Defense under Dr. Hollis Godfrey, Commissioner of Engineering and Education, as chairman, to serve as the medium through which the engineering societies might serve the Government. In September last this committee was reorganized as the "General Engineering Committee," with Prof. C. A. Adams as chairman.

Besides acting on minor questions referred to it by the Government, this committee was active on four important tasks: 1. the preparation of a scheme of organization for war purchases, which has since partly been put into operation; 2. electric welding in shipbuilding; 3. development of cast steel anchor chain; 4. specifications for shipboard cable for the Navy Department. (For details regarding the three latter items see statement herein relating to the Standards Committee.)

The General Engineering Committee dissolved when the Council of National Defense discontinued all of its advisory committees. The work was then reorganized and continued under other agencies.

National Research Council.—The National Research Council was formed at the request of the President of the United States by the President of the National Academy of Sciences for the purpose of furthering scientific research in its broadest aspects. Its field includes educational institutions, technical, scientific and medical organizations, governmental departments and the industries. Upon invitation the Institute designated representatives upon the Engineering Committee of the Council. This committee was organized early in 1917 with offices in Washington and New York. It is in contact with the work of the other committees of the Council and since its organization it has been engaged on a wide range of engineering problems brought to its notice by the several branches of the Government. Weekly meetings have been held and weekly reports of its activities have been sent to each member of the committee. The confidential nature of these reports and of the problems which have been under consideration has rendered it inexpedient to publish detailed statements regarding the work which has been carried on.

Membership Service Classification.—Early in February 1917 the Institute offered its services to the Government in case of emergency. It soon became evident that in order to extend the fullest possible co-operation and to respond promptly and effectively to the numerous demands which were being made upon the Institute by the different governmental departments and industrial establishments engaged in Government work, for technical men of special qualifications, it would be necessary to have available for immediate reference at Institute headquarters detailed information regarding the many members who were willing and anxious to serve the Government. After conferences with the other National Engineering Societies, the matter was brought to the attention of the Board of Directors of the Institute at its meeting of October 8, 1917, at which meeting a proof of a suggested form of questionnaire which had been prepared was submitted with a request for an appropriation sufficient to distribute the form to the membership and to compile an index of the replies. Recognizing the importance of such a classification, the Board granted an appropriation of $1,000 for this purpose and authorized the Secretary to proceed with the work after opportunity had been given for final revision of the form. A special committee was appointed to have supervision of the classification and indexing.

Although primarily intended as an aid in selecting technical men for service in the present emergency, it is proposed to maintain a permanent file of this data at Institute headquarters to be revised from year to year for use after peace has again been restored. Such a classification has been contemplated for several years in connection with the regular work of the Institute and its committees.

Employment.—It is over three years since the Institute first initiated the plan for assisting its members in obtaining employment and employers in obtaining desirable employees. It has not attempted to conduct an employment department in the generally accepted sense of the term, but has simply acted as a medium for placing men in touch with opportunities, through the publication of announcements in the montly Institute PROCEEDINGS, under the heading of "Engineering Service Bulletin."

While the service might have been developed to a greater extent had it been possible to appropriate funds for carrying on the work, the results attained since the plan was inaugurated are, nevertheless, very gratifying. There is no doubt that it offers a wide sphere of usefulness, and that the service is highly appreciated; not only by the individual member who is seeking a position, but also by the many corporations and men in posts of responsibility who have been assisted by the Institute in obtaining the services of high-class technical men. The co-operation of the membership in bringing vacancies to the attention of the Secretary will be very helpful in the future successful conduct of this work.

United Engineering Society.—In the Directors' Annual Report for 1917 reference was made to the addition of three stories to the Engineering Societies Building as a result of an agreement whereby the American Society of Civil Engineers was to be admitted into the fraternity of founder societies and take up its headquarters in the building.

Full details regarding this agreement were published in the PROCEEDINGS for September 1916.

This work was completed in the Fall of 1917 and on December 7, 1917, a joint meeting was held under the auspices of the Institute, the American Society of Mechanical Engineers, and the American Institute of Mining Engineers, at which the American Society of Civil Engineers was formally welcomed into the fraternity of the founder societies and the occupancy of its quarters in the enlarged Engineering Societies Building. These four great National Engineering Societies are thus brought together under one roof with all of the obvious advantages for closer cooperative action, realizing more fully the purpose of the donor of the building, Mr. Andrew Carnegie, that it should be the home and headquarters of the engineering profession of America.

Engineering Council.—The Engineering Council represents the result of an organized effort inaugurated in the latter part of 1916 by four of the leading national engineering societies—the American Society of Civil Engineers, American Society of Mechanical Engineers, American Institute of Mining Engineers and the American Institute of Electrical Engineers, to establish a central body to deal with matters of common interest to engineers and to serve as a connecting medium between the engineer on the one hand and the public welfare on the other so far as such matters relate to the engineering profession, in order that united action may be possible.

The first meeting of the Council was held on June 27, 1917. The Annual Meeting of the Council was held on February 21, 1918. Officers were elected and the following committees were appointed: Executive, Finance, Rules, Public Affairs, American Engineering Service, War, Fuel Conservation and Patents.

For details regarding future plans of the Council, its field, and aims, members are referred to the abstracts from a statement issued by the Council in February 1918, published in the Institute PROCEEDINGS for April 1918.

Sections Committee.—The encroachment of the many demands of these unprecedented times has been reflected in the activities of the Sections and Branches. Many of the leading men have been called away on war duties, while others have assumed additional burdens at home. The number of meetings held during the past year has for these reasons been reduced, yet it is gratifying to observe that the total attendance in the Section meetings has actually increased.

This is undoubtedly due to the fact that these meetings have been arranged to present topics of current interest. It has been evident to any one who has seen the notices of these various meetings over the country that the thought of service and knowledge of present day problems were uppermost in the mind of the engineer.

The list of Sections has been increased by the addition of Erie, Pa., which qualified on January 11, 1918, under conditions which inspire confidence in its future.

The Branches have, however, been severely affected by the influences of war. There our young engineers have forfeited their present educational advantages and have gone into the ranks of the service in large numbers. Several Branches have suspended activity, temporarily, while two, the Queens University Branch, Kingston, Ontario, and the Michigan Agricultural College Branch, East Lansing, Michigan, were added, respectively, on January 11 and March 15, 1918. Two other Branches, the Iowa State College Branch, and the Rhode Island State College

Branch, were terminated by the Board of Directors at their request. The tabulation following shows the reduction in Branch activity.

	For Fiscal Year Ending					
	May 1 1913	May 1 1914	May 1 1915	May 1 1916	May 1 1917	May 1 1918
SECTIONS						
Number of Sections..........	29	30	31	32	32	34
Number of Section meetings held....................	244	233	246	251	265	245
Total Attendance...........	22,825	22,626	23,507	28,553	31,299	34,614
BRANCHES						
Number of Branches........	47	47	52	54	59	59
Number of Branch meetings held....................	357	306	328	360	368	268
Attendance................	11,808	11,617	12,712	15,166	16,107	10,683

On the whole the situation in the Sections and Branches is one for encouragement rather than otherwise. It augurs well for the participation of the engineer in the country's large affairs and insures the continued maintenance of the activities of the Institute.

Standards Committee.—The Standards Committee has held monthly meetings throughout the year except in January and the summer months.

Owing to the numerous demands made upon many members of the committee by war work, all non-essential subcommittees were allowed to mark time thus reducing the number of active subcommittees from over 40 to about 20.

Subcommittee on Wires and Cables.—The Subcommittee on Wires and Cables of the Standards Committee was appointed by the Board of Directors in response to the request of Rear-Admiral Griffith, Chief of the Bureau of Steam Engineering, to assist the Navy Department in the solution of problems relating to wires and cables with special reference to the high-tension cables to be used on the new electrically-driven warships.

This committee, after conducting a considerable number of investigations and much experimental research, has sent reports to the Navy Department on several subjects. The committee is still co-operating with the Navy Department and expects to extend its activities to other departments of the Government.

Electric Welding in Ship Construction. Another important piece of war work which originated in the Standards Committee is the application of electric welding to shipbuilding. This was started in August 1917 as the Electric Welding Subcommittee of the Standards Committee, and was adopted by the General Engineering Committee of the Council of National Defense in September. Finally, after the Council of National

Defense had dropped all of its advisory committees, this subcommittee was appointed in February by the U. S. Shipping Board, Emergency Fleet Corporation, as its Electric Welding Committee, with C. A. Adams as chairman and with ample financial support. An enormous amount of valuable work has been and is being done by this committee, which includes representatives of: The Emergency Fleet Corporation, the Classification Societies, (Lloyds Register of Shipping and the American Bureau of Shipping), U. S. Navy, Bureau of Standards, electric welding manufacturers and users, electrical engineers and metallurgists.

The Research Subcommittee of the Electric Welding Committee is also attached to the Engineering Division of the National Research Council.

The principal object of the Electric Welding Committee is to save time, labor, material and expense by the extension of the application of electric welding to shipbuilding. This is being done not only by the extension of the application to minor parts of ships, but also by demonstrating its suitability and economy on the capital parts of ships.

British Conference. As a result of an invitation from the Engineering Standards Committee of Great Britain, Mr. H. M. Hobart was sent to London in September 1917 as a delegate from the Standards Committee to a joint conference on standards for electrical machinery. Mr. Hobart brought back numerous suggestions which have been acted upon by the Standards Committee.

Mr. Hobart also investigated the electric welding situation in England and rendered a report which has proved very satisfactory in the work of the Electric Welding Committee.

Another result of this visit was that the Emergency Fleet Corporation, engaged Captain James Caldwell, in charge of the electric welding work for the British Admiralty, to spend a couple of months here and to give the Electric Welding Committee the benefit of the experience of Great Britain. This visit has proved most helpful all around.

Standardization Rules. The result of the year's work of the Standards Committee as far as the Standardization Rules are concerned, does not involve many radical changes, although some of the additions are distinctly valuable. The changes and additions will be published in a revised supplement. A complete revision of form and arrangement is under way and will be completed for a new edition in 1919.

American Engineering Standards Committee.—In January 1917 a joint committee was appointed with three representatives from each of the Four National Engineering Societies, and the American Society for Testing Materials, to prepare a plan of organization for an American Engineering Standards Committee. This committee completed its work on June 19, 1917, and rendered its report to the five societies interested.

The governing boards of the four National Engineering Societies, A. S. C. E., A. S. M. E., A. I. M. E., and A. I. E. E., approved the plan of organization and appointed three representatives each upon the proposed American Engineering Standards Committee. The executive committee of the A. S. T. M., presented certain suggestions and additions. The organization committee was therefore continued and proceeded to consider the suggestions of the A. S. T. M.

A revision of the constitution and rules of procedure governing the proposed American Engineering Standards Committee, satisfactory to the representatives of all the societies, has finally been completed and a report has been submitted to the governing boards of the respective societies for approval.

Meetings and Papers Committee.—The Meetings and Papers Committee has held six monthly meetings during the past year, at which the programs of the Institute meetings and conventions during the season were arranged. One meeting was held in Philadelphia in October 1917, four in New York during the winter, and one in Cleveland in March 1918.

The policy of the Institute of holding a number of its regular monthly meetings outside of New York City was thoroughly discussed at the opening meeting and while there was considerable sentiment in favor of continuing this policy as in the previous year, the difficulty of railroad travel and the fact that the present year was an extremely busy one for a large proportion of the Institute membership led the committee to limit the meetings outside of New York to two; one of which was held in Philadelphia in October, and the other in Cleveland in March.

About 80 papers have been considered by the committee during the year and owing to the abnormally high cost of printing and publishing at the present time the selection of papers has been made with unusual care. Every paper which has been offered during the past year has been reported upon by one or two members of the technical committee to which it belonged before final action has been taken by the Meetings and Papers Committee.

In order to bring the papers presented at regular meetings before as large a number of the Institute members as possible, a new arrangement for meetings has been devised which has been called Inter-section Meetings, in which one paper is presented as nearly simultaneously as possible before several different Sections. Two of these meetings have been held this year with considerable success; one in January 1918 in Boston, New York and Chicago, and another in April in Pittsburgh and New York. The meeting at Cleveland was held under the joint auspices of the Cleveland, Pittsburgh, Toledo, Toronto and Detroit Sections, and a joint technical session was held with the Association of Iron and Steel Electrical Engineers. This participation by the different Sections in regular Institute meetings is believed to have added considerable stimulus to the activities of the Sections participating and has been generally beneficial to the Institute.

Editing Committee.—The Editing Committee, which now has entire supervision of the TRANSACTIONS, announced last year that future issues of TRANSACTIONS would be published semi-annually. Owing to war considerations, however, which resulted in a considerable reduction in the amount of material published, it was found that the papers for the entire year would make only one volume of the usual size and it was subsequently decided to publish the 1917 TRANSACTIONS in one volume. This plan was followed to avoid the publication of two small books instead of one of the usual size, and it also results in a very considerable reduction in the expense of binding and distribution.

The volume is now completed and is being distributed approximately five months earlier than in previous years.

The system of handling papers and discussions for the TRANSACTIONS has been gradually evolved from experience with different methods. It has now been practically standardized for several years and appears to meet with general satisfaction.

All authors and discussors have the opportunity of revising their contributions both in the manuscript of the stenographer and in the proof. Each author has the privilege of reading the proof of all discussion before the final revision of his closure.

All papers published in the PROCEEDINGS are not necessarily published in the bound TRANSACTIONS. The responsibility for a decision in this matter now rests with the Editing Committee.

The committee suggests that papers of the character that are accepted for publication without presentation be printed only in the Annual TRANSACTIONS.

Public Policy Committee.—The Public Policy Committee held one formal meeting during the year at which a communication regarding the bill to incorporate the American Academy of Engineers, referred to the committee by a vote of the Board of Directors, was considered. A report upon this matter was subsequently made to the Board of Directors by the committee.

A majority of the members of the Public Policy Committee, being members also of the Engineering Council, official action by the committee on matters other than that above referred to has not seemed desirable, as such matters now come within the scope of the Engineering Council, thus resulting in a large measure in cooperation between the four National Engineering Societies.

Code Committee.—The Code Committee has continued to represent the Institute on the Electrical Committee of the National Fire Protection Association and representatives of the committee attended the biennial meeting of the National Fire Protection Association. The name of this committee was recently changed by an amendment to the by-laws of the Institute from "Code Committee" to "Committee on Safety Codes."

U. S. National Committee, International Electrotechnical Commission.—Two meetings were held by this committee during the year, both at Institute headquarters; the first on December 14, 1917, the second on March 13, 1918, the latter meeting being held jointly with the Institute's Standards Committee.

The International Electrotechnical Commission has been prevented by the War from holding any international gathering, but it has been the expressed desire of the U. S. National Committee that an attempt should be made to hold such a meeting as soon as conditions will permit, as the lack of intercommunication between the various national committees of the allied countries since 1913 has naturally served to retard generally electrotechnical development.

At the March meeting attention was given to certain inquiries made by the French National Committee concerning such changes as may have taken place in the A. I. E. E. Standardization Rules since the last convention of the International Electrotechnical Commission in 1913. The necessity for such explanations would not have arisen if the work of the Commission had not been held in abeyance by the world-war. Some progress, however, has been made independently among the various individual national committees.

Committee on Code of Professional Conduct.—The only matter which this committee acted upon during the year was the formulation of definitions of an "Electrical Engineer." A report embodying several definitions has been prepared and filed at Institute headquarters.

Board of Examiners.—The Board of Examiners held 18 meetings during the year, averaging about three hours each. It considered and referred to the Board of Directors with its recommendations a total of 1889 applications of all kinds. In addition to these the Board reviewed or reconsidered 33 applications for a second and third time.

The demand upon the time of the Board has been greater during the past year than in any previous year, owing to the large number of applications filed for admission or transfer to the higher grades. Such applications are considered in great detail and all of the evidence submitted by the applicants, including the record and communications from references and others, is read by the Board.

The result of the Board's work for the year is given in the following tabulated statement:

APPLICATIONS FOR ADMISSION.

Recommended for grade of Associate	1036	
Not recommended for grade of Associate	10	1046
Recommended for grade of Member	74	
Not recommended for the grade of Member	50	124
Recommended for grade of Fellow	11	
Not recommended for Fellow	5	16
Recommended for enrolment as students	576	576

APPLICATIONS FOR TRANSFER.

Recommended for grade of Member	66	
Not recommended for grade of Member	41	107
Recommended for grade of Fellow	11	
Not recommended for grade of Fellow	9	20
Total number of applications considered		1889
Applications reconsidered		33
Total		1922

Membership Committee.—The work of the committee has resulted in the filing of 1235 new applications. Much credit for the success of the committee's work is due to the loyal support of the Sections and more particularly the chairmen of the local membership committees.

The following tabulated statement shows the number of members in each grade, the total membership of the Institute on April 30, and the additions and deductions which have been made during the year. It is not intended, however, to show the number of applications received, as a considerable number is still in the preliminary stages and cannot, therefore, be embodied in a list of the members of the Institute.

	Honorary Member	Fellow	Member	Associate	Total
Membership, April 30, 1917.	4	455	1223	7028	8710
Additions:					
Transferred	....	14	76	....	....
New Members Qualified	2	3	66	937	....
Reinstated	....	2	8	54	....
Deductions:					
Died	....	4	7	26	....
Resigned	....	1	3	111	....
Transferred	....	....	4	84	....
Dropped	....	5	27	318	....
Membership, April 30, 1918	6	464	1332	7480	9282

Net increase in membership during the year.............................. 572

Deaths.—The following deaths have occurred during the year:

Fellows.—Albert F. Ganz, F. B. H. Paine, John K. Robinson, Karl Von Krogh.

Members.—Harry Bottomley, Eugene F. Roeber, Henry R. Ford, J. G. Lorrain, Osborn P. Loomis, E. W. Stevenson, E. P. Warner.

Associates.—L. R. Pomeroy, S. H. Harvey, Stuart A. Nims, W. K. Kretsinger, P. H. Goodwin, W. S. Horry, Arthur Gunn, William G. Bee, T. Ohta, John Sachs, W. H. Peberdy, George Scharfe, Percy L. Cobb, John Gilmartin, R. C. Carter, Charles O. Smith, William Duddell, John Hesketh, John H. Goehst, Robert L. Stevenson, O. Zell Howard, Bernard W. Capen, Cyril F. Mickler, St. John Chilton, James A. Barkley, Henry N. Brooks.

Total deaths, 37.

Edison Medal.—The Edison Medal for 1916, which had been awarded to Nikola Tesla by the Edison Medal Committee in December of that year "for his early original work in polyphase and high frequency electric currents" was presented to Mr. Tesla with appropriate ceremonies at the Annual Meeting of the Institute held in New York on May 18, 1917.

The Edison Medal for 1917 has been awarded to Col. John J. Carty "for his work in the science and art of telephone engineering," and the presentation will take place at the Annual Meeting of the Institute which will be held in New York on May 17, 1918.

John Fritz Medal.—The John Fritz Medal for 1917 was awarded to Dr. Henry Marion Howe "for his investigations in metallurgy, especially in the metallography of iron and steel." The presentation was made in the Engineering Societies Building in New York on May 10, 1917.

The John Fritz Medal for 1918 was presented to Mr. J. Waldo Smith in the Engineering Societies Building, New York, at a joint meeting held on April 17, 1918, for "achievement as engineer in providing the City of New York with a supply of water."

Finance Committee.—During the year the committee has held monthly meetings, has passed upon the expenditures of the Institute for various purposes, and otherwise performed the duties prescribed for it in the Constitution and By-laws.

Haskins and Sells, certified public accountants, have audited the books, and their report is included herein. It will be noted that in the report a readjustment of the Institute's equity in the property held in trust by the United Engineering Society has been made, due to the admission of a fourth founder society to the United Engineering Society.

In company with the Secretary, the Treasurer, and a representative of Haskins and Sells, the chairman of the committee examined the securities held by the Institute and found them to be as stated in the accountants' report.

NEW YORK
CHICAGO
DETROIT
ST. LOUIS
CLEVELAND
BALTIMORE
PITTSBURGH

HASKINS & SELLS
CERTIFIED PUBLIC ACCOUNTANTS
CABLE ADDRESS "HASKSELLS"
30 BROAD STREET
NEW YORK

SAN FRANCISCO
LOS ANGELES
SEATTLE
DENVER
ATLANTA
WATERTOWN
LONDON

AMERICAN INSTITUTE OF ELECTRICAL ENGINEERS

CERTIFICATE

We have audited the books and accounts of the American Institute of Electrical Engineers for the year ended April 30, 1918, and

WE HEREBY CERTIFY that the accompanying General Balance Sheet properly sets forth the financial condition of the Institute on April 30, 1918, that the Statement of Income and Profit & Loss for the year ended that date is correct, and that the books of the Institute are in agreement therewith.

HASKINS & SELLS,
Certified Public Accountants.

NEW YORK,
May 14, 1918.

AMERICAN INSTITUTE OF

EXHIBIT A.

GENERAL BALANCE SHEET

ASSETS.

REAL ESTATE:			
One-fourth Interest in United Engineering Society's Real Estate, No. 25 to 33 West 39th Street:			
Land and Building		$472,500.00	
Real Estate Equipment, etc.		14,292.79	
Total Real Estate			$486,792.79
EQUIPMENT:			
Library—Volumes and Fixtures		$ 40,031.55	
Works of Art, Paintings, etc.		3,001.35	
Office Furniture and Fixtures		12,274.65	
Total		$ 55,307.55	
Less Reserve for Depreciation		9,419.77	
Remainder—Equipment			$ 45,887.78
INVESTMENTS:			
Bonds—City of Wilmington, Delaware, 4½%, 1934, Par Value $15,000.00		$ 15,834.19	
United States Liberty Loan, 4¼% Bonds		10,000.00	
Total Investments			25,834.19
WORKING ASSETS:			
Publications Entitled "Transactions," etc.		$ 14,049.00	
Paper and Cover Paper		1,046.28	
Badges		676.70	
Total Working Assets			15,771.98
CURRENT ASSETS:			
Cash		$ 12,270.07	
Accounts Receivable:			
Members for Past Dues		9,559.82	
Advertisers		769.70	
Miscellaneous		470.76	
Accrued Interest on Investments		56.25	
Accrued Interest on Bank Balances		358.66	
Total Current Assets			23,485.26
FUNDS:			
Life Membership Fund:			
Cash	$ 438.67		
Chicago, Burlington & Quincy Railroad Company Bonds, 4%, 1958, Par Value $5,000.00	4,868.75		
Accrued Interest	33.33	$ 5,340.75	
International Electrical Congress of St. Louis—Library Fund:			
Cash	$ 943.99		
New York City Bonds, 4½%, 1957, Par Value $2,000.00	2,248.71		
Accrued Interest	45.00	3,237.70	
MAILLOUX FUND:			
Cash	$ 167.35		
New York Telephone Company Bond, 4½%, 1939	1,000.00		
Accrued Interest	22.50		
		1,189.85	
Midwinter Convention Fund—Cash		163.58	
International Electrical Congress of San Francisco—Cash		40.50	
Total Funds			$ 9,972.38
DUES PAID IN ADVANCE—INTERNATIONAL ELECTROTECHNICAL COMMISSION, LONDON, ENGLAND			500.00
Total			$608,244.38

ELECTRICAL ENGINEERS.

APRIL 30, 1918.

LIABILITIES

CURRENT LIABILITIES:		
Accounts Payable—Subject to Approval by the Finance Committee	$ 6,304.01	
Due United Engineering Society Account Building Addition	10,000.00	
Dues Received in Advance	2,382.87	
Entrance Fees and Dues Advanced by Applicants for Membership	179.50	
Total Current Liabilities		$ 18,866.38
FUND RESERVES:		
Life Membership Fund	$ 5,340.75	
International Electrical Congress of St. Louis—Library Fund	3,237.70	
Mailloux Fund	1,189.85	
Midwinter Convention Fund	163.58	
International Electrical Congress of San Francisco	40.50	
Total Fund Reserves		$ 9,972.38
SURPLUS: Per Exhibit "B"		$579,405.62
Total		$608,244.38

AMERICAN INSTITUTE OF ELECTRICAL ENGINEERS.

STATEMENT OF INCOME AND PROFIT AND LOSS.

FOR THE YEAR ENDED APRIL 30, 1918.

EXHIBIT B.

REVENUE:			
Entrance Fees		$ 5,545.00	
Dues		96,897.93	
Student's Dues		3,771.00	
Transfer Fees		940.00	
Advertising		8,189.02	
Subscriptions		3,286.20	
Sales of "Transactions," etc		2,574.24	
Badges Sold	$2,253.75		
Less Cost	1,992.12		
		261.63	
Interest on Investments		675.00	
Interest on Bank Balances		1,027.43	
Exchange		30.74	
Total			$123,198.19
EXPENSES:			
Meetings and Paper Committee:			
Salaries		$ 5,560.00	
Binding and Mailing Proceedings		4,384.97	
Printing Proceedings		6,052.07	
Engraving Proceedings		1,523.22	
Paper and Cover Paper		4,581.05	
Envelopes		723.20	
Stationery and Miscellaneous Printing		177.96	
General Expenses		250.35	
Meetings		3,193.14	
Editing Committee:			
Volume No. 34		136.22	
Volume No. 35		12,370.15	
Volume No. 36		5,372.29	
Total		$ 44,324.62	
Deduct Increase in Inventory of Publications:			
April 30, 1917	$12,884.25		
April 30, 1918	14,049.00	1,164.75	$43,159.87
Executive Department:			
Salaries		$ 17,698.50	
General Expenses		2,737.33	
United Engineering Society—Assessments		4,800.00	
Express		311.13	
Postage		1,980.65	
Advertising		2,623.74	
Stationery and Miscellaneous Printing		3,027.68	
Year Book and Catalgue		3,487.90	
Office Furniture and Fixtures—Discarded		51.20	
Total			36,718.13
Sections Committee:			
Section Meetings		$ 6,162.15	
Branch Meetings		208.85	
Salaries, New York Office		2,340.00	
Stationery and Printing, New York Office		698.70	
Express on Advance Copies		8.09	
Total			$ 9,417.79
FORWARD			$ 89,295.79
REVENUE—(Forward)			$123,198.19

Revenue—(Forward)			$123,198.19
Expenses—(Forward)			$ 89,295.79
General:			
Library Committee		$ 4,000.01	
Membership Committee		1,079.38	
Finance Committee		150.00	
Standards Committee		1,221.72	
Code Committee		30.00	
Annual Dues, International Electrotechnical Commission		250.00	
National Defense—Miscellaneous Expenses		1,488.42	
John Fritz Medal Award		94.00	
Honorary Secretary		4,000.00	
United Engineering Society, Engineering Council		1,600.00	
American Engineering Standards Committee		359.37	
Membership Classification Service		776.65	15,049.55
Total			$104,345.34
Deduct:			
Decrease in Accounts Payable—Subject to Approval by the Finance Committee, Expenses Undistributed at:			
May 1, 1917—As Adjusted		$ 7,869.81	
April 30, 1918		6,304.01	1,565.80
Total Expenses			$102,779.54
Net Revenue			$ 20,418.65
Profit and Loss Credits:			
Accessions to Library Volumes and Fixtures		$ 151.75	
Refund of Unexpended Balance of Contribution of American Institute of Electrical Engineers to proposed International Electrical Congress 1915		321.99	
Total			473.74
Gross Surplus for the Year			$ 20,892.39
Profit and Loss Charges:			
Uncollectible Dues Written Off		$ 4,003.75	
Provision for Depreciation of Furniture and Fixtures		1,393.49	
Amortization of Premium on City of Wilmington, Delaware—4½% Bonds of 1934		52.14	
Total			5,449.38
Net Surplus for the Year			$ 15,443.01
Surplus, May 1, 1917		$623,016.43	
Add—Real Estate Equipment and Preliminary Expenses Credited to the Institute by the United Engineering Society but not Heretofore Shown on the Books		15,710.44	
Total		$638,726.87	
Deduct:			
Value of Equity Adjusted to One-fourth Interest:			
Real Estate Equipment, etc.	$ 4,764.26		
Land and Building	132,500.00		
Total	$137,264.26		
Less Increase in Equity in Building by Addition of Three Stories	62,500.00		
Net Adjustment of Equity Value		74,764.26	
Surplus as Adjusted			563,962.61
Surplus, April 30, 1918			$579,405.62

AMERICAN INSTITUTE OF ELECTRICAL ENGINEERS

STATEMENT OF CASH RECEIPTS AND DONATIONS FOR DESIGNATED PURPOSES. ALSO DISBURSEMENTS, FOR THE YEAR ENDED APRIL 30, 1918.

EXHIBIT C.

RECEIPTS:	
Life Membership Fund	$217.68
International Electrical Congress of St. Louis Library Fund—Interest and Royalties	93.40
Mailloux Fund—Interest	45.00
Midwinter Convention Fund—Contributions and Interest	74.53
Total	$430.61
DISBURSEMENTS:	
Life Membership Fund	217.68
Midwinter Convention Fund	6.75
Total	$224.43

RECEIPTS AND DISBURSEMENTS PER YEAR PER MEMBER.

During each fiscal year for the past eight years.

Year ending April 30	**1911**	**1912**	**1913**	**1914**	**1915**	**1916**	**1917**	**1918**
Membership, April 30, each year	7117	7459	7654	7876	8054	8212	8710	9282
Receipts per Member	$13.37	$13.19	$13.45	$14.08	$14.06	$13.62	$13.30	$13.17
Disbursements per Member	11.03	12.44	15.57	12.86	13.54	13.74	12.75	11.99
Credit Balance per Member	$2.34	$ .75	*$2.12	$1.22	$ .52	*$ .12	$ .55	$1.18

*Deficit.

Respectfully submitted for the Board of Directors,

F. L. HUTCHINSON, *Secretary.*

New York, May 17, 1918.

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SYNOPTICAL AND TOPICAL

INDEX

OF

A. I. E. E. TRANSACTIONS

Vol. XXXVII, Parts I and II

The main headings under which these synopses are classified were arrived at by a careful study of all the papers contributed since the organization of the Institute.

The method of making this classification may be called the automatic method, since it is created by sorting the papers themselves into groups and then naming the groups.

Many papers fall naturally into several different groups and in such cases they are inserted under as many different heads as it is thought they rightfully belong.

The classified synopses are designed for those searching for comprehensive information on any given topic, while the subject index is intended for those looking up specific and definite data or information.

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2. GENERAL THEORY

THE TECHNICAL STORY OF THE FREQUENCIES

B. G. Lamme **Vol. xxxvii—1918, pp. 65-85**

The various frequencies used in alternating-current work in America are first mentioned, and the primary reasons for their introduction are given. This is followed by a discussion of various alternating current applications which were more or less dependent upon frequency.

It is shown that there was an apparent need for two standard frequencies in the region of 60 and 25 cycles, and, further, why 60 and 25 cycles have prevailed. The special fields of application of each one are discussed fully and it is shown why 25 cycles tended to dominate the field.

Discussion, pages 86-89, by Messrs. H. B. Brooks, B. G. Lamme, G. S. Mc Cumber, R. Dalgleish and Mr. Hunt.

A general discussion.

SOME APPLICATIONS OF ELECTROMAGNETIC THEORY TO MATTER

A. C. Crehore **Vol. xxxvii—1918, pp. 261-293**

A Lecture.
No discussion

COMMUTATION IN ALTERNATING-CURRENT MACHINERY

Marius C. A. Latour **Vol. xxxvii—1918, pp. 355-381**

The author introduces into the discussion of the commutating characteristics of alternating-current commutating motors, his theory that perfect commutation in a continuous-current motor depends substantially on the production of a mean resultant neutral field in the region where commutation is taking place, and shows that the production of a perfect revolving field in a polyphase commutator motor assists in insuring perfect commutation at exact synchronism.

In a single-phase commutator motor a "polyphase" revolving field can be produced at synchronism by utilizing supplementary brushes, short-circuited upon themselves, displaced by 90 electrical space degrees from the main single-phase brushes on the commutator.

As in the case of polyphase motors, the problem of securing perfect commutation at synchronism becomes that of producing a perfect rotating field. It is shown by the author that the use of fractional-pitch windings on the rotor and a sinusoidal distribution of conductors on the stator is of much assistance in this connection.

Discussion (including that of papers by W. C. K. Altes and V. Karapetoff), pages 379-389, by Messrs. A. M. Gray, P. M. Lincoln, C. F. Scott, W. C. K. Altes, H. M. Hobart, B. A. Behrend, W. B. Jackson and V. Karapetoff.

A general discussion.

NO-LOAD CONDITIONS OF SINGLE-PHASE INDUCTION MOTORS AND PHASE CONVERTERS

R. E. Hellmund

Vol. xxxvii—1918, pp. 539-626

This paper shows methods and derives formulas for the determination of the fields, the stator and rotor magnetizing currents, and the tertiary voltages for phase converters and single-phase induction motors at no-load. This paper treats a large number of different cases along similar lines, thereby coordinating and explaining many phenomena previously observed, and it should, therefore, form a desirable basis for further investigations and discussion of this subject matter.

The paper is arranged so that it can be read to good advantage without going through those mathematical parts marked by vertical rules.

By reading the conclusions at the end of the paper, a fair idea of the principal points brought out in the paper can be obtained.

Discussion incorporated with that of paper by B. G. Lamme on "A Physical Conception of the Operation of the Single-phase Induction Motor."

A PHYSICAL CONCEPTION OF THE OPERATION OF THE SINGLE-PHASE INDUCTION MOTOR

B. G. Lamme

Vol. xxxvii—1918, pp. 627-658

This paper covers a method of studying the actions of the single-phase induction motor, which the writer has found to be very convenient from the educational standpoint. It is based upon the assumption of two equal and oppositely rotating primary magnetomotive forces combined with a synchronously rotating secondary m. m. f., such as would be produced by direct-current excitation. Diagrams and descriptions are given to illustrate the magnetomotive forces and fluxes, showing how, among other conditions, two oppositely rotating fields of unequal value may be possible.

The next step is a consideration of e. m. f. generation by two oppositely rotating fields, showing how both must be taken into account. The effects upon the counter e. m. f. and excitation, of the reduction or suppression of one field is shown.

The full-load conditions are next considered. A comparison is made between a two-motor unit, consisting of two similar polyphase motors coupled together and connected for opposite rotation, and the straight single-phase induction motor. This is followed by a considerable amount of test data which illustrate the principles and actions described in the paper.

Discussion, (including that of paper by R. E. Hellmund), pages 659-702, by Messrs. J. Slepian, C. Fortescue, C. A. M. Weber, G. H. Garcelon, R. E. Hellmund, A. M. Dudley, B. G. Lamme, M. I. Pupin, E. F. W. Alexanderson, B. A. Behrend, L. W. Chubb, A. M. Gray, C. F. Scott and S. Haar.

A general discussion on theory.

METHOD OF SYMMETRICAL CO-ORDINATES APPLIED TO THE SOLUTION OF POLYPHASE NETWORKS

C. L. Fortescue

Vol. xxxvii—1918, pp. 1027-1115

In the introduction a general discussion of unsymmetrical systems of co-planar vectors leads to the conclusion that they may be represented by symmetrical systems of the same number of vectors, the number of symmetrical systems required to define the given system being equal to its degrees of freedom. A few trigonometrical theorems which are to be used in the paper are called to mind. The paper is subdivided into three parts, an abstract of which follows. It is recommended that only that part of Part I up to formula (33) and the portion dealing with star-delta transformations be read before proceeding with Part II.

Part I deals with the resolution of unsymmetrical groups of numbers into symmetrical groups. These numbers may represent rotating vectors of systems of operators. A new operator termed the sequence operator is introduced which simplifies the manipulation. Formulas are derived for three-phase circuits. Star-delta transformations for symmetrical coordinates are given and expressions for power deduced. A short discussion of harmonics in three-phase systems is given.

Part II deals with the practical application of this method to symmetrical rotating machines operating on unsymmetrical circuits. General formulas are derived and such special cases, as the single-phase induction motor, synchronous motor-generator, phase converters of various types, are discussed.

Discussion, pages 1116-1140, by Messrs. J. Slepian, C. P. Steinmetz, V. Karapetoff, A. M. Dudley, C. F. Scott, C. O. Mailloux and C. L. Fortescue.

A general discussion emphasizing certain mathematical methods of simplifying the author's methods.

SUSTAINED SHORT-CIRCUIT PHENOMENA AND FLUX DISTRIBUTION OF SALIENT-POLE ALTERNATORS

Vol. xxxvii—1918, pp. 1141-1202

N. S. Diamant

It shown in Section IV that with the ordinary field forms met with in practise, the resultant flux wave under s. s. c. (sustained short-circuits) will be extremely distorted, see Figs. 16, (4th wave) 23, 27, 27A, etc., for the simple reason that with the very low voltages obtained under such conditions, the fundamental of the *B*-curve of the field is reduced so much by the armature reaction that the higher harmonics assume a very predominant rôle and become several hundred per cent of the s. s. c. fundamental. See Fig. 26 and Tables XI, XII, and XIII.

As a corollary to the foregoing it is found that the *B*-curve under load will not differ radically from the no-load field form since the fundamental will remain large enough to hold its own. See Fig. 22 and also footnote (4). The cross magnetizing effect of the armature reaction is, of course, to make the *B*-curve unsymmetrical with respect to the mid-pole axis. Compare Figs. 7 and 22.

The magnetic oscillations are studied not only by means of full-pitch stator coils but also by means of rotor coils No. 7 and 8, Fig. 3, and stator coils No. 9, 10, 11, 12, 15, and 16, Figs. 3 and 3A.

Discussion, pages 1203-1208, by Messrs. F. D. Newbury, R. E. Doherty, W. F. Dawson, C. M. Davis and N. S. Diamant.

A discussion involving criticisms of author's solution of short-circuit phenomena.

REACTANCE OF SYNCHRONOUS MACHINES AND ITS APPLICATIONS

R. E. Doherty and O. E. Shirley **Vol. xxxvii—1918, pp. 1209-1297**

Part I treats of the calculation and application of the armature self-inductive reactance of synchronous machines. A short, reliable method is given in the form of curves, making the calculation from design sheet data a matter of a few minutes.

An approximate, but convenient, method of applying the armature reactance in the calculation of field excitation under load is given in Appendix C.

In Part II it is shown that the initial short-circuit current of synchronous machines is determined not only by the armature self-inductive reactance, as is often assumed, but also by the field self-inductive reactance. Neglecting the field reactance in calculation may give a calculated short-circuit current 50 per cent or more, too high. A formula is derived for calculating the field reactance which, added to the armature reactance, gives the total which determines the initial short-circuit current and which it is proposed should be called, as previously recommended by other authors, *transient reactance.*

An attempt is made to describe the apparently complicated physical phenomena of sudden short circuits in terms as free as possible from mathematics. One interesting and important point which the authors establish from the physical interpretation of the problem is that there is a very significant rise in flux at the bottom of the pole at short circuit.

Discussion, pages 1298-1340, by Messrs. C. J. Fechheimer, V. Karapetoff, F. D. Newbury, W. F. Dawson, C. L. Fortescue, H. R. Summerhayes, N. S. Diamant and R. E. Doherty.

A discussion involving a listing of various methods of determining reactance and their feasibility.

SKIN EFFECT IN TUBULAR AND FLAT CONDUCTORS

H. B. Dwight **Vol. xxxvii—1918, pp. 1379-1400**

A method is presented for calculating the skin effect resistance ratio of a tube, which is a form of conductor to be recommended for high-frequency work. A formula is also developed by means of which the asymptote to the curve of the ratio R'/R may be drawn, and thus the magnitude of the skin effect at extremely high frequencies may be obtained.

The values of the ratio R'/R for tubes of various thicknesses, are plotted in a set of curves which may be used for the solution of practical problems.

A similar method is described for the calculation of skin effect in a thin strap. A set of empirical curves for straps is given from which approximate values of R'/R for any case may be read.

Discussion, pages 1401-1403, by Messrs. J. Slepian and H. B. Dwight.

A discussion on the value of authors deductions with further deductions by the author.

CRITICAL REVIEW OF THE BIBLIOGRAPHY ON UNBALANCED MAGNETIC PULL IN DYNAMO-ELECTRIC MACHINES

Alexander Gray and J. G. Pertsch, Jr. **Vol. xxxvii—1918, pp. 1417-1424**

The purpose of this paper is to serve as an introduction to the succeeding comprehensive article by Mr. E. Rosenberg, on the subject of Magnetic

Pull in Electric Machines. The Maxwell equation, upon which all of the formulas for unbalanced magnetic pull are based, is first established. A critical review is then given of the articles pertaining to this subject, which have appeared in the technical press to date. It is pointed out wherein the various expressions which have been proposed differ from one another and to what extent they are consistent.

No discussion.

MAGNETIC PULL IN ELECTRIC MACHINES

Vol. xxxvii—1918, pp. 1425-1469

E. Rosenberg

See abstract of preceding paper.

No discussion.

APPLICATION OF HARMONIC ANALYSIS TO THE THEORY OF SYNCHRONOUS MACHINES

Vol. xxxvii—1918 pp. 1477-1513

Waldo V. Lyon

It is shown that the flux distribution in the airgap of a synchronous machine consists of a series of component distributions that are simple harmonic wave trains, either stationary or moving at constant velocities. Methods are suggested for determining the effects of slots and the saturation of the magnetic circuit on the magnitudes of these component distributions. Expressions for the voltage generated and the power developed thereby are given. The theory is applied to the operation of a three-phase, synchronous machine under different conditions of load, both qualitatively and quantitatively. A table of comparative calculated field currents is appended.

Discussion, pages 1514-1517, by Messrs. N. S. Diamant and W. V. Lyon. General discussion.

3. UNITS, MEASUREMENTS AND INSTRUMENTS

A NEW STANDARD OF CURRENT AND POTENTIAL

Vol. xxxvii—1918, pp. 167-173

Chester T. Allcutt

This paper describes a new secondary standard which is proposed as a substitute for the standard cell in certain classes of d-c. measurements. The device consists of a Wheatstone bridge which will balance for but one value of current.

Various factors affecting the accuracy and permanence of the device are discussed and a number of curves are given showing the characteristics which have been obtained.

Discussion incorporated with that of paper by C. A. Hoxie on "A Thermoelectric Standard Cell."

A THERMOELECTRIC STANDARD CELL

Vol. xxxvii—1918, pp. 175-182

C. A. Hoxie

This paper considers a means of obtaining a secondary standard of e. m. f. by utilizing the e. m. f. of a thermocouple. The construction of the cell is discussed in detail, particularly the use of gas in the bulb.

A review of the characteristics brings out several advantages of the thermoelectric standard cell. The results of permanency tests on a number of cells are shown.

Discussion, (including that of paper by C. T. Allcutt), pages 183-188, by Messrs. C. H. Sharp, F. C. Stockwell, W. B. Kouwenhoven, P. G. Agnew, C. T. Allcutt, C. A. Hoxie and A. C. Campbell.

A discussion covering the making of a practical substitute for the standard cell and other efforts to obtain a standard e. m. f.

THE CHARACTER OF THE THERMAL STORAGE DEMAND METER

P. M. Lincoln **Vol. xxxvii—1918, pp. 189-210**

Following a detailed description of the principle and construction of the thermal storage demand meter the author shows wherein it always indicates what may be called "logarithmic average" rather than "arithmetic average" of power consumption, heretofore indicated by practically all demand meters. The inherent faults of the "arithmetic average" or "block interval" meter are described and examples given demonstrating that the thermal storage meter alone recognizes the true heating effect that fixes size of equipment and therefore cost that should be assessed against the customer.

Discussion, pages 211-219, by Messrs. C. I. Hall, F. V. Magalhaes, G. L. Hoxie, W. H. Pratt, M. G. Lloyd, A. S. Albright, H. D. James, H. L. Wallau and P. M. Lincoln.

This discussion consists of arguments from several viewpoints, for and against the thermal storage demand meter and its method of measuring current.

MEASUREMENT OF POWER LOSS IN DIELECTRICS OF THREE-CONDUCTOR HIGH-TENSION CABLES

F. M. Farmer **Vol. xxxvii—1918, pp. 221-241**

This paper describes the method used at the Electrical Testing Laboratories for measuring the dielectric power losses in 10-foot samples of three-conductor cables with three-phase potential applied to the cable. The difficulties encountered and the methods employed to overcome them are discussed in considerable detail. Typical results are given in the form of data for two specimens of cable, one having a low power loss in the dielectric and one having a high power loss in the dielectric. The data are also presented in the form of curves. Various conclusions are drawn.

Discussion, pages 242-260, by Messrs. H. W. Fisher, R. W. Atkinson, C. W. Davis, D. Du Bois, G. B. Shanklin, J. L. Harper, W. H. Cole, L. L. Elden, J. A. Walton, P. Torchio, F. W. Peek, Jr., M. G. Lloyd, J. R. Craighead, C. A. Adams and F. M. Farmer.

A discussion covering various methods in vogue at laboratories and power companies for measuring and reducing dielectric power losses with consequent increase of cable capacity.

THE SECOMOR

A Kinematic Device Which Imitates the Performance of a Series-Wound Polyphase Commutator Motor.

V. Karapetoff **Vol. xxxvii—1918, pp. 329-354**

The device consists of four bars of adjustable useful length and with adjustable angles. These bars can be set in a combination to represent the vector diagram of voltages in a motor with any desired constants. By moving the bars to vary the load, complete performance characteristics of the motor can be obtained, including the speed, the torque, the power factor, etc. An additional device called the impedometer permits to take into account the impedance drop in the machine. An adjustable saturation curve made of soft wire is used in connection with the secomor, to enable one to investigate the effect of saturation. A brief graphical theory of the motor precedes the description of the secomor to make its action understandable.

Discussion incorporated with that of paper by Marius C. A. Latour on "Commutation in Alternating-Current Machinery."

PRE-CHARGED CONDENSERS IN SERIES AND IN PARALLEL

V. Karapetoff **Vol. xxxvii—1918, pp. 1015-1023**

A condenser is charged from a source of direct voltage, and then is used as a booster in series with this source to charge another condenser. By repeating this process a large number of times the second condenser is finally subjected to twice the voltage of the source. This is the principle of the Delon apparatus for testing cables, and is explained in a numerical example. Then the more general case of two or more "pre-charged" condensers in series is considered, when these condensers are connected to some source of direct voltage; it is shown how to determine the final distribution of voltages among them. A similar problem is solved for pre-charged condensers in parallel. Finally a general network of pre-charged condensers is considered, and equations are derived similar to Kirchoff's laws, from which the final distribution of voltages and charges may be computed knowing the initial distribution.

Discussion, pages 1024-1025, by Messrs. C. O. Mailloux, L. W. Chubb, W. V. Lyon and V. Karapetoff.

A general discussion.

SKIN EFFECT IN TUBULAR AND FLAT CONDUCTORS

H. B. Dwight **Vol. xxxvii—1918, pp. 1379-1400**

A method is presented for calculating the skin effect resistance ratio of a tube, which is a form of conductor to be recommended for high-frequency work. A formula is also developed by means of which the asymptote to the curve of the ratio R'/R may be drawn, and thus the magnitude of the skin effect at extremely high frequencies may be obtained.

The values of the ratio R'/R for tubes of various thicknesses, are plotted in a set of curves which may be used for the solution of practical problems.

A similar method is described for the calculation of skin effect in a thin strap. A set of empirical curves for straps is given from which approximate values of R'/R for any case may be read.

Discussion, pages 1401-1403, by Messrs. J. Slepian and H. B. Dwight.

A discussion on the value of authors deductions with further deductions by the author.

4. INSULATION AND DIELECTRIC PHENOMENA

CORONA TESTS AT HIGH ALTITUDE

B. F. Jakobsen **Vol. xxxvii—1918, pp. 91-108**

This paper describes some corona tests which were made in Peru on a 70-mile transmission line located at an average altitude of 13,300 feet. The voltages range from 40,000 to about 72,000. The method employed in the tests is described and the calculations are given. The results are compared to "Peek's law." Close correspondence is found between these tests and those made by Faccioli on the Shoshone-Leadville line.

It is shown that the results are in good agreement with the formulas deduced by Professor Ryan from extensive laboratory tests.

Finally test data are given for a test made during rainy weather.

Discussion, pages 109-122, by Messrs. F. W. Peek, Jr., H. J. Ryan and B. F. Jakobsen.

A general discussion.

MEASUREMENT OF POWER LOSS IN DIELECTRICS OF THREE-CONDUCTOR HIGH-TENSION CABLES

F. M. Farmer **Vol. xxxvii—1918, pp. 221-241**

This paper describes the method used at the Electrical Testing Laboratories for measuring the dielectric power losses in 10-foot samples of three-conductor cables with three-phase potential applied to the cable. The difficulties encountered and the methods employed to overcome them are discussed in considerable detail. Typical results are given in the form of data for two specimens of cable, one having a low power loss in the dielectric and one having a high power loss in the dielectric. The data are also presented in the form of curves. Various conclusions are drawn.

Discussion, pages 242-260, by Messrs. H. W. Fisher, R. W. Atkinson, C. W. Davis, D. Du Bois, G. B. Shanklin, J. L. Harper, W. H. Cole, L. L. Elden, J. A. Walton, P. Torchio, F. W. Peek, Jr., M. G. Lloyd, J. R. Craighead, C. A. Adams and F. M. Farmer.

A discussion covering various methods in vogue at laboratories and power companies for measuring and reducing dielectric power losses with consequent increase of cable capacity.

APPLICATION OF THEORY AND PRACTISE TO THE DESIGN OF TRANSMISSION LINE INSULATORS

G. I. Gilchrest **Vol. xxxvii—1918, pp. 805-824**

The paper first gives a summation of the items that are apparently the main causes of pin-type insulator failures in service. Each item is briefly discussed and the opinions of operating men are cited.

A brief description is given of the method used to determine the form of the dielectric field about porcelain insulators under normal line voltage. Diagrams of the dielectric field and photographs of flash over tests of theoretical designs are shown. Necessary modifications of the theoretical designs, in order to meet operating and manufacturing conditions, are discussed.

In the latter part of the paper, diagrams and illustrations are shown of a proposed type of commercial insulator design which has been evolved by linking together the theoretical and practical phases of the problem. A comparison is then made between the older types of design and the proposed type.

A summary is made of the advantages it is believed that the new type of design has over the present commercial insulator designs.

Discussion, pages 825-832, by Messrs. C. Fortescue, C. F. Scott, V. Karapetoff, L. W. Chubb, S. Barfoed and G. I. Gilchrest.

A general discussion on insulator design with special emphasis on porosity, refraction of lines of force, etc.

LIGHTNING ARRESTER SPARK GAPS

Their Relation to the Problem of Protecting Against Impulse Voltages

Chester T. Allcutt **Vol. xxxvii—1918, pp. 833-854**

This paper describes a new form of high-voltage lightning arrester gap which has been called the "impulse protective gap".

The paper opens with a brief resumé of some of the results of previous investigations of the subject of impulse voltages. A discussion of the points involved in securing adequate protection against transient voltages of steep wave front follows.

Methods employed in testing these gaps are described and the results of a large number of experiments are tabulated. Tests on the action of a high-frequency impulse combined with a 60-cycle wave are included in the experimental work. From the experimental data a number of curves are plotted showing the discharge characteristics of the impulse protective gap under many different conditions.

Discussion, pages 855-869, by Messrs. P. M. Lincoln, F. W. Peek, Jr., P. H. Thomas, L. W. Chubb and C. T. Allcutt.

A general discussion of the apparatus designed by Allcutt and also the theory of over-voltages that cause insulation failures.

THE OXIDE FILM LIGHTNING ARRESTER

Charles P. Steinmetz **Vol. xxxvii—1918, pp. 871-880**

A short history of lightning protection of electric systems is given, as relating to the three successive types of electric circuits; the communication circuits, the power circuits of negligible electrostatic capacity, and the high power circuits containing distributed capacity and inductance and capable of electric oscillation, leading to the three problems of discharging over-voltage to ground, opening the power current which follows the discharge and discharging so that no power current follows even for a fraction of a half wave. It is shown that these problems are solved by the spark gap to ground, by the use of non-arcing metals in the multigap arrester, which opens the circuit at the end of a half wave of current, and by the so-called "counter e. m. f." type of arrester, represented by the aluminum cell and the oxide film arrester.

In the oxide film arrester a type of arrester is presented which has the same characteristics and therefore the same advantages as the aluminum cell arrester, but does not require daily attendance and contains no liquids.

Its method of operation is explained, and its difference from the aluminum cell arrester.

A short description of the construction of the oxide film arrester is given, a record of its operation in industrial service for over three years, and oscillograms showing the performance of this arrester under recurrents, oscillations and under high-power impulses.

Discussion incorporated with that of paper by Crosby Field on "The Oxide Film Lightning Arrester."

THE OXIDE FILM LIGHTNING ARRESTER

Crosby Field

Vol. xxxvii—1918, pp. 881-890

The oxide film arrester is a new type of lightning arrester made up of a film of insulation in contact with a conducting powder. Upon the application of over-voltage, the insulation will be pierced, but the powder will very rapidly turn into insulation and plug any holes punctured in the original insulation by the over-voltage, thus forming in substance a resealing insulation. A brief description of this arrester is given together with the principles underlying its action and a comparison with other types of lightning arresters. Mention is also made of other characteristics of this combination which are not used in the present arrester, but which are being applied in other developments. A few notes of tests on the commercial arrester complete this paper. With the exception of the basic patents issued to the author this is the first time any disclosure has been made of this arrester.

Discussion (including that of paper by C. P. Steinmetz), pages 891-896, by Messrs. C. P. Steinmetz, N. A. Lougee, L. W. Chubb and C. T. Allcutt. A general discussion.

5. ELECTRIC CONDUCTORS

MEASUREMENT OF POWER LOSS IN DIELECTRICS OF THREE-CONDUCTOR HIGH-TENSION CABLES

F. M. Farmer

Vol. xxxvii—1918, pp. 221-241

This paper describes the method used at the Electrical Testing Laboratories for measuring the dielectric power losses in 10-foot samples of three-conductor cables with three-phase potential applied to the cable. The difficulties encountered and the methods employed to overcome them are discussed in considerable detail. Typical results are given in the form of data for two specimens of cable, one having a low power loss in the dielectric and one having a high power loss in the dielectric. The data are also presented in the form of curves. Various conclusions are drawn.

Discussion, pages 242-260, by Messrs. H. W. Fisher, R. W. Atkinson, C. W. Davis, D. Du Bois, G. B. Shanklin, J. L. Harper, W. H. Cole, L. L. Elden, J. A. Walton, P. Torchio, F. W. Peek, Jr., M. G. Lloyd, J. R. Craighead, C. A. Adams and F. M. Farmer.

A discussion covering various methods in vogue at laboratories and power companies for measuring and reducing dielectric power losses with consequent increase of cable capacity.

DESIGN OF UNDERGROUND DISTRIBUTION FOR ELECTRIC LIGHT AND POWER SYSTEMS

G. J. Newton **Vol. xxxvii—1918, pp. 399-449**

In a previous paper presented at the 10th Annual Convention of the Association of Iron and Steel Electrical Engineers the author treated the subject only in a general manner; the object of this article is to show, assuming average conditions, each step necessary in the design of an underground distribution system, such as is usually required in a medium size city.

Where costs are given they are based on normal conditions and should not be taken as being the present costs, they are used simply for comparison.

Owing to lack of space no tables have been printed in this article as they can be found in electrical handbooks.

Discussion, pages 450-461, by Messrs. A. A. Meyer, C. W. Rakestraw, H. L. Wallau, E. Friedlaender, E. B. Meyer, W. Sykes, F. M. Hibben and G. J. Newton.

A general discussion involving description of variations from practise as described by the author and arguments on the question of varnished cambric vs. paper insulated cables.

SPLIT-CONDUCTOR CABLE—BALANCED PROTECTION

W. H. Cole **Vol. xxxvii—1918, pp. 757-784**

Primarily, this paper is intended to be a brief history of the principal experiences of the Edison Electric Illuminating Company of Boston in the design and application of selective balanced-protection schemes to parallel connected transmission conductors.

Split-conductor cables are discussed at considerable length, both as to design and operation. Paired ordinary conductors also are discussed, and their relation to so-called split-conductors pointed out.

Special apparatus and devices required in connection with current-balancing schemes are illustrated and discussed.

A partial nomenclature is proposed, to assist in clearing the way for intelligent discussion and a uniform understanding of the general subject of current-balance protection for paired conductors.

A schedule of installations in the Boston system is given.

No general conclusions are drawn since the paper is of the nature of a report on progress; specific conclusions are drawn, however, in a number of cases where the evidence or experience appears to be reasonably conclusive.

A mathematical discussion of a number of reactive end-impedance devices, by Professor C. A. Adams, is appended.

Discussion, pages 785-790, by Messrs. J. B. Taylor, W. H. Cole, W. A. Del Mar, P. M. Lincoln, E. B. Meyer, R. W. Atkinson, J. R. Craighead and O. C. Traver.

A discussion on various factors involved in cable installations.

AERIAL CABLE CONSTRUCTION FOR ELECTRIC POWER TRANSMISSION

E B. Meyer **Vol. xxxvii—1918, pp. 791-800**

This paper deals with the problem of supplying high-tension electric service where conditions do not permit of the use of open wire or underground circuits.

The methods of overcoming difficulties incidental to providing for high-tension service are discussed in detail, together with a description of the types of cable used and methods of installation.

The experience of a large central station company operating several hundred miles of overhead and underground cable is given and the paper brings out the fact that the type of construction described may be used advantageously for both 13,200- and 26,400-volt service.

Discussion, pages 801-803, by Messrs. W. F. Dawson, E. B. Meyer, H. L. Wallau, W. A. Del Mar, J. B. Taylor, P. Torchio, J. A. Johnson and H. R. Summerhayes.

A general discussion.

SKIN EFFECT IN TUBULAR AND FLAT CONDUCTORS

H. B. Dwight **Vol. xxxvii—1918, pp. 1379-1400**

A method is presented for calculating the skin effect resistance ratio of a tube, which is a form of conductor to be recommended for high-frequency work. A formula is also developed by means of which the asymptote to the curve of the ratio R'/R may be drawn, and thus the magnitude of the skin effect at extremely high frequencies may be obtained.

The values of the ratio R'/R for tubes of various thicknesses, are plotted in a set of curves which may be used for the solution of practical problems.

A similar method is described for the calculation of skin effect in a thin strap. A set of empirical curves for straps is given from which approximate values of R'/R for any case may be read.

Discussion, pages 1401-1403, by Messrs. J. Slepian and H. B. Dwight.

A discussion on the value of authors deductions with further deductions by the author.

7. BATTERIES

A NEW STANDARD OF CURRENT AND POTENTIAL

Chester T. Allcutt **Vol. xxxvii—1918, pp. 167-173**

This paper describes a new secondary standard which is proposed as a substitute for the standard cell in certain classes of d-c. measurements. The device consists of a Wheatstone bridge which will balance for but one value of current.

Various factors affecting the accuracy and permanence of the device are discussed and a number of curves are given showing the characteristics which have been obtained.

Discussion incorporated with that of paper by C. A. Hoxie on "A Thermoelectric Standard Cell."

A THERMOELECTRIC STANDARD CELL

C. A. Hoxie **Vol. xxxvii—1918, pp. 175-182**

This paper considers a means of obtaining a secondary standard of e. m. f. by utilizing the e. m. f. of a thermocouple. The construction of the cell is discussed in detail, particularly the use of gas in the bulb.

A review of the characteristics brings out several advantages of the thermoelectric standard cell. The results of permanency tests on a number of cells are shown.

Discussion, (including that of paper by C. T. Allcutt), pages 183-189, by Messrs. C. H. Sharp, F. C. Stockwell, W. B. Kouwenhoven, P. G. Agnew, C. T. Allcutt, C. A. Hoxie and A. C. Campbell.

A discussion covering the making of a practical substitute for the standard cell and other efforts to obtain a standard e. m. f.

9. ELECTRICAL MACHINERY AND APPARATUS

THE TECHNICAL STORY OF THE FREQUENCIES

B. G. Lamme **Vol. xxxvii—1918, pp. 65-85**

The various frequencies used in alternating-current work in America are first mentioned, and the primary reasons for their introduction are given. This is followed by a discussion of various alternating current applications which were more or less dependent upon frequency.

It is shown that there was an apparent need for two standard frequencies in the region of 60 and 25 cycles, and, further, why 60 and 25 cycles have prevailed. The special fields of application of each one are discussed fully and it is shown why 25 cycles tended to dominate the field.

Discussion, pages 86-89, by Messrs. H. B. Brooks, B. G. Lamme, G. S. Mc Cumber, R. Dalgleish and Mr. Hunt.

A general discussion.

RATING AND SELECTION OF OIL CIRCUIT BREAKERS

E. M. Hewlett, J. N. Mahoney and G. A. Burnham **Vol. xxxvii—1918, pp. 123-138**

On account of the variable conditions in systems on which circuit breakers are used, it is impossible to give a simple rule which will cover the selection of circuit breakers for all cases. The authors discuss the interpretations of the A. I. E. E. Standardization Rules covering the rating of oil circuit breakers and consider the variable factors which are involved in the selection of circuit breakers for various systems. A method is suggested whereby short-circuit characteristics of various systems can be used for determining the proper selection of oil circuit breakers for average systems. The method does not apply to very large systems or unusual conditions.

Discussion, pages 139-165, by Messrs. E. M. Hewlett, J. N. Mahoney, H. R. Summerhayes, W. W. Willard, B. Jones, F. C. Hanker, P. Torchio, P. M. Lincoln, R. E. Doherty, F. L. Hunt, H. H. Dewey, H. D. James, N. L. Pollard, E. G. Merrick, P. H. Adams, I. M. Cushing, C. Lichtenberg, A. Collins, C. C. Garrard and G. A. Burnham.

A general discussion largely from the viewpoint of the importance of arriving at a standard method of rating circuit breakers as to capacity.

THE POLYPHASE SHUNT MOTOR

W. C. Korthals Altes **Vol. xxxvii—1918, pp. 295-327**

There exists a demand for a reliable, adjustable-speed, alternating-current motor, suitable for operation at a large number of speeds. The neutralized motor with shunt field control is analyzed and it is shown that it is not practical for commercial frequencies. The induction motor with commutator on the secondary side is discussed. The induction motor with commutator on the primary side offers the best solution for machine-tool motors. Its theory is discussed in detail, and a complete description is given of the mechanism required to shift the brushes and the new type of armature winding used.

Discussion incorporated with that of paper by Marius C. A. Latour on "Commutation in Alternating-Current Machinery."

COMMUTATION IN ALTERNATING-CURRENT MACHINERY

Marius C. A. Latour **Vol. xxxvii—1918, pp. 355-381**

The author introduces into the discussion of the commutating characteristics of alternating-current commutating motors, his theory thae perfect commutation in a continuous-current motor depends substantially on the production of a mean resultant neutral field in the region whert commutation is taking place, and shows that the production of a perfect revolving field in a polyphase commutator motor assists in insuring perfect commutation at exact synchronism.

In a single-phase commutator motor a "polyphase" revolving field can be produced at synchronism by utilizing supplementary brushes, short-circuited upon themselves, displaced by 90 electrical space degrees from the main single-phase brushes on the commutator.

As in the case of polyphase motors, the problem of securing perfect commutation at synchronism becomes that of producing a perfect rotating field. It is shown by the author that the use of fractional-pitch windings on the rotor and a sinusoidal distribution of conductors on the stator is of much assistance in this connection.

Discussion (including that of papers by W. C. K. Altes and V. Karapetoff), pages 379-389, by Messrs. A. M. Gray, P. M. Lincoln, C. F. Scott, W. C. K. Altes, H. M. Hobart, B. A. Behrend, W. B. Jackson and V. Karapetoff.

A general discussion.

SOME CONSIDERATIONS IN DETERMINING THE CAPACITY OF ROLLING-MILL MOTORS

Robert F. Hamilton **Vol. xxxvii—1918, pp. 463-484**

A consideration in detail of electric drive for rolling mills, including classification of mills and motors, mathematical determinations of energy required for rolling, relation of speed to tonnage, motor capacity and flywheel application.

Discussion, pages 485-489, by Messrs. A. M. Dudley, C. A. Menk, N. W. Storer and W. Sykes.

A discussion on relative importance of the requirements specified in rolling mill motor selection.

SELECTION OF STEEL MILL AUXILIARY MOTORS AND CONTROL AS AFFECTED BY MECHANICAL FEATURES OF THE DRIVE

J. D. Wright **Vol. xxxvii—1918, pp. 491-496**

The author describes manipulators for blooming mills, which consist of side guards and lifting fingers, the former being used to guide the bloom into the proper groove in the rolls while the latter are used for turning the bloom over. The functions, mechanical layout and operation of these manipulators are described, from which conclusions are drawn as to the size and type of motors as well as the type of control best suited for driving these auxiliaries.

Discussion, pages 497-501, by Messrs. E. Friedlaender, C. A. Menk. T. E. Tynes, H. S. Richardson, B. G. Beck, G. E. Stoltz and J. D. Wright,

A description of variations from authors practise in equipment and control.

NO-LOAD CONDITIONS OF SINGLE-PHASE INDUCTION MOTORS AND PHASE CONVERTERS

R. E. Hellmund **Vol. xxxvii—1918, pp. 539-626**

This paper shows methods and derives formulas for the determination of the fields, the stator and rotor magnetizing currents, and the tertiary voltages for phase converters and single-phase induction motors at no-load. This paper treats a large number of different cases along similar lines, thereby coordinating and explaining many phenomena previously observed, and it should, therefore, form a desirable basis for further investigations and discussion of this subject matter.

The paper is arranged so that it can be read to good advantage without going through those mathematical parts marked by vertical rules.

By reading the conclusions at the end of the paper, a fair idea of the principal points brought out in the paper can be obtained.

Discussion incorporated with that of paper by B. G. Lamme on "A Physical Conception of the Operation of the Single-phase Induction Motor."

A PHYSICAL CONCEPTION OF THE OPERATION OF THE SINGLE-PHASE INDUCTION MOTOR

B. G. Lamme **Vol. xxxvii—1918, pp. 627-658**

This paper covers a method of studying the actions of the single-phase induction motor, which the writer has found to be very convenient from the educational standpoint. It is based upon the assumption of two equal and oppositely rotating primary magnetomotive forces combined with a synchronously rotating secondary m. m. f., such as would be produced by direct-current excitation. Diagrams and descriptions are given to illustrate the magnetomotive forces and fluxes, showing how, among other conditions, two oppositely rotating fields of unequal value may be possible.

The next step is a consideration of e. m. f. generation by two oppositely rotating fields, showing how both must be taken into account. The effects upon the counter e. m. f. and excitation, of the reduction or suppression of one field is shown.

The full-load conditions are next considered. A comparison is made between a two-motor unit, consisting of two similar polyphase motors coupled together and connected for opposite rotation, and the straight single-phase induction motor. This is followed by a considerable amount of test data which illustrate the principles and actions described in the paper.

Discussion, (including that of paper by R. E. Hellmund), pages 659-702, by Messrs. J. Slepian, C. Fortescue, C. A. M. Weber, G. H. Garcelon, R. E. Hellmund, A. M. Dudley, B. G. Lamme, M. I. Pupin, E. F. W. Alexanderson, B. A. Behrend, L. W. Chubb, A. M. Gray, C. F. Scott and S. Haar.

A general discussion on theory.

LIGHTNING ARRESTER SPARK GAPS

Their Relation to the Problem of Protecting Against Impulse Voltages

Chester T. Allcutt **Vol. xxxvii—1918, pp. 833-854**

This paper describes a new form of high-voltage lightning arrester gap which has been called the "impulse protective gap".

The paper opens with a brief resumé of some of the results of previous investigations of the subject of impulse voltages. A discussion of the points involved in securing adequate protection against transient voltages of steep wave front follows.

Methods employed in testing these gaps are described and the results of a large number of experiments are tabulated. Tests on the action of a high-frequency impulse combined with a 60-cycle wave are included in the experimental work. From the experimental data a number of curves are plotted showing the discharge characteristics of the impulse protective gap under many different conditions.

Discussion, pages 855-869, by Messrs. P. M. Lincoln, F. W. Peek, Jr., P. H. Thomas, L. W. Chubb and C. T. Allcutt.

A general discussion of the apparatus designed by Allcutt and also the theory of over-voltages that cause insulation failures.

THE OXIDE FILM LIGHTNING ARRESTER

Charles P. Steinmetz **Vol. xxxvii—1918, pp. 871-880**

A short history of lightning protection of electric systems is given, as relating to the three successive types of electric circuits; the communication circuits, the power circuits of negligible electrostatic capacity, and the high power circuits containing distributed capacity and inductance and capable of electric oscillation, leading to the three problems of discharging over-voltage to ground, opening the power current which follows the discharge and discharging so that no power current follows even for a fraction of a half wave. It is shown that these problems are solved by the spark gap to ground, by the use of non-arcing metals in the multigap arrester, which opens the circuit at the end of a half wave of current, and by the so-called "counter e. m. f." type of arrester, represented by the aluminum cell and the oxide film arrester.

In the oxide film arrester a type of arrester is presented which has the same characteristics and therefore the same advantages as the aluminum cell arrester, but does not require daily attendance and contains no liquids.

Its method of operation is explained, and its difference from the aluminum cell arrester.

A short description of the construction of the oxide film arrester is given, a record of its operation in industrial service for over three years, and oscillograms showing the performance of this arrester under recurrents, oscillations and under high-power impulses.

Discussion incorporated with that of paper by Crosby Field on "The Oxide Film Lightning Arrester."

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Vol. xxxvii—1918, pp. 881-890

Crosby Field

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Discussion (including that of paper by C. P. Steinmetz), pages 891-896, by Messrs. C. P. Steinmetz, N. A. Lougee, L. W. Chubb and C. T. Allcutt. A general discussion.

PRE-CHARGED CONDENSERS IN SERIES AND IN PARALLEL

Vol. xxxvii—1918, pp. 1015-1023

V. Karapetoff

A condenser is charged from a source of direct voltage, and then is used as a booster in series with this source to charge another condenser. By repeating this process a large number of times the second condenser is finally subjected to twice the voltage of the source. This is the principle of the Delon apparatus for testing cables, and is explained in a numerical example. Then the more general case of two or more "pre-charged" condensers in series is considered, when these condensers are connected to some source of direct voltage; it is shown how to determine the final distribution of voltages among them. A similar problem is solved for pre-charged condensers in parallel. Finally a general network of pre-charged condensers is considered, and equations are derived similar to Kirchoff's laws, from which the final distribution of voltages and charges may be computed knowing the initial distribution.

Discussion, pages 1024-1025, by Messrs. C. O. Mailloux, L. W. Chubb, W. V. Lyon and V. Karapetoff.

A general discussion.

PROTECTION FROM FLASHING FOR DIRECT-CURRENT APPARATUS

J. J. Linebaugh and J. L. Burnham **Vol. xxxvii—1918, pp. 1341-1352**

The equipment developed for the protection of direct-current apparatus as described in this paper is applicable to all direct-current apparatus and all methods of operation. Special means of protection for use only with particular apparatus or conditions of operation have not been mentioned. The principal steps in the experimental development of high-speed circuit breakers and flash barriers are briefly given.

Complete protection for *any* direct-current apparatus and service requires both the high-speed breaker and flash barriers. Attention is directed to the importance of arranging the connections to the brush rigging so that the magnetic action on the arc will be a minimum, and properly directed, so the flash will do the least damage.

Discussion, pages 1353-1365, by Messrs. C. L. Fortescue, F. C. Hanker, H. B. Dwight, F. Wunsch, F. D. Newbury, W. F. Dawson, J. F. Tritle, H. E. Trent and J. L. Burnham.

Description of various types of high-speed breakers and installations.

A DIRECT-CURRENT GENERATOR FOR CONSTANT POTENTIAL AT VARIABLE SPEED

S. R. Bergman **Vol. xxxvii—1918, pp. 1405-1412**

The standing of this problem in the past is briefly reviewed. As far as is known to the author the type of machine described presents a new solution.

The theory of the machine is described and diagrams of connections given. Performance curves obtained from tests are shown.

A method is described whereby instantaneous regulation of the voltage is obtained, which method also secures approximately a constant voltage independent of the heating.

Finally there follows a discussion of the efficiency of the new machine as compared with a standard machine of the same speed and output.

Discussion, pages 1413-1415, by Messrs. J. G. De Remer and S. R. Bergman.

General discussion.

ELECTRIC WELDING—A NEW INDUSTRY

H. A. Hornor **Vol. xxxvii—1918, pp. 1519-1529**

This paper covers a brief review of the uses of electric spot and arc welding in this country prior to the formation of the Electric Welding

Committee of the Emergency Fleet Corporation. It compares the status of the art at the present time and emphasizes the developments that have been made in apparatus in the last six months. It treats of the activities of the Welding Committee in applying electric welding processes to the ship-building industry and points the way to the general applicability of this method to other industries. It shows that the results obtained by investigation and physical tests prove that the applications of this process to heavy work are satisfactory.

No discussion.

ELECTRIC MOTORS IN THE CEMENT INDUSTRY

R. B. Williamson **Vol. xxxvii—1918, pp. 1531-1567**

This paper has been compiled by the Committee on Industrial and Domestic Power to give general information regarding the different classes of machinery used in the cement industry and the sizes and types of motor best adapted for the work. The paper gives, first, a brief description of the process. This is followed by an outline of the various kinds of machinery used together with data as to power requirements. The types of motor best suited to each application together with starting characteristics, overload capacity, torque and other features are indicated.

Discussion, pages 1568-1605, by Messrs. A. M. Dudley, A. Simon, W. C. Kalb, L. Bradley, W. L. Merrill, H. D. James, F. J. Burd, E. Friedlaender, W. T. Snyder, L. E. Underwood, S. Haar, J. N. Mahoney, R. J. Dearborn, J. Dixon, A. L. Hadley, C. H. Sonntag, D. B. Rushmore, T. E. Simpers, H. Weichsel, S. H. Harrison, G. H. Rowe, W. I. Slichter and R. B. Williamson.

A discussion mainly on particular points to be emphasized in selection of motors for the cement industry.

11. POWER PLANTS AND CENTRAL STATIONS

EFFECTS OF WAR CONDITIONS ON COST AND QUALITY OF ELECTRIC SERVICE

Lynn S. Goodman and William B. Jackson **Vol. xxxvii—1918, pp. 1-17**

This paper comprises a consideration of the effects of war conditions as they apply to the electric light and power service of the country.

It is shown that the increased cost for fuel and other supplies, labor and taxes alone, occasioned by the war conditions, would amount to an increase of more than $116,000,000 over the operating expenses that should have been expected for the electric light and power companies of the country under normal conditions for 1917.

The important advantages which are inherent in the central electric power stations for supplying power for war manufactures and the advisability for the government to make every reasonable endeavor to encourage the development of the central electric stations is pointed out. The way in which the problems involved are worked out will have a bearing upon the cost of electric light and power service not only during the war but for long after its termination.

Discussion, pages 18-63, by Messrs. B. A. Behrend, Mr. Sykes, J. W.

Cowles, C. A. Adams, W. B. Jackson, F. A. Bryan, R. G. Hudson, M. Freund, B. J. Arnold, P. Torchio, P. Betts, W. S. Gorsuch, H. M. Hobart, D. C. Jackson, P. H. Bartlett, W. N. Smith, J. J. Harold, P. R. Moses, R. F. Schuchardt, F. A. Coffin, D. W. Roper, C. W. Pendell, C. A. Keller, F. H. Bernhard, J. W. Mabbs, A. Honegger.

A general discussion on the various phases of the central station problems.

AMERICA'S ENERGY SUPPLY

Charles P. Steinmetz **Vol. xxxvii—1918, pp. 985-1009**

The gist of the paper is to demonstrate that the economical utilization of the country's energy supply requires generating electric power wherever hydraulic or fuel energy is available, and *collecting the power electrically, just as we distribute it electrically.*

In the first section a short review of the country's energy supply in fuel and water power is given.

In the second section it is shown that the modern synchronous station is necessary for large hydraulic powers, but the solution of the problem of the economic development of the far more numerous smaller waterpowers is the adoption of the induction generator.

The third section considers the characteristics of the induction generator and the induction-generator station, and its method of operation, and discusses the condition of "dropping out of step of the induction generator" and its avoidance.

In the appendix the corresponding problem is pointed out with reference to fuel power, showing that many millions of kilowatts of potential power are wasted by burning fuel and thereby degrading its energy that could be recovered by interposing simple steam turbine induction generators between the boiler and the steam heating systems, and collecting their power electrically.

Discussion, pages 1010-1014, by Messrs. A. M. Schoen, B. A. Behrend, S. Barfoed, R. Bennett and C. P. Steinmetz.

A general discussion.

THE AUTOMATIC HYDROELECTRIC PLANT

J. M. Drabelle and L. B. Bonnett **Vol. xxxvii—1918, pp. 1367-1377**

The automatic hydroelectric generating station of the Iowa Railway and Light Company at Cedar Rapids, Iowa, is a radical step in advance in the elimination of operator's wages in a station of appreciable size without sacrificing complete control.

Discussion, page 1378, by Messrs. H. R. Summerhayes, J. J. Linebaugh, Mr. Bump, O. C. Traver, J. M. Drabelle and L. B. Bonnett.

General discussion.

ECONOMIC PROPORTION OF HYDROELECTRIC AND STEAM POWER

Frank G. Baum **Vol. xxxvii—1918, pp. 1471-1475**

This paper describes a new method of determining for any power system, what proportion of generation should be hydroelectric, and what steam, from the standpoint of economics.

A method is outlined for obtaining a curve showing "Total cost per kilowatt-year for hydroelectric and steam power", for any percentage combination of generation.

With system load curve, fixed charges on steam and hydroelectric plants, cost of fuel and other steam energy charges all known quantities, this curve can readily be calculated, and one can see at a glance the limiting economical percentage of steam power for the given conditions.

No discussion.

ELECTRIC POWER GENERATION IN ONTARIO ON SYSTEMS OF HYDROELECTRIC POWER COMMISSION

Arthur H. Hull **Vol. xxxvii—1918, pp. 1607-1630**

Discussion, pages 1631-1636, by Messrs. H. U. Hart, W. G. Hewson, A. H. Hull, T. A. Worcester, E. M. Ashworth, H. R. Summerhayes, W. Mac Lachlan, P. A. Borden and F. A. Gaby.

Questions and answers on details of the system.

13. TRANSMISSION LINES

CORONA TESTS AT HIGH ALTITUDE

B. F. Jakobsen **Vol. xxxvii—1918, pp. 91-108**

This paper describes some corona tests which were made in Peru on a 70-mile transmission line located at an average altitude of 13,300 feet. The voltages range from 40,000 to about 72,000. The method employed in the tests is described and the calculations are given. The results are compared to "Peek's law." Close correspondence is found between these tests and those made by Faccioli on the Shoshone-Leadville line.

It is shown that the results are in good agreement with the formulas deduced by Professor Ryan from extensive laboratory tests.

Finally test data are given for a test made during rainy weather.

Discussion, pages 109-122, by Messrs. F. W. Peek, Jr., H. J. Ryan and B. F. Jakobsen.

A general discussion.

MEASUREMENT OF POWER LOSS IN DIELECTRICS OF THREE-CONDUCTOR HIGH-TENSION CABLES

F. M. Farmer **Vol. xxxvii—1918, pp. 221-241**

This paper describes the method used at the Electrical Testing Laboratories for measuring the dielectric power losses in 10-foot samples of three-conductor cables with three-phase potential applied to the cable. The difficulties encountered and the methods employed to overcome them are discussed in considerable detail. Typical results are given in the form of data for two specimens of cable, one having a low power loss in the dielectric and one having a high power loss in the dielectric. The data are also presented in the form of curves. Various conclusions are drawn.

Discussion, pages 242-260, by Messrs. H. W. Fisher, R. W. Atkinson,

C. W. Davis, D. Du Bois, G. B. Shanklin, J. L. Harper, W. H. Cole, L. L. Elden, J. A. Walton, P. Torchio, F. W. Peek, Jr., M. G. Lloyd, J. R. Craighead, C. A. Adams and F. M. Farmer.

A discussion covering various methods in vogue at laboratories and power companies for measuring and reducing dielectric power losses with consequent increase of cable capacity.

SPLIT-CONDUCTOR CABLE—BALANCED PROTECTION

W. H. Cole **Vol. xxxvii—1918, pp. 757-784**

Primarily, this paper is intended to be a brief history of the principal experiences of the Edison Electric Illuminating Company of Boston in the design and application of selective balanced-protection schemes to parallel connected transmission conductors.

Split-conductor cables are discussed at considerable length, both as to design and operation. Paired ordinary conductors also are discussed, and their relation to so-called split-conductors pointed out.

Special apparatus and devices required in connection with current-balancing schemes are illustrated and discussed.

A partial nomenclature is proposed, to assist in clearing the way for intelligent discussion and a uniform understanding of the general subject of current-balance protection for paired conductors.

A schedule of installations in the Boston system is given.

No general conclusions are drawn since the paper is of the nature of a report on progress; specific conclusions are drawn, however, in a number of cases where the evidence or experience appears to be reasonably conclusive.

A mathematical discussion of a number of reactive end-impedance devices, by Professor C. A. Adams, is appended.

Discussion, pages 785-790, by Messrs. J. B. Taylor, W. H. Cole, W. A. Del Mar, P. M. Lincoln, E. B. Meyer, R. W. Atkinson, J. R. Craighead and O. C. Traver.

A discussion on various factors involved in cable installations.

AERIAL CABLE CONSTRUCTION FOR ELECTRIC POWER TRANSMISSION

E. B. Meyer **Vol. xxxvii—1918, pp. 791-800**

This paper deals with the problem of supplying high-tension electric service where conditions do not permit of the use of open wire or underground circuits.

The methods of overcoming difficulties incidental to providing for high-tension service are discussed in detail, together with a description of the types of cable used and methods of installation.

The experience of a large central station company operating several hundred miles of overhead and underground cable is given and the paper brings out the fact that the type of construction described may be used advantageously for both 13,200- and 26,400-volt service.

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A general discussion.

APPLICATION OF THEORY AND PRACTISE TO THE DESIGN OF TRANSMISSION LINE INSULATORS

G. I. Gilchrest **Vol. xxxvii—1918, pp. 805-824**

The paper first gives a summation of the items that are apparently the main causes of pin-type insulator failures in service. Each item is briefly discussed and the opinions of operating men are cited.

A brief description is given of the method used to determine the form of the dielectric field about porcelain insulators under normal line voltage. Diagrams of the dielectric field and photographs of flash over tests of theoretical designs are shown. Necessary modifications of the theoretical designs, in order to meet operating and manufacturing conditions, are discussed.

In the latter part of the paper, diagrams and illustrations are shown of a proposed type of commercial insulator design which has been evolved by linking together the theoretical and practical phases of the problem. A comparison is then made between the older types of design and the proposed type.

A summary is made of the advantages it is believed that the new type of design has over the present commercial insulator designs.

Discussion, pages 825-832, by Messrs. C. Fortescue, C. F. Scott, V. Karapetoff, L. W. Chubb, S. Barfoed and G. I. Gilchrest.

A general discussion on insulator design with special emphasis on porosity, refraction of lines of force, etc.

ELECTRIC POWER GENERATION IN ONTARIO ON SYSTEMS OF HYDROELECTRIC POWER COMMISSION

Arthur H. Hull **Vol. xxxvii—1918, pp. 1607-1630**

Discussion, pages 1631-1636, by Messrs. H. U. Hart, W. G. Hewson, A. H. Hull, T. A. Worcester, E. M. Ashworth, H. R. Summerhayes, W. Mac Lachlan, P. A. Borden and F. A. Gaby.

Questions and answers on details of the system.

110,000-VOLT TRANSMISSION LINE OVER THE ST. LAWRENCE RIVER

S. Svenningson **Vol. xxxvii—1918, pp. 1653-166**

The paper deals with some remarkable construction recently completed by the Shawinigan Water & Power Company near Three Rivers, Quebec. The St. Lawrence river is crossed by transmission line wires on a span of 4800 feet, being the longest span in the world. Due to necessities of navigation clearance, the towers are 350 feet high.

The preliminary investigation leading to the adoption of this construction is outlined, and a general description is given of the design and construction of the towers, insulators and cables. The provisions for protection from ice and the method of sag calculations are also given.

Discussion, pages 1666-1670, by Messrs. C. W. Baker, S. Svenningson, E. V. Pannell, H. B. Dwight, H. C. Don Carlos, T. A. Worcester, F. J. Wyman and W. P. Dobson.

A general discussion on details of installation.

14. ELECTRIC SERVICE DISTURBANCES AND PROTECTION

CORONA TESTS AT HIGH ALTITUDE

B. F. Jakobsen **Vol. xxxvii—1918, pp. 91-108**

This paper describes some corona tests which were made in Peru on a 70-mile transmission line located at an average altitude of 13,300 feet. The voltages range from 40,000 to about 72,000. The method employed in the tests is described and the calculations are given. The results are compared to "Peek's law." Close correspondence is found between these tests and those made by Faccioli on the Shoshone-Leadville line.

It is shown that the results are in good agreement with the formulas deduced by Professor Ryan from extensive laboratory tests.

Finally test data are given for a test made during rainy weather.

Discussion, pages 109-122, by Messrs. F. W. Peek, Jr., H. J. Ryan and B. F. Jakobsen.

A general discussion.

RATING AND SELECTION OF OIL CIRCUIT BREAKERS

E. M. Hewlett, J. N. Mahoney and G. A. Burnham **Vol. xxxvii—1918, pp. 123-138**

On account of the variable conditions in systems on which circuit breakers are used, it is impossible to give a simple rule which will cover the selection of circuit breakers for all cases. The authors discuss the interpretations of the A. I. E. E. Standardization Rules covering the rating of oil circuit breakers and consider the variable factors which are involved in the selection of circuit breakers for various systems. A method is suggested whereby short-circuit characteristics of various systems can be used for determining the proper selection of oil circuit breakers for average systems. The method does not apply to very large systems or unusual conditions.

Discussion, pages 139-165, by Messrs. E. M. Hewlett, J. N. Mahoney, H. R. Summerhayes, W. W. Willard, B. Jones, F. C. Hanker, P. Torchio, P. M. Lincoln, R. E. Doherty, F. L. Hunt, H. H. Dewey, H. D. James, N. L. Pollard, E. G. Merrick, P. H. Adams, I. M. Cushing, C. Lichtenberg, A. Collins, C. C. Garrard and G. A. Burnham.

A general discussion largely from the viewpoint of the importance of arriving at a standard method of rating circuit breakers as to capacity.

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LIGHTNING ARRESTER SPARK GAPS

Their Relation to the Problem of Protecting Against Impulse Voltages

Chester T. Allcutt **Vol. xxxvii—1918, pp. 833-854**

This paper describes a new form of high-voltage lightning arrester gap which has been called the "impulse protective gap".

The paper opens with a brief resumé of some of the results of previous investigations of the subject of impulse voltages. A discussion of the points involved in securing adequate protection against transient voltages of steep wave front follows.

Methods employed in testing these gaps are described and the results of a large number of experiments are tabulated. Tests on the action of a high-frequency impulse combined with a 60-cycle wave are included in the experimental work. From the experimental data a number of curves are plotted showing the discharge characteristics of the impulse protective gap under many different conditions.

Discussion, pages 855-869, by Messrs. P. M. Lincoln, F. W. Peek, Jr., P. H. Thomas, L. W. Chubb and C. T. Allcutt.

A general discussion of the apparatus designed by Allcutt and also the theory of over-voltages that cause insulation failures.

THE OXIDE FILM LIGHTNING ARRESTER

Charles P. Steinmetz **Vol. xxxvii—1918, pp. 871-880**

A short history of lightning protection of electric systems is given, as relating to the three successive types of electric circuits; the communication circuits, the power circuits of negligible electrostatic capacity, and the high power circuits containing distributed capacity and inductance and capable of electric oscillation, leading to the three problems of discharging over-voltage to ground, opening the power current which follows the discharge and discharging so that no power current follows even for a fraction of a half wave. It is shown that these problems are solved by the spark gap to ground, by the use of non-arcing metals in the multigap arrester, which opens the circuit at the end of a half wave of current, and by the so-called "counter e. m. f." type of arrester, represented by the aluminum cell and the oxide film arrester.

In the oxide film arrester a type of arrester is presented which has the same characteristics and therefore the same advantages as the aluminum cell arrester, but does not require daily attendance and contains no liquids.

Its method of operation is explained, and its difference from the aluminum cell arrester.

A short description of the construction of the oxide film arrester is given, a record of its operation in industrial service for over three years, and oscillograms showing the performance of this arrester under recurrents, oscillations and under high-power impulses.

Discussion incorporated with that of paper by Crosby Field on "The Oxide Film Lightning Arrester."

THE OXIDE FILM LIGHTNING ARRESTER

Crosby Field **Vol. xxxvii—1918, pp. 881-890**

The oxide film arrester is a new type of lightning arrester made up of a film of insulation in contact with a conducting powder. Upon the application of over-voltage, the insulation will be pierced, but the powder will very rapidly turn into insulation and plug any holes punctured in the original insulation by the over-voltage, thus forming in substance a resealing insulation. A brief description of this arrester is given together with the principles underlying its action and a comparison with other types of lightning arresters. Mention is also made of other characteristics of this combination which are not used in the present arrester, but which are being applied in other developments. A few notes of tests on the commercial arrester complete this paper. With the exception of the basic patents issued to the author this is the first time any disclosure has been made of this arrester.

Discussion (including that of paper by C. P. Steinmetz), pages 891-896, by Messrs. C. P. Steinmetz, N. A. Lougee, L. W. Chubb and C. T. Allcutt.

A general discussion.

PROTECTION FROM FLASHING FOR DIRECT-CURRENT APPARATUS

J. J. Linebaugh and J. L. Burnham **Vol. xxxvii—1918, pp. 1341-1352**

The equipment developed for the protection of direct-current apparatus as described in this paper is applicable to all direct-current apparatus and all methods of operation. Special means of protection for use only with particular apparatus or conditions of operation have not been mentioned. The principal steps in the experimental development of high-speed circuit breakers and flash barriers are briefly given.

Complete protection for *any* direct-current apparatus and service requires both the high-speed breaker and flash barriers. Attention is directed to the importance of arranging the connections to the brush rigging so that the magnetic action on the arc will be a minimum, and properly directed, so the flash will do the least damage.

Discussion, pages 1353-1365, by Messrs. C. L. Fortescue, F. C. Hanker, H. B. Dwight, F. Wunsch, F. D. Newbury, W. F. Dawson, J. F. Tritle, H. E. Trent and J. L. Burnham.

Description of various types of high-speed breakers and installations.

15. DISTRIBUTION SYSTEMS

DESIGN OF UNDERGROUND DISTRIBUTION FOR ELECTRIC LIGHT AND POWER SYSTEMS

G. J. Newton **Vol. xxxvii—1918, pp. 399-449**

In a previous paper presented at the 10th Annual Convention of the Association of Iron and Steel Electrical Engineers the author treated the subject only in a general manner; the object of this article is to show, assuming average conditions, each step necessary in the design of an underground distribution system, such as is usually required in a medium size city.

Where costs are given they are based on normal conditions and should not be taken as being the present costs, they are used simply for comparison.

Owing to lack of space no tables have been printed in this article as they can be found in electrical handbooks.

Discussion, pages 450-461, by Messrs. A. A. Meyer, C. W. Rakestraw, H. L. Wallau, E. Friedlaender, E. B. Meyer, W. Sykes, F. M. Hibben and G. J. Newton.

A general discussion involving description of variations from practise as described by the author and arguments on the question of varnished cambric vs. paper insulated cables.

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A general discussion.

METHOD OF SYMMETRICAL CO-ORDINATES APPLIED TO THE SOLUTION OF POLYPHASE NETWORKS

C. L. Fortescue **Vol. xxxvii—1918, pp. 1027-1115**

In the introduction a general discussion of unsymmetrical systems of co-planar vectors leads to the conclusion that they may be represented by symmetrical systems of the same number of vectors, the number of symmetrical systems required to define the given system being equal to its degrees of freedom. A few trigonometrical theorems which are to be used in the paper are called to mind. The paper is subdivided into three parts, an abstract of which follows. It is recommended that only that part of Part I up to formula (33) and the portion dealing with star-delta transformations be read before proceeding with Part II.

Part I deals with the resolution of unsymmetrical groups of numbers into symmetrical groups. These numbers may represent rotating vectors of systems of operators. A new operator termed the sequence operator is introduced which simplifies the manipulation. Formulas are derived for three-phase circuits. Star-delta transformations for symmetrical coordinates are given and expressions for power deduced. A short discussion of harmonics in three-phase systems is given.

Part II deals with the practical application of this method to symmetrical rotating machines operating on unsymmetrical circuits. General formulas are derived and such special cases, as the single-phase induction motor, synchronous motor-generator, phase converters of various types, are discussed.

Discussion, pages 1116-1140, by Messrs. J. Slepian, C. P. Steinmetz, V. Karapetoff, A. M. Dudley, C. F. Scott, C. O. Mailloux and C. L. Fortescue.

A general discussion emphasizing certain mathematical methods of simplifying the author's methods.

16. CONTROL, REGULATION AND SWITCHING

SOME CONSIDERATIONS IN DETERMINING THE CAPACITY OF ROLLING-MILL MOTORS

Robert F. Hamilton **Vol. xxxvii—1918, pp. 463-484**

A consideration in detail of electric drive for rolling mills, including classification of mills and motors, mathematical determinations of energy required for rolling, relation of speed to tonnage, motor capacity and flywheel application.

Discussion, pages 485-489, by Messrs. A. M. Dudley, C. A. Menk, N. W. Storer and W. Sykes.

A discussion on relative importance of the requirements specified in rolling mill motor selection.

SELECTION OF STEEL MILL AUXILIARY MOTORS AND CONTROL AS AFFECTED BY MECHANICAL FEATURES OF THE DRIVE

J. D. Wright **Vol. xxxvii—1918, pp. 491-496**

The author describes manipulators for blooming mills, which consist of side guards and lifting fingers, the former being used to guide the bloom into the proper groove in the rolls while the latter are used for turning the bloom over. The functions, mechanical layout and operation of these manipulators are described, from which conclusions are drawn as to the size and type of motors as well as the type of control best suited for driving these auxiliaries.

Discussion, pages 497-501, by Messrs. E. Friedlaender, C. A. Menk, T. E. Tynes, H. S. Richardson, B. G. Beck, G. E. Stoltz and J. D. Wright.

A description of variations from authors practise in equipment and control.

THE AUTOMATIC HYDROELECTRIC PLANT

J M. Drabelle and L. B. Bonnett **Vol. xxxvii—1918, pp. 1367-1377**

The automatic hydroelectric generating station of the Iowa Railway and Light Company at Cedar Rapids, Iowa, is a radical step in advance in the elimination of operator's wages in a station of appreciable size, without sacrificing complete control.

Discussion, page 1378, by Messrs. H. R. Summerhayes, J. J. Linebaugh, Mr. Bump, O. C. Traver, J. M. Drabelle and L. B. Bonnett.

General discussion.

17. TRACTION

ELECTRIFICATION OF THE MONTREAL TUNNEL ZONE

William G. Gordo **Vol. xxxvii—1918, pp. 1637-1648**

The author describes the electrification of the tunnel through Mt. Royal at Montreal which was built to give the Canadian Northern Railway entrance into the heart of the city. The tunnel is 3.1 miles long and the method of construction is described in detail.

The power is purchased and delivered to a substation near the west portal of the tunnel. The equipment of the substation is described. Details of the equipment and dimensions of both the locomotives and motor cars are given.

The catenary system, which is described in detail, has a number of unusual features due to special local conditions and the extremely low temperatures which sometimes prevail in Montreal.

Discussion, pages 1649-1652, by Messrs. W. G. Hewson, W. G. Gordon, W. A. Bucke, A. H. Hull, H. U. Hart, W. H. Mulligan, W. Mac Lachlan, W. M. Gifford and R. R. Stevenson.

A general discussion on details of the installation.

20. MISCELLANEOUS APPLICATIONS OF ELECTRICITY

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ELECTRIC POWER FOR NITROGEN FIXATION

E. Kilburn Scott **Vol. xxxvii—1918, pp. 957-974**

Reference is made to propaganda against processes for making nitrates from air by those interested in keeping the Allies dependant on supplies of nitrate from Chili.

A tabular comparison is made of the operations involved in the *indirect* method and the *direct* method of fixing nitrogen. The indirect method involves the manufacture of carbide of calcium and its combination with nitrogen to form calcium cyanimid, from which ammonia and in turn nitric acid are obtained. The direct method merely consists in combining nitrogen and oxygen of the air in the electric arc.

A diagram is given showing the layout of a battery of by-product coke ovens with an electric power house worked by the surplus gas and a nitrate-from-air plant to use the electricity. Figures are given showing that the nitric acid made by such a plant is about the right amount to combine with the ammonia to form ammonium nitrate, a compound in great demand at the present time for explosives.

Discussion, pages 975-983, by Messrs. C. P. Steinmetz, W. A. Del Mar, S. Barfoed, F. W. Sperr, Jr., and E. K. Scott.

A general discussion largely in support of the authors plan for cooperation between nitrogen fixation industry and by-product coke industry.

PRE-CHARGED CONDENSERS IN SERIES AND IN PARALLEL

V. Karapetoff **Vol. xxxvii—1918, pp. 1015-1023**

A condenser is charged from a source of direct voltage, and then is used as a booster in series with this source to charge another condenser. By repeating this process a large number of times the second condenser is finally subjected to twice the voltage of the source. This is the principle of the Delon apparatus for testing cables, and is explained in a numerical example. Then the more general case of two or more "pre-charged" condensers in series is considered, when these condensers are connected to some source of direct voltage; it is shown how to determine the final distribution of voltages among them. A similar problem is solved for pre-charged condensers in parallel. Finally a general network of pre-charged condensers is considered, and equations are derived similar to Kirchoff's laws, from which the final distribution of voltages and charges may be computed knowing the initial distribution.

Discussion, pages 1024-1025, by Messrs. C. O. Mailloux, L. W. Chubb, W. V. Lyon and V. Karapetoff.

A general discussion.

ELECTRIC WELDING—A NEW INDUSTRY

H. A. Hornor

Vol. xxxvii—1918, pp. 1519-1529

This paper covers a brief review of the uses of electric spot and arc welding in this country prior to the formation of the Electric Welding Committee of the Emergency Fleet Corporation. It compares the status of the art at the present time and emphasizes the developments that have been made in apparatus in the last six months. It treats of the activities of the Welding Committee in applying electric welding processes to the ship-building industry and points the way to the general applicability of this method to other industries. It shows that the results obtained by investigation and physical tests prove that the applications of this process to heavy work are satisfactory.

No discussion.

ELECTRIC MOTORS IN THE CEMENT INDUSTRY

R. B. Williamson

Vol. xxxvii—1918, pp. 1531-1567

This paper has been compiled by the Committee on Industrial and Domestic Power to give general information regarding the different classes of machinery used in the cement industry and the sizes and types of motor best adapted for the work. The paper gives, first, a brief description of the process. This is followed by an outline of the various kinds of machinery used together with data as to power requirements. The types of motor best suited to each application together with starting characteristics, overload capacity, torque and other features are indicated.

Discussion, pages 1568-1605, by Messrs. A. M. Dudley, A. Simon, W. C. Kalb, L. Bradley, W. L. Merrill, H. D. James, F. J. Burd, E. Friedlaender, W. T. Snyder, L. E. Underwood, S. Haar, J. N. Mahoney, R. J. Dearborn, J. Dixon, A. L. Hadley, C. H. Sonntag, D. B. Rushmore, T. E. Simpers, H. Weichsel, S. H. Harrison, G. H. Rowe, W. I. Slichter and R. B. Williamson.

A discussion mainly on particular points to be emphasized in selection of motors for the cement industry.

THE USE OF ELECTRIC POWER IN THE MINING OF ANTHRACITE COAL

J. B. Crane

Vol. xxxvii—1918, pp. 1671-1676

This paper gives figures as to the power cost and current consumption of anthracite mines and the reasons for these being in excess of the requirements of bituminous mines.

Estimates are also given as to the additional coal that will be released by the electrification of the anthracite mines.

Illustrations are included showing representative installations of electric drive.

Discussion incorporated with that of paper by F. L. Stone on "Drum Shapes as Affecting the Mine Hoist Duty Cycle and Motor Rating."

DRUM SHAPES AS AFFECTING THE MINE HOIST DUTY CYCLE AND MOTOR RATING

F. L. Stone **Vol. xxxvii—1918, pp. 1677-1695**

The standardization of mine hoists, from an engineering standpoint is considered impossible on account of the wide variation in the conditions and methods under which anthracite coal is mined.

The problem of drum shape consists in varying the diameter of different parts of the winding drum so that the load may be accelerated and retarded at the beginning and end of its travel with the minimum consumption of power.

Numerical examples of the performance of various drum shapes under assumed conditions are given.

Discussion (including that of paper by J. B. Crane), pages 1696-1708, by Messrs. G. Bright, C. W. Parkhurst, F. L. Stone, E. J. Cheney, W. I. Slichter, P. Kain, J. B. Crane, L. H. Rittenhouse, C. A. Adams and L. S. Randolph.

A general discussion with particular emphasis on figures for kw. hr. per ton mined of both anthracite and bituminous coal.

21. TELEPHONY AND TELEGRAPHY

INDUCTIVE EFFECTS OF ALTERNATING CURRENT RAILROADS ON COMMUNICATION CIRCUITS

H. S. Warren **Vol. xxxvii—1918, pp. 503-532**

A brief discussion of inductive interference in general is first given, including reference to the work of the Joint Committee on Inductive Interference in California. Inductive interference due to electrified railroads is then taken up and various possible means for reducing such interference considered. A description is given of four important installations of railroad electrification and the specific means adopted in each case for preventing interference, with the degree of success which has been met with.

Discussion, pages 533-538, by Mr. M. Latour.

Description of system proposed by Latour which allows telegraphic line to be operated in spite of strong disturbing alternating e. m. fs.

THE DESIGN OF TRANSPOSITIONS FOR PARALLEL POWER AND TELEPHONE CIRCUITS

Harold S. Osborne **Vol. xxxvii—1918, pp. 897-936**

This paper presents the results obtained in a recent design of transposition systems for telephone circuits exposed to induction from circuits of other kinds, particularly from three-phase power circuits, and the coordinate arrangements of transpositions in the power circuits.

In presenting the results obtained, a discussion is first given of the requirements which must be met by systems of transpositions for telephone circuits in general, and by the new "exposed line" system in particular. An outline is given of the methods used in the design work and the theory upon which it is based. The diagrams in the paper show the arrangements of transpositions for all circuits on eight crossarms of telephone line. The results to be obtained from the use of these diagrams are outlined, and the suitable locations of coordinate transpositions in parallel power circuits are discussed.

Discussion, pages 937-944, by Messrs. V. Karapetoff, C. Fortescue, A. G. Chapman, H. Mouradian and H. S. Osborne.

A discussion on various methods of measuring "cross-talk."

22. MISCELLANEOUS TOPICS AND INSTITUTE AFFAIRS

EFFECTS OF WAR CONDITIONS ON COST AND QUALITY OF ELECTRIC SERVICE

Lynn S. Goodman and William B. Jackson **Vol. xxxvii—1918, pp. 1-17**

This paper comprises a consideration of the effects of war conditions as they apply to the electric light and power service of the country.

It is shown that the increased cost for fuel and other supplies, labor and taxes alone, occasioned by the war conditions, would amount to an increase of more than $116,000,000 over the operating expenses that should have been expected for the electric light and power companies of the country under normal conditions for 1917.

The important advantages which are inherent in the central electric power stations for supplying power for war manufactures and the advisability for the government to make every reasonable endeavor to encourage the development of the central electric stations is pointed out. The way in which the problems involved are worked out will have a bearing upon the cost of electric light and power service not only during the war but for long after its termination.

Discussion, pages 18-63, by Messrs. B. A. Behrend, Mr. Sykes, J. W. Cowles, C. A. Adams, W. B. Jackson, F. A. Bryan, R. G. Hudson, M. Freund, B. J. Arnold, P. Torchio, P. Betts, W. S. Gorsuch, H. M. Hobart, D. C. Jackson, P. H. Bartlett, W. N. Smith, J. J. Harold, P. R. Moses, R. F. Schuchardt, F. A. Coffin, D. W. Roper, C. W. Pendell, C. A. Keller, F. H. Bernhard, J. W. Mabbs, A. Honegger.

A general discussion on the various phases of the central station problems.

THE TECHNICAL STORY OF THE FREQUENCIES

B. G. Lamme

Vol. xxxvii—1918, pp. 65-85

The various frequencies used in alternating-current work in America are first mentioned, and the primary reasons for their introduction are given. This is followed by a discussion of various alternating current applications which were more or less dependent upon frequency.

It is shown that there was an apparent need for two standard frequencies in the region of 60 and 25 cycles, and, further, why 60 and 25 cycles have prevailed. The special fields of application of each one are discussed fully and it is shown why 25 cycles tended to dominate the field.

Discussion, pages 86-89, by Messrs. H. B. Brooks, B. G. Lamme, G. S. Mc Cumber, R. Dalgleish and Mr. Hunt.

A general discussion.

SOME APPLICATIONS OF ELECTROMAGNETIC THEORY TO MATTER

A. C. Crehore **Vol. xxxvii—1918, pp. 261-293**

A Lecture.
No discussion

THE SECOMOR

A Kinematic Device Which Imitates the Performance of a Series-Wound Polyphase Commutator Motor

V. Karapetoff **Vol. xxxvii—1918, pp. 329-354**

The device consists of four bars of adjustable useful length and with adjustable angles. These bars can be set in a combination to represent the vector diagram of voltages in a motor with any desired constants. By moving the bars to vary the load, complete performance characteristics of the motor can be obtained, including the speed, the torque, the power factor, etc. An additional device called the impedometer permits to take into account the impedance drop in the machine. An adjustable saturation curve made of soft wire is used in connection with the secomor, to enable one to investigate the effect of saturation. A brief graphical theory of the motor precedes the description of the secomor to make its action understandable.

Discussion incorporated with that of paper by Marius C. A. Latour on "Commutation in Alternating-Current Machinery."

COMMUTATION IN ALTERNATING-CURRENT MACHINERY

Marius C. A. Latour **Vol. xxxvii—1918, pp. 355-381**

The author introduces into the discussion of the commutating characteristics of alternating-current commutating motors, his theory that perfect commutation in a continuous-current motor depends substantially on the production of a mean resultant neutral field in the region where commutation is taking place, and shows that the production of a perfect revolving field in a polyphase commutator motor assists in insuring perfect commutation at exact synchronism.

In a single-phase commutator motor a "polyphase" revolving field can be produced at synchronism by utilizing supplementary brushes, short-circuited upon themselves, displaced by 90 electrical space degrees from the main single-phase brushes on the commutator.

As in the case of polyphase motors, the problem of securing perfect commutation at synchronism becomes that of producing a perfect rotating field. It is shown by the author that the use of fractional-pitch windings on the rotor and a sinusoidal distribution of conductors on the stator is of much assistance in this connection.

Discussion (including that of papers by W. C. K. Altes and V. Karapetoff), pages 379-389, by Messrs. A. M. Gray, P. M. Lincoln, C. F. Scott, W. C. K. Altes, H. M. Hobart, B. A. Behrend, W. B. Jackson and V. Karapetoff.

A general discussion.

ADDRESS BY PRESIDENT E. W. RICE, JR. AT A. I. E. E. MIDWINTER CONVENTION FEBRUARY, 1918

Vol. xxxvii—1918, pp. 391-397

A REVIEW OF ELECTRICAL ENGINEERING PROGRESS

E. W. Rice, Jr. **Vol. xxxvii—1918, pp. 703-714**

President's Address.

ENGINEERS AND THE WAR

Major General William M. Black **Vol. xxxvii—1918, pp. 945-956**

An Address.

No discussion.

ELECTRIC POWER FOR NITROGEN FIXATION

E. Kilburn Scott **Vol. xxxvii—1918, pp. 957-974**

Reference is made to propaganda against processes for making nitrates from air by those interested in keeping the Allies dependant on supplies of nitrate from Chili.

A tabular comparison is made of the operations involved in the *indirect* method and the *direct* method of fixing nitrogen. The indirect method involves the manufacture of carbide of calcium and its combination with nitrogen to form calcium cyanimid, from which ammonia and in turn nitric acid are obtained. The direct method merely consists in combining nitrogen and oxygen of the air in the electric arc.

A diagram is given showing the layout of a battery of by-product coke ovens with an electric power house worked by the surplus gas and a nitrate-from-air plant to use the electricity. Figures are given showing that the nitric acid made by such a plant is about the right amount to combine with the ammonia to form ammonium nitrate, a compound in great demand at the present time for explosives.

Discussion, pages 975-983, by Messrs. C. P. Steinmetz, W. A. Del Mar, S. Barfoed, F. W. Sperr, Jr., and E. K. Scott.

A general discussion largely in support of the authors plan for cooperation between nitrogen fixation industry and by-product coke industry,

AMERICA'S ENERGY SUPPLY

Charles P. Steinmetz **Vol. xxxvii—1918, pp. 985-1009**

The gist of the paper is to demonstrate that the economical utilization of the country's energy supply requires generating electric power whereever hydraulic or fuel energy is available, and *collecting the power electrically, just as we distribute it electrically.*

In the first section a short review of the country's energy supply in fuel and water power is given.

In the second section it is shown that the modern synchronous station is necessary for large hydraulic powers, but the solution of the problem of the economic development of the far more numerous smaller waterpowers is the adoption of the induction generator.

The third section considers the characteristics of the induction generator and the induction-generator station, and its method of operation, and discusses the condition of "dropping out of step of the induction generator" and its avoidance.

In the appendix the corresponding problem is pointed out with reference to fuel power, showing that many millions of kilowatts of potential power are wasted by burning fuel and thereby degrading its energy that could be recovered by interposing simple steam turbine induction generators between the boiler and the steam heating systems, and collecting their power electrically.

Discussion, pages 1010-1014, by Messrs. A. M. Schoen, B. A. Behrend, S. Barfoed, R. Bennett and C. P. Steinmetz.

A general discussion.

METHOD OF SYMMETRICAL CO-ORDINATES APPLIED TO THE SOLUTION OF POLYPHASE NETWORKS

C. L. Fortescue **Vol. xxxvii—1918, pp. 1027-1115**

In the introduction a general discussion of unsymmetrical systems of co-planar vectors leads to the conclusion that they may be represented by symmetrical systems of the same number of vectors, the number of symmetrical systems required to define the given system being equal to its degrees of freedom. A few trigonometrical theorems which are to be used in the paper are called to mind. The paper is subdivided into three parts, an abstract of which follows. It is recommended that only that part of Part I up to formula (33) and the portion dealing with star-delta transformations be read before proceeding with Part II.

Part I deals with the resolution of unsymmetrical groups of numbers into symmetrical groups. These numbers may represent rotating vectors of systems of operators. A new operator termed the sequence operator is introduced which simplifies the manipulation. Formulas are derived for three-phase circuits. Star-delta transformations for symmetrical coordinates are given and expressions for power deduced. A short discussion of harmonics in three-phase systems is given.

Part II deals with the practical application of this method to symmetrical rotating machines operating on unsymmetrical circuits. General formulas are derived and such special cases, as the single-phase induction motor, synchronous motor-generator, phase converters of various types, are discussed.

Discussion, pages 1116-1140, by Messrs. J. Slepian, C. P. Steinmetz, V. Karapetoff, A. M. Dudley, C. F. Scott, C. O. Mailloux and C. L. Fortescue.

A general discussion emphasizing certain mathematical methods of simplifying the author's methods.

SUSTAINED SHORT-CIRCUIT PHENOMENA AND FLUX DISTRIBUTION OF SALIENT-POLE ALTERNATORS

N. S. Diamant **Vol. xxxvii—1918, pp. 1141-1202**

It shown in Section IV that with the ordinary field forms met with in practise, the resultant flux wave under s. s. c. (sustained short-circuits) will be extremely distorted, see Figs. 16, (4th wave) 23, 27, 27A, etc., for the simple reason that with the very low voltages obtained under such conditions, the fundamental of the *B*-curve of the field is reduced so much by the armature reaction that the higher harmonics assume a very predominant rôle and become several hundred per cent of the s. s. c. fundamental. See Fig. 26 and Tables XI, XII, and XIII.

As a corollary to the foregoing it is found that the *B*-curve under load will not differ radically from the no-load field form since the fundamental will remain large enough to hold its own. See Fig. 22 and also footnote (4). The cross magnetizing effect of the armature reaction is, of course, to make the *B*-curve unsymmetrical with respect to the mid-pole axis. Compare Figs. 7 and 22.

The magnetic oscillations are studied not only by means of full-pitch stator coils but also by means of rotor coils No. 7 and 8, Fig. 3, and stator coils No. 9, 10, 11, 12, 15, and 16, Figs. 3 and 3A.

Discussion, pages 1203-1208, by Messrs. F. D. Newbury, R. E. Doherty, W. F. Dawson, C. M. Davis and N. S. Diamant.

A discussion involving criticisms of author's solution of short-circuit phenomena.

REACTANCE OF SYNCHRONOUS MACHINES AND ITS APPLICATIONS

R. E. Doherty and O. E. Shirley **Vol. xxxvii—1918, pp. 1209-1297**

Part I treats of the calculation and application of the armature self-inductive reactance of synchronous machines. A short, reliable method is given in the form of curves, making the calculation from design sheet data a matter of a few minutes.

An approximate, but convenient, method of applying the armature reactance in the calculation of field excitation under load is given in Appendix C.

In Part II it is shown that the initial short-circuit current of synchronous machines is determined not only by the armature self-inductive reactance, as is often assumed, but also by the field self-inductive reactance. Neglecting the field reactance in calculation may give a calculated short-circuit current 50 per cent or more, too high. A formula is derived for calculating the field reactance which, added to the armature reactance, gives the total which determines the initial short-circuit current and which it is proposed should be called, as previously recommended by other authors, *transient reactance.*

An attempt is made to describe the apparently complicated physical phenomena of sudden short circuits in terms as free as possible from mathematics. One interesting and important point which the authors establish from the physical interpretation of the problem is that there is a very significant rise in flux at the bottom of the pole at short circuit.

Discussion, pages 1298-1340, by Messrs. C. J. Fechheimer, V. Karapetoff, F. D. Newbury, W. F. Dawson, C. L. Fortescue, H. R. Summerhayes, N. S. Diamant and R. E. Doherty.

A discussion involving a listing of various methods of determining reactance and their feasibility.

CRITICAL REVIEW OF THE BIBLIOGRAPHY ON UNBALANCED MAGNETIC PULL IN DYNAMO-ELECTRIC MACHINES

Alexander Gray and J. G. Pertsch, Jr. **Vol. xxxvii—1918, pp. 1417-1424**

The purpose of this paper is to serve as an introduction to the succeeding comprehensive article by Mr. E. Rosenberg, on the subject of Magnetic Pull in Electric Machines. The Maxwell equation, upon which all of the formulas for unbalanced magnetic pull are based, is first established. A critical review is then given of the articles pertaining to this subject, which have appeared in the technical press to date. It is pointed out wherein the various expressions which have been proposed differ from one another and to what extent they are consistent.

No discussion.

MAGNETIC PULL IN ELECTRIC MACHINES

E. Rosenberg **Vol. xxxvii—1918, pp. 1425-1469**

See abstract of preceding paper.

No discussion.

ECONOMIC PROPORTION OF HYDROELECTRIC AND STEAM POWER

Frank G. Baum **Vol. xxxvii—1918, pp. 1471-1475**

This paper describes a new method of determining for any power system, what proportion of generation should be hydroelectric, and what steam, from the standpoint of economics.

A method is outlined for obtaining a curve showing "Total cost per kilowatt-year for hydroelectric and steam power", for any percentage combination of generation.

With system load curve, fixed charges on steam and hydroelectric plants, cost of fuel and other steam energy charges all known quantities, this curve can readily be calculated, and one can see at a glance the limiting economical percentage of steam power for the given conditions.

No discussion.

ELECTRIC MOTORS IN THE CEMENT INDUSTRY

R. B. Williamson **Vol. xxxvii—1918, pp. 1531-1567**

This paper has been compiled by the Committee on Industrial and Domestic Power to give general information regarding the different classes of machinery used in the cement industry and the sizes and types of motor best adapted for the work. The paper gives, first, a brief description of the process. This is followed by an outline of the various kinds of machinery used together with data as to power requirements. The types of motor best suited to each application together with starting characteristics, overload capacity, torque and other features are indicated.

Discussion, pages 1568-1605, by Messrs. A. M. Dudley, A. Simon, W. C. Kalb, L. Bradley, W. L. Merrill, H. D. James, F. J. Burd, E. Friedlaender, W. T. Snyder, L. E. Underwood, S. Haar, J. N. Mahoney, R. J. Dearborn, J. Dixon, A. L. Hadley, C. H. Sonntag, D. B. Rushmore, T. E. Simpers, H. Weichsel, S. H. Harrison, G. H. Rowe, W. I. Slichter and R. B. Williamson.

A discussion mainly on particular points to be emphasized in selection of motors for the cement industry.

DRUM SHAPES AS AFFECTING THE MINE HOIST DUTY CYCLE AND MOTOR RATING

F. L. Stone **Vol. xxxvii—1918, pp. 1677-1695**

The standardization of mine hoists, from an engineering standpoint is considered impossible on account of the wide variation in the conditions and methods under which anthracite coal is mined.

The problem of drum shape consists in varying the diameter of different parts of the winding drum so that the load may be accelerated and retarded at the beginning and end of its travel with the minimum consumption of power.

Numerical examples of the performance of various drum shapes under assumed conditions are given.

Discussion (including that of paper by J. B. Crane), pages 1696-1708, by Messrs. G. Bright, C. W. Parkhurst, F. L. Stone, E. J. Cheney, W. I. Slichter, P. Kain, J. B. Crane, L. H. Rittenhouse, C. A. Adams and L. S. Randolph.

A general discussion with particular emphasis on figures for kw. hr. per ton mined of both anthracite and bituminous coal.

AMERICAN ENGINEERING RESEARCH

W. R. Whitney **Vol. xxxvii—1918, pp. 1709-1721**

Discussion incorporated with that of paper by R. A. Millikan on "Research in America after the War."

RESEARCH IN AMERICA AFTER THE WAR

R. A. Millikan **Vol. xxxvii—1918, pp. 1723-1734**

Discussion (including that of paper by W. R. Whitney), pages 1735-1746, by Messrs. C. E. Skinner, E. E. F. Creighton, P. G. Agnew, C. H. Sharp, E. P. Hyde, L. T. Robinson, R. A. Millikan, M. W. Franklin, C. A. Adams, J. B. Taylor and G. F. Gray.

A discussion including various suggestions on the solution of the research problem.

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